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Abstract

Policy makers have developed different forms of policy intervention for stopping, or preventing runs on financial firms. This paper provides a general framework to characterize the types of policy intervention that indeed lower the run-propensity of investors versus those that cause adverse investor behavior, which increases the run-propensity. I employ a general global game to analyze and compare a large set of regulatory policies. I show that common policies such as Emergency Liquidity Assistance, and redemption (withdrawal) fees either exhibit features that lower firm stability ex ante, or have offsetting features rendering the policy ineffective.

Key words: financial regulation, bank runs, global games, policy effectiveness, bank resolution, withdrawal fees, emergency liquidity assistance, lender of last resort policies, money market mutual fund gates, suspension of convertibility
JEL Classification: G28,G21,G33, G38, D82, D81, E61

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1 Introduction

L'enfer est plein de bonnes volontés ou désirs
[*The road to hell is paved with good intentions*]
- Bernard of Clairvaux (1090 – 1153)

The prevention of runs on financial institutions such as banks, money market mutual funds, and, more recently, stablecoins and central bank digital currency (CBDC) concerns a vast academic literature[1] and policy institutions today (McCrank, 2022). This paper contributes to a critical debate on financial regulation aimed at reducing a firm's run-propensity and its unintended consequences. The paper develops a flexible framework for analyzing the effectiveness of a large class of financial policy interventions at preventing runs on firms. Because the framework is general, I can identify features of regulation, and ultimately classify common policy regulation according to types that improve versus reduce firm stability.

The paper makes three contributions. The main contribution stems from characterizing policy interventions that improve versus deteriorate firm stability based on how the policy acts on the investors' withdrawal-contingent payoffs to roll over versus withdraw their funds. Policy-driven changes in the relative payoffs alter the investors' ex ante run-propensity, and thus the firm's proneness to runs. I determine two large classes of policy, "smooth" and "harsh." Both smooth and harsh policies can increase or lower the run-propensity of investors but do so in distinct ways. Among regulation that possibly worsens stability is emergency liquidity assistance because it may benefit the "wrong" investor group, that is, those that decide to withdraw.

To classify policy and determine how policy impacts firm stability, I consider the relative investor payoffs to roll over versus withdraw as a function of aggregate withdrawals, where high aggregate withdrawals implicate a run on the firm. Absent regulation and intervention (the status quo), the payoff difference (PI) to roll-over versus withdrawal is generally a continuous function of the aggregate withdrawals, see Goldstein and Pauzner (2005). I define "smooth policy intervention" as a regulation that acts on the PI by shifting relative incentives gradually while preserving the continuity of the PI in the withdrawals. In contrast, "harsh policy" *causes* discontinuities (jumps) in the PI at certain withdrawal thresholds, and possibly shifts these jump points as policy intensity picks up. For intuition on the difference between smooth and harsh intervention, any policy intervention needs to start and finish at some aggregate withdrawal threshold. These thresholds have an

[1] See Diamond and Dybvig (1983); Allen and Gale (2000, 2004); Goldstein and Pauzner (2005); Rochet and Vives (2004); Andolfatto, Nosal, and Sultanum (2017); Ennis and Keister (2009); Diamond and Kashyap (2016); Keister (2015); Green and Lin (2003); Peck and Shell (2003); Gorton and Metrick (2012, 2009); Morris and Shin (2016); Ennis and Keister (2006); Garratt and Keister (2009); Kacperczyk and Schnabl (2013); Allen, Carletti, and Gale (2014); Schmidt, Timmermann, and Wermers (2016); Schilling (2019, 2023); Fernández-Villaverde, Sanches, Schilling, and Uhlig (2021); Schilling, Fernández-Villaverde, and Uhlig (2020)

interpretation as withdrawal-contingent entry and exit points to intervention. A smooth policy may set or shift these entry and exit points to intervention but only in a way that preserves the continuity of the PI in the withdrawals. That is, entry and exit to intervention shall not be too abrupt with regard to its impact on investor payoffs, otherwise a discontinuity in the PI arises, and policy intervention becomes harsh.

A smooth policy is not always beneficial to firm stability, and a harsh policy is not always detrimental. If smooth policy strictly raises the PI to roll over versus withdraw over an interval of aggregate withdrawals (intervention interval) and nowhere lowers the PI, the run-propensity drops and firm stability increases. If harsh policy causes at least one upwards jump and no downwards jump of the PI, meaning that there exists a withdrawal threshold at which the policy intervention increases the favorability of roll-over versus withdraw in an ad-hoc way, then likewise firm stability increases. A change in harsh policy can occur in two forms: Either in the form of a "piecewise smooth policy" that shifts payoffs gradually between jump points without causing additional jumps, and acts prudently if the PI is shifted upwards. Alternatively, a change in harsh policy can occur due to a shift in the jump-point. "Jump-shifts" raise stability ex ante, only if the policy shifts a down-jump point of the PI up to a higher withdrawal level or an up-jump point down to a lower withdrawal level. Stability declines either if smooth policy shifts the PI down or if harsh policy causes a down-jump in the PI. A shift in the jump point lowers stability if it shifts a down-jump down to a lower or an up-jump upwards to a higher withdrawal level. These abstract concepts are brought to life in the application section 5 where I assess existing policy methods.

As the second contribution, the generality of the framework allows me to study conditions under which different policy types offset each other. Many common policies belong to multiple classes, exhibiting harsh and piecewise smooth features such as Emergency Liquidity Assistance. Within the class of smooth policies the paper points out that both bail-ins and bail-outs can either increase or lower firm stability depending on the investor group they benefit. As an application of this result, Schilling (2024) constructs "stability-equivalent" but less costly bailin policies for a given bailout policy, and analyzes the welfare difference between bailout and stability-equivalent bailin policies. Therefore, generically a bailout does not improve firm stability, moreover, it lowers firm stability when paid to the withdrawing agent group. As an application of these results, I demonstrate that imposing and raising a fee on withdrawals is not an effective policy because it gives rise to both stability-improving and stability-deteriorating effects that partially offset each other. There, I also show that lowering the entry threshold to the withdrawal fee is more effective, because it avoids these offsetting effects. Likewise, the provision of Emergency Liquidity Loans to a bank during a fire sale can in fact lower instead of raise bank stability ex ante because the loan constitutes a transfer from the roll-over to the withdrawing agent group, acting like a bail-in to roll-over agents and a bail-out of withdrawing agents, as discussed in section 5.2. As the second result I point out the possibility of a smoke and mirrors effect of policy ("smoke policy"): I show that policy that acts harshly (causes or shifts jumps) can undo smooth policy and vice versa. As an application of this smoke and mirrors effect, section 5.2 shows that providing and raising Emergency Liquidity Loans to a bank can lower instead of raise bank stability ex ante because the ELA provision causes a jump in the payoff difference function if the lender of last resort charges interest on the loan. Likewise, section 5.3 discusses a result of Schilling (2023) which shows that the suspension of convertibility of deposits is a smoke policy since it can lower stability, creating harsh changes in relative investor payoffs (jumps) that offset the stabilizing effect of the intervention. The stability analysis of imposing and raising withdrawal fees or granting and raising an ELA loan at a varying entry threshold are contributions of their own in section 5.1 and 5.2.

Last, this paper makes a technical contribution by extending the jump-free Goldstein and Pauzner (2005) model to general payoff functions with finitely many jump points.

This framework is widely applicable. The firm I consider can be any institution that is exposed to its investors' decision whether to roll over funds or withdraw. Therefore, the firm can be a bank, a money market mutual fund (MMF), a central bank, a stablecoin, or a start-up that requires the roll-over of seed money. Funds can be short-term debt, long-term debt, commercial papers, seed money of start-ups, cryptocurrency and stablecoins, CBDC, or money market mututal fund shares. The framework solely requires that the payoffs to roll over versus withdraw be denominated in the real unit of account (consumption units). Therefore, payoffs need to be pinned down after an adjustment for inflation or an exchange rate.

The framework is general in that the types of regulation and contracts that are studied here solely need a description of the ex post payoffs to investors after the contract, asset returns, and regulation have been applied. More specifically, for the classification of regulation into classes that do improve versus those that lower firm stability, it is sufficient to observe how regulation acts on the investors' payoffs to roll over versus withraw funds, depending on the aggregate withdrawals of all firm investors. The types of regulation and policy interventions that are included in this framework are, though not limited to, bail-ins, bail-outs, emergency liquidity assistance (ELA) by a lender of last resort, suspension of convertibility of deposits or gates or withdrawal fees for money market mutual fund redemptions, and deposit insurance (guarantees), all possibly in a withdrawal contingent way.

The framework is specific in that it imposes sufficient structure on the payoffs to guarantee the selection of a unique equilibrium of the investor's coordination game in a global games framework (Carlsson and Van Damme, 1993; Morris and Shin, 2001), and thus a unique, model-implied ex ante run probability on the firm. For this purpose, I generalize the Goldstein and Pauzner (2005) framework to a setting that considers general withdrawal-contingent payoffs to investors, and allows for jumps in the payoff differences.

1.1 Literature

The paper contributes to three strands of literature, namely the literature on runs on financial firms, the literature on global games, and the literature on financial regulation to improve the resilience of the financial sector. The closest related papers are the bank run global game model in Goldstein and Pauzner (2005), the run model with a lender of last resort application in Rochet and Vives (2004), the firm-regulator interaction with subsidies and runs in Frankel (2017), and the book chapter on global games in Morris and Shin (2001).

This paper adds to the literature on bank and money market mutual fund runs and their prevention. In Diamond and Dybvig (1983), a sufficiently conservative suspension policy deters runs completely. Chari and Jagannathan (1988) study the prevention of panic runs via suspension policies when depositors have asymmetric information. Rochet and Vives (2004) study bank runs with and without lender of last resort policies. Ennis and Keister (2009) consider ex-post optimal intervention delay when a run happens. He and Manela (2016) study dynamic rumor-based bank runs with endogenous information acquisition. Andolfatto, Nosal, and Sultanum (2017) study the prevention of runs by allowing agents to report that a run is happening. Zeng (2017) studies mutual fund runs in a dynamic model but does not consider intervention or run prevention. Schilling (2019, 2023) studies the impact of suspension of convertibility policies on bank stability. Zhong and Zhou (2021) study the impact of bankruptcy code design on run incentives in a dynamic setting. Unlike all these papers, this paper analyzes a very general framework that allows for a wide range of policy interventions and contracts.

Unlike the majority of the mentioned papers, this paper employs a global games information environment (Carlsson and Van Damme, 1993; Morris and Shin, 2001, 1998; Frankel, Morris, and Pauzner, 2003; Angeletos, Hellwig, and Pavan, 2006; Szkup and Trevino, 2015; Inostroza and Pavan, 2018; Morris and Yang, 2022) for attaining a unique equilibrium which enables me to conduct unique comparative statics in the ex ante run likelihood under policy changes. In the context of runs on firms, global games have been employed as an equilibrium selection device by Goldstein and Pauzner (2005); Rochet and Vives (2004); Morris and Shin (2016, 2004); Vives (2014); Frankel (2017); Eisenbach (2017); Allen, Carletti, Goldstein, and Leonello (2018); Schilling (2019, 2023, 2018). This paper deviates from the existing global games literature by analyzing a general global games environment into which I build different types of regulation that impact firm stability. In doing so I build on the general structure in Morris and Shin (2001) to generalize the payoff functions of the classic run model by Goldstein and Pauzner

(2005). I then define types of policy by how they act on the payoff difference function under the constraint of maintaining the global games equilibrium selection. In doing so I explicitly allow for regulation that causes jumps in the payoff difference function while maintaining action single-crossing and (one-sided) strategic complementarity. In the global game by Frankel (2017), a regulator can set transfers to investors to implement the efficient equilibrium whereas the firm can shirk the transfer by altering the contract with its investors. This paper differs from Frankel (2017) by focussing on different types of transfers to and across the coordinating investors. I show, depending on whether transfers are continuity-preserving or discontinuity-causing, positive or negative, they impact stability differently, either improving or deteriorating stability. I show that different types of transfers can, nevertheless, have equivalent effects on stability and I show that commonly applied intervention methods such as emergency loans, and the imposition of withdrawal fees are policies that exhibit mixed features, some improving and some lowering stability ex ante. Frankel (2017) explicitly allows for moral hazard whereas I abstract from that. Similar to Angeletos, Hellwig, and Pavan (2006), this paper studies how a firm's proneness to runs changes with policy. In Angeletos et al. (2006), however, the policy maker observes a payoff-relevant state realization which is not observed by the coordinating investors. Therefore, the policy conveys additional information which gives rise to equilibrium multiplicity. Here, in contrast, the policy does not serve as a signal, and a unique equilibrium attains. Morris and Shin (2016) and Vives (2014) consider the regulation of intermediary balance sheets to impact insolvency and illiquidity risk. I study regulation in a broader sense where I do not pin down balance sheets, contracts and regulation explicitly but rather consider very general payoffs to investors ex post of asset returns, contracts, seniority and regulation. This allows me to nest many common bank run models and regulation, and characterize stability improving regulation on a more abstract level without pinning down the regulation and contracts in detail.

With regard to the literature on unintended consequences of financial regulation, in a setting of self-fulfilling runs, Keister (2015) shows that if financial intermediaries expect bailouts in times of crises, the anticipation of bailouts causes intermediaries to choose illiquid and fragile asset positions. In the context of sovereign debt crises, Fink and Scholl (2016) show that the prevention of sovereign default via bailouts in the short run may come at the cost of a higher default probability in the long run. Farhi and Tirole (2012) show that private leverage choices of banks become strategic complements if the policy response during crises is imperfectly targeted. This model features a simultaneous-move game, as in Diamond and Dybvig (1983) and Goldstein and Pauzner (2005). The withdrawal-contingent intervention policies considered here, however, resemble the literature on random and sequential withdrawals where each arriving depositor obtains a distinct allocation (Wallace et al., 1988; Chari, 1989; Peck and Shell, 2003; Green and Lin, 2003).

2 Model

I first introduce the model, and then discuss its assumptions in section 2.1.

There are three time periods, $t = 0, 1, 2$, and no explicit discounting. Implicitly, a discount factor can be accommodated via the payoffs to investors, as described below. There exists a firm, a regulator and a continuum of investors $i \in [0, 1]$. All of them are risk-neutral. The firm can be a bank, a money market mutual fund, a stablecoin, a central bank issuing CBDC, or a start-up that requires investors and the roll-over of funding seeds. Likewise one can think of the investors as depositors, investors in a money market mutual fund or general investors who at a future point in time need to decide whether to roll-over funds or withdraw. The regulator can represent the FDIC, the government or the lender of last resort (central bank). There is a single good in the economy that agents value for consumption. All payoffs are denominated in terms of that good.

