Resolving the Discounting Dilemma

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RESOLVING THE DISCOUNTING DILEMMA

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Abstract: Social Time Preference (STP) and Social Opportunity Cost (SOC) discounting differ in their objectives, but STP discounting measures capital costs incorrectly. The two-rate discounting method proposed here corrects this error, which current methods of shadow pricing capital (SPC) don't. Thereafter project choice discrepancies between alternative methods decrease substantially and the choice between them becomes unambiguous. The SOC rate is the hurdle feasibility rate either way. The marginal cost of public funds (MCF) correction is not an alternative to SPC correction; both must be used in conjunction when warranted. The Ramsey equation is a tautology that cannot predict the STP rate.

Keywords: Social discount rate; Prescriptive discounting; Descriptive discounting; STP discounting; SOC discounting; Two-rate discounting; Shadow Price of Capital; Marginal Cost of Funds; Declining discount rates; Ramsey rule.

JEL classification: D61; H43

1. Introduction

Referring to the familiar descriptive and prescriptive classification of approaches to discounting in benefit-cost analysis (BCA), William Nordhaus (2019) observed that the debate about discounting is "just as unsettled as it was when first raised three decades ago." The prescriptive approach defines a social time preference rate (STPR), while the descriptive approach seeks to measure the social opportunity cost rate (SOCR). The above classification coined by Arrow *et al* (1995) was a common shorthand in the literature, but more recently the prescriptive approach has come to be called the STP (social time preference) approach and the descriptive approach is more often called the SOC (social opportunity cost) approach. The difference between these rates is due to capital market taxes, which insert a wedge between interest rates paid and received. The lower rate can be taken to express social time preference because savers adjust their consumption path to it, while the higher rate expresses the opportunity coat of capital because entrepreneurs adjust to it.

The cause of the dilemma is that discounting by the alternative rates leads to very different conclusions. Because discounting performs two functions: imputation of capital costs and establishment of intertemporal weights, discounting at low rates undervalues the cost of capital, to which adherents of the SOC approach object, while discounting at high rates values future benefits lower, to which adherents of the STP approach object. High stakes attach to the choice between them, and advocates of both approaches often hold their views strongly. Freeman and Groom (2010) feel that these disagreements "raise the spectre of the near impossibility of reconciling" the STP and the SOC approaches to discounting. Spackman (2020) is blunter: "The divide between advocates of social opportunity cost and social time preference (STP) frameworks seems unbridgeable."

To partially bridge the divide, this paper shows how the two functions of discounting can be performed in distinct steps. In the first step the costs of capital are calculated using the SOCR, which, in the case of feasible projects, leaves only positive net benefits that need to be assigned intertemporal

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weights. As generally there is no dispute that capital has a cost, this step brings the two approaches much closer together because there will no longer be a difference in calculating capital costs.

The question that remains for a second step is what rate should be used to assign intertemporal weights to net benefits after capital costs? We argue that the choice of rate depends on the purpose of the analysis being performed. If the objective is to measure the welfare impact of projects as perceived by consumers themselves, following the principle of willingness to pay, then the appropriate STPR should be used. If the objective is to maximize the welfare impact of a given budget, then the SOCR should be used.

This resolves the dilemma, as the appropriate discounting method can be chosen unambiguously. The choice of STPR and SOCR values to use are empirical questions for each jurisdiction.

Our arguments will be presented in the context of what is often called efficiency analysis, in which benefits are defined by willingness to pay, costs by willingness to supply and income distribution effects or other preferences of social planners are not considered. Any such additional considerations, including indirect secondary effects, should be included in the specification of project flows, but even then, the question of how to account for the costs of capital when it is different from the rate of fall of the numeraire of the analysis will remain.

The paper has eight Sections. The logic behind two-rate discounting is explained in Section 2. Section 3 presents an alternative path to the same conclusions. Section 4 describes the corroboration of the assertions made in Section 2 by reference to a computable agent-based general equilibrium capital-market model. Section 5 discusses a commonly used way of defining the SOCR and reviews related issues, including the use of the marginal cost of funds (MCF). Section 6 reviews ways of defining the STPR. Section 7 examines the source of the discounting dilemma and proposes how to select the appropriate discount rate or rates. Section 8 presents conclusions.

2. Two-rate discounting

We start by recognizing Baumol's (1968) admonition that no single discount rate will yield correct NPV results. "We see now that no optimal rate exists. The rate that satisfies the one requirement [inter-temporal valuation] cannot possibly meet the conditions of the other [measuring the opportunity cost of capital]." We propose, instead, to simultaneously use both the STPR and the SOCR, but only in the role that they are suited to play.

Marglin (1963) already demonstrated that two rates are needed to correctly compute NPVs, but this remained unrecognized because it was not in the focus of his argument. "The answer is remarkably straightforward. […] we plan public projects to maximize their net present value at the marginal social rate of discount, but, in evaluating the social cost of public investment, an opportunity cost reflecting the social value of utilizing resources in private investment *replaces* the money cost of the portion of the resources that comes from the private investment sector." (Emphasis added.)

Assuming that the SOCR measures the opportunity cost of capital, the following simple example shows why both the STPR and the SOCR are needed to define a project's NPV. Take a project with infinite life that has capital costs K and yearly net operational benefits b accruing in perpetuity. In computing the present value of this project, Marglin's key insight was to "replace the money cost" K of the project by its future yearly opportunity cost $K \times SOCR$. The project's NPV therefore is:

$$NPV = -\frac{K \times SOCR}{STPR} + \frac{b}{STPR} \quad (1)$$

The above expression shows that both STPR and SOCR are needed to calculate correctly the NPV of the project. Neither rate can do it by itself.

We can interpret the ratio *SOCR/STPR* as the shadow price of capital (SPC) correction, which is a factor in this case. After having applied it by multiplication of *K*, it appears that the STPR is sufficient by itself, because it is the sole rate used for discounting benefits. This ignores the often-unrecognized fact that the condition for *NPV* being positive is:

$$\frac{b}{STPR} > \frac{K \times SOCR}{STPR} \tag{2}$$

which reduces to $b > K \times SOCR$. This means that the feasibility hurdle rate of return for projects is the SOCR. This is logical, as projects that fail to cover the opportunity costs of their invested capital are welfare destroying, a fact that is unaffected by the value of STPR, because both benefits and opportunity costs of capital lie in the future, so discounting them does not change their relative value. But meeting or exceeding the hurdle rate only ensures that the project is worth undertaking, to measure the present value of its welfare impact we need the intertemporal weighing implicit in the STPR.

Could a weighted average of the two rates compute a correct conventional single discount rate NPV? The equivalent weighted average rate, called *a*, can be derived from the following:

$$-K + \frac{b}{a} = -\frac{K \times SOCR}{STPR} + \frac{b}{STPR} \tag{3}$$

It is easy to solve for *a* in (3), but computing it is not useful because it is not just a function of *STPR* and *SOCR*, but also of *K* and *b* and is therefore only valid for a specific project and will not calculate the correct NPV for other projects.

Take as an example $K = 1$; $b = 0.07$; $STPR = 2\%$; $SOCR = 5\%$. Then $NPV = 1$, as computed by (1) or by the right-hand side of (3). The equivalent conventional weighted average discount rate obtained by solving for *a* is 3.5%, which, when used in the left-hand side of (3), also results in $NPV = 1$. However, should $b = 0.06$, then $NPV = 0.5$ and the weighted average discount rate that would yield the same result is 4%. It is impossible to find a generic weighted average rate that could be used to discount all projects conventionally.

The implicit SPC correction factor can also be computed from the assumed social discount rates (SDR). It is SPC = SOCR / STPR = 5% / 2% = 2.5. The corrected initial investment will be K = 2.5, and as the present value of future benefits is 0.07 / 0.02 = 3.5, NPV = 1. Discounting an SPC corrected net flow with the STPR gives the same NPV as the two-rate calculation method.

