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29 January 2024

Online at https://mpra.ub.uni-muenchen.de/119993/
MPRA Paper No. 119993, posted 08 Feb 2024 14:50 UTC

# Market Size, Trade, and Productivity Reconsidered: Poverty Traps and the Home Market Effect* 

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January 29, 2024


#### Abstract

To investigate questions related to migration and trade, a model of regional or international development is created by altering Melitz and Ottaviano (2008) to include a labor market. The model is then applied to analyze poverty traps and the home market effect. We find that in the spatial economics context of migration but no trade, poverty can persist unless population in one region of many is pushed past a threshold. Then growth commences. In the context of trade but no migration, the home market effect holds for a range of parameters, similar to previous literature. However, unlike previous literature, we find that if populations in countries are highly asymmetric, the home market effect can be reversed.


JEL Codes: F12, R11
Keywords: Monopolistic competition; Poverty trap; Home market effect; Labor market clearing

[^0]
## 1 Introduction

Can an insufficient labor supply cause a poverty trap?
We build on Ottaviano et al. (2002) and Melitz and Ottaviano (2008). Our simplest economy comprises one country and involves two sectors, a manufacturing sector that produces a differentiated product under monopolistic competition, and an agricultural sector that produces under constant returns to scale and perfect competition. Our twist on Melitz and Ottaviano (2008) is that we introduce a simple labor market clearing condition, absent there, to close the model. Surprisingly, this twist yields equilibrium behavior, including firm selection, that is different from and more complex than the earlier models even when there is only one location.

The model is then extended to multiple locations in order to examine the following applications.

We address the issue of poverty traps in a version this model where workers can migrate into a city from the hinterlands or other cities if it increases their utility. In contrast, most models of poverty traps, as surveyed in Azariadis (1996) for example, are based on aspatial models of growth.

We find that if the population is small, there is an equilibrium with an active agricultural sector but no manufacturing sector. Utility of consumers is relatively small. But if labor supply is pushed upward past a threshold, for example by subsidizing in-migration, utility in the region increases and more workers migrate into it. The manufacturing sector initiates production and grows. With an even larger population, wage increases and the agricultural sector ceases production, so there is a big manufacturing sector in the city but no agriculture. Eventually, wage is reduced to its original level, and agricultural production appears in conjunction with manufacturing. Above the lowest population threshold, indirect utility is monotonically increasing in population, which is the same as labor supply. So workers will continually migrate into the region once population is pushed past the lowest threshold. (In contrast, Melitz and Ottaviano (2008) only consider the case where the agricultural good and the manufactured good are always produced in the equilibrium of their basic model.)

This represents a poverty trap in the following sense. Intervention by an entity such as a government is necessary to improve welfare if the agents are myopic in that
they take the utility level in a region as given. If they are forward looking, they will not know which region or regions will have expanding population, so they will wait to migrate and the economy will experience a hold up problem. Either way, a region can become caught in a low utility poverty trap that can be avoided only by encouraging immigration past a threshold.

Next we turn to the home market effect (HME) for two countries in our context. The home market effect states in our model that the larger country should have a larger ratio of goods or firms to population. The model has two countries where trade but no migration is possible. We find that the home market effect holds for some parameter values where the populations in the two countries are relatively balanced, whereas it does not hold for parameter values where the populations are unbalanced. This is different from standard models of trade with increasing returns and trade cost, such as Krugman (1980), where the HME always holds. And as detailed in Medin (2017, p. 304), "The empirical evidence of the HME is also ambiguous."

Another important issue that we raise is that Melitz and Ottaviano (2008) implicitly use the homogeneous agricultural good as the input in the production function for manufactured goods that covers the fixed investment of setting up a factory. Instead, we use labor, as in Krugman (1980). Now for the firm, what the fixed setup cost is paid in (e.g. numéraire) is immaterial, as it's just money. We must, however, digress to primitives of the model in order to be precise about this important difference. The firm production function takes inputs to outputs, so it makes a difference there, as labor is used to cover building of a factory instead of agricultural good. Using agricultural good to build a factory makes no sense. But of most importance, the material balance or market clearance conditions for agricultural good and labor are affected by the choice of which commodity is used to build factories. This, in turn, affects the equilibrium price of labor or wage. That is why our results differ form Melitz and Ottaviano (2008), and why the introduction of a labor market matters.

Finally, in this entire class of models (including ours), notice that it is possible to reinterpret the agricultural sector as a service sector with a homogeneous good and a constant returns to scale production technology, provided that it is not traded.