At time zero, the investors are symmetric, and each is endowed with one unit to invest. All investors enjoy consumption at both $t = 1$ and $t = 2$. The firm requires funding for investment, and for that purpose collects endowments from the investors in $t = 0$. I assume that investing is individually rational to investors. Returns to scale are constant. The initial firm investment and thus funding via investors is normalized to one unit.

I do not model the firm and the regulator separately but rather think of them as one entity that jointly provides payoffs to investors. Therefore, in the benchmark model I do not model the firm's investment payoff structure, the contracts between investors and the firm, and the regulator's subsidy explicitly. Rather, I pin down investor payoffs conditional on the choice of action, *ex post* of firm revenue, contract payments and regulatory intervention. On an abstract level, I can collapse payoffs because, as I will outline below in the analysis, optimal investor behavior does not depend on the origin of payoffs rather than joint payoffs provided by the firm and the regulator conditional on an action. This stark abstraction has pros and cons. On the positive side, it allows me to analyze a very general policy framework that nests many common intervention methods, contracts, and asset payoff structures. But the collapse of firm-regulator payoffs requires me to abstract from moral hazard from the side of the firm towards its investors or between the firm and the regulator. The firm-regulator entity has aligned incentives to maximize firm stability, to be defined below. This set-up does nest a model where the firm and the regulator are modeled separately, as long as the firm faces no moral hazard problem towards the regulator or its investors, see the application section 5 for examples. For a nice example where the firm can shirk a regulatory intervention, see Frankel (2017).

I follow and outline the information structure in Goldstein and Pauzner (2005) but generalize firm (bank) and investor payoffs.

State Let $\theta \sim U[0,1]$ denote the unobservable, random state of the economy. Generalizing Goldstein and Pauzner (2005), as stated above, I do not impose a particular state-dependent firm asset payoff structure. Yet, I assume that the state realization is payoff relevant to investors. One may think of θ as parametrizing the payoff probability of a risky firm asset or a random asset return.

Contract and payoffs In $t = 0$, the firm offers a contract to the investors to raise funds for investment in the risky asset. All investors invest their endowment in the contract with the firm. At $t = 1$, an investor needs to decide on her *action*. She either "withdraws" her investment and thus opts for the short-term payoff $u_1(n, \theta)$ payable in $t = 1$, or she "rolls over" her investment until $t = 2$, opting for the payoff $u_2(n, \theta)$ payable in $t = 2$ where $n \in [0, 1]$ denotes the endogenous share of investors who withdraw in $t = 1$ (aggregate withdrawals). One should think about the payoffs $u_1(n, \theta)$ and $u_2(n, \theta)$ not only as functions of firm asset payoffs, the contract and withdrawals but also ex post of firm profits and regulatory intervention, that is, the payment of bail-outs, bailins, suspension or withdrawal fees. The payoffs u_1 and u_2 are denominated in real terms. Therefore, if the firm is a stablecoin or a CBDC-issuing central bank, then u_1 and u_2 are ex post of a correction for the exchange rate and the price level (inflation). The payoff u_2 can be thought of incorporating a discount factor. The reason for why this generality is possible is because the investors' only care for final per period consumption and due to rational expectations. For roll-over incentives, only final real payoffs matter. The firm and the regulator jointly have deep pockets so that payoffs $u_1(n, \theta)$ and $u_2(n, \theta)$ at a given state θ and aggregate withdrawal level n are feasible, and this is common knowledge among all investors. Observe that the payoffs are not necessarily hard claims but can be state- and withdrawal-contingent. Therefore, the contract I am considering here is not necessarily a demand-deposit or debt contract. The payoffs satisfy monotonicity conditions in the state θ, and the aggregate withdrawals n, as summarized below in assumption 2.1. The functional forms of $u_1(n, \theta)$ and $u_2(n, \theta)$ are known to the depositors ex ante.

Signals Before the investors choose actions in $t = 1$, they observe noisy, private signals about the state θ,

$$\theta_i = \theta + \varepsilon_i. \tag{1}$$

The idiosyncratic noise term ε_i is independent of the state θ and is distributed iid according to the uniform distribution $\varepsilon_i \sim U[-\varepsilon, +\varepsilon]$.

Policy and Regulatory Intervention I assume that at $t = 0$, the regulator sets and commits to a policy parameter $p \in [0, \infty)^M$ where M is the dimension of the policy, where the regulator can steer each dimension independently of the other.[2] For that same reason, in the remainder of the theory part I set $M = 1$, assuming that every policy dimension corresponds to its own policy p. The case $p = 0$ corresponds to a committment to not interfere, or alternatively the absence of a regulatory institution. One can think of p as a policy intensity that is raised under policy intervention. The policy parameter is common knowledge among all investors. A change in p is supposed to act on the investors' payoffs u_1 and u_2 which is why, from now on, I subindex investor payoffs with p. For the first part of the paper, I study investor behavior for a general, given policy intensity $p \in [0, \infty)$, and then characterize different types of policy and policy changes by how they act on the investors' payoffs. Define the payoff difference of rolling over versus withdrawing as

$$v_p(n, \theta) = u_{2,p}(n, \theta) - u_{1,p}(n, \theta). \tag{2}$$

Note, that the aggregate withdrawals n and the state θ are random in $t = 0$. Following Morris and Shin (2001) section 2.2.2. and 2.2.3., I impose monotonicity conditions on the investor's relative payoffs that guarantee equilibrium existence and uniqueness. That is, this model tries to attain maximum generality with regard to the payoffs u_2 and u_1 and thus possible regulatory interventions but within the class of global games.

Assumption 2.1. *Fix policy intensity $p \in [0, \infty)$. It holds*
(1) (Strict state Monotonicity:) $v_p(n, \theta)$ is non-decreasing in θ, and strictly increasing in θ for all $\theta \in [\underline{\theta}_p, \overline{\theta}_p]$.
(2a) (Action single crossing:) For every state $\theta \in [\underline{\theta}_p, \overline{\theta}_p]$, there exists $n^*(p) \in (0, 1)$ such that $v_p(n, \theta) > 0$ for all $n < n^*(p)$ and $v_p(n, \theta) < 0$ for all $n > n^*(p)$.
(2b) (One-sided strategic complementarity:) For every state $\theta \in [\underline{\theta}_p, \overline{\theta}_p]$, whenever n is such that $v_p(n, \theta) > 0$, then $v_p(n, \theta)$ is strictly decreasing in n.
(3) (Uniform limit dominance:) There exist upper and lower regions of action dominance: There exist $\underline{\theta}_p, \overline{\theta}_p \in (0, 1)$ and $\epsilon > 0$ such that: if $\theta \in [0, \underline{\theta}_p]$, then withdraw is dominant, $v_p(n, \theta) < -\epsilon$, for all $n \in [0, 1]$ while for $\theta \in [\overline{\theta}_p, 1]$, roll-over is dominant $v_p(n, \theta) > \epsilon$, for all $n \in [0, 1]$.

Note that assumption 2.1 includes global strategic complementarity in actions. But the assumption imposes sufficiently strong additional structure to also guarantee equilibrium existence and uniqueness under one-sided strategic complementarity which is common in games of runs on financial institutions.

Timing

[2] For instance, a regulation of imposing harcuts needs to pin down the haircut and the withdrawal entry threshold to the haircut. An emergency loan needs to pin down the loan amoun, the interest rate on the loan, and the withdrawal entry threshold.

- In $t = 0$, the regulator sets and fully commits to her policy p without observing the state. The policy p is common knowledge among all agents, and the policy choice conveys no information. Then, the state θ realizes unobservably to all agents. All investors invest in the firm contract.

- In $t = 1$, all investors observe their private signal θ_i. Based on the signal and the policy, they decide whether to request withdrawal. The firm and the regulator jointly observe the aggregate withdrawal requests $n \in [0, 1]$, and depending on the policy p, allocate payoffs $u_1(n, \theta)$ to depositors who withdraw, where the state realization $\theta \in [0, 1]$ remains unobserved by all agents until $t = 2$.

- In $t = 2$, θ is revealed, and payoff $u_2(n, \theta)$ is paid to investors that chose roll-over.

The equilibrium concept is perfect Bayes Nash. Proofs that are not in the main text can be found in the appendix.

2.1 Discussion of model assumptions

Generically, I allow the payoff to withdrawal, $u_1(n, \theta)$, to depend on state θ since the payoff may be paid in $t = 2$ due to regulatory intervention even though the choice to withdraw was made in $t = 1$. One may consider here a mandatory deposit stay where a depositor chooses to withdraw but an intervention in $t = 1$ prevents her from doing so. If the payoff to withdraw is paid in $t = 1$, it cannot depend on θ since the state is revealed only later in $t = 2$. The payoff to roll over is paid in $t = 2$ and therefore can always depend on the state. I allow the payoffs to depend on aggregate withdrawals since in classic bank run models (Diamond and Dybvig, 1983; Ennis and Keister, 2006; Goldstein and Pauzner, 2005), regulatory intervention is triggered by high withdrawals, thus, altering the payoffs to all agents.

To gain intuition for Assumption 2.1, state monotonicity means that the action to "roll over" becomes relatively more favorable than withdraw for high state realizations.

One-sided strategic complementarity and single-crossing mean that, unless the state realizes in either of the dominance regions, for low withdrawals, roll over is optimal, but the optimality of roll over strictly declines in the withdrawals until a critical withdrawal level $n^*(p)$ is reached where the optimal response flips to "withdraw." For all higher withdrawals, withdraw is optimal, and the critical withdrawal level $n^*(p)$ is unique. To put these assumptions in context, in the bank run literature, at policy intensity p, the aggregate investor withdrawals determine whether a run occurs or not. The critical withdrawal level $n^*(p)$ is known as the critical withdrawal level at which the bank becomes illiquid, meaning for higher withdrawals $n \geq n^*(p)$ the bank is unable to fully serve depositors who roll over and withdrawal becomes optimal to depositors. To understand the single-crossing condition, note that policy impacts the relative favorability of roll-over versus withdrawal by altering the payoffs $u_{2,p}$ and $u_{1,p}$. Thus, changes in policy can or are supposed to cause changes in optimal behavior by investors. Alternatively, the threshold $n^*(p)$ can be understood as a regulatory intervention that occurs once withdrawals exceed $n^*(p)$, which may cause optimal investor behavior to switch at the intervention threshold, see section **??**. In applications, the threshold $n^*(p)$ depends on the asset payoffs, budget constraints, the contracted investor's payoffs, the discount factor, and in case of nominal contracts, the price level or an exchange rate.[3]

The assumption on action single-crossing, introduces a coordination game among the investors.[4] The existence of dominance regions is important for the equilibrium selection argument. The subscript p clarifies that policy intensity impacts not only investor payoffs but can also determine the regions of states, $[0, \underline{\theta}_p]$ and $[\overline{\theta}_p, 1]$, for which investors have dominant actions.

Allover, assumption 2.1 is important to attain a unique coordination equilibrium and later, for maintaining equilibrium uniqueness under policy changes.

3 Equilibrium Existence and Uniqueness with Jumps

Any policy intervention is relative to a prevailing status quo. This benchmark status quo needs to be clearly defined so that I can compare equilibrium outcomes before and after a policy intervention or a change in policy. A comparison of outcomes, in particular, requires that the status quo yields a unique equilibrium of the investors' coordination game, and that the equilibrium remains unique as the policy changes.

The status quo environment "p" should comprise the standard setting where a policy or regulation are absent, and , for instance, no regulator exists to intervene during a run ($p = 0$). In such settings, the investors' payoffs to roll-over versus withdraw are typically continuous in the aggregate withdrawals; see (Goldstein and Pauzner, 2005). As a regulator puts a policy in place, though, it affects the shape of the investors' payoff-difference

[3] In classic bank run appliations, for instance, a run occurs if aggregate cash withdrawals $nu_1(n, p)$ exceed a budget $B_1(p)$ available to early withdrawing agents. For attaining equilibrium uniqueness of the coordination game, a classic assumption yielding action single-crossing is that the product $nu_1(n, p)$ be strictly increasing in the aggregate withdrawals n. Therefore, at fixed policy p there exists a unique critical withdrawal level $\hat{n}(p)$ such that if and only if $n \geq \hat{n}(p)$ then $nu_1(n, p) \geq B_1(p)$. In that case, there exists a unique $n^*(p) \leq \hat{n}(p)$ for which $n^*(p)u_1(n^*(p), p) \leq B_1(p)$ and the payoff difference changes sign in $n^*(p)$.

[4] As one interpretation for $n^*(p)$ one can imagine depositors that finance a bank's investment in illiquid assets. The depositors have the possibility to withdraw from the bank at the interim stage if they believe that the asset quality θ will realize low. If the state θ realizes above the lower dominance region $\theta \in [\underline{\theta}, 1]$ and as long as the aggregate withdrawals are sufficiently low, $n < n^*$, the bank can finance all withdrawals by selling assets, and rolling over yields a higher payoff than withdraw. Therefore, $\upsilon_p(n, \theta) > 0$ and "roll over" is the best response to the aggregate action $n < n^*$. If however the withdrawals pick up, the bank needs to liquidate many illiquid assets, and the remaining investment is insufficient to pay a high payoff to depositors who roll over. That is, "withdraw" is the optimal response to high withdrawals $n > n^*(p)$, $\upsilon_p(n, \theta) < 0$.

function. Because I want to consider a large range of possible policy interventions, the status quo environment should also comprise more extreme ("harsh") cases of policy that have strong local and global effects on the investors' incentives to roll-over versus withdraw. Such local effects can take the form of discontinuities (jumps) in the payoff difference function at particular withdrawal thresholds, so that small changes in policy can cause shifts in the discontinuities, and thus large changes in expected payoffs. At the status quo, a payoff difference has jumps, if, for instance, an intervention mechanism is already implemented, $p > 0$, and the regulator contemplates about adjusting the policy.

Assumption 3.1 (Status quo: Discontinuous payoff difference)**.** *Fix policy $p \in [0, \infty)$.*
(i) The payoff difference function has at most finitely many jump points (if any): There exist withdrawal thresholds (jump points) $\{n_1, \ldots, n_k\}$ with $n_1 < \cdots < n_k \in [0,1]$, $0 \leq k < \infty$, $k \in \mathbb{N}_0$, such that $\upsilon_p(n, \theta)$ is continuous in (n, θ) on $[0,1] \setminus \{n_1, \ldots, n_k\} \times [0,1]$, and differentiable in θ on $(\underline{\theta}_p, \overline{\theta}_p)$.
(ii) Every jump is finite: For all jump-points $\{(n)_i\}_{i=1,\ldots,k}$ the left- and right-sided limits of the payoff difference function exist (are finite)

$$|\lim_{n \nearrow (n)_i} \upsilon_p(n, \theta(n, \theta_p^*))| =: c_{i,l} < \infty, \quad |\lim_{n \searrow (n)_i} \upsilon_p(n, \theta(n, \theta_p^*))| =: c_{i,r} < \infty \tag{3}$$

Possible causes of or reasons for these preference jumps are (multiple) entry or exit points to some staggered intervention in the form of threshold levels to intervention. For instance, an Emergency liquidity loan can be granted at a withdrawal threshold n_1 when withdrawals realize above that withdrawal level. Altenatively, a suspension of convertibility, or a bail-in can cause such jumps.

