Endogenous Quality Choice: Price and Quantity Competition

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Abstract

The present paper is concerned with providing a core model to address the issue of firms simultaneously competing in both prices and quantities (capacity levels) within a simple duopoly market setting where products are asymmetrically differentiated by endogenous quality location. A three-stage competitive framework is introduced such that non-collusive firms compete in quality location, followed by choice of fixed capacity, and finally, they compete in prices. There is a continuum of consumers uniformly distributed along a vertical quality street of product locations. In general, a Bertrand-Nash equilibrium with fixed Cournot quantities (fixed capacity) is achieved. Firms tend to move away from minimum differentiation as quality cross-effects in fixed cost investments become less severe, but not necessarily choosing quality locations which would lead towards an outcome of maximum differentiation. An output asymmetry always exists at equilibrium such that the high quality firm always carries excess production capacity relative to the low quality firm. Total production capacity, however, may not fully cover market demand for an incumbent duopoly.

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1. Introduction

Central to the problem of providing an adequate solution to the differentiation issue under the analysis of imperfect competition, such as that of a duopoly, where products are differentiated by quality location, is whether the analysis is structured along the lines of price *or* quantity competition in the specification of the model proposed. There has been a general consensus in the literature that price competition tends toward the "principle of maximum differentiation" whereas quantity competition tends toward the "principle of minimum differentiation"[1].

This understanding has emerged from a number of classical findings such as those in Hotelling (1929), Salop (1979), Shaked and Sutton (1982), Singh and Vives (1984), Motta (1993), and Tirole (1996), among others. Such an assessment, however, restricts the analysis towards firms choosing either prices or quantities as their strategic variables, but not both. As previously mentioned by Leontief (1934) and later utilized by Benoit and Krishna (1987), we believe that any model which relies on either variable exclusively is partially flawed[2]. It seems logical that any theory of oligopolistic behavior ought to take the more reasonable view of firms choosing both prices *and* quantities in anticipated market competition, with pricing strategies constrained by capacity choices and degree of quality location (product differentiation). This is in a hopeful

[1] Price competition is seen to be relaxed by firms maximizing their relative quality locations through maximum differentiation (as proven in D'Aspremont, C., J. Gabszewicz, and J.F. Thisse. 1979, and further in Shaked and Sutton 1982, and in contrast to the original work of Hotelling 1929). On the other hand, quantity (capacity) competition is seen to converge towards minimum differentiation due to the added burden of enforcement costs within Cournot-Nash equilibrium (with insights by Dixit (1979), Salop (1979), Singh and Vives (1984), and Motta (1993), among others).

[2] This view has actually been originally noted by Leontief (1934), and later utilized by Benoit and Krishna (1987) for a homogenous product market. Due to quantity enforcement costs related to inflexible capacity choices, Benoit and Krishna (1987) find that firms generally carry excess capacity in equilibrium.

attempt to create a more refined structural conception of market equilibrium including the dynamics of market behavior under oligopolistic competition.

In adopting this position, we seek to define a stable (i.e. Nash) market equilibrium structured on a three-stage duopoly model as follows: firms choose quality location at the first stage of competition, followed by choice of (fixed) capacities at the second stage, and finally they compete in prices at the third stage. Such sequence of imposed events is not due without merit. Particularly, it is seen that quantity setting decisions are medium to long-term decisions with capacity choices relatively inflexible - or “fixed” - in the short run. On the other hand, prices are considered relatively more flexible. Prices can be easily manipulated in the short run as compared to changing production capacity. This is considered a logical assessment since changing production capacity is generally fixed factor employed (additional production lines must be bought, additional labor trained, production plans rescheduled, etc.) whereas changing product price is considered easier to implement on short notice[3]. Hence, in adopting this view, we assume that firms choose inflexible capacity levels *followed by* choice of flexible prices. The choice of both capacities and prices, however, is then presumed dependent on the initial (endogenous) choice of quality location dictated at the first stage of competition.

The present paper is concerned with providing a core model to address the issue of firms simultaneously competing in both prices and quantities (capacity levels) within a simple duopoly market setting where products are differentiated by quality location. The proposed analysis is of a simple differentiated duopoly with asymmetric quality differentiation. Firms are faced with both price and quantity (capacity) competition after simultaneously choosing their product specifications through an optimal choice of quality location early on. Using backward induction, price competition is first analyzed, followed by the analysis of capacity choice and optimum quality location. In the general case, market equilibrium arises with production capacity not necessarily fully covering the entire market (i.e. market demand is not fully covered). This finding is proven in Proposition 1. Under a fixed range of capacity choices, a Bertrand-Nash equilibrium with fixed Cournot quantities is established such that firms move away from minimum differentiation as quality cross-effects in fixed cost investments become less severe (Proposition 3). The traditional outcomes of minimum and maximum differentiation, however, are not found to

[3] In real life, this can be seen to be the case in most industries e.g. in heavy manufacturing, automotive, fast food, oil production, retail etc.

be locally stable. This is thought to be the case due to the complex nature of market competition faced by firms. Firms are faced with conflicting strategies in anticipation of competition in fixed capacities (quantity choices) and price competition into the future. Since price competition is generally relaxed through maximum differentiation whereas quantity competition is generally relaxed by minimum differentiation, firms choose to differentiate their product offerings by *endogenous quality differentiation* dependent on the choice of production capacity and cost structure of the market (as given in Proposition 2). Quality location for each firm is then found to depend on a weighted average of the anticipated choice of fixed capacity levels, with the high quality firm always carrying excess production capacity relative to the low quality firm (Lemma 2 and Lemma 5).

Consequently, firms are found to ultimately choose a relative degree of endogenous quality differentiation in an attempt to combat the anticipation of both price and quantity competition into the future. As a result, both firms will endogenize their relative quality offerings by choosing quality locations which will induce less fierce competition in capacity levels at the 2nd stage and a more relaxed price competition at the 3rd stage.

As compared to previous findings in the literature, our core model results contrast those of D'Aspremont et.al. (1979), Shaked and Sutton (1982), Motta (1993) and Tirole (1996). In particular, we do not find an outcome of minimum differentiation nor an optimum outcome of maximum differentiation as those models have suggested. In addition, contrary to Kim (1987), we find that the introduction of asymmetric fixed costs in production greatly affect the nature of quality differentiation in a duopoly market. Moreover, the proposed model in this research is seen to expand on the findings established in Benoit and Krishna (1987) and Motta (1993) through the introduction of both price and quantity (capacity) competition into vertical quality differentiation. A clear disparity, however, rests in our finding that production capacity does not necessarily fully cover market demand. Another difference in model results is due to the nature of firm revenues as an outcome of our different equilibrium solutions (as established in Lemma 6 and Proposition 2).

Our general findings are in partial agreement with Wolinsky (1983), Cheng (1985), Cremer and Thisse (1992) and Vives (1999) in that the nature of endogenous quality differentiation always carries a quality premium in Bertrand prices with persistent demand for higher quality locations.

2. Background

Within the theoretical science of industrial economics, endogenous quality choice in differentiated markets have been the focus of a number of research inquiries in the literature, most notably that of Hotelling (1929), Chamberlin (1933), Leontief (1936), Dixit and Stiglitz (1977), Salop (1979), Shaked and Sutton (1982), Wolinsky (1983), Singh and Vives (1984), Kim (1987), Motta (1993), Tirole (1996) and Vives (1999). However, most of the research has been confined towards Cournot *or* Bertrand competition (but not both), and most of the established models are limited to a symmetric cost structure in production where quality choice is fixed by technology or regulation.

In general, there has been an agreement in the literature that the equilibrium outcome of endogenous quality choice within a differentiated market depends upon whether *price* or *quantity* competition have been imposed on the proposed model. Under certain assumptions[4], there is a consensus that firms tend to differentiate *more* in order to soften anticipated price competition whereas they tend to differentiate *less* in order to soften anticipated quantity competition. In extreme form, endogenous quality choice under price competition leads to maximum product differentiation and under quantity competition leads to minimum product differentiation.

Various forms of the above argument have been formally proven by Dixit and Stiglitz (1977), D'Aspremond et. al. (1979), Singh and Vives (1984), Vives (1985), and Shaked and Sutton (1982), as direct extensions of the works of Cournot (1838), Bertrand (1883) and Hotelling (1929). In particular, under *Cournot* (quantity) competition, Singh and Vives (1984) find that best response (reaction functions) tend to be *downward sloping*[5], in contrast to the upward sloping reaction functions characterized by price (Bertrand) competition. Moreover, the *degree of product differentiation* has a large effect on the slope of those reaction functions. As available products in the market become more differentiated, quantity reaction functions become more elastic (horizontal). Vives (1985), Cheng (1985) and Bulow et.al. (1985) extend the result of Singh and Vives (1984) to study multimarket oligopolies, equilibrium stability, and market efficiency under Cournot (quantity) competition. Disregarding their differences, they all agree that a differentiated

[4] That is, the assumptions of symmetric costs and perfect information. In addition, discrete choice preferences must be structured on a bounded quality 'street'.

[5] A downward sloping reaction function under quantity competition implies that a firm's optimal response to an increase in rival output level is to decrease its own output level. As product characteristics become more differentiated in the eyes of the consumer, each producer gains a quasi-monopoly power over other brands in the market, and this ultimately increases own profits.

Cournot market always yield *higher* prices and *lower* output levels than a differentiated Bertrand market, given a *fixed* level of product differentiation[6]. This hypothesis breaks down when firms are allowed to change their degree of product differentiation relative to other firms in the market.

When *endogenous quality choice* is considered, where firms are allowed to endogenize their level of quality differentiation relative to other firms in the market depending on the market environment and future anticipated competition; numerous results are found. Of the many insights into this topic, the works of Vany and Saving (1983), Srinagesh and Bradburd (1989), Gabszewicz, Pepall and Thisse (1992), and Motta (1993) stand as the most forceful. Their analysis of *differentiated market competition* with endogenous quality choice is almost always confined to a multi-stage setting where firms first choose whether or not to enter the market, followed by choice of differentiation (quality levels), and then they compete in either prices (Bertrand competition) or quantities (Cournot competition). The general assessment in those models is that price competition leads towards more differentiation and a higher level of social welfare, but limits the equilibrium number of firms upon entry, relative to quantity competition. There is an informal consensus that duopoly firms fully cover the market of quality locations if firms choose prices as strategies. Motta (1993) also extends this result to several cases of quantity competition.

