Navigating extreme market fluctuations: asset allocation strategies in developed vs. emerging economies.

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ABSTRACT

This paper contributes to the literature on portfolio allocation by assessing how assets from emerging and developed stock markets can be allocated efficiently during crisis periods. Towards this end, the paper proposes an approach to portfolio allocation that combines traditional portfolio theory with extreme value theory (EVT) based on Generalised Pareto Distributions (GPDs) and Generalised Extreme Values (GEVs). The results of the empirical analysis show that for the mean-variance portfolio constructed from GPD, the emerging market portfolio outperforms both the international portfolio, the combination of emerging and developed market assets, and the developed market portfolio. However, the developed market portfolio outperforms the emerging market portfolio for the mean-variance portfolio constructed from GEV distribution. The paper attributes these different outcomes to the intended objectives of these extreme-value approaches in the context of portfolio selection. These results offer essential guidance for investors and asset managers during the construction of portfolios in times of crisis. They highlight that the effectiveness of a portfolio is significantly influenced by its predefined objectives. Ultimately, these objectives are crucial in deciding the most suitable approach for portfolio construction.

Keywords: Extreme Value Theory, General Pareto Distribution, Emerging and developed markets, portfolio optimisation, mean-variance.

1. INTRODUCTION

One of the most important tasks of asset managers is to efficiently select assets that could prevent large portfolio losses, especially during periods of financial distress. Studies show that asset managers and investors have been experiencing difficulties because of the significantly turbulent crises over the past decades, which have spread from country to country. These crises have negatively affected world equity markets and portfolio investments (Afzal & Ali, 2012; Khoon and Lim, 2010 & Samarakoon, 2017).

Several countrywide and regional financial and economic crises have had considerably contagious effects globally, with negative effects in equity markets. For example, the Asian financial Crisis, which occurred in 1997-1998, had negative effects on many Asian economies . The capital outflow that ensued saw their currencies depreciating by approximately 38% and international stocks declining by nearly 60% (Corsetti, Pesenti & Roubini, 1999 and Cohen and Benjamin, 2008).

The dot-com bubble was a historic period of rapid increase in U.S. technology stock equity valuations fuelled by investment in internet based companies. During the dot-com bubble the values of equity markets grew exponentially. However March 2001 marked the burst of the dot-com companies resulting in the NASDAQ Composite declining by 78%. In the same year on September 11, the biggest terrorist attack was perpetrated against the United States. The terrorist attack led to approximately 3,000 deaths. The event prompted closure of the New York Stock Exchange and the NASDAQ composite saw a significant decline. (Junior and Franca, 2012).

The global financial crisis that started in the US in 2007 led to tightening of credit lines across the world and eventually wiped out global equity markets. For example, the Asian stock markets fell between ranges of 38% to 62%, with the largest market declines came from Singapore (27%), Thailand (21%) and the Philippines (21%) (Guinigundo & Paulson, 2010 and Junior & Franca, 2012).

During the initial months of the COVID-19 pandemic, stock markets globally experienced significant losses. Studies indicate that at the height of the pandemic in

March 2020, major stock indices witnessed substantial declines: the S&P 500 index in the United States fell by 27%, Germany's DAX dropped 38%, and Japan's Nikkei decreased by 29% (Cepoi, 2020; Uddin, et al., 2021). This downturn in the stock market, driven by the pandemic, led governments worldwide to implement a range of stimulus packages. These measures aimed to mitigate the economic impacts of the pandemic and to restore investor confidence, addressing the financial uncertainties and disruptions caused by the global health crisis.

This reality shows that most financial and economic crises are contagious and mostly affect global stock markets. Positive correlation between stock markets due to the contagious nature of financial crisis continue to present a challenge for asset diversification and efficient portfolio allocation. It is therefore imperative for investors and asset managers to engineer the best strategy for portfolio allocation and optimisation during crisis periods.

In the literature, Risk-based approaches to portfolio allocation have been suggested to account for financial turbulances. For example, Briec, Kerstens, and Jokung (2007) suggest the use of a higher-order moment portfolio optimisation that creates a mean-variance skewness objective for portfolio allocation during turmoil periods. Naqvi et al. (2017) have expanded the traditional portfolio optimization approach by integrating higher moments of risk, specifically focusing on mean-variance-skewness-kurtosis. This inclusion of additional risk factors, especially fat-tail risk, marks a significant departure from the classic Markowitz mean-variance optimization model. By incorporating these higher moments into the optimization process, their proposed framework aims to enhance decision-making in portfolio selection, steering clear of sub-optimal choices and reducing exposure to more complex risk elements. Kshatriya and Prasanna (2018) explore the effects of incorporating higher moments in risk estimation during international portfolio diversification. The study utilizes a dataset comprising thirty-three globally traded stock market indexes, encompassing both emerging and developed markets, spanning from 2000 to 2012. By integrating skewness and kurtosis into the portfolio optimization process, it transforms into a complex, non-linear, non-convex, and multi-objective problem. To address this complexity, a genetic algorithm has been employed. The empirical findings of the study indicate that the higher moments model surpasses the traditional mean-variance model in performance throughout the examined period.

While the studies mentioned earlier have taken into account the skewness property of stock market returns in their portfolio optimization models, however, studies applying Extreme Value Theory (EVT) in portfolio selection is relatively scarce. For instance, Lin and Ko (2009) present a Genetic Algorithm (GA)-based model for forecasting portfolio volatility. This model is designed to identify the optimal portfolio set and dynamically determine an appropriate peak threshold for each asset within the portfolio. These determined peak thresholds play a critical role in estimating the portfolio's Value at Risk (VaR) through the application of EVT. Bedoui et al. (2023) investigate the potential benefits of using the Conditional Value at Risk (CVaR) portfolio optimization approach with a GARCH model, Extreme Value Theory (EVT), and Vine Copula to obtain the optimal allocation decision for a portfolio consisting of Bitcoin, gold, oil, and stock indices. Their investigation begins by fitting an appropriate GARCH model to the return series of each asset. This is followed by the application of the Generalized Pareto Distribution (GPD) for modeling the tail innovations. Subsequently, a Vine Copula-GARCH-EVT model is constructed to effectively capture the dependencies among the assets. Mainik et al. (2015) employ the Extreme Risk Index (ERI) in a backtesting analysis of a portfolio optimization strategy, using a selection of 400 stocks from the S&P 500. The objective of the study is to evaluate the performance of the ERI strategy in comparison to both the minimum variance portfolio and the equally weighted portfolio. Key comparison metrics include annualized portfolio returns, maximum drawdowns, transaction costs, portfolio concentration, and asset diversity. Their results indicate that the ERI strategy substantially outperforms both the minimum variance and equally weighted portfolios, particularly in the context of assets characterized by heavy tails.