While k is the total number of jump points of the PI, I allow that only some of these jump points depend on the policy p.[5] Let m with $0 \leq m \leq k$ the number of policy-dependent withdrawal jump-thresholds, that change with policy p. For $m > 0$, without loss of generality, I reorder the policy-dependent jump points by $(n)_1 < \ldots, < (n)_m$. Set $(n)_0 = 0$ and $(n)_{k+1} = 1$. The renaming of jump points to $(n)_1, \ldots, (n)_m$ allows me to directly address all of the policy-dependent jump-points.

I need to establish further conditions to attain equilibrium existence and uniqueness of the trigger equilibrium under jumps. I maintain assumption 2.1 but need to adopt the one-sided strategic complementarity assumption.

Assumption 3.2 (Preserving single-crossing with jumps)**.** *The payoff difference function $\upsilon_p(n, \theta)$ is strictly decreasing in the aggregate withdrawals n whenever $\upsilon_p(n, \theta)$ is non-negative:*
(i) As long as the payoff difference function is positive, it is strictly decreasing in n:

[5] This captures that policy can be multi-dimensional, $M > 1$, where differend policy dimensions affect the jump points differently. For instance, an increase in the emergency loan does not affect the jump point whereas a shift in the entry threshold shifts the jump point.

For withdrawals between adjacent jump points $n \in (n_i, n_{i+1})$, $i = 0, \ldots, k$ it holds: whenever $\upsilon(n, \theta) \geq 0$ then $\upsilon(n, \theta) > \upsilon(n + h, \theta)$ for all $h > 0$ with $n + h < n_{i+1}$
(ii) As long as the payoff difference function is weakly positive, only downwards jumps may occur: If the left-sided limit of the payoff difference function towards a jump point n_i, $i = 1, \ldots, k$ is non-negative, $\lim_{n \nearrow n_i} \upsilon_p(n, \theta) \geq 0$, then that jump point must be a downwards jump, $c_{i,l} - c_{i,r} > 0$.

The assumptions (i) and (ii) of assumption 3.2 imply single-crossing of the payoff difference function while allowing for discontinuities. The requirement (ii) imposes that the payoff difference may jump upwards only across negative values of the payoff difference function. Jumps across positive values or from a positive to a negative value must be downwards jumps.

The Goldstein and Pauzner (2005) model is nested in the environment described in this section. When setting $k = 0$ in assumption 3.1, the payoff difference function is continuous at p (has no jumps). Assumptions 3.2 and 3.1 generalize the Goldstein and Pauzner (2005) environment to allow for general investor payoffs with discontinuities of the payoff difference function. Both assumptions are necessary for preserving the equilibrium existence and uniqueness of a trigger equilibrium when allowing for the jumps.[6]

The next result states that under the right monotonicity assumptions, the existence and uniqueness of equilibrium is preserved under harsh policy intervention.

Proposition 3.1 (Equilibrium existence and uniqueness under jumps)
Fix the status quo $p \geq 0$. Assume the payoff difference function $\upsilon_p(n, \theta)$ exhibits at most finitely many jump-points $\{(n)_i\}_{i=1,\ldots,k}$, where each jump is finite, satisfying assumptions 2.1, 3.1 and 3.2. As noise vanishes, $\varepsilon \to 0$, there exists a unique equilibrium and the equilibrium is in threshold strategies $\theta^*(p)$ where all investors withdraw if they observe a signal below the trigger and otherwise roll over.

For tie-breaking reasons, I assume that an investor rolls over the investment whenever observing the equilibrium trigger, $\theta_i = \theta^*(p)$. Given an equilibrium trigger $\theta^*(p)$, the equilibrium withdrawals are described by a deterministic function of the state, $n(\theta, \theta^*)$, given in the appendix, equation (**??**).

The proof to Proposition 3.1 is a contribution to the global games literature beyond the general characterization in Morris and Shin (2001), and is given in the appendix. It generalizes the existence and uniqueness proof of the model in Goldstein and Pauzner (2005) to allow for finitely many jumps in the payoff difference function in addition to having general payoffs, subject to assumptions 3.1 and 3.2. Essentially the proof

[6] By assumption 3.1, the payoff difference $\upsilon_p(n, \theta)$ is bounded in n over the interval $[0, 1]$ because the jumps are finite and because the payoff difference is continuous over the intervals (n_i, n_{i+1}), $i = 0, \ldots, k$. Therefore, and because the discontinuities have measure zero, the payoff difference function remains integrable over $[0, 1]$.

amounts to showing that expected relative payoffs conditional on a signal realization remain continuous, and strictly increasing in the signal when having finitely many jumps in withdrawal points of the payoff difference function.

For an intuition on how the equilibrium trigger is attained, fix the policy parameter $p \in [0, \infty)$ and jump points $n_1, \dots n_k \in [0, 1]$. Consider the marginal investor's expected payoff difference to roll-over versus withdraw when observing the trigger signal $\theta_i = \theta^*$. Because of the finitely many finite jumps in the PI, the expected payoff difference equation can be written as the sum of multiple integrals, where the jump points define the integral bounds,

$$H(p, \theta^*) = \int_0^{n_1} v_p(n, \theta(n, \theta^*))\, dn + \cdots + \int_{n_k}^{1} v_p(n, \theta(n, \theta^*))\, dn. \tag{4}$$

and where $\theta(n, \theta^*(p))$ is the inverse of $n(\theta, \theta^*)$, that is, the state consistent with measure n withdrawals if all depositors play the equilibrium trigger strategy around θ^*,

$$\theta(n, \theta^*) = \theta^* + \varepsilon(1 - 2n), \theta^* \in [\underline{\theta} - \varepsilon,\ \overline{\theta} + \varepsilon] \tag{5}$$

As is standard in global games theory, the equilibrium trigger signal $\theta^*(p)$ is implicitly characterized as the zero, $H(p, \theta^*(p)) = 0$. The formula (4) incorporates that from the perspective of the marginal investor, the aggregate withdrawals are uniformly distributed on $[0, 1]$ ("Laplacian Belief"), because aggregate withdrawals equal, by a law of large numbers, the share of investors who observe signals below $\theta^*(p)$, and given the observation of the marginal investor, $\theta_i = \theta^*(p)$, the true state is located only ε away from θ^*. Note that this property holds independently of policy p, so that policy changes leave the marginal investor's posterior beliefs on the aggregate withdrawals unchanged but affect her payoffs v_p and possibly the jump-points $n_1, \dots n_k$.

By the single-crossing assumption in 2.1, the optimality of roll-over versus withdraw switches as the aggregate withdrawals $n(\theta, \theta^*)$ exceed the critical withdrawal level $n^*(p)$, where n^* is the unique withdrawal level where the PI crosses zero. In the remaining part of the paper, I say that "a run on the firm occurs" if the withdrawals exceed the critical withdrawal level $n^*(p)$.[7] Given the trigger signal $\theta^*(p)$, a unique cut-off state $\theta_b(p) \in [\underline{\theta}, \overline{\theta}]$, the *critical state*, exists at which the aggregate withdrawals push the firm to the edge of a run:

$$n(\theta_b(p), \theta^*(p)) = n^*(p). \tag{6}$$

If and only if $\theta < \theta_b(p)$, a run occurs because sufficiently many investors receive a signal below the trigger $\theta^*(p)$, and withdraw from the firm. Because the state is uniformly distributed, the ex-ante probability of a run equals θ_b. But as noise vanishes, $\varepsilon \to 0$, the

[7] In contrast, the bank run literature often defines a run as the incident where withdrawals reach the level at which the bank runs out of assets to liquidate, that is, as $u_2(n)$ hits zero. This however occurs at a withdrawal level $n > n^*$ where the optimal response has already switched to "withdraw".

equilibrium trigger $\theta^*(p)$ converges to the critical state $\theta_b(p)$. I therefore write:

Definition 3.1 (Firm stability)**.** *Firm stability increases in policy p if the ex-ante probability of a run $\theta_b(p)$ or equivalently[8] the equilibrium trigger $\theta^*(p)$ decline in p.*

Generically, the regulator wants to design a policy p in a way that reduces the ex ante run-likelihood, that is, reduces (and never raises) the trigger $\theta^*(p)$.

The main focus of this paper is to determine what types of policy lower versus raise the probability of runs and thus firm stability. Note, generically, the objective to maximize stability is different from efficiency maximization.[9]

4 Smooth and Harsh Policy Intervention

In this section I introduce different policy classes, characterized by the way they act on the payoff difference function.

4.1 Policy classes

The first policy class contains policies that preserve continuity of the payoff difference function as a regulatory intervention takes place (going from $p = 0$ to $p > 0$), or, as a policy intervention changes intensity (increasing p).

Definition 4.1 (Smooth policy intervention)**.** *Let $p \geq 0$ describe the status quo. A regulator conducts "(piecewise) smooth policy" on the intervention interval $\mathcal{N}(p) \subset [0,1]$ via increasing policy intensity p if:*
(i) a marginal increase in policy p alters the investors' payoffs ro roll-over $u_2(n,\theta)$ versus withdrawal $u_1(n,\theta)$ for aggregate withdrawals (run size) $n \in \mathcal{N}(p)$ in a way that creates no additional discontinuities of the payoff difference function, $v_p(n,\theta)$, in the aggregate withdrawals $n \in [0,1]$ for all $\theta \in [0,1]$.
(ii) the change in payoffs due to the marginal change in policy p preserves the properties of $v_p(n,\theta)$ stated in assumption 2.1.

The intervention interval $\mathcal{N}(p)$ may not contain any jump points. A smooth policy intervention changes the investors' payoffs to roll-over versus withdraw gradually along the withdrawal-interval $\mathcal{N}(p)$, and preserves the contintuity of the PI in the withdrawals

[8] As noise vanishes, $\varepsilon \to 0$, the trigger and the critical state are undistinguishable and their derivatives coincide.

[9] See for instance (Schilling, 2019) where the provision of high deposit insurance can lead to inefficient losses to the deposit insurance fund because the depositors roll over their deposits for bad signals. For analyzing efficiency, one would need to explicitly model the asset's state-contingent payoffs and liquidation values which would impose additional structure on the investor payoffs and the economy. I prefer to keep the payoffs more general for now. An efficiency analysis can be reintroduced once the policy is explicit such as in the application section 5.

over that interval, including the boundary points. In particular, a smooth policy does not cause harsh local changes in the investors' incentives at single withdrawal points, in particular not at the entry and exit points to intervention, that is, the boundary points of $\mathcal{N}(p)$. Requirement (i) says that either the PI is continuous before and after the smooth policy change in p, or the PI has exactly as many jump points before and after the change in p, which are located outside of $\mathcal{N}(p)$.

The second requirement (ii) means that smooth policy intervention should preserve the payoff properties stated in assumption 2.1 which is important for maintaining equilibrium uniqueness as the policy changes. Generically, a policy intervention must be carefully designed. Consider, for instance, increasing the relative favorability to "roll over" versus "withdraw" via the provision of a bailout. If roll over becomes as favorable as withdrawal for low but also for high aggregate withdrawals, equilibrium uniqueness is lost because the payoff difference function is no longer strictly decreasing in the withdrawals when positive, or lacks the single-crossing property. In that case, the global games equilibrium selection approach is no longer applicable, and the impact of policy on firm stability is undetermined since multiple equilibria arise.

In contrast to smooth policy, the following policy class captures the possibility of harsh policy intervention that may initiate or finish abruptly at some entry or exit threshold, thus, causing (additional) discontinuities of the payoff difference function.

Definition 4.2 (Harsh policy intervention)**.** *Fix the status quo policy $p \geq 0$. A policy intervention is "harsh" if its implementation causes (additional) discontinuities of the payoff difference function $v_p(n, \theta)$ in the aggregate withdrawals $n \in [0, 1]$, exhibiting at least one up- or downwards jump point $n_i \in (0, 1)$, $i = 1, \ldots, k$, $k \geq 1$. I call a policy intervention "adverse harsh" if it causes a downwards jump and "prudent harsh" if it causes an upwards jump of the payoff difference function in some withdrawal level.*

I call downward jumps adverseley and upward jumps prudent, because by Proposition 4.1 (i) down-jumps lower firm stability ex ante whereas up-wards jumps increase stability. One might be tempted to call this harsh intervention type "threshold intervention." Note, however that smooth policy intervention likewise starts at a threshold (left endpoint of $\mathcal{N}_p$) and ends at a threshold (right endpoint of $\mathcal{N}_p$) if $\mathcal{N}_p$ does not comprise the full interval $[0, 1]$.

In section 5, I show that the imposition of withdrawal fees, an ELA provision via the lender of last resort, or the suspension of convertibility of deposits are examples of harsh intervention.

The difference between smooth and harsh intervention boils down to continuous versus discontinuous effects on the payoff difference function. This distinction may appear artificial at first sight because functions with a finite number of finite discontinuities, as studied here, can be approximated with continuous functions. I leave the distinction between smooth and harsh policy nevertheless because, first, the existence and uniqueness proof of an equilibrium in the coordination game is more involved in the case of discontinuities rather than the continuous case. Second, the harshness of policy is often an unwanted side-effect of the main regulation which targets a gradual shift of the PI over an interval $\mathcal{N}_p$ but, perhaps unintendedly, also causes a jump at the entry point to the intervention interval, that is, the left boundary point of $\mathcal{N}_p$. Third, in the context of harsh policy, a change in such a policy can, but does not have to, cause a shift in the according jump-points with large consequences for ex ante roll-over incentives.