The outline of the remainder of the paper is as follows. In Section 2 we provide our basic model, altering the Melitz and Ottaviano (2008) model to account for a
labor market. In Section 3 we provide our applications to poverty traps and the home market effect. Each requires that we modify our basic model slightly. Section 4 gives our conclusions and suggestions for future research. Appendix A contains a discussion of our basic model with no endowment of homogeneous agricultural good.

## 2 The Closed Model

### 2.1 Melitz and Ottaviano (2008)

We build on Melitz and Ottaviano (2008). The economy comprises one country and involves two sectors. The mass of consumers (or workers) is $L$. Each worker supplies exactly one unit of labor.

The preferences of a typical consumer are represented by the following utility function:

$$
\begin{equation*}
U=q_{0}+\alpha \int_{0}^{N} q_{i} \mathrm{~d} i-\frac{\gamma}{2} \int_{0}^{N}\left(q_{i}\right)^{2} \mathrm{~d} i-\frac{\eta}{2}\left(\int_{0}^{N} q_{i} \mathrm{~d} i\right)^{2}, \tag{1}
\end{equation*}
$$

where $q_{0}$ is the consumption of the homogeneous agricultural good, $q_{i}$ is the consumption of a differentiated manufacturing good of variety $i, N$ is the mass of varieties, whereas $\alpha>0, \gamma>0$, and $\eta>0$ are fixed utility parameters. ${ }^{1}$ Each individual maximizes her utility subject to the budget constraint:

$$
\begin{equation*}
q_{0}+\int_{0}^{N} p_{i} q_{i} \mathrm{~d} i=\widehat{q}_{0}+w \tag{2}
\end{equation*}
$$

where $p_{i}$ represents the price of the differentiated manufactured good $i, w$ is the wage of a consumer, and $\widehat{q}_{0}$ is an endowment of the homogeneous agricultural good, which is chosen as the numéraire. The endowment is supposed to be sufficiently large for the equilibrium consumption of the numéraire to be positive for each worker. The purpose of this assumption is to avoid corner solutions to the consumer optimization problem, and will be relaxed in the Appendix.

The first-order condition to maximize individual utility subject to the budget yields market demand for each variety $i$ of a manufactured good:

$$
\begin{equation*}
q_{i} L=\frac{\alpha L}{\gamma+\eta N}-\frac{L}{\gamma} p_{i}+\frac{\eta N L \bar{p}}{\gamma(\gamma+\eta N)}, \tag{3}
\end{equation*}
$$

[^1]where
$$
\bar{p} \equiv \frac{1}{N} \int_{i \in \Omega^{*}} p_{i} \mathrm{~d} i
$$
is the average price and $\Omega^{*}$ is the set of varieties with nonnegative demand $q_{i} \geq 0$.
Variety $i$ has nonnegative demand if and only if $q_{i} \geq 0$ in (3), or
\[

$$
\begin{equation*}
p_{i} \leq \frac{\alpha \gamma+\eta N \bar{p}}{\gamma+\eta N} \equiv p_{\max } \tag{4}
\end{equation*}
$$

\]

holds. Because product differentiation ensures a one-to-one relation between firms and varieties, the mass of firms and varieties is the same and is endogenously determined in equilibrium by free entry and exit of firms. Due to ex-post symmetry between varieties, we drop subscript $i$ hereafter.

Turning next to the production side of the model, firms in the numéraire or agricultural sector produce a homogenous agricultural good using labor under perfect competition and constant returns to scale. Units are chosen such that one unit of output requires one unit of labor. Assuming costless trade of the homogenous agricultural good, the equilibrium wage of workers is equal to 1 . However, for later use we will retain notation $w$ as the wage paid in the manufacturing sector, for example when agricultural good is not produced.

A monopolistically competitive firm produces one variety of the differentiated good under a technology that requires a fixed cost followed by constant returns to scale. The production technology of any variety requires labor input $c$ per unit and fixed labor input $f_{E}$ following Krugman (1980). ${ }^{2}$ Each firm, after payment of their entry cost, draws their marginal cost $c$ from a Pareto distribution

$$
G(c ; k)=\left(\frac{c}{c_{M}}\right)^{k}
$$

with support $\left[0, c_{M}\right]$, where $k>1$ is an exogenous parameter and $1 / c_{M}$ is the lower productivity bound. Firms that cannot cover their marginal cost exit, whereas all other firms survive.

Let $c_{D}$ represent the (endogenous) marginal cost of the type of firm that is indifferent between exiting and staying in the industry, namely the type of firm that earns

[^2]exactly zero profit. All firms who draw a higher marginal cost exit. Calculation of equilibrium proceeds exactly as in Melitz and Ottaviano (2008).

