The Value of Anonymous Option

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Abstract. Personal data protection regulations typically require a seller to obtain consumers' explicit consent before processing their information. We model this requirement as an *anonymous option*, allowing consumers to maintain their anonymity when purchasing a product from a seller. We analyze a monopolist's incentive to offer such an option in a model of repeated purchases and limited commitment. Although collecting information implies full consumer surplus extraction in the second period, the seller is better off by offering the anonymous option. This is because it enables the seller to commit to a high second-period price for unrecognized consumers and prevents the consumers' strategic delay of consumption in the first period. In contrast, consumers are worse off because of increased prices and reduced demand. Consequently, privacy regulations mandating a compulsory anonymous option may actually fail to protect consumers' welfare.

Keywords: anonymous option; personalized pricing; privacy concern

JEL Codes: D4, D8, L1

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1 Introduction

In repeated interactions, sellers typically collect consumers' data in the initial period and subsequently utilize this data for price discrimination in later periods.[1] It is widely observed that online shopping platforms offer discounts to encourage first-time consumers to create an account and offer varied prices to repeated consumers based on their historical purchases. On the demand side, consumers are increasingly expressing concerns about the potential economic exploitations they may face when disclosing information to the seller (*instrumental privacy concerns*). Moreover, many consumers have an intrinsic aversion for disclosing their personal data (*intrinsic privacy concerns*).[2] Due to these privacy concerns, consumers act strategically in the initial interaction and may be reluctant to disclose their personal data in the first place.

To address consumers' prevalent privacy concerns, policymakers worldwide have enacted personal data protection regulations, such as the EU's General Data Protection Regulation, the Personal Information Protection Law of PRC, among others. These regulations mandate that sellers must obtain consumers' explicit consent before processing their personal data. In response, many sellers have begun allowing consumers to customize their data collection settings by offering an anonymous option. Under this option, consumers can choose between opting-in for personal data collection or opting-out for no data collection when purchasing products or services from the seller. Opting for no data collection enables consumers to maintain their anonymity, and the seller remains uninformed about their information, including purchasing records and payment methods. For instance, Amazon allows users to purchase items with a guest account that does not require a personal account. Taobao gives consumers the option of "buying anonymously" and "making payment through a third-party".

In this paper, we explore the tradeoff between the consumers' incentive to disclose personal information and a monopoly seller's incentive to engage in personalized pricing after acquiring such information. Our focus is on whether the seller has an incentive to pro-

[1] See, e.g., Villas-Boas 1999, 2004; Taylor 2004 for classic contributions on such settings.

[2] Potential information leakage and wrongful usage of data further intensify such intrinsic privacy concerns.

vide an anonymous option, and whether such an option can effectively protect consumers' interests. We set up a two-period model in which consumers repeatedly purchase a non-durable good from a monopoly seller. If a consumer discloses her information in period 1, the seller learns her valuation and utilizes this information to implement personalized pricing in period 2 when the consumer makes a repeat purchase.[3] For consumers who maintain their anonymity, either by choosing the anonymous option or by not purchasing the product in period 1, the seller does not recognize these consumers and consequently charges them a uniform price in period 2.[4]

The expected usage of consumers' personal information for price discrimination influences a consumer's willingness to disclose such information initially. Moreover, consumers inherently dislike disclosing information and suffer a direct utility loss, denoted as $K \geq 0$ when they opt in for disclosure. Consequently, racheting forces determine the level of indirect compensation that consumers require for the information they disclose. Consumers refrain from disclosing their information unless adequately compensated in period 1.[5]

To analyse the effects of anonymous option, we first analyse a benchmark in which such an option is unavailable. In this scenario, the seller either sells the product to the consumers without collecting information (*no-disclosure mode*), or sells to consumers only if they disclose information (*disclosure mode*). Under the no-disclosure mode, the game degenerates into a repeated static monopoly problem and the seller charges consumers a uniform price equal to the static monopoly price in both periods. Under the disclosure mode, consumers with high valuations purchase and disclose information, while consumers with low valuations do not purchase, thereby keeping their information undisclosed to the seller in period 1. The period-2 market is then divided into a recognized segment where consumers' valuations have been revealed to the seller and an unrecognized segment where consumers' valuations remain unknown to the seller. The seller infers that consumers in

[3] We assume that the seller practices first-degree price discrimination after learning the consumers' valuations, providing the seller with the strongest incentive to collect consumer data in period 1.

[4] The profitability of using consumer information to facilitate price discrimination raises the issue of the endogenous availability of such information. Specifically, information is rarely purchased directly from a consumer in exchange for a monetary payment, it is often the case that information must be sourced indirectly, by recording the consumer's actions, such as their purchase histories. This is how our paper differs from the recent contribution by Choe, Matsushima, and Shekhar (2023).

[5] In a later section, we discuss the case of a negative K.

the unrecognized segment have low valuations and offers them a relatively low uniform price in period 2. In period 1, anticipating a relatively low uniform price for unrecognized consumers and considering the utility loss from information disclosure, consumers strategically delay their consumptions even if their valuations exceed the posted first-period price.

In the benchmark case without anonymous option, consumers need to receive a higher utility than that from not purchasing to disclose information. Moreover, consumers need to be compensated for their intrinsic loss when disclosing information. Under the disclosure mode, the racheting effect pushes down the first-period price, leading to a larger demand in both period 1 and period 2. In comparison to the static monopoly case, the seller obtains a larger profit in period 2. However, his profit is lower than the static monopoly profit in period 1. When K is small, the positive effect on the second-period profit dominates and the seller chooses the disclosure mode. Conversely, when K is large, the strategic delay effect dominates and it is optimal for the seller to choose the no-disclosure mode.

We then proceed to analyse the main model with anonymous option, in which the seller offers the consumers a menu consisting of three options: *anonymous option*, *disclosure option*, and *not purchasing*. If the anonymous option and the disclosure option specify the same prices for the product, all consumers will choose the anonymous option because it spares them from the intrinsic utility loss from disclosure and the instrumental utility loss from future price discrimination. Thus, for consumers to choose the disclosure option, they must be better off than choosing the anonymous option and not purchasing, and this imposes more constraints for consumers to disclose information relative to the benchmark.

We show that when K is small, the game has a unique partial disclosure equilibrium. In this equilibrium, consumers with high valuations choose the anonymous option, those with intermediate valuations choose the disclosure option, and consumers with low valuations do not purchase in period 1. The second-period market is then divided into two segments: a recognized segment consisting of consumers with intermediate valuations who have chosen the disclosure option, and an unrecognized segment consisting of consumers with high valuations who have chosen the anonymous option, along with low-valuation consumers who have not purchased in period 1. Since high- and low- valuation consumers pool together in the unrecognized segment, it is optimal for the seller to choose a high uniform price to maximize second-period profit.[6]

Since information disclosure leads to maximal rent extraction in period 2, one would naturally expect that offering the anonymous option hurts the seller. Surprisingly, the opposite turns out to be true: offering the option benefits the seller. The main driving force is a commitment effect: the anonymous option serves the seller as an effective commitment device that helps to sustain a high second-period uniform price. Anticipating a high uniform price for unrecognized consumers in period 2, consumers have no incentive to strategically delay their consumptions in period 1. The removal of consumers' racheting incentives increases the seller's profit strictly above that in the benchmark scenario, regardless of whether the seller adopts the disclosure mode or the no-disclosure mode.

How does the anonymous option affect consumer privacy and consumer surplus? We show that with the option, more (fewer) consumers disclose their information in equilibrium under an intermediate (low) K, and the same amount of information is disclosed under a high K in comparison with the benchmark. In the partial disclosure equilibrium, consumers with high valuations benefit from the option by receiving a positive rent in period 2 and avoiding the intrinsic utility loss from disclosing information. However, the presence of high-valuation consumers keeping anonymity drives up the uniform price for unrecognized consumers in period 2. This imposes a negative externality on consumers with intermediate and low valuations: they can no longer benefit from a lower second-period price by strategically delaying their consumption. In aggregate, the anonymous option reduces consumers surplus because the average equilibrium prices are higher and the consumed quantities are lower.

Since the anonymous option affects the seller's profit and the consumer surplus in opposite directions, social welfare may decrease or increase, depending on the magnitude of K. With anonymous option, some high-valuation consumers increase their demand, while some relatively low-valuation consumers reduce their demand. The aggregate intrinsic utility loss depends both on the mass of consumers disclosing information and on the

[6]When K is large, the unique subgame perfect equilibrium is a no-disclosure one in which consumers choose either the anonymous option or not purchasing. The equilibrium outcome is the same as that under no anonymous option.

magnitude of K. When the benchmark equilibrium is disclosure, the anonymous option decreases social welfare due to a lower aggregate demand. When the benchmark equilibrium is no disclosure, for K not too large, the aggregate demand is higher with anonymous option. This demand expansion effect dominates the negative effect of increased intrinsic utility loss, and social welfare increases. This benefits on social welfare diminish as K gets larger. Thus, incremental social welfare exhibits an inverted U-shape with K when the pricing regime moves from no anonymous option to anonymous option.

Our analysis suggests that privacy regulations aiming to protect consumers' interests through mandating anonymous options may actually backfire and impact both consumer surplus and total welfare. The partial disclosure equilibrium offers a compelling explanation for the privacy paradox: some consumers express concern about privacy, but choose to disclose personal information even when they have an option to keep their anonymity. Anticipating a high future uniform price which exceeds their affordability, consumers see no benefits by strategically delaying their consumptions or hiding their identity. Thus, weighing the benefits from consuming the product against the potential loss from disclosing information, consumers opt in to disclose information when offered a sufficiently large discount in the initial interaction.

The paper proceeds as follows. In the remaining part of this section, we relate our work to the existing literature. In section 2, we describe the model setup. In section 3, we derive the equilibrium in a benchmark where the anonymous option is not available. In section 4, we fully characterize the equilibrium outcome of the game with anonymous option. A comparison of the equilibrium outcomes with and without anonymous option provides welfare implications. Section 5 contains some concluding remarks and discussions.

Related Literature. Our paper relates to the literature that studies the trade-off between consumer recognition and price discrimination in repeated interactions.[7] In this literature, consumers' instrumental privacy concerns motivate consumers to act strategically in their initial interaction with the seller. Taylor (2004) provides an early analysis of consumer privacy and the market for customer information. He shows that recogni-

[7] See Acquisti, Taylor, and Wagman(2016) and Goldfarb and Tucker (2019) for comprehensive reviews of the literature.

tion may either benefit or harm consumers depending on whether consumers are naive or sophisticated. Acquisti and Varian (2005) show that a monopoly seller will never find it optimal to price discriminate consumers conditional on their purchasing history. Doval and Skreta (2022) study the trade-off between a seller's choice of product lines which reveals more consumer information and rent extraction from second-period price discrimination, and show that by restricting the number of product lines in period 1, the seller can commit not to learn the consumers' information. These papers do not consider the usage of an anonymous option.

Our paper contributes to the fast-growing literature on consumer privacy management. Conitzer, Taylor and Wagman (2012) set up a model where consumers can hide their identity at a cost in period 2 and show that when all consumers can freely maintain their anonymity, they all individually choose to do so and this leads to the highest profit for the seller. Dengler and Prufer (2021) provide a micro-foundation for consumers' privacy choices when facing a seller with information technologies that enables perfect price discrimination. They show that total welfare is maximal at zero anonymization cost. Thus, eliminating anonymization costs by providing an anonymous option can benefit consumers. Lagerlöf (2023) analyses the efficiency level of consumers' hiding behavior when a monopoly seller bases prices on consumers' purchasing histories. He shows that increasing the cost of anonymity benefits consumers up to a certain threshold. We differ from these works in two respects: 1) in our setting the seller uses the anonymous option to screen the consumers; 2) in period 2, the seller is able to charge personalized prices which provides the seller the highest incentive to collect consumer information. Our framework allows for an explicit analysis of the welfare impact of the anonymous option.

