Spatial Competition in a Differentiated Market with Asymmetric Costs

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Abstract

Spatial quality choice is introduced, where consumers are horizontally differentiated by taste and firms vertically differentiated by quality location, within an equilibrium model of duopoly competition characterized by asymmetric fixed and variable costs. Firms choose quality location followed by prices but then may vertically re-locate their quality offerings based on changing horizontal consumer taste. A monopolistic equilibrium solution arises with firms achieving positive economic profits through price-quality markups exceeding marginal costs. Under strict inequality conditions, each firm acts as a monopolistic competitor within a range of quality choices governed by multiple relative differentiation outcomes. On the other hand, vertical re-location exhibits a resistance to change on the part of vertically located firms such that firms dislike quality re-location and prefer stable preferences in quality. Such resistance to change is overcome by firms re-locating their quality offerings to maximize monopolistic brand-space gains. It is argued that more horizontal differentiation may force more product differentiation by vertical quality relocation. A relative change in quality preferences may result in wider quality spreads in the market through vertical quality re-locations, even though the resistance to change arguments may still hold good.

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I. SPATIAL QUALITY CHOICE

The analysis of endogenous quality choice under fixed and variable costs of quality by Beath and Y. Katsoulacos (1991) Reitman (1991), Tirole (1996), Vives (1999), and Motta (1993), as an extension of the works of Bonnano (1986), Champsaur and Rochet (1989), Vives (1985), and Shaked and Sutton (1982) – as initially introduced by the original works of Hotelling (1929) and Chamberlin (1933); sets an interesting exposure into the area of differentiated quality choice under different cost assumptions of quality and output (capacity), given flexible preferences for consumer taste. Under the finiteness property of natural oligopolies set to two firms in the market (as proven by Shaked and Sutton (1983, 1984) and utilized by Vives (1985)), the analysis in the literature suggests that a stable differentiation argument always arises for an optimal choice of quality levels, with the degree of product differentiation measured by the differential quality spread in the market given an optimal pricing schedule or an optimal revenue path. Several extensions of the traditional quality differentiation model[1] to include variable consumer taste based on a uniform distribution of willingness to pay in consumption, results in an equilibrium outcome such that firm profits become a function of the differential quality spread in the market *and* the maximum willingness-to-pay for a single differentiated good (resulting in optimal product positioning based on endogenous quality location). Other findings in the literature notably suggest that under both fixed and variable costs of quality improvement, the market outcome is actually segmented according to the ratio of quality choice by the existing two firms in the market, and that

[1] Traditional models of quality differentiation with fixed preferences (fixed consumer taste based on a utility function which differentiates consumers by different levels of income rather than by differential preferences in quality choice) include Shaked and Sutton (1982, 1984) and Vives (1985). Also, Motta (1993) extends these models to account for Cournot and Bertrand competition with endogenous quality choice using a more general utility function which exhibits continuous preferences for quality, and by extending (utilizing) additional cost assumptions related to fixed and variable costs of quality.

such a market segmentation contains three layers of "equilibrium-chosen preferences" based on the distribution of quality taste in consumption (some consumers will buy the high-quality brand, others will buy the low-quality brand, and still others will buy neither brand). For a fully covered market with positive reservation utility, it has been proven that all consumers will buy either the high-quality brand or the low-quality brand contingent on their surplus value functions and/or depending on their relative income levels (in Jun 1996, Motta 1993, Vives 1985, Dixit 1979, and Dixit and Stiglitz 1977). Another dimension is due to the nature of quality differentiation. Most of the analysis in the literature suggests that vertical product differentiation by quality choice always arises at equilibrium, regardless of the level of fixed or variable costs of quality incurred by both firms in the market (as illustrated by Shaked and Sutton 1982 and Kim 1987), and regardless of the nature of the distribution of consumer taste (preferences) for quality (such as the analysis in Tirole 1996, Jun 1996, and Motta 1993 among others)[2]. Locational models of horizontal differentiation, on the other hand, do not have such an advantage (see Cremer and Thisse 1991, and Vives 1999). Yet, in general, there has been a consensus in the literature that quality competition usually leads to minimum or maximum product differentiation depending on the nature of consumer taste and the inclusion or exclusion of price competition in the models henceforth proposed (see Schmalensee 1979, Scherer 1980, Singh and Vives 1984, and Boyer and Moreaux 1987).

It is the objective of this research to introduce the concept of spatial quality choice as an integrative differentiation model that deals with both horizontal differentiation and vertical location simultaneously.

The specific representation outlined in the analysis assumes consumer taste as a form of horizontal differentiation while, at the same time, proposes vertical location by differentiated firms. Hence, the concept of “spatial quality choice” is merely defined as a product positioning model where consumers are horizontally differentiated by taste while firms are vertically differentiated by quality location. The current analysis is confined to a duopoly market setting with asymmetric fixed and variable costs of quality in production.

[2] Vertical product differentiation in Bertrand competition always arises at equilibrium in the sense that equilibrium profits are a function of the ratio of quality choice chosen by the two firms. The distribution of consumer taste (willingness to pay for quality improvement) does not enter the equilibrium profit function, but rather only the maximum willingness to pay parameter [$\overline{v}$] does.

Several assessments within the literature of industrial economics are then extended for the newly established concept of spatial quality choice to see if the same results hold good once horizontal differentiation is introduced into vertical quality location. These assessments constitute the core of our analysis. The impact of vertical location and re-location on equilibrium quality are first examined within the specified presentation of spatial quality choice and Nash stability in equilibrium location is then investigated for a spatial quality model. Additionally, several cost assumptions are further relaxed to include non-symmetric fixed and variable costs of quality. The sensitivity of firm quality re-location to horizontal consumer taste is investigated as well. Strategic effects and cross-effects within a spatial quality choice model with vertical location and horizontal differentiation is henceforth developed and analyzed.

We establish monopolistic inequality conditions that deal with equilibrium stability and then establish a more general monopolistic brand-space equilibrium solution with asymmetric costs. A thorough assessment of vertical re-location on spatial quality equilibrium results in a resistance to change argument by differentiated firms, such that firms dislike quality relocations and prefer stable horizontal preferences in consumer taste. Flexible choice is then introduced as a consequence of this effect.

The following hypotheses constitute the central theme of our analysis:

(i) How is "spatial quality choice" defined and what are the equilibrium stability conditions for "spatial quality equilibrium"?

(ii) Formally, what type(s) of differentiation outcome(s) could develop within a stable equilibrium position of spatial quality choice? Are there multiple differentiation outcomes, and if so, under what conditions are they able to develop?

(iii) How does a given change in horizontal differentiation and/or flexible choice affect vertical location and product positioning in spatial quality equilibrium?

Theoretical investigations into these interesting hypotheses constitute the remainder of this paper. Before proceeding to the formal analysis, however, it is thought vital to define spatial quality choice as a product positioning model where consumers are horizontally differentiated by taste while firms are vertically differentiated by quality location, as follows:

***Definition*. Spatial quality choice is defined as a product positioning model where consumers are horizontally differentiated by taste while firms are vertically differentiated by quality location. For a duopoly market structure, the following model framework is assumed:**

(A). Two firms, denoted by *A* and *B*, each producing a single product. Both firms located on a vertical quality street bounded by [0,L].

(B). Consumers reside on a horizontal street of taste preferences; with surplus disutility $U(\tau, x) = \begin{cases} -P_a - \tau(x-a)^2 \\ -P_b - \tau(L-b-x)^2 \end{cases}$ **;**

(C). Products are positioned at quality locations *a* and *b* on the vertical scale, with $(0 \le a \le b \le L)$**, and by** (x^S, y^S) **on the horizontal scale.**

(D). Competition: quality (first stage) then prices (second stage).

(E) Profits:
$$\pi_a[P_a, C_a, K(a)] = \{P_a - C_a(a)\}x^S(\tau) - K(a)$$
$$\pi_b[P_b, C_b, K(b)] = \{P_b - C_b(b)\}(1 - x^S(\tau)) - K(b)$$

with $K'(a) > 0, K'(b) > 0, C'_a(a) > 0, C'_b(b), C''(\cdot) < 0, x^S(\tau) > 0.$

(F). The duopoly market is fully covered: $x^S(a,b,\tau) + y^S(a,b,\tau) = L$.

(G). Maximum vertical location coincides with maximum horizontal taste.

The proposed model is therefore that of an incumbent duopoly market where consumers are differentiated by horizontal taste while firms are differentiated by vertical quality location[3].

II. THE MONOPOLISTIC INEQUALITY CONDITIONS

[3] Spatial quality choice, as defined here, is not synonymous with spatial competition. Whereas spatial competition deals with geocentric and/or geographical differences between markets or between different products, spatial quality choice on the other hand is considered a form of product differentiation which deals with product positioning along horizontal consumer taste and vertical product location. It should be also noted that the proposed spatial quality model analyzed herein is structured to be of an introductory nature, and that it is a hopeful attempt to open a new area of research within the science of industrial economics. More precisely, it is not a comprehensive assessment of the topic.

Consider an asymmetric duopoly market with product positioning in spatial quality choice such that firms compete in vertical quality location followed by prices; whereas consumers are horizontally differentiated by changing taste preferences based on unit transportation costs. Spatial quality choice[4], in essence, therefore refers to the introduction of horizontal quality differentiation into vertical location, and refers to modeling quality choice via two main axes of product positioning: *horizontal* differentiation (as governed by consumer preferences along a quality street), and *vertical* location (as governed by firm-specific quality location and re-location).

Assume consumers behave according to a dis-utility function of the form[5]:

$$U(\tau, x) = \begin{cases} -P_a - \tau(x-a)^2 \\ -P_b - \tau(L-b-x)^2 \end{cases} \tag{1}$$

where τ signifies unit transportation cost for the surplus disutility associated with purchasing a non-ideal brand, or equivalently, the linear marginal disutility from purchasing a non-ideal brand, and *L* represents the spectrum of quality choice offered in the market, with $a, b \in [0, L]$. It follows that the demand for the low-quality brand (x) can then be found by equating the disutilities given in (1) above, as:

$$\frac{P_b - P_a}{\tau} = (x-a)^2 - (L-b-x)^2 \tag{2}$$

This results in (see Appendix A):

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[4] Horizontal differentiation is based on Hotelling's (1929) –style quality street, and the term *spatial quality differentiation* comes from modeling quality along two axes of quality choice: horizontal (differentiation by consumer preference or taste) and vertical (differentiation by product location). This terminology is also in line with those in Singh and Vives (1984), Shy (1995), Tirole (1998), and Vives (1999).

[5] There is a continuum of consumers uniformly distributed along the horizontal quality street of product locations. Demand is differentiated by quality preferences based on unit transportation costs and the quality street is bounded by [0,L]. A representative indifferent consumer between low-quality and high-quality location (i.e. the two quality locations offered by the incumbent duopoly firms) is located at $\hat{x}$; where $a \le \hat{x} \le b$; such that all consumers whose preferences lie within [0,x) prefer the low-quality brand and all consumers whose preferences lie within (x,L] prefer the high-quality brand. Moreover, consumer demand fully covers all quality locations.

$$D_a(P_a, P_b) = x^S = \frac{P_b - P_a}{2\tau(L-b-a)} + \frac{L-b+a}{2}. \tag{3}$$

The demand for the high-quality brand (*B*) can similarly be derived to be:

$$D_b(P_a, P_b) = \frac{P_a - P_b}{2\tau(L-b-a)} + \frac{(2-L)+b-a}{2} \tag{4}$$

For an incumbent duopoly, the derived demands for the low and high-quality brands in (3) and (4) imply $\frac{\partial D_a}{\partial \tau} < 0$ and $\frac{\partial D_b}{\partial \tau} > 0$. An increase in the unit transportation cost, requiring a higher disutility from purchasing a non-ideal brand, causes a decline in consumer demand for the low quality brand and an increase in consumer demand for the high quality brand. Consequently, as consumer preferences become more biased against purchasing a non-ideal brand, actual consumption tends to favor the high quality brand.