The total operating profit of a firm is given by

$$
\begin{equation*}
\pi(N, c)=(p(c)-w c) q(c) \tag{5}
\end{equation*}
$$

where $q(c)=q_{i} L$ is determined by (3).
The equilibrium operating profit of a firm with marginal cost $c$ is

$$
\begin{equation*}
\pi(c)=\frac{w^{2} L}{4 \gamma}\left(c_{D}-c\right)^{2} \tag{6}
\end{equation*}
$$

The variables whose equilibrium values are most important to us are $c_{D}$ and $N$ :

$$
\begin{align*}
c_{D}^{*} & =\left(\frac{\phi}{w L}\right)^{\frac{1}{k+2}}  \tag{7}\\
N^{*} & =\frac{2(k+1) \gamma}{\eta} \frac{\alpha-w c_{D}^{*}}{w c_{D}^{*}} \tag{8}
\end{align*}
$$

where $\phi \equiv 2(k+1)(k+2) \gamma c_{M}^{k} f_{E}$.
Since free entry is assumed, in equilibrium the firms must have zero expected profit. This condition is

$$
\begin{equation*}
\int_{0}^{c_{D}} \pi(c) \mathrm{d} G(c)=w f_{E} \tag{9}
\end{equation*}
$$

Melitz and Ottaviano (2008) assume $c_{M}>\left.c_{D}^{*}\right|_{w=1}$, or

$$
c_{M}>\sqrt{2(k+1)(k+2) \gamma f_{E} / w L} .
$$

So far, the derivations parallel those in Melitz and Ottaviano (2008).

### 2.2 Introduction of the Labor Market Clearing Condition

Melitz and Ottaviano (2008) use a fixed wage and thus, at least implicitly, a supply of labor that is infinitely elastic.

Plugging (7) into (8), the equilibrium number of firms is

$$
\begin{equation*}
N^{*}=\frac{2 \gamma(k+1)}{\eta}\left[\frac{\alpha}{w}\left(\frac{w L}{\phi}\right)^{\frac{1}{k+2}}-1\right] \tag{10}
\end{equation*}
$$

if $L>L_{0} \equiv \phi w^{k+1} / \alpha^{k+2}$. Whereas the equilibrium number of active firms is $N^{*}$, the equilibrium number of entrants is given by $N_{E}^{*}=N^{*} / G\left(c_{D}\right)$. This differs from
the number of active firms because some firms enter and then find that their draw of marginal cost is too high to produce.

Using (10) and (7), the aggregate demand for the labor in the manufacturing sector is computed as

$$
\begin{equation*}
L_{\text {demand }} \equiv \frac{N^{*}}{G\left(c_{D}^{*}\right)} \int_{0}^{c_{D}^{*}} c q(c) \mathrm{d} G(c)+N_{E}^{*} f_{E}=\frac{(k+1) L\left(\frac{\phi}{w L}\right)^{\frac{1}{k+2}}\left[\alpha-w\left(\frac{\phi}{w L}\right)^{\frac{1}{k+2}}\right]}{(k+2) \eta}, \tag{11}
\end{equation*}
$$

which is positive if $L>L_{0}$, i.e., both $L_{\text {demand }}$ and $N^{*}$ are positive if $L>L_{0}$.
When both agriculture and manufactured goods are produced, the equilibrium wage is equal to 1 . Since $L$ is the total supply of labor in the economy, the equilibrium number of agricultural workers is given by

$$
\left.L_{a}(L)\right|_{w=1} \equiv L-L_{\text {demand }}
$$

if it is positive. This condition does not appear in Melitz and Ottaviano (2008). Thus, we will be quite explicit below when we use it.

Solving $\left.L_{a}(L)\right|_{w=1}=0$ yields two solutions

$$
\begin{aligned}
& L_{1}=\left[\frac{(k+1) \alpha-\sqrt{(k+1)\left[(k+1) \alpha^{2}-4(k+2) \eta\right]}}{2(k+2) \eta}\right]^{k+2} \phi, \\
& L_{2}=\left[\frac{(k+1) \alpha+\sqrt{(k+1)\left[(k+1) \alpha^{2}-4(k+2) \eta\right]}}{2(k+2) \eta}\right]^{k+2} \phi,
\end{aligned}
$$

which are real if $\alpha \geq \alpha_{\min } \equiv 2 \sqrt{(k+2) \eta /(k+1)}$. The agricultural good is not produced in equilibrium if and only if $\left.L_{a}(L)\right|_{w=1} \leq 0$, which holds if and only if $L \in\left[L_{1}, L_{2}\right]$ and $\alpha \geq \alpha_{\text {min }}$. Otherwise, the agricultural good is produced. From (10), manufactured goods are produced in equilibrium if and only if $L>L_{0}$. Hence, we have the following proposition concerning a comparative static in $L$. At this point, it is exogenous, but it will be endogenous later. For the purpose of comparing the following Proposition with Melitz and Ottaviano (2008), both agricultural and manufactured goods are always produced in that model.

