Borrowing to Finance Public Investment: A Politico-Economic Analysis of Fiscal Rules

Uchida, Yuki and Ono, Tetsuo

Seikei University, Osaka University

21 December 2023

Borrowing to Finance Public Investment: A Politico-Economic Analysis of Fiscal Rules *

Yuki Uchida†
Seikei University

Tetsuo Ono‡
Osaka University

January 6, 2024

Abstract

This study examines the golden rule of public finance, distinguishing between public investment and consumption spending when borrowing, allowing for exclusively debt-financed public investment. Explored within an overlapping-generations model that encompasses the accumulation of physical and public capital, the rule and its associated fiscal policy emerge internally, selected by short-lived governments that represent current generations. The model is calibrated for Germany, Japan, and the United Kingdom, demonstrating adherence to the rule in Germany and deviation from it in Japan and the United Kingdom, which aligns with the available evidence. The evaluation from the perspective of a long-term planner reveals the government's excessive choices in public debt across these countries.

- Keywords: Fiscal Rule; Golden Rule of Public Finance; Probabilistic Voting; Overlapping Generations; Political Distortions
- JEL Classification: D70, E62, H63

*The authors gratefully acknowledge financial support from Grant-in-Aid for Early-Career Scientists from the Japan Society for the Promotion of Science (No. 18K12802, Uchida) and Grant-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (No. 21K01539, Ono).

†Yuki Uchida: Faculty of Economics, Seikei University, 3-3-1 Kichijoji-Kitamachi, Musashino, Tokyo 180-8633, Japan E-mail: yuchida@econ.seikei.ac.jp.

‡Tetsuo Ono: Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan E-mail: tono@econ.osaka-u.ac.jp.

1 Introduction

Many developed countries have incurred large budget deficits, thereby accumulating debt in the aftermath of the global financial crisis of 2008/9 (OECD, 2021). On the one hand, budget deficits enable governments to spend more than they receive in tax revenue, thereby providing citizens with higher levels of public goods and services in the short run. On the other hand, budget deficits might reduce economic growth by inhibiting capital accumulation, thus lowering the provision of public goods and services in the long run. In addition, fiscal deficits raise the issue of intergenerational equity in fiscal burdens. This is because they imply a transfer of the burden of public spending from the current generation to future generations.

To cope with large budget deficits and their associated problems, many developed countries have implemented various types of fiscal rules (Budina et al., 2012; Schaechter et al., 2012; Wyplosz, 2013; Lledó et al., 2017; Caselli et al., 2018). A well-known fiscal rule is the balanced budget rule (BB). BB is a constitutional requirement that ensures that tax revenues must be sufficient to cover expenditures and interest payments on debt, implying zero or negative deficits (Azzimonti et al., 2016). BB has the advantage of maintaining fiscal discipline each year but presents a disadvantage in terms of efficiency and intergenerational equity. In terms of efficiency, BB does not allow for smoothing tax rates over time (Stockman, 2001, 2004). In terms of intergenerational equity, BB requires current generations to finance the entire public investment burden, even though its benefits would be received only by future generations (Bom and Ligthart, 2014).

The golden rule of public finance (GR) is a fiscal rule that addresses the abovementioned problems of BB (Buiter et al., 1993; Corsetti and Roubini, 1996; Robinson, 1998). GR, which allows budget deficits to finance only public investment but not current expenditure, has been in place in Germany, Japan, and the United Kingdom (Kumar et al., 2009). Following the development of this alternative rule, many researchers analyzed GR in growth models with public capital. They showed that GR could gradually improve growth and welfare across generations (e.g., Greiner and Semmler, 2000; Ghosh and Mourmouras, 2004a,b; Greiner, 2008; Yakita, 2008; Minea and Villieu, 2009; Agénor and Yilmaz, 2017; Ueshina, 2018). These studies consider tax rates and/or expenditure as given, and thus, their conclusions rely on the assumption that fiscal policy instruments are independent of changes in the fiscal rule. However, the fiscal rule greatly influences the governments' choice of fiscal policy (e.g., Bisin et al., 2015; Halac and Yared, 2018; Coate and Milton, 2019; Bouton et al., 2020; Dovis and Kirpalani, 2020, 2021; Piguillem and Riboni, 2020). Therefore, considering the endogenous response of fiscal policies to the fiscal rule would provide a new perspective on the consequences of the fiscal rule for growth and welfare.

Several approaches have been attempted to investigate the effects of fiscal rules on fiscal policy formation and the resulting impacts on growth and welfare. They include Barseghyan and Battaglini (2016), Arai et al. (2018), Andersen (2019), and Uchida and Ono (2021). The analysis of Arai et al. (2018) is based on a model without public capital, and thus, their analysis does not focus on GR. Barseghyan and Battaglini (2016), Andersen (2019) and Uchida and Ono (2021) presented models with public investment in human capital or infrastructure that works as productive capital, but their main focus was on the austerity program or debt ceiling.[1] A notable exception is Bassetto and Sargent (2006), who presented a multi-period overlapping-generations model with durable public goods. They calibrated the model to the US economy and showed that GR may approximate the Pareto-efficient allocation.

Our study differs from Bassetto and Sargent (2006) in the following two aspects. First, fiscal rules are endogenously determined for each period through voting. Fiscal rules are generally set by law or constitution in many countries and cannot be easily changed. However, exemptions from the rules are granted in Japan and the United Kingdom with the approval of the Diet and Parliament, respectively. We approximate the political mechanism of such rule exemptions by modeling the determination of the fiscal rule via voting patterns in each period and call it a politically preferred fiscal rule. We show that the politically preferred fiscal rule is affected by structural parameters representing preferences for public goods and the elasticity of public capital with respect to public investment. The differences in these parameters might explain why some countries follow the GR while others break it.

Second, we assume a logarithmic utility function. Under this assumption, income and substitution effects offset each other, but a general equilibrium effect of policy choices through the interest rate still remains. This general equilibrium effect influences the determination of fiscal rules and policies through voting. Notably, Bassetto and Sargent (2006) abstract this type of effect by employing a quasilinear utility function, neglecting the general equilibrium effect despite its well-documented importance in political economy literature.(e.g., Gonzalez-Eiras and Niepelt, 2008, 2012; Song et al., 2012)

To execute our analysis, we employ a three-period overlapping-generations model with physical and public capital. Each generation comprises many identical individuals who live over three periods: the young who make no economic decision, the middle-aged who work, and older adults who are retired. Public investment and the public capital inherited from the past are inputs in the public capital formation process, contributing to the productivity in output per worker. Governments, as elected representatives, finance public investment in public capital and public goods provision through taxes on capital and labor income as well as through public debt issuance. When expenditure is constrained by fiscal rules, public goods provision must be financed solely by tax revenues, while a certain proportion of the public investment, denoted by $\phi \geq 0$, can be financed by public debt issuance. In particular, the rule does not allow for deficit when $\phi = 0$; all public investments in public capital are allowed to be financed with public debt issuance when $\phi = 1$; and all public investments plus a part of current expenditures are debt-financed and GR is broken when $\phi > 1$.

Under this framework, we begin our analysis by considering the politics of fiscal policy

[1] Andersen (2019) mentioned GR as an alternative rule, but their analysis is limited to a brief sketch as a possible extension.

formation for a given fiscal rule, ϕ. In particular, we assume probabilistic voting to demonstrate the extent to which generations face conflicts over such policies. In each period, middle-aged and older adult individuals vote on candidates.[2] The government, represented by elected politicians, maximizes the political objective function of the weighted sum of utilities of the middle-aged and older adult populations. In this voting environment, the current policy choice affects the decision on future policy via physical and public capital accumulation. This intertemporal effect creates the three driving forces that shape fiscal policy, namely, a general equilibrium effect through the interest rate, a disciplining effect through the capital income tax rate in the next period, and a disciplining effect through public goods provision in the next period. The three effects induce the government to finance part of its expenditure via public debt issuance.

We calibrate the model economy to Germany, Japan, and the United Kingdom, where the GR has been in place (Kumar et al., 2009). We study how changes in the fiscal rule affect the government's choice of fiscal policies and the resulting allocation of physical and public capital. We show that a higher ϕ, implying a larger share of public investment financed by public debt issuance, lowers physical capital accumulation through the crowding-out effect. This in turn raises the marginal cost of the labor income tax, and thus induces the government to choose a lower labor income tax rate. We also show that a higher ϕ has two conflicting effects on the choice of public investment: a negative effect through the crowding-out effect on physical capital, and a positive effect through a reduced tax burden. Given these conflicting effects, there is a threshold value of ϕ at which the two effects are balanced, and an increase in ϕ has an inverse U-shaped effect on the ratio of public investment to GDP around the threshold value of ϕ.

We then move on to the analysis of the politically preferred ϕ, that is, the debt-financed proportion of public investment that realizes the maximization of the political objective function. We show that the politically preferred ϕ depends on the two structural parameters, θ, representing the preferences for public goods, and η, representing the elasticity of public capital with respect to public investment. A higher θ yields a stronger incentive for the government to cut public debt issuance, and a higher η implies a higher marginal benefit of public investment, incentivizing the government to lower the debt-financed proportion of public investment. Given this property, we compute the politically preferred ϕ for the three countries and show that it is less than one in Germany that has the highest θ and η. We also show that the politically preferred ϕ is greater than one in Japan and the United Kingdom, which implies that these two countries break the GR. These model predictions are consistent with the evidence; a waiver of the GR has been approved by the Diet almost every year since 1975 in Japan (Kumar et al., 2009), and the GR adopted in 1997 was met only for the first few years in the United Kingdom (Wyplosz, 2013).

[2] The young may also have an incentive to vote since they would benefit from public investment financed by taxing capital and labor income. However, for the tractability of analysis, we assume that politicians do not care about their preferences following Saint-Paul and Verdier (1993), Bernasconi and Profeta (2012), and Lancia and Russo (2016). This assumption is supported in part by the fact that a significant portion of the young are below the voting age.

The abovementioned politically preferred fiscal rule is a choice that only considers the generations existing at the time of voting. Because public investment generates spillover effects that impact future generations, evaluating the rule from an intergenerational perspective becomes essential. To emphasize the effects, we describe the optimal allocation chosen by a long-lived planner, called a Ramsey planner. The planner can commit to all its choices and set the fiscal policy sequences in the initial period so as to maximize the discounted sum of utilities of current and future generations subject to the same set of constraints faced by the elected governments. Assuming such a hypothetical planner, we focus on the deviations between the political equilibrium allocation and the allocation of the Ramsey planner in terms of fiscal policy variables. Consequently, we demonstrate that, within a political equilibrium, the ratio of public debt to public investment surpasses that observed in the Ramsey allocation, implying a tendency for short-lived governments to excessively issue public debt and to pass excess fiscal burden to future generations.

The remainder of this paper is organized as follows. Section 2 introduces the model and characterizes an economic equilibrium that describes the behavior of households and firms for a given set of fiscal policies. Section 3 presents the politics of fiscal policy formation, characterizes a political equilibrium for a given fiscal rule, and investigates the effects of the fiscal rule on fiscal policy formation. It also demonstrates the political determination of the fiscal rule and investigates its property. Section 4 characterizes the Ramsey planner's allocation and evaluates the politically preferred fiscal policy and rule from the viewpoint of the Ramsey planner. Section 5 provides the concluding remarks. All proofs are presented in the appendix.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live for three periods: youth, middle age, and older adult age. Each middle-aged individual has $1+n$ children. The middle-aged population for period t is N_t and the population grows at a constant rate of $n(>-1) : N_{t+1} = (1+n)N_t$.

2.1 Individuals

Individuals display the following economic behavior over their life cycles. During youth, they make no economic decisions and live under the financial support and protection of their parents. In middle age, individuals work, receive market wages, and pay taxes. They use after-tax income for consumption and savings. Individuals retire in their older adult years and receive and consume returns from savings.[3]

Consider an individual born in period $t-1$. In period t, the individual is middle-aged and endowed with one unit of labor. He/she supplies it inelastically in the labor market, and obtains

[3]The role of the young is not directly incorporated into the subsequent analysis. Nonetheless, their presence is assumed as we calibrate the model economy to align with the key statistics of the sample countries outlined in Section 3.

labor income w_t, where w_t is the wage rate per unit of labor in period t. After paying tax $\tau_t w_t$, where $\tau_t \in (0,1)$ is the period t labor income tax rate, the individual distributes the after-tax income between consumption c_t and savings invested in physical capital s_t. Therefore, the period t budget constraint for the middle-aged becomes $c_t + s_t \leq (1-\tau_t)w_t$.

The period $t+1$ budget constraint in older adult age is $d_{t+1} \leq \left(1-\tau_{t+1}^k\right) R_{t+1} s_t$, where d_{t+1} is consumption, τ_{t+1}^k is the period $t+1$ capital income tax rate, $R_{t+1}(>0)$ is the gross return from investment in physical capital, and $R_{t+1}s_t$ is the return from savings. The results are qualitatively unchanged if the capital income tax is on the net return from savings rather than the gross return from savings.

The preferences of the middle-aged in period t are specified by the following expected utility function in the logarithmic form, $U_t^M = \ln c_t + \theta \ln g_t + \beta\left(\ln d_{t+1} + \theta \ln g_{t+1}\right)$, where g is per capita public goods provision, $\beta \in (0,1)$ is the discount factor, and $\theta(>0)$ is the degree of preferences for public goods. The preferences of older adults in period t are given by $U_t^O = \ln d_t + \theta \ln g_t$.

We substitute the budget constraints into the utility function of the middle-aged, U_t^M, to form the following unconstrained maximization problem:

$$\max_{\{s_t\}} \ln\left[(1-\tau_t)w_t - s_t\right] + \theta \ln g_t + \beta\left[\ln\left(1-\tau_{t+1}^k\right) R_{t+1}s_t + \theta \ln g_{t+1}\right], \tag{1}$$

where the factor prices, $\{w_t, R_{t+1}\}$, and the fiscal policy variables, $\{\tau_t, \tau_{t+1}^k, g_t, g_{t+1}\}$, are taken as given. By solving the problem in (1), we obtain the following savings and consumption functions:

$$s_t = \frac{\beta}{1+\beta}(1-\tau_t)w_t, \tag{2}$$

$$c_t = \frac{1}{1+\beta}(1-\tau_t)w_t, \tag{3}$$

$$d_{t+1} = \left(1-\tau_{t+1}^k\right) R_{t+1}\frac{\beta}{1+\beta}(1-\tau_t)w_t. \tag{4}$$

2.2 Firms and Public Capital

Each period contains a continuum of identical firms that are perfectly competitive profit maximizers. According to Cobb–Douglas technology, the firms produce a final good Y_t using two inputs: aggregate physical capital K_t and aggregate labor N_t. Aggregate output is given by $Y_t = A_t\left(K_t\right)^\alpha\left(N_t\right)^{1-\alpha}$, where A_t represents a period-t technology and $\alpha \in (0,1)$ denotes the capital share. The production function in intensive form is $y_t = A_t\left(k_t\right)^\alpha$ where $y_t \equiv Y_t/N_t$ and $k_t \equiv K_t/N_t$ denote per capita output and physical capital, respectively.

The first-order conditions for profit maximization with respect to N_t and K_t are

$$w_t = (1-\alpha)A_t\left(k_t\right)^\alpha, \tag{5}$$

$$R_t = \alpha A_t\left(k_t\right)^{\alpha-1}, \tag{6}$$

where w_t and R_t are labor wages and the gross return on physical capital, respectively. The conditions state that firms hire labor and physical capital until the marginal products are equal to the factor prices. Capital is assumed to depreciate fully within each period.