While existing studies highlight the significance of Extreme Value Theory (EVT) in addressing extreme events, they haven't proposed a method to integrate portfolio theory with EVT for optimally distributing assets between emerging and developed stock markets. This paper's dual contribution begins with enhancing the traditional Markowitz mean-variance approach by introducing an EVT mean-variance framework. This method selects assets for an efficient portfolio based on the Generalized Pareto

Distribution (GPD) and Generalized Extreme Value (GEV) distributions, particularly considering their left-tail distributions. The second contribution demonstrates the efficient allocation of assets from both emerging and developed markets within international portfolio diversification, specifically during times of crisis or turmoil. Addressing this gap could clarify the contentious debate over the value of assets from developed versus emerging markets in crisis situations. Research indicates that emerging markets offer higher yields and potentially better risk-adjusted returns than developed markets (Bartram & Bodnar, 2012). Omoshoro-Jones and Bonga-Bonga (2020) found that many emerging markets remained detached from developed markets during recent financial and economic crises, suggesting they may be a better choice for investors and asset managers in such times. Conversely, other studies argue that developed markets serve as safe havens during global financial crises, recommending increased investment in these markets to mitigate crisis-related risks (Min et al., 2016; Tachibana, 2022; Gurdgiev and Petrovskiy,2023). This contradictory evidence presents a challenge in determining how to effectively combine assets from developed and emerging economies to form an efficient portfolio during crises, necessitating thorough investigation.

The remainder of the paper is structured as follows: section 2 explains the methodology used. section 3 presents the data used, the estimation and discuss the results obtained. Section 4 concludes the paper.

2. METHODOLOGY

In order to construct an efficient international portfolio by combining the mean-variance portfolio and EVT theory with stock market assets from developed and emerging economies we use the following steps; first, we filter each return series by fitting an –ARMA-GARCH process to remove serial correlation , such as ;

$$Y_t = w + \sum_{i=1}^{p} \alpha_i Y_{t-i} + \varepsilon_t + \sum_{j=1}^{q} \beta_j \varepsilon_{t-j} \qquad \varepsilon_t \sim i.i.d(0,1) \tag{1}$$

$$\sigma_t^2 = w + \sum_{i=1}^{p} a_i \varepsilon_{t-1}^2 + \sum_{j=1}^{q} b_j \sigma_{t-1}^2 + \sum_{i=1}^{p} \gamma \psi_{t-i} \varepsilon_{t-1}^2 \tag{2}$$

Where Equation 1 is the mean equation expressed as ARMA model and Equation 2 is the variance equation, expressed as a GJR-GARCH model. $a, b, \alpha, \beta and\ \ \gamma$ are parameters, γ Indicates the leverage effect, if $\gamma = 0$ there is no asymmetric volatility; if $\gamma < 0$ negative shocks increase volatility if $\gamma > 0$ positive shocks increase volatility and ψ represents parameter affected by shocks.

Second, peak over threshold (POT) and block maxima (minima) model (BMM) are applied to obtain minimum extreme value. Given the distributions of the POT and BMM, namely the General Pareto Distribution (GPD) and Generalised Extreme Value (GEV) distribution, respectively, we fit the minima based on the two distributions. The general mathematical formula of GPD is written as follows:

$$G_{\xi,\beta} = \begin{cases} 1 - \left(1 + \frac{\xi_x}{\beta}\right), & \xi \neq 0 \\ 1 - \exp\left(-\frac{x}{\beta}\right), & \xi = 0 \end{cases} \tag{17}$$

Where ξ denotes the shape parameter and β denotes the scale parameter. When $\xi < 0$ it represents a pareto distribution of type 2, when $\xi = 0$ it represents exponential distribution and when $\xi > 0$ it represents a reparametrised type of pareto distribution. The general mathematical formula of GEV may be written as follows:

$$G_{\xi}\,(x) = \begin{cases} \exp(-(1+\ \xi_{\ x})^{\frac{-1}{\xi}}), & \xi \neq 0 \\ \exp(-e^{-x})\,, & \xi = 0 \end{cases} \tag{18}$$

Where ξ denotes the shape parameter. When $\xi < 0$ it represents the Weibull distribution, when $\xi = 0$ it represents the Gumbel distribution and when $\xi > 0$ it represents the Frechet distribution. The factor $(1+\ \xi_{\ x})$ is always positive.

Third, the simulated return series derived from these distributions are utilized to create various mean-variance portfolios. This process primarily aims at determining the weights of the optimal and tangent portfolios within the framework of mean-variance portfolio theory. It's crucial to understand that the optimal portfolio represents the most suitable portfolio for an individual investor, reflecting their risk tolerance and investment objectives. This portfolio is identified by locating the point on the efficient frontier that provides either the maximum expected return for a given level of risk or the minimum risk for a given level of expected return. In contrast, the tangent portfolio is acknowledged as the optimal combination of risky assets. This is because it yields the highest expected return for a specified level of risk among all the portfolios on the efficient frontier. The tangent portfolio is a theoretical construct in portfolio theory, employed to demonstrate market equilibrium (Tarrazo and Úbeda, 2012).

Last, we employed the Sharpe ratio and Sortino ratio to evaluate the performance of each of the constructed portfolios. The superior model should produce higher Sharpe and/or Sortino ratios.

3. DATA AND EMPIRICAL RESULTS

3.1 Data

The paper utilises daily closing prices of key equity indices from five developed and five emerging markets. These include France (CAC 40), Canada (S&P/TSX), the United Kingdom (FTSE 100), Japan (NIKKEI 225), the United States (S&P500) representing developed markets, and Brazil (BOVESPA), China (SHCOMP), India (S&P BSE SENSEX), Indonesia (IDX Composite) and Turkey (BIST 100) from emerging markets. The analysis covers the period from August 1997 to August 2022. This timeframe was chosen due to the availability of consistent data, particularly at the beginning of the period. It encompasses various significant economic events, including the Dotcom crisis, Asian financial crisis, global financial crisis, and other notable financial and economic downturns.

Returns series are obtained as follows:

$$r_t = (\ln P_t - lnP_{t-1}) * 100 \tag{22}$$

where r_t is the daily rate of return, *P* are the closing prices. *ln* is the natural logarithm. Preliminary and descriptive statistics of the daily equity returns series are reported below starting with Figure 1.