A simple numerical example of a finitely lived project shows how to compute an NPV with two rates and that the two-rate calculation method is again equivalent to conventionally discounting with the STPR after having shadow priced capital. Consider the following project flow:

Table 1

PROJECT FLOW OF A FIVE-YEAR PROJECT WITH CONSTANT BENEFITS

Years	0	1	2	3	4	5
Project flow	-1	0.4	0.4	0.4	0.4	0.4

To compute the two-rate NPV we replace the initial capital expenditure by its yearly opportunity cost, which is the annuity necessary to repay $K = 1$ with a yield of $SOCR = 5\%$ over five years. In effect this combines the required return on the committed stock of capital and its repayment. This is equal to 0.23 per year. The modified project flow is the following (notice that there are no values in Year 0):

Table 2
TWO RATE NET FLOW OF THE PROJECT IN TABLE 1

Years	0	1	2	3	4	5
Benefits	0	0.4	0.4	0.4	0.4	0.4
Opportunity costs of capital	0	0.23	0.23	0.23	0.23	0.23
Net flow	0	0.17	0.17	0.17	0.17	0.17

The NPV of the above Net flow is 0.80, which equals the difference between the present value of the benefits (1.89) and the present value of the opportunity costs of capital (1.09), all discounted at *STPR* = 2%.

To compute the NPV using the shadow price of capital (SPC) method, we can use the elegant calculation expression proposed by Cline (1992) and cited in the OECD CBA Manual (2018:207), which will calculate the present value of the opportunity costs of $1 invested for 5 years with the assumed SDR values.

$$SPC = \frac{SOCR}{STPR} \cdot \frac{1-(1+STPR)^{-5}}{1-(1+SOCR)^{-5}} = 1.09 \tag{4}$$

Notice that the computed SPC is the same as the present value of the opportunity costs of capital calculated from the figures shown in Table 2. The project flow using the SPC calculation method is the following:

Table 3
PROJECT FLOW OF TABLE 1 WITH SPC ADJUSTMENT

Year	0	1	2	3	4	5
Project flow	–1.09	0.4	0.4	0.4	0.4	0.4

The NPV of this flow discounted at the STPR is 0.80, the same value obtained using the two-rate method.

The ratio of the present value of the opportunity costs of capital to the nominal capital expenditure can be viewed as the SPC correction factor. Interestingly, there is another way to perform the same correction in an additive form. The present value of the financing flow (investment of 1 for five yearly repayments of 0.23) discounted at the STPR is 0.09, which is an equivalent additive SPC correction, which should be subtracted from the nominal cash flow in period 0. The present value of the financing flow is not the present value of the opportunity costs of capital, but rather the present value of the difference in opportunity costs attributable to the difference between the SOCR and the STPR.

Is the SPC correction computed in (4) useful for other projects as well? No, for it is also dependent on project-specific information: the useful life of the project and the assumption that capital is repaid in constant annuities. The following example shows how the two-rate NPV can be computed for a project with different annual flows.

Table 4

SIMPLE GENERIC PROJECT

Years	0	1	2	3	4	5
Capital expenses	0.5	0.5				
Operating costs		0.2	0.2	0.7	0.8	0.9
Benefits				2.5	2.8	3.1
Project net flow	–0.5	–0.7	–0.2	1.8	2	2.2

In this case calculating the net project flow after the opportunity costs of capital is more complex:

Table 5

FUNDING OF THE PROJECT IN TABLE 4

Years	0	1	2	3	4	5
Capital: starting stock	0.000	0.500	1.225	1.486	0.000	0.000
Capital: returns due	0.000	0.025	0.061	0.074	0.000	0.000
Capital: change in stock	0.500	0.700	0.200	–1.561	0.000	0.000
Capital: ending stock	0.500	1.225	1.486	0.000	0.000	0.000

Capital is needed to keep the project solvent, that is, to cover the initial negative cash flow, which includes not just capital expenditures, but also operational and funding costs, until the capital is fully amortized. The negative elements of the last line of Table 4, the project net flow, constitute the positive additions to capital stock in the third line of Table 5. Decreases in capital stock from benefits appear as negative values in the same line, and their values in absolute terms are the lesser of positive net flows (line 4 Table 4) or the outstanding capital balance plus returns due (sum of lines 1 and 2 of Table 5). The returns due are calculated by multiplying the starting capital stock of each year by the SOCR. The ending capital stock for each year, consisting of starting value, returns due and increases or decreases of stock, is shown in the fourth line and their values are transferred as the starting stock to the first line of the next year. Capital has been fully repaid and serviced when the ending stock is zero.

From Table 5 we observe:

- The opportunity costs of capital are not just a function of the investments made but also of the timing of project benefits. The longer it takes for the initial capital investment to be amortized, the longer capital will still be committed to the project, and the higher will the present value of its opportunity costs be.
- We can see why the SOCR is necessarily the feasibility hurdle rate, as there will be no net project benefits until capital has been fully amortized.
- Capital is not tied down infinitely in a project, but only for the limited time for which it is needed.
- Because SOCR>STPR, it makes sense to devote project benefits first to capital amortization, for that will maximize NPV. In fact, it is following this practice that makes our results agree with the calculation that is implicit in ordinary discounting. The NPV of the project of Table 1 would be higher if the fastest possible repayment rate were used, instead of assuming a repayment of capital in equal installments, as in Table 2.

Table 6 presents the calculation of the net flow after capital costs.

Table 6

NET PROJECT FLOW AFTER CAPITAL COSTS

Years	0	1	2	3	4	5
Project net flow	–0.500	–0.700	–0.200	1.800	2.000	2.200
Funding flow	–0.500	–0.700	–0.200	1.561	0.000	0.000
Net flow after capital costs	0.000	0.000	0.000	0.239	2.000	2.200

The first line of Table 6 is identical to the project net flow from the last line of Table 4. The funding flow is taken from Table 5 by multiplying line 3, changes in capital stock, by –1. The internal rate of return (IRR) of this flow is 5%, as it should be, as that is the value of the SOCR. The third line, which results from subtracting the second from the first, is the project net flow after capital costs. Discounting this at the STPR gives the project's NPV, which is 4.066.

A two-rate NPV can also be computed by compounding forward the conventional net flow of the project (benefits minus operating costs minus investments) at the SOCR until the future value ceases to be negative, thereafter doing so at the STPR until reaching the time horizon. Discounting the future value (FV) so obtained at the STPR yields the two-rate NPV. A project will have a positive welfare impact (evaluated at the STPR) if its FV is positive, or, equivalently, if its NPV is positive:

$$NPV = \frac{\sum_{i=0}^{t} b_i \prod_{j=i}^{t}(1+r_j)}{(1+STPR)^t} > 0 \tag{5}$$

where i, j, k are time period indexes ranging between 0 and time horizon t.

b_x = conventional net flow for period x (i or j)

r_j = SOCR if $\sum_{j=0}^{i-1} b_j \prod_{k=j}^{i-1}(1+r_k) < 0$

r_j = STPR if $\sum_{j=0}^{i-1} b_j \prod_{k=j}^{i-1}(1+r_k) \geq 0$

$r_0 = 0$

See a sample calculation in the Appendix. A simpler, recursive formulation of the above that will yield the same result is also shown in the Appendix.

The SPC correction that implements Marglin's admonition is shown in Table 7. We replace the money value of the investments, that is, the additions to capital stock, with the present value of the opportunity costs of capital of the project (–1.471 is the present value of the negative values of line 3, Table 5, in this case of – 1.561).