The productivity A_t is assumed to be proportional to the per labor public capital: $A_t \equiv A\left(K_t^p/N_t\right)^{1-\alpha}$, where K_t^p denotes the aggregate public capital in period t. Thus, public capital involves a technological externality of the type often used in endogenous-growth theories. Under this assumption, the first-order conditions in (5) and (6) are rewritten as follows:

$$w_t = w\left(k_t, k_t^p\right) \equiv (1-\alpha)A\left(k_t\right)^{\alpha}\left(k_t^p\right)^{1-\alpha}, \tag{7}$$

$$R_t = R\left(k_t, k_t^p\right) \equiv \alpha A\left(k_t\right)^{\alpha-1}\left(k_t^p\right)^{1-\alpha}, \tag{8}$$

and the associated production function in intensive form becomes:

$$y_t = y\left(k_t, k_t^p\right) \equiv A\left(k_t\right)^{\alpha}\left(k_t^p\right)^{1-\alpha}. \tag{9}$$

The public capital evolves in the following way. The aggregate public capital in period $t+1$, K_{t+1}^p, is a function of public investment, x_t, and the public capital inherited from the past, K_t^p. In particular, K_{t+1}^p is formulated using the following equation:

$$K_{t+1}^p = \tilde{D}\cdot\left(N_{t+1}x_t\right)^{\eta}\left(K_t^p\right)^{1-\eta},$$

where $\tilde{D}\,(>0)$ is a scale factor, $N_{t+1}x_t$ is the aggregate public investment evaluated on a generation $t+1$ population basis, and $\eta \in (0,1)$ denotes the elasticity of public capital with respect to public investment. Let $k_t^p \equiv K_t^p/N_t$ denote per capita public capital. Then the equation is reformulated as

$$k_{t+1}^p = D\cdot\left(x_t\right)^{\eta}\left(k_t^p\right)^{1-\eta}, \tag{10}$$

where $D \equiv \tilde{D}\cdot(1+n)^{\eta-1}$. This formulation is based on that of Barseghyan and Battaglini (2016), but discarding private investment.

2.3 Government Budget Constraint and Fiscal Rule

Government expenditure items are an investment in public capital and expenditure on (unproductive) public goods provision. They are financed by capital and labor taxes, as well as public debt issues. Let B_t denote the aggregate inherited debt. The aggregate government budget constraint in period t is

$$B_{t+1} + \tau_t^k R_t s_{t-1} N_{t-1} + \tau_t w_t N_t = N_{t+1}x_t + G_t + R_t B_t, \tag{11}$$

where B_{t+1} is newly issued public debt; $\tau_t^k R_t s_{t-1} N_{t-1}$ is the aggregate capital tax revenue; $\tau_t w_t N_t$ is the aggregate labor tax revenue; $N_{t+1}x_t$ is the aggregate public investment; G_t is aggregate public goods provision, and $R_t B_t$ is debt repayment. We assume a one-period debt structure to derive analytical solutions from the model. We also assume that the government in each period is committed to not repudiating the debt. The associated per capita government budget constraint in period t is

$$(1+n)b_{t+1}+\tau_t^k R_t \frac{s_{t-1}}{1+n}+\tau_t w_t=(1+n)x_t+\frac{2+n}{1+n}g_t+R_t b_t. \tag{12}$$

Consider a situation in which government expenditures are constrained by fiscal rules. In particular, following the literature on GR, we focus on fiscal rules that impose constraints on how to finance public investment. Specifically, the fiscal rule in the present study is given by $B_{t+1}=\phi_t N_{t+1} x_t$, or:

$$(1+n)b_{t+1}=\phi_t(1+n)x_t, \tag{13}$$

where $b_{t+1}\equiv B_{t+1}/N_{t+1}$ is per-capita public debt and $\phi_t\,(\geq 0)$ represents the period-t fiscal rule that determines the percentage of public investment to be financed by public debt issuance.

While this paper's fiscal rule, as denoted by (13), serves as our foundational assumption, the GR takes on a more generalized form as $B_{t+1}-B_t\leq N_{t+1}x_t$. When this is expressed on a per capita basis, it is represented as

$$(1+n)\,b_{t+1}-b_t\leq(1+n)\,x_t. \tag{14}$$

A comparison between (13) and (14) reveals two key deviations in our assumption. Firstly, (13) focuses on targeting public debt issuance, $(1+n)b_{t+1}$, rather than the budget deficit, $(1+n)\,b_{t+1}-b_t$. This choice is primarily rooted in its ability to provide a closed-form solution for the policy function within the context of political equilibrium—a matter that we will delve into later in our analysis. To circumvent the technical complexities that arise when targeting the budget deficit, we opt to center our attention on the public debt issuance.

Secondly, our analysis accommodates the possibility of government deviation from the fiscal rule. In essence, we allow for the scenario where the government issues more public debt than the level of public investment. This possibility is represented by the parameter ϕ_t in (13), with ϕ_t taking a value greater than or equal to 0. When $\phi_t\leq 1$, it signifies adherence to the fiscal rule, whereas $\phi_t>1$ indicates a decision to deviate from the rule.

Building on the aforementioned framework, the timing in period t can be delineated as follows. Initially, with k_t, k_t^p, and b_t as given, the government determines ϕ_t with the aim of maximizing the political objective function (which will be introduced later). Subsequently, based on the ϕ_t determined in the initial stage and the state variables k_t, k_t^p, and b_t, the government makes determinations regarding τ_t, τ_t^k, x_t, and g_t within the context of maximizing the political objective function.

Substitution of (13) into (11) leads to the associated per capita form of the government budget constraint:

$$\tau_t^k R_t \frac{s_{t-1}}{1+n}+\tau_t w_t=(1-\phi_t)(1+n)x_t+\frac{2+n}{1+n}g_t+R_t b_t. \tag{15}$$

When $\phi_t=0$, the rule does not allow for a deficit, and requires a balanced budget. All government expenditures must be financed using tax revenues. When $\phi_t=1$, all public investments can be financed by public debt issuance. This rule allows a budget deficit only to finance public investment, and prohibits the financing of current expenditures (i.e., public goods provision) by budget deficit. When $\phi_t > 1$, all public investments plus a part of public goods provision are financed by public debt issuance. As ϕ_t increases, the fiscal burden of public investment is increasingly deferred to future generations.

2.4 Economic Equilibrium

Public debt is traded in the domestic capital market. The market clearing condition for capital is $B_{t+1} + K_{t+1} = N_t s_t$, which expresses the equality of total savings by the middle-aged population in period t, $N_t s_t$ to the sum of the stocks of aggregate public debt and aggregate physical capital at the beginning of period $t+1$, $B_{t+1} + K_{t+1}$. Using $k_{t+1} \equiv K_{t+1}/N_{t+1}$, $k_t^p = K_t^p/N_t$, the profit-maximization condition in (5), and the savings function in (2), we can rewrite the abovementioned condition as

$$(1+n)\left(k_{t+1} + b_{t+1}\right) = s\left(\tau_t; k_t, k_t^p\right) \equiv \frac{\beta}{1+\beta}(1-\tau_t) w\left(k_t, k_t^p\right). \tag{16}$$

The following defines the economic equilibrium in the present model.

Definition 1 Given a sequence of policies and rules, $\left\{\tau_t^k, \tau_t, x_t, g_t, \phi_t\right\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations $\left\{c_t, d_t, s_t, k_{t+1}, b_{t+1}, k_{t+1}^p\right\}_{t=0}^{\infty}$ and prices $\{w_t, R_t\}_{t=0}^{\infty}$ with initial conditions $k_0 (> 0), b_0 (\geq 0)$ and $k_0^p (> 0)$, such that (i) given $\left(w_t, R_{t+1}, \tau_{t+1}^k, \tau_t, x_t, g_t, g_{t+1}\right)$, (c_t, d_{t+1}, s_t) solves the utility-maximization problem; (ii) given (w_t, R_t), k_t solves the firm's profit maximization problem; (iii) given (w_t, k_t^p, R_t, b_t) and ϕ_t, $\left(\tau_t^k, \tau_t, x_t, b_{t+1}\right)$ satisfies the government budget constraint; and (iv) the capital market clears: $(1+n)\left(k_{t+1} + b_{t+1}\right) = s_t$.

Definition 1 allows us to reduce the economic equilibrium conditions to a system of difference equations that characterizes the motion of (k_t, b_t, k_t^p). The system includes the capital market clearing condition in (16), the public capital formation function in (10), and the government budget constraint in (15) with the optimality conditions of firms in (5) and (6). The government budget constraint in (15) is reformulated as

$$TR^K\left(\tau_t^k; k_t, k_t^p, b_t\right) + TR\left(\tau_t; k_t, k_t^p\right) = (1-\phi_t)(1+n)x_t + \frac{2+n}{1+n} g_t + R\left(k_t, k_t^p\right) b_t, \tag{17}$$

where $TR^K\left(\tau_t^k; k_t, k_t^p, b_t\right)$ and $TR\left(\tau_t; k_t, k_t^p\right)$, representing the tax revenues from capital and labor income, respectively, are defined as follows:

$$\begin{aligned} TR^K\left(\tau_t^k; k_t, k_t^p, b_t\right) &\equiv \tau_t^k R\left(k_t, k_t^p\right)\left(k_t + b_t\right), \\ TR\left(\tau_t; k_t, k_t^p\right) &\equiv \tau_t w\left(k_t, k_t^p\right). \end{aligned}$$

In economic equilibrium, the indirect utility of the middle-aged population in period t, V_t^M, and that of the older adult population in period t, V_t^O, can be expressed as functions of fiscal policy, physical and public capital, and public debt as follows:

$$
\begin{aligned}
V_t^M &= V^M\left(\tau_t, x_t, g_t, \tau_{t+1}^k, g_{t+1}; k_{t+1}, k_{t+1}^p, b_{t+1}, k_t, k_t^p\right) \\
&\equiv \ln c\left(\tau_t; k_t, k_t^p\right) + \theta \ln g_t + \beta\left[\ln d\left(\tau_{t+1}^k; k_{t+1}, k_{t+1}^p, b_{t+1}\right) + \theta \ln g_{t+1}\right], \qquad (18)\\
V_t^O &= V^O\left(\tau_t^k, g_t; k_t, b_t, k_t^p\right) \equiv \ln d\left(\tau_t^k; k_t, k_t^p, b_t\right) + \theta \ln g_t, \qquad (19)
\end{aligned}
$$

where $c\left(\tau_t; k_t, k_t^p\right)$ and $d\left(\tau_t^k; k_t, k_t^p, b_t\right)$, representing consumption in middle and older adult ages, respectively, are defined as follows:

$$
\begin{aligned}
c\left(\tau_t; k_t, k_t^p\right) &\equiv \frac{1}{1+\beta}\left(1-\tau_t\right) w\left(k_t, k_t^p\right), \\
d\left(\tau_t^k; k_t, k_t^p, b_t\right) &\equiv \left(1-\tau_t^k\right) R\left(k_t, k_t^p\right)(1+n)\left(k_t+b_t\right).
\end{aligned}
$$

3 Politics

Based on the characterization of the economic equilibrium in Subsection 2.4, we consider the politics of fiscal policy formation. In particular, we employ probabilistic voting à la Lindbeck and Weibull (1987). In this voting scheme, there is electoral competition between two office-seeking candidates. Each candidate announces a set of fiscal policies subject to the government budget constraint. As demonstrated by Persson and Tabellini (2000), the platforms of the two candidates converge in the equilibrium to the same fiscal policy that maximizes the weighted average utility of voters (i.e., the middle-aged and older adults).

In the present framework, the young, the middle-aged, and older adults have the incentive to vote. While the young may benefit from public investment through public capital accumulation, we assume that their preferences are not considered by politicians. We impose this assumption, which is often used in the literature (e.g., Saint-Paul and Verdier, 1993; Bernasconi and Profeta, 2012; Lancia and Russo, 2016), for tractability reasons. However, this assumption could be supported in part by the fact that a significant proportion of the young are below the voting age.

The political objective is defined as the weighted sum of the utility of the middle-aged and older adults, given by $\tilde{\Omega}_t \equiv \omega V_t^O + (1+n)(1-\omega)V_t^M$, where $\omega \in (0,1)$ and $1-\omega$ are the political weights placed on older adults and middle-aged, respectively. The weight on the middle-aged is adjusted by the gross population growth rate, $(1+n)$, to reflect their share of the population. We divide $\tilde{\Omega}_t$ by $(1+n)(1-\omega)$ and redefine the objective function as follows:

$$
\Omega_t = \frac{\omega}{(1+n)(1-\omega)} V_t^O + V_t^M, \qquad (20)
$$

where the coefficient $\omega/(1+n)(1-\omega)$ of V_t^O represents the relative political weight on older adults.

The political objective function suggests that the current policy choice affects the decision on future policy via physical and public capital accumulation. In particular, the period t choices of τ_t^k, x_t, g_t, and b_{t+1} affect the formation of physical and public capital in period $t+1$. This, in turn, influences the decision making on the period-$t+1$ fiscal policy. To demonstrate such an intertemporal effect, we employ the concept of a Markov-perfect equilibrium under which fiscal policy in the present period depends on the current payoff-relevant state variables.

In our framework, the payoff-relevant state variables are physical capital k_t, public debt b_t, and public capital k_t^p. Thus, the expected rate of capital income tax for the next period, τ_{t+1}^k, is given by the function of the period-$t+1$ state variables, $\tau_{t+1}^k = T^k\left(k_{t+1}, b_{t+1}, k_{t+1}^p\right)$. We denote the arbitrary lower limits of τ and τ^k by $-\underline{\tau}(<0)$ and $-\underline{\tau}^k(<0)$, respectively. By using recursive notation with z' denoting the next period z, we can define a Markov-perfect political equilibrium in the present framework as follows.

Definition 2 Given ϕ, a *Markov-perfect political equilibrium* is a set of functions, $\langle \hat{T}, \hat{T}^k, \hat{X}, \hat{G}, \hat{B}\rangle$, where $\hat{T} : \Re_+^3 \to (-\underline{\tau}, 1)$ is the labor income tax rule, $\tau = \hat{T}(k, b, k^p)$, $\hat{T}^k : \Re_+^3 \to \left(-\underline{\tau}^k, 1\right)$ is a capital income tax rule, $\tau^k = \hat{T}^k(k, b, k^p)$, $\hat{X} : \Re_+^3 \to \Re_+$ is a public investment rule, $x = \hat{X}(k, b, k^p)$, and $\hat{G} : \Re_+^3 \to \Re_+$ is a public goods provision rule, $g = \hat{G}(k, b, k^p)$, so that (i) given k, b, and k^p, $\langle \hat{T}(k, b, k^p), \hat{T}^k(k, b, k^p), \hat{X}(k, b, k^p), \hat{G}(k, b, k^p)\rangle$ is a solution to the problem of maximizing Ω in (20), subject to the public capital formation function in (10), the capital market clearing condition in (16), and the government budget constraint in (17); and (ii) $\hat{B} : \Re_+^3 \to \Re_+$ is a public debt rule, $b' = \hat{B}(k, b, k^p)$, that follows the fiscal rule in (13).[4]

3.1 Political Equilibrium

We derive the political equilibrium policy functions and the associated sequence of per capita physical and public capital. To obtain the set of policy functions, we conjecture the capital income tax rate and public goods provision in the next period as follows:

$$\tau^{k\prime} = 1 - T^k \cdot \frac{1}{\alpha\left(1 + \frac{b'}{k'}\right)}, \tag{21}$$

$$g' = G \cdot y\left(k', k^{p\prime}\right), \tag{22}$$

where $T^k(>0)$ and $G(>0)$ are constant. The conjectures in (21) and (22) suggest that at the aggregate level, the after-tax capital income and the government expenditure on public goods provision are linearly related to GDP.