In a two-market model where two firms collect data in one market and make use of the data for price setting in a second market, Cong and Matsushima (2023) study the welfare effect of consumers' withdrawal of personal data and show that allowing consumers to withdraw data can harm consumers.[8] In a concurrent work, Choe, Matsushima and

[8] In a multi-product and Bayesian persuasion setting, Ichihashi (2020) shows that consumers may be better off by pre-committing to withholding some information when purchasing from a multi-product monopolist. Ali, Lewis and Wasserman (2023) show that richer and more sophisticated information disclosure can benefit consumers. Rhodes and Zhou (2021) study consumer privacy choice in a general oligopoly model, and show that too much data sharing occurs in equilibrium by comparison with the consumer

Shekhar (2023) analyse the welfare effect of opt-in versus opt-out options in a monopoly setting when consumers differ in both valuations and privacy costs. Similar to our work, they also model the consent requirement as a screening device. However, our paper differs from theirs in two important respects. First, in their model, the seller's benefits from using consumer data are exogenous, while in our model, both the costs and the benefits of collecting consumer information are endogenous. Second, in their analysis, the main driving force is that the anonymous option expands demand by allowing consumers with high privacy costs to buy services without sharing data. In our analysis, the main driving force is consumers' strategic interaction in response to the seller's discriminatory pricing, and including an opt-in option provides the seller with commitment power. Due to these key differences, we draw drastically different results about the welfare implications of the anonymous option.

By considering a repeat purchase setting in which the seller cannot commit to future prices, our paper is also related to the dynamic pricing literature under limited commitment. In a sequential screening framework, Courty and Hao (2000) show that optimal mechanisms depend on the informativeness of consumers' initial knowledge about their valuations. It can be optimal to subsidize consumers with smaller valuation uncertainty to reduce the rent for those who face greater uncertainty. Skreta (2006) shows that posting a price in each period is a revenue-maximizing allocation mechanism in a finite period model without commitment. Our analysis suggests that the optimal mechanism in a repeated purchase setting under limited commitment may involve allowing consumers to have an anonymous option in the initial period.

2 The Model

We consider a two-period monopoly market where the firm produces and sells a non-durable good to a unit mass of consumers in each period. A consumer demands at most one unit of the product in each period. A consumer's valuation for the product, v_i, is uniformly distributed on $[0, 1]$ and does not change across periods. The seller can not

optimal level.

commit to future prices, thus the prices he chooses must be time-consistent. Production has zero fixed cost and constant marginal cost, which is normalized to zero. There is no discounting.

The firm possesses a technology that reveals a consumer's valuation (v_i) if she allows the firm to track her personal data. If a consumer consents to tracking, she suffers a direct utility loss $K \in [0, 1]$, which captures her intrinsic privacy concerns.[9] At the beginning of the game, all consumers are anonymous and the seller offers the same menu to all consumers. At the second period, the seller posts discriminatory prices using his updated information about the consumers' valuations.

The timing of the game is as follows:

1. At $t = 1$, the seller offers a menu $M \equiv \{(A, p_a), (D, p_d), (\emptyset, 0)\}$, and consumers make their purchase decisions after observing the menu. A consumer pays p_a and keeps her anonymity if she chooses the *anonymous option* (A, p_a), pays p_d and discloses her personal data if she chooses the *disclosure option* (D, p_d), and pays 0 and keeps her anonymity if she chooses *not to purchase* $(\emptyset, 0)$.

2. At $t = 2$, the seller learns the valuations of the consumers who have chosen (D, p_d) at $t = 1$ and posts personalized prices $p_2^i = v_i$ for them. The seller also posts a uniform price p_2^N for the unrecognized consumers who have chosen (A, p_a) or $(\emptyset, 0)$ at $t = 1$. Consumers make their second-period purchasing decisions after observing the prices.

A consumer is fully rational and maximizes her expected utility when making purchase decisions in each period. In period 2, the seller posts personalized prices for consumers who have disclosed their valuations in period 1. Anticipating this, consumers strategically adjust their first-period purchasing behavior, reflecting their instrumental privacy concern — concern about future price discrimination.

The consumers' first-period choices divide the second-period market into two segments: a *recognized segment* consisting of consumers who have chosen (D, p_d) and an *unrecognized segment* consisting of consumers that remain anonymous to the seller, the consumers who

[9] In Section 5, we discuss the case of a negative K.

have chosen (A, p_a) and $(\emptyset, 0)$ in the first period. Note that no consumer chooses (D, p_d) if $p_d > p_a$ because by disclosing valuation to the seller, a consumer suffers triple losses: 1) she pays a higher price for the same product at $t = 1$, 2) she suffers a privacy loss K, and 3) her consumer surplus will be fully extracted at $t = 2$ when she makes a repeat purchase. A consumer needs to be compensated through a low p_d to allow tracking. In the subsequent analysis, it is without loss of generality to focus on menus with $p_d \leq p_a$.

3 Benchmark without Anonymous Option

In this section we analyse the benchmark case without anonymous option, in which the seller chooses between the *disclosure mode* and the *no-disclosure mode*. Under the no-disclosure mode, the seller offers $\{(A, p_a), (\emptyset, 0)\}$ and does not collect any consumer data. Under the disclosure mode, the seller offers $\{(D, p_d), (\emptyset, 0)\}$, and a consumer must disclose her data if she makes a purchase.

Under the *no-disclosure* mode, all consumers remain anonymous at $t = 2$, and the second-period market is identical to the first-period market. The game degenerates into a repeated static monopoly problem, and it is optimal for the seller to post the static monopoly price in each period. We summarize the equilibrium outcome in the next remark.

Remark 1 *Under the no-disclosure mode, the seller chooses the static monopoly price in each period, $p_a^{nd} = p_2^{nd} = 1/2$. Consumers with $v_i \in [1/2, 1]$ purchase the product in both periods and consumers with $v_i < \frac{1}{2}$ do not consume. The seller's profit is $\Pi^{nd} = 1/2$ and total consumer surplus is $CS^{nd} = 1/4$.*

Under the *disclosure mode*, a consumer pays price p_d and discloses information at $t = 1$ and faces $p_2^i = v_i$ in the second period when she makes a repeat purchase. If she does not purchase at $t = 1$, she remains anonymous and faces the price p_2^N at $t = 2$. Observing p_d, a consumer makes a purchase if and only if $v_i \geq \hat{v}$, where $\hat{v}$ is the valuation of the marginal consumer who is indifferent between purchasing and not purchasing in period 1. Given $\hat{v} \in [0, 1]$, the seller optimally sets $p_2^N = \arg\max_{p_2} (\hat{v} - p_2) p_2$, resulting in $p_2^N = \frac{\hat{v}}{2}$ in period 2.

When $p_d > \frac{1}{2} - K$, $\hat{v} = 1$ and no consumer makes a purchase in the first period. When $p_d \leq \frac{1}{2} - K$, $\hat{v}$ is uniquely defined as the solution to

$$\hat{v} - p_d - K + \max\{v_i - p_2^i, 0\} = \max\{\hat{v} - \frac{\hat{v}}{2}, 0\}$$

thus, $\hat{v} = 2(p_d + K)$. In the first period, the seller chooses p_d to maximize his expected profit from the two periods:

$$\max_{p_d} \ \Pi^d(p_d) = (1 - \hat{v})p_d + \int_{\hat{v}}^{1} v_i \mathrm{d}v_i + (\hat{v} - p_2^N)p_2^N, \tag{1}$$

where the first term is the seller's profit from selling to a mass of $1 - \hat{v}$ consumers at price p_d in the first period, the second term is the seller's profit from selling products to recognized consumers at their valuations at $t = 2$, and the third term is the seller's profit from the unrecognized consumers at $t = 2$. Plugging $p_2^N = \frac{\hat{v}}{2}$ and $\hat{v} = 2(p_d + K)$ into (1), we obtain

$$\Pi^d(p_d) = p_d - 3(p_d)^2 + 1/2 - 4Kp_d - K^2, \tag{2}$$

which leads to the optimal choice of p_d for $K < \frac{1}{2}$: $p_d = (1 - 4K)/6$. Note that the seller may need to subsidize consumers in the first period to encourage them to disclose information.

Comparing the seller's profits under the no-disclosure and the disclosure mode leads to the equilibrium outcome when an anonymous option is not available, which is summarized in the next proposition.

Proposition 1 *Without anonymous option, there exists a unique subgame perfect equilibrium.*

1. *When $K < \frac{2-\sqrt{3}}{2}$, the seller chooses the disclosure mode and offers $\{(D, p_d), (\emptyset, 0)\}$ with $\tilde{p}_d = \frac{1-4K}{6}$ at $t = 1$. Consumers with $v_i \in [\hat{v}, 1]$, where $\hat{v} = \frac{1+2K}{3}$, purchase at price $\tilde{p}_d$ and consumers with $v_i < \hat{v}$ do not purchase at $t = 1$. At $t = 2$, the seller charges personalized price $\tilde{p}_2^i = v_i$ to repeat purchasers, and posts a uniform price $\tilde{p}_2^N = \frac{1+2K}{6}$ to the remaining consumers. The seller's profit and consumer surplus*

are, respectively,

$$\tilde{\Pi} = \frac{7 - 8K + 4K^2}{12}, \qquad \tilde{CS} = \frac{(5 - 2K)^2}{72}.$$

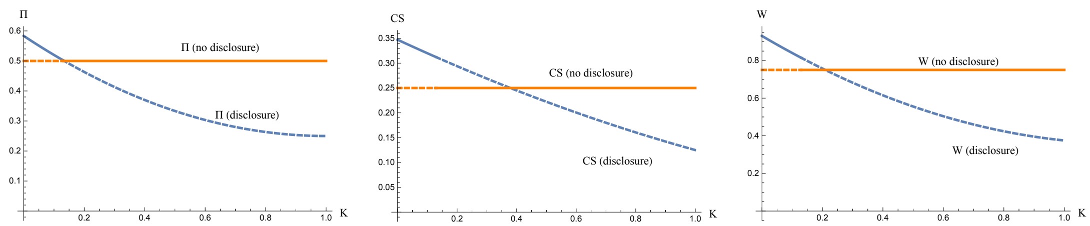
2. *When $K \geq \frac{2-\sqrt{3}}{2}$, the seller chooses the no-disclosure mode and offers $\{(A, p_a), (\emptyset, 0)\}$ with $\tilde{p}_a = \frac{1}{2}$ at $t = 1$. At $t = 2$ the seller offers uniform price $\tilde{p}_2 = \frac{1}{2}$ to all consumers. The seller's profit and consumer surplus are, respectively,*

$$\tilde{\Pi} = \frac{1}{2}, \qquad \tilde{CS} = \frac{1}{4}.$$

It is optimal for the seller to choose the disclosure mode when K is small and choose the no-disclosure mode when K is large. Under the disclosure mode, consumers rationally anticipate that if they purchase and disclose their data at $t = 1$, their surplus will be fully extracted at $t = 2$. However, if they postpone their first purchase to the second period, they will face the price p_2^N and avoids the direct utility loss K at the same time, which may bring them a positive surplus in period 2. Consumers are willing to purchase and disclose information at $t = 1$ if and only if the first-period price p_d is sufficiently low.