Proposition 1 If $\alpha>\alpha_{\min }$, then there are $0<L_{0}<L_{1}<L_{2}$ such that:
(i) only the agricultural good is produced for $L \leq L_{0}$;
(ii) both the agricultural and manufactured goods are produced for $L_{0}<L<L_{1}$;
(iii) only the manufactured goods are produced for $L_{1} \leq L \leq L_{2}$;
(iv) both the agricultural and manufactured goods are produced for $L>L_{2}$.

If $\alpha \leq \alpha_{\min }$, then there is $L_{0}$ such that:
(i) only the agricultural good is produced for $L \leq L_{0}$;
(ii) both the agricultural and manufactured goods are produced for $L>L_{0}$.

Remark 2 We interpret this result as follows. If the marginal utility of manufactured goods is sufficiently high, then there are 4 phases of production as labor supply increases upward from zero. First, only agricultural good is produced, then both types of goods are produced, followed by only manufactured goods, and finally, both goods. The transition between the last two phases is driven by keener competition in the manufacturing sector, which leads to reappearance of the agricultural sector. ${ }^{3}$ We may say that the increasing share of manufacturing labor in phase (ii) is industrialization, whereas the decreasing share in phase (iv) is deindustrialization. If marginal utility of manufactured goods is low, then there are only two phases as labor supply increases. First, only agricultural good is produced, then both goods are produced.

Remark 3 In contrast with Melitz and Ottaviano (2008), our use of labor in place of agricultural good to build factories combined with the introduction of a labor market makes a difference in results (iii) and (iv) of Proposition 1.

Therefore, the equilibrium use of manufacturing labor is given by

$$
L_{m}^{*}= \begin{cases}0 & \text { for } L \leq L_{0} \\ L_{\text {demand }} & \text { for } L_{0}<L<L_{1} \text { or } L>L_{2} \\ L & \text { for } L_{1} \leq L \leq L_{2}\end{cases}
$$

When we analyze equilibrium in applications, we will have to consider the case $L \in\left[L_{1}, L_{2}\right]$, which occurs when the homogeneous good, typically agriculture, is not produced due to various endogenous factors, such as a high wage in the manufacturing sector. In this case, the demand system is slightly different from the one in previous literature.

[^3]When agricultural good in not produced in equilibrium, wage $w$ is no longer equal to 1 and there is an additional equilibrium condition, $L=L_{m}$, that determines equilibrium $w^{*}(>1)$. The equilibrium $c_{D}^{*}$ and $N^{*}$ for $L \in\left[L_{1}, L_{2}\right]$ are obtained by substituting $w^{*}$ into (7) and (10), respectively.

## Comparative statics

Following the tradition of growth theory, we examine comparative statics with respect to an exogenous change in population $L$. We also consider comparative statics with respect to an exogenous change in manufacturing productivity; see Appendix B.

We can show that

$$
\begin{equation*}
\frac{\partial c_{D}^{*}}{\partial L}<0, \quad \frac{\partial N^{*}}{\partial L}>0, \quad \frac{\partial L_{m}^{*}}{\partial L}>0, \quad \frac{\partial U^{*}}{\partial L}>0 \tag{12}
\end{equation*}
$$

for all $L$.
On the interval $\left[L_{1}, L_{2}\right]$, equilibrium $w^{*}$ as a function of $L$ has an inverted U -shape. This is shown as follows.

Let $w=h(L)$ be the implicit function defined by the equation $L_{a}(L)=0$. We know that $w=h\left(L_{1}\right)=h\left(L_{2}\right)=1$ and that there are at most two solutions $L$ of $L_{a}(L)=0$, given $w$. This implies that $w=h(L)$ is either U-shaped or inverted Ushaped on the interval $\left[L_{1}, L_{2}\right]$. Since manufacturing labor demand $L_{m}$ is higher than labor supply $L$ when $w=1, w>1$ holds on the interval $\left[L_{1}, L_{2}\right.$ ]. Thus, $w=h(L)$ has an inverted U-shape.