Given these conjectures, we consider the optimization problem in Definition 2, and obtain the following first-order derivatives:

[4] The state variables do not line up in compact sets because they grow across periods. To define the equilibrium more precisely, we need to redefine the equilibrium as a mapping from a compact set to a compact set by introducing the following notations: $\hat{x}_t \equiv x_t/y\left(k_t, k_t^p\right)$, $\hat{g}_t \equiv g_t/y\left(k_t, k_t^p\right)$, and $\hat{b}_{t+1} \equiv b_{t+1}/y\left(k_t, k_t^p\right)$. However, for simplicity, we define the equilibrium as in Definition 2.

$$\tau^k : \tfrac{\omega}{(1+n)(1-\omega)} \tfrac{d_{\tau^k}}{d} + \lambda TR^K_{\tau^k} = 0, \tag{23}$$

$$\tau : \frac{c_\tau}{c} + \beta \left(\frac{d'_\tau}{d'} + \theta \frac{g'_\tau}{g'} \right) + \lambda TR_\tau = 0, \tag{24}$$

$$g : \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} - \lambda \frac{2+n}{1+n} = 0, \tag{25}$$

$$x : \beta \left(\frac{d'_x}{d'} + \theta \frac{g'_x}{g'} \right) - \lambda (1-\phi)(1+n) = 0, \tag{26}$$

where $\lambda\,(\geq 0)$ is the Lagrangian multiplier associated with the government budget constraint in (17). The choice of x determines the level of public debt issuance according to the rule in (13). The impact of debt issuance decisions through such a rule is contained in the term $\left(\frac{d'_\tau}{d'} + \theta \frac{g'_\tau}{g'} \right)$ of (24) and the terms $\beta \left(\frac{d'_x}{d'} + \theta \frac{g'_x}{g'} \right)$ and $\lambda (1-\phi)(1+n)$ of (26).

The first-order conditions in (23)–(26) are summarized as follows, focusing on the marginal benefit of public goods provision appearing on the left-hand side of (25):

$$\left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} = -\frac{2+n}{1+n} \cdot \frac{\frac{\omega}{(1+n)(1-\omega)} \frac{d_{\tau^k}}{d}}{TR^K_{\tau^k}}, \tag{27}$$

$$\left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} = -\frac{2+n}{1+n} \cdot \frac{\frac{c_\tau}{c} + \beta \left(\frac{d'_\tau}{d'} + \theta \frac{g'_\tau}{g'} \right)}{TR_\tau}, \tag{28}$$

$$\left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \frac{\theta}{g} = \frac{2+n}{1+n} \cdot \frac{\beta \left(\frac{d'_x}{d'} + \theta \frac{g'_x}{g'} \right)}{1+n}. \tag{29}$$

According to the expressions in (27)–(29), the government chooses policies to equate the marginal benefits of public goods provision that appear on the left-hand side with the marginal benefits of capital income tax cut, marginal net benefits of the labor income tax cut, and marginal benefits of public investment that appear on the right-hand side, respectively.

A detailed interpretation of these conditions is as follows. The intuition for the marginal benefit of public goods provision appearing on the left-hand side of each equation is straightforward. An increase in public goods leads to an increase in the marginal utility of public goods for older adults and for the middle-aged. The right-hand side of (27) represents the marginal benefit of capital income tax cut. A decrease in the capital income tax rate raises the consumption of older adults, and thus makes them better off. We evaluate this effect based on the change in tax revenue through capital income taxation represented by the term $TR^K_{\tau^k}$ in the denominator. The right-hand sides of (28) and (29) include the following three inter-temporal effects: the general equilibrium effect of capital through the interest rate, R'; the disciplining effect through the capital income tax rate, $\tau^{k\prime}$; and the disciplining effect through public goods provision, g'. These effects play crucial roles in shaping fiscal policy, which are explained in turn below.

First, consider the right-hand side of (28), showing the marginal benefit of the labor income tax cut. A cut of the labor income tax rate causes disposable income to rise, thereby increasing the consumption of the middle-aged, as represented by the term c_τ/c. The term

$\beta\left(d'_{\tau}/d' + \theta g'_{\tau}/g'\right)$ includes the marginal costs and benefits of the labor income tax cut that the current middle-aged population are expected to receive when they are older adults. The cut of the labor income tax rate increases the disposable income of the middle-aged, and thus expands their savings, which in turn raises their consumption when they are older adults. In addition, the increase in savings works to lower the return from savings, $R^{'}$, and thus reduces their consumption when they are older adults. Simultaneously, the increase in savings promotes capital accumulation, lowers the interest rate and thus reduces the burden of debt repayment. This enables the government to lower the capital income tax rate in the next period, $\tau^{k\prime}$, which raises the consumption of the middle-aged when they are older adults. These effects are represented by the term d'_{τ}/d'. The term $\theta g'_{\tau}/g'$ shows that the labor income tax cut stimulates savings and capital accumulation, and thus promotes public goods provision in the next period. The right-hand side evaluates these effects based on the change in the tax revenue through the labor income tax cut, as represented by the term TR_{τ} in the denominator.

Next, consider the right-hand side of (29), showing the marginal net benefit of public investment. The investment promotes public capital accumulation and thus, raises the return from savings, $R^{'}$. This leads to an increase in consumption for the older adults. Simultaneously, the increase in the public capital level leads to an increase in public goods provision in the next period. In addition, under the fiscal rule in (13), an increase in public investment expands public debt issuance, which slows down physical capital accumulation. This raises the interest rate R', and increases consumption in older adult age. At the same time, hindered accumulation of physical capital serves to elevate the interest rate, consequently amplifying the burden of debt repayment. This prompts the government to raise the capital income tax rate in the subsequent period, $\tau^{k\prime}$, ultimately leading to a decline in the consumption of older adults. Furthermore, impeded physical capital accumulation lowers public goods provision in the next period, g'. The right–hand side of (29) includes these five marginal costs and benefits, which affect the middle-aged when they are older adults.

In deriving the policy functions using the first-order conditions in (23)–(26), alongside the fiscal rule in (13) and the government budget constraint in (17), we further conjecture that the policy functions of τ and x are given by

$$\tau = T, \tag{30}$$

$$(1+n)x = X \cdot y(k, k^p), \tag{31}$$

where T and X are constant. We can verify the conjectures in (21), (22), (30), and (31) and obtain the following result:

Proposition 1 *Given a fiscal rule in (13), there is a Markov-perfect political equilibrium such* *that the policy functions of τ^k, g, x, τ, and b' are given by*

$$1-\tau^k = T^k \cdot \frac{1}{\alpha \frac{k+b}{k}},$$
$$\frac{2+n}{1+n} g = G \cdot y(k, k^p),$$
$$(1+n)x = X \cdot y(k, k^p),$$
$$\tau = T,$$
$$(1+n)b' = \phi X \cdot y(k, k^p),$$

where T^k, G, X, and T, defined in Appendix A.1, are constant and dependent on the fiscal rule, ϕ.

Proof. See Appendix A.1.

Proposition 1 implies that the policy functions have the following features. First, the capital income tax rate is increasing in public debt but decreasing in physical capital. A higher level of public debt increases the burden of debt repayment. The government responds to the increased burden by raising the capital income tax rate. By contrast, a higher level of physical capital lowers the interest rate and thus reduces the burden of debt repayment. This enables the government to lower the capital income tax rate. Second, the labor income tax rate is independent of the state variables and is constant across periods, whereas the levels of public goods provision and public investment are linear functions of output, y. Given the fiscal rule in (13), this property, with the first one, is necessary for the government budget constraint in (17) to be satisfied in each period.

Having established the policy functions, we are now ready to present physical and public capital accumulation. We substitute the policy functions presented in Proposition 1 into the capital market clearing condition in (16) and the public capital formation function in (10), and obtain

$$\frac{k'}{k} = \frac{1}{1+n}\left(\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X\right) A \left(\frac{k^p}{k}\right)^{1-\alpha}, \tag{32}$$
$$\frac{k^{p\prime}}{k^p} = D\left[\frac{1}{1+n} X A \left(\frac{k}{k^p}\right)^{\alpha}\right]^{\eta}. \tag{33}$$

Given the initial condition, $\{k_0, k_0^p\}$, the sequence $\{k_t, k_t^p\}$ is characterized by the above two equations in (32) and (33). A *steady state* is defined as a political equilibrium with $k^{p\prime}/k' = k^p/k$. In other words, the ratio of public to physical capital is constant across periods. Eqs. (32) and (33) lead to:

$$\frac{k^{p\prime}}{k'} = \Psi\left(\frac{k^p}{k}\right) \equiv \frac{D\left[\frac{1}{1+n} X A\right]^{\eta}}{\frac{1}{1+n}\left(\frac{\beta}{1+\beta}(1-T)(1-\alpha) - \phi X\right) A} \left(\frac{k^p}{k}\right)^{\alpha(1-\eta)},$$

with the property of $\Psi'(\cdot) > 0$ and $\Psi''(\cdot) < 0$. This property shows that there is a unique stable steady state for the path of $\{k^p/k\}$.

3.2 Effects of Fiscal Rules

The result in Subsection 3.1 indicates that the fiscal rule ϕ, taken as an institutional parameter at this stage, has a decisive effect on the formation of fiscal policies. In particular, the rule has direct impacts on τ (the labor income tax rate) via the term $\beta\left(d'_\tau/d'/+\theta g'_\tau/g'\right)$ in (24) and on x (the public investment) via the terms $\beta\left(d'_x/d'+\theta g'_x/g'\right)$ and $\lambda\left(1-\phi\right)\left(1+n\right)$ in (26). The rule has no such direct impact on τ^k (the capital income tax rate) and g (public goods provision) although they are affected by the rule via the choice of τ and x. Thus, we focus on the first-order conditions with respect to τ in (24) and x in (26) to investigate the direct effects of the fiscal rule. For this aim, we reformulate (24) and (26) as follows:

$$\tau:\underbrace{\frac{-1}{1-\tau}}_{\equiv\frac{c_\tau}{c}}+\underbrace{\frac{-\alpha\beta\left(1+\theta\right)\frac{\beta}{1+\beta}w\left(k,k^p\right)}{\frac{\beta}{1+\beta}\left(1-\tau\right)w\left(k,k^p\right)-\phi\left(1+n\right)x}}_{\equiv\beta\left(\frac{d'_\tau}{d'}+\theta\frac{g'_\tau}{g'}\right)}+\underbrace{\lambda w\left(k,k^p\right)}_{\equiv\lambda TR_\tau}=0, \tag{34}$$

$$x:\underbrace{\frac{-\alpha\beta\left(1+\theta\right)\phi\left(1+n\right)}{\frac{\beta}{1+\beta}\left(1-\tau\right)w\left(k,k^p\right)-\phi\left(1+n\right)x}+\frac{\beta\eta\left(1+\theta\right)\left(1-\alpha\right)}{x}}_{\equiv\beta\left(\frac{d'_x}{d'}+\theta\frac{g'_x}{g'}\right)}-\lambda\left(1-\phi\right)\left(1+n\right)=0. \tag{35}$$

Equations (34) and (35) show that the fiscal rule, represented by ϕ, affects the formation of the labor income tax rate and the ratio of public investment to GDP, respectively. Given the difficulty of showing its qualitative impacts, we clarify the effects quantitatively based on numerical methods. In particular, we calibrate the model economy such that the steady-state political equilibrium allocation and policies match some key statistics of each sample country. The sample countries are Germany, Japan, and the United Kingdom, which have adopted GR as a fiscal rule over the past several decades (Kumar et al., 2009).

Our strategy is to calibrate the model economy in such a manner that for each sample country, the political equilibrium demonstrated in Proposition 1 matches some key statistics over the time period 1995–2016. We assume that each period of the present model lasts 30 years; this assumption is standard in quantitative analyses of two- or three-period overlapping-generations models (e.g., Gonzalez-Eiras and Niepelt, 2008; Lancia and Russo, 2016). The population growth rate, n, is obtained from the average of each sample country during the 1995–2016 period. We assume the share of capital, α, is common to the three countries and fix at $\alpha=1/3$, in line with Song et al. (2012) and Lancia and Russo (2016).

We calibrate the country-specific parameters β, ω, η, and θ to simultaneously match the average statistics of the labor income tax rate (τ), the ratios of general government final consumption expenditure (G/Y), public investment ($N'X/Y$), and government deficit (B'/Y) for each country during the period 1995–2016. Table 1 reports the average statistics and the estimated parameter values for each sample country. Appendix A.2 provides details on calibration.[5]

[5]Strictly speaking, B'/Y represents the ratio of government debt to GDP within the current framework. However, given the focus on examining the Golden Rule within the context of the real-world economy, we opt

Country	POP	TAX	GOV	INV	DEF
Germany	1.0004	0.3925	0.1919	0.0223	0.0210
Japan	1.0006	0.2236	0.1786	0.0456	0.0588
United Kingdom	1.0059	0.2270	0.1892	0.0230	0.0384
Country	n	β	ω	η	θ
Germany	0.0117	$0.0976(\approx (0.9808)^{120})$	0.5242	0.7426	0.2489
Japan	0.0178	$0.2409(\approx (0.9882)^{120})$	0.4361	0.5480	0.2433
United Kingdom	0.1921	$0.1463(\approx (0.9841)^{120})$	0.4518	0.4202	0.2488

Table 1: Data and calibrated parameters for Germany, Japan, and the United Kingdom. The first five columns are the annual gross population growth rate (POP), the labor income tax rate (TAX), the ratio of general government consumption expenditure to GDP (GOV), the ratio of public investment to GDP (INV), and the ratio of deficit to GDP (DEF), during 1995–2016. The last five columns are the calibrated parameter values of n, β, ω, η, and θ.
Note: In the table, the quarterly values of β are presented for reference, based on the values of β obtained from the calibration. For example, the quarterly β for Germany is about 0.9808.
Sources: World Development Indicators (`https://databank.worldbank.org/source/world-development-indicators`, accessed on September 9, 2022) for data on POP and GOV; the data archive of Professor McDaniel (`https://www.caramcdaniel.com/research`, accessed on April 6, 2021) for data on TAX; OECD gross domestic product (GDP) indicator (`doi:10.1787/dc2f7aec-en`, accessed on April 12, 2022), OECD investment ($GFCF$) indicator (`doi:10.1787/b6793677-en`, accessed on April 12, 2022), and OECD investment by sector indicator (`doi:10.1787/abd72f11-en`, accessed on April 12, 2022) for data on INV; OECD library, General government deficit indicator (`https://doi.org/10.1787/cc9669ed-en`, accessed on February 25, 2021) for data on DEF.

Figure 1 illustrates the numerical result of the effects of an increased ϕ on the labor income tax rate, T (Panel (a)), and the ratio of public investment to GDP, X (Panel (b)), for the three countries. On the basis of the result in Figure 1, we can interpret Eqs. (34) and (35) as follows. First, Eq. (34) shows that the fiscal rule, represented by ϕ, affects the formation of the labor income tax rate. The greater ϕ is, the greater the share of public investment financed by public debt issuance. Debt issuance lowers physical capital accumulation through crowding-out effects, reduces the next-period public goods provision, and generates the following two opposing effects on older adult consumption: a positive effect via an increase in the interest rate, and a negative effect via an increase in the next-period capital income tax rate. These effects are observed in the second term of the left-hand side in (34). In the present framework, the latter effect dominates the former one; thus, an increase in ϕ leads to an increase in the marginal cost of τ. To offset this increase in the marginal cost, the government has the incentive to lower the labor income tax rate as ϕ increases. Therefore, an increase in ϕ leads to a decrease in the labor income tax rate, as we can see from Panel (a) of Figure 1.