Table 7

PROJECT FLOW OF TABLE 4 WITH SPC ADJUSTMENT

Year	0	1	2	3	4	5
Project net flow	–1.471	0	0	1.800	2.000	2.200

The NPV of this project net flow is again 4.066. It is not easy to express this SPC correction as a factor in this case, however. To which element of Table 4 should we relate the present value of the opportunity costs of capital (1.471) to define an SPC factor? Any choice would be arbitrary. The opportunity costs of capital are the sum of the returns payable periodically on the stock of capital tied-up, in addition to repayments of principal. Their present value is not naturally measurable by a meaningful factor that multiplies some or all additions to the capital stock. However, computing an SPC factor is not necessary, for we already have the present value of the opportunity costs of capital, which is all that we need for the NPV calculation.

Rather than creating an adjusted flow such as that of Table 7, it is easier to use the additive correction defined by the NPV of the financing flow. We can either subtract it from the conventional NPV discounted at the STPR, or equivalently, subtract it from the time-period zero value of the cash flow. For the example of Table 4, the present value of the funding flow (Table 6, line 2) discounted at the STPR equals 0.092. The conventional NPV discounted at the STPR is 4.158. Subtracting 0.092 from this value gives the two-rate NPV of 4.066[1]. Equivalently, we can modify the Year 0 value of the net flow of Table 4 by subtracting 0.092 from it:

Table 8

PROJECT FLOW OF TABLE 4 WITH SPC ADJUSTMENT

Year	0	1	2	3	4	5
Project flow	–0.592	–0.700	–0.200	1.800	2.000	2.200

The NPV of this flow discounted at the STPR is also 4.066, the value obtained using the two-rate method.

We can see that the SPC correction is project specific. Using expression (4) to compute the SPC factor would be wrong in this case, because its assumptions of a single initial investment and subsequent even amortization of capital costs are not met. Capital needed is not the same as the initial capital expenditure but is defined by the amount and duration of the external financing required by the project. As with the conventional weighted average discount rate, there is no point in computing the SPC correction, because it will be project specific and could not be used for any other project.

The foregoing shows that Marglin's (1963) attempt to define a generic SPC correction based on the idea of an infinitely lived alternative investment is not the solution that BCA needs. This was recognized by Cline (1992), who developed a way to compute an approximate correction that takes the useful lives of projects into account. Generic SPC factors, such as the one proposed by Newell *et al* (2023), are not useful to quantify the opportunity costs of capital of specific projects. The cited study goes about measuring the opportunity costs of capital in a different way: "the SPC is defined as the change in immediate consumption equivalent to the present value of the stream of consumption losses associated with the immediate displacement of $1 of capital, discounted at the consumption discount rate. Computing this requires considering the effect that such an immediate displacement of capital would have on consumption over time, given savings rates, investment returns, and depreciation." "The degree of the displacement's persistence is determined by broad economic equilibrium dynamics, including depreciation and savings, suggesting that the SPC should be guided by macro-derived models of savings and investment."

The conceptual problem with the Newell *et al* (2023) approach is that it creates a fictitious alternative investment that has a time profile of capital commitment that is different from that of the project being appraised, which makes the two incomparable. The Newell *et al* (2023) method is complicated and fraught with estimation problems. It attempts to derive a universally valid degree of persistence of a generic investment of $1, whereas the duration for which capital is committed to a given project can be calculated from its forecast cash flow, as in Table 5. It should be noted that depreciation and savings rates are irrelevant to the determination of the capital committed to a project. Capital is not the same as capital goods, and depreciation is not a cost. BCA is based on cash flows. If a capital asset needs replacing, the replacement cost will be seen in the cash flow, as will its residual value, should it have any useful life left at the end of the period of analysis. The savings rate is not needed either, because as much capital remains committed to the project as has not been amortized.

By making the capital needs of projects explicit, two-rate discounting treats capital like any other input, valuing it at the SOCR, which measures its welfare cost. This makes the SPC correction unnecessary.

[1] Rounding errors occur when operating with rounded numbers.

Two-rate discounting greatly simplifies choice of technique calculations, which Spackman (2020) treats as a special case, calling it " 'Like with like' comparisons." To deal with such cases Feldstein proposed the following calculation approach: "Benefits and costs should be discounted at the social time preference rate after revaluing foregone private investment with a social opportunity cost shadow price equal to the discounted value of the future consumption that is lost per dollar of forgone investment." Noting that two-rate discounting calculates the opportunity cost of capital much more accurately that the original Marglin (1963) models to which Feldstein refers in the last sentence of the quote, we can assert that two-rate discounting can solve choice of technique questions with ease, following Feldstein's conceptual approach[2]. All we need to do is define a new project such that its net flow is the difference between the net flows of the two project versions between which a choice is to be made. Computing the NPV of this incremental project with the two-rate discounting method will identify the more advantageous option. It will have done so accounting for the opportunity cost of capital at the SOCR and performing the intertemporal weighting of net benefits at the STPR.

In conclusion, when performing a BCA using the consumption numeraire, the two-rate discounting method offers an easy way to reach accurate results. An equivalent conventional discount rate can be found for any project, but it is project specific and not useful for other projects. An equivalent SPC correction can be found as well, but it is also project specific and cannot usefully be expressed as a factor except in rare circumstances. Only the two-rate method will compute correct NPVs for any project, in a manner consistent with the core demands of both discounting approaches, using the universally applicable STPR and SOCR values[3].

It is worth recalling, further, that the feasibility hurdle rate is the SOCR, as shown in (2) above. Table 5 and Table 6 help to visualize the reason. If the benefits of a project are not sufficient to amortize the capital used, the project will have no net benefits while having a negative residual value (the unpaid balance on the capital account), which will then result in a negative NPV, no matter what STPR is used for discounting. The same conclusion holds when the SPC correction approach is followed instead of the two-rate method. If a project's IRR is below the SOCR, the present value of the opportunity costs of capital will be greater than the present value of the net operational benefits, regardless of the discount rate used.

3. An alternative path to the same conclusions

There is an alternative path to the conclusions of the previous section. Baumol's admonition that no single rate can simultaneously provide the correct intertemporal weighting and calculate the correct cost of capital is true because the process of discounting accomplishes both tasks with whatever rate is being used. (This is why project flows never include interest, because it would double count them, as discounting imputes the interest costs implicit in the discount rate.) Consequently, the only solution to the problem is to calculate the NPV in two separate steps, using the correct rate in each.

This is commonly done in financial analysis to compute return to equity. The problem to be solved in that case is analogous to the one we face. A firm borrows to finance an investment, incurring interest costs. To calculate the return to the owners of the firm, we must convert the project flow into the flow to equity, namely the cash-flow of the owners. For simplicity, assume that the owners finance the entire capital expenditure. To convert the project flow, we *replace* the initial capital expenditure by the debt service flow of the loan that finances it. The result is the flow to equity, which contains no initial capital expenditure but has benefits reduced by the debt service of the loan.

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[2] I thank David Burgess for pointing out that the Feldstein calculation approach would fail if conventional SPC correction were applied instead of two-rate discounting.

[3] Szekeres (2022) was not the first to propose using both SDRs simultaneously. So did Liu (2003), but the latter's method seriously underestimates operating costs, as well as the opportunity cost of capital, which makes it unsuitable for use in BCA. See Szekeres (2023), which explores the interaction between alternative discounting methods and the choice of numeraire in BCA.

It is the project flow less the funding flow. This is the equity holders' cash flow, from which a present value can be computed using the equity holders' opportunity cost of funds, which is different from the interest rate on the loan.

The calculations presented for the last example of the previous Section are analogous. The equity holder role is played by society, which finances its capital requirements at the SOCR and discounts its cash flow at the STPR. The NPV so computed is correct from a welfare point of view, as the method uses the correct rate in each step:

- The SOCR is used to calculate the opportunity costs of capital but is not used for discounting.
- The STPR is used to assign intertemporal weights but is not used to compute the opportunity cost of capital. (It could not compute one because the after-capital-costs net flow contains no negative values, save possibly in the last period if the residual value is sufficiently negative.)