Lemma 1. For spatial quality choice with vertical location and horizontal differentiation, consumer demand heavily depends on revealed preferences for unit transportation costs and price-quality differentials such that:

$$D_a(P_a, P_b) = \frac{P_b - P_a}{2\tau(L-b-a)} + \frac{L-b+a}{2}\ ;\ D_b(P_a, P_b) = \frac{P_a - P_b}{2\tau(L-b-a)} + \frac{(2-L)+b-a}{2};$$

with $\frac{\partial D_a}{\partial \tau} < 0$ and $\frac{\partial D_b}{\partial \tau} > 0$; thereby creating a consumer bias against low-quality brands in favor of high-quality brands for an increase in τ.

For the two-stage Bertrand-quality game in spatial quality choice, where firms choose vertical location (quality levels) at the first stage of the game based on horizontal preferences for consumer taste, and then compete in prices at the second stage, a profit function with non-symmetric fixed and variable costs of quality is assumed to be[6]:

[6] Here, the finiteness property of oligopoly pricing is still assumed, where at most two firms can co-exist at equilibrium for a two-stage Bertrand game in spatial quality choice. Following Vives (1985) and Motta (1993), the market is also assumed to be fully covered by the two chosen quality levels, as is given by the surplus disutility function in (1).

$$\pi_a[P_{a,}C_a, K(a)] = \{P_a - C_a(a)\}x^S(\tau) - K(a)$$
$$\pi_b[P_b, C_b, K(b)] = \{P_b - C_b(b)\}(1 - x^S(\tau)) - K(b) \tag{5}$$

with $K'(a) > 0, K'(b) > 0, C_a'(a) > 0, C_b'(b), C''(\cdot) < 0, x^S(\tau) > 0.$

Profits are a function of unit variable costs of quality, $C_a(a)$ and $C_b(b)$, and fixed costs of quality (investments in quality location), $K(a)$ and $K(b)$. There is no collusion among firms and there are no quality cross-effects in fixed cost investments.

Using backward induction, the first-order conditions for equilibrium profit levels for both firms at the second stage of the game, given flexible horizontal preferences for quality choice at the first stage, can be written to be:

$$\partial\pi_a / \partial P_a = x^S(\tau) + \frac{\partial x^S(\tau)}{\partial P_a}[P_a - C_a(a)] = 0$$
$$\partial\pi_b / \partial P_b = (1 - x^S(\tau)) - \frac{\partial x^S(\tau)}{\partial P_b}[P_b - C_b(b)] = 0 \tag{6}$$

Solving for prices (see Appendix B), we get:

$$P_a = \frac{1}{3}\left[2C_a + C_b + \tau(L - b - a)(2 + L - b + a)\right]$$
$$P_b = \frac{1}{3}\left[2C_b + C_a + \tau(L - b - a)(4 - L + b - a)\right] \tag{7}$$

Equilibrium profits are therefore:

$$\pi_a = \frac{1}{3}\left[C_b - C_a + \tau(L - b - a)(2 + L - b + a)\right]x^S(\tau) - K(a)$$
$$\pi_b = \frac{1}{3}\left[C_a - C_b + \tau(L - b - a)(4 - L + b - a)\right][1 - x^S(\tau)] - K(b) \tag{8}$$

The second-order conditions from (6) are satisfied for maximum profits, since $\partial^2\pi_a \big/ \partial P_a^2 = \partial^2\pi_b \big/ \partial P_b^2 = \dfrac{-1}{\tau(L-b-a)} < 0$. Therefore, equilibrium prices as given in (7) are a function of own and competing variable costs, unit disutility costs, in addition to the vertical positioning of quality levels introduced by the two competing firms in the market. This is due to *positive price correlations* in quality choice such that if the variable cost of the competing quality level increases, this will induce the competing price level to increase, also causing own price levels to increase. Price sensitivity due to a change in horizontal consumer taste is higher for the high-quality brand as compared to that of the low-quality brand, since $\dfrac{\partial P_b}{\partial \tau} > \dfrac{\partial P_a}{\partial \tau}$. This may be due to actual consumption behavior favoring the high-quality brand for a change in disutility costs, a consequence of our earlier discussion from the fact that $\dfrac{\partial D_a}{\partial \tau} < 0$ and $\dfrac{\partial D_b}{\partial \tau} > 0$. Additionally, from (7), the price spread is $\left(P_b - P_a\right) = \dfrac{1}{3}\left(C_b - C_a\right) + \dfrac{2}{3}\tau(L-b-a)\left(1-L+b-a\right)$. This signifies a weighted average between the cost difference of the two chosen quality locations and a combination of quality spreads and unit transportation cost inducing a quality premium in prices. It is yet clear that larger cost differences and/or a higher unit transportation cost in consumption, will create a larger price spread in production.

The realization of equilibrium profits, as given in (8), is contingent on realizing equilibrium location based on horizontal consumer preferences at the first stage of the two-stage game. Such an equilibrium location can be found by choosing horizontal quality levels that maximize profits given the vertical configuration of prices in (7). For the case of the low-quality firm (*A*), with $\pi_a = (P_a - C_a)x^S - K(a)$, the first-order condition for an optimal location choice is found to be[7]:

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[7] For the quality spectrum normalized to unity, i.e. with $L=1$, prices for the low and high-quality brands become:

$P_a = \dfrac{1}{3}\left[2C_a + C_b + \tau(1-b-a)(3-b+a)\right]$ and $P_b = \dfrac{1}{3}\left[2C_b + C_a + \tau(1-b-a)(3+b-a)\right]$, respectively. From these, we can also get $\dfrac{\partial P_a}{\partial(a)} = \dfrac{-2\tau(1+a)}{3} < 0$ and $\dfrac{\partial P_a}{\partial(b)} = \dfrac{2\tau(b-2)}{3} < 0$.

$$\frac{d\pi_a}{d(a)} = \left(\frac{\partial P_a}{\partial a} - C_a'\right)x^S(\tau) + \frac{\partial x^S(\tau)}{\partial (a)}(P_a - C_a) - K'(a) = 0 \tag{9}$$

This can be easily translated to:

$$K'(a) = \frac{\partial x^S}{\partial (a)}(P_a - C_a) + \left(\frac{\partial P_a}{\partial (a)} - C_a'(a)\right)x^S \tag{10}$$

The expression in (10) above basically states that fixed capital investment "return" on quality improvements, $K'(a)$, has to equal, at equilibrium, the sum of its associated higher price markup due to a higher quality choice, $\frac{\partial x^S}{\partial (a)}(P_a - C_a)$, plus the marginal cost effect on net profits through indirect consumer demand as given by $\left(\frac{\partial P_a}{\partial (a)} - C_a'(a)\right)x^S$.

As a result, for:

$$\frac{\partial P_b}{\partial \tau} > \frac{\partial P_a}{\partial \tau},\ C_a' > 0,\ P_a > C_a,\ \frac{\partial x^S}{\partial (a)} = \frac{\left(-\frac{\partial P_a}{\partial (a)}\right)(2\tau)(L-b-a) + 2\tau(P_b - P_a)}{4\tau^2(L-b-a)^2} + \frac{1}{2} > 0;$$

the first term on the right-hand side of (10), which signifies the *quality markup*, is actually positive, while the second term which captures the *marginal cost* effect on profits, is actually negative. An increase in vertical quality location through additional investments to improve vertical quality choice is only optimal if the investment return on quality equals the sum of the positive quality markup associated with those investments plus the (negative) marginal cost effect as a consequence of additional investments in quality.

The *quality markup* is intuitively *positive* due to higher demand and higher prices associated with a better quality choice based on consumer preferences, while the *marginal cost* effect is intuitively *negative* due to reduction in profits caused by increased marginal costs as a consequence of a higher quality product.

This leads to the following Lemma:

Lemma 2. Spatial differentiation in quality choice yield capital investment returns composed of positive quality markups and negative marginal cost effects, of:

$$K'(a) = \frac{\partial x^S}{\partial(a)}(P_a - C_a) + \left(\frac{\partial P_a}{\partial(a)} - C'_a(a)\right)x^S \quad ;$$

$$K'(b) = \left\{\left[\frac{\partial P_b}{\partial(b)} - C'_b(b)\right](1 - x^S(\tau))\right\} - \frac{\partial x^S(\tau)}{\partial(b)}(P_b - C_b),$$

given vertical configuration of equilibrium prices:

$$P_a = \frac{1}{3}\left[2C_a + C_b + \tau(L - b - a)(2 + L - b + a)\right] \quad ;$$

$$P_b = \frac{1}{3}\left[2C_b + C_a + \tau(L - b - a)(4 - L + b - a)\right].$$

A positive investment outlay in horizontal quality choice at the first stage of the two-stage Bertrand-quality game, as given by $K'(a) > 0$ and $K'(b) > 0$, is evident only if the following conditions hold true (from (10) and Lemma 2, also see Appendix C):

$$\begin{aligned} \left|\frac{\partial x^S(\tau)}{\partial(a)}(P_a - C_a)\right| &> \left|\left(\frac{\partial P_a}{\partial(a)} - C'_a(a)\right)x^S\right| \\ \left|\frac{\partial x^S(\tau)}{\partial(b)}(P_b - C_b)\right| &> \left|\left(\frac{\partial P_b}{\partial(b)} - C'_b(b)\right)(1 - x^S(\tau))\right| \end{aligned} \tag{11}$$

The above inequalities imply that a positive investment return on quality choice is only feasible if the absolute value of the quality markup (due to higher marginal pricing and higher indirect consumer demand captured by horizontal preferences in quality) exceeds the absolute value of the marginal cost effect. In essence, such an argument is in favor of *brand-specific monopolistic competition*, where each firm monopolizes itself within a brand range of the quality spectrum and achieves positive profits through price-quality markups exceeding marginal costs. Such an argument inherently does not deny the idea of maximum product differentiation, as a feasible differentiation outcome within the multiple differentiation outcomes of brand-specific monopolistic competition, such that the monopolistic inequality conditions in (11) always hold true.

Notice also that the price of the high-quality brand, P_b, is implicitly included within the indirect demand function of the low-quality brand, since $x^S = x^S(P_a, P_b; a, b, L, \tau)$ and $\frac{\partial x^S}{\partial P_b} > 0$. An increase in the price of the high-quality brand has a *negative* effect on low-quality investment returns, and therefore pushes the low-quality firm to increase its investment outlay in order to achieve higher quality markups and achieve a more determined monopolistic position within the industry's quality spectrum. Formally, with $P_b \uparrow$, $x^S \uparrow$, we have $\left(\frac{\partial P_a}{\partial(a)} - C_a'(a)\right)x^S \downarrow$, resulting in $K'(a) \downarrow$ from (10), and therefore requiring *a* and $K(a)$ to increase in order to offset the reduction in low quality investment returns induced by an increase in the price of the competing high-quality product.