Solving equation (10) for $w$ yields

$$
w=\left[\frac{2 \alpha}{\frac{\eta N^{*}}{\gamma(k+1)}+2}\right]^{\frac{k+2}{k+1}}\left(\frac{L}{\phi}\right)^{\frac{1}{k+1}},
$$

and thus,

$$
\frac{d w}{d L}=\frac{\partial w}{\partial L}+\frac{\partial w}{\partial N_{-}^{*}} \frac{\partial N^{*}}{\partial L}
$$

Therefore, the sign of $d w / d L$ depends on the two terms on the right hand side. The first term is the size effect. Keeping $N^{*}$ constant, a larger market size $L$ leads to higher profits and a higher wage. The second term is the procompetitive effect. A larger market size intensifies competition and reduces profits and the wage. As $L$ increases, the former effect dominates the latter in the initial phase of $\left[L_{1}, L_{2}\right]$ so that $\partial w / \partial L$ is positive, whereas the opposite is true in the later phase of $\left[L_{1}, L_{2}\right]$.

The excess demand for manufacturing labor is given by $-L_{a}(L)$. The inverted U-shaped relationship implies that as $L$ gets larger, the excess demand $-L_{a}(L)$ at $w=1$ initially increases and raises the wage $w$. In contrast, as $L$ gets even larger, excess demand decreases and lowers the wage.

The intuition for the comparative static of wage on labor or population is as follows. The exogenous labor supply can be represented by a vertical supply curve. As $L$ increases, the supply curve shifts to the right. Demand for labor by the manufacturing sector is given by a downward sloping derived demand curve. As $L$ increases, it shifts to the right. Whether wage increases or decreases depends on how fast demand shifts to the right relative to supply. At levels of population just over $L_{1}$, demand shifts to the right faster than the supply curve, so wage increases. At levels of population just below $L_{2}$, supply shifts faster than demand, so wage decreases with $L$.

### 2.3 Numerical Simulations

Here we present numerical simulations of the model detailed in this section.
Set $\alpha=2.4, c_{M}=f_{E}=\gamma=\eta=1$, and $k=2$. In this case, the calculated values of the thresholds are: $L_{0}=0.72, L_{1}=4.4$, and $L_{2}=41$. We put labor usage $L$ on the horizontal axis in Figure 1, and suppose that we increase $L$ monotonically from 0. Since $\alpha \geq \alpha_{\min }$, the first case in Proposition 1 applies. As $L$ increases beyond $L_{0}$, both $N^{*}$ and $U^{*}$ continuously increase whereas $c_{D}^{*}$ decreases regardless of whether the agricultural good is produced or not. The manufacturing wage $w^{*}$ is equal to 1 for $L \in\left(0, L_{1}\right] \cup\left[L_{2}, \infty\right)$, whereas it is larger than 1 and inverted $U$-shaped for $L \in\left(L_{1}, L_{2}\right)$. For $L<L_{0}$, only agricultural good is produced, whereas wage and utility are constant. Beginning at $L_{0}$, the manufacturing sector initiates production. Then, at $L_{1}$, agricultural production ceases and wage rises. Eventually, wages reach a maximum and begin to decline. At $L_{2}$, wage returns to its original level, and agricultural production is re-initiated, joining manufacturing. Throughout, utility is (weakly) increasing in $L$.

## 3 Applications

### 3.1 Poverty Traps

Until this point, we have taken $L$ to be an exogenous parameter. Next we consider the case of many regions where $L$ is endogenously determined by the utility level available to consumers in the region, who are free to migrate to the region that offers the highest utility level to them. ${ }^{4}$ It is typical in this variety of model to limit migration at a given time to be proportional to the utility differences between regions; see for example Krugman (1991). There is no trade. Consider first the situation where all regions have the same population $L \leq L_{0}$. Then it is an agricultural economy. Indirect utility as a function of population is the same for all population levels below $L_{0}$. To obtain higher utility, any given region must be pushed past the $L_{0}$ population threshold by encouraging immigration. This could be achieved by subsidizing immigration into the region from the others. Once population size $L_{0}$ is passed, utility is higher in the target region, and utility is strictly increasing in $L$. Then population and utility growth are self-sustaining for this region, and manufacturing is initiated. Thus, the poverty trap is a result of paucity of population in regions or cities.

The preceding discussion presumes that consumers are myopic, in that they migrate to where utility is highest, without foresight. This is common in the urban economics literature. We can account for foresight as follows. Informally, if consumers cannot predict which region will grow, they will wait until they observe growth empirically before moving or, alternatively, they wait for the government to choose the region that will be subsidized. So without an explicit government policy, they will wait until other consumers select a region and migrate. Thus, there can be a bad equilibrium where every consumer is waiting for others to migrate.

### 3.2 The Home Market Effect

Next we consider a model of trade, where consumers are completely immobile in the context of two regions or countries. In this subsection, the thresholds will differ from those discussed previously.

[^4]The countries will be denoted by $l=H, F$ for Home and Foreign. Their respective populations are $L^{l}(l=H, F)$, assumed to be fixed in this subsection. The barriers to imports will be denoted by $\tau>1$, where 1 unit of a commodity arrives at its destination when $\tau$ units are shipped.