4. Corroboration

Even though there is no theoretical reason to doubt the conclusions reached thus far, Szekeres (2022) built a computable capital-market general equilibrium model to experimentally corroborate them. It is a two-period agent-based model, with 58 agents. All have constant elasticity of substitution utility functions with coefficients of risk aversion equal to 1, and preset endowments (income) for Year 0 and Year 1. For half of the agents the Year 1 endowment is larger than that of Year 0, for the other half it is less. The agents optimize their consumption path by borrowing, lending, or investing in an equity with stochastic yield, the supply of which is infinitely elastic. Given the time profile of their endowments, half of the agents are inclined to borrow, and the other half are inclined to lend. The model finds the equilibrium market rate of interest at which lending equals borrowing and finds the amount invested in equities. There is a tax to be paid on interest income and dividends.

The model calculates endogenously both the STPR and the SOCR, and it can simulate the effect of undertaking a public sector project. The model assumes that the entire capital expenditure of the project is financed by issuing bonds that the agents subscribe, and that the net benefits of the project will be distributed equally among all agents (or if negative, then collected as a lump sum tax).

The model was used to compute the NPV of several sample projects using the two-rate discounting method described in Section 2. The corroboration of the results obtained derives from the gold standard of welfare analysis: the computation of an equivalent compensating variation. The model recorded the level of expected utility attained by each agent after consumption path optimization for both the with-project and the without-project situations. After this, taking the without-project expected utilities as a benchmark, the model calculated how much additional Year 0 income would make each agent's expected utility equal to that of the with-project situation. The aggregate welfare improvement attributable to the project is the aggregation of these compensating variations. For all tested sample projects, the aggregate welfare gain (or loss) coincided with the NPV's computed using the two-rate NPV calculation method, which corroborates that the method computes the welfare consequences of implementing projects correctly.

The calculations also confirm that the endogenous SOCR correctly measures the welfare cost of public borrowing and that the endogenous STPR computes present values correctly. The model offers interesting insights into the computation of both the STPR and the SOCR, which will be mentioned as relevant in the following Sections, in which references to "the model" are to the model mentioned in this Section.

5. The SOCR

5.1 Estimating the SOCR

Harberger (1972), in writing on measuring the social opportunity cost of public funds, defines what we call the SOCR as the weighted average of the net-of-tax yield of savings and the gross-of-tax cost of borrowing paid by investors, with the weights being proportional to their respective elasticities of supply and demand. This is consistent with how BCA calculates shadow prices. Raising capital for a public sector project will cause a slight increase in the market rate structure, resulting (1) in a small increase in savings that is valued at the rate that savers require to part from their money and (2) in a small decrease of borrowing by investors, valued at what they would have been willing to pay to borrow. The welfare cost of funding a public sector project can be calculated as a weighted average of the two cited rates and constitutes the SOCR. Burgess and Zerbe (2011) expanded Harberger's formulation by adding foreign borrowing as a third source of funds, thereby adding a third rate and a third weight to the formula.

This weighted average computes the SOCR, which is not the ephemeral project-specific average of the STPR and the SOCR that was found not to be very useful in Section 2 above. The Harberger weighted average is relatively stable, changing only with market conditions, but it only computes the SOCR. As we have seen the SOCR is the correct feasibility hurdle rate, but it will not compute a welfare correct NPV by itself because it will undervalue future net benefits when consumption is the numeraire. For accurate welfare measurement, the Harberger SOCR must be used in conjunction with the STPR, either using the two-rate discounting method or by performing a project specific SPC correction prior to calculating the NPV at the STPR.

It is quite interesting that the SOCR, defined to measure a marginal welfare cost, happens to coincide with the public sector's financial cost of funds. Burgess and Zerbe (2013) stated: "the [SOCR] criterion measures the impact of the project on the government's budget..." In the model, the sum of the effects of the classical components of the SOCR calculation (displaced productive investments, displaced consumer borrowing and induced savings) nearly exactly corresponds to the sum of two financial items: the interest paid by the public sector on the amount it borrowed, and the amount of taxes forgone because of its intervention in the capital market. Harberger (2007) explains and illustrates this phenomenon with a numerical example.

Burgess and Zerbe (2011) stated that "according to the [SOC] approach the marginal source of funding for all projects is the capital market, thus keeping the issue of tax reform separate from project evaluation. If a particular tax is being proposed to finance a particular project, the revenue from the tax could be used to pay down the debt instead of funding the project, so an alternative use of funds for any project is to pay down the debt." In the same vein, Harberger and Jenkins (2015) stated that "for most governments, all over the world, the capital market is indeed the marginal source or destination of funds. Governments make budgets once a year projecting in some detail their expected outlays and receipts. But rarely does the outcome match the expectations. The actual deficit or surplus ends up larger or smaller, often quite a bit larger or smaller than what was originally budgeted. And nearly all governments, it seems are perennially in debt."

Using the model, we simulated the effects of retiring debt in the same amount that was borrowed in the debt financing case. The result was a welfare gain virtually identical to the welfare cost of the financing alternative. This is not surprising because the two cases are mirror images of one another. When the public sector raises financing, consumer borrowing and investment in equities decline, whereas when the public sector retires debt, nearly the same changes take place with the opposite sign. This means that the SOCR, derived from the welfare effects on capital market participants that public borrowing has, also measures the value of retiring debt.

5.2 The SOCR in the long run

The paper has not yet considered the possibility of the SOCR changing over time. The relevance of this question depends on the benefits of the project, however. For economically feasible projects, capital might not be needed for all that long. If IRR>SOCR, then the initial capital investment will be paid off before the end of the life of the project. If this happens in reasonable time, comparable to the maturities that market players typically handle, then the SOCR derived from their actions is a good guide that does not need revising. If a project takes much longer than that to break even, then obviously explicit or implicit refinancing of the initially committed capital stock will be necessary, and a forecast of the SOCR for the appropriate time will be needed. Computing NPVs with variable SOCRs is trivial. Projects that never break even, however (IRR<SOCR), should not be undertaken to begin with.

Because of its prominence in the discounting literature, it is worth mentioning Martin L Weitzman's claim that certainty equivalent interest rates decline over time. Weitzman (1998) asserted that when uncertain market interest rates show positive autocorrelation, their certainty equivalent rate is a declining function of time. Deriving certainty equivalent rates from the expected value of all possible discount factors yields declining discount rates (DDR). Gollier (2004) came to the opposite conclusion by treating compound factors likewise, and this became the Weitzman-Gollier Puzzle. It turns out that Weitzman's assertion is based on a fallacy against which statisticians warn: the inverse of an expected value is not the expected value of the possible inverses[4]. Weitzman discounting violates the definition of present value. Correcting this error results in increasing certainty equivalent discount rates, rather than declining ones. See Szekeres (2020).

5.3 Empirical estimations of the SOCR

The foregoing addressed questions related to the estimation of the SOCR. The best way to elucidate remaining doubts is through actual empirical studies. These studies do exist (and are not onerously expensive, as they serve all projects in the country for which results are calculated). An estimation for the US was presented in Burgess and Zerbe (2011). The World Bank prepared one for Mexico in 2014, also following Harberger's methodology. See Coppola *et al* (2014).

5.4 Is the marginal cost of public funds an alternative to the SOCR?

While both the SOCR and the SPC are estimated assuming that public sector projects are funded by borrowing, there is a large body of literature devoted to the opportunity cost of public funds that focuses on the costs of taxation instead. This is also called the marginal cost of public funds (MCF). This literature largely ignores future opportunity costs. In a survey article Massiani and Picco (2013) listed twenty articles on the subject, of which only five mentioned the crowding out of private investments. In commenting on this they added "Eventually, this implies that the question about crowding out is not to know how much it counts but whether it should be taken into account. … Generally, the actual relevance of the crowding out in the OCPF's analysis has to be carefully questioned."