Since $\hat{v} = \min\{2(p_d + K), 1\}$ is an increasing function in p_d, a lower p_d leads to an expansion of both the first- and second- period markets, reducing the seller's first-period profit but increasing the seller's second-period profit. On the other hand, for any $K \geq 0$, $\hat{v} > p_d$ and some consumers with $v_i < \hat{v}$ strategically delay their consumption until the second period even though their valuations are above $p_d + K$. Such delay of consumption hurts the seller's first-period profit but increases the seller's second-period profit. When K is small, the market expansion effect dominates, and the seller benefits from tracking consumers. When K is large, the strategic delay effect and consumers' utility loss from information disclosure dominate, and the seller refrains from collecting consumers' information.

Note that the equilibrium outcome exhibits under-disclosure of consumer information from the perspective of social welfare. When $K \in [\frac{2-\sqrt{3}}{2}, \frac{17-3\sqrt{22}}{14}]$, it is socially efficient to adopt the disclosure mode, but the seller chooses not to: the seller's profit decreases at a higher speed than the consumer surplus increases, and the seller needs to lower p_d well below the static monopoly price to incentivize consumers to purchase and disclosure information in period 1. Figure 1 illustrates how the seller's profit, consumer surplus, and social welfare vary with K in equilibrium when there is no anonymous option.



(a) Π (b) CS (c) W

Figure 1: Seller profit, consumer surplus and total welfare w/o anonymous option.

4 Anonymous Option

In this section, we analyse the equilibria when the seller uses an anonymous option. Note that any menu M posted by the seller in period 1 starts a proper subgame. Thus, the equilibrium concept is Subgame Perfect Equilibrium. We use backward induction to solve for the seller's optimal prices in the two periods in sequence. To break ties, we assume that when a consumer is indifferent between multiple options, she chooses the one that is preferred by the seller.

Given a menu $M = \{(A, p_a), (D, p_d), (\emptyset, 0)\}$, a consumer can either choose the anonymous option (A, p_a), get the product at price p_a and keep her anonymity, or choose the disclosure option (D, p_d), get the product at price p_d, and disclose her data, or choose not to purchase in the first period. A consumer's expected utility when her valuation is v_i and she chooses option $m \in M$ in the first period is

$$u_i(v_i, m) = \begin{cases} v_i - p_a + \max\{v_i - p_2^N, 0\} & \text{if } m = (A, p_a) \\ v_i - p_d - K + \max\{v_i - p_2^i, 0\} & \text{if } m = (D, p_d) \\ \max\{v_i - p_2^N, 0\} & \text{if } m = (\emptyset, 0) \end{cases} .$$

Note that a consumer with a higher v_i has a stronger incentive to choose (A, p_a) over (D, p_d). This is because a marginal increase in the consumer's valuation increases her benefit from choosing (D, p_d) only through her first-period consumption, while by choosing (A, p_a), her payoff increases both from her first-period consumption and from her second-period consumption if she anticipates the second-period uniform price, p_2^N, to be lower than her valuation v_i. We first establish the following observations on the ranking of consumers' valuations when they make distinct choices. This observation proves useful in the subsequent characterization of equilibria.

Lemma 1 *Suppose in equilibrium, some consumers with v_1, v_2 and v_3 choose respectively, (A, p_a), (D, p_d), and $(\emptyset, 0)$ at $t = 1$. It holds that 1) $p_2^N \geq p_d + K$, 2) $v_1 > v_2 > v_3$.*

Let the set of consumers who choose the anonymous option, disclosure option, and not purchasing be denote by

$$\mathcal{S}_A \equiv \big\{v_i | u_i(v_i, A) \geq \max\{u_i(v_i, D), u_i(v_i, \emptyset)\}\big\}, \quad \mathcal{S}_D \equiv \big\{v_i | u_i(v_i, D) \geq \max\{u_i(v_i, A), u_i(v_i, \emptyset)\}\big\},$$
$$\mathcal{S}_\emptyset \equiv \big\{v_i | u_i(v_i, \emptyset) > \max\{u_i(v_i, D), u_i(v_i, A)\}\big\},$$

respectively. Define $v_m \equiv \sup(\mathcal{S}_\emptyset)$ and $v_h \equiv \inf(\mathcal{S}_A)$. Then v_m is the highest valuation of a consumer that chooses not to purchase among the three options, and v_h is the lowest valuation of a consumer that chooses (A, p_a). By definition, $\mathcal{S}_\emptyset$ and $\mathcal{S}_A$ are disjoint sets. Moreover, if $v_1 \in \mathcal{S}_A$, it holds that $v_i \in \mathcal{S}_A$ for all $v_i > v_1$; and if $v_2 \in \mathcal{S}_\emptyset$, it holds that $v_i \in \mathcal{S}_\emptyset$ for all $v_i < v_2$. Thus, v_m is the valuation of the marginal consumer who is indifferent between not purchasing and the disclosure option, and v_h is the valuation of the marginal consumer who is indifferent between the disclosure option and the anonymous option. Moreover, $v_m \leq v_h$.

Lemma 1 implies that in any equilibrium with nonempty sets of $\mathcal{S}_\emptyset$, $\mathcal{S}_A$ and $\mathcal{S}_D$, the market is divided into two segments with two thresholds $v_m, v_h \in (0, 1)$ and $v_m < v_h$ at the end of the first period: 1) an unrecognized segment with $v_i \in [0, v_m) \cup (v_h, 1]$. Consumers with $v_i \in (v_h, 1]$ who chose the anonymous option and consumers with $v_i \in [0, v_m)$ who do not consume in period 1 are pooled together and remain anonymous to the seller; 2) a recognized segment with $v_i \in [v_m, v_h]$. These consumers chose the disclosure option in period 1 and the seller recognizes their valuations in period 2. Moreover, for such an equilibrium to exist, the period-2 uniform price for unrecognized consumers, p_2^N, must exceed $p_d + K$. Otherwise all consumers will prefer to purchase their first unit in period-2 instead of period-1, destroying the equilibrium.

In period 2, the seller charges personalized price $p_2^i = v_i$ to consumers in the recognized segment, and posts a uniform price p_2^N for the consumers in the unrecognized segment. There are two candidates for the optimal p_2^N: i) $p_2^N \geq v_h$, at which the seller excludes consumers in set $\mathcal{S}_\emptyset$ from period 2; or ii) $p_2^N < v_m$, at which the seller serves both consumers from set $\mathcal{S}_\emptyset$ and $\mathcal{S}_A$. In case i), the optimal price is uniquely given by $p_2^N = \max\{v_h, \frac{1}{2}\}$, and in case ii) the optimal price solves $\max_p (1 - v_h + v_m - p)p$, leading to $p_2^N = \frac{1-v_h+v_m}{2}$. In the next Lemma, we show that in any equilibrium with nonempty $\mathcal{S}_\emptyset$, $\mathcal{S}_D$ and $\mathcal{S}_A$, the seller's optimal price choice is $\hat{p}_2^N = \max\{v_h, \frac{1}{2}\}$.

Lemma 2 *In any equilibrium with nonempty $\mathcal{S}_\emptyset$, $\mathcal{S}_D$ and $\mathcal{S}_A$, it holds that 1) $0 < v_m < v_h < 1$; 2) the optimal period-2 uniform price is given by $\hat{p}_2^N = \max\{v_h, \frac{1}{2}\}$.*

To understand the logic behind Lemma 2, suppose a price menu M with p_a and p_d induces an equilibrium in which $0 < v_m < v_h < 1$ and $p_2^N = \frac{1-v_h+v_m}{2} = \delta < v_m$. Anticipating the uniform price p_2^N, consumers with valuations $v_i \in (p_2^N, v_m)$ will not purchase in period 1 even if $v_i > p_d + K$. These consumers will wait until period 2 to consumer their first unit, hurting the seller's profit.

Now consider an alternative price menu M' with p_a' such that $v_h' = v_h$ and $v_m' = \delta$. Then the optimal period-2 uniform price is $p_2^N = \max\{v_h, \frac{1}{2}\}$ and consumers with valuations $v_i < \delta$ can not afford the product in period 2, and it is not worthwhile for them to strategically delay their consumptions. As a result, these consumers will purchase their first unit in period 1 as long as $v_i > p_d + K$.

Note that by lowering p_a to induce $p_2^N = \max\{v_h, \frac{1}{2}\}$ in period 2, the seller faces a tradeoff: 1) a lower p_a induces more consumers to purchase anonymously in the first period, this reduces the seller's period-1 profit from consumers with $v_i \in [v_h, 1]$; 2) consumers with $v_i \in [p_d + K, v_h]$ will now consume at price p_d, increasing the seller's profit; 3) a high price for unrecognized consumers in period 2 increases the seller's revenue from consumers in segment $\mathcal{S}_A$ in period 2. The two positive effects in 2) and 3) reinforce each other and dominate the negative effect in 1). Overall, for the seller, any price menu that induces $p_2^N < v_m$ is dominated by an alternative menu that induces $p_2^N = \max\{v_h, \frac{1}{2}\}$, while excluding low-valuation consumers in the unrecognized segment in period 2.

In the next Lemma we establish that $v_m > 0$ and $v_h < 1$ hold in any equilibrium, however, either $v_m < v_h$ or $v_m = v_h$ can hold. This implies that in any equilibrium $\mathcal{S}_\emptyset$ and $\mathcal{S}_A$ are always nonempty. Nevertheless, $\mathcal{S}_D$ is nonempty if $v_m < v_h$, and is empty if $v_m = v_h$.

Lemma 3 *In any equilibrium,* $0 < v_m \leq v_h < 1$. *When* $v_m < v_h$, $\mathcal{S}_\emptyset$, $\mathcal{S}_D$ *and* $\mathcal{S}_A$ *are nonempty. When* $v_m = v_h$, $\mathcal{S}_\emptyset$ *and* $\mathcal{S}_A$ *are nonempty.*

Lemma 3 implies that in equilibrium there is always a positive mass of consumers who choose the anonymous option and not purchase. However, there might be no consumers who choose the disclosure option. Using this observation, we can divide all equilibrium candidates in a subgame following a price menu M into two categories:

1. Equilibrium with no disclosure ($0 < v_m = v_h < 1$). Consumers with $v_i \in [v_h, 1]$ choose (A, p_a), consumers with $v_i \in [0, v_h)$ choose $(\emptyset, 0)$, and no consumers choose (D, p_d) in period 1.

2. Equilibrium with partial disclosure ($0 < v_m < v_h < 1$). Consumers with $v_i \in [v_h, 1]$ choose (A, p_a) , consumers with $v_i \in [v_m, v_h)$ choose (D, p_d), and consumers with $v_i \in [0, v_m)$ choose $(\emptyset, 0)$ in the first period.

4.1 No Disclosure vs Partial Disclosure

In an equilibrium with no disclosure, since all consumers remain anonymous in period 2, it is optimal for the seller to set $p_2^N = \frac{1}{2}$. Moving backwards to period 1, it is optimal for the seller to sell the products to consumers with $v_i \in [\frac{1}{2}, 1]$ at price $p_a = \frac{1}{2}$. Moreover, $v_i - p_d - K \leq v_i - p_a$ must hold for all $v_i \in [p_d + K, 1]$ so that no consumer prefers the disclosure option, leading to $p_d \geq p_a - K$. We summarise this outcome in the next lemma.