Figure 1 below illustrates the descriptive statistics of daily equity returns series of all the markets. From the figure it can be deduced that all series depict volatility clustering and heteroscedasticity. Moreover, periods of crises are characterised by high volatility translated by high spikes.

[Insert Figure 1]

Table 1 presents the descriptive statistics for the returns of all the stock markets included in the study. The data in Table 1 reveal that all developed markets experienced a negative average return, each hovering close to zero. This indicates a general trend of minimal gains or slight losses in these markets over the study period. In contrast, among the emerging markets, only Brazil's BOVESPA showed a negative mean return of -0.03. The remaining emerging markets recorded positive mean returns, with Turkey's BIST 100 exhibiting the highest average return of 0.06, suggesting a more robust performance in these markets.

Furthermore, all markets displayed negative skewness, indicating a higher likelihood of witnessing negative returns rather than positive ones. This skewness suggests that investors in these markets may have experienced more frequent losses. Additionally, the kurtosis values for all markets exceeded 3, signifying the presence of leptokurtic distributions. Leptokurtic distributions are characterized by fatter tails and a higher peak compared to a normal distribution. This implies that during financial crises, these markets are prone to experiencing significant price drops, leading to extreme losses. Such behaviour underscores the higher risk associated with these markets, especially during periods of economic instability.

[Insert Table 1]

[Insert Figure 2]

Figure 2 illustrates the risk-reward plot of selected emerging and developed stock markets. The plot shows that the Turkish BIST 100 has the highest return and risk. The BOVESPA reports high risk with negative returns, similar to the FTSE 100, CAC 40, and Nikkei 225. The S&P/TSX and S&P 500 show moderate risk with low returns, while the SHCOMP and JSE have moderate risk and returns. Emerging market indices are generally riskier but offer higher returns compared to developed market indices. Additionally, Figure 2 indicates that for a similar level of risk, the South African stock market (JSI) provides a higher average return than the Japanese market (Nikkei 225), suggesting that emerging market assets can outperform developed markets in terms of the risk-return tradeoff.

Regarding the steps for modeling the proposed EVT-Mean-Variance portfolio optimization approach, as detailed in the methodology section, our empirical analysis begins by filtering the return series data using the ARMA(p,q)-GJR-GARCH (t,n) process. The orders of the mean and variance equations are determined based on the Akaike Information Criterion and vary for each specific stock market, as shown in Table A1 in the appendix. The choice of the asymmetric GARCH model for the variance equation is supported by the news impact curves obtained for all stock market series. For instance, Figure 3 illustrates the news impacts for the CAC 40 and DAX 30, demonstrating the asymmetric effects of volatility shocks and justifying the use of an asymmetric model.

[Insert Figure 3]

In the second step, we obtain the left tail of the distribution of the filtered returns from the POT and BMM methods. For the POT method, we set the threshold by taking the 95th percentile of the filtered returns at the left tail. Moreover, for the BMM method, we set the block to 30 days and calculate the number of blocks needed to cover the entire dataset. We then extract the left tail of the distribution by computing the minimum value within each block. As a sample of the outcomes of this step, Figure 4 presents the histogram of the left tail returns of the CAC 40[1] obtained from the POT and BMM methods. The figure shows that the series comprises observations with lower returns, thus, identifying the left tail of the distribution of return series.

[Insert Figure 4]

[1] Other figures can be obtained on request,

It is crucial to observe from Figure 4 that the frequencies of the Peaks Over Threshold (POT) distribution are notably higher than those of the Block Maxima method. This significant difference suggests that the Generalized Pareto Distribution (GPD), as applied in the POT approach, is particularly adept at capturing the magnitude and frequency of extreme deviations from the norm. The higher frequency in the POT distribution indicates its effectiveness in identifying and analysing more extreme events that exceed a predefined threshold.

Conversely, the Block Maxima (Minima) approach, which encompasses all observations in identifying maxima/minima within designated blocks, seems especially beneficial when the focus is on understanding the overall behaviour of extremes. This method is inclusive of all extreme values within a block, providing a comprehensive view of the extremities, regardless of whether they surpass a specific threshold. This aspect makes the Block Maxima method valuable for analyzing the complete range of extreme behaviours in a dataset.

The choice between these two methods depends on the specific objectives of the analysis. If the interest lies in scrutinizing the most extreme deviations and their characteristics, the POT approach is more suitable. However, for a broader analysis that includes all extreme values within a dataset, the Block Maxima method is preferable.

This issue of selecting the appropriate method based on the research objectives and the nature of the data will be further discussed in the paper, highlighting the strengths and limitations of each approach in different contexts.

Thirdly, we applied the Generalized Pareto Distribution (GPD) and the Generalized Extreme Value (GEV) distributions to fit the left-tail returns obtained from the Peaks Over Threshold (POT) and Block Maxima Method (BMM), respectively. As an illustrative example, Figure 5 displays the Quantile-Quantile (QQ) plots for the residuals of the GPD model fits for the left tails of the CAC 40 and NIKKEI 225 series. These plots are particularly insightful as they demonstrate the GPD model's accuracy in capturing extreme value behaviour in both series. This is evident from the alignment of the plotted points along a straight line in the QQ plots. Such an alignment suggests a good fit, indicating that the GPD model accurately represents the distribution of the extreme values in these series. Deviations from this straight line in the QQ plot would have been indicative of discrepancies between the observed and modelled quantiles, signalling a potential mismatch between the empirical data and the theoretical model.

[Insert Figure5]

After fitting the left tail returns with GPD and GEV distributions, we simulate the returns series from these distributions and used them for portfolio selection and construction.

3.2 PORTFOLIO SELECTION AND CONSTRUCTION

Having simulated left-tail series from various Extreme Value Theory (EVT) distributions, we now aim to evaluate the efficiency of portfolios constructed using mean-variance portfolio theory enhanced with EVT. Specifically, we will analyze the performance of the mean-variance-GPD portfolio, which incorporates series obtained from the POT method, and the mean-variance-GEV portfolio, derived from series obtained using the BMM method.

Our assessment will cover three types of portfolios: international portfolios (comprising a combination of developed and emerging economies), as well as separate portfolios for developed economies and emerging economies. The effectiveness of these portfolios will be measured based on the Sharpe and Sortino ratios, which are key indicators of risk-adjusted return performance.