Massiani and Picco (2013) concluded that most definitions of the cost of public funds concentrate on the following effects: distortions in agents' behavior, deadweight loss caused by taxation, and to a lesser extent, corruption and tax collection costs. All these effects are contemporaneous with the collection of tax revenues, so a common way to express the result is one plus the ratio of these costs to tax revenues, resulting in a factor that is meant to multiply the

[4] As shown in Szekeres (2020) a Weitzman present value is equal to the correct present value times one minus the covariance between the compound and discount factors. As this covariance is always negative, because the factors are each other's reciprocals, the Weitzman present value is always larger than the correct present value.

investment flow of projects. The reported range of factors observed lies between 1.02 and 2.43, with the bulk being under 1.5.

Perhaps because the MCF is a factor and the SPC correction defined by Marglin (1963) is also a factor, some believe that these are alternative ways of calculating the welfare costs of public investments depending on the source of funds employed. We have seen in Section 2, however, that the correct SPC correction cannot be defined as a factor that multiplies investment flows, because the cost of capital depends on the time profile of the stock of capital used. The nature of this cost is quite different from that of the MCF.

The MCF measures the welfare costs of raising tax revenues. It is not just for valuing investments that this factor needs to be considered, if germane, but also to adjust operating costs if they are funded from tax revenues, as well as possibly to adjust monetary revenues, if they exist and are collected by a public entity,

The MCF adjustment is therefore not an alternative to shadow pricing capital; both corrections must be used simultaneously if the project is such that the MCF adjustment is required. The MCF will adjust the values of relevant flows, whereas the SOCR will define the cost of the committed capital stock.

This is very easily done using two-rate discounting. Multiplying investment flows by (MCF-1) will quantify the added welfare costs that funding a project through taxes entails. These amounts should be kept separate from investments when defining capital needs, because they do not need financing.

An important caveat must be added: we assumed that the MCF reflects only welfare losses that are contemporaneous with the raising of taxes. Capitalizing the future effects of taxing induced savings changes into the MCF is best avoided for two reasons: (1) it would require the use of the SOCR for capitalization, with the risk that the rate used when estimating the MFC was different from that used when analyzing a specific project, and (2), most importantly, because generic capitalization into the MCF could not take into account the time profile of capital utilization of specific projects. To obtain a clean analysis, the contemporaneous welfare loss of taxation should be measured by the MCF while the opportunity cost of capital should be measured by the SOCR.

6. The STPR

Harberger (1972) assumed that “the 'social rate of time preference' refers to an appropriately weighted average of the different marginal rates of time preference applicable to the individuals who compose the society.” The model referred to in this paper computes the STPR by finding the compensating variation payment that must be given to agents in Year 0 so that their utilities are the same as when they receive a very small payment in Year 1. The aggregation of these compensating variations indicates the aggregate value that agents place in Year 0 on a small sum received in Year 1. From this the STPR can be computed. This rate turns out to be equal to the weighted average of the rates implicit in the agents’ MRS between consumption in Year 0 and Year 1. For those that borrow it is the market interest rate, whereas for those that lend it is the market interest rate net of taxes. The STPR so calculated, which corresponds exactly to Harberger’s (1972) cited description, could be called the revealed preference STPR and is the only one that will compute welfare correct NPVs. Any other STPR value would either underestimate or overestimate future benefits from the point of view of aggregate welfare measured by the affected people’s revealed preferences.

This is not how Arrow *et al* (1995) defined the STP approach to finding the STPR, however. Their stance was rather based on normative or ethical considerations. Perhaps all definitions that depart from Harberger’s (1972) description should be called authoritarian, because they replace the revealed preferences of individuals by the preference of someone with authority to know better, be they politicians, social planners, or anyone else with the authority to decide. Two surveys of the field of social discounting provide a comprehensive overview: Greaves (2017) and Groom *et al* (2022).

The Ramsey Rule is one method of defining the STPR that is often used according to both surveys. Groom *et al* (2022) characterized the simple Ramsey Rule as a "workhorse" model for social discounting. Greaves (2017) stated that "The standard approach to determining the discount rate is via *the Ramsey equation*." Because of its prominence in literature the Ramsey equation merits a closer look. It is derived from a Constant Elasticity of Substitution utility function of the following form:

$$U = \frac{C^{1-\eta} - 1}{1-\eta} \tag{6}$$

where U is utility, C is consumption and η is the constant elasticity of marginal utility of consumption, which can also be interpreted as the coefficient of risk aversion. The first derivative of U with respect to C is:

$$U' = \frac{1}{C^{\eta}} \tag{7}$$

The first order condition for consumption path optimization is:

$$\frac{1}{C_0^{\eta}} = exp\ (r - \delta) \frac{1}{C_1^{\eta}} \tag{8}$$

The left-hand side is the marginal utility of consumption in time-period 0, the right-hand side is the marginal utility of consumption in time-period 1. Interest rate r for one period reflects the rate at which consumption can be transferred between periods and the pure rate of time preference δ indicates the loss of value of consumption in the next period. Agents achieve equality in (8) by changing C_0 and C_1 until the two sides of the equation become equal, while respecting their budget constraints. Rearranging (8) we obtain:

$$\frac{C_1^{\eta}}{C_0^{\eta}} = exp\ (r - \delta) \tag{9}$$

Taking natural logarithms of both sides and rearranging we reach the Ramsey formula, in which g is the growth rate of consumption per period implicit in C_0 and C_1:

$$r = \delta + \eta g \tag{10}$$

Economists have used this expression to estimate the STPR. Greaves (2017) reported that Stern assumed δ = 0.1%, η =1 and g = 1.3% to arrive at r = 1.4%, whereas Nordhaus (2019) states that "I adopt a time discount rate of 1.5 percent per year with a consumption elasticity of 2. These yield an equilibrium real interest rate of 5.5 percent per year with the consumption growth that is projected over the next century by the DICE-2007 model."

The Ramsey formula may have become the "workhorse" model for social discounting, but it is problematic because it is a circular definition. Notice that expression (10) is derived from the basic assumption underlying expression (8), namely that the agent has optimized his consumption path by reference to a *known r*. Expression (10) should not be used to *find r*, therefore.

The Ramsey formula is a tautology that is true for any point along a saver's indifference curve, provided it is tangent to an exogenously determined r, for only then will expression (8) be true. Therefore, the Ramsey formula is not a behavioral function that can be used to determine an unknown r, because if r is not known, then expression (8) cannot be assumed to be true, and therefore neither can expression (10). This is confirmed by the model. Reducing the pure rate of time preference of all agents by half a percentage point does not reduce the endogenous STPR of the model by a like amount, as it should if the Ramsey formula were a behavioral function. The STPR does fall, but by much less. The modification causes a change in the equilibrium interest rate, and new optimal consumption values are chosen by all agents, for which the Ramsey expression is again true, as it would be for any possible equilibrium point. Obviously, the Ramsey formula cannot deal with a change in taxes payable on interest revenues that will directly affect r. As expression (8) will hold for any conceivable market equilibrium, it cannot be used to find the actual market equilibrium r.

Further, the g variable of the Ramsey equation is not an exogenous variable because it is the result of optimization. The incomes of agents in Year 0 and Year 1 can be set exogenously (and therefore can be forecast), but g, computed from C_0 and C_1, can only be calculated once the model has found the market clearing equilibrium r, by which time all agents will have reached their optimal consumption paths at the equilibrium rate found. Increasing Year 1 incomes of all agents in the model by 0.5% only resulted in a growth of consumption of 0.03% because consumption smoothing by agents caused them to also increase their Year 0 consumption. The actual value of g turned out to be much lower than that of the income growth that induced it.