Benoit and Krishna (1987) analyze the dynamics of an *undifferentiated* quantity-setting market where firms compete in prices in the short-run (short-run Bertrand competition) while competing in distinct quantities in the long-run (long-run Cournot competition). They find that firms generally carry excess (idle) capacity in equilibrium. This creates an imposed constraint on firm behavior such that firms are unable to sustain a monopoly (monopolistic) position in the long run. In the short run, however, firms can behave with some degree of monopolistic power due to flexible pricing strategies. On the other hand, Vany and Saving (1983) develop conditions for a Nash equilibrium outcome with quality differentiation being modeled as a function of output, firm capacity, and a Poisson wait function to obtain specific product characteristics desired by consumers. A quality characteristic, z, is defined on demand, output, capacity, and production costs. The model abstracts completely from individual price-signaling arguments, yet obtains the same conclusions as that of *monopolistic competition* such as those of Chamberlin (1933) and Spence (1976). Kim (1987) extends the work of Vany and Saving (1983) to include *asymmetric*

[6] A fixed level of product differentiation is equivalent to fixed quality locations. Here, a fixed degree of product differentiation is a necessary condition for equilibrium stability, see Vives (1985).

fixed costs and finds that the inclusion of additional production capacities do *not* alter the equilibrium outcome if fixed costs are somehow correlated with the expected range of quality characteristics offered by the firm[7]. Also, Srinagesh and Bradburd (1989), as an extension of the work of Prescott and Vischer (1977), show that *quality discrimination* (with, or without, product positioning) leads to profitable market segmentation. A long-run equilibrium may exist where a monopolistic outcome per segment prevails, each enhancing the level of quality to achieve a separating consumer choice - the ultimate result of maximal quality differentiation profitable for all. Firms maximize market share per segment using alternative entry deterrence strategies, but in the long-run, an equilibrium pattern of locations is achieved which, on some assumptions, is also shown to yield a Nash equilibrium outcome[8]. Gabszewicz, Pepall and Thisse (1992), and subsequently Tirole (1996), attempt to present dynamic models of quality choice in an duopoly/oligopoly market with varying assumptions and starkly different model approaches. In particular, Gabszewicz et.al. (1992) provide a model of dynamic quality choice where the equilibrium outcome is governed by brand loyalty as quality products move up the goodwill ladder from experience to search goods based on a converging consumer *learning-by-using* exponential function. Consumers are shown to favor differentiated products supplied at high capacities in comparison to those supplied at low capacities, so long as the price-signaling argument holds good[9]. Tirole (1996), on the other hand, argues that firms invest in *collective reputations* and that entry firms may collect additional quality perception (i.e. goodwill) in the eyes of the consumer by borrowing an already invested quality attribute while achieving lower costs in present-value terms. The model argues of a persistence in quality attributes initially offered by historical firms which generate differentiated products at higher capacities than others

[7] A stable equilibrium outcome develops if consumers are "patient enough". If, however, consumers are impatient in their desire to consume certain product characteristics, then the equilibrium stability conditions break down and the schedule of optimal quality choices becomes unsustainable.

[8] In addition to the above models, Jun and Vives (1996), Bulow (1986) and Allen (1988) analyze the dynamics of quality differentiation in the presence of demand shifts, product obsolescence and the resulting changes of differential quality through time. Allen (1988) studies price-quality dynamics when supply is faced with quantity adjustments in response to stochastic changes in demand. The presence of demand uncertainty forces supply to adjust outputs every period (random walk) and shifts consumer focus from prices to a more exact choice of quality through time. Bulow (1986) argues for planned obsolescence as a result of present-value profit maximization in which firms desire to produce differentiated goods with uneconomically short useful lives, in order to extract a large amount of profit surplus early on, and to trap consumers for repeat purchases in the future (later augmented in Fishman, Gandal and Shy (1993)). Bulow's argument sharply contrasts that of the invariance result due to Swan's optimum durability theorem (Swan 1970). Although Swan's analysis is more general, the planned obsolescence hypothesis due to Bulow (1986) and Fishman et.al. (1993) provide a valid market dependence argument for the dynamic choice of quality in differentiated markets.

[9] The price-signaling argument here implies that prices are true signals of product quality. In other words, higher prices signal a higher quality product to an uninformed consumer.

in equilibrium[10]. Vives (1999) approaches dynamic quality choice from an *oligopoly pricing* point of view and argues that the long-run solution for a quality differentiated market is found by an aggregate of short-run price-quality signaling competition, regardless of capacity choices[11].

From the literature, product differentiation models with endogenous quality choice are mostly studied under price (Bertrand) or quantity (Cournot) competition but not both. Results always seem to point towards a maximum or minimum differentiation outcome, with the latter typically tied to multi-stage quantity (capacity) competition and the former usually tied to a monopolistic outcome of price competition with some degree of market segmentation.

3. Model & Analysis

Consider a duopoly market setting characterized by asymmetric quality differentiation. Firms choose quality location at the first stage of competition, followed by a choice of fixed capacities at the second stage, and finally they compete in prices at the third stage. Accordingly, the proposed model framework is based upon three stages of market competition where firms are differentiated by endogenous quality choice. This amounts to a market equilibrium where prices and capacity levels are dependent on the endogenous choice of quality location throughout all three stages of competition. Producer profits obey a non-collusive maximization of revenues with differentiated capacity levels and observable price-quality offerings; while consumer surplus obeys a separable discrete-choice behavior along a differentiated quality street of surplus value functions. The incumbent duopoly market is presumed non-symmetric and demand is fully covered by the available quality spectrum of quality locations, given that consumer preferences are structured along a vertical quality street of length $L > 0$. We now present the necessary preliminaries and develop the required terminology before proceeding to the central analysis.

[10] Tirole (1996) shows that firms which historically persistent in high production capacities generally work in collaborative networks and achieve a higher grade of quality perception to consumers. This perception, however, does not necessarily result in actual product durability due to asymmetric information and advertising bias. It is also shown that there may exist a persistence in product failures even under a stable equilibrium solution.

[11] Here, price-quality signals may not necessarily reveal the true level of quality to an uninformed consumer. Vives (1999) argues that price-quality signals (whether under perfect or imperfect information) yield a pattern of short-run equilibria which, on aggregate, develop into a long-run stable market outcome.

A. PRELIMINARIES

Firms

There are two firms, denoted by *A* and *B*, each producing a single differentiated product. Firm *A* produces the low-quality brand while firm *B* produces the high-quality brand. Product location is structured on a vertical quality street bounded by [0,L]; $(L \equiv 1)$. Quality location is at points *a* and *b* $(0 \le a \le b \le L)$ from the origin, respectively. Demand fully covers the available spectrum of quality choices, normalized to unity $(L \equiv 1)$. There are no exit strategies out of the market. Denoting production capacity levels as $x(a,b,C_a)$ for the low-quality firm and $y(a,b,C_b)$ for the high-quality firm, firms maximize revenue (profit) functions of the form:

$$\pi_a(a,b,x,C_a) = (P_a - C_a)x(a,b,C_a) - K_a(a,b) \tag{1}$$
$$\pi_b(a,b,y,C_b) = (P_b - C_b)y(a,b,C_b) - K_b(a,b)$$

where C_a and C_b are unit variable costs for low-quality and high-quality location respectively; while $K_a(a,b)$ and $K_b(a,b)$ are fixed costs associated with corresponding choice of quality location. Firms invest according to their initial chosen level of quality location, and subsequently incur variable costs in production. There exist quality cross-effects in fixed investments. Specifically, investing in a particular level of quality location affects the choice of quality location by the competing firm (this behavior will be studied by using a specific functional form for fixed investments as given by $K_a(a,b) = \frac{1}{2}(a - \gamma b)^2$ and $K_b(a,b) = \frac{1}{2}(b - \gamma a)^2$; $0 \le \gamma \le 1$).

There is no collusion among firms.

Consumers

There is a continuum of consumers uniformly distributed along the vertical quality street of product locations. Demand is differentiated by exact quality preferences[12] bounded by [0,1]. A

[12] Since there is no disutility from purchasing a non-ideal brand, the assumed quality street is considered a form of vertical differentiation under exact preferences. In general, there are two types of vertical differentiation: one under exact preferences and another under non-exact (probabilistic) preferences. Vertical differentiation under exact preferences is usually associated with discrete choice behavior in consumption. Consumers know their exact preferences and will not purchase a product unless that product's quality characteristics exactly matches their preferences. This, however, does not negate the fact that those

representative indifferent consumer between low-quality and high-quality location (i.e. the two quality locations offered by the incumbent duopoly firms) is located at $\hat{x}$; where $a \leq \hat{x} \leq b$; such that all consumers whose preferences lie within [0,x) prefer the low-quality brand and all consumers whose preferences lie within (x,1] prefer the high-quality brand. Consumer demand fully covers all quality locations: D(x(a,b))+D(y(a,b))=1; whereas production capacity may or may not fully cover available quality locations: x(a,b)+y(a,b) $\leq$ 1. All consumers located on [0,x) gain a higher utility from purchasing brand A (lower quality) than from purchasing brand B. Similarly, all consumers located on (x,1] gain a higher utility from purchasing brand B (higher quality) than from purchasing brand A[13]. Consumers behave according to the following surplus value functions[14]:

$$\begin{aligned} V_x(a) &= a[x(a,b)]^2 - P_a(a,b) \\ V_y(b) &= b[y(a,b)]^2 - P_b(a,b) \end{aligned} \tag{2}$$

where $a, b \in [0, L]$; $V(\bullet) > -\max\{P_a, P_b\}$; $x(a,b) \geq 0$; $y(a,b) \geq 0$; $b \geq a$.

Competition

Stage 1:	Quality Location	[firms compete in quality and invest accordingly]
Stage 2:	Capacity Selection	[firms choose production capacity levels]
Stage 3:	Price Competition	[firms compete in prices]

Firms choose quality location with *a* and *b* as variables in stage 1; and then select capacity levels (x, y) in stage 2, given their choice of (a, b) from stage 1; and finally engage in price competition in stage 3, given their choice of capacity levels (x, y) from stage 2 and quality location (a, b) from stage 1. The endogenous choice of quality is embodied in the parameters *a* and *b* such that *a* represents low quality location and *b* represents high quality location (for a duopoly setting); with

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consumers also weigh in the tradeoff between their preferences and their willingness to pay for a certain quality attribute as observed through product price.

[13] Therefore, the number of consumers willing to buy from firm A is x, whereas the number of consumers willing to buy from firm B is (1-x).

[14] The surplus value functions in (2) imply that consumer utility exhibits a "love for quality" consumption behavior and indicates an implication towards substitute experience goods. Also, (2) exhibits the property that all consumers purchase either product A or product B without the generation of a group of "reservation consumers" who do not purchase any brand; thus allowing the assumption of a fully covered market.

an industry quality spectrum of $a, b \in [0,\text{L}]$, where L (normalized to unity) is the maximum possible quality attainable using current levels of production technology; and with $b \geq a$.