Lemma 4 *In an equilibrium with no disclosure,* $p_a = p_2^N = \frac{1}{2}$*,* $p_d \in [\max\{\frac{1}{2} - K, 0\}, \frac{1}{2}]$*. Consumers with* $v_i \in [\frac{1}{2}, 1]$ *choose the anonymous option in period 1 and purchase again in period 2. Consumers with* $v_i \in [0, \frac{1}{2})$ *do not purchase in either period 1 or 2. No consumer chooses the disclosure option. The seller's profit is* $\Pi^{nd} = \frac{1}{2}$*.*

Next we characterise an equilibrium with partial disclosure in which $\mathcal{S}_\emptyset$, $\mathcal{S}_D$ and $\mathcal{S}_A$ are all nonempty. By Lemma 2, $p_2^N = \max\{v_h, \frac{1}{2}\}$, which implies that in period 2 the seller serves only the high-valuation consumers with $v_i \in (v_h, 1]$ in the unrecognized segment. Should the seller deviate to a lower price to serve some consumers from $[0, v_m)$ as well, the seller would choose $p_2^N = \frac{1-v_h+v_m}{2} < v_m$, which requires $v_h + v_m > 1$. Thus, for $p_2^N = \max\{v_h, \frac{1}{2}\}$ to be optimal, the induced valuations of the marginal consumers, v_m and v_h, must satisfy:

$$i)\ v_h + v_m \leq 1, \quad \text{or} \quad ii)\ v_h + v_m > 1 \quad \text{and} \quad (1 - v_h)v_h \geq \frac{(1 - v_h + v_m)^2}{4}. \tag{3}$$

In period 1, observing p_a and p_d, anticipating that $p_2^i = v_i$ and $p_2^N = \max\{v_h, \frac{1}{2}\}$, a marginal consumer with v_m is indifferent between (D, p_d) and $(\emptyset, 0)$, and a marginal consumer with v_h is indifferent between (A, p_a) and (D, p_d). Thus

$$v_m - p_d - K + 0 = 0; \quad v_h - p_a + \max\{v_h - p_2^N, 0\} = v_h - p_d - K + 0.$$

Plugging in $p_2^N = \max\{v_h, \frac{1}{2}\}$, we have $v_m = p_a = p_d + K$. Making use of $v_m = p_a$ and (3), we arrive at the following outcome following a price menu M with p_a and p_d.

Lemma 5 *In an equilibrium with partial disclosure, following price menu* M*,* $v_m = p_a$ *and* $p_d = p_a - K$ *hold. Moreover, any* $v_h \in (p_a, 1)$ *that satisfies*

$$(i) \quad v_h \leq 1 - p_a; \quad or \quad (ii) \quad v_h > 1 - p_a \quad and \quad (1 - v_h)v_h \geq \frac{(1 - v_h + p_a)^2}{4} \tag{4}$$

can be supported in a subgame perfect equilibrium.

Lemma 5 implies that following price menu M, any v_h that satisfies condition (4) can be supported in a partial disclosure equilibrium. Among the multiple equilibria with different v_h, and thus different $p_2^N = \{v_h, \frac{1}{2}\}$, we focus on the seller's optimal equilibrium that maximizes the seller's expected profit.[10] Making use of Lemma 5, we transform the seller's maximization problem into one of choosing p_a to maximize

$$\begin{aligned}\Pi(p_a) =&(v_h - v_m)p_d + (1 - v_h)p_a + \int_{v_m}^{v_h} v_i \mathrm{d}v_i + \Big(1 - \max\{v_h, \frac{1}{2}\}\Big) \max\{v_h, \frac{1}{2}\} \\ =&(v_h - p_a)(p_a - K) + (1 - v_h)p_a + \int_{p_a}^{v_h} v_i \mathrm{d}v_i + \Big(1 - \max\{v_h, \frac{1}{2}\}\Big) \max\{v_h, \frac{1}{2}\}\end{aligned} \tag{5}$$

subject to constraint (4). Note that the last inequality in (4) is equivalent to

$$\underline{v}_h(p_a) \equiv \frac{3 + p_a - 2\sqrt{1 - p_a - p_a^2}}{5} \leq v_h \leq \frac{3 + p_a + 2\sqrt{1 - p_a - p_a^2}}{5} \equiv \bar{v}_h(p_a). \tag{6}$$

Analysing the set of v_h that is induced by price menu M with p_a and satisfies (4), we arrive at the following result.

Lemma 6 *An equilibrium with partial disclosure exists if and only if $p_a \leq \frac{\sqrt{5}-1}{2}$. In the seller-optimal equilibrium,*

$$v_h(p_a) = \begin{cases} 1 - K & \text{if } K < \frac{1}{2},\ p_a \leq 2\sqrt{K(1-K)} - K \\ \bar{v}_h(p_a) & \text{if } K < \frac{5-\sqrt{5}}{10},\ p_a \in (2\sqrt{K(1-K)} - K, \frac{\sqrt{5}-1}{2}] \\ \max\{\underline{v}_h(p_a), \frac{1}{2}\} & \text{otherwise} \end{cases}. \tag{7}$$

Solving for p_a that maximizes the seller's objective (5) subject to the constraint (7) gives us the outcome in a subgame equilibrium with partial disclosure. Let

$$p_a^* \equiv \arg\max_{p_a} \Pi(p_a) \quad \text{subject to} \quad (7). \tag{8}$$

In the next lemma, we show that an equilibrium with partial disclosure exists if and only if $K < \frac{1}{2}$.

[10] Given a price menu M with $p_a = p_d + K$, although different v_h, and thus different $p_2^N = \{v_h, \frac{1}{2}\}$, can be supported in subgame perfect equilibrium, the seller can coordinate on a particular one by announcing an unbinding uniform price p_2^N as a coordinating device, and such an announcement is indeed self-sustainable.

Lemma 7 *An equilibrium with partial disclosure exists if and only if* $K < \frac{1}{2}$*. In the seller optimal equilibrium,*

1. *when* $K \in [\frac{1}{26}, \frac{1}{2})$*,* $p_a^* = \frac{1+K}{3}$*,* $p_d^* = \frac{1-2K}{3}$*,* $v_h^* = 1 - K > v_m^* = \frac{1+K}{3}$*,* $p_2^{N,*} = v_h^*$*. The seller's profits are given by* $\Pi^* = \frac{2}{3}(1 - K + K^2)$*;*

2. *when* $K < \frac{1}{26}$*,* $p_a^* = \hat{p}_a < \frac{1+K}{3}$ *in which* $\hat{p}_a$ *is the unique solution to*

$$(1 + K - 3p_a) + \Big(1 - K - \bar{v}_h(p_a)\Big)\frac{1}{5}\Big(1 - \frac{1 + 2p_a}{\sqrt{1 - p_a - p_a^2}}\Big) = 0,$$

   and $p_d^* = \hat{p}_a - K$*,* $v_h^* = \bar{v}_h(\hat{p}_a) = \frac{3+\hat{p}_a+2\sqrt{1-\hat{p}_a-\hat{p}_a^2}}{5} > v_m^* = \hat{p}_a$*,* $p_2^{N,*} = v_h^*$*. The seller's profits are given by* $\Pi^* = -\frac{v_h^{*2}}{2} + (1-K)v_h^* + (1+K)\hat{p}_a - \frac{3}{2}\hat{p}_a^2$*.*

4.2 Equilibrium with Anonymous Option

From Lemma 4, the seller can induce an equilibrium with no disclosure through a price menu M with $p_a = \frac{1}{2}$ and $p_d \in [\max\{\frac{1}{2} - K, 0\}, \frac{1}{2}]$, and such an outcome can be supported as a subgame equilibrium for $K \in [0, 1]$. On the other hand, when $K < \frac{1}{2}$, the seller can choose a price menu with a low p_a to induce an equilibrium with partial disclosure, as shown in Lemma 7.

Since the seller's expected payoff from the seller-optimal partial-disclosure equilibrium is strictly higher than the payoff from the no-disclosure equilibrium, we arrive at the unique subgame perfect equilibrium of the game: when $K < \frac{1}{2}$, it is optimal for the seller to induce an equilibrium with partial disclosure; when $K \geq \frac{1}{2}$, it is optimal for the seller to induce the equilibrium with no disclosure. In the next proposition, we summarise the equilibrium outcome when the seller can use an anonymous option.

Proposition 2 (Equilibrium with Anonymous Option) *When the seller uses an anonymous option, there exists a unique subgame perfect equilibrium.*

1. *When* $K \in [0, \frac{1}{2})$*, there is a unique equilibrium with partial disclosure. In this equilibrium,* $p_a^* = p_d^* + K \leq \frac{1+K}{3}$*, following which* $v_m^* = p_a^*$*,* $v_h^* = p_2^{N,*} > \frac{1}{2}$*. Consumers with* $v_i \in [v_m^*, v_h^*]$ *choose* (D, p_d^*)*, and consumers with* $(v_h^*, 1]$ *choose* (A, p_a^*)*.*

2. *When $K \geq \frac{1}{2}$, there is a unique equilibrium with no disclosure. In this equilibrium, $p_a^* = \frac{1}{2}$, $p_d^* \in [0, \frac{1}{2}]$, following which consumers with $v_i \in [\frac{1}{2}, 1]$ choose (A, p_a^*), and $p_2^{N,*} = \frac{1}{2}$.*

4.3 Welfare Effects of Anonymous Option

From Propositions 1 and 2, when $K \geq \frac{1}{2}$, the seller adopts the no-disclosure mode without anonymous option; with anonymous option the equilibrium is the no-disclosure one. Thus, there is no information disclosure in either regime, and the seller's expected profits are also the same. When $K < \frac{1}{2}$, without anonymous option, the seller adopts the disclosure mode when $K < \frac{2-\sqrt{3}}{2}$ and adopts the no-disclosure mode when $K \geq \frac{2-\sqrt{3}}{2}$; with anonymous option, the equilibrium outcome is the one with partial disclosure. Comparison of the seller's profits shows that the seller is strictly better off with anonymous option. We summarise the result in the next proposition.

Proposition 3 *When $K \geq \frac{1}{2}$, there is no information disclosure in equilibrium with or without anonymous option, and the seller's profits are the same in the two regimes. When $K < \frac{1}{2}$, the seller is strictly better off with anonymous option.*

When K is sufficiently large, the seller does not collect any consumer data despite the availability of tracking. Thus, the seller's profit is the same regardless of whether the anonymous option is available or not. When K is relatively small, offering the anonymous option strictly increases the seller's profit. By inducing some consumers with high valuations to purchase anonymously, the seller alters the second-period market segmentation and makes a high price optimal for unrecognized consumers in period 2. This removes consumers' incentive to strategically delay consumption in period 1: anticipating a high uniform price for unrecognized consumers in period 2, delaying consumption does not lead to a lower price as in the case without anonymous option.

Discussion of $K = 0$. A positive K is not essential for the anonymous option to be profitable for the seller. When $K = 0$, the seller adopts the disclosure mode in the benchmark case. With anonymous option, there exists an equilibrium with partial disclosure in which $p_a^* = p_d^* = 0.33$, following which $v_m^* = 0.33 < v_h^* = 0.96$, and $p_2^{N,*} = v_h^* = 0.96$. Consumers with $v_i > v_h^*$ are strictly better off from choosing (A, p_a^*) over (D, p_d^*). Consumers with $v_i \in [v_m^*, v_h^*]$ choose (D, p_d^*). If they choose (A, p_a^*) instead, they will only consume in period 1. If they choose (D, p_d^*), they consume two units. Consumers with $v_i < v_m^*$ do not consume in either period. If they choose the disclosure mode instead, they receive a negative utility.