Definition 4.3 (Jump-shifts). *A change in harsh policy p causes a "jump-shift" if it shifts at least one jump point of the payoff difference function $v_p(n,\theta)$ up or down, meaning the jump occurs at a higher or lower withdrawal level of the PI. That is, it holds $m > 0$ and $\frac{\partial}{\partial p}(n)_i \neq 0$ for $i \in \{1, \ldots, m\}$, $m \leq k$*

Jump-shifts cannot occur with smooth policy since, by its definition, jumps are absent from the PI. A jump-shift should be interpreted as a modification of a harsh policy that is already in place. Section 5, for instance, shows that granting an emergency loan at a positive interest rate at some withdrawal threshold constitutes harsh policy since it causes a jump in the payoff difference at that withdrawal threshold. ELA constitutes a 3-dimensional policy, pinning down the loan amount, the interest rate and the entry threshold to intervention. Going from a loan amount of zero to $\varepsilon > 0$ causes a jump at the entry threshold to intervention if the interest rate on the loan is positive. Likewise, at a loan amount of $\varepsilon > 0$ going from a zero interest rate to an interest rate $r > 0$ causes a policy shift from smooth to harsh including a jump point at the entry to ELA. Once this harsh policy is in place, a change in the entry threshold constitutes a jump-shift whereas at an interest rate $r = 0$, a change of the entry threshold leaves the PI continuous (but extends the intervention interval $\mathcal{N}_p$).

4.2 Effects of Policy on Stability

A change in policy can affect the payoff difference function and thus ex ante firm stability in two kinds of ways: either via a gradual shift of the function by altering payoffs over an entire interval $\mathcal{N}_p$ (piecewise smooth policy)[10] and / or via shifting at least one of the jump-points.

In practice, a policy change can involve several smooth shifts of the PI that go in different directions as well as shifts of jump points. The following Proposition captures the effect of an isolated policy change on ex ante firm stability, holding every other feature of the PI fixed. The next proposition is my second main result.

[10] In that case, it is necessary that the intervention interval is located inbetween jump points.

Proposition 4.1 (Policy effects on stability)
Fix policy $p \geq 0$, and let assumption 3.1 hold.

1. *Assume the payoff difference function $\upsilon_p(n, \theta)$ exhibits at least one jump point, $k > 0$, where $\{n_1, \ldots, n_k\}$ denote all of the jump points of the payoff difference function.*

   (a) *Stability-impairing jump-shift: Consider a change in a single jump point n_i, leaving all other jump points and values of the payoff difference function constant. Firm stability declines ex ante (the trigger $\theta^*(p)$ and the critical state $\theta_b(p)$ strictly increase in p) if and only if*

   - *either jump point $i \in \{1, \ldots, k\}$ constitutes a down-jump in n_i, $c_{i,l} - c_{i,r} > 0$ and the policy change shifts the jump point n_i to a lower withdrawal level (left-shift), $\frac{\partial}{\partial p}(n)_i < 0$, or*
   - *jump point i is an up-jump, $c_{i,l} - c_{i,r} < 0$, and the policy shifts n_i to a higher withdrawal level (to the right), $\frac{\partial}{\partial p}(n)_i > 0$.*

   (b) *Stability-improving jump-shift: Firm stability improves ex ante (the trigger $\theta^*(p)$ and the critical state $\theta_b(p)$ strictly decline in p.) if the policy*

   - *shifts a down-jump n_i to a higher withdrawal level (to the right) or*
   - *an up-jump to a lower withdrawal level (to the left).*

2. *Consider an open interval of aggregate withdrawals $\mathcal{N}(p) \subset [0, 1]$ on which the payoff difference function is continuous in n, and differentiable in p. Assume a change in policy p is piecewise smooth, that is, alters the payoffs to withdrawal or roll-over gradually along $n \in \mathcal{N}(p)$ in a way that preserves the continuity of the payoff difference on $\mathcal{N}(p)$, and leaves all of the possible jump-points outside of $\mathcal{N}(p)$ constant.*

   (a) *Stability-improving (piecewise) smooth policy: A piecewise smooth policy improves firm stability ex ante if it pushes the PI upwards (an nowhere downwards) with*
   (i) $\frac{\partial}{\partial p}\upsilon_p(n, \theta) \geq 0$, for all withdrawals $n \in [0, 1] \setminus \{n_1, \ldots, n_k\}$ and
   (ii) $\frac{\partial}{\partial p}\upsilon_p(n, \theta) > 0$ for withdrawals $n \in \mathcal{N}(p)$.

   (b) *Stability-impairing (piecewise) smooth policy: A piecewise smooth policy lowers firm stability ex ante if it pushes the PI downwards (an nowhere upwards) with*
   (i) $\frac{\partial}{\partial p}\upsilon_p(n, \theta) \leq 0$, for all withdrawals $n \in [0, 1] \setminus \{n_1, \ldots, n_k\}$ and
   (ii) $\frac{\partial}{\partial p}\upsilon_p(n, \theta) < 0$ for withdrawals $n \in \mathcal{N}(p)$.

The policy effects above are evaluated in isolation, assuming that the policy solely affects either one jump point or the PI over a particular range of withdrawals $\mathcal{N}(p)$.

To gain insight in how a policy change affects firm stability, recall that a policy change alters the relative payoffs to roll-over versus withdraw conditional on the aggregate withdrawal realization n. Recall that the investors' withdrawal decisions take place simultaneously, so that an individual withdrawal decision cannot be made contingent on the realization n. Ex ante, before the investors make their withdrawal decision, the aggregate withdrawals are therefore random. For the marginal investor though, the posterior belief on the aggregate withdrawals is uniformly distributed on $[0, 1]$, meaning that policy changes affect the marginal investors' expected payoffs to roll-over versus withdraw. As a consequence, the equilibrium trigger signal $\theta^*(p)$ that makes the marginal investor indifferent between actions needs to adjust. The change in the investors' trigger equilibrium, in return, alters firm stability ex ante.

A stability-improving smooth policy intervention marginally raises the relative favorability of "roll-over" versus "withdraw" by gradually shifting the according payoffs over the interval of withdrawals $\mathcal{N}(p)$ in a way that preserves the continuity of the payoff difference function $\upsilon_p(n)$.
A stability-improving smooth policy can be attained by either increasing the payoffs to roll-over (bailout to investors who roll over), u_2, or equivalently by reducing the payoffs to withdraw (bail-in of investors that withdraw, suspension or withdrawal fee), u_1. A stability-impairing smooth policy, perhaps by mistake, does the opposite, increasing u_1 via, for instance, a lender of last resort emergency liquidity provision, see section 5.2 or lowers u_2 via a bail-in of investors that roll over. The equivalence demonstrates the power of this general approach[11]: To the investors, only relative payoffs matter for roll-over incentives, meaning a raise in u_2 and a lowering of u_1 have equivalent effects on firm stability.

With regard to smooth policy, technically, a smooth policy can act on several disjoint and disconnected intervals $\mathcal{N}_1(p)$ and $\mathcal{N}_2(p)$ simultaneously, meaning that the intervention interval becomes a disconnected intervention set $\mathcal{N}(p) = \mathcal{N}_1(p) \cup \mathcal{N}_2(p)$ in the form of the union of several disjoint smaller open intervention intervals. The intervention set $\mathcal{N}(p)$ may not contain any jump points. Likewise, $\mathcal{N}(p)$ cannot be a single point threshold since this creates a discontinuity (jump), rendering it a harsh policy.

Important for the definition of a stability-improving smooth intervention is that there exists no subinterval of $[0, 1]$, on which that same policy acts adverselely via $\frac{\partial}{\partial p}\upsilon_p(n) < 0$. Obviously, there can exist policy interventions that are mixtures between stability-improving and stability-deteriorating smooth policies in the sense that there exist intervals $\mathcal{N}_a(p)$ and $\mathcal{N}_b(p)$ such that the payoff difference function of roll-over versus withdrawal, $\upsilon_p(n, \theta)$, strictly increases in p along $\mathcal{N}_a(p)$ but strictly declines on $\mathcal{N}_b(p)$. These cases are not clear-cut, and require a more thorough analysis, see the proof to

[11] This equivalence is not suprising within the global games literature where payoff differences are the bread and butter of every comparative statics anaysis.

Proposition 4.1 for the general treatment below, as well as section ?? and the application section 5.

I allow the intervention interval $\mathcal{N}(p)$ to depend on p, meaning that a change in policy p can widen the intervention interval, see for instance sections 5.1 and 5.2 where a change in policy lowers the entry threshold to imposing a withdrawal fee respectively an emergency liquidity loan. The intervention interval cannot depend on the state since otherwise the regulator's announcement of the interval in $t = 0$ would convey information on the state, which would give rise to equilibrium multiplicity, see (Angeletos et al., 2006).

To see the similarity between a stability impairing jump-shift and a stability impairing smooth policy, both of these policies reduce the payoffs to roll-over versus withdraw at some or several withdrawal levels n, thus reducing the favorability of the action "roll-over." Even though these policies have broadly the same effect on the payoff difference, the exact way how they act on the PI is very different. The proof to Proposition 4.1 shows, for instance, that the length of the intervention interval $\mathcal{N}_p$ of a smooth policy plays a similar role as the depth of a jump in the payoff difference when shifting that jump-point. That is, the range of withdrawals $\mathcal{N}_p$ in the case of smooth policy has a comparable role to the payoff difference in a jump point.

Commonly, a policy change has more than one effect, smooth and or harsh, on the PI, which can lead to ambiguous effects on stability (mixtures). For instance, a policy change can act smoothly over several intervals $\mathcal{N}(p)$, potentially stability improving over one interval and stability-impairing over another interval. Also, a policy can act smoothly and harshly at the same time, shifting a jump-point, and simultaneously causing a gradual shift in the PI over some intervention interval. A policy may also shift several jump points, possible in opposite directions. To evaluate the overall effect on stability, one can either think about the impact of concatenated policy change on stability, using the Proposition above. If the effects go in opposite directions, a general statement on the impact on stability is tricky, but can be derived along the lines of the proof in Proposition 4.1. Consider, for instance, harsh combination policy: if a downward (upward) jump point declines (increases) fast in the policy and if the payoff difference makes large jumps in the policy-dependent thresholds $\{(n_i)\}_{i=1}^m$, then the equilibrium trigger increases and stability drops in policy p even though the policy may simultaneously act in a prudent piecewise smooth way over some interval $\mathcal{N}(p)$.

A jump-shift alters the payoff difference function only at discrete points, whereas a (piecewise) smooth policy intervention shifts the payoff difference function gradually over an entire intervals of withdrawals. Every jump-shift is either stability-improving or impairing. In contrast, there exist mixtures of stability-improving and -impairing piecewise smooth policies. While I do not formally define these, their analysis is included in Proposition 4.1 and its proof below.

Smoothness of policy requires that entry and exit to an intervention do not occur too sudden at the boundaries to the intervention interval $\mathcal{N}(p)$. But there are intervention types where the immediacy of the intervention cannot preserve continuity. A simple intuitive example of smooth-harsh combination policy is the case where the regulatory policy intervention starts harsh at a withdrawal threshold n_1, causing a jump, but simultaneously raises the payoff difference function on the interval $\mathcal{N}(p) = (n_1, 1]$, see subsection 5.3.

Section 5 demonstrates that ELA provision via a lender of last resort or the suspension of convertibility of deposits constitute smooth-harsh combination policy that causes and shifts a jump point while also conducting piecewise smooth policy on subintervals of withdrawals.

Policy changes that trigger a combination of jump-shifts and piecewise smooth policy are common in applied settings, see section 5.

Proposition 4.1 not only concerns stability changes when altering existing harsh policy. It also covers stability changes when a harsh policy is imposed for the first time, transitioning from a continuous payoff difference (absent intervention, $p = 0$) to a discontinuous PI, under harsh policy. Generically, in bank run settings absent of regulatory policy ($p = 0$) the payoff difference function is continuous, that is, without jumps, see Goldstein and Pauzner (2005). For analyzing the change in stability under the transition from no policy (continuity) to harsh policy (with jumps) one would study the framework above (with jumps), where the jump point is shifted from the boundary $n_i = 1$ (no policy intervention and no jumps) towards the interior, $n_i \in (0, 1)$ (harsh policy with jumps). We know that left-shifts of downwards jumps constitute stability-deteriorating harsh policy. Therefore, the imposition of harsh policy that causes a downwards jump in a previously continuous PI constitutes adverse harsh policy, and rationalizes the Definition 4.2. The other way around, the imposition of harsh policy that causes an upwards jump in a previously continuous PI constitutes stability-improving harsh policy.

The proof to Proposition 4.1 gives insight into why shifts in the jump points of the payoff difference function cause preemptive investor behavior. Therefore, I prove the proposition here in the text.

Proof. [Proposition 4.1] Consider the payoff difference equation (4). To prove Proposition 4.1, recall that the equilibrium trigger θ^* is implicitly defined as the zero of the payoff indifference equation $H(\theta^*, p) = 0$, (4). It holds $\frac{\partial H}{\partial \theta^*} > 0$ by assumption 2.1 (1). Using the implicit function theorem, we know that the trigger declines in p if and only if the change in the payoff difference equation due to a change in p is positive, $\left\{\frac{\partial \theta^*}{\partial p} < 0\right\} \Leftrightarrow \{\frac{\partial H}{\partial p} > 0\}$. By the Leibniz rule for parameter integrals, the change in the payoff difference equation due to a change in p equals

$$\frac{\partial}{\partial p}H(p,\theta^*) = \int_{[0,n_1]\cap\mathcal{N}(p)} \frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))\,dn + \cdots + \int_{[n_k,1]\cap\mathcal{N}(p)} \frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))\,dn \tag{7}$$

$$+\sum_{i=1}^{m}\frac{\partial(n)_i}{\partial p}\Big(\lim_{n\nearrow(n)_i}\upsilon_p(n,\theta(n,\theta^*)) - \lim_{n\searrow(n)_i}\upsilon_p(n,\theta(n,\theta^*))\Big) \tag{8}$$

The integrals in (7) describe how a change in policy affects the payoff difference function over the intervention intervals $\mathcal{N}_i(p) = [n_{i-1}, n_i] \cap \mathcal{N}(p),\ i = 1, \ldots k+1, n_0 \equiv 0, n_{k+1} \equiv 1$ (adverse versus prudent piecewise smooth), while the summation term (8) describes how the jump points, e.g. entry and exit points, shift in the policy and whether jumps are up- or downwards jumps.

Concerning the proof of part (ii), under a pure piecewise smooth policy, a change in the policy either leaves all jump points constant or no jump points exist, so that the summation term (8) equals zero. The sign of the derivative $\frac{\partial}{\partial p}H(p,\theta^*)$ is, thus, solely determined by the sign of the terms in (7), and is positive only if the piecewise smooth policy is prudent. Further, if the piecewise smooth policy is prudent, then $\frac{\partial\theta^*}{\partial p} < 0$, and bank stability increases.

