On Salesforce Compensation with Inventory Considerations

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Abstract

This study critically examines Dai and Jerath's (2013) influential paper on incentive schemes in inventory management, revealing a substantial flaw: an equilibrium fails to exist for a broad set of parameters allowed by the paper. Illustrated through a specific example, we identify fundamental economic reasons behind this issue and propose discretization as a solution. Addressing the non existence problem reshapes key findings, clarifying and correcting counterintuitive notions the author had highlighted in the paper. This resolution ensures that inventory considerations no longer impact compensation, while preserving the informational value crucial to incentive schemes.

Keywords: salesforce compensation; inventory; salesperson

1 Introduction

Dai and Jerath (2013) is an influential paper that studies how incentive schemes are affected by the need to keep inventories. Unfortunately, the model as stated has no equilibrium for a relevant set of parameters (that the authors allow in their setting). I provide an example that shows the non-existence. Towards the end of the note, I propose two alternative solutions.

The non-existence problem is not solely technical; there is a fundamental economic reason why the model has no solution, as stated. I will discuss the problem after the example. Also, importantly, when one fixes the non existence problem, some of the main results in the paper are no longer true. For example, the salesperson payment is no longer contingent on the inventory cost.

2 Model

A risk neutral firm hires a risk neutral agent, who must choose between two levels of effort e_H or $e_L < e_H$; he either "works" or "shirks". The demand is uncertain and the realization of the demand, denoted by ξ, can be H (high), M (medium), or L (low). The effort of the salesperson influences the demand distribution as follows: for $s \in \{L, M, H\}$,

$$\Pr(\xi = s) = \begin{cases} p_s & \text{if } e = e_{\mathrm{H}}, \\ q_s & \text{if } e = e_{\mathrm{L}}, \end{cases}$$

where,

$$p, q \in \Delta\{H, M, L\} \equiv \left\{ x \in \mathbf{R}^3_+ : \sum_{i=1}^{3} x_i = 1 \right\},$$

and the distributions satisfy the monotone likelihood ratio property (MLRP); i.e.,

$$\frac{p_L}{q_L} < \frac{p_M}{q_M} < \frac{p_H}{q_H}.$$

The authors also assume $p_L > 0$. The costs of exerting effort are $\psi > 0$ for e_H, and 0 for e_L.

The principal must choose a contract to incentivize the salesperson (a triplet of salesforce compensations S_L, S_M and S_H corresponding to each output level) subject to limited liability ($S_i \geq 0$ for $i = L, M, H$) and expected utility greater than that of the outside option, $\overline{u} = 0$. At the same time, the principal must choose the inventory level Q to stock: sales will be equal to the minimum of demand and how much is in stock; so if demand is H but $Q < H$, sales are Q. The price of the good is r per unit, and the marginal cost of stocking the good is $c < r$. To eliminate the trivial case of the firm choosing $Q < M$ (making motivation redundant), the authors assume

$$c < C \equiv (p_H + p_M)r - \frac{\psi}{(M-L)\left(1 - \frac{(q_H+q_M)}{(p_H+p_M)}\right)}.$$

This leaves open three possibilities. The first is stocking $Q = H$ where the realization of the demand is perfectly observed, in which case it is optimal to pay $S_H = \frac{\psi}{p_H - q_H}$ and $S_M = S_L = 0$ (as is standard, it is cheapest to induce incentives using the value of demand with the highest likelihood ratio, and 0 otherwise). The second is producing $Q = M$, here if demand is not L, it cannot be known if demand was H or M, so it is optimal to pay $S_{HM} = \frac{\psi}{1-(q_M+q_M)/(p_H+p_M)}$ and $S_L = 0$. The third case is producing $Q \in (M, H)$, where it is optimal to pay $S_H = \frac{\psi}{p_H - q_H}$ if demand is H and 0 otherwise (in this case, demand is also observable).

The paper argues that only the first two cases could be optimal, depending on parameters. The problem is that if $p_H r < c$, there exists no equilibrium. The authors argue that since profits in this case are