Repeating the previous calculations for this slightly modified model, the upper cutoff of marginal cost for firms that will produce differentiated good for the domestic market in country $l$ at equilibrium is

$$
c_{D}^{l}=\left[\frac{\gamma \phi\left(w^{h}-w^{l} \tau^{-k}\right)}{L^{l}\left(w^{l} w^{h}\right)^{2}\left(1-\tau^{-2 k}\right)}\right]^{\frac{1}{k+2}}
$$

and that for the export market is $c_{X}^{h}=c_{D}^{l} / \tau$. The number of firms or varieties active in country $l$ is

$$
N^{l}=\frac{2(k+1) \gamma\left(\alpha-w^{l} c_{D}^{l}\right)}{\eta w^{l} c_{D}^{l}} .
$$

The first threshold, called $L_{0}$ above, is the lowest population where manufacturing occurs. It solves $\alpha-w^{l} c_{D}^{l}=0$ with $w^{l}=1$. In the current context with trade, it will be a function of the (fixed) population in the other country.

Turning next to the other thresholds, $L_{1}$ is the lowest population where only manufacturing is active, whereas $L_{2}$ is the highest population where only manufacturing is active. The analogs here will be functions of the (fixed) population in the other country. Define:

$$
L_{m}^{l} \equiv N^{l} \frac{\int_{0}^{c_{D}^{l}} c q_{i}^{l} L^{l} \mathrm{~d} G(c)+\int_{0}^{c_{X}^{l}} c q_{i}^{h} L^{h} \mathrm{~d} G(c)}{G\left(c_{D}^{l}\right)}=N^{l} \frac{k\left(1+\tau^{-k}\right) c_{D}^{l}}{k+1} .
$$

The analogs of $L_{1}$ and $L_{2}$ are now solutions of $L^{l}=L_{m}^{l}$ and $L^{h}=L_{m}^{h}$ with $w^{l}=w^{h}=1$. Calculating utility levels in the two countries,

$$
U^{l}=1+\frac{1}{2 \eta}\left(\alpha-w^{l} c_{D}^{l}\right)\left(\alpha-\frac{k+1}{k+2} w^{l} c_{D}^{l}\right) .
$$

For further analysis, please refer to Figure 2, using the same parameter values as for the previous numerical simulations with $\tau=10$. In Figure 2, the exogenous population of country $H, L^{H}$, is on the horizontal axis, whereas the exogenous population of country $F, L^{F}$, is on the vertical axis. The lines represent the the thresholds discussed above, that are functions of both countries' population.

According to equation (15) in Behrens et al. (2009), the home market effect in this model holds if and only if:

$$
L^{l}>L^{h} \text { implies } \frac{N^{l}}{L^{l}}>\frac{N^{h}}{L^{h}} .
$$

In words, the larger country has a larger ratio of firms to population. For populations that are relatively balanced, in other words in the middle of the Figure, the home market effect holds, since no agricultural good is produced in either country. However, if populations in the two countries are imbalanced, for example $L^{H}=30$ and $L^{F}=60$, then $N^{H} / L^{H}=0.28>0.20=N^{F} / L^{F}$, implying that the home market effect no longer holds. That is because in equilibrium, the smaller country $H$ produces no agricultural good, only manufactured goods, whereas the larger country $F$ produces both agricultural and manufactured goods.

## 4 Conclusions

We have reexamined a standard model of monopolistic competition in the frameworks of regional economics and international trade, introducing a simple labor market. In the context of regional economics, namely of free migration but no trade, complex behavior in the form of a poverty trap is a result, and policies that encourage immigration can overcome the trap. In the context of international trade, namely of costly trade but no migration, the home market effect can disappear if populations are imbalanced.

Future work should consider both costly trade and migration in the same model, as well as normative questions such as optimal trade and migration policy. Moreover, dynamic versions of the model with capital accumulation and endogenous technological progress should be examined.

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## 5 Appendix A: The Model with No Endowment of Agricultural Good

In this appendix, we examine our model with no endowment of agricultural commodity, as in Arkolakis (2008) and Demidova (2017). The purpose is twofold. First, it seems like a reasonable assumption relative to the real world. Second, we wonder how robust the model is to such a small alteration.