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3.2.1 The Mean-Variance GPD portfolio

We evaluate the efficiency of portfolio allocation among various types - international, developed markets, and emerging markets portfolios - during tumultuous periods, using left-tail series derived from the Generalized Pareto Distribution (GPD). Table 2 presents the results for mean-variance GPD portfolios that constitute an international portfolio, which blends assets from both emerging and developed stock markets. Our assessment focuses on two specific types of mean-variance portfolios: the optimal portfolio, tailored for the best balance of risk and return, and the tangent portfolio, which aligns with the market portfolio on the efficient frontier.

It is worth noting that the optimal portfolio is a portfolio that offers the highest level of expected return for a given level of risk or the lowest level of risk for a given level of expected return. It represents the set of portfolios that lie on the efficient frontier, which is a curve that shows the trade-off between risk and return. However, the tangent portfolio, also called the market portfolio, is a specific portfolio that lies on the efficient frontier and offers the highest expected return for a given level of risk, or equivalently, a portfolio that has the highest Sharpe ratio. Given that investors and asset managers' risk tolerance differ, those whose risk tolerance is high may prefer the optimal portfolio. Nonetheless, risk-averse investors may prefer the tangent portfolio.

Table 2 reveals that in the case of the optimal portfolio, a significant portion, 66.3%, is allocated to developed market indices. Within this allocation, the S&P 500 and the FTSE 100 each receive substantial weights of 18.58% and 18.65%, respectively. This distribution reflects a preference for the stability and lower volatility often associated with developed market indices.

[Insert Table 2]

In contrast, the tangent portfolio, while still favouring developed markets, allocates a notable 44.94% to emerging markets. This allocation strategy indicates a more balanced approach, leveraging the potential higher returns from emerging markets while maintaining a substantial commitment to the more stable developed markets.

Notably, the tangent portfolio demonstrates a higher Sharpe ratio of 2.07773 compared to the optimal portfolio. The Sharpe ratio, a measure of risk-adjusted return, being higher for the tangent portfolio suggests that it offers a more favorable balance of return for each unit of risk taken. This is typically expected as the tangent portfolio is designed to lie on the efficient frontier where the highest return per unit of risk is achieved.

The differences in portfolio composition and resulting Sharpe ratios underscore the distinct strategies and risk-return profiles of the optimal and tangent portfolios. The optimal portfolio tends to focus on minimizing risk for a given level of expected return, while the tangent portfolio aims to maximize returns for a given level of risk, as evidenced by its higher Sharpe ratio. This distinction is particularly relevant in the context of market turmoil, where the balance between risk and return becomes even more critical.

[Insert Figure 6]

The efficient portfolio frontier for generalised Pareto distribution is depicted above. It shows the mean-variance efficient frontier with a negatively sloped Sharpe Ratio (orange line). The negative slope implies that as targeted returns increase, the ratio of the mean return to risk decreases inversely. The equally weighted portfolio (EWP) indicates a return of 3.5, greater than the tangent portfolio-blue circle on efficient frontier. However, the EWP is less optimal as it is slightly out of the efficient frontier.

Table 3 below reports the mean-variance GPD portfolios, which comprise only of developed country indices. The results reported in Table 3 show that the FTSE 100 and S&P500 are allocated the largest portions compared to other stock market assets. The Sharpe Ratio and Sortino Ratio of the developed market portfolio are 2.288 and 2.3132. they are higher than in the case of international portfolio.

[Insert Table 3]

[Insert Table 4]

Table 4 reports on the mean-variance GPD portfolios of emerging countries indices. In this portfolio, more weight is allocated to S$P BSE SENEX, with the lowest weight allocated to the JSE, the South African stock market. The Sharpe ratio for the optimal portfolio made of emerging market stocks is 2.336, higher than the Sharpe ratio of international portfolio and developed market portfolio. However, the Sharpe ratio of the tangent portfolio for emerging market is 2.294, lower than the Sharpe ratio of developed economies. The Sortino ratio confirms that the emerging market portfolio performs better than the developed market portfolio, both for efficient and tangent portfolios.

These findings indicate that for investors or asset managers with a high tolerance for risk, emerging market portfolios represent a more favorable investment option compared to international or developed market portfolios. This conclusion is drawn from the mean-variance GPD portfolio analysis. The superior investment potential of emerging market assets over international and developed market portfolios can be attributed to several factors. First, during crises, international portfolios might not offer better diversification opportunities due to the financial contagion observed between developed and emerging markets, as discussed in various studies (e.g., Boubaker et al., 2016; Baur, 2012). Second, despite their increased volatility, many emerging markets tend to decouple from developed economies during significant crises, as highlighted in Bonga-Bonga (2018). This decoupling suggests that emerging markets are often shielded from the adverse impacts of global crises, particularly those originating in developed economies. Consequently, they could present more advantageous investment opportunities during periods of crisis.

3.2.2 The Mean-Variance GEV distribution portfolio

Table 5 presents the composition of mean-variance portfolios under the GEV distribution for an international portfolio. The data in Table 5 reveal a higher allocation of weights to indices of developed markets relative to those of emerging markets within this portfolio. Furthermore, it's notable that the weights assigned to emerging markets in the GEV distribution scenario are lower than those in the GPD distribution scenario. Additionally, when comparing the GEV and GPD distributions, the Sharpe ratios of both the optimal and tangent portfolios are lower in the case of the GEV distribution.

[Insert Table 5]

[Insert Table 6]

Tables 6 and 7 detail the outcomes of the mean-variance portfolio analysis under the GEV distribution for developed and emerging economies, respectively. The results, particularly the Sharpe and Sortino ratios, clearly demonstrate that portfolios composed of assets from developed economies have superior performance compared to both international and emerging market portfolios. This finding is in stark contrast to the results observed under the GPD distribution, where portfolios from emerging markets exhibited better performance than those from developed and international markets.