Concerning the proof of part (i), under a harsh policy change that is purely due to jump-shifts, there exists no interval of withdrawals $\mathcal{N}(p)$ over which the payoff difference changes gradually, and the integrals in (7) are all zero. Moreover, the payoff difference jumps in the withdrawal points $(n)_i$. Therefore, the left- and right-sided limits in each jump point are distinct, implying that the differences

$$\lim_{n\nearrow(n)_i}\upsilon_p(n,\theta(n,\theta^*)) - \lim_{n\searrow(n)_i}\upsilon_p(n,\theta(n,\theta^*)) \tag{9}$$

are non-zero. A difference is positive if the according jump point $(n)_i, i = 1, \ldots, k$, implies a down jump, whereas a difference is negative if the jump point implies an up-jump. The boundary derivatives in (8) are, thus, non-zero if at least one jump point is shifted by the policy.

If a difference (9) is positive (down jump), the boundary derivative in (8) is negative if the jump point strictly declines in the policy $\frac{\partial(n)_i}{\partial p} < 0$. If a difference (9) is negative (up jump), the boundary derivative in (8) is negative if the jump point strictly increases in the policy $\frac{\partial(n)_i}{\partial p} > 0$.

Therefore, a jump-shift strictly increases the trigger θ_p^* (lowers stability) if either all down-jump points (weakly) decline and or all up- jump points (weakly) increase in the policy parameter p, with at least one jump-shift being strict.

Concerning (iii), under a harsh combination policy, the intervention intervals $\mathcal{N}_i(p)$ are non-empty. Further, the gradual change in payoffs $\frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))$ over at least one of the intervention intervals is positive under a prudent piecewise smooth policy. Thus, at least one of the integrals in (7) is positive. Therefore, the trigger may decline (stability can improve) in the policy p, if the change in payoffs is stronger than the change in the jump point. If the jump points alter fast in the policy and if the intervention causes harsh changes in incentives (deep jumps) at the intervention points, stability can decline in the policy under an adverse jump-shift even though the relative incentives to roll over improve over the set of withdrawals $\mathcal{N}(p)$. □

5 Applications

This section discusses several common policy interventions to provide examples of smooth, and harsh policies. To construct the examples, I need to define a status quo where policy is absent. For that purpose, I next describe a risk-neutral version of the banking model in Goldstein and Pauzner (2005) (GP) which serves as the benchmark model before policy intervention is introduced.

Benchmark before policy intervention (Goldstein-Pauzner)

There exists a continuum of depositors $[0, 1]$. Unlike in GP, all depositors are risk-neutral and can consume in $t = 1$ and $t = 2$ (are "patient"). Let $\theta \sim U[0, 1]$ parametrize the random, unobservable state of the economy, and let $\overline{\theta} \in (0, 1)$ an upper threshold state close to 1. Besides storage, there exists a risky asset in the economy to shift consumption across time. For every unit investment, if the state realizes in $\theta \in [0, \overline{\theta})$ the asset pays $R > 2$ in $t = 2$ with probability $p(\theta)$ and otherwise zero, and in case of liquidation in $t = 1$ pays 1 like storage. If the state realizes high in $\theta \in [\overline{\theta}, 1]$, the asset pays R already in $t = 1$ and with probability $p(\theta) = 1$. The function $p(\theta)$ is positive, strictly increasing, and differentiable in θ for $\theta \in [0, \overline{\theta})$ and is constant at 1 for $\theta \in [\overline{\theta}, 1]$. The bank offers a demand-deposit contract to depositors to raise funds for investment in the risky asset. Following GP, assume the contract offers a short-term coupon $r_1 > 1$ in the case a depositor withdraws the deposit[12] in $t = 1$, and offers a long-term coupon $\frac{R(1-nr_1)}{1-n}$ in the case a depositor rolls over the deposit to $t = 2$, where $n \in [0, 1]$ is the endogenous measure of depositors who withdraw in $t = 1$. Risk-sharing imposes a payoff externality: As long as withdrawals are low, $n < 1/r_1$, the bank can service all withdrawal requests by liquidating assets. But if the withdrawals reach the threshold $n_{Ill} := 1/r_1$, the bank can no longer finance all withdrawals by liquidation, and becomes illiquid (bank run). In that case, the depositors who roll over receive zero. The depositors who withdraw queue in front of the bank. With probability $\frac{1}{nr_1}$, a withdrawing depositor is early in the

[12] GP show that risk-sharing, that is, setting $r_1 > 1$ is socially optimal with risk-averse, and some impatient depositors even though it gives rise to runs. I impose risk-sharing even though agents are risk-neutral and patient here to keep the possibility of runs alive.

queue and receives the face value of the deposit r_1, whereas with probability $1 - \frac{1}{nr_1}$ she is late in the queue and receives zero. The payoff difference function in the liquid case $n \in [0, n_{Ill})$ equals $\upsilon_L(n) = p(\theta)\ \frac{R(1-nr_1)}{1-n} - r_1$ whereas in the illiquid case $n \in [n_{Ill}, 1]$, $\upsilon_{Ill}(n) = 0 - \big(\frac{n_{Ill}}{n} \times r_1 + (1 - \frac{n_{Ill}}{n}) \times 0\big)$.

5.1 Redemption (withdrawal) fees

The following example is to the best of my knowledge new to the literature[13], and analyzes the marginal change of firm stability when the regulator imposes a withdrawal fee $c \in (0, r_1)$ as soon as the aggregate withdrawals $n \in [0, 1]$ exceed a cutoff $n_c \geq 0$. The firm can be a bank, a money market mutual fund (MMF) or a stablecoin. Henceforth, I call the firm a bank.

Assume the imposition of the withdrawal fee attains before the bank becomes illiquid, $n_c < 1/r_1$. The imposition of a withdrawal fee constitutes a 2-dimensional policy tool (n_c, c) because the intervention threshold and the fee can be move independently of one another. I discuss changes in either policy variable. As long as the endogenous aggregate withdrawals realize below the intervention threshold n_c, no fee is imposed and the payoff difference function equals

$$\upsilon_L(n) = \underbrace{p(\theta)\ \frac{R(1-nr_1)}{1-n}}_{u_2(n)} - \underbrace{r_1}_{u_1},\ n \in [0, n_c). \tag{10}$$

As soon as the withdrawals are high enough to trigger the fee, $n \geq n_c$, the claim of a withdrawing investor is reduced by the amount of the fee. Importantly, in my example, the reduced claim allows the bank to reduce the speed of its asset liquidation for servicing withdrawals.[14] The reduced speed of asset liquidations pushes the illiquidity threshold of the bank up from $n = 1/r_1$ (when never imposing a fee) to

$$n_{Ill}(c) \equiv n_c + \frac{(1 - r_1 n_c)}{(r_1 - c)}, \tag{11}$$

meaning the bank can now survive larger runs, that is, stays liquid for a greater range of withdrawals. If the withdrawals are high enough to trigger the fee but low enough so that the bank remains liquid, $n \in [n_c, n_{Ill})$, a withdrawing investor receives the face value r_1 if she is sufficiently early in the queue so that she is served before the fee is imposed. The

[13] The imposition of fees to prevent MMF runs has previously been studied in Cipriani et al. (2014) and Voellmy (2021) in a Diamond-Dybvig (1983) style model. Voellmy (2021) studies first best implementation via gates and fees when investors can incur liquidity shocks. There, the probability of a run is, however, not uniquely determined so that a marginal change in bank stability due to a marginal change in the fee or the threshold cannot be analyzed.

[14] One could alternatively design payoffs to instead redistribute the fee from the withdrawing depositors to depositors who roll over but the original idea of withdrawal fees is to reduce asset liquidations.

probability of that event is n_c/n. If she is late in the queue, with probability $1 - n_c/n$, she is served after the fee is imposed, and receives the face value reduced by the fee. The payoff difference for $n \in [n_c, n_{Ill})$, thus, becomes

$$\upsilon_{L,c}(n) = \underbrace{p(\theta)\ \frac{R(1 - n_c r_1 - (n - n_c)(r_1 - c))}{1 - n}}_{u_2(n,\theta)} - \underbrace{\left(\frac{n_c}{n} r_1 + \frac{n - n_c}{n}(r_1 - c)\right)}_{u_1(n)}. \qquad (12)$$

As soon as the bank becomes illiquid, $n \in [n_{Ill}, 1]$, investors who roll-over receive zero. Investors who withdraw receive the face value r_1 if they are early in the queue before the withdrawal fee is triggered, they receive the reduced face value $r_1 - c$ if they withdraw after the fee is imposed but before the bank becomes illiquid, and otherwise receive zero. The payoff difference becomes

$$\upsilon_{Ill}(n) = \underbrace{0}_{u_2} - \underbrace{\left(\frac{n_c}{n} r_1 + \frac{n_{Ill}(c) - n_c}{n}(r_1 - c) + \frac{n - n_{Ill}(c)}{n} \times 0\right)}_{u_1(n)}. \qquad (13)$$

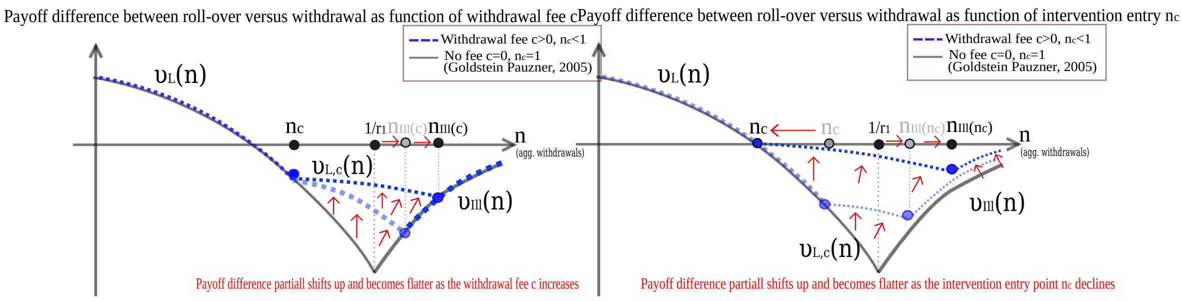
5.1.1 Analysis: Raising the withdrawal fee

I first consider a change in the withdrawal fee, holding the intervention threshold constant, and consider a change in the intervention threshold in the next subsection. The imposition of the constant withdrawal fee constitutes smooth intervention: the payoff difference function jumps neither at the intervention threshold $n = n_c$, where the imposition of the fee is triggered, nor at the illiquidity threshold $n = n_{Ill}$. To determine whether this smooth intervention acts prudent or adverse, consider the withdrawal range over which the fee is imposed but the bank is not yet illiquid, $n \in [n_c, n_{Ill})$. An increase in the withdrawal fee raises the payoff difference to roll over versus withdraw, $\frac{\partial}{\partial c}\upsilon_{L,c}(n) > 0$, for two reasons. First, the fee reduces the payoff to withdraw directly and, second, it slows down the required asset liquidation for servicing further withdrawals which increases the roll-over payoff at the margin. The fee, thus, simultaneously acts like a bail-in of investors that withdraw and a bail-out to investors that roll-over, in comparison to the benchmark where no intervention exists. Next consider the withdrawal range where the bank is illiquid, $n \in [n_{Ill}, 1]$. The allover impact on payoffs is zero in this withdrawal range, $\frac{\partial}{\partial c}\ \upsilon_{Ill}(n) = 0$, but, there are two effects at play here that cancel each other out: First, as in the case of the range $[n_c, n_{Ill})$, increasing the fee reduces the payoff to withdraw directly. On the other hand, the fee pushes the illiquidity threshold $n_{Ill}(c)$ up because the additional slow down of asset liquidations allows the bank to survive larger runs. Perhaps surprisingly, the latter effect acts *against* bank stability because it increases the expected payoff to withdraw[15] because the positive payoff upon withdrawing $r_1 - c$ is

[15]Here, expectation is taken over the range of possible withdrawals $n \in [0, 1]$.

attained with a greater probability. In fact, this latter effect exactly undoes the stability improving first effect, both effects offset each other such that the payoff difference stays exactly constant.

Consequentially, the interval on which intervention is effective is not $[n_c, 1]$ but the smaller interval $\mathcal{N}_c = [n_c, n_{Ill}]$, meaning the imposition of the withdrawal fee is not effective for preventing runs on $n \in [n_{Ill}, 1]$ even though the fee is imposed in this range, see Figure 1a versus 1b. As a Corollary of Proposition ??(i), I obtain:



(a) The payoff difference function $v(n)$ shifts up in the range $[n_c, n_{Ill}]$ the larger the withdrawal fee c.

(b) The payoff difference function $v(n)$ shifts up in the range $[n_c, 1]$ as the intervention entry point n_c is lowered.

Figure 1

Corollary 5.1 (Raising the withdrawal fee)
Assume the regulator imposes a fee on withdrawals, $c \in [0, r_1)$, if the aggregate withdrawals exceed threshold $n_c \in (0, 1/r_1)$. A policy change that raises the withdrawal fee c holding n_c constant constitutes prudent smooth intervention on $\mathcal{N}_c = [n_c, n_{Ill}]$, and, thus, increases bank stability ex ante monotonically. The larger fee allows the bank to survive greater runs which is, however, a feature that acts against bank stability by increasing the expected payoff to withdraw, which reduces the effectiveness of the intervention.

The feature that an increased survival range acts against bank stability is not unique to withdrawal fees, see section 5.2 on raising the Emergency Liquidity Assistance by a lender of last resort. To study the stability change when transitioning from not imposing to imposing a withdrawal fee at n_c, one can study the case $c \to 0$, because the PI under fees converges to the PI of the Goldstein-Pauzner setting when not imposing a fee, where convergence is in $\mathcal{L}^1$.

5.1.2 Altering the entry point to policy intervention

Next, I consider a change in policy by lowering the intervention point n_c while leaving the fee constant. Lowering the intervention point allows the bank to reduce asset liquidations sooner, and consequentially, the bank can survive larger runs. Therefore, the illiquidity threshold $n_{Ill}(n_c)$ rises as the intervention point n_c declines. Because the payoff difference is continuous in n_c and $n_{Ill}(n_c)$, a change in n_c will not create or shift any jumps so that a decline in n_c constitutes smooth policy intervention, if at all.

To evaluate how a change in the intervention threshold n_c effects bank stability, consider the change in the payoff differences due to an increase in n_c. It holds $\frac{\partial}{\partial n_c}\upsilon_{L,c}(n) < 0$ because as the fee is imposed later overall more asset liquidation is required which lowers the roll-over payoff. Further, $\frac{\partial}{\partial n_c}\upsilon_{Ill}(n) < 0$ because as the intervention is delayed, a withdrawing depositor is served with a higher probability which increases the expected payoff to withdraw. The intervention interval when altering the intervention entry threshold equals $\mathcal{N}_p = [n_c, 1]$ and is thus larger than the intervention interval when raising the fee. As a consequence, altering the intervention entry point is potentially the more effective prudent smooth policy in comparison to raising the withdrawal fee. Thus, as a Corollary to Proposition **??**,

Corollary 5.2 (Lowering the entry point to impose the withdrawal fee)
Assume the regulator imposes a fee on withdrawals, $c \in [0, r_1)$, if the aggregate withdrawals exceed threshold $n_c \in (0, 1/r_1)$. A policy that lowers the intervention entry threshold n_c constitutes prudent smooth policy, and, thus, raises bank stability ex ante monotonically. The intervention interval equals $\mathcal{N}_p = [n_c, 1]$, and is larger than the intervention interval of a policy that raises the withdrawal fee.