B. PRICE COMPETITION

Using backward induction, we begin the analysis at the third stage of competition where firms compete in prices. Assuming rational choice and utility-maximizing behavior for an equilibrium solution, the equilibrium level of demand for quality levels *a* and *b*, respectively, for a differentiated duopoly market is (*see* Appendix A):

$$\begin{aligned} D(x(a,b)) &= \left(\frac{P_b - P_a}{b-a}\right)^{1/2} \\ D(y(a,b)) &= 1 - D(x(a,b)) \end{aligned} \tag{3}$$

where the spectrum of quality choice (L) is normalized to unity; and $b > a$[15].

Given (2) and (3), the maximization of profits in (1) with respect to prices yield:

$$\begin{aligned} \left(\frac{\partial \pi_a}{\partial P_a}\right) &= \left(\frac{P_b - P_a}{b-a}\right)^{1/2} + P_a \frac{1}{2}\left(\frac{P_b - P_a}{b-a}\right)^{-1/2}\left(\frac{1}{a-b}\right) = 0 \\ \left(\frac{\partial \pi_b}{\partial P_b}\right) &= 1 - \left(\frac{P_b - P_a}{b-a}\right)^{1/2} - \frac{1}{2} P_b \left(\frac{P_b - P_a}{b-a}\right)^{-1/2}\left(\frac{1}{b-a}\right) = 0 \end{aligned} \tag{4}$$

Solving simultaneously; it is easy to verify that $\left[P_b - P_a\right] = \dfrac{q}{4}$; with $q \equiv (b-a)$. Hence, there always exists a "quality premium", *q>0*, at equilibrium.

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[15] Demand is undefined if $b = a$. This is typical for a vertical differentiation model since there are no unit transportation costs between quality levels. More specifically, there is no indifferent consumer for such a case, since all products become homogenous with no degree of quality differentiation between product locations for the case of $b = a$. This constraint has also been troubling for D'Aspremont et.al. 1979, Shaked and Sutton 1982, and Motta 1993. Intuitively, $b = a$ may imply the traditional Cournot competition with costless production for a homogenous product market, and therefore has no place in a vertically differentiated market structure.

Equilibrium prices suggest (*see* Appendix B):

$$P_a = \frac{b-a}{4} \quad ; \quad P_b = \frac{b-a}{2} > P_a \tag{5}$$

with $b > a$ retained for positive prices and feasible demand.

The second-order conditions are satisfied as given in Appendix B.

Let the minmax revenues of firm A and firm B be implicitly defined by the least amount of demand firm $j \neq i$ can hold firm *i*'s demand down to; where $i, j \in (A, B)$:

$$\begin{aligned} z_a\big(D(x(a,b)\big) &= \min_{P_b} \max_{P_a} D\big(x(a,b)\big) \\ z_b\big(D(y(a,b)\big) &= \min_{Pa} \max_{Pb} D\big(x(a,b)\big) \end{aligned} \tag{6}$$

This occurs when firm *B* charges a price of zero, and similarly for firm *A*. By direct substitution of $P_a = 0$ and $P_b = 0$ into (3), we get:

$$\begin{aligned} z_a\big(D(x(a,b)\big) &= D(x(a,b) \big| P_b = 0) = 0 \\ z_b\big(D(y(a,b)\big) &= D(y(a,b \big| P_a = 0) = \left(\frac{P_b}{b-a}\right)^{1/2} > 0 \end{aligned} \tag{7}$$

Thus, there is positive minmax revenues for high-quality location and zero minmax revenues for low-quality location.

We may now proceed to establish:

LEMMA 1. (i) *Bertrand prices require a quality premium of* $[P_b - P_a] = \left(\frac{b-a}{4}\right) > 0$ *;* (ii) *minimum differentiation is not locally stable.*

Proof: (i) see Appendix B. (ii) see Appendix A (also, demand in (3) is undefined for $b = a$).

LEMMA 2. *There is persistent demand (positive minmax revenues) for high quality location, due to* $D(y(a,b)|P_a = 0) > 0$ *; but not vice-versa for low-quality location, i.e.* $D(x(a,b)|P_b = 0) = 0$.

Proof: This follows directly from (6) and (7).

Intuitively, if consumers are faced with a higher quality product at zero prices, then they will not choose to purchase the low-quality product. On the other hand, if consumers are faced with zero prices for the low-quality product, then some consumers will still choose to consume the high quality product provided that the surplus value associated with high-quality location is larger than that associated with low-quality location. This provides the high-quality firm (the firm producing product B) a level of "persistent demand", as given in (7) and mentioned in Lemma 2, whereas such an advantage is not given to the low quality firm (the firm producing product A).

C. CHOICE OF FIXED CAPACITIES

Given that Bertrand prices are governed by (5), we now turn our attention to the second stage of competition where firms choose fixed capacity levels given initial choice of quality location. Firms take *a* and *b* as given and choose capacity levels $x(a,b,C_a)$ and $y(a,b,C_b)$ which would ensure maximum profits with the anticipation of Bertrand equilibrium in the 3rd stage. Firms also incur variable costs of quality in accordance with their selection of production capacities.

Accordingly, at the 2nd stage, choosing optimal production capacity levels require:

$$\left[\frac{\partial \pi_a}{\partial x(a,b,C_a)}\middle| P_a = \frac{b-a}{4}\right] = 0 \; ; \; \left[\frac{\partial \pi_b}{\partial y(a,b,C_b)}\middle| P_b = \frac{b-a}{2}\right] = 0 \qquad (8)$$

Production capacity, anticipating the Bertrand equilibrium in the 3rd stage, is therefore given by (*see* Appendix C):

$$x(a,b,C_a)=\left[\frac{P_b-C_a}{3(b-a)}\right]^{1/2}=\left(\frac{\frac{b-a}{2}-C_a}{3(b-a)}\right)^{1/2}=\left(\frac{b-a-2C_a}{6(b-a)}\right)^{1/2}$$

$$y(a,b,C_b)=\left[\frac{P_a-C_b+b-a}{3(b-a)}\right]^{1/2}=\left(\frac{\frac{b-a}{4}-C_b+b-a}{3(b-a)}\right)^{1/2}=\left(\frac{5(b-a)-4C_b}{12(b-a)}\right)^{1/2} \qquad (9)$$

We are now in a position to strengthen our model result.

LEMMA 3. *Suppose profits obey (1) and demand obeys (3). Then, the optimum choice of fixed capacities is* $x(a,b,C_a)=\left(\frac{b-a-2C_a}{6(b-a)}\right)^{1/2}$ *and* $y(a,b,C_b)=\left(\frac{5(b-a)-4C_b}{12(b-a)}\right)^{1/2}$.

Proof: see Appendix C.

LEMMA 4. *Given* $0<C_a<\left(\frac{b-a}{4}\right)$ *and* $0<C_b<\left(\frac{b-a}{2}\right)$; *the choice of production capacities is limited to the fixed range of:* $\sqrt{\frac{1}{12}}<x^*(a)<\sqrt{\frac{1}{6}}$ *and* $\frac{1}{2}<y^*(b)<\sqrt{\frac{5}{12}}$.

Proof: see Appendix C.

It follows from Lemma 3 and Lemma 4 that an asymmetric choice of production capacities exists at equilibrium[16]:

$$y^*(a,b,C_b)>x^*(a,b,C_a) \qquad (10)$$

which requires

$$\sqrt{\frac{b-a-2C_a}{6(b-a)}}<\sqrt{\frac{5(b-a)-4C_b}{12(b-a)}}\ ,$$

[16] From Lemma 4; the low quality firm producing brand A roughly covers 30 per cent to 40 per cent of the market whereas the high quality firm producing brand B roughly covers 50 per cent to 60 per cent of the market. As given in Lemma 3, the market share of the respective firms depends on their corresponding unit variable costs in production and on the existing quality spread in the market.

whence, for $0 < C_a < \left(\frac{b-a}{4}\right)$ *and* $0 < C_b < \left(\frac{b-a}{2}\right)$;

$$\left(\frac{1}{2} - \sqrt{\frac{1}{6}}\right) < y^*(a,b,C_b) - x^*(a,b,C_a) < \left(\sqrt{\frac{5}{12}} - \sqrt{\frac{1}{12}}\right) \quad . \tag{11}$$

This suggests that the "market share" differential between high-quality location and low-quality location is always in excess of $\left(\frac{1}{2} - \sqrt{\frac{1}{6}}\right)L$, and is always less than $\left(\frac{\sqrt{5}-1}{2\sqrt{3}}\right)L$.

The above analysis leads to the following important result:

LEMMA 5. *The high-quality firm always carries excess production capacity relative to the low-quality firm:* $y^*(a,b,C_b) > x^*(a,b,C_a)$; $\left(\frac{1}{2} - \sqrt{\frac{1}{6}}\right) < y^*(a,b,C_b) - x^*(a,b,C_a) < \left(\frac{\sqrt{5}-1}{2\sqrt{3}}\right)$.

Lemma 5 basically states that the high quality firm always produces in excess capacity relative to the low quality firm, regardless of the initial choice of quality locations.

Hence, high quality location generates "over-production".

Intuitively, this may be a consequence of the "persistent demand" argument in Lemma 2 where there is positive minmax revenues for high quality location but not for low quality location. Another explanation is due to product availability. The high quality firm may desire to make its products more "available" to consumers than its own demand may warrant, lest one or more consumers are driven towards a product characteristic of high quality location not available in the substandard quality characteristics of low quality location.

Furthermore, it is easy to verify (*see* Appendix C):

$$\left[\frac{\partial x}{\partial(b-a)}\middle|\bar{y}\right] > 0 \ ; \ \left[\frac{\partial y}{\partial(b-a)}\middle|\bar{x}\right] > 0 \tag{12}$$

and

$$\left[\frac{\partial x}{\partial C_a}\right] < 0 \ ; \ \left[\frac{\partial y}{\partial C_b}\right] < 0 \ . \tag{13}$$

The choice of production capacity levels depend on the existing quality spread in the market (constrained by the choice of fixed capacities of the competing firm) and on the current level of unit variable cost in production (given fixed quality locations)[17].

Having characterized the choice of production capacity by (10), (11) and (12); we now turn our attention to the extent of market coverage by the incumbent duopoly firms.

Given the range of fixed capacity choices in Lemma 3 and Lemma 4, it follows that for any given market demand L>0 (*see* Appendix D):

$$x\left\{a\middle|C_a \in [0,\frac{b-a}{4})\right\} > \frac{1}{2\sqrt{3}}L \tag{14}$$

and

$$y\left\{b\middle|C_b \in [0,\frac{b-a}{2})\right\} > \frac{1}{2}L \ . \tag{15}$$

[17] The choice of production capacity is increasing with the spread of quality location (given fixed quality locations at the 1st stage of competition) and is clearly decreasing with the current level of unit variable cost at the 2nd stage of competition, given fixed quality locations.