Second, Eq. (35) shows that the fiscal rule, represented by ϕ, has two conflicting effects on the choice of public investment, x. The first is a negative effect on x observed in the first term on the left-hand side; the effect is qualitatively similar to the negative effect on τ, as mentioned

to utilize the ratio of government deficit, as opposed to government debt, in relation to GDP as the target for calibration.

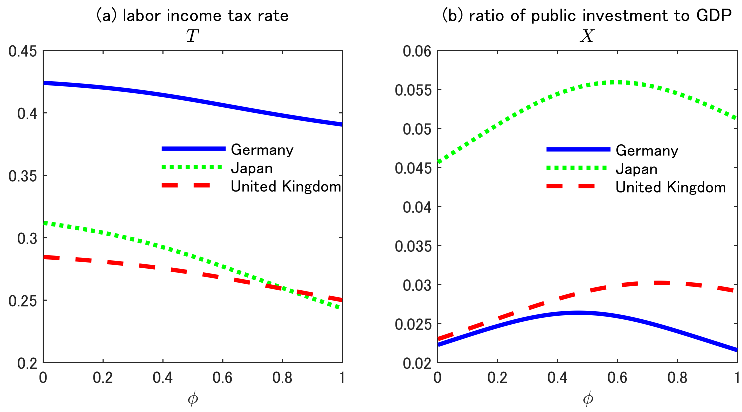

Figure 1: Changes in the labor income tax rate, T and the ratio of public investment to GDP, X against changes in ϕ.

previously. The second is a positive effect on x, as observed in the third term on the left-hand side. This effect arises because the tax burden associated with public investment, x, decreases as the debt-financed proportion of public investment, ϕ, increases. Given these conflicting effects, there is a threshold value of ϕ at which the two effects are balanced, and an increase in ϕ has an inverse U-shaped effect on the ratio of public investment to GDP around the threshold value, as illustrated in Panel (b) of Figure 1.

3.3 Preferences of Voters for Fiscal Rules

Thus far, we have considered fiscal rules as institutionally given and investigated the impact of the rules on fiscal policy decisions. However, fiscal rules themselves are generally introduced after deliberation and voting in parliaments. Thus, it is natural to assume that they are also determined through the political process of voting. In this subsection, we introduce such a process into the model and show how the process and the resulting fiscal rules are affected by the structural parameters of the model economy.

The timing of the fiscal rule and fiscal policy decisions is as follows. (i) First, the office-seeking two candidates, say O^A and O^B, simultaneously and non-cooperatively, announce their fiscal rules, denoted by ϕ^A and ϕ^B, respectively. (ii) Given ϕ^A and ϕ^B, the two candidates, simultaneously and non-cooperatively, announce their policy platforms, $\left(\tau^A, \tau^{kA}, x^A, g^A\right)$ and $\left(\tau^B, \tau^{kB}, x^B, g^B\right)$, respectively, as described at the beginning of this section. The elected candidate implements their announced fiscal rule and policy platform. This two-stage game is solved by backward induction. Thus, the fiscal rule determined through voting, called a *politically preferred fiscal rule*, is defined as ϕ, which maximizes the political objective function subject to the policy functions presented in Proposition 1.

In writing down the political objective function at the first stage, recall the indirect utility functions of the middle-aged and older adults derived in Section 2. Using the policy function obtained in Proposition 1 and the equilibrium condition of the capital market in (16), the

Country	$\phi_{political}$
Germany	0.944
Japan	1.289
United Kingdom	1.669

Table 2: The politically preferred fiscal rule, $\phi_{political}$.

political objective function in (20) at the first stage is written as follows

$$\Omega = \frac{\omega}{(1+n)(1-\omega)}\left[\ln d\left(\tau^k; k, k^p, n\right) + \theta \ln g\right] \\ + \ln c\left(\tau; k, k^p\right) + \theta \ln g + \beta\left[\ln d'\left(\tau, x, b'; k, k^p\right) + \theta \ln g'\left(\tau, x, b'; k, k^p\right)\right], \tag{36}$$

where the derivation of (36) is given in Appendix A.3. Thus, the problem of the government at the first stage is to choose ϕ to maximize Ω in (36) subject to the fiscal rule in (13), the government budget constraint in (17), and the policy functions derived in Proposition 1. The solution to the problem, denoted by $\phi_{political}$, is provided in the following proposition.

Proposition 2 *A politically preferred fiscal rule, $\phi_{political}$, along the Markov-perfect political equilibrium characterized in Proposition 1 is*

$$\phi_{political} = \frac{1-\alpha(1+\theta)}{(1+\theta)\,\eta\,(1-\alpha)}. \tag{37}$$

Proof. See Appendix A.3.

The result demonstrated in Proposition 2 suggests that θ, representing the preferences for public goods, and η, representing the elasticity of public capital with respect to public investment, are crucial to the determination of the politically preferred fiscal rule. A greater θ implies that voters attach a larger weight to public goods expenditure, and thus provide a stronger incentive for the government to cut public debt issuance. A greater η implies a greater marginal benefit of public investment, thus, incentivizing the government to lower the debt-financed proportion of public investment, ϕ, from the viewpoint of balancing the marginal costs and benefits of public investment, as we can see in Eq. (35). Thus, the politically preferred fiscal rule, $\phi_{political}$, decreases as θ and η increase.

Based on the result in Proposition 2 with the calibration reported in Table 1, we compute the politically preferred fiscal rule for the three countries, as reported in Table 2. Germany obtains the lowest politically preferred ϕ because it attains the highest θ and η among the three countries. As for the politically preferred ϕ of Japan and the United Kingdom, the relative degree of the effects of θ and η is important because θ is higher but η is lower in the United Kingdom than in Japan. Table 2 reports that the politically preferred ϕ of Japan is lower than that of the United Kingdom, suggesting that the effect of η overcomes the effect of θ in determining the politically preferred ϕ of these two countries.

The result in Table 2 also reports that among the three countries, the politically preferred ϕ is less than one in Germany, whereas it is greater than one in Japan and the United Kingdom. This suggests that the latter two countries broke GR during the period covered. This model prediction is consistent with the evidence. Wyplosz (2013) reports that in the United Kingdom, GR, which was adopted in 1997, was met only for the first few years. In Japan, Article 4 of Public Finance Law 1947 stipulates: "Expenditures of the State shall be financed by revenue other than public bonds or borrowings."[6] However, the following proviso states: "However, public bonds may be issued, or borrowings may be made to finance public works expenditure, capital contributions, and loans, within the amount approved by the Diet." Based on this proviso, a waiver of this rule has been approved by the Diet every year since 1975, except for the period 1990-1993 (Kumar et al., 2009).

4 Political Distortions

In Subsection 3.3, we have demonstrated the politically preferred fiscal rule, $\phi_{political}$, set through voting. We have also shown that the rule is in accordance with GR in Germany, but not in Japan or the United Kingdom. This contrasting result is due to the differences in structural parameters across countries. Furthermore, even in Germany, where GR is abided by, the debt-financed proportion of public investment, reported in Table 2, is high and close to one. These results may be because the politically preferred fiscal rule is a choice that considers only the generations existing at the time of voting and does not take into account future generations.

To evaluate the politically preferred fiscal rule from an intergenerational perspective, we begin by assuming a Ramsey planner who sets the fiscal policy sequence in the initial period so as to maximize the discounted sum of utilities of current and future generations subject to the same set of constraints faced by the elected governments. Assuming such a planner, we characterize the allocation chosen by the Ramsey planner and compare it to the political equilibrium allocation, particularly focusing on the debt-financed proportion of public investment. This comparison should provide us with some insight into the setting of fiscal rules that take future generations into account.

The Ramsey planner is assumed to value the welfare of all generations. In particular, its objective is to maximize a discounted sum of the lifecycle utility of all current and future generations, $SW = \gamma^{-1}V_0^O + \sum_{t=0}^{\infty}\gamma^t V_t^M$, $0 < \gamma < 1$, subject to the public capital function in (10), the government budget constraint in (12), and the capital market clearing condition in (16), where k_0, k_0^p, and b_0 are given. The individuals' optimal saving and consumption in (2), (3), and (4) and firms' optimal demand for labor and capital in (7) and (8) are included in V_0^O and V_t^M. The solution to this problem is called a "Ramsey allocation." The parameter $\gamma \in (0,1)$ in the objective function is the discount factor of the Ramsey planner.

Solving the problem leads to the following characterization of the policy functions in the Ramsey allocation.

[6]Source: Ministry of Internal Affairs and Communications, e-Gov Japan (Hourei Kensaku, in Japanese): https://elaws.e-gov.go.jp/document?lawid=322AC0000000034 (accessed on October 12, 2022).

Proposition 3 *Given k_0, k_0^p, and b_0, the policy functions in the Ramsey allocation are given by the following:*

$$1-\tau_t^k = \frac{\bar{T}^k}{\alpha}\frac{1}{1+k_t/b_t}, \tag{38}$$

$$\frac{2+n}{1+n}g_t = \bar{G}\cdot y\left(k_t, k_t^p\right), \tag{39}$$

$$(1+n)x_t = \bar{X}\cdot y\left(k_t, k_t^p\right), \tag{40}$$

$$\tau_t = \bar{T}, \tag{41}$$

$$(1+n)\, b_{t+1} = \bar{B}\cdot y\left(k_t, k_t^p\right), \tag{42}$$

where $\bar{T}^k$, $\bar{G}$, $\bar{X}$, $\bar{T}$, and $\bar{B}$, defined in Appendix A.4, are constant and dependent on γ.

Proof. See Appendix A.4.

The results of Proposition 3 highlight two essential points. Firstly, the parameters $\bar{T}^k$ $\bar{G}$, $\bar{X}$, $\bar{T}$, and $\bar{B}$ in the policy functions of Proposition 3 are contingent on the discount factor γ of the Ramsey planner. In simpler terms, the policy maximizing social welfare depends on the importance the Ramsey planner places on future generations relative to the current generation. Secondly, although the policy functions in Proposition 3 may initially seem akin to those in the political equilibrium from Proposition 1, the parameters in the policy functions chosen by the Ramsey planner differ from those in the political equilibrium.

To delve into these points further, we present the parameters of the policy function in Ramsey allocations for Germany, Japan, and the United Kingdom, based on the calibrations in Subsection 3.2. Figure 2 illustrates $\bar{T}$, $\bar{T}^k$, $\bar{B}$, $\bar{X}$, and $\bar{G}$, respectively, against the Ramsey planner's discount factor γ on the horizontal axis for the three countries. The figure reveals the following trends. First, as γ increases, the ratio of public investment to GDP ($\bar{X}$) rises, while the policy parameter $\bar{T}^k$ and the ratio of public goods to GDP ($\bar{G}$) decrease. Second, the labor income tax rate ($\bar{T}$) displays a U-shaped pattern, decreasing initially and then rising with an increase in γ. Conversely, the ratio of public debt to GDP ($\bar{B}$) follows an inverse U-shaped pattern, initially rising and then falling as γ increases. When γ reaches a certain level, the ratio of public debt to GDP ($\bar{B}$) becomes zero due to zero issuance of public debt. Subsequently, we discuss the mechanisms behind these findings.

Beginning with the impact of the discount factor γ on $\bar{X}$ and $\bar{G}$, an increase in public investment incurs three costs or benefits. First, the current supply of public goods decreases because of limited government budgets, causing a utility loss for the current older adults. However, this loss is less significant for the Ramsey planner as γ increases, encouraging public investment. Second, increased public investment raises wages and interest rates for both current and future generations, promoting overall welfare. Third, the wage increase effect leads to higher household savings and increased capital in successive periods. This, in turn, boosts production and the supply of public goods in future periods, benefiting all future generations. Consequently, an increase in γ promotes public investment and thus the ratio of public investment to GDP ($\bar{X}$),

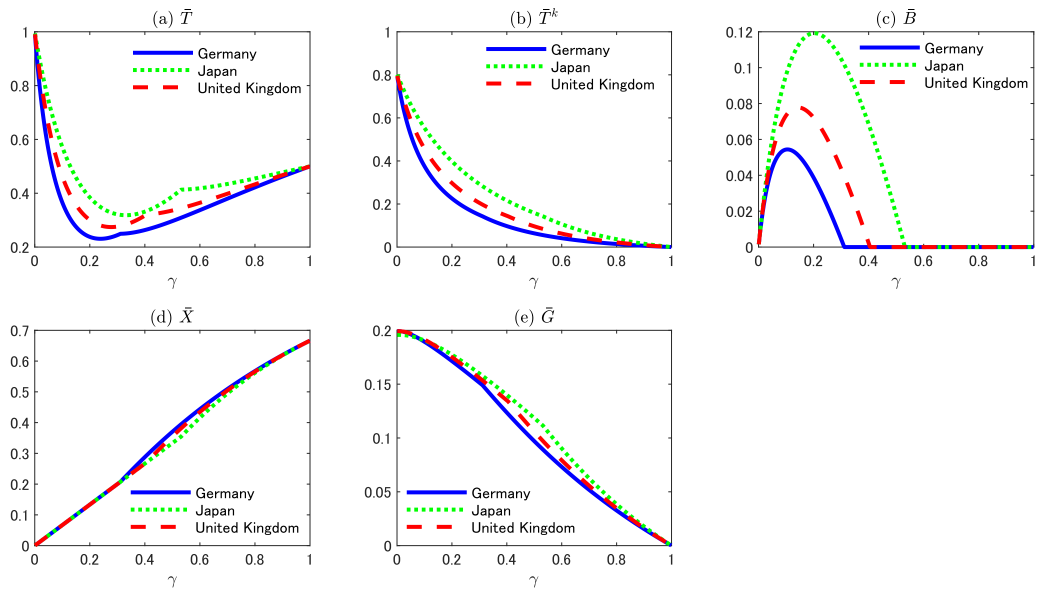

Figure 2: Changes in $\bar{T}$, $\bar{T}^k$, $\bar{B}$, $\bar{X}$, and $\bar{G}$ against changes in γ.

while it reduces public goods provision and thus the ratio of public goods to GDP ($\bar{G}$) due to limited budget constraint.

Next, when considering the impact of an increase in γ on $\bar{T}^k$, it is important to acknowledge that private savings remain unaffected by the capital tax rate (τ_t^k) due to the logarithmic utility function. This independence implies that τ_t^k does not generate spillover effects on future generations through physical and public capital accumulation. Consequently, the effects of τ_t^k are static in the following manner. As τ_t^k rises, the consumption of the current older adults decreases. However, the Ramsey planner values this utility loss less as γ increases. Simultaneously, the increased tax revenues from a higher capital tax rate enhance the supply of public goods for the current generation. This benefit is relatively less valued with a larger γ. Consequently, a larger γ leads to a higher τ_t^k and thus a lower $\bar{T}^k$.

Finally, upon assessing the impact of γ on variables $\bar{B}$ and $\bar{T}$, an increase in debt issuance entails distinct costs and benefits. Regarding the cost of debt issuance, an increased debt issuance yields two adverse effects on future generations. Firstly, it triggers a negative spillover effect on physical capital through crowding out, and secondly, it constricts the government's budget due to increased debt repayment costs. These effects gain greater significance with a higher γ, signifying a stronger consideration by the Ramsey planner toward future generations. Conversely, the benefit of debt issuance lies in the government's ability to alleviate the tax burden on the middle-aged by funding public expenditures through debt issuance. However, this positive effect diminishes in value with a larger γ. Furthermore, the reduced tax burden on the middle-aged mitigates the negative spillover effect on physical capital via increased savings, a consequence that gains prominence as γ increases. Initially low γ results in an augmented debt-GDP ratio ($\bar{B}$), while higher initial γ levels lead to a decrease in $\bar{B}$. Beyond a certain threshold of γ, $\bar{B}$ becomes zero due to a cessation of debt issuance. Conversely, the impact on the labor income tax rate ($\bar{T}$) contrasts that on $\bar{B}$. This discrepancy arises from the increase (or decrease) in debt issuance revenue being offset by a decrease (or increase) in labor income tax revenue.