The seller's total profit with and without anonymous option is respectively $\Pi^* = 0.66$ and $\tilde{\Pi} = \frac{7}{12}$. The inclusion of an anonymous option increases the seller's expected profit by 14.18%. This example shows that a small mass of consumers in set $\mathcal{S}_A$ is sufficient to sustain a high uniform price for the unrecognized consumers in period 2.

Discussion of $K = \frac{1}{4}$. In the benchmark, the seller chooses the no-disclosure mode and receives a profit equal to $\frac{1}{2}$. With anonymous option, there exists an equilibrium with partial disclosure in which $p_a^* = \frac{5}{12}$, $p_d^* = \frac{1}{6}$, and following such prices $v_m^* = \frac{5}{12} < v_h^* = \frac{3}{4} = p_2^{N,*}$. The seller's equilibrium profit is $\Pi^* = \frac{13}{24}$, which is 8.33% higher than the profit without anonymous option.

Consumer Privacy and Consumer Surplus. The intention of anonymous option stipulated in the EU's GDPR and privacy regulations in other areas is to protect consumer privacy and consumer surplus. As we show in the next proposition, it only achieves the first goal when the seller collects personal data to engage in personalized pricing in repeated interaction.

Proposition 4 *1) When K is small, anonymous option decreases information disclosure; when K is intermediate, anonymous option increases information disclosure; and when K is large, anonymous option has no effect on the intensity of information disclosure. 2) The usage of anonymous option lowers consumer surplus.*

When K is large, the seller does not collect consumer information with or without anonymity. Thus consumer surplus is not affected. When K is low the seller collects consumer data, and when K is intermediate, the seller does not collect consumer data without anonymous option. With anonymous option, the seller offers a price menu that leads to the partial-disclosure equilibrium.

In the partial disclosure equilibrium, consumers with very high valuations are better off relative to the benchmark of no anonymous option: although they pay a higher price p_a^* in period 1, they avoid the privacy loss K and enjoy positive surplus from period 2. However, these high-valuation consumers generate a negative externality on consumers with intermediate valuations: consumers with intermediate valuations pay a higher period-1 price, and if they do not consume in period 1 they will face a high price in period 2, which is not affordable to them. The loss from consumers with intermediate valuations is higher than the benefits for consumers with high valuations, and overall consumer surplus decreases due to the usage of anonymous option.

Social Welfare. Does anonymous option improve social welfare? By comparing the sum of the seller's profit and consumer surplus in the two scenarios, we arrive at the following corollary.

Corollary 1 *1) When $K \geq \frac{2-\sqrt{3}}{2}$, the benchmark equilibrium is no disclosure. Anonymous option improves social welfare if and only if $K \leq \frac{5}{22}$.*

2) When $K < \frac{2-\sqrt{3}}{2}$, the benchmark equilibrium is disclosure. Anonymous option decreases social welfare.

The anonymous option has three effects on total welfare. We illustrate the intuitions with $K \in (1/26, (2-\sqrt{3})/2)$ under which the seller chooses the disclosure mode without anonymous option.

1. Demand expansion effect. The anonymous option induces some intermediate-value consumers to increase their demand from one unit to two units. Consumers with $v_i \in [(1+K)/3, (1+2K)/3]$ purchase only once in period 2 without anonymous option, and consume two units with anonymous option. The anonymous option expands demand by $\frac{K}{3}$, which increases with K.

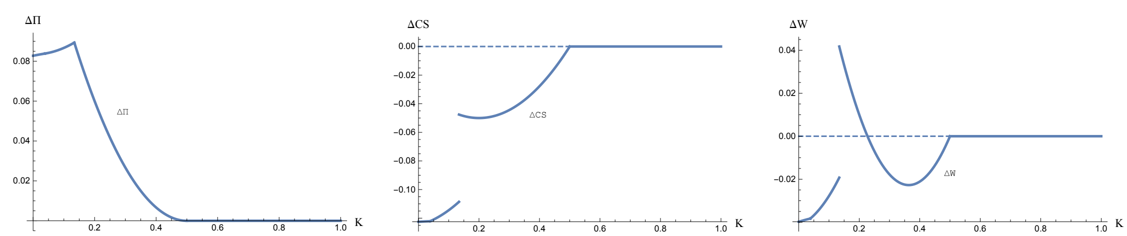
2. Demand reduction effect. The anonymous option reduces consumption by some low-value consumers from one unit to zero unit. Consumers with $v_i \in [(1+2K)/6, (1+$

$K)/3]$ consume one unit without anonymous option, and do not consume with anonymous option. The anonymous option deduces demand by $1/6$.

3. Privacy loss. Consumers with $v_i \in [(1+2K)/3, 1]$ disclose information without anonymous option, whereas consumers with $v_i \in [(1+K)/3, 1-K]$ disclose information. Fewer consumers disclose information, and the anonymous option thus reduces privacy loss by $2K^2/3$, which increases with K.

Overall, when K is low, the second negative effect dominates and total welfare is lower with anonymous option. However, when $K \geq \frac{2-\sqrt{3}}{2}$, the benchmark equilibrium shifts to the no-disclosure mode. In this case, the anonymous option increases both trade frequency and privacy loss. In particular, for $K \in [\frac{2-\sqrt{3}}{2}, \frac{1}{2}]$, consumers either purchase twice or do not purchase under both the benchmark and the main model. The anonymous option expands demand by $\frac{1-2K}{6}$ (consumers in $[\frac{1+K}{3}, \frac{1}{2}]$) in each period, which increases total welfare in two periods by $\frac{5-8K-4K^2}{36}$. Meanwhile, the anonymous option also increases privacy loss by $\frac{2K-4K^2}{3}$ from consumers in $[\frac{1+K}{3}, 1-K]$. When $K \leq \frac{5}{22}$, the first effect dominates and the anonymous option increases total welfare. Conversely, when $K > \frac{5}{22}$, the second effect dominates and the anonymous option harms total welfare. Therefore, the anonymous option increases social welfare if and only if K takes intermediate values.

We close this section with Figure 2 illustrating the change in the seller's profit, consumer surplus, and total welfare when the pricing regime moves from no anonymous option to the case with anonymous option.



(a) $\Delta\Pi$ (b) ΔCS (c) ΔW

Figure 2: Welfare Effects of Anonymous Option

5 Conclusion

We study a monopolist's incentive to provide an anonymous option when the seller can collect consumer data and engage in personalized pricing in repeated interactions. We obtain a set of surprising results. 1) Despite the benefits of using consumer data for future price discrimination, the seller is better off offering the anonymous option. The option changes the second-period market segmentation and provides the seller with endogenous commitment power to a high uniform price for unrecognized consumers in period 2, removing the "ratchet effect". 2) The anonymous option lowers consumer surplus. Consumers with high valuations benefit from the option because they avoid the privacy loss and obtain positive surplus from their second-period assumption. However, there is a negative externality on consumers with intermediate valuations. Although they may pay a lower first-period price, their rents are either fully extracted or they face a high uniform price, which is not affordable to them. In aggregate, consumer surplus decreases due to the anonymous option. 3) The anonymous option increases social welfare only when K takes intermediate values. The outcome depends on the trade-off among a demand expansion effect, a demand reduction effect, and privacy loss.

In the analysis, we assume positive K to capture consumers' intrinsic losses from disclosing information. However, some consumers may enjoy sharing data which implies a negative K.[11] Without anonymous option, the seller would optimally choose the disclosure mode since the profit is strictly higher than the no-disclosure mode when $K < 0$. The equilibrium exhibits a similar pattern to that in part (1) of Proposition 1. Although strategic delay of consumption still exists in equilibrium, it is alleviated by the consumers' utility gain from information disclosure. When the seller is able to utilize a menu with an anonymous option, it is harder to encourage consumers to choose the anonymous option. For a sufficiently large $|K|$, thus a high utility gain from information disclosure, it is optimal for the seller to adopt the disclosure mode, tracking all consumers' information when they make the first purchase. However, for a relatively small $|K|$, consumers gain from information disclosure is not pivoting. The seller is able to strictly increase his profit

[11] Some consumers may experience direct psychological benefits from sharing data. See, for example, Tamir and Mitchell (2012).

by incentivizing some high-valuation consumers to purchase anonymously, consistent with the result in part (1) of Proposition 2.

One extension of our work is to consider unobservable privacy types. In the two-dimensional screening problem, consumers with high valuations may also have low privacy cost. To serve this group of consumers, the seller needs to lower the price that accompanies the anonymous option, which increases the imitation incentives of consumers with relatively lower valuations. The overall effect of offering the anonymous option is thus more subtle. Another natural extension is to examine the effect of the anonymous option in a competitive environment. When the seller is a monopoly, consumers' preference for the anonymous option and the disclosure option can be monotonically ranked. However, when there is competition, consumers have the choice of buying from other sellers, and this will adversely affect the seller's incentive to set a high uniform price in the second period. This may change the seller's incentive of including an anonymous option in their price menu.

Appendix

Proof of Proposition 1. Plugging the optimal first-period price, $\tilde{p}_d = \frac{1-4K}{6}$, into (2), we obtain the seller's profit under the disclosure mode

$$\Pi^d = \frac{7 - 8K + 4K^2}{12}.$$

Note that Π^d monotonically decreases in K. From Remark 1, the seller's profit under no disclosure is given by $\Pi^{nd} = 1/2$. It follows that $\Pi^d > \Pi^{nd}$ if and only if $K < (2 - \sqrt{3})/2$. Thus, the seller chooses the disclosure mode if and only if $K < (2 - \sqrt{3})/2$.

When $K < (2 - \sqrt{3})/2$, since the seller chooses the disclosure mode, the consumer surplus is

$$\tilde{CS} = \int_{\hat{v}}^{1} (v_i - \tilde{p}^d - K)dv_i + \int_{\tilde{p}_2^N}^{\hat{v}} (v_i - \tilde{p}_2^N)dv_i = \frac{(5 - 2K)^2}{72}.$$

When $K \geq (2-\sqrt{3})/2$, the seller chooses the no-disclosure mode and the consumer surplus is given by $\tilde{CS} = 1/4$, as in Remark 1. ∎

Proof of Lemma 1. 1) Suppose $p_2^N < p_d + K$ instead. Then no purchase always dominates purchase with disclosure at $t = 1$, and no consumer will disclose information, contradicting the assumption. Thus, for a positive mass of consumers to choose (D, p_d) in equilibrium, it must hold that $p_2^N \geq p_d + K$.

2) Anticipating a uniform price $p_2^N \geq p_d + K$ for unrecognized consumers at $t = 2$, the consumer with v_2 chooses (D, p_d) instead of (A, p_a) and $(\emptyset, 0)$, implying that

$$v_2 - p_d - K \geq \max\left\{v_2 - p_a + \max\{v_2 - p_2^N, 0\}, \max\{v_2 - p_2^N, 0\}\right\}.$$

It follows that

$$\max\{v_2 - p_2^N, 0\} \leq p_a - p_d - K.$$

Suppose $v_1 \leq v_2$, we have

$$\max\{v_1 - p_2^N, 0\} \leq \max\{v_2 - p_2^N, 0\} \leq p_a - p_d - K,$$

which implies

$$v_1 - p_a + \max\{v_1 - p_2^N, 0\} \leq v_1 - p_d - K. \tag{9}$$

If $v_1 - p_d - K \geq 0$, then $v_1 - p_d - K \geq \max\{v_1 - p_2^N, 0\}$, thus

$$v_1 - p_d - K \geq \max\left\{v_1 - p_a + \max\{v_1 - p_2^N, 0\}, \max\{v_1 - p_2^N, 0\}\right\},$$

and the consumer with v_1 must also choose (D, p_d) instead of (A, p_a), which is a contradiction. If $v_1 - p_d - K < 0$, then $v_1 - p_d - K < \max\{v_1 - p_2^N, 0\}$, and the consumer with v_1 must choose not to purchase. This leads to a contradiction to the assumption that the consumer with v_1 chooses (A, p_a). Thus $v_1 > v_2$ holds.