A moment of reflection should suffice to realize that no expression containing data only about one side of a market, the supply of savings, carries enough information to determine the market interest rate (or set of distorted market rates). For that, information about the demand for loanable funds is also necessary, not least of which is the rate of technological progress that dominates the productivity of investments.

Groom *et al* (2022) reported that there is an extended Ramsey rule that adds an additional term to expression (8) because "the social planner is concerned that future consumption levels may be below their expected value and therefore is prepared to save out of precaution in projects that pay off in the future." The extended formulation would not be able to predict market rates either, not just for the cited reason of circularity, but also because adding a term to expression (10) implies a violation of expression (8), from which it was derived. The reference to the social planner, however, seems to indicate that the Ramsey rule is viewed not as a method of estimating a real-world discount rate, but rather as an intellectual construct useful in devising an authoritarian STPR.

Whether risks should affect the value of the STPR has not been addressed in this paper. It is standard BCA practice to assume risk neutrality, but for a description of this topic see Spackman (2020).

Unfortunately, empirical estimations of revealed preference STPRs do not seem to exist, even though there is no reason to think that they should be difficult to make. Having such estimates would be useful to benefit-cost analysts interested in calculating the welfare impact of projects and to social planners to measure costs imposed on the present generation by their choice of authoritarian STPRs.

7. The discounting dilemma

7.1 The need for discounting

BCA needs discounting because investment projects transform present day resources into future benefits, so costs and benefits accruing at different times need to be made comparable to reduce the dimensionality of a complex set of effects into a single number that will facilitate a yes or no decision. Discounting achieves intertemporal weighting (present value of a future sum) by producing a set of weights that typically decline through time.

Two distinct rationales can be found for declining intertemporal weights. One could be called an intrinsic valuation or *time preference*, which might derive its values from considerations such as the following.

- As income levels rise, future consumption is worth less than present consumption because of declining marginal utilities of income.
- Pure time preference for the present, which might be linked to the declining probability of being able to enjoy future consumption.
- Ethical considerations concerning the value attributed to the consumption of future generations. These might even result in non-declining weights.

The other rationale for discounting derives from the classical definition of present value. The present value of $1 at future time *t* is the amount that the allocator of resources would need to devote to an alternative investment opportunity available in the present that will yield $1 at time *t*. This is not intrinsic valuation, as the rule exhibits no a-priori time preference, it is just the result of a rational consideration: why value something higher than an alternative way of obtaining it? We will call this rationale *opportunity cost valuation*, which focuses on the yield of possible alternative investments.

It is these alternative rationales that gave rise to the alternative approaches to discounting in BCA.

7.2 Discounting methods

7.2.1 Social Time Preference Discounting

How members of society value future consumption relative to present consumption can be detected from how they allocate their own consumption between the present and the future. They do so by optimizing their consumption path, considering the rate at which they can transfer resources between time periods. After consumption path optimization their marginal rate of substitution (MRS) between present and future consumption will reflect the interest rate available to them. The weighted average of such rates can be viewed as the revealed preference STPR. Even though this rate is derived from the opportunity costs of funds of members of society, it plays the role of an intrinsic time preference in BCA.

The STPR so defined should be viewed as the shadow price of future consumption relative to present consumption. The fact that the value of the STPR reflects market distortions is immaterial. It is like the shadow price of taxi rides in a regulated market. The regulated fare *is* the shadow price of a ride because it corresponds both to the value and the cost of the marginal ride, by virtue of the optimizations performed by riders and drivers[5].

When BCA selects consumption as a numeraire, STPR is the appropriate discount rate, as it measures how society values future consumption.

As the cited reviews of the literature show, however, most proposed STPR definitions are not derived from people's preferences, but from other considerations, such as ethical concerns, or a host of other rationales that a social planner or entity endowed with the authority to decide may choose to define. In that case the STPR could have a very low value, be declining in time, or even be zero. It would then be a purely intrinsic time preference that does not depend on any opportunity cost nor on anything else outside the designs of the decider.

[5] Arnold Harberger offered this illuminating example at a lecture in Washington, DC, sometime in the 1980s.

7.2.1 Social Opportunity Cost Discounting

This discounting method derives from the rational consideration, which is fundamental in economics generally, that one should not assign a higher present value to any future \$1 than the present amount for which the future \$1 could be alternatively obtained. This method is indifferent to whatever time preference the decision maker might hold for that future \$1, because the same preference adjustment would also apply to the future \$1 obtained as a yield of an alternative investment. Therefore, assuming rational behavior, opportunity cost valuation always trumps time preference.

Consider a project that for each \$1 invested creates a benefit at future time t of $(1+p)^t$, where p is the IRR of the project. Each \$1 could also be invested alternatively, yielding a return of a, to obtain $(1+a)^t$. If the decision maker had a time preference rate of ρ, the feasibility criterion would be the following:

$$\frac{(1+p)^t}{(1+\rho)^t} > \frac{(1+a)^t}{(1+\rho)^t} \tag{11}$$

It is easy to see that time preference ρ is irrelevant to the decision and that the hurdle feasibility rate is the alternative yield a, showing that opportunity cost valuation always trumps time preference.

When BCA adopts a public funds numeraire, the role of the alternative investment rate is played by the SOCR.

7.3 The genesis of the discounting dilemma

STP discounting and SOC discounting yield different NPVs, of course. Even though it is generally recognized that conventional STP discounting undervalues the cost of capital, only lip service is paid to it generally and there appears to be a generalized belief that SPC corrections, if done, would not have large impacts. The discount rates themselves receive far more attention.

Many considered that opportunity cost valuation undervalues future benefits. Pigou (1932) famously characterized this as having “defective telescopic faculty.” Chichilnisky (1997) thought that it embodies the “tyranny of the present over the future.” Groom *et al* (2005) stated that "The deleterious effects of exponential discounting ensure that projects that benefit generations in the far distant future at the cost of those in the present are less likely to be seen as efficient, even if the benefits are substantial in future value terms."

Those who focus instead on opportunity cost could retort that if Pigou’s telescope were to train not just on project benefits, but also on their alternative, the view would change. They could also argue that exponential discounting does not have deleterious effects, it just ensures that proposed projects are at least as efficient in transforming present day resources into future benefits as other available investments.

The discounting dilemma stems from the seemingly irreconcilable approaches to intertemporal valuation. Weitzman was moved to state "There does not now exist within the economics profession, nor has there ever existed, anything remotely resembling a consensus, even—or, perhaps one should say, especially—among the 'experts' on this subject. (Actually, with very little exaggeration or cynicism, an 'expert' here might be defined as an economist who knows the literature well enough to be able to justify any reasonable social discount rate by some internally consistent story.)" Groom *et al* (2005) added "Ultimately, the practitioner is faced with a potentially confusing array of rationales and a sense that almost any discount rate can be applied." This is a prediction that the already cited surveys of the literature tend to bear out.

7.3 The cost of capital

Discounting is not just an intertemporal valuation method, however, but also a way to impute capital costs, and it is well known that unadjusted STP discounting fails at the task. Section 2 has shown, however, that it is possible to separate the intertemporal valuation function of discounting from its capital costing role. Because the cost of capital is a cash project expense, totally unrelated to time preference, the cost of capital calculation should be the same regardless of which intertemporal valuation method is chosen.

Valuing capital costs correctly is a *sine qua non* requirement of performing a good BCA. It is hard to overstate the importance of correctly computing capital costs. Some may feel that the opportunity cost of capital is the cost of a hypothetical alternative investment. It is not. Once a project is undertaken, it will be an actual resource cost, to be paid by someone, just like the costs of material inputs, energy, or labor. Incurred capital costs are also subject to relatively little uncertainty in the short or medium term, as the cost of funds allocated to a project is relatively easy to ascertain.