This may also have some local entry repercussions, in the sense that a high-quality price increase causes investment "sufferings" for the low-quality brand, making it more difficult for the low quality brand to enter the monopolistic brand-space of the high quality brand.

It can be easily shown that higher unit variable costs reduce the quality markup, thereby reducing low-quality investment returns, and hence require an increase in fixed investments for a better "positioning strategy" across the industry's differentiation range. More formally, if $C_a(a) \uparrow$, then $(P_a - C_a) \downarrow$, inducing $\frac{\partial x^S}{\partial(a)}(P_a - C_a) \downarrow$ and $K'(a) \downarrow$, thus requiring $K(a) \uparrow$.

The above analysis can also be extended for the case of the high-quality brand with similar results (see Appendix C)[8].

It is also important to note that the second-order condition for equilibrium location is found to be:

[8] The analysis for the high quality brand holds similar results in terms of the brand-space monopolistic inequality conditions given in (11), see Appendix C. However, the low quality firm cannot cause investment "sufferings" for the high quality firm, in contrast to the earlier discussion.

$$\frac{\partial^2 \pi_a}{\partial (a)^2} = \left(\frac{\partial^2 P_a}{\partial (a)^2} - C''_a\right) x^S(\tau) + \frac{\partial x^S(\tau)}{\partial (a)}\left(\frac{\partial P_a}{\partial (a)} - C'_a\right) + \frac{\partial^2 x^S(\tau)}{\partial (a)^2}(P_a - C_a) - K''(a)$$

$$\frac{\partial^2 \pi_b}{\partial (b)^2} = \left(\frac{\partial^2 P_b}{\partial (b)^2} - C''_b\right)(1 - x^S(\tau)) - \left[\frac{\partial x^S(\tau)}{\partial (b)}\left(\frac{\partial P_b}{\partial (b)} - C'_b\right) + \frac{\partial^2 x^S(\tau)}{\partial (b)^2}(P_b - C_b)\right] - K''(b) \tag{12}$$

With $\frac{\partial^2 P_a}{\partial (a)^2} = \frac{-2\tau}{3} < 0$, $\frac{\partial x^S}{\partial (a)} > 0$, and $\frac{\partial^2 x^S}{\partial (a)^2} < 0$, coupled with the retained assumptions of $\frac{\partial P_a}{\partial (a)} < 0$, and $C'_a > 0$, the second-order condition of equilibrium location for the spatial quality model is negative (i.e. satisfied for maximal profits) if and only if the following two conditions hold:

(1) $K''(a) \geq 0$; or $K''(a) < 0$ but not too sufficiently negative (to offset the other three terms)

(2) $\left|\frac{\partial^2 P_a}{\partial (a)^2}\right| > \left|C''_a(a)\right|$

for the low quality brand, and similarly for the high quality brand:

(1)' $K''(b) \geq 0$; or $K''(b) < 0$ but not too sufficiently negative (to offset the other three terms)

(2)' $\left|\frac{\partial^2 P_b}{\partial (b)^2}\right| > \left|C''_b(b)\right|$.

The first condition basically states that fixed investments (fixed entry costs) should either be convex, linear, or not too concave for an optimal solution. The second condition, on the other hand, states that the marginal price effect for vertical quality location has to dominate the marginal cost effect in absolute value. Thus, if unit variable costs are linear in quality, the second condition always stands true.

The *monopolistic inequality conditions* for price-quality equilibrium with spatially differentiated products are summarized in Proposition 1, below (see Appendix C):

Proposition 1. Spatial quality choice, at equilibrium, yield positive economic profits through price-quality markups exceeding marginal costs subject to the following monopolistic inequality conditions:

$$\left|\frac{\partial x^S(\tau)}{\partial(a)}(P_a - C_a)\right| > \left|\left(\frac{\partial P_a}{\partial(a)} - C_a'(a)\right)x^S\right| \text{;}$$

$$\left|\frac{\partial x^S(\tau)}{\partial(b)}(P_b - C_b)\right| > \left|\left(\frac{\partial P_b}{\partial(b)} - C_b'(b)\right)(1 - x^S(\tau))\right| \text{;}$$

such that a high-quality firm could shield its monopolistic brand-space and cause investment "sufferings" for the low-quality firm, but not vice versa.

Accordingly, spatial quality choice with vertical location and horizontal differentiation leads to a monopolistic competitive outcome whereby firms achieve positive profits at equilibrium, with prices governed by choices of low and high quality location, own and competing unit variable costs of quality, in addition to horizontal preferences for consumer taste captured by unit transportation costs.

The "victim" of the spatial quality model, as developed in Proposition 1 above (see Appendix B and Appendix C), is that firm choosing to produce a low-quality product. Although still achieving positive economic profits at equilibrium, the low-quality producer is more vulnerable to attack from price fluctuations of the high-quality brand, due to positive price correlations in equilibrium location and due to high-quality price increases inducing a negative effect on low-quality investment returns. This creates a more costly brand-space monopolistic position for brands produced by the low-quality firm as compared to those produced by the high-quality firm[9].

[9] The conclusion that the low-quality firm is the "victim" of the proposed spatial quality model should be weighted against the assumptions of that model. If some of the model's assumptions are relaxed, for example, those pertaining to the cost structure of the industry, and if we assume that variable costs for the high-quality firm are more convex with quality choice than those of the low-quality firm (the latter, for example, described by linearity), then such a conclusion may not hold. However, given the assumptions of the proposed model, the low-quality firm does have a "competitive dis-advantage" in the industry as compared to the high-quality firm because of its costly monopolistic position within the quality spectrum offered by the industry, and due to its reactive nature in price correlations compared to the active monopolistic price position of the high quality firm.

III. STRATEGIC EFFECTS AND CROSS EFFECTS

The monopolistic outcome of the spatial quality model, as basically presented in Lemma 2 and Proposition 1 above, yield non-specific product differentiation locations for the low quality and high quality brands. Yet, from Proposition 1, the low-quality brand has an additional burden of reactive investment sufferings to a high-quality price increase. In addition, the vertical location of the low quality firm seems to be relatively unstable (or reactive) to the location-space of the high quality firm, even though both firms achieve positive economic profits at equilibrium. Turning back to Lemma 2, the composition of (positive) capital investment returns to quality choice, at equilibrium, contain positive quality markups and negative marginal cost effects, an argument in favor of brand-specific monopolistic competition.

Having said that, it is worth investigating the effect of *internalizing* capital investments of quality location within *both* profit functions, specifically for the low-quality brand, in order to observe how the endogenous choice of vertical quality location behaves with different investment choices of an originally prescribed horizontal quality attribute (for *fixed horizontal consumer taste*). The analysis has the objective of investigating vertical quality location given fixed horizontal taste at the stage of price competition with capital investments internalized within the marginal cost functions of firm profits. In other words, we would like to investigate the *direct* (strategic) effect of capital investments with regards to the low-quality and high-quality brands, in addition to investigating the *indirect* (cross) effects associated with such investments; by expanding on the monopolistic inequality conditions in (11) to include direct and indirect effects on equilibrium profits given *fixed* horizontal taste. The analysis here is confined to the second stage (i.e. the stage of price competition) with the assumption that the original level of fixed capital investments in quality location have already been incurred (i.e. as a sunk cost) at the first stage of competition. Additional investments in quality improvements are then internalized within marginal costs.

Let the low-quality cost function be formulated as $C(a) = C\{K(a), a\}$ with $C'_K < 0$ and $C'_a > 0$, and symmetrically for high quality location. Hence, with capital investments in quality improvements internalized within the (variable) cost function of quality choice, additional capital investments in more *efficient* quality choices are assumed to lower variable costs of quality

($C'_K < 0$), whereas an increase in quality location (i.e. a *level* change in quality location, desiring higher quality characteristics) is assumed to increase variable costs of quality ($C'_a > 0$)[10].

With this assumption, the profit functions (at the stage of price competition) can be written as:

$$\pi_a(K(a),a) = [P_a - C_a(K,a)]x^S(\tau) \tag{13}$$
$$\pi_b(K(b),b) = [P_b - C_b(K,b)](1 - x^S(\tau))$$

A more efficient quality choice, given a fixed sum of capital investments, will make production reliability and process efficiency actually higher, thus causing lower variable costs in quality. A change in quality level, on the other hand, requiring added costs of quality improvements and increased marginal costs due to higher-quality product characteristics, will cause an increase in variable costs associated with a higher quality location. As an extension of the spatial quality model developed earlier, the first-order conditions for an optimal location choice with profit maximization yield:

$$\frac{d\pi_a}{d(a)} = \left[\frac{\partial P_a}{\partial(a)} - C'_a - C'_K \frac{\partial K(a)}{\partial(a)}\right]x^S(\tau) + \frac{\partial x^S}{\partial(a)}(P_a - C_a(K)) = 0 \tag{14}$$
$$\frac{d\pi_b}{d(b)} = \left[\frac{\partial P_b}{\partial(b)} - C'_b - C'_K \frac{\partial K(b)}{\partial(b)}\right](1 - x^S(\tau)) - \frac{\partial x^S}{\partial(b)}(P_b - C_b(K)) = 0$$

For the case of low quality location, this amounts to:

$$\left[\frac{\partial P_a}{\partial(a)} - C'_a(a)\right]x^S(\tau) - C'_K[K(a),a]\frac{\partial K}{\partial(a)}x^S(\tau) = -\frac{\partial x^S(\tau)}{\partial(a)}[P_a - C_a(a)] \tag{15}$$

Therefore,

[10] See Appendix D.

$$\frac{\partial K}{\partial(a)} = \frac{\left(\frac{\partial P_a}{\partial(a)} - C_a'(a)\right)}{C_K'[K(a),a]} + \frac{\partial x^S(\tau)}{\partial(a)}(P_a - C_a(K))\frac{1}{C_K'}\frac{1}{x^S(\tau)} \tag{16}$$

The first term on the right-hand-side of (16) is a negative divided by a negative, generating a positive term. The second term, on the other hand, is actually negative due to $C_K' < 0$. The first (positive) term is associated with marginal markup revenues due to higher quality returns from additional investments in quality improvement, whereas the second (negative) term is associated with the normal default losses due to indirect demand effects on added quality characteristics. As a result, $\frac{\partial K}{\partial(a)} > 0$ is true if and only if marginal profit-seeking quality returns to additional investments in quality improvement strictly exceed the normal default losses associated with horizontal demand effects on increased vertical quality location.

Consequently, $\frac{\partial K}{\partial(a)} > 0$ as $\left|\frac{\frac{\partial P_a}{\partial(a)} - C_a'(a)}{C_K'[K(a),a]}\right| > \left|\frac{\partial x^S(\tau)}{\partial(a)}(P_a - C_a(K))\frac{1}{C_K'}\frac{1}{x^S(\tau)}\right|$.

In more simple terms, $\frac{\partial K}{\partial(a)} >> 0$ as $\left|\frac{\partial P_a}{\partial(a)} - C_a'\right| >> \left|C_K'\right|$.

The expressions in (15) and (16) constitute the *strategic effects* of vertical quality location to an internalized capital investment expenditure facing price competition, given fixed horizontal taste (with no change in unit transportation costs in consumer demand).