Consumers with v_2 and v_3 choosing (D, p_d) and $(\emptyset, 0)$ respectively implies

$$v_2 - p_d - K \geq 0, \qquad \text{and} \quad v_3 - p_d - K < 0,$$

leading to $v_2 > v_3$. ∎

Proof of Lemma 2. Suppose a price menu M with p_a and p_d induces an equilibrium in which $\mathcal{S}_\emptyset$, $\mathcal{S}_D$ and $\mathcal{S}_A$ are nonempty. Since v_m is the highest valuation in set $\mathcal{S}_\emptyset$ and v_h

is the lowest valuation in set $\mathcal{S}_A$, $0 < v_m < v_h < 1$ follows directly.

In the following, we prove part 2) of the claim. Suppose $p_2^N = \frac{1-v_h+v_m}{2} \equiv \delta < v_m$ in equilibrium. This implies that consumers with $v_i \in [\delta, v_m)$ purchase their first units in period 2. Given the definitions of v_m and v_h, we have

$$v_h - p_a + v_h - p_2^N = v_h - p_d - K, \quad v_m - p_d - K = v_m - p_2^N,$$

which imply $\delta = p_d + K$ and $v_h = p_a$.

Now consider an alternative menu M' with $p'_a = \delta$, $p'_d = p_d = \delta - K$. Following price menu M', $p_2^{N'} = v_h$ is supported as a second-period equilibrium price. Then the valuations of the marginal consumers, v'_h and v'_m induced by M', now satisfy

$$v'_h - p'_a + v'_h - v_h = v'_h - p_d - K, \quad v'_m - p_d - K = 0.$$

Thus, $v'_h = v_h$, $v'_m = \delta > 0$. Now in period 2, the recognized segment contains consumers with $v_i \in [0, \delta) \cup (v_h, 1]$, and $p_2^{N\prime} = \delta$ is obviously dominated by $p_2^{N'} = v_h$: the demand is the same but the uniform price charged to unrecognized consumers is strictly higher.

Finally, we show that the seller's profit in the constructed equilibrium with menu M' is strictly higher than the profit under the original price menu M:

$$\Pi(M) = (v_h - v_m)p_d + (1 - v_h)p_a + (v_m - \delta)\delta + \int_{v_m}^{v_h} v_i \mathrm{d}v_i + (1 - v_h)\delta;$$
$$\Pi(M') = (v_h - \delta)p_d + (1 - v_h)p'_a + \int_{\delta}^{v_h} v_i \mathrm{d}v_i + (1 - v_h)v_h;$$
$$\Pi(M') - \Pi(M) = (v_m - \delta)p_d - (v_m - \delta)\delta + \int_{\delta}^{v_m} v_i \mathrm{d}v_i = (v_m - \delta)p_d + \frac{(v_m - \delta)^2}{2} > 0.$$

Thus, for any price menu M that induces nonempty sets of $\mathcal{S}_\emptyset$, $\mathcal{S}_D$ and $\mathcal{S}_A$, and leads to optimal $p_2^N < v_m$ in the subgame, there exists an alternative price menu M' which induces a subgame with $p_2^N = \max\{v_h, \frac{1}{2}\}$ and achieves strictly higher profit than price menu M. ∎

Proof of Lemma 3. We prove the claim in two steps. Step one, we first show that if $v_m = 0$, then $v_h \in (0, 1)$ holds. Following this, we show that an alternative menu that induces v_m marginally above zero while keeping v_h at the same level leads to a strictly higher profit for the seller, and thus $v_m > 0$ hold in equilibrium. In step two, we show that $v_h = 1$ leads to a contradiction and thus $v_h < 1$ holds in equilibrium.

(i) Suppose a price menu M with p_a and p_d induces $v_m = 0$ in equilibrium. Since consumer with $v_i = 0$ makes a purchase at price p_d, this implies $p_d + K \leq 0$. We show that $v_h < 1$ must hold. Suppose $v_h = 1$ instead. Then all consumers choose (D, p_d) in the first period and the seller's total profit is bounded by $\Pi \leq 1/2 - K$. Inducing $v_h = 1$ is clearly dominated by selling to consumers at prices $p_a = p_2^N = \frac{1}{2}$, leading to a total profit $\tilde{\Pi} = \frac{1}{2}$.

Moreover, $v_h = 0$ is not an equilibrium outcome either. Suppose $v_h = 0$ instead, the seller earns zero profit in the first period, and the seller's maximal profit is $\frac{1}{4}$ by setting $p_2^N = \frac{1}{2}$ in the second period, leading to a total profit $\Pi = \frac{1}{4}$. Such an outcome is clearly dominated by selling to consumers at prices $p_a = p_2^N = \frac{1}{2}$. This implies that if $v_m = 0$, $0 < v_h < 1$ holds, and consumers with $v_i \in [0, v_h)$ choose the disclosure option while consumers with $v_i \in [v_h, 1]$ choose the anonymous option.

Now we show that a price menu M inducing $v_m = 0$ is dominated by an alternative price menu inducing $v_m > 0$. Consider price menu M' with $p'_d = \epsilon$ and $p'_a = p_a + \epsilon$ in which ϵ is a infinitely small positive number. Now $v'_m = \epsilon$ and $v'_h = v_h > 0$. Price menu M' brings the seller a strictly higher profit because

$$\Pi(M') - \Pi(M) \geq \epsilon(v_h - \epsilon) - \int_0^{\epsilon} v\mathrm{d}v = v_h\epsilon - \frac{3}{2}\epsilon^2 > 0,$$

where $\epsilon(v_h - \epsilon)$ is the lower bound of profit increase in period 1, and $\int_0^{\epsilon} v\mathrm{d}v$ is the profit decrease in period 2 when the price menu changes from M to M'. For $v_h > 0$, $\frac{3}{2}\epsilon^2$ is of second order, thus we have $\Pi(M') - \Pi(M) > 0$. Therefore, $v_m > 0$ holds in equilibrium.

(ii) Suppose some price menu M induces $v_h = 1$ in equilibrium. We show that there exists an alternative price menu M' that induces $v_h < 1$ and brings the seller strictly higher profit.

With $v_h = 1$, no one chooses the anonymous option in period 1. Since $v_m > 0$, consumers with $v_i \in [v_m, 1]$ choose (D, p_d) and consumers with $v_i \in [0, v_m)$ choose $(\emptyset, 0)$ in period 1. It is optimal for the seller to choose $p_2^N = \frac{v_m}{2}$ in period 2. Consumers with $v_i \in [\frac{v_m}{2}, v_m)$ purchase their first units in period 2. It follows that v_m, the valuation of the marginal consumer who is indifferent between (D, p_d) and $(\emptyset, 0)$ in period 1, satisfies

$$v_m - p_d - K + 0 = v_m - \frac{v_m}{2},$$

thus $p_d = \frac{v_m}{2} - K$.

Now consider an alternative menu M' with $p'_a = \frac{v_m}{2}$, $p'_d = p_d = \frac{v_m}{2} - K$. Then $v'_m = \frac{v_m}{2}$ and $v'_h = 1 - \frac{v_m}{2}$. Now $p_2^{N\prime} = 1 - \frac{v_m}{2}$ is supported as an equilibrium in period 2. To see this, note that anticipating $p_2^{N\prime} = 1 - \frac{v_m}{2}$, the valuations of marginal consumers, v'_h and v'_m under price menu M', satisfy the following:

$$v'_h - p'_a + v'_h - p_2^{N\prime} = v'_h - p_d - K; \quad v'_m - p_d - K = 0.$$

Thus, under M', consumers with $v_i > 1 - \frac{v_m}{2}$ choose (A, p_a) and consumers with $v_i \in [\frac{v_m}{2}, 1 - \frac{v_m}{2}]$ choose (D, p_d). For consumers in the unrecognized segment, $v_i \in [0, \frac{v_m}{2}) \cup (1 - \frac{v_m}{2}, 1]$, the optimal second-period price is indeed $p_2^{N\prime} = 1 - \frac{v_m}{2}$, leading to second-period profit $p_2^{N'}(1 - p_2^{N'}) = \frac{v_m}{2}(1 - \frac{v_m}{2})$, which is larger than setting $p_2^{N\prime} < \frac{v_m}{2}$ and serving some consumers with $v_i \in [0, \frac{v_m}{2})$.

We now compare the seller's profits from menu M and M':

$$\begin{aligned}
\Pi(M) &= (1 - v_m)p_d + (v_m - p_2^N)p_2^N + \int_{v_m}^{1} v_i \mathrm{d}v_i; \\
\Pi(M') &= (p_2^{N'} - p'_a)p'_d + (1 - p_2^{N'})p'_a + \int_{p'_a}^{p_2^{N'}} v_i \mathrm{d}v_i + (1 - p_2^{N'})p_2^{N'} \\
&= (1 - v_m)p_d + (v_m - p_2^N)p_2^N + \int_{v_m}^{1} v_i \mathrm{d}v_i - (1 - v_m)(v_m - p_2^N) + (1 - p_2^{N'})p_2^{N'}; \\
\Pi(M') - \Pi(M) &= (1 - p_2^{N'})p_2^{N'} - (1 - v_m)(v_m - p_2^N) = (v_m - p_2^N)p_2^N > 0.
\end{aligned}$$

Thus, $v_h < 1$ must hold in equilibrium. ∎

Proof of Lemma 6. We first show that $v_h < \frac{1}{2}$ can not occur in an equilibrium with partial disclosure so that $p_2^N = \max\{v_h, \frac{1}{2}\} = v_h$ holds. We then characterize the optimal p_a that maximizes the seller's profit (5) subject to constraint (4), and analyse the set of v_h that can be induced by a price p_a.

Suppose $v_h < \frac{1}{2}$ holds. It follows that $p_2^N = \max\{v_h, \frac{1}{2}\} = \frac{1}{2}$. Then objective function

(5) simplifies to

$$\Pi(p_a) = (v_h - p_a)(p_a - K) + (1 - v_h)p_a + \int_{p_a}^{v_h} v_i \mathrm{d}v_i + \frac{1}{4}. \tag{10}$$

Since $\frac{\partial \Pi(p_a)}{\partial v_h} = -K + v_h$, $\Pi(p_a)$ in (10) decreases with v_h when $v_h < K$ and increases with v_h when $v_h > K$. Thus, on the interval $v_h \in (p_a, \frac{1}{2})$, the seller's profit $\Pi(p_a)$ is bounded above by either $\Pi(p_a) \mid_{v_h \to p_a}$, or $\Pi(p_a) \mid_{v_h \to \frac{1}{2}}$. When $v_h \to p_a$, the seller's profit is $\Pi(p_a) \mid_{v_h \to p_a} = (1 - p_a)p_a + \frac{1}{4} \leq \frac{1}{2}$. Such outcome is obviously dominated by the one in the no-disclosure equilibrium characterized in Lemma 4. Moreover, for $v_h < \frac{1}{2}$ satisfying (4), $v_h + \epsilon$ also satisfies (4) for infinitely small positive ϵ, thus can also be supported in an equilibrium. Therefore, $v_h < \frac{1}{2}$ can not be the seller's optimal equilibrium.