In two-rate discounting the SOCR can be combined with any intertemporal valuation method and will result in the correct measurement of capital costs while keeping the chosen intertemporal weighting for remaining benefits. (Combining SOCR with SOCR is, of course, the same as conventional SOC discounting).

Two-rate discounting greatly reduces the discrepancies in results deriving from choosing alternative discounting methods, for the following reasons:

- The common rejection of projects that fail to cover their capital cost, a decision that is independent of the valuation of future benefits. Recall from Section 2 that the hurdle rate of return is the SOCR.
- Intertemporal valuation will only apply to benefits remaining after full capital amortization, and its only job will be alternative selection between feasible projects. While computed NPV values will always be higher with STP discounting than with SOC discounting, this effect will apply equally to all alternative projects.

7.4 Choice of discounting method

Two-rate discounting will greatly reduce the discrepancies in results between the alternative discounting methods but will not eliminate them entirely. The remaining differences, attributable to divergent intertemporal weighing, will affect the selection between alternative feasible projects. The discount rate selection should suit the objectives of the analysis.

- If the objective of BCA is to select projects that maximize the present value of attainable benefits, then net benefits after capital costs should be discounted at the SOCR, because that is the *opportunity cost of the available funds*. This is the method to be used by public entities interested in optimizing the allocation of a given investment budget. In this case conventional discounting using the SOCR will compute correct NPVs. Notice that what this discounting method measures is the cost of matching the net benefits of projects through alternative investments.
- Alternatively, if the objective of BCA is to measure the impact of projects on consumers' welfare, as perceived by themselves, *i.e.*, to forecast the welfare consequences of the with-project state of the world, then net benefits after capital costs should be discounted at the STPR because that is the *consumers' intertemporal time preference*[6]. In this case there is no

[6] As explained in Section 4, the NPV so computed would equal a present payment received by consumers such that they would be indifferent between it and the benefits of the project being analyzed.

shortcut, two-rate discounting must be used, first to measure capital costs and then to separately value future consumption.
- The same applies in the case of an authoritarian STPR, which would not measure welfare, however, but the social planner's preference.

SOC discounting is consistent with the definition of present value, because the computed NPVs will compound back to project benefits at the SOCR, which is the decision maker's opportunity cost of funds.

Two-rate discounting, which is effectively STP discounting applied to net benefits after capital costs, is internally consistent if the STPR reflects the revealed preferences of the representative consumer, because the NPV will compound back to quantified benefits. However, the method will not be consistent from the point of view of the resource allocating authority, because the NPV will overstate the amount that is necessary to match project benefits through alternative investments at the authority's opportunity costs of funds, which is the SOCR.

Discounting always assumes that all benefits will be fully reinvested, otherwise the congruence between discounting and compounding would be lost. In the case of SOC discounting correcting project flows to reflect any induced savings would have no effect on the computed NPVs because the social value of savings equals the implicit time preference. In the case of STP discounting, however, adjusting net benefit flows to reflect the value of any induced savings will have an effect, as shown in the Appendix, because the social value of savings (SOCR) is higher than the social time preference rate (STPR).

7.5 Resolution of the dilemma

The discounting dilemma was caused by the following factors:

- The spread between the values of possible STPR and SOCR rates created opposing camps strongly attached to conflicting views, without any resolution method in sight.
- The expectation that there should be "one" discount rate. Not only is this impossible, as observed by Baumol (1968), but it is also something that shouldn't even be tried, because the alternative rates measure the rates of fall of two different numeraires. A seen in section 2, no generic weighted average of the two exists that will yield correct results.
- Willingness to forgo the SPC correction in STP discounting and the acceptance of the necessarily flawed results that follow.

As we have seen, the resolution of the dilemma lies in:

- Correctly computing capital costs through the two-rate discounting method proposed in this paper.
- Choosing the intertemporal valuation method that suits the objective of the analysis.

7.6 Current practices

According to Groom *et al* (2022, Table 1) thirteen of twenty jurisdictions or entities selected use SOC discounting. BCAs performed in these will compute capital costs correctly, except for the six among them that use declining discount rates, because declining discount rates will likely understate the costs of capital that approved projects will incur.

In the case of those that use STP discounting, capital costs are very likely underestimated, as there is no report of shadow pricing of capital being performed. The OECD CBA Manual (2018:221) states that this is an all-too-common practice: "Using the Shadow Price of Capital approach (SPC) is advisable when using the [STPR], so that the opportunity cost of public capital can be reflected in the NPV calculation. This rarely happens in practice due to onerous informational requirements."

Even if shadow pricing of capital were conducted, it would be of the hypothetical investment alternative kind that was shown to be inadequate in Section 2.

Whenever capital costs are incorrectly valued, for any reason, the likelihood of investing in welfare destroying projects is real, which would saddle future generations with debt repayments in excess of benefits generated by projects. That would truly be a case of enslavement of future generations. Adherents of STP discounting must realize that failing to fully account for all resource costs makes their BCAs flawed. They must use two-rate discounting to avoid committing this grave error.

8. Conclusions

The discounting dilemma caused uncertainty about which discounting method to use. The two-rate discounting method proposed in this paper corrects the undervaluation of capital costs that results from conventional STP discounting. Having corrected that, both discounting methods would agree on which projects are economically feasible. The only question that remains then is how to discount net benefits after capital cost. We propose the following:

- If the objective is to maximize the present value of benefits, then the discounting method to use in BCA is SOC discounting, for that is the method that identifies projects that convert present resources into future benefits in the most efficient way possible. In this case conventional discounting at the SOCR will give the same results as two rate discounting.
- If the objective is to measure the impact of projects on consumers' welfare, as perceived by themselves, then two-rate discounting must be used (with SOCR and STPR) to both correctly measure capital costs and to value consumption at its shadow price.

When performing BCAs using a consumption numeraire, the best way to compute NPVs is to use the STPR and the SOCR jointly as proposed in this paper. As Marglin (1963) stated, "the answer is remarkably straightforward," replace the money costs of the initial investments by their opportunity costs (measured by the SOCR) and discount the resulting project flow by the STPR. A two-rate NPV can be computed easily by compounding forward the conventional net flow of the project (benefits minus operating costs minus investments) at the SOCR until the future value ceases to be negative, thereafter doing so at the STPR until reaching the time horizon. Discounting the future value (FV) so obtained at the STPR yields the two-rate NPV. A project will have a positive welfare impact (evaluated at the STPR) if the FV is positive, or, equivalently, if the NPV is positive.

The proposed two-rate discounting method shares with Marglin (1963) the recognition that the opportunity costs of capital lie in the future and that their present value should replace the money cost of investments. However, it differs in that two-rate discounting precisely measures the capital requirements of projects, whereas Marglin (1963) traced the likely welfare impact of an alternative hypothetical investment that is not comparable with specific projects because of differing capital requirement time profiles. Further, the calculation of the opportunity cost of capital of the alternative hypothetical project is fraught with estimation uncertainties.

Using the two-rate method is instructive, for by showing explicitly the time profile of the capital needs of projects it allows capital to be treated as just another project input that is valued by its shadow price, the SOCR. It also highlights an important fact that is not evident when the opportunity costs of capital are only implicitly calculated through the act of discounting: that capital is a stock, the returns to which are a flow defined by a rate per unit of time.

The following are further conclusions worthy of mention:

- The two-rate method will correctly compute the social welfare impact of any project, using the universally applicable STPR and SOCR values.