[Insert Table 7]

Our results offer several noteworthy observations. First, when it is believed that international portfolio, which combines assets from developed and emerging assets and dominated by developed market assets should offer an efficient and diversified portfolio, this paper shows that this portfolio is not necessary the most performing one. The reason for the poor performance of this portfolio may be attributed to possible contagion during crisis periods that impedes possible diversification between emerging and developed economies assets due to their possible positive correlation. Next, we find that a portfolio consisting exclusively of emerging market assets outperforms those from developed and international markets when applying GDP distribution in extreme value simulation. This superior performance likely stems from the Peak Over Threshold (POT) method's unique approach to modelling extreme values in GDP distributions, which involves selecting extreme events based on a specific threshold. This selection criterion enables investors and asset managers to implement strategies like stop-loss or portfolio rebalancing, mitigating potential losses. Such a strategy is particularly advantageous for emerging economies, which, despite facing losses during crises, can occasionally yield returns higher than those in developed markets. Last, the reason why developed markets outperform emerging markets when GEV distribution is used for extreme values is that the block maxima (minima for our case) approach used in GEV distribution models the distribution of minimum values taken from a wide range of extreme value behaviour. This approach is important when investors or asset managers are interested on the overall behaviour of the extremes, not just those beyond a specific threshold. For this reason, developed markets may be preferred to emerging markets as the magnitude of negative returns during crisis periods is limited compared to emerging markets. studies show that emerging stock markets exhibit heightened vulnerability to external shocks and internal instability, particularly during periods of crisis (Roni, 2018; Bhowmik et al., 2022; Younis, 2023;). This vulnerability is underpinned by the argument that, during such turbulent times, stocks in emerging economies display greater volatility compared to their counterparts in developed countries. The nature of this volatility in emerging markets during stock crises is distinct: prices typically plummet rapidly and sharply. However, the recovery phase for these markets is notably protracted (see Cevik, 2016). Unlike developed markets, where recovery mechanisms and investor confidence might be quicker to rebound, emerging markets often grapple with prolonged periods of uncertainty. This extended recovery is attributed to factors such as weaker economic foundations, less mature market mechanisms, and heightened sensitivity to both global and domestic economic fluctuations (Mlachila and Sanya, 2016). Consequently, investors in emerging markets must navigate a landscape marked by steeper declines and more gradual recoveries, underscoring the need for cautious and well-informed investment strategies in these regions during crisis periods.

4. CONCLUSION

The paper aimed to assess how assets from emerging and developed stock markets can be allocated efficiently during crisis periods. To this end, it proposed a methodology that combines portfolio theory and extreme value theory (EVT), namely the mean-variance-GPD (Generalised Pareto Distribution) and the mean-variance - GEV (Generalised Extreme Value) models. The performance of these models are compared based on the Sharpe and Sortino ratios. The application of these models followed a number of steps. First, return series of the different stock markets are filtered using the ARMA-GJR-GARCH process. Second, from these series, peak over threshold (POT) and block maxima (minima) model (BMM) distribution are applied to obtain minimum extreme value related to GPD and GEV models , respectively. Third, the simulated return series derived from these distributions are utilised to comstruct various mean-variance portfolios. Last, we employed the Sharpe ratio and Sortino ratio to evaluate the performance of each of the constructed portfolios. The superior model is supposed to produce higher Sharpe and/or Sortino ratios.

The results of the empirical analysis show that for the mean-variance portfolio constructed from GPD, the emerging market portfolio outperforms both the international portfolio, the combination of emerging and developed market assets, and the developed market portfolio. However, the developed market portfolio outperforms the emerging market portfolio for the mean-variance portfolio constructed from GEV distribution. The paper attributes these different outcomes to the intended objectives of these extreme-value approaches in the context of portfolio selection. The GPD is particularly effective when portfolio selection focuses on the magnitude and frequency of extreme values of assets from a given threshold. In the meantime, the GEV distribution is ideal for investors and asset managers who are interested in the behaviour of assets at extremes in general, not just those above a certain threshold. These results offer essential guidance for investors and asset managers during the construction of portfolios in times of crisis. They highlight that the effectiveness of a portfolio is significantly influenced by its predefined objectives. Ultimately, these objectives are crucial in deciding the most suitable approach for portfolio construction.

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Competing Interests

There is no conflict of interest related to this paper

Data Availability

Data will be provided on request

Ethical Approval

Not applicable. Paper made use of secondary data

Informed consent

Not applicable

Appendix

Table A1. Estimation of the ARMA-GJR-GARCH model

	CAC 40	S&P/TSX	FTSE 100	NIKKIE 225	S&P 500	BOVESPA	SHCOMP	S&P BSE SENEX	JSI	BIST 100
$ar1$	0.157	1.270	0.233	-0.773	2.276	1.285	0.994	-0.544	1.050	1.349
	(17.23)	(6.04)	(13.56)	(-3.55)	(101.9)	(13.41)	(405.33)	(-3.91)	(1044)	(16.94)
$ar2$	1.012	-0.645	-0.250	-0.341	-2.426	-0.823	-0.983		-0.216	-0.897
	(31.55)	(-4.43)	(-9.97)	(-2.02)	(-44.29)	(-4.59)			(-3830)	(-9.39)
$ar3$	0.275	-	0.263		1.375		0.029		0.104	
	(6.18)		(5.40)		(21.41)				(265)	
$ar4$	-0.859	-	-0.947		-0.279	-			0.986	
	(-88.08)		(-201.1)		(-9.182)				(-2046)	
$ar5$		-				-			-	
$ma1$	-0.169	-1.242	-0.226	0.749	-2.316	-1.28	-0.983	0.610	-0.986	-1.329
	(-59.27)	(-5.58)	(-7105)	(3.44)	(-605)	(-13.15)	(-8371)	(4.64)	(-2316)	(-14.54)
$ma2$	-1.039	0.593	0.250	-	2.487	0.799	.		0.144	0.867
	(-33.73)	(3.78)	(4754)		(129)	(4.29)			(1297)	(7.86)
$ma3$	-0.886	-	-0.279	-	-1.401	-			-0.11	
	(-8.01)		(-2175)		(-423)				(-2815)	
$ma4$	0.886	-	0.942	-	0.245	-			0.502	
	(-538.5)		(9623)		(22.02)				(2014)	
$ma5$		-		-	0.041	-			-	
					(3.72)					
ω	0.355	0.014	0.023	0.061	0.021	0.106	0.029	0.043	0.049	0.048
	(4.89)	(4.38)	(5.089)	(5.14)	(4.50)	(3.79)	(3.58)	(4.41)	(2.07)	(3.59)
α	0.111	0.028	0.004	0.044	0.002	0.036	0.060	0.046	0.075	0.078
	(1.25)	(2.63)	(0.41)	(4.19)	(4.50)	(3.45)	(6.14)	(4.21)	(4.76)	(5.16)
β	0.896	0.909	0.887	0.875	0.891	0.883	0.907	0.874	0.862	0.865
	(76.76)	(78.62)	(73.42.)	(59.31)	(3.32)	(47.65)	(90.85)	(62.07)	(26.45)	(43.35)
γ	0.146	0.090	0.175	0.107	0.184	0.095	0.056	0.128	0.095	0.085
	(7.72)	(5.42)	(8.48)	(5.82)	(7.33)	(3.92)	(3.25)	(5.78)	(3.0)	(3.81)
φ	10.38	9.045	11.75	8.906	7.191	10.08	4.251	7.139	5.084	5.181
	(6.71)	(7.50)	(5.86)	(8.43)	(8.65)	(5.87)	(13.45)	(9.58)	(11.24)	(11.81)