Figure 3 depicts the optimal ratio of public debt to public investment ($\bar{B}/\bar{X}$), for the Ramsey planner, with the discount factor, γ, on the horizontal axis. As previously noted, an increase in the discount factor of the Ramsey planner, γ, initially leads to a rise and subsequent fall in the ratio of public debt to GDP ($\bar{B}$), while the ratio of public investment to GDP ($\bar{X}$) consistently increases. Due to the strong monotonic increase in $\bar{X}$, the $\bar{B}/\bar{X}$ ratio decreases monotonically with increasing γ. Beyond a certain γ level, the $\bar{B}/\bar{X}$ ratio reaches zero, indicating that the Ramsey planner ceases to issue public debt.

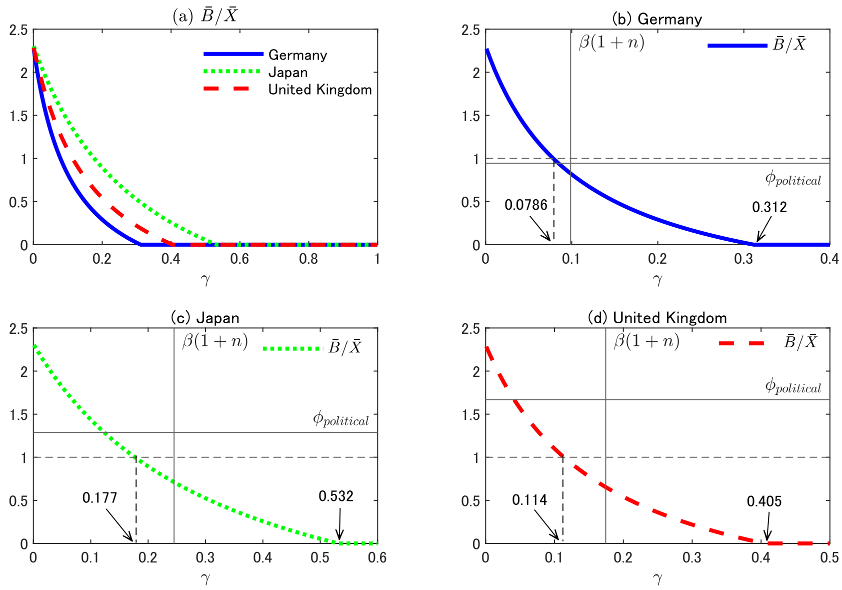

Figure 3: Changes in $\bar{B}/\bar{X}$ against changes in γ.

To assess the ratio of public debt to public investment in the political equilibrium from the Ramsey planner's perspective, we focus on the case of natural weight, where γ equals $\beta(1+n)$. The natural weights for Germany, Japan, and the United Kingdom are 0.099, 0.245, and 0.174, respectively. As illustrated in Panels (b) – (d) of Figure 3, the ratio of public debt to public investment in the Ramsey allocation under natural weights is lower than that in political equilibrium for each country. In essence, the ratio of public debt to public investment in the political equilibrium proves excessive as long as the Ramsey planners adhere to the natural weight.

The relationship between the discount factor (γ) and the ratio of public debt to public investment in the Ramsey allocation ($\bar{B}/\bar{X}$), as depicted in Figure 3, holds implications for fiscal policy. For small γs ($\gamma < 0.0786$, 0.177, and 0.114 in Germany, Japan, and the United

Kingdom, respectively), the Ramsey planner, who assigns low value to future generations, find $\bar{B}/\bar{X} > 1$ desirable. This implies a willingness to finance unproductive government spending, i.e., the provision of public goods, through the issuance of public debt. However, within a reasonable γ range ((0.0786, 0.3127), (0.177, 0.572), and (0.114, 0.405), for Germany, Japan, and the United Kingdom, respectively), the Ramsey planner prefers the $\bar{B}/\bar{X}$ ratio to fall between $(0, 1)$. This signifies the desirability of the golden rule from the standpoint of social welfare maximization. Lastly, if γs are sufficiently high ($\gamma > 0.312$, 0.532, and 0.405 in Germany, Japan, and the United Kingdom, respectively), the Ramsey planner opts for $\bar{B}/\bar{X} = 0$, indicating the desirability of the balanced budget rule for maximizing social welfare. These implications underscore the dependence of the desirability of a fiscal rule on societal emphasis on future generations.

5 Conclusion

In this study, we consider the golden rule of public finance (GR), which states that budget deficit is allowed only to finance public investment but not current expenditure. We investigate what proportion of public investment should be financed by public debt issuance from a generational perspective and what proportion is politically preferred by successive short-sighted governments. To address these questions, we develop a politico-economic overlapping-generations model where fiscal policy is determined for each period via probabilistic voting and calibrate the model economy to Germany, Japan, and the United Kingdom, where GR has been in place. We show that in politics, Germany sets the proportion that follows the GR while Japan and the United Kingdom set the proportions that break the GR, which are consistent with the literature. We subsequently compute the optimal proportion taking into account the spillover effects of public investment on future generations and show that it is lower than the politically preferred one in each country.

The novelty of the present study is twofold. First, we quantitatively show whether each country follows the GR depends on structural parameters such as preferences for public goods and the elasticity of public capital with respect to public investment. Differences in structural parameters successfully explain differences in the responses of the countries to fiscal rule. Second, we point out that in the context of maximizing social welfare, the selected fiscal rule in the political equilibrium disregards the spillover effects on future generations via physical and public capital accumulation. This leads to an excessive allowance of public debt issuance within the chosen fiscal rule in the political equilibrium. Our results with these two characteristics are expected to provide important implications for policy makers in designing fiscal rules from an intergenerational perspective.

Our model can be expanded in several directions. For instance, it would be straightforward to incorporate other fiscal rules such as revenue and expenditure rules, both of which have been widely introduced in developed countries. This would enable us to compare and evaluate several types of fiscal rules in terms of minimizing political distortions. Additionally, our model can be used to explore a wide variety of policy questions: to what extent do changes in fiscal rules affect the fiscal burden on each generation? What fiscal rules are optimal from the perspective of maximizing economic growth? Which fiscal rule is most efficient from the perspective of minimizing political distortions? Addressing these questions would provide policymakers with richer information.

A Proofs and Supplementary Explanations

A.1 Proof of Proposition 1

Given the conjecture in (30) and (31), we can reformulate the first-order condition with respect to x in (26) as follows:

$$\lambda = \frac{\beta(1+\theta)}{(1-\phi)\, y(k,k^p)} \left[(-1) \frac{\alpha\phi(1+\theta,)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X} + \frac{\eta(1-\alpha)}{X} \right]. \tag{A.1}$$

We can also reformulate the first-order condition with respect to τ in (24) as follows:

$$\lambda = \frac{1}{(1-\alpha)\, y(k,k^p)} \left[\frac{1}{1-T} + \frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X} \right]. \tag{A.2}$$

With (A.1) and (A.2), we obtain

$$\frac{\beta(1+\theta)\eta(1-\alpha)}{(1-\phi)} \cdot \frac{1}{X} = \frac{1}{(1-\alpha)(1-T)} + \frac{\alpha\beta(1+\theta)\left[\frac{\beta}{1+\beta}+\frac{\phi}{1-\phi}\right]}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}, \tag{A.3}$$

where (A.3) includes two undetermined constants, T and X. Denote the left-hand (right-hand) side of (A.3) by $LHS^{(A.3)}$ ($RHS^{(A.3)}$). They have the following properties: $\partial LHS^{(A.3)}/\partial X < 0$, $\lim_{X\to 0} LHS^{(A.3)} = +\infty$, $\lim_{X\to+\infty} LHS^{(A.3)} = 0$, $\partial RHS^{(A.3)}/\partial X > 0$, $\lim_{X\to 0} RHS^{(A.3)} \in (0,+\infty)$, and $\lim_{X\to\frac{\beta}{1+\beta}(1-\alpha)(1-T)/\phi} RHS^{(A.3)} = +\infty$. These properties imply that given T, there is a unique X that satisfies (A.3): $X = X(T)$.

Given T and X, the first-order condition with respect to τ^k in (23) is reformulated as

$$1-\tau^k = \frac{\omega}{(1+n)(1-\omega)}(1-\alpha) \left[\frac{1}{1-T} + \frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X} \right]^{-1} \frac{1}{\alpha\frac{k+b}{k}}. \tag{A.4}$$

Eq. (A.4) shows that the conjecture of τ^k in (21) is correct as long as T and X are constant.

Next, given T and X, the first-order condition with respect to g in (25) is reformulated as

$$\frac{2+n}{1+n} g = \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \theta \left[\frac{1}{1-T} + \frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X} \right]^{-1} (1-\alpha)\, y(k,k^p). \tag{A.5}$$

Eq. (A.5) shows that the conjecture of g in (22) is correct as long as T and X are constant.

The remaining task is to show that the conjectures of T in (30) and X in (31) are correct. Consider the government budget constraint in (17). We substitute the policy functions derived thus far into the constraint and rearrange the terms to obtain

$$\begin{aligned} &\alpha - \frac{\omega}{(1+n)(1-\omega)} \left[\frac{1}{1-T} + \frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X} \right]^{-1} (1-\alpha) + T(1-\alpha) \\ &= \left(\frac{\omega}{(1+n)(1-\omega)} + 1 \right) \theta \left[\frac{1}{1-T} + \frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X} \right]^{-1} (1-\alpha) + (1-\phi)X. \end{aligned} \tag{A.6}$$

The expression in (A.6) is independent of the state variables, k, k^p, and b, and times. Thus, we can verify that the two unknown parameters, X and T, are constant and solved for using (A.3) and (A.6). They are given by

$$X \equiv \frac{I(\phi)}{(1-\phi)\phi}\left\{1+\left[\frac{\omega}{(1+n)(1-\omega)}(1+\theta)+\theta\right]\frac{\frac{\beta}{1+\beta}-\frac{I(\phi)}{1-\phi}}{\frac{\beta}{1+\beta}\left[1+\alpha\beta(1+\theta)\right]-\frac{I(\phi)}{1-\phi}}+\frac{I(\phi)}{\phi}\right\}^{-1},$$

$$T \equiv 1-\frac{1}{1-\alpha}\left\{1+\left[\frac{\omega}{(1+n)(1-\omega)}(1+\theta)+\theta\right]\frac{\frac{\beta}{1+\beta}-\frac{I(\phi)}{1-\phi}}{\frac{\beta}{1+\beta}\left[1+\alpha\beta(1+\theta)\right]-\frac{I(\phi)}{1-\phi}}+\frac{I(\phi)}{\phi}\right\}^{-1},$$

where $I(\phi)$ appeared in the expressions of X and T is

$$I(\phi) \equiv \frac{H_1(\phi)-\sqrt{(H_1(\phi))^2-4H_2(\phi)}}{2},$$

and $H_1(\phi)$ and $H_2(\phi)$ are

$$H_1(\phi) \equiv \beta(1+\theta)(\alpha+\eta(1-\alpha))\phi+\frac{\beta}{1+\beta}\left[1+\alpha\beta(1+\theta)\right](1-\phi),$$
$$H_2(\phi) \equiv (1-\phi)\phi\beta(1+\theta)\eta(1-\alpha)\frac{\beta}{1+\beta}.$$

Substitution of X and T into (A.4) and (A.5) lead to the policy functions of $1-\tau^k$ and $\frac{2+n}{1+n}g$ as presented in Proposition 1, where T^k and G are given by

$$T^k \equiv \frac{\omega}{(1+n)(1-\omega)}(1-\alpha)\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1},$$
$$G \equiv \left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\left[\frac{1}{1-T}+\frac{\alpha\beta(1+\theta)\frac{\beta}{1+\beta}(1-\alpha)}{\frac{\beta}{1+\beta}(1-\alpha)(1-T)-\phi X}\right]^{-1}(1-\alpha).$$

■

A.2 Calibration

A.2.1 Calibration on n, β, ω, η, and θ

Country specific parameters, n, β, ω, η, and θ, are calibrated in the following way. The population growth rate, n, is obtained from the average of each sample country during the 1995—2016 period. Let POP_j denote the annual gross population growth rate of country j. The net population growth rate for 30 years is $(POP_j)^{30}-1$.

To determine the remaining four parameters, β, ω, η, and θ, we focus on the labor income tax rate, τ, the ratio of the public goods provision to GDP, G/Y, the ratio of public investment

Country	estimation	data
Germany	0.4981	0.1826
Japan	0.6056	0.2235
United Kingdom	0.6118	0.2771

Table 3: Estimated and actual capital income tax rates of the sample three countries.
Source: Actual capital income tax rates are obtained from Professor McDaniel's data archive: `https://www.caramcdaniel.com/` (accessed on February 17, 2022).

to GDP, $N'x/Y$, and the ratio of public debt to GDP, B'/Y. These four variables are given by

$$\tau_j = T_j \equiv TAX_j, \tag{A.7}$$

$$\left.\frac{G}{Y}\right|_j = G_j \equiv GOV_j, \tag{A.8}$$

$$\left.\frac{N'x}{Y}\right|_j = X_j \equiv INV_j, \tag{A.9}$$

$$\left.\frac{B'}{Y}\right|_j = \phi X_j \equiv DFF_j, \tag{A.10}$$

where the subscript j is the country code. We use the data of TAX_j, GOV_j, INV_j, and DFF_j for each sample country during 1995–2016 and solve the four equations in (A.7) – (A.10) for β_j, ω_j, η_j, and θ_j. The result is presented in Table 1.

The capital income tax rate is not a target for the calibration. Using the values of the parameters estimated based on the calibration, we need to compute the capital income tax rate and check whether it is consistent with the data of the three sample countries. Table 3 reports the capital income tax rates we compute on the basis of the calibration and those obtained from Professor McDaniel's data archive.[7] The result in Table 3 shows that the computed capital income tax rates are higher than the actual rates for the three countries. This suggests that there is room for improvement in the calibration, but the computed capital income tax rates fall within the range $(0, 1)$. We, therefore, use the calibration result in Table 1 to carry out the analysis in the main text.

A.2.2 Data on Public Investment

To estimate the ratio of public investment to GDP, we use the following three sorts of the data: the gross domestic product (GDP), gross fixed capital formation ($GFCG$), and the ratio of the investment by sector, general government ($ISGG$). Using this data, we first compute the ratio of public investment to GDP of country i in year t, denoted by $INV_{i,t}$, as follows:

$$INV_{i,t} = \frac{(ISGG_{i,t}/100) \cdot GFCF_{i,t}}{GDP_{i,t}}.$$

[7] https://www.caramcdaniel.com/ (accessed on February 17, 2022).