- A weighted average of the SOCR and STPR can be found that will compute a conventional single discount rate NPV that equals the computed two-rate NPV, but it is project specific and cannot be used for any other project. For this reason, it is not worth calculating. This ephemeral weighted average rate is not the weighted average rate proposed by Harberger (1971), however, which instead defines the SOCR.
- The feasibility hurdle rate of return is the SOCR because projects must cover their opportunity costs of capital, which consist of the sum of required returns payable periodically on the stock of capital tied-up, in addition to its repayment. Society will only receive net benefits from a project after its capital has been amortized. If the present value of operational benefits does not exceed the present value of costs of capital, the NPV will be negative, irrespective of the value of the STPR, as both operational benefits and costs of capital lie in the future and discounting them does not change their relative magnitudes.
- Two-rate discounting is equivalent to performing an SPC correction tailored to the exact capital requirements of each project. The additive form of the correction, the NPV of the financing flow discounted at the STPR, is a convenient way of adjusting the capital costs implicit in the STPR to reflect those implicit in the SOCR[7].
- An SPC correction can be found for any project that will yield a correct single rate NPV discounted at the STPR, but this adjustment is project specific, as it depends on the time profile of the benefits and the consequent capital requirements. Therefore, no single SPC factor can be universally used for all projects. Further, the SPC correction is difficult to apply as a factor, unless only the first element of a project's net flow is negative. Because capital is a stock, the opportunity costs of capital are not meaningfully measurable by a factor multiplying some or all increments of that stock. Because two-rate discounting values capital at its opportunity cost, using it is equivalent to a well conducted SPC correction, which therefore becomes unnecessary.
- There is a widely held perception that the STPR should be the hurdle rate, for once an SPC correction has been performed, only the STPR is used for discounting. This perception might be responsible for the fact that the discounting literature focuses largely on the STPR. Those holding this view should realize, however, that if a project's IRR<SOCR then the SPC correction will assume a value such that it makes the project's NPV negative.
- Conventionally discounting project flows by the STPR without making a SPC correction seriously biases the result of the calculation by understating the opportunity costs of capital.
- The MCF, as commonly defined, is not a measure of the opportunity costs of capital, but of the welfare costs of raising taxes. Therefore, it represents a necessary correction that is additional to that of the opportunity cost of capital.
- Choice of technique calculations are quite easy to do with two-rate discounting. The NPV of an incremental project defined by the difference between the net flows of the two alternative versions of the project to consider will determine which is better.

How to calculate the welfare impact of projects correctly using the SPTR, SOCR and MCF should be clear, but disputes remain regarding the values of these parameters.

Most proposed STPRs are authoritarian, including those that are derived from the Ramsey equation, which is unsuitable for predicting real world rates because it is a tautology that will hold for all conceivable market equilibria. It would be useful to have empirically measured revealed preference STPR values so that policymakers could assess the welfare impact of their investment decisions.

The Harberger (1971) method of SOCR estimation is widely accepted. Because the SOCR is the feasibility hurdle rate in all cases, the discounting literature should perhaps redirect its attention

[7] A second funding flow might have to be specified for certain projects. For a nuclear powerplant with high decommissioning costs a sinking fund must be modeled, the interest revenues of which might differ from both the STPR and the SOCR. This might involve a third rate.

to its empirical estimation. The SOCR is not a discretionary policy variable and should be measured as accurately as possible. This is important to avoid undertaking projects that are destructive of future generations' welfare. As Becker, Murphy and Topel (2011) stated: "Future generations would not thank us for investing in a low-return project."

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APPENDIX

1. Computing Two-Rate NPVs

The explicit expression defining two-rate NPV as a function of a project's net flow was shown in Section 2 to be the following:

$$NPV = \frac{\sum_{i=0}^{t} b_i \prod_{j=i}^{t}(1+r_j)}{(1+STPR)^t} > 0 \qquad (11)$$

where i, j, k are time period indexes ranging between 0 and time horizon t.

b_x = conventional net flow for period x (i, or j)

r_j = SOCR if $\sum_{j=0}^{i-1} b_j \prod_{k=j}^{i-1}(1+r_k) < 0$

r_j = STPR if $\sum_{j=0}^{i-1} b_j \prod_{k=j}^{i-1}(1+r_k) \geq 0$

$r_0 = 0$

The logic behind this expression is that we take every element b_i of the net flow and compound it forward to time horizon t, doing so stepwise, one period at a time. The compounding rate r_j used in each step depends on the sign of the result of the same expression (11) computed up to the previous period.

The following is a calculation example using the data of the project shown in Table 4, replicated in Table 9 for convenience.

Table 9

SIMPLE GENERIC PROJECT

Years	0	1	2	3	4	5
Project net flow	–0.5	–0.7	–0.2	1.8	2	2.2

The calculations performed using expression (11) are the following. Recall that STPR = 2% and SOCR = 5%.

Table 10

FUTURE VALUE OF THE SIMPLE GENERIC PROJECT

1+r	1	1.05	1.05	1.05	1.05	1.02
Year	0	1	2	3	4	5
0	–0.500	–0.525	–0.551	–0.579	–0.590	–0.602
1		–0.700	–0.735	–0.772	–0.787	–0.803
2			–0.200	–0.210	–0.214	–0.218
3				1.800	1.836	1.873
4					2.000	2.040
5						2.200
Total	–0.500	–1.225	–1.486	0.239	2.244	4.489

The second row of Table 10 shows the years, the first row shows the corresponding compounding factors, the values of which depend on the sign of the previous period's value in the last line of Table 10 that contains totals. Each row shows the calculations performed for net flow element b_i that is identified in the first column. The FV of the project can be found in the lower right corner of the table and is 4.489.

The two-rate NPV of the project is the PV of the FV, namely 4.489/(1.02)^5 = 4.066.

A simpler calculation is possible if we define a recursive compounded cumulative value function of the project net flow values as follows:

$$C_i = C_{i-1}\,(1+r_i) + b_i \qquad (12)$$

where i is the time period index ranging between 0 and time horizon t.

b_i = conventional net flow for period i
r_i = SOCR if $C_{i-1} < 0$
r_i = STPR if $C_{i-1} \geq 0$
$r_0 = 0$
$C_{-1} = 0$

The calculations performed using expression (12) are the following:

Table 11
FUTURE VALUE OF THE SIMPLE GENERIC PROJECT

Years	0	1	2	3	4	5
Project net flow	–0.5	–0.7	–0.2	1.8	2	2.2
C_i	–0.500	–1.225	–1.486	0.239	2.244	4.489

The FV value is C_t, or C_5 in this case, equaling 4.489, just like before, which discounted at the SRTP yields 4.489/(1.02)^5 = 4.066.

The recursive compound cumulative method is the easiest way to compute two-rate NPVs.

These calculation methods will yield accurate results for projects that have a single change of sign in their net cash-flow. When this condition is not met, adjustments might be needed.

2. Accounting for savings out of project benefits

To illustrate how to account for induced savings, we reproduce Table 6, but splitting net benefits into consumption (80%) and savings (20%), as follows:

Table 12
ACCOUNTING FOR THE EFFECT OF SAVINGS

Years	0	1	2	3	4	5
Project net flow	–0.500	–0.700	–0.200	1.800	2.000	2.200
Funding flow	–0.500	–0.700	–0.200	1.561	0.000	0.000
Net flow after capital costs	0.000	0.000	0.000	0.239	2.000	2.200
Consumed benefits	0.000	0.000	0.000	0.192	1.600	1.760
Saved benefits	0.000	0.000	0.000	0.048	0.400	0.440
Compounding of savings	0.000	0.000	0.000	0.048	0.450	0.913
Adjusted net benefit flow	0.000	0.000	0.000	0.192	1.600	2.673

Saved benefits are compounded forward at the SOCR in line 6 of Table 12, to obtain a final value of 0.913. This is added as a residual value to the adjusted net benefit flow of the last line of the table. As this project has a short life, the effect is not spectacular. The SOCR is used for compounding because that is the opportunity cost of capital, and therefore the social value of savings. The adjusted net benefit flow is the sum of the consumed benefits plus the final value of the compounded savings. The NPV of the adjusted net benefit flow is 4.079, slightly higher than the unadjusted NPV of 4.066 found for the net flow of Tables 6.