5.2 Emergency Liquidity Assistance

The following example is, to the best of my knowledge, also new to the literature, and complements the analysis of lender of last resort policies given in Rochet and Vives (2004), section 6.

5.2.1 Motivation

Lender of last resort policies exist because banks can be solvent and yet illiquid, suffering from a bank run that pushes the bank into default (Rochet and Vives). The reaon for such incidents is that banks invest in illiquid assets to conduct maturity transformation. If agents panic and withdraw even though they have no instantaneous consumption needs, the bank is forced to liquidate illiquid assets, which forces the bank into default even though the bank assets are of high quality (Diamond and Dybvig). In the analysis of Rochet and Vives (and later Morris and SHin), a lender of last resort observes the quality of the bank's assets perfectly, and serves the bank with unbounded liquidity if a run occurs. In that case, the run is always stopped, and the bank is rescued.

In this analysis, I take a different approach: The LOLR does not perfectly observe the bank's asset quality, and thus does not lend unboundedly but lends a bounded amount B which should be sufficient if the bank was only illiquid but truly solvent. As a second difference, I model the depositors' payoffs direcly instead of considering payoffs of fund managers. This has the consequence that depositors who stay with the bank need to repay the ELA with interest which gives rise to inventives that have not been observed in the literature before.

5.2.2 Model

Assume there exists a lender of last resort (LOLR) that is willing to lend a bounded amount of emergency liquidity assistance (ELA) $B > 0$ at gross rate $r > 1$ once the bank is perceived as facing a run, and before the bank becomes illiquid. Assume the bank is perceived as facing a run if the withdrawals exceed a threshold $n_B \in (0, 1/r_1)$. Akin to the imposition of a withdrawal fee, ELA provision is a 3-dimensional policy tool (n_B, B, r). Until ELA is triggered, the bank services withdrawals by liquidating assets. Once ELA is active, the bank no longer needs to liquidate assets, but can draw on the liquid resources B to repay the face value r_1 to withdrawing depositors. The borrowed amount B needs to be repaid with interest in $t = 2$ by the depositors who roll over. Assume that the asset's return is high enough to repay ELA as long as withdrawals are sufficiently low, $R > B(r - 1)$, see the discussion on insolvency below. The borrowed funds allow the bank to fully repay withdrawing depositors for a larger range of withdrawals, meaning the ELA provision defers the illiquidity of the bank, pushing the illiquidity threshold up from threshold $n = 1/r_1$ to $n_{Ill}(B) = \frac{(1+B)}{r_1}$. If the ELA provision is sufficiently large, with $B \geq r_1 - 1$, then illiquidity of the bank is ruled out, $\frac{1+B}{r_1} \geq 1$. I henceforth assume that the ELA provision is partial, $B < r_1 - 1$, because I want to understand how the withdrawal incentives of depositors change as the ELA provision increases from zero onwards. Because ELA is partial, the bank is forced to resume the liquidation of assets once the resources B are used up, that is, for $nr_1 > n_B r_1 + B$. I call the withdrawal threshold at which all funds B are used up and liquidation resumes $n_{res}(B) \equiv n_B + \frac{B}{r_1}$. The payoff difference before ELA is triggered equals

$$v_L(n) = \underbrace{p(\theta)\,\frac{R(1 - nr_1)}{1 - n}}_{u_2(n)} - \underbrace{r_1}_{u_1},\ n \in [0, n_B). \tag{14}$$

Once the ELA intervention starts, asset liquidation is halted as long as ELA is sufficient to serve withdrawals. The payoff difference[16] on $[n_B, n_{res})$ becomes

$$v_{L,B}(n) = \underbrace{p(\theta)\ \max\left(\frac{R(1 - n_B r_1) + B - (n - n_B)r_1 - rB}{1 - n}, 0\right)}_{u_2(n)} - \underbrace{r_1}_{u_1}. \tag{15}$$

I assume that the bank borrows the entire funds B, and cannot borrow a withdrawal-contingent amount. Borrowed funds that are not utilized to repay withdrawing agents in $t = 1$ are invested

[16]Observe, if the ELA intervention threshold n_B is chosen too high, then the bank is insolvent before all funds are utilized.

in storage, and jointly with the returns on the asset are used to repay the loan to the LOLR in $t = 2$. I apply the max operator in (15) and following because the bank has limited commitment, and because the bank becomes insolvent before it becomes illiquid in $t = 1$. This is an observation that the literature has made before, see also Rochet and Vives (2004): The ELA loan allows more withdrawals at the expense of agents who roll-over, meaning the loan is a transfer from the roll-over depositors whom need to repay the loan to withdrawing depositors. Ultimately, this is the reason why the ELA provision is a double-edged sword, lowering illiquidity risk in the short-run at the expense of raising credit risk in the long run. A policy that imposes withdrawal fees, in contrast, constitutes a transfer from the withdrawing to the roll-over agent group. Thus, its impact on stability will turn out to be very different from ELA's impact.

I henceforth assume $n_B < \frac{R-B(r-1)}{Rr_1-B(r-1)}$ so that the acceptance of ELA, $n \geq n_B$, does not cause the bank's insolvency right away. This assumption can be rationalized by demanding that ELA is provided only to illiquid but solvent banks, as in Rochet and Vives (2004). If the withdrawals are so high that the ELA funds B are insufficient to cover all withdrawals, $n \geq n_{res}$, the bank is forced to resume the liquidation of assets and the payoff difference becomes

$$\upsilon_{L,B+}(n) = \underbrace{p(\theta)\ \max\left(\frac{R(1-nr_1+B)-rB}{1-n}, 0\right)}_{u_2(n)} - \underbrace{r_1}_{u_1},\ n \in [n_{res}, n_{Ill}). \tag{16}$$

until the bank becomes illiquid for $nr_1 \geq 1 + B$. As soon as the bank becomes illiquid, the payoff difference becomes

$$\upsilon_{Ill}(n) = \underbrace{0}_{u_2} - \underbrace{\left(\frac{n_{Ill}(B)}{n} \times r_1 + \left(1 - \frac{n_{Ill}(B)}{n}\right) \times 0\right)}_{u_1(n)},\ n \in [n_{Ill}(B), 1] \tag{17}$$

because a withdrawing depositor is served the face value only if she is early in the queue. The expected payoff difference equals

$$H(B, n_B, r, \theta^*) = \int_0^{n_B} \left(p(\theta(n,\theta^*))\frac{R(1-nr_1)}{1-n} - r_1\right) dn \tag{18}$$

$$+ \int_{n_B}^{n_{res}(B,n_B)} \left(p(\theta)\ \max\left(\frac{R(1-n_Br_1)+B-(n-n_B)r_1-rB}{1-n}, 0\right) - r_1\right) dn \tag{19}$$

$$+ \int_{n_{res}(B,n_B)}^{n_{Ill}(B)} \left(p(\theta)\ \max\left(\frac{R(1-nr_1+B)-rB}{1-n}, 0\right) - r_1\right) \tag{20}$$

$$- \int_{n_{Ill}(B)}^{1} \left(\frac{n_{Ill}(B)}{n} \times r_1 + \left(1 - \frac{n_{Ill}(B)}{n}\right) \times 0\right) dn \tag{21}$$

5.2.3 ELA where interest is only paid on used liquidity

The formulation above assumes that a loan B is paid in full, so that interest rB is owed even if not the entire liquidity provision was necessary. Now assume instead, that there is a finite liquidity window B. The bank pays interest on the amount $\min((n-n_B)r_1, B)$. Then the investors' payoff in the range $[n_B, n_{res}]$ changes. In that range ELA is not fully used up, $(n-n_B)r_1 < B$ so that interest $(n-n_B)r_1 r$ accrues instead of rB.

$$H(B,n_B,r,\theta^*) = \int_0^{n_B} \left(p(\theta(n,\theta^*))\frac{R(1-nr_1)}{1-n} - r_1\right) dn \tag{22}$$

$$+ \int_{n_B}^{n_{res}(B,n_B)} \left(p(\theta)\ \max\left(\frac{R(1-n_Br_1)-(n-n_B)r_1r}{1-n}, 0\right) - r_1\right) dn \tag{23}$$

$$+ \int_{n_{res}(B,n_B)}^{n_{Ill}(B)} \left(p(\theta)\ \max\left(\frac{R(1-nr_1+B)-rB}{1-n}, 0\right) - r_1\right) \tag{24}$$

$$- \int_{n_{Ill}(B)}^{1} \left(\frac{n_{Ill}(B)}{n} \times r_1 + \left(1 - \frac{n_{Ill}(B)}{n}\right) \times 0\right) dn \tag{25}$$

Then the result on raising the ELA provision is robust. It holds

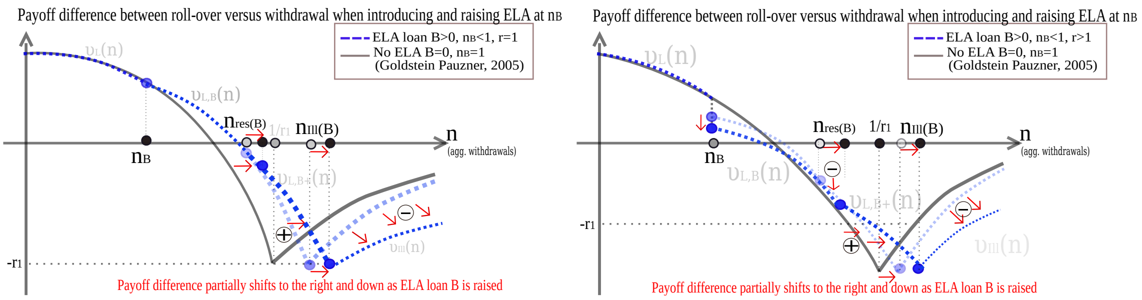
$$\frac{\partial}{\partial B} H(B,\theta^*) = + \int_{n_{res}}^{n_{Ill}} p(\theta(n,\theta^*))\ \left(\frac{R-r}{1-n}\right) dn - \int_{n_{Ill}}^{1} \frac{1}{n}\, dn \tag{26}$$

the stabilizing first effect though vanishes as $n_B \to 1/r_1$ so that granting and raising ELA reduces stability if it is granted too late.

5.2.4 Analysis: Raising the ELA provision

First, observe that the ELA provision $B > 0$ causes a downward jump of the payoff difference function as the withdrawals hit the ELA entry point n_B if the LOLR charges interest on the loan, $r > 1$, see Figure 2b: $\lim_{n \nearrow n_B} \upsilon_L(n) - \lim_{n \searrow n_B} \upsilon_{L,B}(n,\gamma) = p(\theta)\frac{(r-1)B}{1-n_B} > 0$. Thus, ELA constitutes harsh policy intervention if the LOLR charges positive net interest $r > 1$. The depth of the jump increases with the ELA loan B because more interest becomes due in $t = 2$. If the LOLR charges no interest, $r = 1$, then no jump occurs in the entry threshold n_B, see Figure 2a. There is no jump at the ELA exit point n_{res} where the funds are used up. Therefore, raising the ELA provision constitutes piecewise smooth policy when holding the jump point n_B constant.

I next analyse how an increase in B affects the payoff difference function, holding the ELA entry point n_B fixed. I discuss shifting the ELA entry point in the next subsection. Once ELA is triggered, $n \in [n_B, n_{res})$, the payoff to roll-over declines in the ELA provision because depositors who roll-over need to repay more funds with interest to the LOLR given survival of the bank, $\frac{\partial}{\partial B}\upsilon_{L,B}(n,\gamma) < 0$, see Figure 2b. This effect negatively affects bank stability, it, however, becomes void if the LOLR charges zero interest, $r = 1$. Thus, a rise in the ELA provision impacts the roll-over incentives adversely piecewise smooth in the range $n \in [n_B, n_{res})$ if interest is charged, $r > 1$, and otherwise has no impact.



(a) Zero interest Ela loan $r = 1$ (b) Interest on ELA loan, $r > 1$

Figure 2: When an ELA loan B is provided at threshold n_B, the payoff difference function $\upsilon(n)$ shifts to the right, allowing the bank to survive larger runs as B increases (the illiquidity threshold n_{Ill} rises). The payoff difference function $\upsilon(n)$ shifts up for all $n \in [n_{res}, n_{Ill}]$, but the PI shifts down over the range $n \in [n_B, n_{res}]$ and $[n_{Ill}, 1]$ because ELA is expensive and because given bank illiquidity, the payoff to withdraw increases with the ELA provision because the likelihood of getting served in the queue goes up. If the LOLR charges interest on the ELA loan, $r > 1$, a jump in the PI occurs at n_B. The depth of the down-jump increases with B.

In the withdrawal range for which the ELA provision is used up but the bank is not illiquid yet, $n \in [n_{res}, n_{Ill})$ the change in relative payoffs due to an increase in ELA $\frac{\partial}{\partial B}\upsilon_{L,B+}(n,\gamma)$ can go in either direction: On the one hand, as the lender of last resort (LOLR) raises the ELA provision, the depositors who roll-over need to repay more funds and interest to the LOLR given survival. This negative effect does not vanish if the LOLR charges zero interest. On the other hand, as more ELA is provided, the liquidation of assets can be deferred for longer. Overall, whether the payoff to roll over increases or declines with the ELA provision in this withdrawal range depends on whether the return on the asset R exceeds the cost of the ELA loan r. A sufficient and reasonable condition for the latter, $\frac{\partial}{\partial B}\upsilon_{L,B+}(n,\gamma) > 0$, is that the LOLR charges lower interest on the ELA loan than the return on the asset, $r \leq R$, for instance, $r = 1$ (zero net interest). The withdrawal threshold at which the bank needs to resume the asset liquidations, n_{res}, shifts upwards as more ELA is provided, see Figures 2b and 2a. Likewise, the bank's illiquidity is deferred: the threshold n_{Ill} increases, as the LOLR provides more ELA. That is, the withdrawal interval for which the ELA funds are used up but the bank is not illiquid yet, $n \in [n_{res}(B), n_{Ill}(B)) = \mathcal{N}_B$, shifts upwards with the ELA funds B but maintains its length constant. Even though the payoff difference is continuous at n_{Ill}, the rise in the illiquidity threshold n_{Ill} matters directly for incentives because it increases the probability that a depositor is served the face value when withdrawing, once the bank is illiquid: Because the ELA provision pushes the illiquidity point $n_{Ill}(B)$ upwards, it holds $\frac{\partial}{\partial B}\upsilon_{Ill}(n) < 0$. That is, the increase in the ELA provision constitutes adverse piecewise smooth policy and has a negative effect on the roll-over incentives, acting like a bail-in of depositors that roll over in this withdrawal range. The intervention interval equals $\mathcal{N}_B = [n_B, 1]$ for positive net interest $r > 1$, and equals $\mathcal{N}_B = [n_{res}, 1]$ for zero net interest, $r = 1$. Allover by Proposition 4.1 (ii),

Corollary 5.3 (Increasing the ELA funds)
Assume the LOLR provides an ELA loan $B > 0$ at interest rate $r \in [1, R)$ if the withdrawals

realize above a threshold $n_B \in (0, 1/r_1)$. A policy that raises the ELA provision B, holding the entry threshold n_B constant, has ambiguous effects on stability. A rise in ELA lowers ex ante bank stability if the entry threshold n_B is set too close to the illiquidity point $1/r_1$, even if the ELA loan is granted at zero net interest $r = 1$.