Assume that, unlike Melitz and Ottaviano (2008), $\widehat{q}_{0}=0$. That is, the preferences of a typical consumer are represented by (1), but the budget constraint (2) is replaced with

$$
\begin{equation*}
q_{0}+\int_{0}^{N} p_{i} q_{i} \mathrm{~d} i=w+\frac{N_{E} \cdot \Pi}{L}, \tag{13}
\end{equation*}
$$

where $\Pi$ is the average profit of a firm and $N_{E}$ is the number of firms that enter, i.e. pay the fixed cost. As is apparent, we assume that consumers in the one region or country are endowed with equal profit shares in the firms. ${ }^{5}$ The firms with $c \in\left[c_{D}, c_{M}\right]$ earn negative profit after sinking the fixed cost and exiting. Hence the free entry condition (9) and the law of large numbers implies that $\Pi=0$.

The free entry condition is rewritten as

$$
\begin{equation*}
\int_{0}^{c_{D}} p(c) q(c) \mathrm{d} G(c)-w \int_{0}^{c_{D}} c q(c) \mathrm{d} G(c)=w f_{E} \tag{14}
\end{equation*}
$$

Using the budget constraint $q_{0}+\int_{0}^{N} p_{i} q_{i} \mathrm{~d} i=w$, the total demand for agricultural good is given by

$$
\begin{equation*}
Q_{0}^{\text {demand }}=q_{0} L=\left(w-\int_{0}^{N} p_{i} q_{i} \mathrm{~d} i\right) L \tag{15}
\end{equation*}
$$

The total supply of agricultural good is given by

$$
Q_{0}^{\text {supply }}=L-L^{\text {demand }}=L-N\left[\frac{\int_{0}^{c_{D}} c q(c) \mathrm{d} G(c)}{G\left(c_{D}\right)}\right]-N_{E} f_{E} .
$$

Using (14), this is rewritten as

$$
\begin{equation*}
Q_{0}^{\text {supply }}=L-\frac{N}{w} \frac{\int_{0}^{c_{D}} p(c) q(c) \mathrm{d} G(c)}{G\left(c_{D}\right)} . \tag{16}
\end{equation*}
$$

[^5]Since $\int_{0}^{c_{D}} p(c) q(c) \mathrm{d} G(c) / G\left(c_{D}\right)$ is the average revenue per firm and $\int_{0}^{N} p_{i} q_{i} \mathrm{~d} i$ is the expenditure for the manufactured goods per consumer,

$$
N \frac{\int_{0}^{c_{D}} p(c) q(c) \mathrm{d} G(c)}{G\left(c_{D}\right)}=L \int_{0}^{N} p_{i} q_{i} \mathrm{~d} i
$$

Plugging the left hand side of this equation into (16) and noting that $N_{E}=N / G\left(c_{D}\right)$ yields

$$
\begin{equation*}
Q_{0}^{\text {supply }}=L-\frac{L}{w} \int_{0}^{N} p_{i} q_{i} \mathrm{~d} i . \tag{17}
\end{equation*}
$$

Plugging $w=1$ into (15) and (17), we confirm that $Q_{0}^{\text {supply }}=Q_{0}^{\text {demand }}$.
(a) When the agricultural good is produced and consumed, $Q_{0}^{\text {supply }}=Q_{0}^{\text {demand }}$ should be positive. This is true in phase (ii) of Proposition 1 when $L_{0}<L<L_{1}$ and phase (iv) when $L>L_{2}$. Insofar as the agricultural good is produced and consumed, all the derivations and equilibrium values in the model with sufficiently large endowment are the same as those in the model with no endowment. This means that as long as the agricultural good is produced, we don't need the assumption of a sufficiently large endowment.
(b) However, this does not apply when the agricultural good is not produced for $L_{1} \leq L \leq L_{2}$. In the model with a sufficiently large endowment, the endowment of the agricultural good is always consumed even when the agricultural good is not produced. In contrast, with no endowment of agricultural good, when the agricultural good is not produced, it cannot be consumed. The latter case is analyzed in Arkolakis (2008).

## 6 Appendix B: Comparative statics on the upper productivity bound $c_{M}$

As technology improves with innovation, the upper bound $c_{M}$ on the distribution of marginal cost in manufacturing decreases. Then, we can show that

$$
\frac{\partial c_{D}^{*}}{\partial c_{M}}>0, \quad \frac{\partial N^{*}}{\partial c_{M}}<0
$$

for all $c_{M}$. That is, the impact of reducing the upper productivity bound $c_{M}$ is opposite that of increasing population $L$. Fixing the same population size $L_{1}=$
$L_{2}=L$, we can also show that the marginal change in labor usage by manufacturing firms is given by

$$
\frac{\partial L_{m}^{*}}{\partial c_{M}} \begin{cases}=0 \text { for } c_{M} \geq c_{M_{0}} & \text { phase (i) }  \tag{18}\\ <0 \text { for } c_{M_{0}}>c_{M}>c_{M_{1}} & \text { phase (ii) } \\ =0 \text { for } c_{M_{1}} \geq c_{M} \geq c_{M_{2}} & \text { phase (iii) } \\ >0 \text { for } c_{M}<c_{M_{2}} & \text { phase (iv) }\end{cases}
$$