Then, INV_i, representing the average ratio of public investment to GDP of country i during 1995–2016, is computed based on the following equation:

$$INV_i = \frac{1}{22}\sum_{t=1995}^{2016} INV_{i,t}.$$

■

A.3 Supplement to Subsection 3.3

A.3.1 Derivation of Ω in (36)

Recall the indirect utility function of the middle in (18), which is reformulated using the recursive notation as follows:

$$V^M\left(\tau, x, g, \tau^{k\prime}, g'; k, k^p, k', k^{p\prime}, b'\right) = \ln c\left(\tau; k, k^p\right) + \theta \ln g + \beta\left[\ln d'\left(\tau^{k\prime}; k', k^{p\prime}, b'\right) + \theta \ln g'\right]. \tag{A.11}$$

The term d' in (A.11) is reformulated as follows:

$$\begin{aligned} d'\left(\cdot\right) &= \left(1-\tau^{k\prime}\right) R\left(k', k^{p\prime}\right)(1+n)\left(k'+b'\right) \\ &= T^k \frac{k'}{\alpha\left(k'+b'\right)} \alpha A\left(k'\right)^{\alpha-1}\left(k^{p\prime}\right)^{1-\alpha}(1+n)\left(k'+b'\right) \\ &= (1+n)T^k \cdot y\left(k', k^{p\prime}\right), \end{aligned} \tag{A.12}$$

where the equality in the first line comes from the budget constraint for older adults, $d' = \left(1-\tau^{k\prime}\right) R's$ and the capital market clearing condition, $(1+n)(k'+b') = s$; the equality in the second line comes from the conjecture of $\tau^{k\prime}$ in (21) and the first-order condition with respect to capital in (6), and the equality in the third line comes from the production function, $y(k,k^p) = A\left(k\right)^{\alpha}\left(k^p\right)^{1-\alpha}$. The term $g'(\cdot)$ is reformulated, using the conjecture of g' in (22), as

$$g'(\cdot) = G \cdot y\left(k', k^{p\prime}\right). \tag{A.13}$$

The term $y\left(k', k^{p\prime}\right)$, which appeared in (A.12) and (A.13), is further reformulated as follows:

$$\begin{aligned} y\left(k', k^{p\prime}\right) &= A\left[\frac{1}{1+n}\frac{\beta}{1+\beta}(1-\tau)\left(1-\alpha\right) y(k,k^p) - b'\right]^{\alpha}\left[D\left(k^p\right)^{1-\eta}\left(x\right)^{\eta}\right]^{1-\alpha} \\ &= y\left(\tau, x, b'; k, k^p\right) \end{aligned} \tag{A.14}$$

where we use the capital market clearing condition in (16) and the public capital formation function in (10) in deriving (A.14). Thus, the terms $d'\left(\cdot\right)$ in (A.12) and $g'(\cdot)$ in (A.13) are now given by

$$\begin{aligned} d' &= d'\left(\tau, x, b'; k, k^p\right) \equiv (1+n)T^k \cdot y\left(\tau, x, b'; k, k^p\right), \\ g' &= g'\left(\tau, x, b'; k, k^p\right) \equiv G \cdot y\left(\tau, x, b'; k, k^p\right), \end{aligned}$$

respectively, and the indirect utility function of the middle-aged in (A.11) is reformulated as follows:

$$V^M\left(\tau,x,g,b';k,k^p\right)=\ln c\left(\tau;k,k^p\right)+\theta\ln g \\ +\beta\left[\ln d'\left(\tau,x,g,b';k,k^p\right)+\theta\ln g'\left(\tau,x,g,b';k,k^p\right)\right]. \tag{A.15}$$

We substitute the indirect utility function of older adults in (19) and that of the middle-aged in (A.15) into the political objective function in (20) and then obtain (36).

A.3.2 Proof of Proposition 2

The problem of the government is to choose ϕ to maximize Ω in (36) subject to the government budget constraint in (17), the fiscal rule in (13), and the policy functions presented in Proposition 1. Substituting (17) and (13) into (36) and rearranging the terms, we have

$$\begin{aligned}\Omega\approx{}&\frac{\omega}{(1+n)(1-\omega)}\ln d\left(\tau^k;k,k^p,n\right)+\ln c\left(\tau;k,k^p\right)\\&+\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\ln\underbrace{\frac{1+n}{2+n}\left[TR^K\left(\tau^k;k,k^p\right)+TR\left(\tau;k,k^p\right)-(1-\phi)(1+n)x-R\left(k,k^p\right)b\right]}_{=g}\\&+\beta\left(1+\theta\right)\alpha\ln\underbrace{\left[\frac{\beta}{1+\beta}\left(1-\tau\right)\left(1-\alpha\right)y(k,k^p)-\phi(1+n)x\right]}_{=(1+n)k'}+\beta\left(1+\theta\right)\eta\left(1-\alpha\right)\ln x,\end{aligned} \tag{A.16}$$

where the politically unrelated terms are omitted from the expression in (A.16).

We write the policy functions of τ^k, τ, and x derived in Proposition 1 in the following implicit form:

$$\begin{aligned}\tau^k&=\tau^k\left(\phi;k,k^p\right),\\\tau&=\tau\left(\phi\right),\\x&=x\left(\phi;k,k^p\right).\end{aligned}$$

We substitute these policy functions into (A.16) and obtain, by using the envelope theorem, the first-order condition with respect to ϕ as follows:

$$\frac{\partial\Omega}{\partial\phi}=0\Leftrightarrow\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\frac{\frac{1+n}{2+n}}{g}=\frac{\beta\left(1+\theta\right)\alpha}{(1+n)k'}. \tag{A.17}$$

To solve (A.17) for ϕ, recall the policy functions of g presented in Proposition 1,

$$\frac{2+n}{1+n}g=\left(\frac{\omega}{(1+n)(1-\omega)}+1\right)\theta\left[\frac{1}{1-T}+\frac{\alpha\beta\left(1+\theta\right)\frac{\beta}{1+\beta}\left(1-\alpha\right)}{\frac{\beta}{1+\beta}\left(1-\alpha\right)\left(1-T\right)-\phi X}\right]^{-1}\left(1-\alpha\right)y(k,k^p), \tag{A.18}$$

and the capital market clearing condition in (16) with the fiscal rule in (13),

$$(1+n)k'=\frac{\beta}{1+\beta}\left(1-\tau\right)\left(1-\alpha\right)y(k,k^p)-\phi Xy(k,k^p). \tag{A.19}$$

We substitute (A.18) and (A.19) into (A.17) and rearrange the terms to obtain:

$$\frac{1}{1-T} = \frac{\alpha (1+\theta) \frac{\beta}{1+\beta} (1-\alpha)}{\frac{\beta}{1+\beta} (1-\alpha) (1-T) - \phi X},$$

or,

$$\frac{\beta}{1+\beta} (1-\alpha (1+\theta)) = \frac{I(\phi)}{1-\phi}, \tag{A.20}$$

where we use the definition of T and X in Proposition 1 in deriving (A.20).

Substituting the definition of $I(\phi)$ into (A.20), we obtain

$$\frac{\beta}{1+\beta} (1-\alpha (1+\theta)) = \frac{H_1(\phi) - \sqrt{(H_1(\phi))^2 - 4H_2(\phi)}}{2(1-\phi)},$$

or,

$$-H_2(\phi) = -\frac{\beta}{1+\beta} (1-\alpha (1+\theta)) (1-\phi) H_1(\phi) + \left[\frac{\beta}{1+\beta} (1-\alpha (1+\theta)) (1-\phi)\right]^2. \tag{A.21}$$

With the use of the definition of $H_1(\phi)$ and $H_2(\phi)$, (A.21) is further reformulated as in (37).

■

A.4 Proof of Proposition 3

Recall the indirect utility of the middle-aged population in period t in (18) and that of older adult population in period 0 in (19), both of which are restated as follows:

$$\begin{aligned} V_t^M &= \ln \frac{1}{1+\beta} (1-\tau_t) w(k_t, k_t^p) + \theta \ln g_t \\ &\quad + \beta \ln \left(1-\tau_{t+1}^k\right) R\left(k_{t+1}, k_{t+1}^p\right) \frac{\beta}{1+\beta} (1-\tau_t) w(k_t, k_t^p) + \beta\theta \ln g_{t+1}, && \text{(A.22)} \\ V_0^O &= \left(1-\tau_0^k\right) R(k_0, k_0^p) (1+n) (k_0+b_0) + \theta \ln g_0. && \text{(A.23)} \end{aligned}$$

With (A.22) and (A.23), we can reformulate $SW = \gamma^{-1} V_0^O + \sum_{t=0}^{\infty} \gamma^t V_t^M$ as follows:

$$\begin{aligned} SW \approx\ & \frac{1}{\gamma} \ln \left(1-\tau_0^k\right) + (1+\beta) \ln (1-\tau_0) + \alpha (1+\beta) \ln k_0 + (1-\alpha)(1+\beta) \ln k_0^p + \theta \left(\frac{1}{\gamma}+1\right) \ln g_0 \\ & + \sum_{t=1}^{\infty} \gamma^t \left[\frac{\beta}{\gamma} \ln \left(1-\tau_t^k\right) + (1+\beta) \ln (1-\tau_t) + \left(\frac{\beta(\alpha-1)}{\gamma} + \alpha(1+\beta)\right) \ln k_t \right. \\ & \left. + (1-\alpha) \left(\frac{\beta}{\gamma} + 1 + \beta\right) \ln k_t^p + \theta \left(\frac{\beta}{\gamma}+1\right) \ln g_t \right]. \end{aligned} \tag{A.24}$$

The government budget constraint in period t in (12) is rewritten as:

$$g_t = \frac{1+n}{2+n} \left[R(k_t, k_t^p)(k_t+b_t)\tau_t^k + w(k_t, k_t^p)\tau_t + (1+n) b_{t+1} - (1+n) x_t - R(k_t, k_t^p) b_t\right]. \tag{A.25}$$

Substitution of (A.25) into (A.24) leads to:

$$
\begin{aligned}
SW \approx\ & \frac{1}{\gamma} \ln \left(1-\tau_0^k\right) + (1+\beta) \ln (1-\tau_0) + \alpha (1+\beta) \ln k_0 + (1-\alpha)(1+\beta) \ln k_0^p \\
& + \theta \left(\frac{1}{\gamma}+1\right) \ln \left[R\left(k_0, k_0^p\right)(k_0+b_0)\tau_0^k + w\left(k_0,k_0^p\right)\tau_0 + (1+n) b_1 - (1+n) x_0 - R\left(k_0,k_0^p\right) b_0\right] \\
& + \sum_{t=1}^{\infty} \gamma^t \left\{ \frac{\beta}{\gamma} \ln \left(1-\tau_t^k\right) + (1+\beta)\ln(1-\tau_t) \right. \\
& + \left(\frac{\beta(\alpha-1)}{\gamma} + \alpha(1+\beta)\right) \ln k_t + (1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right) \ln k_t^p \\
& \left. + \theta\left(\frac{\beta}{\gamma}+1\right) \ln \left[R\left(k_t,k_t^p\right)(k_t+b_t)\tau_t^k + w\left(k_t,k_t^p\right)\tau_t + (1+n) b_{t+1} - (1+n) x_t - R\left(k_t,k_t^p\right) b_t\right] \right\}.
\end{aligned}
\tag{A.26}
$$

The problem of the Ramsey planner is to choose a sequence $\left\{\tau_t, \tau_t^k, b_{t+1}, x_t\right\}_{t=0}^{\infty}$ that maximizes SW in (A.26). The associated sequence $\{g_t\}_{t=0}^{\infty}$ is determined by substituting the solution of $\left\{\tau_t, \tau_t^k, b_{t+1}, x_t\right\}$ into the government budget constraint in (A.25). Hereafter, for simplicity of notation, $w\left(k_t, k_t^p\right)$ and $R\left(k_t, k_t^p\right)$ are denoted as w_t and R_t, respectively. The first order conditions with respect to τ_t, τ_t^k, b_{t+1}, and x_t are as follows:

$$\begin{aligned}\tau_t : 0 = &-\frac{1+\beta}{1-\tau_t} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{R_t\left(k_t+b_t\right)\tau_t^k + w_t\tau_t + \left(1+n\right)b_{t+1} - \left(1+n\right)x_t - R_t b_t}\\ &+\sum_{j=1}^{\infty}\gamma^j\left\{\left(\frac{\beta\left(\alpha-1\right)}{\gamma}+\alpha\left(1+\beta\right)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\right.\\ &\left.\times\frac{\frac{\partial R_{t+j}}{\partial k_{t+j}}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + R_{t+j}\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}}b_{t+j}}{R_{t+j}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + w_{t+j}\tau_{t+j} + \left(1+n\right)b_{t+j+1} - \left(1+n\right)x_{t+j} - R_{t+j}b_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial \tau_t},\end{aligned} \tag{A.27}$$

$$\tau_t^k : 0 = -\frac{\beta}{\gamma}\frac{1}{1-\tau_t^k} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{R_t\left(k_t+b_t\right)}{R_t\left(k_t+b_t\right)\tau_t^k + w_t\tau_t + \left(1+n\right)b_{t+1} - \left(1+n\right)x_t - R_t b_t}, \tag{A.28}$$

$$\begin{aligned}b_{t+1} : 0 = &\,\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{R_t\left(k_t+b_t\right)\tau_t^k + w_t\tau_t + \left(1+n\right)b_{t+1} - \left(1+n\right)x_t - R_t b_t}\\ &+\gamma\theta\left(\frac{\beta}{\gamma}+1\right)\frac{R_{t+1}\tau_{t+1}^k - R_{t+1}}{R_{t+1}\left(k_{t+1}+b_{t+1}\right)\tau_{t+1}^k + w_{t+1}\tau_{t+1} + \left(1+n\right)b_{t+2} - \left(1+n\right)x_{t+1} - R_{t+1}b_{t+1}}\\ &+\sum_{j=1}^{\infty}\gamma^j\left\{\left(\frac{\beta\left(\alpha-1\right)}{\gamma}+\alpha\left(1+\beta\right)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\right.\\ &\left.\times\frac{\frac{\partial R_{t+j}}{\partial k_{t+j}}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + R_{t+j}\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}}b_{t+j}}{R_{t+j}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + w_{t+j}\tau_{t+j} + \left(1+n\right)b_{t+j+1} - \left(1+n\right)x_{t+j} - R_{t+j}b_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial b_{t+1}},\end{aligned} \tag{A.29}$$

$$\begin{aligned}x_t : 0 = &-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{R_t\left(k_t+b_t\right)\tau_t^k + w_t\tau_t + \left(1+n\right)b_{t+1} - \left(1+n\right)x_t - R_t b_t}\\ &+\sum_{j=1}^{\infty}\gamma^j\left\{\left(1-\alpha\right)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k_{t+j}^p}+\theta\left(\frac{\beta}{\gamma}+1\right)\right.\\ &\left.\times\frac{\frac{\partial R_{t+j}}{\partial k_{t+j}^p}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}^p}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}^p}b_{t+j}}{R_{t+j}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + w_{t+j}\tau_{t+j} + \left(1+n\right)b_{t+j+1} - \left(1+n\right)x_{t+j} - R_{t+j}b_{t+j}}\right\}\frac{\partial k_{t+j}^p}{\partial x_t}\\ &+\sum_{j=2}^{\infty}\gamma^j\left\{\left(\frac{\beta\left(\alpha-1\right)}{\gamma}+\alpha\left(1+\beta\right)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\right.\\ &\left.\times\frac{\frac{\partial R_{t+j}}{\partial k_{t+j}}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + R_{t+j}\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}}b_{t+j}}{R_{t+j}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + w_{t+j}\tau_{t+j} + \left(1+n\right)b_{t+j+1} - \left(1+n\right)x_{t+j} - R_{t+j}b_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial x_t}.\end{aligned} \tag{A.30}$$

To simplify notation, we present the following definition:

$$I_{t+j} \equiv R_{t+j}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + w_{t+j}\tau_{t+j} + \left(1+n\right)b_{t+j+1} - \left(1+n\right)x_{t+j} - R_{t+j}b_{t+j}, \tag{A.31}$$

$$J_{t+j} \equiv \frac{\partial R_{t+j}}{\partial k_{t+j}^p}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}^p}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}^p}b_{t+j}, \tag{A.32}$$

$$L_{t+j} \equiv \frac{\partial R_{t+j}}{\partial k_{t+j}}\left(k_{t+j}+b_{t+j}\right)\tau_{t+j}^k + R_{t+j}\tau_{t+j}^k + \frac{\partial w_{t+j}}{\partial k_{t+j}}\tau_{t+j} - \frac{\partial R_{t+j}}{\partial k_{t+j}}b_{t+j}. \tag{A.33}$$

By employing (A.31) – (A.33), we can streamline the expressions of the first-order conditions in (A.34) – (A.37) as follows:

$$\begin{aligned}
\tau_t : 0 =& -\frac{1+\beta}{1-\tau_t} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t} \\
&+ \sum_{j=1}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial \tau_t}, \qquad \text{(A.34)} \\
\tau_t^k : 0 =& -\frac{\beta}{\gamma}\frac{1}{1-\tau_t^k} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{R_t(k_t+b_t)}{I_t}, \qquad \text{(A.35)} \\
b_{t+1} : 0 =& \ \theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_t} + \gamma\theta\left(\frac{\beta}{\gamma}+1\right)\frac{R_{t+1}\tau_{t+1}^k - R_{t+1}}{I_{t+1}} \\
&+ \sum_{j=1}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial b_{t+1}}, \qquad \text{(A.36)} \\
x_t : 0 =& -\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_t} \\
&+ \sum_{j=1}^{\infty}\gamma^j\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k_{t+j}^p} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}^p}{\partial x_t} \\
&+ \sum_{j=2}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}} + \theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial x_t}. \qquad \text{(A.37)}
\end{aligned}$$

The process for identifying policy functions that satisfy the conditions in (A.34), (A.35), (A.36) and (A.37) involves the following steps.