The Corollary implies that a commitment to a "too late" ELA provision is an unfortunate policy because it implies making things worse ex ante. Intuitively, as ELA is granted late, much of the assets are already liquidated, leaving only few incentives to the depositors to roll over. If the emergency loan is granted on top of that, the burden of repaying that loan at an interest rate accrues additionally to depositors that roll-over, causing them to withdraw as well.

The result is alarming also for a second reason. The LOLR does not perfectly observe the asset quality $p(\theta)$. A bounded amount of emergency liquidity is granted in the belief that the run should stop at some point if the bank only faces liquidity issues but is solvent. This result shows that the granting of the loan can increase the run incentive ex ante. That is, to the LOLR is becomes undistinguishable whether a run continues because the bank is insolvent or whether the run goes on *because* ELA was granted. The question that arises how to grant emergency liquidity without making runs more likely ex ante.

Proof. [Corollary 5.3] I need to consider the expected change in the payoff difference for determining the overall impact of policy on stability. Via equation (7), and with a policy variable $p = B$ it holds

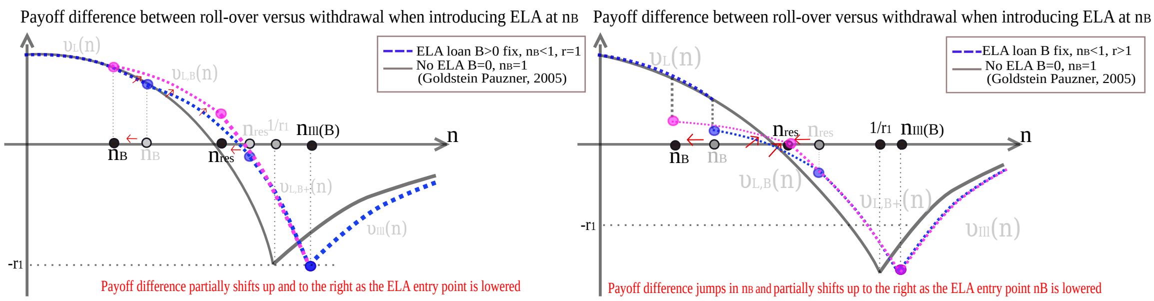
$$\frac{\partial}{\partial B} H(B, \theta^*) = -\int_{n_B}^{n_{res}} p(\theta(n, \theta^*)) \, \frac{(r-1)}{1-n} \, dn + \int_{n_{res}}^{n_{Ill}} p(\theta(n, \theta^*)) \, \left(\frac{R-r}{1-n}\right) dn - \int_{n_{Ill}}^{1} \frac{1}{n} \, dn \tag{27}$$

Observe that the raise in B does not shift the jump point. If the ELA entry threshold n_B is chosen below but close to the original illiquidity threshold absent intervention, $1/r_1$, then by Lebesgue's dominated convergence theorem, the only stability-improving effect on stability via the ELA provision vanishes, $\lim_{n_B \to 1/r_1} \int_{n_B + B/r_1}^{(1+B)/r_1} p(\theta(n, \theta^*)) \left(\frac{R-r}{1-n}\right) dn \to 0$, whereas all the adverse effects on stability remain: It holds $\lim_{n_B \to 1/r_1} \frac{\partial}{\partial B} H(B, \theta^*) < 0$ and thus, with the implicit function theorem, $\frac{\partial \theta^*}{\partial B} = -\frac{\partial}{\partial B} H(B, \theta^*) / \frac{\partial}{\partial \theta^*} H(B, \theta^*) > 0$, meaning the trigger increases, and range of miscoordination becomes larger, increasing the ex ante run probability. This holds even for net interest zero $r = 1$. Thus, bank stability strictly declines as the LOLR increases the ELA provision B. □

5.2.5 Lowering the ELA entry point

From the previous result, one key take-away for the LOR should be that ELA intervention should not occur "too late." That is, the intevention should be triggered at a lower withdrawal threshold. I next discuss how an ELA policy that commits to intervening at a lower entry point, n_B, affects the investors' behavior and thus bank stability ex ante, holding the liquidity provision B fixed.

To determine the overall change in incentives, I need to consider the shift in the jump point as well as changes in the payoff difference function $v_B(n)$ due to changes in n_B. First, we know that the payoff difference function jumps down in the ELA entry point n_B if the LOLR charges interest $r > 1$ because as ELA is granted, the depositors that roll over additionally owe the interest on the ELA loan. Lowering the entry point, thus, affects the roll-over incentives adversely by Proposition 4.1(i). The ELA exit point n_{res} depends on the ELA entry point but the payoff difference function is continuous in n_{res}, so its boundary derivative vanishes. Concerning changes in the payoff difference function $v_B(n)$, when ELA is active and asset liquidation has not resumed yet, $[n_B, n_{res})$, lowering the ELA entry point raises the PI because the bank can stop costly asset liquidations sooner, it holds $\frac{\partial}{\partial n_B} v_{L,B}(n, \gamma) < 0$, independently of interest r. Thus, *lowering* the entry point acts prudent piecewise smooth on the intervention interval $[n_B, n_{res})$. For $r > 1$, the downwards shift of the down-jump point n_B and the prudent piecewise smooth effect on $[n_B, n_{res})$ act against one another.



(a) Zero interest Ela loan $r = 1$: no jump at threshold n_B

(b) Interest on ELA loan, $r > 1$, causes a jump at ELA entry point n_B

Figure 3: When lowering the ELA entry point n_B holding the loan amount B constant, the interval $[n_B, n_{res}]$ over which ELA is active shifts down but maintains its length. The PI over $[n_B, n_{res}]$ declines slower and thus shifts up as n_B shifts down. The illiquidity threshold is unchanged. If $r > 1$, the depth of the down-jump increases with n_B and lowering n_B causes an adverse jump-shift which acts against bank stability, lowering the effectiveness of ELA.

Corollary 5.4 (Lowering the ELA entry point)
Consider the provision of an ELA loan B at entry point n_B at interest rate $r \geq 1$. Lowering the ELA entry threshold n_B while holding B fixed raises bank stability ex ante independently of whether interest is charged on the loan or not. But if the LOLR charges interest, $r > 1$, lowering the ELA entry point n_B is not as effective in improving stability because it gives rise to a stability-deteriorating side-effect, a jump-shift, at the ELA entry point n_B.

Proof. [Corollary 5.4] To determine the overall effect on stability when lowering the entry threshold to ELA, I need to consider the expected change in the PI when raising n_B which is given by

$$\frac{\partial}{\partial n_B} H(B, n_B, r, \theta^*) = p(\theta(n_B, \theta^*)) \left(\frac{B(r-1)}{1 - n_B} \right) - \int_{n_B}^{n_{res}(B, n_B)} p(\theta) \, \frac{r_1(R-1)}{1-n} \, dn \qquad (28)$$

where $p(\theta(n_B, \theta^*)) \left(\frac{B(r-1)}{1-n_B}\right)$ is the change in the PI due to the shift in the jump point. The two terms have opposite sign. It turns out that the effect due to the jump point is always weaker than the smooth effect due to the shift in the PI over the interval $[n_B, n_{res}]$: Because $R > r$, it holds $\int_{n_B}^{n_B+B/r_1} p(\theta(n, \theta^*)) \frac{r_1(R-1)}{1-n} dn > p(\theta(n_B + B/r_1, \theta^*)) \frac{B(r-1)}{1-n_B} \to p(\theta(n_B, \theta^*)) \frac{B(r-1)}{1-n_B}$ as $\varepsilon \to 0$. Therefore, the overall change in the expected PI when raising n_B is negative for any ELA interest rate $r \geq 1$: $\frac{\partial}{\partial n_B} H(n_B, \theta^*) \leq 0$. By Proposition 4.1(ii), thus, bank stability strictly decreases as the threshold n_B increases, and stability increases as the entry threshold to ELA, n_B is *lowered*. □

We can compare the policy that raises the ELA loan B to a policy that lowers the entry threshold n_B.[17]

It is clear from result X that, if at all, ELA should not be provided "too late."

5.3 Suspension of convertibility and Budget Interdependence

The imposition of withdrawal fees or an ELA provision, discussed above, provide examples where the intervention threshold and the policy-implied budget transfer across agent groups can be set independently of one another. I next present an intervention type, the suspension of convertibility followed by the bank's resolution under receivership (in short "receivership resolution"), where the policy jointly pins down the intervention threshold and the transfer. As a consequence, receivership intervention is particularly tricky to handle when it comes to designing stability-maximizing policy. The following example is based on the analysis of suspension interventions followed by resolution under receivership in Schilling (2019, 2023) for the special case of zero deposit insurance.

As previously, the depositors can withdraw the face value of their deposit r_1 at the interim period, and the bank finances withdrawals by liquidating assets. As the standard bank run externality, high withdrawals reduce the remaining bank investment and thus the payoffs to depositors that roll over. This payoff externality via the withdrawals creates interdependence of budgets available to the withdrawing and the not withdrawing agent group which in return leads to a reduction of the policy variables: A regulator observes withdrawals at the bank level, and has the authority to stop runs by suspending the convertibility of deposits before the bank becomes illiquid. The regulator sets the intervention delay $p \in [0, 1]$ as the policy variable, where $1 - p$ denotes the measure of cash withdrawals the regulator tolerates until intervention. The regulator intervenes to stop the run once the cash withdrawals reach $1 - p \in [0, 1]$, and thus imposes the $t = 1$ budget constraint $nr_1 \leq 1 - p$. Policy p pins down the critical suspension entry threshold

$$n_c(p) := \frac{1-p}{r_1} \in (0, 1/r_1) \tag{29}$$

at which the regulator intervenes. Absent regulatory intervention, $p = 0$, the bank is illiquid if the cash withdrawals reach the liquidation value of the asset, $nu_1 \geq 1$. Therefore, 1 is

[17] We can do that because the policy-driven change in ex ante bank stability θ^* due to a change in policy $p = B$ versus $p = n_B$ only differs in the numerator $\frac{\partial}{\partial B} H(B, \theta^*)$ versus $\frac{\partial}{\partial n_B} H(n_B, \theta^*)$ and not in the denominator $-\frac{\partial}{\partial \theta^*} H(B, \theta^*) = -\frac{\partial}{\partial \theta^*} H(n_B, \theta^*)$, recall the proof to Proposition 4.1.

the maximum budget to early withdrawing investors. The policy contingent budget available to early withdrawing investors is given as $G_1(p) = \max(1-p, 0)$. The remaining investment in the asset accrues interest until $t = 2$. The budget to late withdrawing agents is given as $G_2(p) = H\ (1 - \min(nr_1, 1-p))$. The budgets to early and late withdrawing agents are interdependent: As policy intensity p increases, the regulator tolerates fewer withdrawals until intervention, thus, the budget to early withdrawing agents $G_1(p)$ declines whereas the budget to agents that roll over, $G_2(p)$, increases. As I will explain next, this budget interdependence makes the suspension policy a harsh combination policy. To determine the payoff difference function: If the aggregate cash withdrawals remain below the policy dependent budget, $nr_1 \leq G_1(p)$, then no policy intervention occurs. In that case, investors who withdraw receive r_1, and the investors who roll over receive an equal share of the budget in $t = 2$, u_2 satisfies $(1-n)u_2(n,\theta) = G_2(p)$. Thus whenever $n \leq n_c(p)$ (no policy intervention "np"), the payoff difference equals

$$v_{np}(n,\theta) = \underbrace{p(\theta)\frac{H(1-nu_1)}{1-n}}_{u_2(n)} - \underbrace{r_1}_{u_1}. \tag{30}$$

If the cash withdrawals however reach or exceed the budget G_1, the regulator intervenes, stops the run, takes over control of the remaining assets, and continues the investment of the remaining asset share p at a reduced return $r \in (0, H)$ that is likewise subject to aggregate risk, $p(\theta)$. The regulator's reduced effectiveness in managing assets implies a costliness of intervention, which in return creates a jump of the payoff difference function in the intervention threshold n_c, see below. Withdrawals that would exceed budget G_1 are no longer served. Instead, these agents enter a regulatory procedure, a "mandatory deposit stay," jointly with the agents that rolled over. Under a mandatory deposit stay, all these investors share the proceeds of remaining investment. The proportion p of the asset that was protected by intervention matures, and yields a policy-dependent, risky pro rata share to agents under the mandatory stay $u_p = p(\theta)\,\frac{rp}{1-G_1(p)/r_1}$. where $G_1(p)/r_1 = 1 - n_c(p)$ is the share of depositors that may withdraw before policy intervention occurs. Conditional on policy intervention, $n > n_c(p)$, the payoff difference equals

$$v_p(n,\theta) = \underbrace{p(\theta)\,\frac{pr}{1-G_1(p)/r_1}}_{u_2} - \underbrace{\left(\frac{G_1(p)/r_1}{n}\,r_1 + \left(1 - \frac{G_1(p)/r_1}{n}\right)\,p(\theta)\,\frac{pr}{1-G_1(p)/r_1}\right)}_{u_1(n)} \tag{31}$$

where $\frac{G_1(p)/r_1}{n}$ is the probability that an investor who requests withdrawal is served the face value r_1 and thus does not enter the mandatory stay. The payoff difference conditional on intervention is always negative because for states for which withdrawal is not dominant, $\theta \in (\underline{\theta}, 1]$, it must hold $r_1 - \frac{p(\theta)\,pr}{1-G_1(p)/r_1} > 0$.[18]

[18]For all states in the lower dominance region $\theta \in [0, \underline{\theta})$ withdrawal (by definition) is dominant, meaning the payoff difference is negative for all realizations of n. For all states between the upper and lower dominance region $\theta \in [\underline{\theta}, \overline{\theta}]$ the sign of the payoff difference function depends on the realization of the aggregate withdrawals n. If for some $\theta \in (\underline{\theta}, 1]$ it held $r_1 - \frac{p(\theta)\,pr}{1-G_1(p)/r_1} < 0$, then also $r_1 - \frac{p(\theta)\,H(1-nr_1)}{1-n} < 0$ for all $n < n_c(p)$, contradicting that withdrawal is not dominant, see (Schilling, 2019, 2023) for the construction of the lower dominance region for this example.