where $c_{M_{0}}, c_{M_{1}}$, and $c_{M_{2}}\left(c_{M_{0}}>c_{M_{1}}>c_{M_{2}}\right)$ correspond to $L_{0}, L_{1}$, and $L_{2}\left(L_{0}<L_{1}<\right.$ $L_{2}$ ). When the upper productivity bound $c_{M}$ falls to $c_{M_{0}}$ (phase (i)), manufacturing labor demand $L_{m}^{*}$ starts increasing until it hits the labor supply constraint $L$ at $c_{M}=c_{M_{1}}$ (phase (ii)). Manufacturing labor demand $L_{m}^{*}$ is equal to $L$ until $c_{M}$ falls to $c_{M_{2}}$ (phase (iii)). Then, a further fall in $c_{M}$ decreases manufacturing labor demand $L_{m}^{*}$ (phase (iv)). In sum, $L_{m}^{*}$ is inverted U-shaped, which is in accord with the empirical findings; see Herrendorf et al. (2014).

Phase (ii) of increasing $L_{m}^{*}$ (the third line in (18)) occurs because faster productivity growth in the manufacturing sector induces more workers to abandon the agricultural sector. This conforms with the size effect in the comparative statics on population $L$. In contrast, phase (iv) of decreasing $L_{m}^{*}$ (the first line in (18)) occurs because productivity gains in the manufacturing sector push workers out of the manufacturing sector (Matsuyama, 2008). This is consistent with the procompetitive effect in the comparative statics on population.

To make this concrete, consider a developed country $H$ and developing country $F$, with different production technologies $c_{M}^{H}<c_{M}^{F}$. When both are large and $c_{M}^{F} \geq$ $c_{M_{0}}>c_{M}^{H}$, the developing country produces the agricultural good only (phase (i)), whereas the developed country produces both agricultural and manufacturing goods (phase (ii)). This implies interindustry trade. When $c_{M_{0}}>c_{M}^{F}>c_{M_{1}}>c_{M}^{H}$, the developing country produces the agricultural and manufacturing goods (phase (ii)), whereas the developed country produces the manufacturing good only (phase (iii)). This is both interindustry and intraindustry trade (Grubel and Lloyd, 1975). When $c_{M_{1}}>c_{M}^{F}>c_{M_{2}}>c_{M}^{H}$, the developing country produces the manufacturing good only (phase (iii)), whereas the developed country produces both the agricultural and manufacturing goods (phase (iv)). This is both interindustry and intraindustry trade. It is worth noting that the transition from phases (iii) to (iv) is deindustrialization in
the developed country, which is not only due to the shift of manufacturing production from developed to developing countries, but also due to technological progress in artificial intelligence and robotization of manufacturing production in the developing country (Mayer, 2018). Thus, the model in Melitz and Ottaviano (2008) can describe complex and detailed stylized facts in economic development once a labor market clearing condition is introduced, as we have done in Section 2.2.

Figure 1: The equilibrium number of firms, utility, cost cut-off, agricultural labor, and wage



Figure 2: The home market effect
(blue: $L_{0}^{H}=0$, orange: $L_{0}^{F}=0$, left green: $L_{1}^{H}=L_{m}^{H}$, right green: $L_{2}^{H}=L_{m}^{H}$, lower red: $L_{1}^{F}=L_{m}^{F}$, upper red: $L_{2}^{F}=L_{m}^{F}$ )



[^0]:    *The authors thank Carsten Eckel, Kohei Takeda, and Pierre-Olivier Weill for helpful comments, but retain responsibility for any errors.
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[^1]:    ${ }^{1}$ We cannot obtain analytical results if we alter the utility function, for example to a Stone-Geary subutility.

[^2]:    ${ }^{2}$ Melitz (2003) and Melitz and Ottaviano (2008) assume that $f_{E}$ is paid in the numéraire, which is the agricultural good. However, as explained in the introduction, it does not make sense to use the agricultural good to build a factory.

[^3]:    ${ }^{3}$ That is, as $L$ gets large, the cut-off $c_{D}^{*}$ goes down, which decreases the profit of each manufacturing firm, and thus decreases the manufacturing wage, so that the agricultural sector reappears for sufficiently large $L$.

[^4]:    ${ }^{4}$ For our purposes, either a finite number of regions or an open city model can be used here.

[^5]:    ${ }^{5}$ This is actually irrelevant, since we prove in the next few lines that $\Pi=0$.