Therefore, for a fully covered market of $L > 0$ with $a, b \in [0, L]$, the low-quality firm is able to cover at least $\frac{1}{2\sqrt{3}}$ of total market demand, whereas the high-quality firm is able to cover at least one-half of total market demand. Thus, when firms compete under price competition with fixed capacities, both firms are able to *collectively* cover at least $\left[\frac{1}{2} + \frac{1}{2\sqrt{3}}\right] < 1$ of the market.

This leaves the possibility of at most $\left[\frac{\sqrt{3}-1}{2\sqrt{3}}\right] > 0$ of market demand left uncovered.

We may now proceed to establish:

PROPOSITION 1. *Production capacity may not cover market demand* :

(i) $\left(x^* + y^*\right)^{\min} = \left[\frac{1}{2} + \frac{1}{2\sqrt{3}}\right] L < L$,

since

(ii) $x^{\min}\left\{a \middle| C_a \in [0, \frac{b-a}{4})\right\} = \frac{1}{2\sqrt{3}} L$ *and* $y^{\min}\left\{b \middle| C_b \in [0, \frac{b-a}{2})\right\} = \frac{1}{2} L$,

then:

(iii) *maximum uncovered demand is* $\left[\frac{\sqrt{3}-1}{2\sqrt{3}}\right] L > 0$.

Proof: see Appendix D.

Proposition 1 indicates that the choice of fixed capacities does not necessarily cover the full quality spectrum of the market.

A residual demand of $\left[\frac{\sqrt{3}-1}{2\sqrt{3}}\right] L$ could be left uncovered.

This finding sharply contrasts that of Shaked and Sutton (1983, 1982), Kim (1987), and Motta (1993). In particular, Shaked and Sutton (1982) indicate that duopoly firms relax price competition through quality differentiation such that a duopoly market fully covers all available quality locations. Their result has also been confirmed by Kim (1987) with asymmetric fixed costs. Moreover, Motta (1993) also finds a two-firm fully covered equilibrium solution for both price (Bertrand) and quantity (Cournot) competition, with the former attaining a higher level of social welfare. As mentioned by Vives (1999), a duopoly market fully covers available quality locations if vertical differentiation is fixed by technology or location. Within the confines of our model, however, such an assessment breaks down. This is merely because we are not considering either price or quantity competition, but rather an integrative model of capacity constrained price competition for a duopoly market structure. It seems that when duopoly firms compete in both prices and quantities (fixed capacity levels), then they are unable to fully cover the available market of quality locations. Thus, Proposition 1 runs in contrast to duopoly models existing in the literature where firms compete in prices or quantities alone.

D. QUALITY LOCATION

Having established the equilibrium level of Bertrand prices in the 3rd stage as well as the optimum level of production capacity in the 2nd stage, we are now in a position to strengthen our model result by analyzing the choice of quality location at the 1st stage of competition. At the 1st stage, firms invest and compete in quality location, anticipating their fixed choice of capacity levels at the 2nd stage and Bertrand prices in the 3rd stage. If Bertrand prices are given by (5) and capacity levels by (9), then competition in quality location at the 1st stage is essentially based on the following *reduced form profit functions* (*see* Appendix E):

$$\pi_a(a,b) = \left[\frac{b-a-4C_a}{4}\right]\left[\frac{b-a-2C_a(a)}{6(b-a)}\right]^{1/2} - K_a(a,b) \tag{16}$$

$$\pi_b(a,b) = \left[\frac{b-a-2C_b}{2}\right]\left[\frac{5(b-a)-4C_b}{12(b-a)}\right]^{1/2} - K_b(a,b)\,.$$

From (16), we establish profit variations with *competing* quality choice[18] given fixed capacity levels as (*see* Appendix F):

[18] We analyze profit variations with competing quality choice under the added assumption of no fixed costs. This assumption will be relaxed later when the optimum choice of quality location is considered in the general analysis. This assumption helps to portray the significance of quality cross-effects absent any fixed costs.

$$\left(\frac{\partial \pi_a}{\partial (b)}\right) = \frac{x^*}{4} > 0 \tag{17}$$

and

$$\left(\frac{\partial \pi_b}{\partial (a)}\right) = -\frac{y^*}{2} < 0\,. \tag{18}$$

Hence:

$$\left|\frac{\partial \pi_b}{\partial (a)}\right| - \left|\frac{\partial \pi_a}{\partial (b)}\right| = \frac{1}{2}\left(y^* - \frac{x^*}{2}\right) \tag{19}$$

But since $y^* > x^*$ is always true at equilibrium from (10), then it follows that:

$$\left|\frac{\partial \pi_b}{\partial (a)}\right| > \left|\frac{\partial \pi_a}{\partial (b)}\right| \tag{20}$$

We are now ready to establish:

LEMMA 6. *With no fixed costs:*

(i) *The revenue of the low quality firm increase as the quality of the better product improves:* $\left(\frac{\partial \pi_a}{\partial (b)}\right) = \frac{x^*}{4} > 0$ *;*

(ii) *The revenue of the high quality firm decrease as the quality of the lesser product improves:* $\left(\frac{\partial \pi_b}{\partial (a)}\right) = -\frac{y^*}{2} < 0$ *;*

(iii) *The effect of* (ii) *dominates the effect of* (i): $\left|\frac{\partial \pi_b}{\partial (a)}\right| > \left|\frac{\partial \pi_a}{\partial (b)}\right|$.

Proof: see Appendix F.

Accordingly, with no fixed costs, own profit variation with competing quality choice are significant for both firms, such that $\left(\frac{\partial \pi_a}{\partial (b)}\right) > 0$ and $\left(\frac{\partial \pi_b}{\partial (a)}\right) < 0$. Furthermore, $\left|\frac{\partial \pi_b}{\partial (a)}\right| > \left|\frac{\partial \pi_a}{\partial (b)}\right|$. A change in quality location affects the profit of the competing firm, but such an effect is more severe against profits of high-quality location for a given change in low-quality location as compared to profits of low-quality location for a given change in high-quality location.

Lemma 6(i) runs in accordance with Shaked and Sutton (1982) yet its finding is contrary to D'Aspremond et.al. (1979). At the same time, Lemma 6(ii) contrasts the results in Shaked and Sutton (1982) yet is in general agreement with D'Aspremond et.al. (1979)[19]. Also, Lemma 6(iii) expands on the findings established in Benoit and Krishna (1987) and Motta (1993) through the introduction of both price and quantity (capacity) competition into vertical quality differentiation[20].

We are now in a position to relax our earlier assumption of no fixed costs. Rather, we introduce a general form of fixed cost allocations in order to analyze the general form of the model more fully.

Fixed costs are formulated to be:

[19] It is not surprising that this is the case. Most notably, Shaked and Sutton (1982) are involved in quality differentiation with anticipated price competition whereas D'Aspremond are basically involved in a Cournot-Nash equilibrium (quantity-setting) model with (horizontal) product differentiation by quality choice. Motta (1993) seems to mostly agree with Shaked and Sutton (1982) while analyzing a vertical differentiation model as a variant to D'Aspremond et.al. (1979). The dispersion of findings in those models have been noted by Vives (1999) as being a difference in model assumptions (i.e. price vs. quantity competition, horizontal vs. vertical differentiation, collusive behavior vs. non-collusive behavior, etc.) but with the exception of fixed cost allocations. Hence, Lemma 6 formally asserts the descriptive arguments put forth earlier by Vives (1999).

[20] Motta (1993) compares aspects of price vs. quantity competition with endogenous quality choice and is in disagreement with D'Aspremond et.al. (1979). Benoit and Krishna (1987) analyze price and quantity competition for a homogenous product market with costless production. Earlier in the literature, however, the critical aspect of profit variation with competing quality choice as evidenced through substitutions in demand, have been noted by Leontief (1936) in his echoing of Stackelberg's notion of imperfect competition for "two monopolistic competitors" (duopoly firms), suggesting : "a real theory of imperfect competition must prove, or disprove, the validity of this postulate rather than introduce it as an assumption." (Leontief 1936, p. 559).

$$K_a(a,b) = K(a^*|b)$$
$$K_b(a,b) = K(b^*|a) \tag{21}$$

In particular, we are interested in finding the necessary level of fixed costs required for an *optimum choice* of quality location, given that duopoly firms invest in quality location, choose fixed capacities, and then compete in prices.

Quality reaction functions are therefore implicit in (21).

We establish:

LEMMA 7. *When duopoly firms invest in quality location, choose fixed capacities, and then compete in prices, their choice of investment (fixed cost) for optimum quality location requires:*

$$K_a'(a^*|b) = \left(\frac{b-a^*-2C_a}{6(b-a^*)}\right)^{1/2}\left[-\frac{1}{4}+\frac{1}{2(b-a^*)}\left(\frac{b-a^*}{4}-C_a\right)\right] = \left[-\frac{1}{4}+\frac{1}{2(b-a^*)}\left(\frac{b-a^*}{4}-C_a\right)\right]\bar{x}(a^*|b)$$
$$K_b'(b^*|a) = \left(\frac{5(b^*-a)-4C_b}{12(b^*-a)}\right)^{1/2}\left[\frac{1}{2}-\frac{1}{3(b^*-a)}\left(\frac{b^*-a}{2}-C_b\right)\right] = \left[\frac{1}{2}-\frac{1}{3(b^*-a)}\left(\frac{b^*-a}{2}-C_b\right)\right]\bar{y}(b^*|a)\ .$$

Proof: see Appendix E.

The optimum choice of quality location at the 1st stage of competition is therefore based on implicit quality reaction functions as represented in Lemma 7. Once fixed costs are more precisely determined, the reduced form profit functions in (16) could then "retrieve" the optimum choice of quality locations, (a^*, b^*), given that Bertrand prices and fixed capacity levels are governed by Lemma 1 and Lemma 3, respectively .

Hence, as a general solution to the model:

At stage 1, firms choose quality location as determined by quality reaction functions derived from Lemma 7. At stage 2, they choose fixed Cournot quantities (fixed capacity levels) as determined by Lemma 3, given the optimum choice of quality locations derived from Lemma 7. At stage 3, they choose Bertrand prices as determined by Lemma 1, given their choice of fixed capacity levels from Lemma 3 and their choice of quality location from Lemma 7. Equilibrium profits are then derived by direct substitution of Bertrand prices, fixed capacity levels, and optimum quality locations; into the reduced form profit functions given in (16).

We are now in a position to strengthen our general model result.