Figure 1. Log returns of equity indices

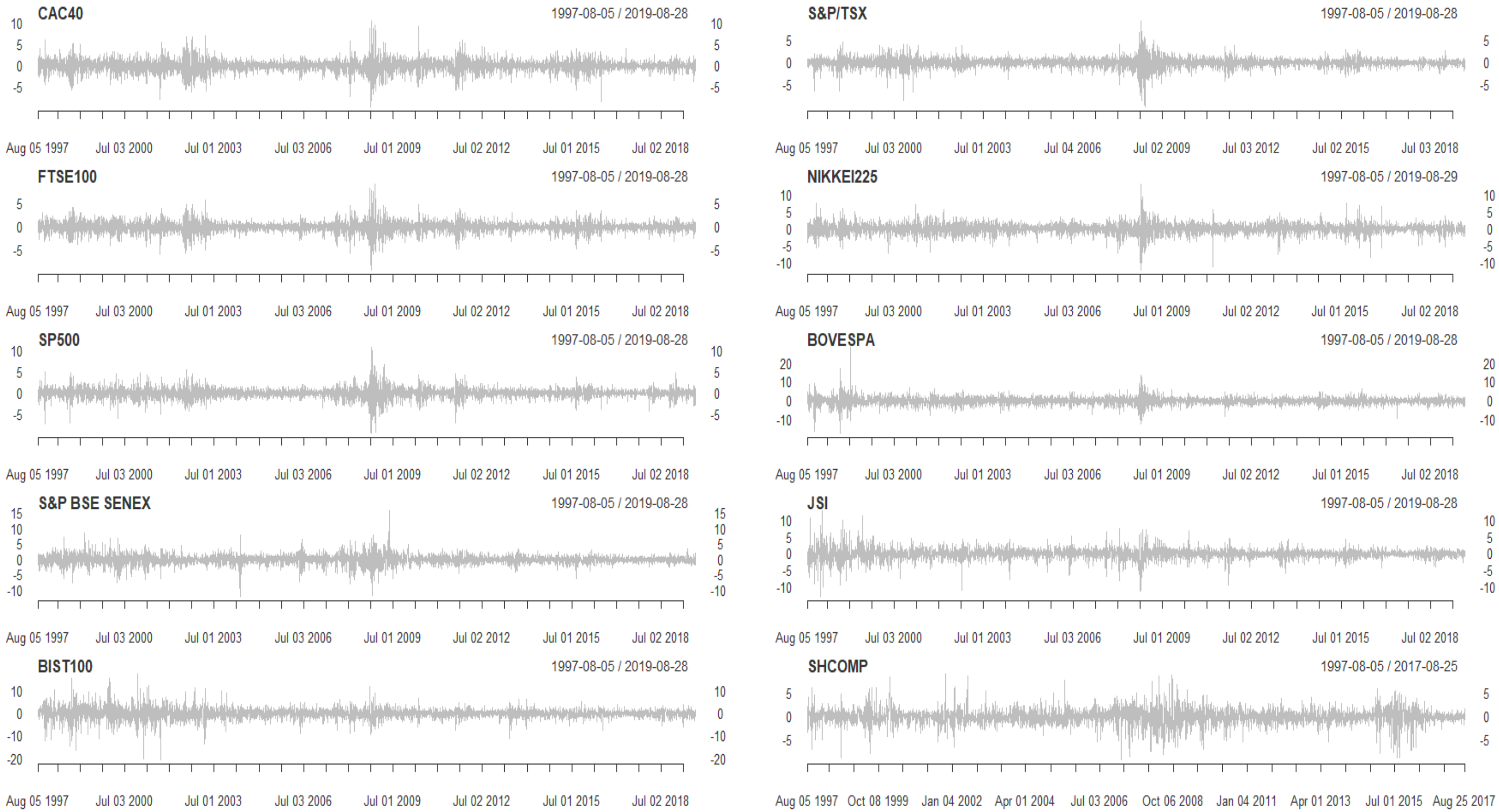


Figure 2. Risk Reward Plot

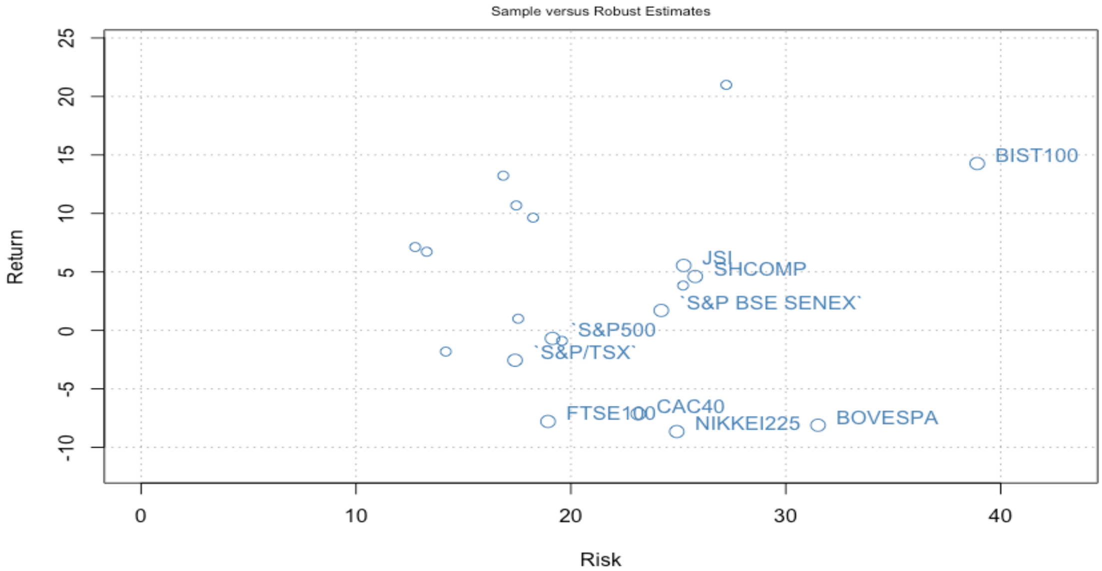
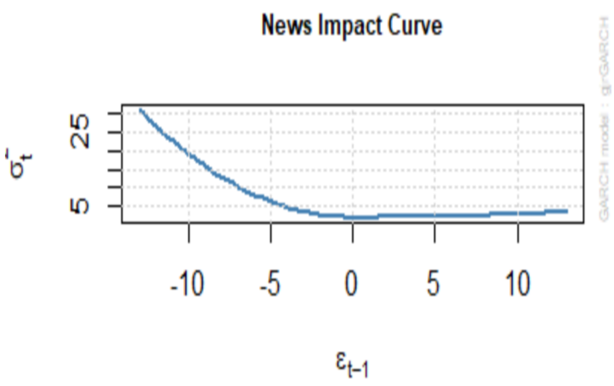
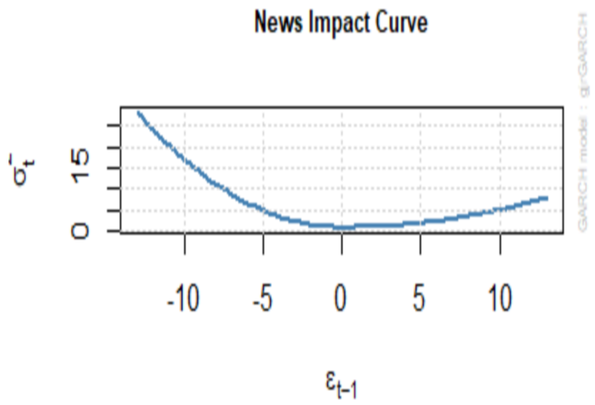