Hence, to illustrate this finding, additional capital expenditures in vertical quality improvements for low quality location are favorable if the direct strategic effect on marginal prices (leading towards additional monopolistic profit gains) exceed the strategic investment effect on default demand losses given fixed horizontal consumer taste. Additional investment returns are higher when an increase in K substantially reduces total variable cost.

The second-order condition for the strategic effect on the low-quality brand[11] given fixed horizontal taste is:

$$\left[\frac{\partial^2 P_a}{\partial(a)^2} - C_a'' - C_K' K''(a)\right] x^S(\tau) + \frac{\partial x^S(\tau)}{\partial(a)}\left[\frac{\partial P_a}{\partial(a)} - C_a' - C_K' \frac{\partial K}{\partial(a)}\right] + \frac{\partial^2 x^S(\tau)}{\partial(a)^2}(P_a - C_a(K)) \tag{17}$$

With the first and last terms being negative, the above expression can only be overall negative if:

$$\left|\frac{\partial P_a}{\partial(a)} - C_a'\right| > \left|C_K' \frac{\partial K}{\partial(a)}\right| \tag{18}$$

A symmetrical analysis for the case of high quality location is presented in Appendix D.

This leads to the following Lemma:

Lemma 3. Strategic investment effects on spatial price-quality equilibrium lead to favorable investments in vertical quality improvements to better capture horizontal consumer demand, if and only if $\left|\frac{\partial P_a}{\partial(a)} - C_a'\right| > \left|C_K' \frac{\partial K}{\partial(a)}\right|$ **and** $\left|\frac{\partial P_b}{\partial(b)} - C_b'\right| > \left|C_K' \frac{\partial K}{\partial(b)}\right|$**.**

The *total strategic effect* on equilibrium profits, with $\pi_a(K) = \pi_a[C_a(K), x^S(\tau, K), K]$, can be found by examining the direct and indirect effects of strategic investment choices on equilibrium location as follows:

$$\frac{d\pi_a}{dK} = \frac{\partial \pi_a}{\partial C_a}\frac{dC_a}{dK} + \frac{\partial \pi_a}{\partial x^S}\frac{dx^S}{dK} + \frac{\partial \pi_a}{\partial K} \tag{19}$$

After substitution, and re-arrangement, the total strategic effect amounts to:

[11] Note that, by symmetry, this argument also holds for the high-quality brand.

$$\frac{d\pi_a}{dK} = -2C'_K x^S(\tau) + (P_a - C_a(K))\frac{dx^S}{dK} \qquad (20)$$

Given that $C'_K < 0$, $x^S(\tau) > 0$, and $P_a > C_a(K)$, we have the following argument:

If $\frac{dx^S}{dK} > 0$, implying strategic investments in vertical quality location ignite more demand in consumption and/or capture more horizontal consumer brand-space, then $\frac{d\pi_a}{dK} > 0$. Thus, in that case, those strategic investments do generate extra profit for the firm. If, on the other hand, $\frac{dx^S}{dK} << 0$ (sufficiently negative), implying strategic investments in quality location decrease potential demand due to excessive pricing (or excessive quality markups not desired by consumer taste), then we can have $\frac{d\pi_a}{dK} < 0$. Thus, in that case, those strategic investments in quality will ultimately generate lower economic profits for the firm and will cause undesirable economic consequences in corresponding market share and profits [12].

In general, it can be verified that strategic effects are favorable leading to $\frac{d\pi_a}{dK} > 0$ for:

(i) $\frac{dx^S}{dK} > 0$; or (ii) $\frac{dx^S}{dK} < 0$ with $\left|\frac{dx^S}{dK}\right| < \left|\frac{2C'_K x^S(\tau)}{P_a - C_a(K)}\right|$.

[12] Those investments are undesirable in the sense that they achieve lower economic profits than otherwise the case (they have a negative net present value). Increased investment in unwanted quality characteristics may generate lower indirect demand as dictated by revealed consumer preference or taste, thus reducing the "competitive advantage" of the firm, and if sufficiently negative (a sufficiently negative demand response to additional strategic investments in quality location) then overall economic profits will fall relative to no strategic investments at all, i.e. $\frac{d\pi_a}{dK} < 0$, for $\frac{dx^S}{dK} << 0$ (sufficiently negative). Also, for $\frac{d\pi_a}{dK} < 0$ and with $\frac{dK}{d(a)} > 0$, this may lead to $\frac{d\pi_a}{d(a)} < 0$, implying that excessive strategic investments in unwanted quality characteristics, or those which induce too high marginal pricing, may actually reduce economic profits. In other words, excessive risk in strategic quality investments when accompanied by undesirable quality characteristics on the part of the consumer may ultimately reduce firm profits.

Hence, $\left|\dfrac{dx^S}{dK}\right| > \left|\dfrac{2C'_K x^S(\tau)}{P_a - C_a(K)}\right|$ is "sufficiently negative" (meaning $\dfrac{dx^S}{dK} << 0$ and $\dfrac{d\pi_a}{dK} < 0$).

A symmetrical result can be verified for the case of the high quality firm.

The above analysis deals with strategic investment choices given fixed horizontal taste with the implicit assumption that competing price levels do not necessarily adjust to additional investments in own quality improvements. However, competing price levels may have indirect demand effects, or *cross-effects*, such that increases in strategic investments for quality improvement are met by adjustments in competing price levels. These cross-effects, for the case of the low quality firm, can be incorporated into own-demand by utilizing $x^S(\tau) = x^S[P_a(a,K), P_b(K), \tau]$ hence suggesting that increases in own strategic investment given fixed horizontal taste (if feasible, i.e. having positive investment returns to quality location), can generate $\dfrac{dP_b}{dK} < 0$. This can also be seen by examining the equilibrium price functions in (7). Cross-effects may also occur due to the fear of the competing firm capturing a greater quality space and gaining a larger monopolistic advantage due to changes in competing price levels, even at fixed horizontal consumer taste. This is especially true if strategic investments can position the low-quality brand towards the high end of its monopolistic brand-space or the high-quality brand towards the lower end of its monopolistic brand-space.

With this argument, economic profits and corresponding *cross-effects* regarding investments in vertical quality location given fixed horizontal taste, for the case of the low quality brand, can be written as:

$$\pi_a(K) = \pi_a\{C_a(K), x^S[\tau, K, P_a(K), P_b(K)], K\} \tag{21}$$

Therefore,

$$\pi_a(K) = [P_a - C_a(K)][x^S[\tau, K, P_a(K), P_b(K)], K] \tag{22}$$

Then, the cross-effect can be incorporated into the strategic effect as:

$$\frac{d\pi_a}{d(K)} = \frac{\partial \pi_a}{\partial C_a}\frac{dC_a}{dK} + \frac{\partial \pi_a}{\partial x^S}\left[\frac{\partial x^S}{\partial P_b}\frac{dP_b}{dK}\right] + \frac{\partial \pi_a}{\partial K} \tag{23}$$

This leads to:

$$\frac{d\pi_a}{d(K)} = -2C_K' x^S[\cdot] + (P_a - C_a(K))\left[\frac{\partial x^S}{\partial P_b}\frac{dP_b}{dK}\right] \tag{24}$$

The last term in parenthesis in (24), $\left[\frac{\partial x^S}{\partial P_b}\frac{dP_b}{dK}\right]$, captures the cross-effect, whereas the other terms are a consequence of direct strategic effects on quality location. For a moderate negative cross-effect, i.e. $\frac{dP_b}{dK} < 0$, we can have $\frac{d\pi_a}{d(K)} > 0$ if the direct effect of lowering total costs (and therefore own prices) due to strategic investments in quality exceed the cross-effect of rival firm lowering its price and capturing a larger market share (re-capturing lost monopolistic advantage over its brand-space). If cross-effects are dominant, on the other hand, requiring a sufficiently negative $\frac{dP_b}{dK} << 0$, then $\frac{d\pi_a}{d(K)} < 0$.

From this, strategic investments in quality location may lower potential economic profits for the case of the low-quality brand[13] if the cross-effect dominates the direct strategic effect. That is, $\frac{d\pi_a}{d(K)} < 0$, if:

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[13] This argument is true for the case of the low-quality brand since its vertical price location is reactive to that of the high-quality brand, given the bias in consumer demand towards the high-quality end of the market for a change in quality preferences, as given by the demand functions in (3) and (4) and by the fact that $\frac{\partial D_a}{\partial \tau} < 0$ and $\frac{\partial D_b}{\partial \tau} > 0$. In addition, equilibrium prices in vertical location given horizontal preferences as in (7) reveal that $\frac{\partial P_b}{\partial \tau} > \frac{\partial P_a}{\partial \tau}$, and with the fact that $\frac{\partial x^S}{\partial P_b} > 0$, then an increase in the price of the high-quality brand

$$\left|\frac{dP_b}{dK}\right| > \left|\frac{-2C'_K x^S(\tau)}{\dfrac{\partial \pi_a}{\partial x^S}\dfrac{\partial x^S}{\partial P_b}}\right| \tag{25}$$

If the response of the rival firm due to strategic investments in quality location are so severe in terms of lowering (competing) prices, such that relative monopolistic brand-space gains of own quality location become very low (or minimal), and such that additional investment returns on quality improvement become irrational (from the fact that $\frac{d\pi_a}{d(K)} < 0$, in response to $\frac{dP_b}{dK} << 0$), *then* those strategic investments become undesirable from an economic point of view. Fierce price competition coupled with excessive cross-effects may therefore limit the monopolistic gains of the low-quality firm over its high-quality rival in quality location.

This leads to the following Lemma:

Lemma 4. Strategic investments in quality improvement for the case of the low quality firm could result in lower economic profits if spatial cross-effects exceed direct monopolistic brand-space gains in quality location, i.e. $\frac{d\pi_a}{d(K)} < 0$ **for** $\left|\frac{dP_b}{dK}\right| > \left|\frac{-2C'_K x^S(\tau)}{\dfrac{\partial \pi_a}{\partial x^S}\dfrac{\partial x^S}{\partial P_b}}\right|$**.**

Although strategic effects were found symmetric across both firms, as given in (18) and Lemma 3, the analysis of cross-effects is not symmetric across both firms and is found pertinent only for the case of the low quality firm.

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produces a negative effect on low-quality investment returns, therefore giving rise to the argument that a high-quality price increase causes investment "sufferings" for the low-quality brand, but not vice-versa.

IV. THE MONOPOLISTIC BRAND-SPACE EQUILIBRIUM SOLUTION WITH ASYMMETRIC COSTS

The analysis thus far assumes fixed horizontal taste with vertical quality location, hence abstaining from a result of optimal vertical location given flexible horizontal taste. The objective of this section is to determine the monopolistic brand-space solution for optimal quality choice based on vertical quality location, in which spatially differentiated duopoly firms can achieve at equilibrium, given flexible horizontal consumer taste.