- Step 1. Simplify the expression of the first-order conditions by substituting the summation notation with alternative expressions.
- Step 2. Conjecture a set of policy functions that fulfill the first-order conditions, guided by Proposition 3, substitute the conjectured solution into first-order conditions, and rigorously verify its correctness.

A.4.1 Step 1

The derivatives of k_{t+j} with respect to τ_t, b_{t+1}, and x_t are, respectively,

$$\frac{\partial k_{t+j}}{\partial \tau_t} = \begin{cases} \frac{\partial k_{t+1}}{\partial \tau_t} & \text{for } j = 1, \\ \prod_{i=1}^{j-1} \frac{\partial k_{t+i+1}}{\partial k_{t+i}} \frac{\partial k_{t+1}}{\partial \tau_t} & \text{for } j \geq 2, \end{cases} \tag{A.38}$$

$$\frac{\partial k_{t+j}}{\partial b_{t+1}} = \begin{cases} \frac{\partial k_{t+1}}{\partial b_{t+1}} & \text{for } j = 1, \\ \prod_{i=1}^{j-1} \frac{\partial k_{t+i+1}}{\partial k_{t+i}} \frac{\partial k_{t+1}}{\partial b_{t+1}} & \text{for } j \geq 2, \end{cases} \tag{A.39}$$

$$\frac{\partial k_{t+j}}{\partial x_t} = \begin{cases} 0 & \text{for } j = 1, \\ \frac{\partial k_{t+2}}{\partial k^p_{t+1}} \frac{\partial k^p_{t+1}}{\partial x_t} & \text{for } j = 2, \\ \begin{aligned} &\frac{\partial k_{t+j}}{\partial k^p_{t+j+1}} \left(\prod_{l=1}^{j-2} \frac{\partial k^p_{t+l+1}}{\partial k^p_{t+l}} \right) \frac{\partial k^p_{t+1}}{\partial x_t} + \sum_{i=2}^{j-2} \left[\left(\prod_{l=j-i+1}^{j-1} \frac{\partial k_{t+l+1}}{\partial k_{t+l}} \right) \frac{\partial k_{t+j-i+1}}{\partial k_{t+j-i}} \right. \\ &\left. \times \left(\prod_{l=1}^{j-i-1} \frac{\partial k^p_{t+l+1}}{\partial k^p_{t+l}} \right) \frac{\partial k^p_{t+1}}{\partial x_t} \right] + \left(\prod_{l=2}^{j-1} \frac{\partial k_{t+l+1}}{\partial k_{t+l}} \right) \frac{\partial k_{t+2}}{\partial k^p_{t+1}} \frac{\partial k^p_{t+1}}{\partial x_t} \end{aligned} & \text{for } j \geq 3, \end{cases} \tag{A.40}$$

$$\frac{\partial k^p_{t+j}}{\partial x_t} = \begin{cases} \frac{\partial k^p_{t+1}}{\partial x_t} & \text{for } j = 1, \\ \prod_{i=1}^{j-1} \frac{\partial k^p_{t+i+1}}{\partial k^p_{t+i}} \frac{\partial k^p_{t+1}}{\partial x_t} & \text{for } j \geq 2. \end{cases} \tag{A.41}$$

Using (A.38), (A.39), and (A.40), we reformulate the first-order conditions in (A.34) – (A.37).

First, we substitute (A.38) into (A.34) and (A.39) into (A.36), and obtain

$$0 = -\frac{1+\beta}{1-\tau_t} + \theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{w_t}{I_t} + \frac{\partial k_{t+1}}{\partial \tau_t} \Lambda_t, \tag{A.42}$$

$$0 = \theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{1+n}{I_t} + \gamma\theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{R_{t+1}\tau^k_{t+1} - R_{t+1}}{I_{t+1}} + \frac{\partial k_{t+1}}{\partial b_{t+1}} \Lambda_t, \tag{A.43}$$

where Λ_t is defined as

$$\begin{aligned} \Lambda_t \equiv \gamma &\left\{ \left(\frac{\beta(\alpha - 1)}{\gamma} + \alpha(1+\beta) \right) \frac{1}{k_{t+1}} + \theta \left(\frac{\beta}{\gamma} + 1 \right) \times \frac{L_{t+1}}{I_{t+1}} \right\} \\ &+ \sum_{j=2}^{\infty} \gamma^j \left\{ \left(\frac{\beta(\alpha - 1)}{\gamma} + \alpha(1+\beta) \right) \frac{1}{k_{t+j}} + \theta \left(\frac{\beta}{\gamma} + 1 \right) \times \frac{L_{t+j}}{I_{t+j}} \right\} \prod_{i=1}^{j-1} \frac{\partial k_{t+i+1}}{\partial k_{t+i}}. \end{aligned} \tag{A.44}$$

Removing the term Λ_t from (A.42) and (A.43) by putting them together, and using $\partial k_{t+1}/\partial \tau_t = -\frac{1}{1+n} \frac{\beta}{1+\beta} (1-\alpha) y_t$ and $\partial k_{t+1}/\partial b_{t+1} = -1$, we obtain the following expression:

$$0 = \theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{1+n}{I_t} - \gamma\theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{R_{t+1}\left(1 - \tau^k_{t+1}\right)}{I_{t+1}} - \frac{-\frac{1+\beta}{1-\tau_t} + \theta \left(\frac{\beta}{\gamma} + 1 \right) \frac{w_t}{I_t}}{\frac{1}{1+n} \frac{\beta}{1+\beta} (1-\alpha) y_t}. \tag{A.45}$$

Second, recall the first-order condition with respect to x_t in (A.37). Taking one-period ahead and rearranging the terms, we obtain

$$\begin{aligned}\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_{t+1}} &= \sum_{j=2}^{\infty}\gamma^j\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k^p_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+j}}{I_{t+j}}\right\}\frac{\partial k^p_{t+j}}{\partial x_{t+i}}\\ &+\sum_{j=3}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial x_{t+i}}. \qquad (A.46)\end{aligned}$$

Using the definition of I_{t+j} in (A.31), J_{t+j} in (A.32), and L_{t+j} in (A.33), we can reformulate the first-order condition with respect to x_t in (A.37) as follows:

$$\begin{aligned}0 &= -\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_t}\\ &+\underbrace{\sum_{j=2}^{\infty}\gamma^j\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k^p_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+j}}{I_{t+j}}\right\}\frac{\partial k^p_{t+j}}{\partial x_t}}_{(\#1\_A.47)}\\ &+\gamma\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k^p_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+1}}{I_{t+1}}\right\}\frac{\partial k^p_{t+1}}{\partial x_t}\\ &+\underbrace{\sum_{j=3}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial x_t}}_{(\#2\_A.47)}\\ &+\gamma^2\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+2}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+2}}{I_{t+2}}\right\}\frac{\partial k_{t+2}}{\partial x_t}. \qquad (A.47)\end{aligned}$$

By using (A.41), we can reformulate the term $\partial k^p_{t+j}/\partial x_t$ (#1_A.47) in (A.47) as

$$\frac{\partial k^p_{t+j}}{\partial x_t}=\frac{\frac{\partial k^p_{t+2}}{\partial k^p_{t+1}}\frac{\partial k^p_{t+1}}{\partial x_t}}{\frac{\partial k^p_{t+2}}{\partial x_{t+1}}}\frac{\partial k^p_{t+j}}{\partial x_{t+1}}. \qquad (A.48)$$

By using (A.40), we can also reformulate the term $\partial k_{t+j}/\partial x_t$ (#2_A.47) in (A.47) as:

$$\frac{\partial k_{t+j}}{\partial x_t}=\left(\frac{\frac{\partial k_{t+j}}{\partial x_{t+1}}}{\frac{\partial k^p_{t+2}}{\partial x_{t+1}}}\frac{\partial k^p_{t+2}}{\partial k^p_{t+1}}+\prod_{i=2}^{j-1}\frac{\partial k_{t+i+1}}{\partial k_{t+i}}\frac{\partial k_{t+2}}{\partial k^p_{t+1}}\right)\frac{\partial k^p_{t+j}}{\partial x_t}. \qquad (A.49)$$

Plugging (A.46), (A.48), and (A.49) into (A.47) and rearranging the terms, we have

$$\begin{aligned}0 &= -\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_t}+\gamma\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k^p_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+1}}{I_{t+1}}\right\}\frac{\partial k^p_{t+1}}{\partial x_t}\\ &+\gamma^2\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+2}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+2}}{I_{t+2}}\right\}\frac{\partial k_{t+2}}{\partial x_t}\\ &+\frac{\frac{\partial k^p_{t+2}}{\partial k^p_{t+1}}\frac{\partial k^p_{t+1}}{\partial x_t}}{\frac{\partial k^p_{t+2}}{\partial x_{t+1}}}\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_{t+1}}\\ &+\frac{\partial k_{t+2}}{\partial k^p_{t+1}}\frac{\partial k^p_{t+1}}{\partial x_t}\cdot\underbrace{\sum_{j=3}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\prod_{i=2}^{j-1}\frac{\partial k_{t+i+1}}{\partial k_{t+i}}}_{(\#1\_A.50)}.\end{aligned}$$

(A.50)

Using the first-order condition with respect to τ_t in (A.34) and the derivative of k_{t+j} with respect to τ_t in (A.38), we can simplify the term (#1_A.50) in (A.50) as follows:

$$\sum_{j=3}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\prod_{i=2}^{j-1}\frac{\partial k_{t+i+1}}{\partial k_{t+i}}$$
$$=\frac{1}{\frac{\partial k_{t+2}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial \tau_t}}\left\{\frac{1+\beta}{1-\tau_t}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}-\sum_{j=1,2}\gamma^j\left[\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right]\frac{\partial k_{t+j}}{\partial \tau_t}\right\}. \tag{A.51}$$

Plugging (A.51) into (A.50), we obtain

$$\begin{aligned}0=&-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_t}+\gamma\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)\frac{1}{k^p_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{J_{t+1}}{I_{t+1}}\right\}\frac{\partial k^p_{t+1}}{\partial x_t}\\
&+\gamma^2\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+2}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+2}}{I_{t+2}}\right\}\frac{\partial k_{t+2}}{\partial x_t}\\
&+\frac{\frac{\partial k^p_{t+2}}{\partial k^p_{t+1}}\frac{\partial k^p_{t+1}}{\partial x_t}}{\frac{\partial k^p_{t+2}}{\partial x_{t+1}}}\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1+n}{I_{t+1}}+\frac{\frac{\partial k_{t+2}}{\partial k^p_{t+1}}\frac{\partial k^p_{t+1}}{\partial x_t}}{\frac{\partial k_{t+2}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial \tau_t}}\\
&\times\left\{\frac{1+\beta}{1-\tau_t}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}-\sum_{j=1,2}\gamma^j\left[\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right]\frac{\partial k_{t+j}}{\partial \tau_t}\right\}.\end{aligned} \tag{A.52}$$

Finally, recall again the first-order condition with respect to τ_t in (A.34). Taking one-period ahead and rearranging the terms, we have

$$\begin{aligned}&\sum_{j=2}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial \tau_{t+1}}\\
&=\frac{1+\beta}{1-\tau_{t+1}}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_{t+1}}{I_{t+1}}.\end{aligned} \tag{A.53}$$

In addition, using (A.38), we can reformulate the first-order condition with respect to τ_t in (A.34) as

$$\begin{aligned}0=&-\frac{1+\beta}{1-\tau_t}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}\\
&+\sum_{j=2}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial \tau_t}\\
&+\gamma\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+1}}{I_{t+1}}\right\}\frac{\partial k_{t+1}}{\partial \tau_t}\\
=&-\frac{1+\beta}{1-\tau_t}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}\\
&+\frac{\frac{\partial k_{t+2}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial \tau_t}}{\frac{\partial k_{t+2}}{\partial \tau_{t+1}}}\sum_{j=2}^{\infty}\gamma^j\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+j}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+j}}{I_{t+j}}\right\}\frac{\partial k_{t+j}}{\partial \tau_{t+1}}\\
&+\gamma\left\{\left(\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)\right)\frac{1}{k_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+1}}{I_{t+1}}\right\}\frac{\partial k_{t+1}}{\partial \tau_t}.\end{aligned} \tag{A.54}$$

We substitute (A.53) into (A.54) and obtain

$$0=-\frac{1+\beta}{1-\tau_t}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_t}{I_t}+\frac{\frac{\partial k_{t+2}}{\partial k_{t+1}}\frac{\partial k_{t+1}}{\partial \tau_t}}{\frac{\partial k_{t+2}}{\partial \tau_{t+1}}}\left[\frac{1+\beta}{1-\tau_{t+1}}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{w_{t+1}}{I_{t+1}}\right]$$
$$+\gamma\left\{\left(\frac{\beta\left(\alpha-1\right)}{\gamma}+\alpha\left(1+\beta\right)\right)\frac{1}{k_{t+1}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{L_{t+1}}{I_{t+1}}\right\}\frac{\partial k_{t+1}}{\partial \tau_t}. \tag{A.55}$$

We have so far derived three conditions, namely (A.45), (A.52), and (A.55), from the first-order conditions (A.34), (A.36), and (A.37) corresponding to the fiscal variables τ, b', and x, respectively. In conjunction with the first-order condition with respect to τ_t^k in (A.35), these four conditions collectively define the optimal fiscal variables, $\left(\tau,\tau^k,b',x\right)$, that maximize social welfare in (A.26).