PROPOSITION 2. *A Bertrand-Nash equilibrium solution with fixed Cournot quantities is achieved via a stable equilibrium configuration of firm strategies; such that*:

(i) (a^*,b^*) *are implicitly determined by quality reaction functions:*

$$K'_a(a^*|b) = \left(\frac{b-a^*-2C_a}{6(b-a^*)}\right)^{1/2}\left[-\frac{1}{4}+\frac{1}{2(b-a^*)}\left(\frac{b-a^*}{4}-C_a\right)\right] = \left[-\frac{1}{4}+\frac{1}{2(b-a^*)}\left(\frac{b-a^*}{4}-C_a\right)\right]\bar{x}(a^*|b)$$

$$K'_b(b^*|a) = \left(\frac{5(b^*-a)-4C_b}{12(b^*-a)}\right)^{1/2}\left[\frac{1}{2}-\frac{1}{3(b^*-a)}\left(\frac{b^*-a}{2}-C_b\right)\right] = \left[\frac{1}{2}-\frac{1}{3(b^*-a)}\left(\frac{b^*-a}{2}-C_b\right)\right]\bar{y}(b^*|a) \;;$$

(ii) *Given* (a^*,b^*) *; the optimum choice of fixed capacities (fixed Cournot quantities) is given by:*

$$x^*(a,b,C_a) = \left(\frac{b^*-a^*-2C_a}{6(b^*-a^*)}\right)^{1/2} \text{ and } y^*(a,b,C_b) = \left(\frac{5(b^*-a^*)-4C_b}{12(b^*-a^*)}\right)^{1/2} \;;$$

(iii) *Given* (x^*,y^*) *and* (a^*,b^*) *; Bertrand prices are of the form:*

$$P_b^* = \frac{b^*-a^*}{2} \text{ and } P_a^* = \frac{b^*-a^*}{4} \;;$$

(iv) G*iven* (a^*,b^*) *,* (x^*,y^*) *,and* (P_a^*,P_b^*) *; equilibrium profits are:*

$$\pi_a(a^*,b^*) = \left[\frac{b^*-a^*-4C_a}{4}\right]\left[\frac{b^*-a^*-2C_a}{6(b^*-a^*)}\right]^{1/2} - K_a(a^*|b^*)$$

$$\text{and } \pi_b(a^*,b^*) = \left[\frac{b^*-a^*-2C_b}{2}\right]\left[\frac{5b^*-4C_b-5a^*}{12(b^*-a^*)}\right]^{1/2} - K_b(b^*|a^*) \;.$$

Proof: (i) and (iv) see Appendix E, (ii) see Appendix C, and (iii) see Appendix B.

The general solution to the model as outlined in Proposition 2 deserves several comments. Since firms choose both prices *and* quantities (capacity levels) in order to optimally locate their quality offerings to consumers, they are faced with two conflicting strategies. The first strategy calls for relaxing their price competition through distant quality locations due to the anticipation of fierce

Bertrand competition. On the other hand, a second strategy calls for reducing their relative quality locations by minimizing their degree of product differentiation in anticipation of fixed Cournot (capacity) competition. It therefore seems that firms are not in a position to choose minimum nor maximum differentiation due to the complex nature of their market competition. This is a consequence of firms competing in both *flexible* Bertrand prices and *fixed* Cournot quantities (fixed production capacity) as constrained by their *initial* choice of quality location. Consequently, as given in Proposition 2, firms will ultimately choose a relative degree of *endogenous quality differentiation* in an attempt to combat the anticipation of both price and quantity competition into the future[21]. Since competition in capacity levels is fixed, whereas price competition is more flexible, firms will mostly have the tendency to diverge away from minimum differentiation, but not necessarily choosing quality locations which would lead towards an outcome of maximum differentiation.

It should be noted here that the above analysis have focused on a general model where the functional form of the profit function in (16) contains a general formulation of the fixed cost term with respect to quality location, as suggested by (21). Having reached a general solution to the problem, however, it seems natural to work out an example which is as close as possible to the general case but avoiding the difficulty of explanation exhibited in the general sense. Accordingly, we next illustrate a specific model solution by examining a special case of the model where fixed costs are more precisely formulated and where quality cross-effects are more formally introduced into the analysis.

Let fixed costs at the 1[st] stage of competition take the following form:

$$K_a(a,b) = \frac{1}{2}(a - \gamma b)^2 \qquad (22)$$

$$K_b(a,b) = \frac{1}{2}(b - \gamma a)^2$$

[21] In anticipation of future market competition in both prices and quantities, firms at the 1[st] stage of competition will engage in endogenous quality differentiation in an attempt to combat the above mentioned conflicting strategies. As a result, both firms will endogenize their relative quality offerings by choosing quality locations which will induce less fierce competition in capacity levels at the 2[nd] stage and a more relaxed price competition at the 3[rd] stage. Such a solution, however, may yield multiple differentiation outcomes based on the functional form of firm revenues (as reflected in the reduced form profit functions in (16)), which is based - in part - by the nature and extent of fixed costs incurred.

where the parameter γ signifies quality cross-effects (i.e. cross-effects in fixed investments with respect to quality location); and with $0 \le \gamma \le 1$. The introduction of quality cross-effects in fixed investments exemplifies the notion of investment competition in quality location. Specifically, *investing* in a particular level of quality location affects the *choice* of quality location by the competing firm.

Suppose further that unit variable costs obey:

$$\begin{aligned} C_a &= \lambda a \\ C_b &= \lambda b \end{aligned} \tag{23}$$

where the parameter λ signifies "unit cost of quality" in production[22].

Given that unit variable costs behave according to (23) and fixed costs according to (22), the profit functions in (16) can now be re-written to be:

$$\begin{aligned} \pi_a(a,b;\lambda,\gamma) &= \left[\frac{b-a-4\lambda a}{4}\right]\left[\frac{b-a-2\lambda a}{6(b-a)}\right]^{1/2} - \frac{1}{2}(a-\gamma b)^2 \\ \pi_b(a,b;\lambda,\gamma) &= \left[\frac{b-a-2\lambda b}{2}\right]\left[\frac{5b-4\lambda b-5a}{12(b-a)}\right]^{1/2} - \frac{1}{2}(b-\gamma a)^2 \end{aligned} \tag{24}$$

Since the optimum choice of quality location requires $\left[\frac{\partial \pi_a}{\partial (a)}\middle| b^*\right] = 0$ and $\left[\frac{\partial \pi_b}{\partial (b)}\middle| a^*\right] = 0$, then direct substitution into Lemma 7 yield the following *quality reaction functions*[23] (*see* Appendix G):

---

[22] Equivalently, $\lambda = \frac{C_a}{a} = \frac{C_b}{b}$. Hence, there is a "unit cost of quality" in variable production dependent on the endogenous choice of quality location, such that the parameter λ in (23) may represent $ *per* production unit *per* unit of quality location. This representation does not negate the fact that unit variable costs could still be asymmetric between firms, i.e. $C_a = \lambda a$ is not necessarily equal to $C_b = \lambda b$. In fact, $C_a = \frac{a}{b} C_b$.

$$
\begin{aligned}
a^* &= \gamma b - \frac{\bar{x}}{4}(1+4\lambda) \\
b^* &= \gamma a + \frac{\bar{y}}{2}(1-2\lambda)
\end{aligned}
\tag{25}
$$

The severity of quality cross-effects is evident in the slope of the reaction functions[24]:

$$
\left(\frac{\partial a^*}{\partial b}\right) = \left(\frac{\partial b^*}{\partial a}\right) = \gamma \tag{26}
$$

The *optimum choice of quality location* is then achieved by simultaneously solving the quality reaction functions in (25) given the representations of an equilibrium configuration in Lemma 7 and Proposition 2 (*see* Appendix G):

$$
\begin{aligned}
b^* &= \frac{1}{1-\gamma^2}\left\{\bar{x}\left(-\frac{\gamma}{4} - \gamma\lambda\right) + \bar{y}\left(\frac{1}{2} - \lambda\right)\right\} \\
a^* &= \frac{1}{1-\gamma^2}\left\{\bar{x}\left(-\frac{1}{4} - \lambda\right) + \bar{y}\left(\frac{\gamma}{2} - \lambda\gamma\right)\right\}
\end{aligned}
\tag{27}
$$

The optimum choice of quality location at the 1st stage of competition is therefore described by (27), in consequence to the quality reaction functions derived in (25), with the presumption of a market cost structure characterized by (22) and (23).

As a consequence to Proposition 2 and (27), the quality spread between the two optimum quality locations is always governed by:

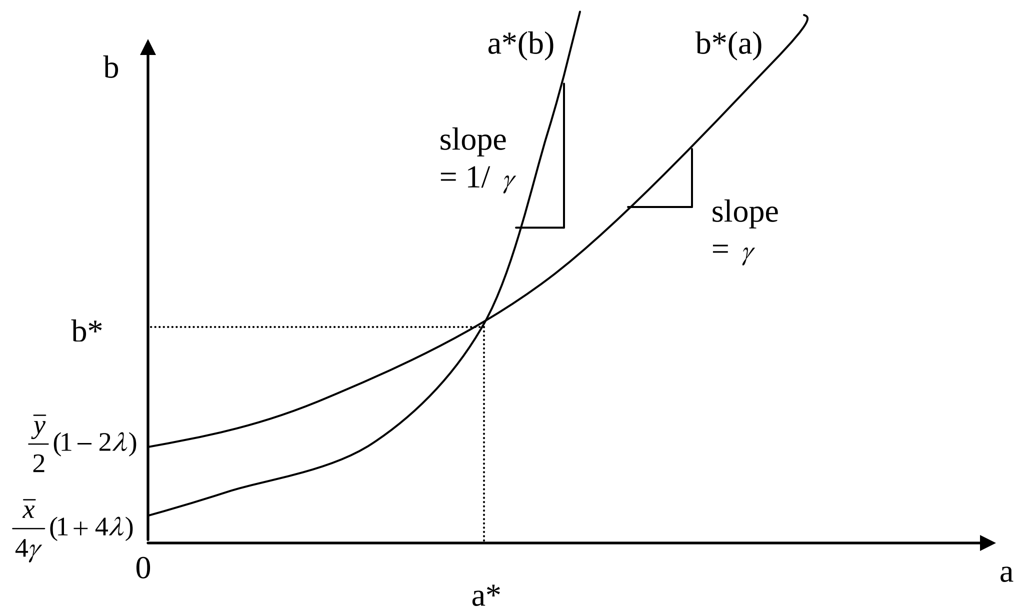
$$
(b^* - a^*) = \frac{1}{1-\gamma^2}\left\{\left(-\frac{1}{4} - \lambda\right)(\gamma - 1)\bar{x} + \left(\frac{1}{2} - \lambda\right)(1-\gamma)\bar{y}\right\} \tag{28}
$$

---

[23] Given fixed capacity levels as portrayed in Lemma 3, quality reaction functions are dependent on the degree of quality cross-effects in fixed cost investments of quality location (as represented by the parameter γ), unit cost of quality as represented by the parameter λ, in addition to fixed capacity levels $\bar{x}$ and $\bar{y}$.