Assuming linear variable costs with quality location and quadratic fixed costs with quality choice[14], such that: $C_a(a) = \mu a$, $C_b(b) = \mu b$, $K(a) = \frac{1}{2}a^2$, and $K(b) = \frac{1}{2}b^2$; the profit functions and equilibrium price locations for the low and high-quality brands amount to be:

$$\begin{aligned} \pi_a &= [P_a(a,b) - \mu a]x^S - \frac{1}{2}a^2 \\ \pi_b &= [P_b(a,b) - \mu b](1 - x^S) - \frac{1}{2}b^2 \end{aligned} \tag{26}$$

$$\begin{aligned} P_a(a,b) &= \frac{1}{3}[2C_a + C_b + \tau(L - b - a)(2 + L - b + a)] \\ P_b(a,b) &= \frac{1}{3}[2C_b + C_a + \tau(L - b - a)(4 - L + b - a)] \end{aligned} \tag{27}$$

Equilibrium profits at the first stage of competition (competition in spatial quality choice), given vertical configuration of equilibrium prices in (27) and given horizontal market demand for low quality location and high quality location respectively obey (3) and (4); are then found to be:

[14] Following Motta (1993), yet in contrast to Shaked and Sutton (1982,1984). There is a constant "unit cost of quality" in the variable cost functions for both quality locations, and fixed costs are quadratic (convex) in quality choice. This is also in line with Boyer and Moreaux (1987).

$$\pi_a(a,b,\tau) = \left[\frac{1}{3}\tau(L-b-a)(2+L-b+a) - \mu a\right]\left[\frac{\frac{1}{3}\left(C_b - C_a\right)}{2\tau(L-b-a)} + \frac{L-b+a}{6} + \frac{1}{3}\right] - \frac{1}{2}a^2$$

$$\pi_b(a,b,\tau) = \left[\frac{1}{3}\tau(L-b-a)(4-L+b-a) - \mu b\right]\left[1 - \frac{\frac{1}{3}\left(C_b - C_a\right)}{2\tau(L-b-a)} - \frac{L-b+a}{6} - \frac{1}{3}\right] - \frac{1}{2}b^2$$

(28)

Solving for $\left(\frac{\partial \pi_a}{\partial (a)}\middle| b^*\right) = 0$ and $\left(\frac{\partial \pi_b}{\partial (b)}\middle| a^*\right) = 0$ simultaneously using (26), (27) and (28) yield multiple solution outcomes, given the monopolistic inequality conditions in (11). However, there is an optimal range of low quality location and an optimal range of high quality location mutually covering the entire spectrum of quality locations; for a given level of horizontal transportation cost[15].

The *monopolistic brand-space equilibrium solution* for the spatial quality choice model amounts to the following simultaneous conditions (see Appendix E):

$$x^S(\tau) = \left(\frac{\frac{\mu}{3}(b-a)}{2\tau(L-b-a)}\right) + \left(\frac{L-b+a}{6}\right) + \left(\frac{1}{3}\right) \tag{29}$$

$$y^S(\tau) = 1 - x^S(\tau) \tag{30}$$

$$a^* = \frac{\frac{1}{3}x^S(\mu + 2\tau)}{1 + \frac{2}{3}\tau x^S} \tag{31}$$

[15] And therefore, for a given level of disutility from purchasing a non-ideal brand in horizontal consumer differentiation. This gives rise to the monopolistic brand-space equilibrium solution attained; such that the optimum choice of vertical quality location is contingent on horizontal consumer demand and unit transportation costs (horizontal consumer taste). This argument has been discussed earlier in lieu of Proposition 1.

$$b^* = \frac{(1-x^S)[\frac{1}{3}\mu - \frac{2}{3}\tau(L-2)]}{\frac{2}{3}\tau(1-x^S)+1} \tag{32}$$

The monopolistic brand-space equilibrium solution imply that the choice of optimum vertical quality location is contingent on horizontal consumer demand. For example, for symmetric horizontal demand, i.e. $x^S = y^S = 1/2$, we get an optimum choice of vertical quality locations given by $a^* = \dfrac{\frac{1}{6}(\mu+2\tau)}{1+\frac{1}{3}\tau}$ and $b^* = \dfrac{\mu - 2\tau(L-2)}{2(\tau+3)}$.

The equilibrium conditions in (29)-(32), if imposed by the monopolistic inequality conditions derived in (11), yield a *Nash-stable cost-effective outcome* for spatial quality choice of[16] $a^* \in [0, \hat{z})$ and $b^* \in [\hat{z}, L]$ where:

$$\hat{z} = \left(\frac{2\tau - L/2}{3\tau}\right)\left(\frac{\mu+2\tau}{2\tau\left(\frac{2\tau - L/2}{3\tau}\right)+1}\right) \equiv \left(\frac{\mu+2\tau}{2\tau + \left(\frac{3\tau}{2\tau - L/2}\right)}\right) \tag{33}$$

or equivalently:

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[16] This is also due to the realization of $x^S > 1/3$ and $y^S < 2/3$ from (29); where:

$x^S = \dfrac{\frac{\mu}{3}(b-a) + (L-b+a)(L-b-a) + 2\tau(L-b-a)}{6\tau(L-b-a)}$; and with (11) and (31) this leads to:

$x^S(\tau,\mu,L) = \dfrac{\frac{\mu}{3}[2\tau(1-L)]}{12\tau L(\tau+3) - 6\tau(\mu+2\tau)(3-L)} + \dfrac{2(\tau+3)(L+2\tau) + 2\tau(L-1)}{12\tau(\tau+3)}$.

$$0 \le a^* < \left\{ \frac{\mu + 2\tau}{2\tau + \left(\frac{3\tau}{2\tau - L/2} \right)} \right\} \quad \text{and} \quad \left\{ \frac{\mu + 2\tau}{2\tau + \left(\frac{3\tau}{2\tau - L/2} \right)} \right\} \le b^* < L \ . \tag{34}$$

Hence, each firm specializes within a range of vertical quality locations given a certain level of horizontal consumer taste. In addition, the range of optimal quality choices covers the entire spectrum of quality locations for various levels of unit transportation cost. The value of the critical parameter $\hat{z}$ affects the "monopolistic brand-space" of both quality locations. A larger monopolistic brand-space for high quality location is only realized by a corresponding lower monopolistic brand-space for low quality location, given a certain level of horizontal consumer taste, τ.

From (33) and (34), we can deduce that the high quality firm can cover a larger monopolistic brand-space (while the low quality firm will then necessarily cover a smaller monopolistic brand-space) if $\mu \downarrow$, $L \uparrow$, $\tau \uparrow$, $x^S \downarrow$; or in qualitative terms, if:

(i) μ is sufficiently small (unit variable costs of quality are not too high)

(ii) L is sufficiently large (a large quality spectrum is available to choose from)

(iii) τ is relatively high (high marginal disutility from purchasing a non-ideal brand)

(iv) x^S is relatively small (insufficient demand for rival firm's quality level).

Given the above arguments, the monopolistic brand-space equilibrium solution yields *multiple differentiation outcomes* through different optimum levels of vertical quality location contingent on a given level of horizontal consumer taste.

In essence, spatial quality choice with vertical quality location given flexible horizontal taste leads to a monopolistic brand-space equilibrium solution of $a^* \in [0, \hat{z})$ and $b^* \in [\hat{z}, L]$ as defined by (33) and (34).

In addition, changes in low-quality vertical location for a change in high-quality locational choice, given flexible horizontal preferences, are found to be:

$$\frac{\partial a^*}{\partial(b)} = \frac{\left\{-\frac{1}{3}(\mu+2\tau)\frac{\partial x^S}{\partial(b)}[1+\frac{2}{3}\tau x^S]\right\} - \left\{\frac{2}{3}\tau\frac{\partial x^S}{\partial(b)}[-\frac{1}{3}x^S(\mu+2\tau)]\right\}}{\left\{1+\frac{2}{3}\tau x^S\right\}^2} \tag{35}$$

After re-arrangement and simplification, this yields:

$$\frac{\partial a^*}{\partial(b)} = \frac{-\frac{1}{3}(\mu+2\tau)\frac{\partial x^S}{\partial(b)}}{\left(1+\frac{2}{3}\tau x^S\right)^2} > 0 \tag{36}$$

On the other hand, changes in high-quality vertical location for a change in low-quality locational choice (given flexible horizontal preferences), are:

$$\frac{\partial b^*}{\partial(a)} = \frac{-\frac{\partial x^S}{\partial(a)}\left(\frac{2}{3}\tau(1-x^S)+1\right)+\frac{2}{3}\tau\frac{\partial x^S}{\partial(a)}\left(1-x^S\right)}{\left(\frac{2}{3}\tau(1-x^S)+1\right)^2}\left(-\frac{1}{3}\mu-\frac{2}{3}\tau\right) \tag{37}$$

After simplification, this amounts to be:

$$\frac{\partial b^*}{\partial(a)} = \frac{-\frac{\partial x^S}{\partial(a)}\left(-\frac{1}{3}\mu-\frac{2}{3}\tau\right)}{\left(\frac{2}{3}\tau(1-x^S)+1\right)^2} > 0 \tag{38}$$

The fact that $\frac{\partial a^*}{\partial(b)} > 0$ and $\frac{\partial b^*}{\partial(a)} > 0$ deserves some attention. An increase in high-quality location, given flexible horizontal preferences, pushes the monopolistic brand-space gains of the low-quality brand towards a higher vertical location. Also, an increase in low-quality location causes a higher vertical quality choice for the high-quality brand. How so? From the cost-effective optimal solution in vertical location, we have that $a^* \in [0, \hat{z})$ and $b^* \in [\hat{z}, L]$, given flexible horizontal consumer taste. A positive shift in low-quality location may give the high-quality firm a price-quality signal that the low-quality firm is gaining monopolistic advantages within a higher quality scale. In retaliation, due to the high-quality firm fearing the low-quality firm capturing higher demand and gaining a higher competitive advantage in the market, the high-quality firm increases its own quality choice, also as an investment in reputation. A positive shift in high-quality location, on the other hand, causes the same rational reaction from the low-quality firm, thus giving rise to both price rivalry and quality wars in the spatial quality choice model with flexible horizontal taste.

Proposition 2. Spatial quality choice gives rise to a monopolistic brand-space equilibrium solution where each firm acts as a monopolistic competitor within a range of quality choices governed by $a^* \in [0, \hat{z})$ and $b^* \in [\hat{z}, L]$, where $\hat{z} = \left(\dfrac{\mu + 2\tau}{2\tau + \left(\dfrac{3\tau}{2\tau - L/2} \right)} \right)$; with the monopolistic inequality conditions of Proposition 1 as a sufficient condition for Nash equilibrium. Spatial quality signals (the slopes of quality reaction functions) are of the following form:

$$\frac{\partial a^*}{\partial (b)} = \frac{-\frac{1}{3}(\mu + 2\tau)\frac{\partial x^S}{\partial (b)}}{\left(1 + \frac{2}{3}\tau x^S\right)^2} > 0 \quad \textbf{and} \quad \frac{\partial b^*}{\partial (a)} = \frac{-\frac{\partial x^S}{\partial (a)}\left(-\frac{1}{3}\mu - \frac{2}{3}\tau\right)}{\left(\frac{2}{3}\tau(1 - x^S) + 1\right)^2} > 0 \textbf{.}$$

Accordingly, the monopolistic brand-space equilibrium solution yields multiple differentiation outcomes through different optimum levels of vertical quality location contingent on a given level of horizontal consumer taste (see Appendix E). Each firm specializes within a range of vertical quality choices for a given level of transportation cost in horizontal consumer demand. In essence, spatial quality choice with vertical quality location given flexible horizontal taste leads to a monopolistic brand-space equilibrium solution where monopolistic inequality conditions act as an implicit stability criterion for Nash equilibrium. The choice of optimum vertical location is contingent on horizontal consumer disutility from purchasing a non-ideal brand.