Figure 3. News impact curves



CAC40



DAX30

Figure 4. Histogram of the left tail of CAC 40 obtained for POT and BMM methods

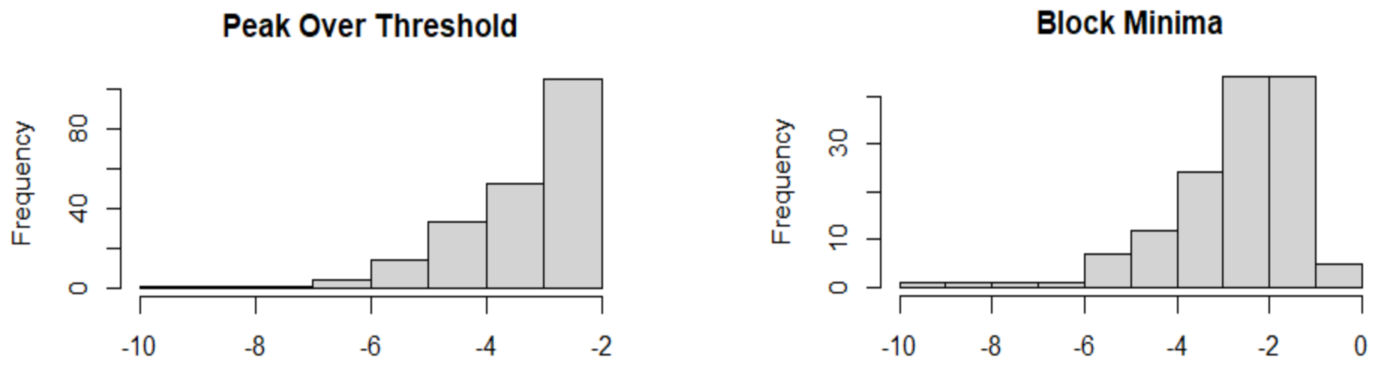


Figure 5. QQ plot of the residuals of selected stock returns

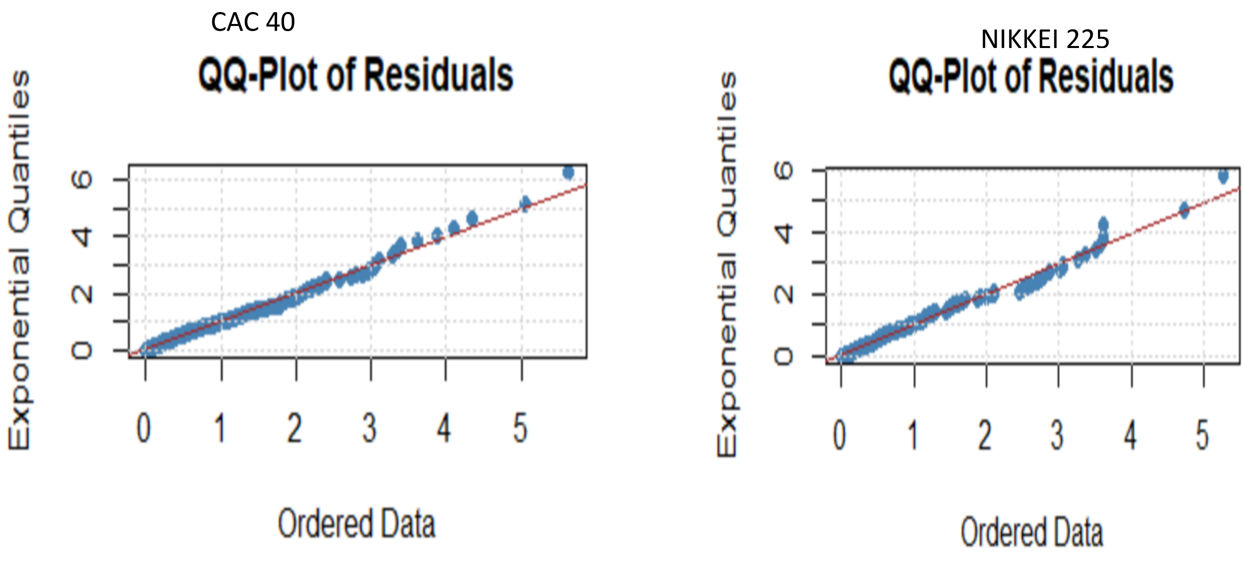


Figure 6 : Efficient Portfolio Frontier for Generalised Pareto Distribution

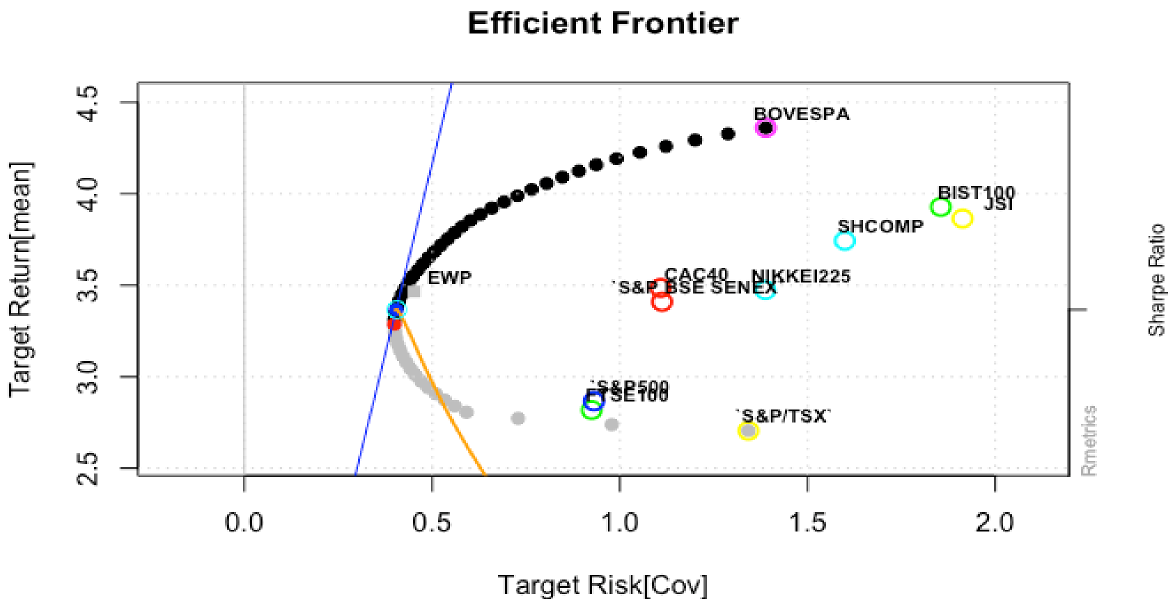


Table 1. Descriptive Statistics

Stock Market	Mean	Standard deviation	Skewness	Kurtosis
Developed Markets Indices				
CAC 40	-0.03	1.46	-0.29	3.57
S&P/TSX	-0.01	1.10	-0.55	8.18
FTSE 100	-0.03	1.19	-0.42	4.89
NIKKEI 225	-0.03	1.57	-0.44	5.88
S&P500	0.00	1.21	-0.53	5.40
Emerging Markets Indices				
BOVESPA	-0.03	1.98	-0.65	5.76
SCHOMP	0.02	1.62	-0.29	5.08
S&P BSE SENEX	0.01	1.52	-0.44	4.69
JSE	0.02	1.59	-0.25	7.83
BIST 100	0.06	2.45	-0.21	7.70

Table 2. Mean-Variance GPD International Portfolio

Portfolio	GENERALISED PARETO DISTRIBUTION	
	Optimal portfolio Weights	**Tangent portfolio weights**
Developed Markets Indices		
CAC 40	0.1233	0.1311
S&P/TSX	0.0844	0.0676
FTSE 100	0.1865	0.1585
NIKKEI 225	0.0836	0.0879
S&P 500	0.1858	0.1614
Emerging Markets Indices		
BOVESPA	0.0783	0.1366
SHOMP	0.0586	0.0745
S&P BSE SENEX	0.1185	0.1243
JSE	0.0402	0.0551
BIST 100	0.0407	0.0589
Expected Return ($E[R]$)	3.2873	3.3670
Risk ($CVaR$)	-2.6836	-2.7534
Sharpe Ratio	**2.028**	**2.0773**
Sortino Ratio(MAR=0)	**3.133**	**3.2093**

Table 3: Mean-Variance developed countries portfolio weights

	MEAN-VARIANCE GPD	
Portfolio	**Optimal portfolio weights**	**Tangent portfolio weights**
Developed Markets Indices		
CAC 40	0.1873	0.2168