When $v_h \geq \frac{1}{2}$, $p_2^N = \max\{v_h, \frac{1}{2}\} = v_h$ holds. The seller's objective (5) becomes

$$\Pi(p_a) = -\frac{v_h^2}{2} + (1 - K)v_h + (1 + K)p_a - \frac{3}{2}p_a^2, \tag{11}$$

which increases in v_h for $v_h < 1 - K$ and decreases in v_h for v_h for $v_h > 1 - K$. Notice that when $p_a > \frac{\sqrt{5}-1}{2}$, constraint (4) reduces to an empty set, and an equilibrium with partial equilibrium does not exist. We differentiate the following cases:

1. If $0 < p_a < \frac{1}{2}$, constraint (4) simplifies to $\frac{1}{2} \leq v_h \leq \bar{v}_h$.

   When $K < \frac{1}{10}$ and $p_a \leq 2\sqrt{K(1-K)} - K < \frac{1}{2}$, or $K \in [\frac{1}{10}, \frac{1}{2}]$ and $p_a \in (0, \frac{1}{2})$, the seller's optimal v_h is uniquely given by $v_h = 1 - K$.

   When $K < \frac{1}{10}$ and $2\sqrt{K(1-K)} - K < p_a < \frac{1}{2}$, the seller's optimal v_h is uniquely given by $v_h = \bar{v}_h$.

   When $K > \frac{1}{2}$ and $p_a \in (0, \frac{1}{2})$, the seller's optimal v_h is uniquely given by $v_h = \frac{1}{2}$.

2. If $\frac{1}{2} \leq p_a \leq \frac{\sqrt{5}-1}{2}$, constraint (4) simplifies to $\underline{v}_h \leq v_h \leq \bar{v}_h$.

   When $K \in [\frac{1}{10}, \frac{1}{2}]$ and $\frac{1}{2} \leq p_a \leq 2\sqrt{K(1-K)} - K$, $v_h = 1 - K$ uniquely maximizes $\Pi(p_a)$ in (11).

   When $K \leq \frac{1}{10}$ and $\frac{1}{2} \leq p_a \leq \frac{\sqrt{5}-1}{2}$, or $\frac{1}{10} < K < \frac{5-\sqrt{5}}{10}$, and $2\sqrt{K(1-K)} - K < p_a \leq \frac{\sqrt{5}-1}{2}$, $v_h = \bar{v}_h$ maximizes $\Pi(p_a)$.

   When $\frac{5-\sqrt{5}}{10} \leq K \leq \frac{1}{2}$ and $2\sqrt{K(1-K)} - K < p_a \leq \frac{\sqrt{5}-1}{2}$, or when $K > \frac{1}{2}$ and $\frac{1}{2} \leq p_a \leq \frac{\sqrt{5}-1}{2}$, $v_h = \underline{v}_h$ maximizes $\Pi(p_a)$.

Combining the above cases leads to the claimed results.

∎

Proof of Lemma 7. We differentiate the following three cases:

1. When $K \in [\frac{1}{26}, \frac{1}{2})$, $\frac{1+K}{3} \leq 2\sqrt{K(1-K)} - K$. From Lemma 6, the global solution $p_a = \frac{1+K}{3}$ uniquely solves the seller's profit maximization problem.

   Thus, when $K \in [\frac{1}{26}, \frac{1}{2})$, $p_a^* = p_d^* + K = \frac{1+K}{3} < \frac{1}{2}$, following which, $v_m^* = \frac{1+K}{3}$ and $v_h^* = 1-K$. In the second period, $p_2^{N,*} = 1-K$ and $p_2^i = v_i$. The seller's profit is $\Pi^* = \frac{2}{3}(1-K+K^2)$.

   Following price menu M^* with $p_a^* = \frac{1+K}{3}$, and $p_d^* = \frac{1-2K}{3}$, the partial disclosure outcome is indeed supported as a subgame perfect equilibrium. Observing the menu, it is optimal for consumers with $v_i \in [p_d^* + K, 1-K]$ to choose (D, p_d^*) and consumers with $v_i \in (1-K, 1]$ to choose (A, p_a^*), anticipating that $p_2^{N,*} = 1-K$ and $p_2^i = v_i$. Then it is optimal for the seller to set $p_2^{N,*} = 1-K$ and $p_2^i = v_i$, respectively, for unrecognized consumers and recognized consumers in period 2.

2. When $K < \frac{1}{26}$, the seller chooses $2\sqrt{K(1-K)} - K < p_a \leq \frac{\sqrt{5}-1}{2}$ to maximize

$$\begin{aligned}\Pi(p_a) &= (\bar{v}_h(p_a) - p_a)(p_a - K) + (1 - \bar{v}_h(p_a))p_a + \int_{p_a}^{\bar{v}_h(p_a)} v_i \mathrm{d}v_i + (1-\bar{v}_h(p_a))\bar{v}_h(p_a) \\ &= -\frac{\bar{v}_h^2(p_a)}{2} + (1-K)\bar{v}_h(p_a) + (1+K)p_a - \frac{3}{2}p_a^2. \end{aligned} \tag{12}$$

   The first-order-condition is

$$\frac{\mathrm{d}\Pi(p_a)}{\mathrm{d}p_a} = (1+K-3p_a) + (1-K-\bar{v}_h(p_a))\frac{1}{5}\Big(1 - \frac{1+2p_a}{\sqrt{1-p_a-p_a^2}}\Big) = 0. \tag{13}$$

   Note that

$$\begin{aligned}&\frac{\mathrm{d}\Pi(p_a)}{\mathrm{d}p_a}\Big|_{p_a \to 2\sqrt{K(1-K)}-K} = 1+K-3p_a > 0, \\ &\frac{\mathrm{d}\Pi(p_a)}{\mathrm{d}p_a}\Big|_{p_a \to \frac{\sqrt{5}-1}{2}} \to -\infty < 0. \end{aligned}$$

   Moreover, (12) is a concave function because $-\frac{3}{2}p_a^2 + (1+K)p_a$ is concave in p_a, and $-\frac{\bar{v}_h^2}{2} + (1-K)\bar{v}_h$ is also concave in p_a since it is increasing and concave in $\bar{v}_h$, and $\bar{v}_h(p_a)$ is concave in p_a. Therefore, there exists a unique p_a which solves (13) and maximizes the seller's object (12). This unique solution is given by $\hat{p}_a$ defined in Lemma 7. Moreover, since $\frac{\mathrm{d}\Pi}{\mathrm{d}p_a}|_{p_a=\frac{1+K}{3}} < 0$, in equilibrium $p_a^* = \hat{p}_a < \frac{1+K}{3}$.

Following $p_a^* = \hat{p}_a$, $p_d^* = \hat{p}_a - K$, the partial disclosure outcome is indeed supported as the subgame equilibrium. Let $v_h^* \equiv \bar{v}_h(p_a) \mid_{p_a=\hat{p}_a}$. Observing a price menu with such p_a^* and p_d^*, it is optimal for consumers with $v_i \in [p_d^* + K, v_h^*]$ to choose (D, p_d^*) and consumers with $v_i \in (v_h^*, 1]$ to choose (A, p_a^*), anticipating that $p_2^{N,*} = v_h^*$ and $p_2^i = v_i$. Then it is optimal for the seller to set $p_2^{N,*} = v_h^*$ and $p_2^i = v_i$, respectively, for unrecognized and recognized customers in period 2.

3. When $K \geq \frac{1}{2}$, the seller chooses $p_a \leq \frac{\sqrt{5}-1}{2}$ to maximize

$$\Pi(p_a) = -\frac{v_h^2}{2} + (1-K)v_h + (1+K)p_a - \frac{3}{2}p_a^2,$$

where $v_h = \max\{\underline{v}_h, \frac{1}{2}\}$. In particular,

$$\Pi(p_a) = \begin{cases} -\frac{1}{8} + (1-K)\frac{1}{2} + (1+K)p_a - \frac{3}{2}p_a^2 & \text{if } p_a < \frac{1}{2} \\ -\frac{\underline{v}_h^2}{2} + (1-K)\underline{v}_h + (1+K)p_a - \frac{3}{2}p_a^2 & \text{if } p_a \in (\frac{1}{2}, \frac{\sqrt{5}-1}{2}] \end{cases}$$

Notice that the objective profit function $\Pi(p_a)$ increases with p_a when $p_a < \frac{1}{2}$ since $\frac{\mathrm{d}\Pi(p_a)}{\mathrm{d}p_a} = 1 + K - 3p_a > 0$. Moreover, $\Pi(p_a)$ decreases with p_a when $p_a > \frac{1}{2}$ since

$$\begin{aligned} \frac{\mathrm{d}\Pi(p_a)}{\mathrm{d}p_a} &= (1+K-3p_a) + (1-K-\underline{v}_h)\frac{1}{5}\Big(1 + \frac{1+2p_a}{\sqrt{1-p_a-p_a^2}}\Big) \\ &< (\frac{3}{2} - 3p_a) + (\frac{1}{2} - \underline{v}_h)\frac{1}{5}\Big(1 + \frac{1+2p_a}{\sqrt{1-p_a-p_a^2}}\Big) < 0 \end{aligned}$$

The first inequality above holds because $\frac{\mathrm{d}\Pi(p_a)}{\mathrm{d}p_a}$ is decreasing in K. The second inequality holds because $p_a > \frac{1}{2}$ and $\underline{v}_h > \frac{1}{2}$. Thus, the unique solution features $p_a \to \frac{1}{2}$, following which $v_h \to \frac{1}{2}$. However, with $v_m = p_a = v_h$, $\mathcal{S}_D$ becomes an empty set, and there exists no equilibrium with partial disclosure.

∎

Proof of Proposition 2. From Lemma 4 and Lemma 7, when $K \geq \frac{1}{2}$, the no-disclosure outcome is the only equilibrium that is supported as subgame perfect equilibrium. When

$K < \frac{1}{2}$, both the no-disclosure equilibrium in Lemma 4 and the partial-disclosure equilibrium in Lemma 7 are supported. We compare the seller's expected profits from the two categories of equilibria to pin down the seller's optimal price choice.

1. When $K \in [\frac{1}{26}, \frac{1}{2})$, from Lemma 7, $p_a^* = \frac{1+K}{3}$ uniquely solves the seller's maximization problem. And the seller's profit is $\Pi^* = \frac{2}{3}(1 - K + K^2) > \Pi^{nd} = \frac{1}{2}$, higher than the seller's expected profit, n the equilibrium with no disclosure as in Lemma 4.