V. FLEXIBLE CHOICE, VERTICAL RE-LOCATION AND RESISTANCE TO CHANGE

Thus far, it has been proven that spatial quality equilibrium yields a brand-specific monopolistic outcome such that firms achieve positive profits through price-quality markups exceeding marginal costs, and such that positive investment returns to vertical quality positioning require monopolistic profit gains (strategic effects) to dominate indirect demand losses (cross-effects) given a certain level of horizontal consumer taste. There are multiple differentiation outcomes under strict monopolistic inequality conditions, but each firm manages to specialize its vertical quality location towards a determined quality brand-space away from its competing firm.

The purpose of this section is to study the effect of horizontal differentiation (a change in horizontal consumer taste with respect to quality characteristics) *on* vertical location, *within* the monopolistic brand-space equilibrium solution outlined in the previous section. We utilize the concept of "marginal profits" as a rent sensitivity parameter of firm profits due to a change in horizontal consumer preferences within the multiple differentiation outcomes established in the monopolistic brand-space equilibrium solution. With profits of $\pi_a = (P_a - C_a(a))x^S(\tau) - K(a)$ for the low-quality firm, and with the retained assumptions of $\frac{\partial D_a}{\partial \tau} < 0$ and $\frac{\partial P_a}{\partial \tau} > 0$, the *marginal profits*[17] for low quality location amount to:

$$\frac{d\pi_a}{d\tau} = \frac{\partial P_a}{\partial \tau} x^S(\tau) + \frac{\partial x^S(\tau)}{\partial \tau}(P_a - C_a(a)) \tag{39}$$

The first term, $\frac{\partial P_a}{\partial \tau} x^S(\tau)$, is the *price effect* (positive), whereas the second term, $\frac{\partial x^S(\tau)}{\partial \tau}(P_a - C_a)$, is the *demand effect* (negative). An increase in unit transportation costs

[17] The term "marginal profit" has been borrowed from several sources in the literature, and basically mean the change in net present value (or changes in investment returns given an opportunity cost of capital with a certain degree of risk aversion) for an additional unit of (inventory) demand or unit cost. For our purpose here, marginal profits imply changes in equilibrium profits for a change in horizontal consumer demand through preferences in the form of unit transportation costs, given the established monopolistic brand-space of vertical quality locations.

(implying a higher consumer disutility from purchasing a non-ideal brand) increases the price of the low-quality brand such that monopolistic gains are higher (per vertical location) and such that economic profits can ultimately be higher, leading to a positive *price effect* on economic profits. On the other hand, higher unit transportation costs also imply lower indirect demand for the low quality brand, thus causing a negative *demand effect* on economic profit. Since marginal profit is an additive function of both the price and demand effects, then it can easily be verified that $\frac{d\pi_a}{d\tau} > 0$ is true only if $\left|\frac{\partial P_a}{\partial \tau} x^S(\tau)\right| > \left|\frac{\partial x^S(\tau)}{\partial \tau}(P_a - C_a)\right|$. In other words, marginal profits are positive for the case of the low-quality brand only if the price effect strongly dominates the indirect demand effect on economic profits. Higher unit transportation costs imply higher economic profits for the case of the low-quality brand only if increases in monopolistic gains through higher prices dominate the decline in indirect consumer demand due to higher marginal disutilities from purchasing a non-ideal brand.

For the case of the high-quality firm, with $\pi_b = (P_b - C_b(b))D_b(\tau) - K(b)$, and with the retained assumptions of $\frac{\partial D_b}{\partial \tau} > 0$ and $\frac{\partial P_b}{\partial \tau} > 0$, *marginal profits* amount to:

$$\frac{d\pi_b}{d\tau} = \frac{\partial P_b}{\partial \tau} D_b(\tau) + \frac{\partial D_b}{\partial \tau}(P_b - C_b(b)) \tag{40}$$

In that case, both the price and demand effects are positive, implying that $\frac{d\pi_b}{d\tau} > 0$ is always true. Therefore, a higher disutility from purchasing a non-ideal brand *always* benefits the high-quality firm in terms of yielding higher economic profits, since higher unit transportation costs cause high-quality monopolistic gains and capture more indirect consumer demand[18].

[18] Higher disutility (higher unit transportation costs) always benefits the profits of the high-quality firm, whereas this is not necessarily the case for the low-quality firm (unless the price effect strongly dominates the demand effect). However, higher marginal disutility also imply an increase in the price of *both* high and low quality brands, with $\frac{\partial P_a}{\partial \tau} > 0$ and $\frac{\partial P_b}{\partial \tau} > 0$.

Note that competing price levels are implicitly included in the marginal profit functions in (39) and (40). For the case of the low-quality brand, we know that $\frac{\partial x^S}{\partial P_b} > 0$ such that the price of the high-quality brand is implicit in the marginal profit formulation. This gives rise to $\frac{d\pi_a}{d\tau(dP_b)} > 0$, implying that high-quality price increases may actually increase marginal profits for the low quality firm. In addition, for the case of the high-quality brand, given $\frac{\partial D_b}{\partial P_a} > 0$, high-quality marginal profits increase with low-quality prices, i.e. $\frac{d\pi_b}{d\tau(dP_a)} > 0$.

We establish:

Lemma 5. Higher transportation costs for horizontal consumer taste in spatial quality choice always benefit high quality location such that $\frac{d\pi_b}{d\tau} > 0$; with $\frac{d\pi_a}{d\tau} > 0$ only true if the monopolistic price effect strongly dominates the indirect demand effect on low quality location, i.e. if $\left|\frac{\partial P_a}{\partial \tau} x^S(\tau)\right| > \left|\frac{\partial x^S(\tau)}{\partial \tau}(P_a - C_a)\right|$.

Vertical quality *re-location*, which can be defined as firms adjusting their vertical quality locations due to changes in horizontal consumer taste within the established monopolistic brand-space solution, can now be examined. Vertical quality re-location may also imply changes in marginal profits, and therefore, firm profits. Vertical quality re-location in spatial quality choice is analyzed based on the marginal profit functions in (39) and (40), given the vertical configuration of prices in (27) and the equilibrium solution in Proposition 2.

It can be verified that the *marginal profit of re-location* (sensitivity of marginal profits with respect to a change in horizontal consumer taste) regarding vertical quality location of the low quality brand always imply:

$$\frac{\partial \pi_a}{\partial(a)\partial\tau} < 0 \tag{41}$$

Thus, an increase in unit transportation costs reduces the marginal profit of re-location for the case of the low-quality brand. The reduction in marginal profits depend on the original level of vertical quality location and on the level of low-quality demand. A higher original level of vertical quality location or a higher level of low-quality demand imply a lower marginal profit of re-location, and therefore imply lower profits in low-quality re-location.

For the case of the high-quality brand, the marginal profit of re-location in simplified reduced form is:

$$\frac{\partial \pi_b}{\partial (b) \partial \tau} = \frac{2}{3}(L - 2 - b)(1 - x^S) \tag{42}$$

For $L > (b+2)$, we have $\frac{\partial \pi_b}{\partial (b) \partial \tau} > 0$; whereas for $L < (b+2)$ we have $\frac{\partial \pi_b}{\partial (b) \partial \tau} < 0$.

Therefore, an increase in unit transportation costs increases the marginal profit of re-location for the case of the high-quality brand if the range of quality choices is sufficiently high: $L > (b+2)$ (i.e. if there is a large quality spectrum offered by the industry). If, however, quality choice is tight, $L < (b+2)$, then the marginal profit of re-location is negative. Intuitively, higher marginal disutilities will favor high-quality re-location only if there is sufficient room for quality improvement implying sufficient monopolistic quality space for additional profit gains from that vertical re-location. If, however, quality choice is tight and the range of offered quality choices is relatively small, then vertical quality re-location due to increased transportation costs will lower economic profits for the case of the high-quality firm, due to insufficient monopolistic space or insufficient room for quality improvements.

The second-order differentials for vertical re-location are found to be:

$$\frac{\partial^2 \pi_a}{\partial (a)^2 \partial \tau} = -\frac{2}{3} x^S < 0 \tag{43}$$

$$\frac{\partial^2 \pi_b}{\partial (b)^2 \partial \tau} = -\frac{2}{3}(1 - x^S) < 0 \tag{44}$$

With both second-order differentials for vertical quality re-location always negative, this implies that the marginal profit of re-location for the low and high-quality firms always decline for increased unit transportation costs (for a given change in horizontal consumer demand implying higher disutility from purchasing a non-ideal brand). Such an argument basically states that there is a *resistance to change* on the part of vertically located firms with respect to changes in consumer taste (due to increased marginal disutility from purchasing a non-ideal brand) such that both firms dislike too much re-location and prefer stable preferences in quality. Both firms prefer stable vertical positioning of their brands in order to maximize their respective brand-space monopolistic gains, and therefore prefer stable preferences in quality characteristics (stable horizontal demand). However, the higher-quality firm may have a marginal profit incentive to change its vertical quality location (i.e. to vertically re-locate) if unit transportation costs increase and if the range of quality choices is sufficiently high.

Lemma 6. Second-order differentials in vertical quality re-location imply a resistance to change on the part of vertically located firms, with $\frac{\partial^2 \pi_a}{\partial (a)^2 \partial \tau} < 0$ **and** $\frac{\partial^2 \pi_b}{\partial (b)^2 \partial \tau} < 0$**; given** $\frac{\partial \pi_a}{\partial (a) \partial \tau} = -\frac{2}{3}(1+a)x^S < 0$ **and** $\frac{\partial \pi_b}{\partial (b) \partial \tau} = \frac{2}{3}(L-2-b)(1-x^S)$**.**

The effect of horizontal consumer taste on vertical quality re-location needs a final assessment. Given that a cost-effective equilibrium solution entails $\frac{d\pi_a}{d\tau} > 0$ (if the price effect dominates the demand effect) and $\frac{d\pi_b}{d\tau} > 0$, and with $\frac{d\pi_a}{d(a)} < 0$ and $\frac{d\pi_b}{d(b)} > 0$ under Nash stability conditions, then we can indirectly deduce the direction of $\frac{da^*}{d\tau}$ and $\frac{db^*}{d\tau}$ from:

$$\frac{d\pi_a}{d\tau} = \frac{d\pi_a}{d(a)} \frac{da^*}{d\tau} \tag{45}$$

$$\frac{d\pi_b}{d\tau} = \frac{d\pi_b}{d(b)} \frac{db^*}{d\tau} \tag{46}$$

Therefore, $\frac{da^*}{d\tau} < 0$ and $\frac{db^*}{d\tau} > 0$. A relative change in quality preferences, through more horizontal differentiation in consumer taste, may force more product differentiation by vertical quality re-location, with the low-quality firm producing a lower quality brand and the high-quality firm producing a higher quality brand, even though both firms prefer stable quality preferences and both firms dislike re-location. This also means that more product differentiation will arise by vertical quality choice only if quality preferences, as dictated by horizontal differentiation in consumer demand, force the re-location and vertical re-positioning of both firms, under the Nash stable conditions for a cost-effective equilibrium solution.