[24] The slope of the reaction function for high quality location $b^*(a)$ is γ; and the slope of the reaction function for low quality location $a^*(b)$ is $1/\gamma \;\; > \gamma$ (for $\gamma < 1$).

A simplified graphical illustration of the optimum solution is shown in Figure 3 below.



$$a^* = \frac{1}{1-\gamma^2}\left\{\bar{x}\left(-\frac{1}{4}-\lambda\right)+\bar{y}\left(\frac{\gamma}{2}-\lambda\gamma\right)\right\}$$

$$b^* = \frac{1}{1-\gamma^2}\left\{\bar{x}\left(-\frac{\gamma}{4}-\gamma\lambda\right)+\bar{y}\left(\frac{1}{2}-\lambda\right)\right\}$$

Figure 3: A Simplified Illustration of the Optimum Solution for Endogenous Quality Location in Capacity Constrained Price Competition

We are now in a leading position to establish:

PROPOSITION 3. *Suppose the differentiated duopoly market has a cost structure characterized by (22) and (23). Then, the optimum choice of quality location is given by:*

$$b^* = \frac{1}{1-\gamma^2}\left\{\bar{x}\left(-\frac{\gamma}{4}-\gamma\lambda\right)+\bar{y}\left(\frac{1}{2}-\lambda\right)\right\}$$

$$a^* = \frac{1}{1-\gamma^2}\left\{\bar{x}\left(-\frac{1}{4}-\lambda\right)+\bar{y}\left(\frac{\gamma}{2}-\lambda\gamma\right)\right\}$$

Proof: see Appendix G.

With market costs described by (22) and (23), endogenous quality location in capacity constrained price competition yields a Bertrand-Nash equilibrium solution with fixed Cournot (capacity) levels as given in Proposition 2, with optimum quality location at the 1st stage of competition determined by Proposition 3. Since Proposition 3 implies $\left(\frac{\partial(b-a)}{\partial\gamma}\right)<0$ (see Appendix G), firms tend to differentiate more as quality cross-effects in fixed costs investments become less severe.

Optimal quality location for the case of costless production is analyzed in Appendix H.

4. Conclusion

We have developed a simple model of endogenous quality location with price and quantity (capacity) competition for a differentiated duopoly characterized by asymmetric quality differentiation. The nature of market competition has been characterized by non-collusive behavior between firms with consumer demand governed by surplus value functions based on vertical quality preferences. Firms possess asymmetric fixed and variable costs of quality with fixed costs characterized by quality cross-effects and variable costs characterized by constant unit cost of quality in production. The dynamics of market competition have been assumed to be in three stages: quality location at the 1st stage, followed by choice of fixed production capacity at the 2nd stage, and finally, competition in prices at the 3rd stage.

The analysis suggests persistent demand for high quality location, with equilibrium prices always carrying a quality premium such that minimum differentiation is not a stable differentiation outcome. The high quality firm always carries excess production capacity relative to the low quality firm but both firms are constrained by a fixed range of capacity choices. More importantly, due to output asymmetry between the two firms, production capacity may not fully cover market demand for an incumbent duopoly.

A change in quality location always affects the profit of the competing firm, but such an effect is more severe against profits of high-quality location for a given change in low-quality location as compared to profits of low-quality location for a given change in high-quality location.

As a general solution, a Bertrand-Nash equilibrium arises with fixed Cournot quantities (fixed capacity levels) such that the optimum choice of quality location is dependent on upward-sloping quality reaction functions. Quality reaction functions, in turn, are largely sensitive to the relative degree of quality cross-effects in fixed cost investments. Consequently, firms are found to ultimately choose a relative degree of endogenous quality differentiation in an attempt to combat the anticipation of both price and quantity competition into the future. Since competition in capacity levels is fixed, whereas price competition is more flexible, our findings indicate that firms mostly have the tendency to diverge away from minimum differentiation as quality cross-effects become less severe, but not necessarily choosing quality locations which would lead towards an outcome of maximum differentiation. Quality location for each firm is also found to depend on a weighted average of the anticipated choice of fixed capacity levels by both firms in the market.

Introducing price and quantity competition in a duopoly model contrasts the tendency towards achieving either minimum differentiation (usually achieved under quantity competition) or maximum differentiation (usually achieved under price competition). It seems that as the nature of market competition become more complex, a different set of multiple differentiation outcomes clearly develop. The model proposed in this research, however, needs to clarify the extent of repeated price competition under fixed capacities, and could introduce market entry towards an assessment of oligopoly competition. Social welfare measures under an oligopoly market framework can then be assessed.

5. Appendix

A. Differentiated Demand Functions

An indifferent consumer between low-quality and high-quality location (i.e. the two quality locations offered by the incumbent duopoly firms) located at $\hat{x}$; where $a \le \hat{x} \le b$; can be depicted as: [0------- a ----- $\hat{x}$ ------- b ----------1] ; such that all consumers whose preferences lie within [0,x) prefer the low-quality brand and all consumers whose preferences lie within (x,1] prefer the high-quality brand. All consumers located on [0,x) gain a higher utility from purchasing brand A (lower quality) than from purchasing brand B. Similarly, all consumers located on (x,1] gain a higher utility from purchasing brand B (higher quality) than from purchasing brand A. Therefore, the number of consumers buying from firm A is x, whereas the number of consumers buying from firm B is (1-x). Given the assumed surplus value functions:

$$\begin{aligned} V_x(a) &= a[x(a,b)]^2 - P_a(a,b) \\ V_y(b) &= b[y(a,b)]^2 - P_b(a,b) \end{aligned} \quad ;$$

an indifferent consumer would have the same surplus value for low and high quality products as follows: $a[x(a,b)]^2 - P_a(a,b) = b[x(a,b)]^2 - P_b(a,b)$. Therefore, $(b-a)[x(a,b)]^2 = P_b - P_a$; and it directly follows that $D(x(a,b)) = x(a,b,P_a,P_b) = \left(\dfrac{P_b - P_a}{b-a}\right)^{1/2}$. Demand for high-quality location is an assumed residual of the demand for low-quality location; hence $D(y(a,b)) = 1 - D(x(a,b))$ for a fully covered market normalized to unity. This leads to a fully covered market (normalized to unity) since $\dfrac{D(x(\bullet)) + D(y(\bullet))}{L} = 1$.

B. Second-Order Conditions for Price Competition

From the first-order conditions:

$$\begin{aligned} \left(\frac{\partial \pi_a}{\partial P_a}\right) &= \left(\frac{P_b - P_a}{b-a}\right)^{1/2} + P_a \frac{1}{2}\left(\frac{P_b - P_a}{b-a}\right)^{-1/2}\left(\frac{1}{a-b}\right) = 0 \\ \left(\frac{\partial \pi_b}{\partial P_b}\right) &= 1 - \left(\frac{P_b - P_a}{b-a}\right)^{1/2} - \frac{1}{2}P_b\left(\frac{P_b - P_a}{b-a}\right)^{-1/2}\left(\frac{1}{b-a}\right) = 0 \end{aligned} \quad ;$$

the choice of optimal prices suggest (solving simultaneously):

$P_a = \frac{b-a}{4} > 0$ and $P_b = \frac{b-a}{2} > P_a$; for $b > a$.

The second-order condition for equilibrium prices is satisfied for maximum profits (with $b > a$) as follows:

$$\frac{\partial^2 \pi_a}{\partial P_a^2} = \frac{1}{2}\left(\frac{P_b - P_a}{b-a}\right)^{-1/2}\left(\frac{1}{a-b}\right) + \frac{1}{2(a-b)}\left[\left(\frac{P_b - P_a}{b-a}\right)^{-1/2} + P_a\left(\frac{P_b - P_a}{b-a}\right)^{-3/2}\right] < 0$$

$$\frac{\partial^2 \pi_b}{\partial P_b^2} = -\left\{\frac{1}{2}\left(\frac{P_b - P_a}{b-a}\right)^{-1/2}\left(\frac{1}{b-a}\right) + \frac{1}{2(b-a)}\left[\left(\frac{P_b - P_a}{b-a}\right)^{-1/2} - \frac{1}{2}P_b\left(\frac{P_b - P_a}{b-a}\right)^{-3/2}\left(\frac{1}{b-a}\right)\right]\right\}$$

$$= -\frac{1}{(b-a)}\left(\frac{P_b - P_a}{b-a}\right)^{-1/2} + \frac{1}{4(b-a)^2}P_b\left(\frac{P_b - P_a}{b-a}\right)^{-3/2}$$

$$= -\frac{1}{(b-a)}\left(\frac{P_b - P_a}{b-a}\right)^{-1/2}\left[1 - \frac{P_b}{4(b-a)}\left(\frac{P_b - P_a}{b-a}\right)^{-1}\right]$$

$$= -\frac{1}{(b-a)}\left(\frac{P_b - P_a}{b-a}\right)^{-1/2}\left[1 - \frac{P_b}{4(b-a)}\left(\frac{(b-a)}{\frac{b-a}{2} - \frac{b-a}{4}}\right)\right]$$

$$= -\frac{1}{(b-a)}\left(\frac{P_b - P_a}{b-a}\right)^{-1/2}\left[1 - \frac{P_b}{(b-a)}\right] = -\frac{1}{(b-a)}\left(\frac{P_b - P_a}{b-a}\right)^{-1/2}\left(\frac{1}{2}\right) = -\frac{1}{2(b-a)}\left(\frac{P_b - P_a}{b-a}\right)^{-1/2} < 0 \ .$$

C. Production Capacity Levels at the Second Stage

Demand and Surplus Value Functions in (1) and (3) imply:

$$P_a[a,b,x(a,b)] = P_b(a,b) + (a-b)[x(a,b)]^2$$

$$P_b[a,b,y(a,b)] = (b-a)[1 - 2y(a,b) + y(a,b)^2] + P_a(a,b)$$

Hence, profit functions are given by:

$$\pi_a(a,b,x,C_a) = (P_a - C_a)x(a,b,C_a) - K(a) = (P_a - C_a)\left(\frac{P_b - P_a}{b-a}\right)^{1/2} - K(a)$$

$$\pi_b(a,b,y,C_b) = (P_b - C_b)y(a,b,C_b) - K(b) = (P_b - C_b)\left[1 - \left(\frac{P_b - P_a}{b-a}\right)^{1/2}\right] - K(b)$$