$$\pi = (p_H r - c)\,Q + r\,(p_M M + p_L L) - \frac{\psi}{p_H - q_H},$$

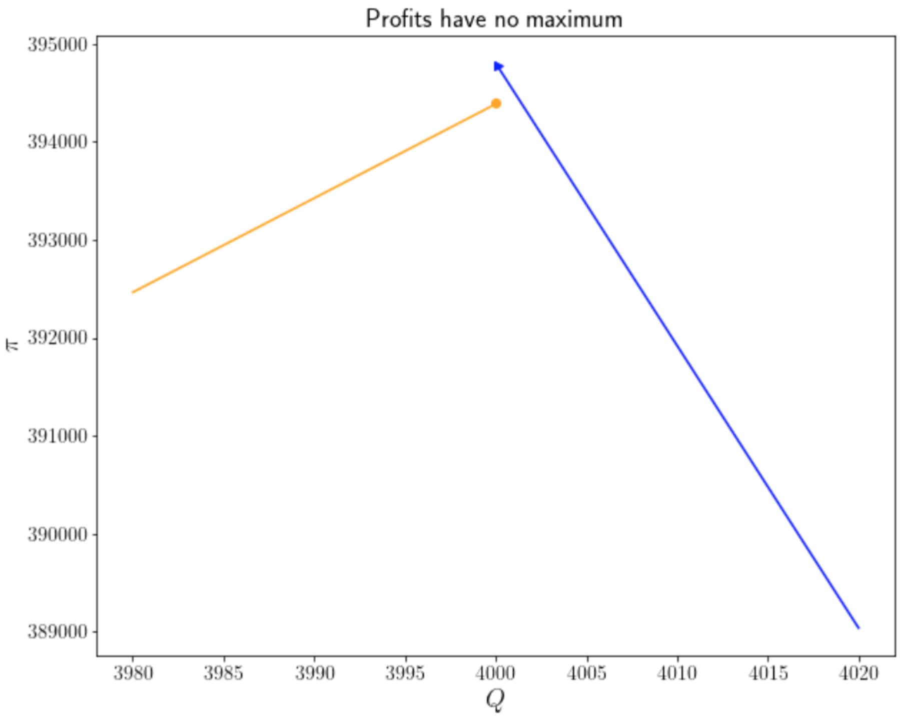
no Q in the range (M, H) can be optimal, as $p_H r - c < 0$ would imply that the firm would be better off with a smaller Q, and $p_H r - c \geq 0$ would imply that $Q = H$ is optimal. Both those arguments are correct. But the problem is that if $p_H r - c < 0$ there is no optimal Q: reducing Q increases profits, but choosing $Q = M$ is strictly worse than choosing $Q + \varepsilon$. The idea is that stocking an additional ε has a very low cost of εc, but a large benefit, since one is able to learn whether M or H happened. This is crucial as its informational value is large: implementing e_H is much cheaper, if one can observe whether M or H happened.

An example to illustrate non-existence follows.

3 Example

Let $p = (p_L, p_M, p_H) = (0.01, 0.96, 0.03)$, $q = (q_L, q_M, q_H) = (0.5, 0.49, 0.01)$, which satisfy all of the assumptions in the paper. Additionally, let $L = 3000$, $M = 4000$, $H = 5000$, $\psi = 800$, $r = 400$, $c = 300$, which satisfies $c < C$ from (2). In this situation the firm's profits if it produces $Q = H$ are 107.700; if it produces $Q = M$ profits are 394.384, whereas if it produces $Q = M + \varepsilon$ are $394.800 - 288\varepsilon$, which is larger than 394.384 if ε is small.

Grafically:



4 Discussion

One solution is to make parametric assumptions in a manner that invalidates the example, i.e., $p_H r \geq c$, ensuring this result is unattainable, then producing H is the only optimal choice, and we are back in a simple principal agent problem, and there is no interaction with inventory management (Proposition 3 is rendered invalid because in benchmark 2, $Q = H$ would also be produced). $M + \varepsilon$ always dominates M, and if $p_H r \geq c$ it is convenient to produce H instead of $M + \varepsilon$. The paper is mistaken in considering that M occurs when $c > p_H r + \frac{\psi}{H-M} \frac{1}{\left(1 - \frac{(q_H + q_M)}{(p_H + p_M)}\right)} \equiv \widehat{c}$ because assuming $p_H r \geq c$ contradicts it.

To address all this, another potential solution is to discretize the decision variable Q to avoid the issue highlighted by the example when $p_H r < c$. In that case, for a fixed set of parameters, one can stipulate that Q belongs to a grid with the distance between adjacent values of Q of less than $\frac{\psi}{c-p_H r}\left(\frac{p_H q_M - p_M q_H}{(q_L-p_L)(p_H-q_H)}\right)$. In that case, the optimal levels of inventory will be either $M+\varepsilon$, or H. as follows:

$$Q^* = \begin{cases} H & \text{if } c < \bar{c}, \\ M+\varepsilon & \text{if } c \geq \bar{c}, \end{cases}$$

where $\bar{c} = p_H r$. In this situation, the reward for achieving the highest possible sales outcome is $S^* = \frac{\psi}{p_H - q_H}$ (it is cheaper to always pay the salary when demand is H) and the expected payment to the salesperson is $E(S^*) = \frac{\psi}{1-\frac{q_H}{p_H}}$. Therefore, the firm's expected profit is

$$\pi^* = \begin{cases} r(p_H H + p_M M + p_L L) - cH - \frac{\psi}{1-q_H/p_H} & \text{if } c < \bar{c}, \\ r(p_H(M+\varepsilon) + p_M M + p_L L) - c(M+\varepsilon) - \frac{\psi}{1-q_H/p_H} & \text{if } c \geq \bar{c}. \end{cases}$$

Taking into account this discretization, the constrained inventory does not distort the signal acquired by the company from the salesperson.

5 Conclusions

Two things are worth noting. First, the counterintuitive notion highlighted in the paper that "as inventory becomes more expensive, leading the firm to reduce its inventory, the firm may nevertheless pay the agent a higher bonus" ceases to be true if Q is discrete, as the compensation would not be affected by inventory considerations. Second, it is important to note that the failure of existence comes from a relevant economic problem: a tiny amount of stock reveals whether H happened, and that has a large informational value. In the example, since the probability of H is low, it doesn't make sense to stock H units, but from an informational point of view, knowing H has happened is very informative regarding whether the agent has exerted effort or not. Therefore, stocking one extra unit is cheap, and it provides useful information.