Thus, in spatial quality equilibrium with flexible horizontal taste and flexible vertical location (vertical re-location), the market may become more differentiated on the basis of both duopoly firms re-locating their product (quality) characteristics along a wider quality range, and such that both firms may enjoy monopolistic brand-space gains through vertical quality re-location given a change in horizontal consumer taste.

In other words, more horizontal differentiation may induce more product differentiation by vertical quality re-location[19].

Proposition 3. In the spatial equilibrium model of quality choice, more horizontal differentiation may force more product differentiation by vertical quality re-location, even though the resistance to change arguments may still hold good; i.e. $\frac{da^*}{d\tau} < 0$ **and** $\frac{db^*}{d\tau} > 0$, **with** $\frac{\partial^2 \pi_a}{\partial (a)^2 \partial \tau} < 0$ **and** $\frac{\partial^2 \pi_b}{\partial (b)^2 \partial \tau} < 0$.

This outcome is binding even though second-order differentials imply a resistance to change on the part of vertically located firms.

[19] In other words, horizontal preferences may force more product differentiation. It should also be noted here that such an outcome may be true for the cost-effective equilibrium solution with monopolistic brand-space gains. However, if the original vertical specification is such that both firms locate their quality choices at the extreme of the quality spectrum, meaning that the market is originally located on the premise of maximum product differentiation, then additional product differentiation may only occur if it is both technically and financially feasible for both firms to relax their differentiation characteristics based on changes in consumer taste. Investments in quality re-location, then, have to weighed against monopolistic gains and changes in indirect demand based on the new level of horizontal consumer taste.

A graphical illustration of the model is outlined in the Figure below.

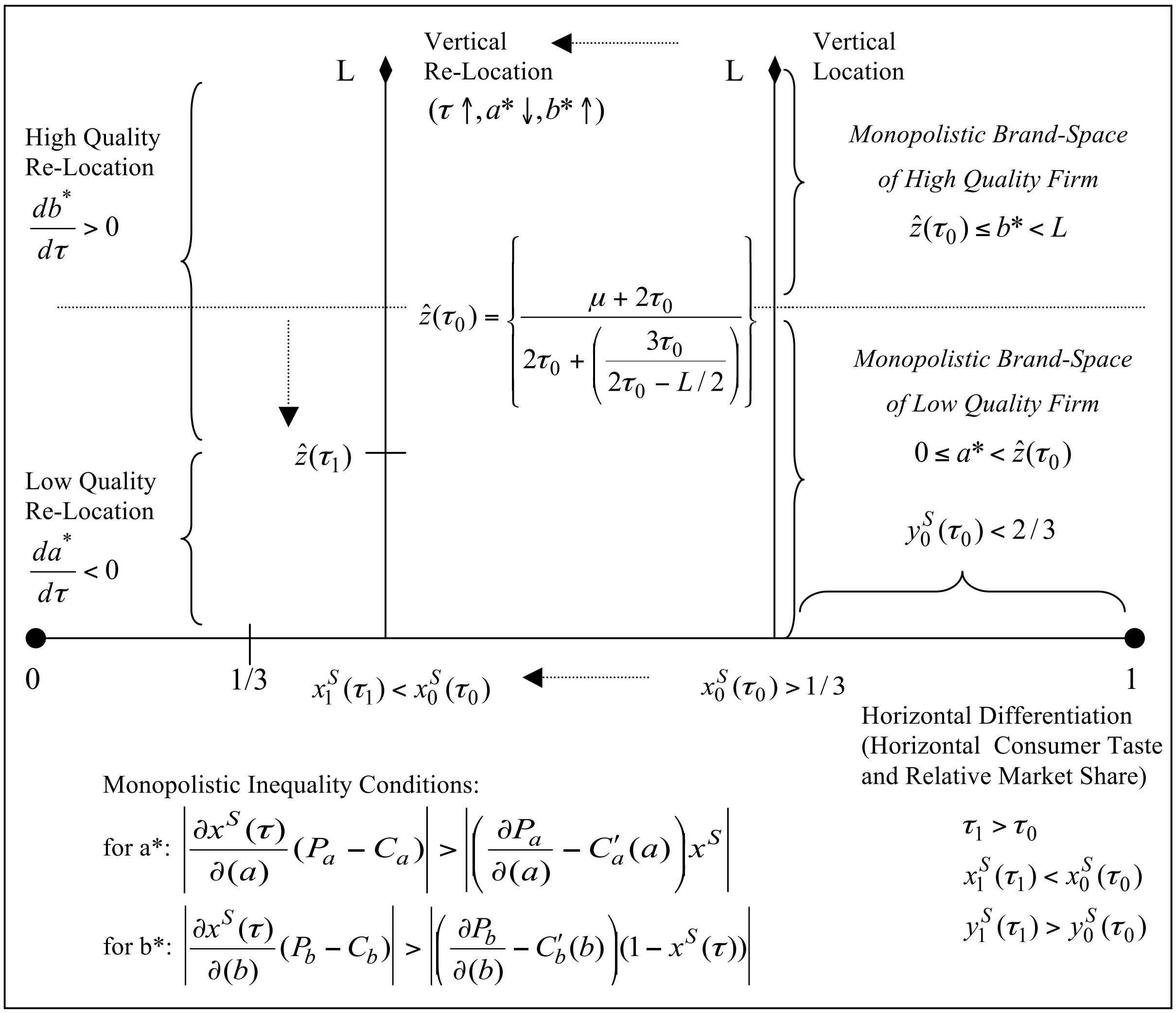

Figure 2: A Simplified Illustration of Spatial Quality Choice with Horizontal Differentiation and Vertical Location yielding the Monopolistic Brand-Space Equilibrium Solution (with Vertical Re-Location)

VI. CONCLUSION

To summarize, we have introduced the concept of "spatial quality choice" as an integrative differentiation model with product positioning in horizontal and vertical quality space where consumers are horizontally differentiated by taste while firms are vertically differentiated by quality location. The analysis has been confined to a duopoly market structure with asymmetric fixed and variable costs of quality. Firms are assumed to compete in quality choice followed by prices. In general, horizontal consumer demand heavily depends on revealed preferences for unit transportation costs and price-quality differentials such that there is a consumer bias against low quality location in favor of high quality location for a given increase in unit transportation cost. Higher disutility in horizontal consumer taste always benefit the high-quality firm while they benefit the low-quality firm only if monopolistic price effects strongly dominate indirect demand effects on economic profits.

Spatial quality equilibrium yields a monopolistic brand-space equilibrium solution such that each firm specializes with a range of quality choices away from its competing firm. The vertical configuration of the equilibrium solution imply a cost-effective outcome with equilibrium capital investment returns composed of positive quality markups and negative marginal cost effects. Equilibrium prices are then determined by a given level of horizontal consumer demand. The horizontal configuration of the equilibrium solution imply strict monopolistic inequality conditions towards a stable differentiation outcome. Moreover, strategic investments may lead towards lower economic profits for the case of the low quality firm if cross-effects due to fierce reduction in competing price levels exceed direct monopolistic brand-space gains.

Spatial quality choice with flexible horizontal taste and flexible vertical location suggest that each firm acts as a monopolistic competitor within a range of quality choices leading towards multiple differentiation outcomes. In analyzing flexible choice with vertical quality re-location, within the monopolistic brand-space solution, second-order differentials imply a resistance to change on the part of vertically located firms such that firms dislike vertical re-location and prefer stable horizontal preferences in quality (stable horizontal consumer taste). More horizontal differentiation may force more product differentiation by vertical quality re-location even though the resistance to change conditions may still hold good. In essence, horizontal differentiation dictates the vertical quality re-location outcome in spatial quality competition.

The results of our model do not match those found in the literature for either vertical or horizontal differentiation, yet are considered a tradeoff between several established principles within different differentiation models of quality competition found in the literature. Most notably, the analysis of horizontal differentiation (such as those in the classic papers of Hotelling 1929, D'Aspremont et.al. 1979, Dixit and Stiglitz 1977, Dixit 1979, and Jun and Vives 1996) usually tend towards an uncovered monopolistic market outcome with quality differentiation heavily dependent on unit transportation costs and consumer surplus value functions, with firms locating their quality offerings based on relative quality differentiation with non-uniqueness of a stable equilibrium solution. On the other hand, the analysis of vertical differentiation (such as those in Leontief 1936, Schmalensee 1979, Shaked and Sutton 1982, Vives 1985, Kim 1987, and Gilbert and Matutes 1993) usually tend towards a covered market outcome with endogenous quality differentiation heavily dependent on price-quality signals and choice of strategic investments in vertical quality location; and with most models achieving a unique differentiation outcome of minimum, center, or maximum differentiation by quality choice.

The analysis of integrative-type models in quality differentiation have been rare; and mostly limited to ideas rather than elaborate models. However, the research of Spence 1974, Boyer and Moreaux 1987, Thisse and Vives 1988, Beath and Katsoulacos 1991, Cremer and Thisse 1991, Motta 1993, and Tirole 1996; among few others, have had a profound impact on the ideas presented in this research. The core agreement between those models and the results achieved herein are basically that of concepts rather than precise economic formulations. In particular, we all agree that the defining elements of the strategy space is most critical in establishing the differentiation outcome. It is also interesting to note that we are still in general agreement with the elementary suggestions of Hotelling 1929, Chamberlin 1933, and Leontief 1936; in their assessment that horizontal consumer taste defines the scope of quality differentiation, whereas vertical differentiation may define the degree of endogenous quality location, within most generalized models of quality competition.

Having said that, several recommendations are useful towards achieving a better end: (i) to study the effect of quantity, versus price, competition in spatial quality equilibrium; (ii) to introduce different formulations of horizontal preferences (e.g. non-linear, exponential, probabilistic, etc.) in horizontal consumer taste; (iii) to investigate the concept of spatial quality choice within an oligopoly market setting, and; (iv) if possible, to formulate a dynamic analysis of the model.

APPENDIX

A. Demand for the Low-Quality Spatially Differentiated Product

From Equation (2): $\frac{P_b - P_a}{\tau} = (x-a)^2 - (L-b-x)^2$,

$$\Rightarrow \frac{P_b - P_a}{\tau} = x^2 - 2ax + a^2 - [(L-b)^2 - 2x(L-b) + x^2]$$
$$\Rightarrow \frac{P_b - P_a}{\tau} = 2x(L-b-a) + a^2 - (L-b)^2$$
$$\Rightarrow x = \frac{P_b - P_a}{2\tau(L-b-a)} + \frac{(L-b)^2 - a^2}{2(L-b-a)} = \frac{P_b - P_a}{2\tau(L-b-a)} + \frac{(L-b-a)(L-b+a)}{2(L-b-a)}$$
$$\Rightarrow x = \frac{P_b - P_a}{2\tau(L-b-a)} + \frac{L-b+a}{2}$$

B. Prices and Equilibrium Profits for the Two-Stage Spatial Model of Quality Choice with Horizontal Preferences

From Equation (6):

$$\partial\pi_a \big/ \partial P_a = x^S(\tau) + \frac{\partial x^S(\tau)}{\partial P_a}[P_a - C_a(a)] = 0$$
$$\partial\pi_b \big/ \partial P_b = (1 - x^S(\tau)) - \frac{\partial x^S(\tau)}{\partial P_b}[P_b - C_b(b)] = 0$$

Utilizing the demand function in Equations (3) & (4):

$$D_a(P_a, P_b) = x^S = \frac{P_b - P_a}{2\tau(L-b-a)} + \frac{L-b+a}{2}$$

$$\text{and } D_b(P_a, P_b) = \frac{P_a - P_b}{2\tau(L-b-a)} + \frac{(2-L)+b-a}{2},$$

we can substitute into the first-order conditions to get:

$$\frac{P_b - P_a}{2\tau(L-b-a)} + \frac{L-b+a}{2} - \left(\frac{1}{2\tau(L-b-a)}\right)(P_a - C_a) = 0$$
$$\frac{P_a - P_b}{2\tau(L-b-a)} + \frac{2-L+b-a}{2} - \left(\frac{1}{2\tau(L-b-a)}\right)(P_b - C_b) = 0$$

Adding the two first-order conditions, and simplifying, we get:

$(P_a - C_a) + (P_b - C_b) = 2\tau(L-b-a)$.