Then, the first-order conditions with respect to quantities, given that $\frac{\partial \pi_a}{\partial K(b)} = \frac{\partial x(a)}{\partial C_b} = 0$ and $\frac{\partial \pi_b}{\partial K(a)} = \frac{\partial y(b)}{\partial C_a} = 0$; lead to:

$$x(a,b,C_a) = \left[\frac{P_b - C_a}{3(b-a)}\right]^{1/2} = \left(\frac{\frac{b-a}{2} - C_a}{3(b-a)}\right)^{1/2} = \left(\frac{b-a-2C_a}{6(b-a)}\right)^{1/2}$$

$$y(a,b,C_b) = \left[\frac{P_a - C_b + b - a}{3(b-a)}\right]^{1/2} = \left(\frac{\frac{b-a}{4} - C_b + b - a}{3(b-a)}\right)^{1/2} = \left(\frac{5(b-a) - 4C_b}{12(b-a)}\right)^{1/2}.$$

Second-order conditions are satisfied for $b > a$ as follows:

$$\left(\frac{\partial^2 \pi_a}{\partial x^2}\right) = 6\left(\frac{b-a}{4(b-a)}\right)^{1/2}(a-b) = 3(a-b) < 0$$

$$\left(\frac{\partial^2 \pi_b}{\partial y^2}\right) = -6(b-a) + 6(b-a)\left(\frac{b-a}{4(b-a)}\right)^{1/2} = -3(b-a) < 0\ .$$

or, equivalently:

$$\frac{\partial^2 \pi_a}{\partial x^2} = 6\left(\frac{P_b - P_a}{b-a}\right)^{1/2}(a-b) = 6\left(\frac{b-a}{4(b-a)}\right)^{1/2}(a-b) = 3(a-b) < 0$$

$$\frac{\partial^2 \pi_b}{\partial y^2} = -6(b-a)\left[1 - \left(\frac{P_b - P_a}{b-a}\right)^{1/2}\right] = -6(b-a) + 6(b-a)\left(\frac{b-a}{4(b-a)}\right)^{1/2} = -3(b-a) < 0\ .$$

Accordingly, optimal Cournot quantities are:

$$x(a,b,C_a) = \left(\frac{b-a-2C_a}{6(b-a)}\right)^{1/2} \text{ and } y(a,b,C_b) = \left(\frac{5(b-a) - 4C_b}{12(b-a)}\right)^{1/2}.$$

Also, given $0 < C_a < \left(\frac{b-a}{4}\right)$ and $0 < C_b < \left(\frac{b-a}{2}\right)$; it easy to verify:

$$x^*(a|C_a = 0) = \left(\sqrt{\frac{1}{6}}\right)\ ; \quad x^*(a|C_a = \frac{b-a}{4}) = \left(\sqrt{\frac{1}{12}}\right)\ ;$$

$$y^*(b|C_b = 0) = \left(\sqrt{\frac{5}{12}}\right) \; ; \;\; y^*\left(b\left|C_b = \frac{b-a}{2}\right.\right) = \left(\sqrt{\frac{1}{4}}\right) = \frac{1}{2}.$$

Hence, the choice of production capacities is limited to the fixed range of:

$\sqrt{\frac{1}{12}} < x^*(a) < \sqrt{\frac{1}{6}}$ and $\frac{1}{2} < y^*(b) < \sqrt{\frac{5}{12}}$ as given in Lemma 4.

Furthermore, from (9) and Lemma 3:

$$\left[\frac{\partial x}{\partial(b-a)}\Big|\bar{y}\right] = \frac{1}{2}\left(\frac{b-a-2C_a}{6(b-a)}\right)^{-1/2}\left(\frac{6(b-a)-6(b-a)+12C_a}{36(b-a)^2}\right) > 0 \; ;$$

$$\left[\frac{\partial y}{\partial(b-a)}\Big|\bar{x}\right] = \frac{1}{2}\left(\frac{5(b-a)-4C_b}{12(b-a)}\right)^{-1/2}\left(\frac{5(12)(b-a)-60(b-a)+48C_b}{144(b-a)^2}\right) > 0$$

and

$$\left[\frac{\partial x}{\partial C_a}\right] = \frac{1}{2}\left(\frac{b-a-2C_a}{6(b-a)}\right)^{-1/2}\left(\frac{-1}{3(b-a)}\right) < 0 \; ;$$

$$\left[\frac{\partial y}{\partial C_b}\right] = \frac{1}{2}\left(\frac{5(b-a)-4C_b}{12(b-a)}\right)^{-1/2}\left(\frac{-48(b-a)}{144(b-a)^2}\right) < 0 .$$

D. Market Coverage

From $x(a,b,C_a) = \left(\frac{b-a-2C_a}{6(b-a)}\right)^{1/2}$ and $y(a,b,C_b) = \left(\frac{5(b-a)-4C_b}{12(b-a)}\right)^{1/2}$ in Lemma 3;

and given $0 < C_a < \left(\frac{b-a}{4}\right)$ *and* $0 < C_b < \left(\frac{b-a}{2}\right)$ as a consequence of (5) and Lemma 1;

then:

$$\frac{x\left\{a \middle| C_a \in [0, \frac{b-a}{4})\right\}}{L} > \left\{ \sqrt{\frac{b-a-2\left(\frac{b-a}{4}\right)}{6(b-a)}} \equiv \sqrt{\frac{b-a}{12(b-a)}} \equiv \sqrt{\frac{1}{12}} \equiv \frac{1}{2\sqrt{3}} \right\}$$

$$\frac{y\left\{b \middle| C_b \in [0, \frac{b-a}{2})\right\}}{L} > \left\{ \sqrt{\frac{5(b-a)-4\left(\frac{b-a}{2}\right)}{12(b-a)}} \equiv \sqrt{\frac{3(b-a)}{12(b-a)}} \equiv \frac{1}{2} \right\}$$

The implication here is that $\left[x(a)^{\min} + y(b)^{\min}\right] = \left[\frac{1}{2} + \frac{1}{2\sqrt{3}}\right] L < L$.

E. Reduced Form Profit Functions and the Optimum Choice of Quality Location

Substituting the production capacity levels of (9) and Lemma 3, in addition to the Bertrand prices established in (5) and Lemma 1, into the profit functions assumed in (1), we get:

$$\pi_a(a,b,x,C_a) = (P_a - C_a)\left(\frac{b-a-2C_a}{6(b-a)}\right)^{1/2} - K_a(a,b) = \left(\frac{b-a}{4} - C_a\right)\left(\frac{b-a-2C_a}{6(b-a)}\right)^{1/2} - K_a(a,b)$$

$$\pi_b(a,b,y,C_b) = (P_b - C_b)\left(\frac{5(b-a)-4C_b}{12(b-a)}\right)^{1/2} - K_b(a,b) = \left(\frac{b-a}{2} - C_b\right)\left(\frac{5b-4C_b-5a}{12(b-a)}\right)^{1/2} - K_b(a,b)$$

.

Re-arranging:

$$\pi_a(a,b) = \left[\frac{b-a-4C_a}{4}\right]\left[\frac{b-a-2C_a(a)}{6(b-a)}\right]^{1/2} - K_a(a,b)$$

$$\pi_b(a,b) = \left[\frac{b-a-2C_b}{2}\right]\left[\frac{5(b-a)-4C_b}{12(b-a)}\right]^{1/2} - K_b(a,b)\,.$$

Upper-bound profits (when $C_a = 0$ and $C_b = 0$) are:

$$\bar{\pi}_a(a,b,K_a(a,b)) = \left(\frac{b-a}{4}\right)\left(\frac{1}{6}\right)^{1/2} - K_a(a,b) = \sqrt{\frac{P_a(a,b)^2}{6}} - K_a(a,b)$$

$$\bar{\pi}_b(a,b,K_b(a,b)) = \left(\frac{b-a}{2}\right)\left(\frac{5}{12}\right)^{1/2} - K_b(a,b) = \sqrt{\frac{5P_b(a,b)^2}{12}} - K_b(a,b)\,.$$

Choice of optimum quality location therefore requires:

$$\left[\frac{\partial \pi_a}{\partial(a)}\right] = -\frac{1}{4}\left(\frac{P_b - C_a}{3(b-a)}\right)^{1/2} + \frac{1}{2}\left(\frac{b-a}{4} - C_a\right)\left(\frac{P_b - C_a}{3(b-a)}\right)^{-1/2}\left(\frac{3(P_b - C_a)}{9(b-a)^2}\right) - K_a'(a,b) = 0$$

$$\left[\frac{\partial \pi_b}{\partial(b)}\right] = \frac{1}{2}\left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{1/2} + \ Z(\bullet) = 0\,;$$

$$where\ \ Z(\bullet) = \frac{1}{2}\left(\frac{b-a}{2} - C_b\right)\left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{-1/2}\left(\frac{3(b-a) - 3(P_a - C_b + b - a)}{9(b-a)^2}\right) - K_b'(a,b).$$

For the case of low quality location:

$$\begin{aligned}
K_a'(a,b) &= -\frac{1}{4}\left(\frac{P_b - C_a}{3(b-a)}\right)^{1/2} + \left(\frac{1}{2(b-a)}\right)\left(\frac{b-a}{4} - C_a\right)\left(\frac{P_b - C_a}{3(b-a)}\right)^{1/2} \\
&= \left(\frac{P_b - C_a}{3(b-a)}\right)^{1/2}\left[-\frac{1}{4} + \left(\frac{1}{2(b-a)}\right)\left(\frac{b-a}{4} - C_a\right)\right] \\
&= \left(\frac{b-a-2C_a}{6(b-a)}\right)^{1/2}\left[-\frac{1}{4} + \left(\frac{1}{2(b-a)}\right)\left(\frac{b-a}{4} - C_a\right)\right]\ ;
\end{aligned}$$

For the case of high quality location:

$$\begin{aligned}
K_b'(a,b) &= \frac{1}{2}\left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{1/2} + \frac{1}{2}\left(\frac{b-a}{2} - C_b\right)\left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{-1/2}\left(\frac{C_b - P_a}{3(b-a)^2}\right) \\
&= \left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{1/2}\left[\frac{1}{2} - \frac{1}{2(b-a)}\left(\frac{b-a}{2} - C_b\right)\left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{-1}\left(\frac{P_a - C_b}{3(b-a)}\right)\right] \\
&= \left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{1/2}\left[\frac{1}{2} - \frac{2}{6(b-a)}\left(\frac{b-a}{2} - C_b\right)\left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{-1}\left(\frac{P_a - C_b}{3(b-a)} + \frac{1}{3}\right)\right] \\
&= \left(\frac{5(b-a) - 4C_b}{12(b-a)}\right)^{1/2}\left[\frac{1}{2} - \frac{1}{3(b-a)}\left(\frac{b-a}{2} - C_b\right)\right].
\end{aligned}$$