S&P/TSX	0.1304	0.1165
FTSE 100	0.2792	0.2596
NIKKEI 225	0.1252	0.1433
S&P 500	0.2779	0.2637
Expected Return ($\boldsymbol{E}[\boldsymbol{R}]$)	3.0240	3.0565
Risk ($\boldsymbol{CVaR}$)	-2.3498	-2.3793
Sharpe Ratio	**2.2885**	**2.3132**
Sortino Ratio(MAR=0)	**3.3638**	**3.4001**

Table 4: Mean-Variance GDP of emerging countries portfolio weights

	MEAN-VARIANCE	
Portfolio	**Optimal portfolio weights**	**Tangent portfolio weights**
Emerging markets		
BOVESPA	0.2275	0.2626
SHCOMP	0.1699	0.1670
S&P BSE SENEX	0.3554	0.3166
JSE	0.1186	0.1207
BIST 100	0.1286	0.1332
Expected Return ($E[R]$)	3.8037	3.8393
Risk ($CVaR$)	-2.8993	-2.9306
Sharpe Ratio	**2.3362**	**2.2942**
Sortino Ratio(MAR=0)	**3.5265**	**3.4633**

Table 5: Mean Variance GEV international portfolio weights

	Optimal portfolio weight	Tangent portfolio weight
Developed Markets Indices		
CAC 40	0.1449	0.1478
S&P/TSX	0.117	0.0919
FTSE 100	0.1863	0.1566
NIKKEI 225	0.1041	0.1191
S&P 500	0.1508	0.1311
Emerging Markets Indices		
BOVESPA	0.0644	0.0907
S&P BSE SENEX	0.0789	0.0789
IPC	0.0729	0.0782
JSI	0.0518	0.0581
BIST 100	0.029	0.0757
Expected Return (E[R])	2.449	2.5445
Risk (CVaR)	-1.6088	-1.6814
Sharpe Ratio	1.5827	1.6447
Sortino Ratio(MAR=0)	2.3923	2.4855

Table 6. Mean Variance GEV Developed market portfolio weights

		MEAN-VARIANCE	
		Optimal Portfolio weights	Tangent Portfolio weights
	Portfolios		
Developed Market Indices			
	CAC 40	0.2165	0.2388
	S&P/TSX	0.1711	0.1494
	FTSE 100	0.2813	0.2563
	NIKKEI 225	0.1659	0.2016
	S&P 500	0.1652	0.1538
Expected returns		2.3275	2.364
Risks (cVaR)		-1.3341	-1.516
Sharpe ratio		1.759	1.787
Sortino ratio		2.585	2.6272