Substituting back into the first first-order condition for P_b, and solving for P_a:

$$P_a - C_a - C_b + 2P_a - C_a - \tau(L-b-a)(L-b+a) = 2\tau(L-b-a)$$

$$\Rightarrow 3P_a - 2C_a - C_b = 2\tau(L-b-a) + \tau((L-b)^2 - a^2)$$
$$\Rightarrow 3P_a = 2C_a + C_b + \tau(L-b-a)(2+L-b+a)$$
$$\Rightarrow P_a = \frac{1}{3}\left\{2C_a + C_b + \tau(L-b-a)(2+L-b+a)\right\}$$

Solving for P_b by utilizing the second first-order condition and by substituting the last equation above for P_a, we get:

$$P_b = \frac{4}{3}C_a + \frac{2}{3}C_b + \frac{2}{3}\tau(L-b-a)(2+L-b+a) - C_a - \tau(L-b-a)(L-b+a)$$
$$\Rightarrow P_b = \frac{C_a}{3} + \frac{2}{3}C_b + \frac{2}{3}\tau(L-b-a)(L-b+a) + \frac{4}{3}\tau(L-b-a) - \tau(L-b-a)(L-b+a)$$
$$\Rightarrow P_b = \frac{C_a}{3} + \frac{2}{3}C_b + \frac{4}{3}\tau(L-b-a) - \frac{1}{3}\tau(L-b-a)(L-b+a)$$
$$\Rightarrow P_b = \frac{1}{3}\left\{2C_b + C_a + 4\tau(L-b-a) - \tau(L-b-a)(L-b+a)\right\}$$
$$\Rightarrow P_b = \frac{1}{3}\left\{2C_b + C_a + \tau(L-b-a)(4-L+b-a)\right\}$$

Substituting the equilibrium prices (in the last equations above for P_a and P_b) into the profit functions of Equation (5), and simplifying, we get equilibrium profits as:

$$\pi_a = \frac{1}{3}\left[C_b - C_a + \tau(L-b-a)(2+L-b+a)\right]x^S(\tau) - K(a)$$
$$\pi_b = \frac{1}{3}\left[C_a - C_b + \tau(L-b-a)(4-L+b-a)\right][1 - x^S(\tau)] - K(b)$$

C. Equilibrium Location for the High-Quality Brand (*B*) in the Spatial Equilibrium Model of Quality Choice

For the case of the high-quality brand, with the retained assumptions of :

$K'(b) > 0, C'_b(b), C''(\cdot) < 0, x^S(\tau) > 0$; and with the proven arguments of $\frac{\partial P_b}{\partial \tau} > \frac{\partial P_a}{\partial \tau}$,

$\frac{\partial D_a}{\partial \tau} < 0$, and $\frac{\partial D_b}{\partial \tau} > 0$; and given that high-quality demand

is $D_b(P_a, P_b) = \frac{P_a - P_b}{2\tau(L-b-a)} + \frac{(2-L)+b-a}{2}$,

and that profits are given by $\pi_b[P_b, C_b, K(b)] = \{P_b - C_b(b)\}(1 - x^S(\tau)) - K(b)$,

and that $\frac{\partial x^S}{\partial(b)} = \frac{\frac{\partial P_b}{\partial(b)}(2\tau)(L-b-a) + (2\tau)(P_b - P_a)}{4\tau^2(L-b-a)^2} - \frac{1}{2}$,

then equilibrium location is governed by:

$$\pi_b = \frac{1}{3}\left[C_a - C_b + \tau(L-b-a)(4-L+b-a)\right][1 - x^S(\tau)] - K(b),$$

with $\partial\pi_b \big/ \partial P_b = (1 - x^S(\tau)) - \frac{\partial x^S(\tau)}{\partial P_b}[P_b - C_b(b)] = 0$ and

$$P_b = \frac{1}{3}\left[2C_b + C_a + \tau(L-b-a)(4-L+b-a)\right],$$

and with the following condition for optimal quality location satisfied:

$$\frac{d\pi_b}{d(b)} = \left\{\left[\frac{\partial P_b}{\partial(b)} - C_b'(b)\right](1 - x^S(\tau))\right\} - \frac{\partial x^S(\tau)}{\partial(b)}(P_b - C_b) - K'(b) = 0.$$

This translates to:

$$K'(b) = \left\{\left[\frac{\partial P_b}{\partial(b)} - C_b'(b)\right](1 - x^S(\tau))\right\} - \frac{\partial x^S(\tau)}{\partial(b)}(P_b - C_b)$$

Similar to the analysis of the low-quality brand, the first-term in the above equation signifies increases in marginal costs (and consequent reduction in profits) while the second term captures the quality markup due to a better quality brand. The monopolistic brand-space inequality for positive investment returns with regards to the high-quality brand is therefore:

$$\left|\frac{\partial x^S(\tau)}{\partial(b)}(P_b - C_b)\right| > \left|\left(\frac{\partial P_b}{\partial(b)} - C_b'(b)\right)(1 - x^S(\tau))\right|.$$

D. Strategic Effects for the High-Quality Firm in the Spatial Price-Quality Equilibrium Model of Location Choice

For the case of the high-quality brand, equilibrium profits are:

$$\pi_b(K(b),b) = [P_b - C_b(K,b)](1 - x^S(\tau))$$

The first-order condition for price-quality equilibrium location, with capital investments internalized are:

$$\frac{d\pi_b}{d(b)} = \left[\frac{\partial P_b}{\partial(b)} - C_b' - C_K'\frac{\partial K(b)}{\partial(b)}\right](1 - x^S(\tau)) - \frac{\partial x^S}{\partial(b)}(P_b - C_b(K)) = 0$$

This leads to:

$$\left[\frac{\partial P_b}{\partial(b)} - C_b'(b)\right](1 - x^S(\tau)) - C_K'[K(b),b]\frac{\partial K}{\partial(b)}(1 - x^S(\tau)) = \frac{\partial x^S(\tau)}{\partial(b)}[P_b - C_b(b)].$$

From this, strategic investment returns on quality, at equilibrium, are:

$$\frac{\partial K}{\partial(b)} = \frac{\left(\frac{\partial P_b}{\partial(b)} - C_b'(b)\right)}{C_K'[K(b),b]} - \frac{\partial x^S(\tau)}{\partial(b)}(P_b - C_b(K))\frac{1}{C_K'}\frac{1}{(1 - x^S(\tau))}.$$

The second-order condition is:

$$\left[\frac{\partial^2 P_b}{\partial(b)^2} - C_b'' - C_K' K''(b)\right](1 - x^S(\tau)) - \frac{\partial x^S(\tau)}{\partial(b)}\left[\frac{\partial P_b}{\partial(b)} - C_b' - C_K'\frac{\partial K}{\partial(b)}\right] - \frac{\partial^2 x^S(\tau)}{\partial(b)^2}(P_b - C_b(K))$$

which is negative for $\left|\frac{\partial P_b}{\partial(b)} - C_b'\right| > \left|C_K'\frac{\partial K}{\partial(b)}\right|$.

E. Monopolistic Brand-Space Equilibrium Solution for Spatial Quality Choice given Flexible Horizontal Taste

From Equations (26) and (27), $\pi_a = [P_a(a,b) - \mu a]x^S - \frac{1}{2}a^2$

$P_a(a,b) = \frac{1}{3}[2C_a + C_b + \tau(L - b - a)(2 + L - b + a)]$

This implies,

$$\frac{d\pi_a}{d(a)} = \left[\frac{1}{3}\{2\mu + \tau[(-2 - L + b - a) + L - b - a]\} - \mu\right]x^S - a$$

given $x^S = \dfrac{\frac{\mu}{3}(b - a) + (L - b + a)(L - b - a) + 2\tau(L - b - a)}{6\tau(L - b - a)}$;

or $x^S(\tau,\mu,L) = \dfrac{\frac{\mu}{3}[2\tau(1-L)]}{12\tau L(\tau+3) - 6\tau(\mu+2\tau)(3-L)} + \dfrac{2(\tau+3)(L+2\tau)+2\tau(L-1)}{12\tau(\tau+3)}$.

The monopolistic inequality conditions of,

$$\left|\frac{\partial x^S(\tau)}{\partial(a)}(P_a - C_a)\right| > \left|\left(\frac{\partial P_a}{\partial(a)} - C_a'(a)\right)x^S\right|$$

$$\left|\frac{\partial x^S(\tau)}{\partial(b)}(P_b - C_b)\right| > \left|\left(\frac{\partial P_b}{\partial(b)} - C_b'(b)\right)(1-x^S(\tau))\right|$$

with vertical configuration of equilibrium prices,

$$P_a(a,b) = \frac{1}{3}[2C_a + C_b + \tau(L-b-a)(2+L-b+a)]$$

$$P_b(a,b) = \frac{1}{3}[2C_b + C_a + \tau(L-b-a)(4-L+b-a)]$$

yield the monopolistic brand-space equilibrium solution:

$$0 \le a^* < \left\{\frac{\mu+2\tau}{2\tau+\left(\frac{3\tau}{2\tau-L/2}\right)}\right\} \quad \text{and} \quad \left\{\frac{\mu+2\tau}{2\tau+\left(\frac{3\tau}{2\tau-L/2}\right)}\right\} \le b^* < L.$$

Horizontal consumer demand obeys:

$$x^S(\tau) = \left(\frac{\frac{\mu}{3}(b-a)}{2\tau(L-b-a)}\right) + \left(\frac{L-b+a}{6}\right) + \left(\frac{1}{3}\right)$$

$$y^S(\tau) = 1 - x^S(\tau);$$

hence vertical quality location given flexible horizontal consumer taste is:

$$a^* = \frac{\frac{1}{3}x^S(\mu+2\tau)}{1+\frac{2}{3}\tau x^S} \quad \text{and} \quad b^* = \frac{(1-x^S)[\frac{1}{3}\mu - \frac{2}{3}\tau(L-2)]}{\frac{2}{3}\tau(1-x^S)+1}$$

with spatial quality signals:

$$\frac{\partial a^*}{\partial(b)} = \frac{-\frac{1}{3}(\mu+2\tau)\frac{\partial x^S}{\partial(b)}}{\left(1+\frac{2}{3}\tau x^S\right)^2} > 0 \quad \text{and} \quad \frac{\partial b^*}{\partial(a)} = \frac{-\frac{\partial x^S}{\partial(a)}\left(-\frac{1}{3}\mu - \frac{2}{3}\tau\right)}{\left(\frac{2}{3}\tau(1-x^S)+1\right)^2} > 0 .$$

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