A.4.2 Step 2

To find the policy functions, τ, τ^k, b', and x that satisfy the four first-order conditions, (A.35), (A.45), (A.52), and (A.55), we conjecture the policy functions as follows:

$$\begin{cases}\tau_t=\bar{T},\\ 1-\tau_t^k=\frac{\bar{T}^k}{\alpha}\frac{1}{1+k_t/b_t},\\ \left(1+n\right)b_{t+1}=\bar{B}\cdot y\left(k_t,k_t^p\right),\\ (1+n)x_t=\bar{X}\cdot y\left(k_t,k_t^p\right),\end{cases} \tag{A.56}$$

where $\bar{T}$, $\bar{T}^k$, $\bar{B}$, and $\bar{X}$ are constant. We substitute these conjectured solutions into the four first-order conditions, (A.35), (A.45), (A.52), and (A.55) and verify their correctness.

For presentation of the analysis, recall w_t in (7) and R_t in (8), which have the following properties:

$$\begin{cases}\frac{\partial w_t}{\partial k_t}=\alpha\left(1-\alpha\right)\frac{y_t}{k_t};\ \frac{\partial w_t}{\partial k_t^p}=\left(1-\alpha\right)^2\frac{y_t}{k_t^p};\\ \frac{\partial R_t}{\partial k_t}=\alpha\left(\alpha-1\right)\frac{y_t}{\left(k_t\right)^2};\ \frac{\partial R_t}{\partial k_t^p}=\alpha\left(1-\alpha\right)\frac{y_t}{k_t k_t^p}\end{cases} \tag{A.57}$$

Given the properties in (A.57) and the conjectured policy functions in (A.56), I_t, J_t, and L_t defined in (A.31), (A.32), and (A.33), respectively, are reformulated as follows;

$$I_t=\left(\alpha-\bar{T}^k+\left(1-\alpha\right)\bar{T}+\bar{B}-\bar{X}\right)y_t, \tag{A.58}$$

$$J_t=\left(1-\alpha\right)\left(\alpha-\bar{T}^k+\left(1-\alpha\right)\bar{T}\right)\frac{y_t}{k_t^p}, \tag{A.59}$$

$$L_t=\left(-\alpha\left(1-\alpha\right)-\left(1-\alpha\right)\bar{T}^k+\alpha+\left(1-\alpha\right)\bar{T}\right)\frac{y_t}{k_t}-\bar{T}^k\frac{y_t}{k_t+b_t}. \tag{A.60}$$

Using the capital market clearing condition in (16) with (A.56) and (A.57), we have

$$\begin{cases}\frac{\partial k_{t+1}}{\partial \tau_t}=-\frac{1}{1+n}\frac{\beta}{1+\beta}\left(1-\alpha\right)y_t;\\ \frac{\partial k_{t+1}}{\partial k_t}=\frac{1}{1+n}\frac{\beta}{1+\beta}\left(1-\bar{T}\right)\alpha\left(1-\alpha\right)\frac{y_t}{k_t};\\ \frac{\partial k_{t+1}}{\partial b_{t+1}}=-1;\\ \frac{\partial k_{t+1}}{\partial k_t^p}=\frac{1}{1+n}\frac{\beta}{1+\beta}\left(1-\bar{T}\right)\left(1-\alpha\right)^2\frac{y_t}{k_t^p}.\end{cases} \tag{A.61}$$

From (10), we also have the following properties of the public capital formation function:

$$\frac{\partial k_{t+1}^p}{\partial k_t}=\eta\frac{k_{t+1}^p}{x_t};\ \frac{\partial k_{t+1}^p}{\partial k_t^p}=\left(1-\eta\right)\frac{k_{t+1}^p}{k_t^p}. \tag{A.62}$$

With (A.56) – (A.62), we reformulate the four conditions, (A.35), (A.45), (A.52), and (A.55) in the following way. First, with (A.56), (A.57), and (A.58), the condition in (A.35) is reduced to:

$$0=-\frac{\beta}{\gamma}\cdot\frac{1}{\bar{T}^{k}}+\frac{\theta\left(\frac{\beta}{\gamma}+1\right)}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}. \tag{A.63}$$

Second, with (A.56), (A.57), (A.58) and (A.60), the condition in (A.45) is reduced to:

$$\begin{aligned}&-\frac{1+\beta}{1-\bar{T}}\left(\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}\right)+\theta\left(\frac{\beta}{\gamma}+1\right)(1-\alpha)\\&=\frac{\beta}{1+\beta}(1-\alpha)\left[\theta\left(\frac{\beta}{\gamma}+1\right)-\gamma\theta\left(\frac{\beta}{\gamma}+1\right)\frac{\bar{T}^{k}}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)}\right].\end{aligned} \tag{A.64}$$

Third, with (A.56), (A.60) and (A.61), the condition in (A.55) is reduced to:

$$\begin{aligned}0=&-\frac{1+\beta}{1-\bar{T}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1-\alpha}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\\&+\frac{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)\alpha(1-\alpha)}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}\left[\frac{1+\beta}{1-\bar{T}}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1-\alpha}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right]\\&-\gamma\frac{\beta}{1+\beta}(1-\alpha)\left\{\frac{\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}\right.\\&\left.+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{\frac{-\alpha(1-\alpha)-(1-\alpha)\bar{T}^{k}+\alpha+\alpha(1-\alpha)\bar{T}}{\frac{\beta}{1+\beta}(1-\bar{T})(1-\alpha)-\bar{B}}-\frac{\bar{T}^{k}}{\frac{\beta}{1+\beta}(1-\bar{T})(1-\alpha)}}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right\}.\end{aligned} \tag{A.65}$$

Finally, with (A.56), (A.58), (A.60) and (A.62), the condition in (A.52) is reduced to:

$$\begin{aligned}0=&-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{\bar{X}}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\\&+\gamma\left\{(1-\alpha)\left(\frac{\beta}{\gamma}+1+\beta\right)+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{(1-\alpha)\left(\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}\right)}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right\}\\&+\gamma^{2}\left\{\frac{\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{\frac{-\alpha(1-\alpha)-(1-\alpha)\bar{T}^{k}+\alpha+\alpha(1-\alpha)\bar{T}}{\frac{\beta}{1+\beta}(1-\bar{T})(1-\alpha)-\bar{B}}-\frac{\bar{T}^{k}}{\frac{\beta}{1+\beta}(1-\bar{T})(1-\alpha)}}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right\}\\&\times\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)^{2}\\&-\frac{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}{\alpha\frac{\beta}{1+\beta}}\left\{\frac{1+\beta}{1-\bar{T}}-\theta\left(\frac{\beta}{\gamma}+1\right)\frac{1-\alpha}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right.\\&+\gamma\frac{\beta}{1+\beta}(1-\alpha)\left[1+\frac{\gamma\frac{\beta}{1+\beta}\left(1-\bar{T}\right)\alpha(1-\alpha)}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}\right]\\&\left.\left[\frac{\frac{\beta(\alpha-1)}{\gamma}+\alpha(1+\beta)}{\frac{\beta}{1+\beta}\left(1-\bar{T}\right)(1-\alpha)-\bar{B}}+\theta\left(\frac{\beta}{\gamma}+1\right)\frac{\frac{-\alpha(1-\alpha)-(1-\alpha)\bar{T}^{k}+\alpha+\alpha(1-\alpha)\bar{T}}{\frac{\beta}{1+\beta}(1-\bar{T})(1-\alpha)-\bar{B}}-\frac{\bar{T}^{k}}{\frac{\beta}{1+\beta}(1-\bar{T})(1-\alpha)}}{\alpha-\bar{T}^{k}+(1-\alpha)\,\bar{T}+\bar{B}-\bar{X}}\right]\right\}.\end{aligned} \tag{A.66}$$

Conditions (A.63), (A.64), (A.65) and (A.66) all include only constant terms, $\bar{T}$, $\bar{T}^k$, $\bar{B}$, and $\bar{X}$, and they are independent of the state variables. Therefore, (A.63), (A.64), (A.65) and (A.66) are solved for $\bar{T}$, $\bar{T}^k$, $\bar{B}$, and $\bar{X}$ when these four variables are constant. This verifies the conjecture in (A.56).

■

References

Agénor, P.-R. and Yilmaz, D. (2017). The simple dynamics of public debt with productive public goods. *Macroeconomic Dynamics*, 21(4):1059–1095. 1

Andersen, T. M. (2019). Intergenerational conflict and public sector size and structure: A rationale for debt limits? *European Journal of Political Economy*, 57:70–88. 1, 2

Arai, R., Naito, K., and Ono, T. (2018). Intergenerational policies, public debt, and economic growth: A politico-economic analysis. *Journal of Public Economics*, 166:39–52. 1

Azzimonti, M., Battaglini, M., and Coate, S. (2016). The costs and benefits of balanced budget rules: Lessons from a political economy model of fiscal policy. *Journal of Public Economics*, 136:45–61. 1

Barseghyan, L. and Battaglini, M. (2016). Political economy of debt and growth. *Journal of Monetary Economics*, 82:36–51. 1, 2, 6

Bassetto, M. and Sargent, T. J. (2006). Politics and efficiency of separating capital and ordinary government budgets. *Quarterly Journal of Economics*, 121(4):1167–1210. 2

Bernasconi, M. and Profeta, P. (2012). Public education and redistribution when talents are mismatched. *European Economic Review*, 56(1):84–96. 3, 9

Bisin, A., Lizzeri, A., and Yariv, L. (2015). Government policy with time inconsistent voters. *American Economic Review*, 105(6):1711–37. 1

Bom, P. R. and Ligthart, J. E. (2014). Public infrastructure investment, output dynamics, and balanced budget fiscal rules. *Journal of Economic Dynamics and Control*, 40:334–354. 1

Bouton, L., Lizzeri, A., and Persico, N. (2020). The political economy of debt and entitlements. *Review of Economic Studies*, 87(6):2568–2599. 1

Budina, M. N., Kinda, M. T., Schaechter, M. A., and Weber, A. (2012). *Fiscal rules at a glance: Country details from a new dataset*. International Monetary Fund. `https://www.imf.org/external/pubs/ft/wp/2012/wp12273.pdf` (accessed June 7, 2021). 1

Buiter, W., Corsetti, G., and Roubini, N. (1993). Excessive deficits: sense and nonsense in the treaty of Maastricht. *Economic Policy*, 8(16):57–100. 1

Caselli, F. G., Eyraud, L., Hodge, A., Kalan, F. D., Kim, Y., Lledó, V., Mbaye, S., Popescu, A., Reuter, W. H., Reynaud, J., et al. (2018). Second-generation fiscal rules: Balancing simplicity, flexibility, and enforceability-technical background papers. *Staff Discussion Notes*, 2018(004). 1

Coate, S. and Milton, R. T. (2019). Optimal fiscal limits with overrides. *Journal of Public Economics*, 174:76–92. 1

Corsetti, G. and Roubini, N. (1996). European versus american perspectives on balanced-budget rules. *American Economic Review*, 86(2):408–413. 1

Dovis, A. and Kirpalani, R. (2020). Fiscal rules, bailouts, and reputation in federal governments. *American Economic Review*, 110(3):860–88. 1

Dovis, A. and Kirpalani, R. (2021). Rules without commitment: Reputation and incentives. *Review of Economic Studies*. `https://doi.org/10.1093/restud/rdab006`. 1

Ghosh, S. and Mourmouras, I. A. (2004a). Debt, growth and budgetary regimes. *Bulletin of Economic Research*, 56(3):241–250. 1

Ghosh, S. and Mourmouras, I. A. (2004b). Endogenous growth, welfare and budgetary regimes. *Journal of Macroeconomics*, 26(4):623–635. 1

Gonzalez-Eiras, M. and Niepelt, D. (2008). The future of social security. *Journal of Monetary Economics*, 55(2):197–218. 2, 14

Gonzalez-Eiras, M. and Niepelt, D. (2012). Ageing, government budgets, retirement, and growth. *European Economic Review*, 56(1):97–115. 2

Greiner, A. (2008). Human capital formation, public debt and economic growth. *Journal of Macroeconomics*, 30(1):415–427. 1

Greiner, A. and Semmler, W. (2000). Endogenous growth, government debt and budgetary regimes. *Journal of Macroeconomics*, 22(3):363–384. 1

Halac, M. and Yared, P. (2018). Fiscal rules and discretion in a world economy. *American Economic Review*, 108(8):2305–34. 1

Kumar, M., Baldacci, E., Schaechter, A., Caceres, C., Kim, D., Debrun, X., Escolano, J., Jonas, J., Karam, P., Yakadina, I., et al. (2009). Fiscal rules–anchoring expectations for sustainable public finances. *IMF Staff Paper*. Washington D.C. `https://www.elibrary.imf.org/view/journals/007/2009/096/article-A000-en.xml` (accessed August 17, 2021). 1, 3, 14, 18

Lancia, F. and Russo, A. (2016). Public education and pensions in democracy: A political economy theory. *Journal of the European Economic Association*, 14(5):1038–1073. 3, 9, 14

Lindbeck, A. and Weibull, J. W. (1987). Balanced-budget redistribution as the outcome of political competition. *Public Choice*, 52(3):273–297. 9

Lledó, V., Yoon, S., Fang, X., Mbaye, S., and Kim, Y. (2017). *Fiscal rules at a glance*. International Monetary Fund. `https://www.imf.org/external/datamapper/FiscalRules/map/map.htm` (accessed August 17, 2021). 1

Minea, A. and Villieu, P. (2009). Borrowing to finance public investment? the ‘golden rule of public finance’ reconsidered in an endogenous growth setting. *Fiscal Studies*, 30(1):103–133. 1

OECD (2021). *Government at a Glance 2021*. OECD Publishing, Paris. `https://doi.org/10.1787/1c258f55-en` (accessed August 17, 2021). 1

Persson, T. and Tabellini, G. (2000). *Political economics: explaining economic policy*. MIT Press, Cambridge, Massachusetts. 9

Piguillem, F. and Riboni, A. (2020). Fiscal Rules as Bargaining Chips. *Review of Economic Studies*. `https://doi.org/10.1093/restud/rdaa080`. 1

Robinson, M. (1998). Measuring compliance with the golden rule. *Fiscal Studies*, 19(4):447–462. 1

Saint-Paul, G. and Verdier, T. (1993). Education, democracy and growth. *Journal of Development Economics*, 42(2):399–407. 3, 9

Schaechter, M. A., Kinda, M. T., Budina, M. N., and Weber, A. (2012). *Fiscal Rules in Response to the Crisis: Toward the" Next-Generation" Rules: A New Dataset*. International Monetary Fund, Washinton, D.C. `http://www.imf.org/external/pubs/ft/wp/2012/wp12187.pdf` (accessed June 7, 2021). 1

Song, Z., Storesletten, K., and Zilibotti, F. (2012). Rotten parents and disciplined children: A politico-economic theory of public expenditure and debt. *Econometrica*, 80(6):2785–2803. 2, 14

Stockman, D. R. (2001). Balanced-budget rules: Welfare loss and optimal policies. *Review of Economic Dynamics*, 4(2):438–459. 1

Stockman, D. R. (2004). Default, reputation, and balanced-budget rules. *Review of Economic Dynamics*, 7(2):382–405. 1

Uchida, Y. and Ono, T. (2021). Political economy of taxation, debt ceilings, and growth. *European Journal of Political Economy*, 68:101996. 1, 2

Ueshina, M. (2018). The effect of public debt on growth and welfare under the golden rule of public finance. *Journal of Macroeconomics*, 55:1–11. 1

Wyplosz, C. (2013). *Fiscal Rules: Theoretical Issues and Historical Experiences*, pages 495–525. University of Chicago Press. In: Alesina, A. and Giavazzi, F.(Eds.), Fiscal Policy after the Financial Crisis, 495–525. University of Chicago Press, Chicago, Illinois. 1, 3, 18

Yakita, A. (2008). Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. *Journal of Public Economics*, 92(3-4):897–914. 1