2. When $K < \frac{1}{26}$, the seller's expected profit is

$$\begin{aligned}\Pi^* &= -\frac{v_h^{*2}}{2} + (1-K)v_h^* + (1+K)\hat{p}_a - \frac{3}{2}\hat{p}_a{}^2 \\ &> -\frac{v_h^{*2}}{2} + (1-\frac{1}{26})v_h^* + (1+\frac{1}{26})\hat{p}_a - \frac{3}{2}\hat{p}_a{}^2 = \frac{217}{338} > \frac{1}{2}.\end{aligned}$$

Since the seller's expected payoff from the partial disclosure equilibrium is strictly higher in both cases, it is optimal for the seller to choose the seller-optimal price menu characterised in Lemma 7 when $K < \frac{1}{2}$. ∎

Proof of Proposition 3. By Propositions 1, the seller's expected profit without anonymous option is

$$\tilde{\Pi} = \begin{cases} \dfrac{7 - 8K + 4K^2}{12} & \text{if } K < \dfrac{2-\sqrt{3}}{2} \\ \dfrac{1}{2} & \text{if } K \geq \dfrac{2-\sqrt{3}}{2} \end{cases}.$$

By Lemma 7 and Proposition 2, the seller's expected profit with anonymous option is

$$\Pi^* = \begin{cases} -\dfrac{v_h^{*2}}{2} + (1-K)v_h^* + (1+K)\hat{p}_a - \dfrac{3}{2}\hat{p}_a^2 & \text{if } K < \dfrac{1}{26} \\ \dfrac{2 - 2K + 2K^2}{3} & \text{if } \dfrac{1}{26} \leq K < \dfrac{1}{2} \\ \dfrac{1}{2} & \text{if } K \geq \dfrac{1}{2} \end{cases}$$

When $K \geq \frac{1}{2}$, $\tilde{\Pi} = \Pi^* = \frac{1}{2}$. When $K \in [\frac{1}{26}, \frac{1}{2})$, it is straightforward to see that $\Pi^* > \tilde{\Pi}$. In the following, we show $\Pi^* > \tilde{\Pi}$ when $K < \frac{1}{26}$. Note that when $K < \frac{1}{26}$

$$\frac{\mathrm{d}(\Pi^* - \tilde{\Pi})}{\mathrm{d}K} = -(v_h^* - \hat{p}_a) + \frac{2}{3} - \frac{2}{3}K = -\frac{3 - 4\hat{p}_a + 2\sqrt{1 - \hat{p}_a - \hat{p}_a{}^2}}{5} + \frac{2}{3} - \frac{2}{3}K.$$

From (13), we have

$$\begin{aligned}\frac{d\Pi(p_a)}{dp_a}\,|_{p_a=0.33} &= 0.01 + K + \frac{1}{5}\Big(1 - K - \frac{3.33 + 2\sqrt{1 - 0.33 - 0.33^2}}{5}\Big)\Big(1 - \frac{1.66}{\sqrt{1 - 0.33 - 0.33^2}}\Big) \\ &= 0.001 + 1.243K > 0,\end{aligned}$$

and this implies $\hat{p}_a > 0.33$ in equilibrium. Thus, $\frac{\mathrm{d}(\Pi^* - \tilde{\Pi})}{\mathrm{d}K}$ is positive since it increases in $\hat{p}_a$ and

$$\frac{\mathrm{d}(\Pi^* - \tilde{\Pi})}{\mathrm{d}K}\Big|_{\hat{p}_a=0.33} = -\frac{3 - 1.32 + 2\sqrt{1 - 0.33 - 0.33^2}}{5} + \frac{2}{3} - \frac{2}{3}K > 0.$$

Using $\hat{p}_a > 0.33$ again, it also holds that when $K \to 0$, $\Pi^* > \Pi(p_a = 0.33)$ and $\bar{v}_h = \frac{3+0.33+2\sqrt{1-0.33-0.33^2}}{5}$. Then we have

$$\begin{aligned}(\Pi^* - \tilde{\Pi})|_{K\to 0} &> (\bar{v}_h - 0.33)(0.33 - 0) + (1 - \bar{v}_h)0.33 + \int_{0.33}^{\bar{v}_h} v_i \mathrm{d}v_i + (1 - \bar{v}_h)\bar{v}_h - \frac{7}{12} \\ &= 0.666 - \frac{7}{12} > 0.\end{aligned}$$

Since $\Pi^* - \tilde{\Pi}$ increases in K and is positive at $K \to 0$, $\Pi^* - \tilde{\Pi} > 0$ holds for $K < \frac{1}{26}$. ■

Proof of Proposition 4. Without anonymous option, the total mass of consumers who disclose information is

$$\tilde{M} = \begin{cases} \dfrac{2 - 2K}{3} & \text{if } K < \dfrac{2 - \sqrt{3}}{2} \\ 0 & \text{if } K \geq \dfrac{2 - \sqrt{3}}{2} \end{cases}.$$

With anonymous option,

$$M^* = \begin{cases} \dfrac{3 + p_a^* + 2\sqrt{1 - p_a^* - (p_a^*)^2}}{5} - p_a^* & \text{if } K < \dfrac{1}{26} \\ \dfrac{2 - 4K}{3} & \text{if } \dfrac{1}{26} \leq K < \dfrac{1}{2} \\ 0 & \text{if } K \geq \dfrac{1}{2} \end{cases}.$$

Since $\frac{3+p_a^*+2\sqrt{1-p_a^*-(p_a^*)^2}}{5} - p_a^* < \frac{2-4K}{3} < \frac{2-2K}{3}$, it follows that the anonymous option lowers information disclosure when $K < \frac{2-\sqrt{3}}{2}$, increases information disclosure when $K \in (\frac{2-\sqrt{3}}{2}, \frac{1}{2})$. When $K \geq \frac{1}{2}$, the mass of consumers who disclose information is 0 regardless of the anonymous option.

Next, we compare consumer surplus in the two regimes. Without anonymous option, consumer surplus is

$$\tilde{CS} = \begin{cases} \dfrac{(5-2K)^2}{72} & \text{if } K < \dfrac{2-\sqrt{3}}{2} \\ \dfrac{1}{4} & \text{if } K \geq \dfrac{2-\sqrt{3}}{2} \end{cases}.$$

With anonymous purchase option, consumer surplus is

$$CS^* = \int_{p_a^*}^{1} (v - p_a^*)\mathrm{d}v + \int_{v_h^*}^{1} (v - v_h^*)\mathrm{d}v = \begin{cases} \dfrac{(1-p_a^*)^2}{2} + \dfrac{(1-v_h^*)^2}{2} & \text{if } K < \dfrac{1}{26} \\ \dfrac{5K^2-2K+2}{9} & \text{if } \dfrac{1}{26} \leq K < \dfrac{1}{2} \\ \dfrac{1}{4} & \text{if } K \geq \dfrac{1}{2} \end{cases}.$$

Since $\frac{5K^2-2K+2}{9} < \frac{1}{4}$ for $K < \frac{1}{2}$, it follows directly that $CS^* \leq \tilde{CS}$ for all $K \geq \frac{1}{26}$, with strictly inequality on $K \in [\frac{1}{26}, \frac{1}{2})$. When $K < \frac{1}{26}$, without anonymous option, consumer surplus comes from consumers with $v_i \geq \frac{1+2K}{6}$ each consuming one unit of the product at price $\frac{1+2K}{6}$.

Note that

$$\begin{aligned} p_a^* - (1 - \bar{v}_h(\hat{p}_a)) &= p_a^* - 1 + \frac{3 + p_a^* + 2\sqrt{1 - p_a^* - (p_a^*)^2}}{5} = \frac{1}{5}\big(6p_a^* - 2 + 2\sqrt{1 - p_a^* - (p_a^*)^2}\big) \\ &> \frac{1}{5}\big(6 \times 0.33 - 2 + 2\sqrt{1 - 0.33 - (0.33)^2}\big) = 0.295 > \frac{1+2K}{6}, \end{aligned}$$

where the first inequality holds because $p_a^* > 0.33$ and the second inequality holds because $K < \frac{1}{26}$. Thus

$$\begin{aligned} CS^* &= \int_{p_a^*}^{1} (v - p_a^*)\mathrm{d}v + \int_{\bar{v}_h(\hat{p}_a)}^{1} (v - \bar{v}_h(\hat{p}_a))\mathrm{d}v \\ &= \int_{p_a^*}^{1} (v - p_a^*)\mathrm{d}v + \int_{p_a^* - (1-\bar{v}_h(\hat{p}_a))}^{p_a^*} \big(v - p_a^* + (1 - \bar{v}_h(\hat{p}_a))\big)\mathrm{d}v \end{aligned}$$

$$< \int_{p_a^* - (1-\bar{v}_h(\hat{p}_a))}^{1} \big(v - p_a^* + (1 - \bar{v}_h(\hat{p}_a))\big)\mathrm{d}v$$
$$< \int_{\frac{1+2K}{6}}^{1} (v - \frac{1+2K}{6})\mathrm{d}v = \tilde{CS}.$$

Therefore, we conclude that the inclusion of the anonymous option harms consumers because $CS^* < \tilde{CS}$ for $K < \frac{1}{2}$ and $CS^* = \tilde{CS}$ for $K \geq \frac{1}{2}$. ∎

Proof of Corollary 1. Without anonymous purchase option, total welfare is

$$\tilde{W} = \begin{cases} \dfrac{7-8K+4K^2}{12} + \dfrac{(5-2K)^2}{72} & \text{if } K < \dfrac{2-\sqrt{3}}{2} \\ \dfrac{1}{2} + \dfrac{1}{4} & \text{if } K \geq \dfrac{2-\sqrt{3}}{2} \end{cases}.$$

Whereas with anonymous purchase option, total welfare is

$$W^* = \begin{cases} 1 - (p_a^*)^2 - (\bar{v}_h(p_a^*) - p_a^*)K & \text{if } K < \dfrac{1}{26} \\ \dfrac{2-2K+2K^2}{3} + \dfrac{5K^2-2K+2}{9} & \text{if } \dfrac{1}{26} \leq K < \dfrac{1}{2} \\ \dfrac{1}{2} + \dfrac{1}{4} & \text{if } K \geq \dfrac{1}{2} \end{cases}.$$

Notice that $W^* = \frac{8-8K+11K^2}{9}$ decreases on $K \in [\frac{1}{26}, \frac{4}{11})$ and increases on $K \in (\frac{4}{11}, \frac{1}{2})$. When $K \in [\frac{1}{26}, \frac{2-\sqrt{3}}{2})$,

$$W^* - \tilde{W} = \frac{8-8K+11K^2}{9} - \frac{67-68K+28K^2}{72}$$
$$= \frac{-3+4K+60K^2}{72} < \frac{-3+4\frac{2-\sqrt{3}}{2}+60(\frac{2-\sqrt{3}}{2})^2}{72} < 0.$$

And when $K \in [\frac{2-\sqrt{3}}{2}, \frac{1}{2})$,

$$W^* - \tilde{W} = \frac{8-8K+11K^2}{9} - \frac{3}{4} = \frac{5-32K+44K^2}{36},$$

which is positive for $K \in [\frac{2-\sqrt{3}}{2}, \frac{5}{22})$, and negative for $K \in (\frac{5}{22}, \frac{1}{2})$. Thus, the option of anonymous option increases total welfare if $K \in [\frac{2-\sqrt{3}}{2}, \frac{5}{22})$, decreases total welfare if $K \in [\frac{1}{26}, \frac{2-\sqrt{3}}{2}) \cup (\frac{5}{22}, \frac{1}{2})$ and does not affect welfare if $K \geq \frac{1}{2}$. To complete the proof, it remains to show that $W^* < \tilde{W}$ when $K < \frac{1}{26}$.

$$W^* - \tilde{W} = 1 - (p_a^*)^2 - (\bar{v}_h - p_a^*)K - \frac{67 - 68K + 28K^2}{72}$$
$$< 1 - (0.33)^2 - \frac{67 - 68 \times \frac{1}{26} + 28 \times (\frac{1}{26})^2}{72} < 0.$$

Therefore, the option of anonymous option increases total welfare if and only if $K \in [\frac{2-\sqrt{3}}{2}, \frac{5}{22})$. ∎

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