Hence,

$$K'_a(a,b) = \left(\frac{b-a-2C_a}{6(b-a)}\right)^{1/2}\left[-\frac{1}{4} + \frac{1}{2(b-a)}\left(\frac{b-a}{4} - C_a\right)\right]$$
$$K'_b(a,b) = \left(\frac{5(b-a)-4C_b}{12(b-a)}\right)^{1/2}\left[\frac{1}{2} - \frac{1}{3(b-a)}\left(\frac{b-a}{2} - C_b\right)\right]$$

This also suggests the following (after re-substituting (9) and rearranging):

$$\left[\frac{K'_a(a,b)}{\bar{x}(a)}\right] = \left[-\frac{1}{4} + \frac{1}{2(b-a)}\left(\frac{b-a}{4} - C_a\right)\right]$$
$$\left[\frac{K'_b(a,b)}{\bar{y}(b)}\right] = \left[\frac{1}{2} - \frac{1}{3(b-a)}\left(\frac{b-a}{2} - C_b\right)\right]$$

and, consequently, quality reaction functions are implicit in:

$$K'_a(a^*|b) = \left(\frac{b-a^*-2C_a}{6(b-a^*)}\right)^{1/2}\left[-\frac{1}{4} + \frac{1}{2(b-a^*)}\left(\frac{b-a^*}{4} - C_a\right)\right] = \left[-\frac{1}{4} + \frac{1}{2(b-a^*)}\left(\frac{b-a^*}{4} - C_a\right)\right]\bar{x}(a^*|b)$$
$$K'_b(b^*|a) = \left(\frac{5(b^*-a)-4C_b}{12(b^*-a)}\right)^{1/2}\left[\frac{1}{2} - \frac{1}{3(b^*-a)}\left(\frac{b^*-a}{2} - C_b\right)\right] = \left[\frac{1}{2} - \frac{1}{3(b^*-a)}\left(\frac{b^*-a}{2} - C_b\right)\right]\bar{y}(b^*|a) \ .$$

F. Proof of Lemma 6

From (16), we establish profit variations with competing quality choice (under no fixed costs) as:

$$\left(\frac{\partial \pi_a}{\partial (b)}\right) = \frac{1}{4}\left(\frac{P_b - C_a}{3(b-a)}\right)^{1/2} + \frac{1}{2}\left(\frac{b-a}{4} - C_a\right)\left(\frac{P_b - C_a}{3(b-a)}\right)^{-1/2}\left(\frac{3C_a}{9(b-a)^2}\right)$$
$$= \frac{1}{4}\left(\frac{P_b - C_a}{3(b-a)}\right)^{1/2} + \frac{1}{2}\left(P_a - C_a\right)\left(\frac{P_b - C_a}{3(b-a)}\right)^{-1/2}\left(\frac{C_a}{3(b-a)^2}\right) > 0.$$

$$\left(\frac{\partial \pi_b}{\partial (a)}\right) = -\frac{1}{2}\left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{1/2} + \frac{1}{2}\left(\frac{b-a}{2} - C_b\right)\left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{-1/2}\left(\frac{-3C_b}{9(b-a)^2}\right)$$
$$= -\frac{1}{2}\left(\frac{P_a - C_b + b - a}{3(b-a)}\right)^{1/2}\left[1 + (P_b - C_b)y(a,b,C_b)^2\left(\frac{C_b}{3(b-a)^2}\right)\right] < 0.$$

With capacity choices fixed [i.e. substituting (9) and (5) back into (16); such that $x(\bullet) = x^*$ and $y(\bullet) = y^*$ as proven in Lemma 4], then after rearranging and simplifying, we get:

$$\left(\frac{\partial \pi_a}{\partial (b)}\right) = \frac{x^*}{4} > 0 \quad \text{and} \quad \left(\frac{\partial \pi_b}{\partial (a)}\right) = -\frac{y^*}{2} < 0 .$$

Hence:

$$\left|\frac{\partial \pi_b}{\partial (a)}\right| - \left|\frac{\partial \pi_a}{\partial (b)}\right| = \frac{1}{2}\left(y^* - \frac{x^*}{2}\right)$$

But since $y^* > x^*$ is always true at equilibrium from (10), then it follows that:

$$\left|\frac{\partial \pi_b}{\partial (a)}\right| > \left|\frac{\partial \pi_a}{\partial (b)}\right| \quad .$$

G. Quality Reaction Functions

From Lemma 7:

$$K_a'(a^* | b) = \left(\frac{b - a^* - 2C_a}{6(b - a^*)}\right)^{1/2}\left[-\frac{1}{4} + \frac{1}{2(b - a^*)}\left(\frac{b - a^*}{4} - C_a\right)\right] = \left[-\frac{1}{4} + \frac{1}{2(b - a^*)}\left(\frac{b - a^*}{4} - C_a\right)\right]\bar{x}(a^* | b)$$

$$K_b'(b^* | a) = \left(\frac{5(b^* - a) - 4C_b}{12(b^* - a)}\right)^{1/2}\left[\frac{1}{2} - \frac{1}{3(b^* - a)}\left(\frac{b^* - a}{2} - C_b\right)\right] = \left[\frac{1}{2} - \frac{1}{3(b^* - a)}\left(\frac{b^* - a}{2} - C_b\right)\right]\bar{y}(b^* | a) \; .$$

Then, given fixed capacity levels as governed by Lemma 3, and continuing the assumption of $K_a(a,b) = \frac{1}{2}(a - \gamma b)^2$ and $K_b(a,b) = \frac{1}{2}(b - \gamma a)^2$ with $C_a = \lambda a$ and $C_b = \lambda b$; then:

$$\pi_a(a,b;\lambda,\gamma) = \left[\frac{b - a - 4\lambda a}{4}\right]\bar{x} - \frac{1}{2}(a - \gamma b)^2$$

$$\pi_b(a,b;\lambda,\gamma) = \left[\frac{b - a - 2\lambda b}{2}\right]\bar{y} - \frac{1}{2}(b - \gamma a)^2$$

and since $\left[\frac{\partial \pi_a}{\partial(a)}\middle| b*\right]=0$ and $\left[\frac{\partial \pi_b}{\partial(b)}\middle| a*\right]=0$ is a necessary requirement for optimal quality location at the 1st stage of competition given fixed capacity levels at the 2nd stage, then it follows that: $\left[\frac{\partial \pi_a}{\partial(a)}\right]=-\frac{\bar{x}}{4}[1+4\lambda]-a+\gamma b=0$ and $\left[\frac{\partial \pi_b}{\partial(b)}\right]=\frac{\bar{y}}{2}[1-2\lambda]-b+\gamma a=0$. This yields:

$a*(b|\bar{x})=\gamma b-\frac{\bar{x}}{4}(1+4\lambda)$ and $b*(a|\bar{y})=\gamma a+\frac{\bar{y}}{2}(1-2\lambda)$; and solving simultaneously requires:

$$b^*=\frac{1}{1-\gamma^2}\left\{\bar{x}\left(-\frac{\gamma}{4}-\gamma\lambda\right)+\bar{y}\left(\frac{1}{2}-\lambda\right)\right\}$$
$$a^*=\frac{1}{1-\gamma^2}\left\{\bar{x}\left(-\frac{1}{4}-\lambda\right)+\bar{y}\left(\frac{\gamma}{2}-\lambda\gamma\right)\right\}$$

The quality spread between the two quality locations is therefore always governed by:

$$(b*-a*)=\frac{1}{1-\gamma^2}\left\{\left(-\frac{1}{4}-\lambda\right)(\gamma-1)\bar{x}+\left(\frac{1}{2}-\lambda\right)(1-\gamma)\bar{y}\right\}\ .$$

Hence, $\left(\frac{\partial(b-a)}{\partial\gamma}\right)<0$; $\left(\frac{\partial a}{\partial x}<\frac{\partial b}{\partial x}<0\right)$; and $\left(\frac{\partial b}{\partial y}>\frac{\partial a}{\partial y}>0\right)$. Also, $\left(\frac{\partial a}{\partial\lambda}<0\right)$ and $\left(\frac{\partial b}{\partial\lambda}<0\right)$.

H. Analysis of Quality Cross-Effects in Capacity Constrained Price Competition with Costless Production

Assume costless production:

$C_a=C_b=0$

Then, from Lemma 3, we have the choice of fixed capacities as $x*=\sqrt{\frac{1}{6}}$ and $y*=\sqrt{\frac{5}{12}}\ >x*$.

From this, we have profit functions in (24) as:

$$\pi_a(a,b,\gamma)=\left(\frac{b-a}{4}\right)\sqrt{\frac{1}{6}}-\frac{1}{2}(a-\gamma b)^2$$

$$\pi_b(a,b,\gamma)=\left(\frac{b-a}{2}\right)\sqrt{\frac{5}{12}}-\frac{1}{2}(b-\gamma a)^2$$

Quality reaction functions from $\left[\frac{\partial \pi_a}{\partial (a)} \middle| b^* \right] = 0$ and $\left[\frac{\partial \pi_b}{\partial (b)} \middle| a^* \right] = 0$ are:

$$a^* = \gamma b - \frac{1}{4}\sqrt{\frac{1}{6}}$$

$$b^* = \gamma a + \frac{1}{2}\sqrt{\frac{5}{12}}$$

Optimum quality location in costless production is therefore
(by solving the quality reaction functions simultaneously):

$$b^* = \frac{1}{1-\gamma^2}\left[\frac{1}{2}\sqrt{\frac{5}{12}} - \frac{\gamma}{4}\sqrt{\frac{1}{6}}\right]$$

$$a^* = \frac{1}{1-\gamma^2}\left[\frac{\gamma}{2}\sqrt{\frac{5}{12}} - \frac{1}{4}\sqrt{\frac{1}{6}}\right]$$

As $\gamma \to 0$ (no quality cross-effects):

which implies $K(a) = \frac{1}{2}a^2$ and $K(b) = \frac{1}{2}b^2$;

we get $b^* = \frac{1}{4}\sqrt{\frac{5}{3}}$ and $a^* = 0$ (minimum location for low-quality firm).

As $\gamma \to 1$ (severe quality cross-effects):

which implies $K_a(a,b) = K_b(a,b) = \frac{1}{2}(b-a)^2$;

we get $a^* \to b^*$ with $b^* = 1$ (minimum differentiation at maximum location).

For $\gamma = 0.5$ (average quality cross-effects):

which implies $K_a(a,b) = \frac{1}{2}\left(a - \frac{b}{2}\right)^2$ and $K_b(a,b) = \frac{1}{2}\left(b - \frac{a}{2}\right)^2$;

we get $b^* = \frac{4}{3}\left[\frac{1}{2}\sqrt{\frac{5}{12}} - \frac{1}{8}\sqrt{\frac{1}{6}}\right] = 0.36$ and $a^* = \frac{4}{3}\left[\frac{1}{4}\sqrt{\frac{5}{12}} - \frac{1}{4}\sqrt{\frac{1}{6}}\right] = 0.08$.

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