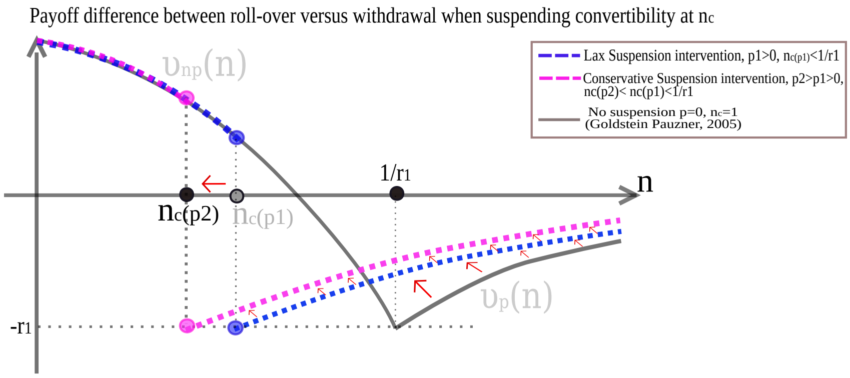


Figure 4: Assume the regulator suspends the convertibility of deposits as withdrawals exceed $n_c(p)$. The intervention causes a down-jump in $n_c(p)$ simultaneously to an upwards shift of the PI in the range $\mathcal{N}_p = [n_c(p), 1]$. As fewer withdrawals are tolerated, p increases from p_1 to p_2, corresponding to a lower intervention entry threshold $n_c(p_2) < n_c(p_1)$, and thus a lower jump point (adverse harsh), as well as an additional upwards shift of the PI on $\mathcal{N}_p = [n_c(p), 1]$ (prudent piecewise smooth).

5.3.1 Analysis

The budget interdependence creates a harsh policy combination: The intervention jump threshold $n_c(p)$ depends on and shifts in policy intensity p, and generically constitutes a discontinuity. By $r < H$, the payoff difference jumps down in $n = n_c$: $\lim_{n \nearrow n_c(p)} \upsilon_{np}(n, \theta) - \lim_{n \searrow n_c(p)} \upsilon_p(n, \theta) = p(\theta) \frac{(H-r)p}{1-\frac{1-p}{r_1}} > 0$. As the regulator tolerates fewer withdrawals until intervention, p increases, and the down-jump point $n_c(p)$ declines (comes forward), implying an adverse harsh effect on bank stability via Proposition 4.1(ia). Simultaneously, a policy that tolerates fewer withdrawals acts prudent piecewise smoothly on the intervention interval $\mathcal{N}(p) = (n_c, 1]$ because it increases the budget $G_2(p)$ to investors that roll-over by lowering the budget available to investor that withdraw. Consequently, the payoff difference function $\upsilon_p(n, \theta)$ shifts upwards in p conditional on intervention.

Corollary 5.5 (Schilling (2019): Suspension of convertibility and receivership)
Assume a regulator sets a policy $p \in (0, 1)$ whereby it stops runs by suspending the convertibility of deposits if the cash withdrawals at the bank exceed the level $1 - p$, that is, for withdrawals above a threshold $n_c(p) \in (0, 1/r_1)$. Lowering the suspension entry threshold n_c constitutes harsh combination policy. If r is large and close to H, lowering the entry threshold improves stability ex ante. But if r is low, lowering the entry threshold can deteriorate stability ex ante.

Similar to an ELA provision, the suspension of convertibility is a policy that can backfire, and a policy that imposes withdrawal fees is the the safer policy with regard to assuring a positive impact on bank stability. The online appendix gives an additional prudent smooth policy example, namely partial deposit insurance where the intervention interval is the full range $\mathcal{N}(\gamma) = [0, 1]$.

6 Conclusion

This paper provides a general framework to analyze the effectiveness of policy interventions with regard to their capacity to prevent or ease runs on firms such as banks, money market mutual funds, or stablecoins. The paper establishes two different classes of policy based on how the policy acts on the investor's payoffs, "smooth" or "harsh". Every real-world policy belongs to at least one class. For each class I determine how it impacts the investors' ex ante run propensity and, thus, firm stability. The range of policies that lower bank stability ex ante is large, and act by either lowering the favorability of roll-over versus withdrawal gradually or in a way that gives rise to discontinuities in the relative payoffs of investors.

I then show that common policies such as emergency liquidity provision (ELA) by a lender of last resort, the imposition of withdrawal fees or the suspension of convertibility belong to multiple classes, and thus have mixed effects on stability. I show that if a policy belongs to multiple classes, and exhibits the according features, it can become ineffective with regard to improving stability since different features can partially offset each other. Bailins can act like bailouts and can both improve or deteriorate stability. An ELA provision can lower stability, and the imposition of withdrawal fees is partially ineffective with regard to lowering firm stability *because* it allows firms to survive greater runs, thus, acting like a bailout to withdrawing investors which is equivalent to a bail-in of investors that roll-over.

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7 Appendix

7.1 Equilibrium Existence and uniqueness with jumps

Proof. [Proposition 3.1] To show existence and uniquenss of a trigger equilibrium, assume again that all investors follow the same strategy that maps signals θ_i to actions. Assume that investors follow a threshold strategy around θ^*. Then the measure of agents that run at each state, $n(\theta, \theta^*)$ is deterministic. Observe that $n(\theta, \theta^*)$ is at one for $\theta < \theta^* - \varepsilon$, because all agents oberve signals below the trigger signal and withdraw. Further, $n(\theta, \theta^*)$ is strictly decreasing in state θ for $\theta \in [\theta^* - \varepsilon, \theta^* + \varepsilon]$, and attains zero for $\theta > \theta^* + \varepsilon$. Therefore, as θ increases in $[\theta^* - \varepsilon, \theta^* + \varepsilon]$, n transitions through all jump points $n_1, \ldots, n_k$ of the payoff difference function.

Consider the inverse of $n(\theta, \theta^*)$, $\theta(n, \theta^*)$, as given in (5). Let $\theta_1, \ldots, \theta_k$ the states for which $n(\theta, \theta^*)$ attains the jump points, that is, $\theta_1 = \theta(n_1, \theta^*), \ldots, \theta_k = \theta(n_k, \theta^*)$. In this proof, I call these states the "jump-states", and address them using the subscript θ_j, not to be confused with signal θ_i. Note, due to $n_1 < \cdots < n_{k-1} < n_k$, I have $\theta_k < \theta_{k-1} < \cdots < \theta_1$. Set $\theta_0 = 1$ and $\theta_{k+1} = 0$. Note that in a trigger equilibrium around θ^*, it holds that $\theta_1, \ldots, \theta_k \in (\theta^* - \varepsilon, \theta^* + \varepsilon)$ because $n(\theta, \theta^*)$ is continuous and because $n(\theta^* - \varepsilon, \theta^*) = 1$, $n(\theta^* + \varepsilon, \theta^*) = 0$. Then $[0,1] = \cup_{j=0}^{k}[\theta_{j+1}, \theta_j]$, and for every signal θ_i and $\varepsilon > 0$, it holds $[\theta_i - \varepsilon, \theta_i + \varepsilon] \subset \cup_{j=0}^{k}[\theta_{j+1}, \theta_j]$. I want to partition the interval $[\theta_i - \varepsilon, \theta_i + \varepsilon]$ by the jump states it contains, by considering $[\theta_i - \varepsilon, \theta_i + \varepsilon] \cap \left(\cup_{j=0}^{k}[\theta_{j+1}, \theta_j]\right)$. Let $n \in \{0, 1, \ldots, k\}$ the number of jump states contained in the interval $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. If $n = 0$, then there exists no partition by jump points and I write $[\theta_i - \varepsilon, \theta_i + \varepsilon] \cap \left(\cup_{j=0}^{k}[\theta_{j+1}, \theta_j]\right) = [\theta_i - \varepsilon, \theta_i + \varepsilon]$.

If $n \geq 1$, I address the jump states in this interval directly by calling them $\theta_{j_1}, \ldots \theta_{j_n}$, where θ_{j_1} is the smallest one among them, and thus, θ_{j_n} the largest, and where because of the reverse numbering of the jump states, it holds $j_1 \leq k$ and $j_n \geq 1$. This yields a partition of $[\theta_i - \varepsilon, \theta_i + \varepsilon]$ according to $[\theta_i - \varepsilon, \theta_i + \varepsilon] \cap \left(\cup_{j=0}^{k}[\theta_{j+1}, \theta_j]\right) = [\theta_i - \varepsilon, \theta_{j_1}] \cup [\theta_{j_1}, \theta_{j_2}] \cup \cdots \cup [\theta_{j_n}, \theta_i + \varepsilon]$.

By assumption 3.1, the payoff difference function is continuous on all open intervals $[\theta_i - \varepsilon, \theta_{j_1})$, $(\theta_{j_1}, \theta_{j_2})$, $\ldots (\theta_{j_n}, \theta_i + \varepsilon]$. Further by assumption 3.1, the right and left sided limits of the payoff difference function exist at each jump state θ_{j_i}, $i = 1, \ldots m$,

$$|\lim_{\theta \nearrow \theta_{j_i}} \upsilon_p(n(\theta, \theta^*), \theta)| = |\lim_{n \searrow n_{j_i} \equiv n(\theta_{j_i}, \theta^*)} \upsilon_p(n, \theta(n, \theta^*))| =: c_{i,r} < \infty \tag{32}$$

$$|\lim_{\theta \searrow \theta_{j_i}} \upsilon_p(n(\theta, \theta^*), \theta)| = |\lim_{n \nearrow n_{j_i} \equiv n(\theta_{j_i}, \theta^*)} \upsilon_p(n, \theta(n, \theta^*))| =: c_{i,l} < \infty \tag{33}$$

Given a signal θ_i, the true state must be located in $[\theta_i - \varepsilon, \theta_i + \varepsilon]$. If this interval contains jump states, $n \geq 1$, an agent's expected payoff difference to roll over versus withdraw when observing signal θ_i can therefore be rewritten as

$$H(\theta_i, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon}\left(\int_{\theta_i - \varepsilon}^{\theta_{j_1}} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta + \int_{\theta_{j_1}}^{\theta_{j_2}} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta + \cdots + \int_{\theta_{jn}}^{\theta_i + \varepsilon} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta\right) \tag{34}$$

If an investor observes the trigger signal $\theta_i = \theta^*$, the interval of possible states $[\theta^* - \varepsilon, \theta^* + \varepsilon]$ contains all jump states, $n = k$, and her expected payoff difference equals

$$H(\theta^*, n(\cdot, \theta^*)) = \frac{1}{2\varepsilon}\left(\int_{\theta^* - \varepsilon}^{\theta_{j_1}} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta + \int_{\theta_{j_1}}^{\theta_{j_2}} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta + \cdots + \int_{\theta_{j_k}}^{\theta^* + \varepsilon} \upsilon_p(n(\theta, \theta^*), \theta)\, d\theta\right) \tag{35}$$

I first argue, there exists a unique θ^*, that satisfies $H(\theta^*, n(\cdot, \theta^*)) = 0$. To see that, note that $H(\theta^*, n(\cdot, \theta^*))$ is strictly increasing in θ^* for $\theta^* < \overline{\theta} + \varepsilon$, because by assumption 2.1 $\upsilon_p(n(\theta, \theta^*), \theta)$ is non-decreasing and is strictly increasing in θ for $\theta \in [\underline{\theta}_p, \overline{\theta}_p]$. Further, $H(\theta^*, n(\cdot, \theta^*)) > 0$ for $\theta^* \in [\overline{\theta}_p + \varepsilon, 1]$, and $H(\theta^*, n(\cdot, \theta^*)) < 0$ for $\theta^* \in [0, \underline{\theta}_p - \varepsilon]$. Last,

Lemma 7.1. *$H(\theta^*, n(\cdot, \theta^*))$ is continuous in θ^**

Because $H(\theta^*, n(\cdot, \theta^*))$ is strictly increasing and continuous in θ^*, exceeding 0 for high values of θ^* and undercutting 0 for low values of θ^*, there exists a unique θ^* with $H(\theta^*, n(\cdot, \theta^*)) = 0$, the candidate for a trigger equilibrium.

It remains to show that θ^* is an equilibrium. That is, one needs to show that for all signals $\theta_i < \theta^*$ it follows $H(\theta_i, n(\cdot, \theta^*)) < 0$ whereas for all $\theta_i > \theta^*$ it follows $H(\theta_i, n(\cdot, \theta^*)) > 0$. By assumption 2.1, $\upsilon_p(n, \theta)$ is positive for high values of θ, negative for low values of θ, and satisfies single-crossing. Therefore, for this part, the existence proof on page 1313 in Goldstein and Pauzner (2005) also applies here. They show, if $\theta_i < \theta^*$, then $H(\theta_i, n(\cdot, \theta^*)) < 0 = H(\theta^*, n(\cdot, \theta^*))$. This holds because $\upsilon_p(n, \theta)$ is positive for high values of θ, negative for low values of θ, crosses zero only once, and because agent i forms expectations about the payoff difference over a lower range of fundamentals than for $\theta_i = \theta^*$. Likewise for $\theta_i > \theta^*$. Allover, there exists a unique threshold equilibrium around trigger θ^*.

No non-threshold equilibria

It remains to show that there are no non-threshold equilibria. I follow the notation in Goldstein and Pauzner (2005): A mixed strategy for investor i is a measurable function $s_i : [-\varepsilon, 1 + \varepsilon] \to [0, 1]$ that maps the investor's private signal into a probability to withdraw. A strategy profile is then denoted by $\{s_i\}_{i\in[0,1]}$. A state realization θ generates random signals $\theta_i = \theta + \varepsilon_i$ in the range $[\theta - \varepsilon, \theta + \varepsilon]$. The signals jointly with the strategy profile $\{s_i\}_{i\in[0,1]}$ generate the aggregate withdrawals $\tilde{n}(\theta)$ at state θ which is a random variable. For a given state θ, define the cumulative distribution function of $\tilde{n}(\theta)$ as

$$F_\theta(n) = \mathbb{P}(\tilde{n}(\theta) \le n|\theta) = \mathbb{P}\left(\int_{i\in[0,1]} s_i(\theta + \varepsilon_i)di \le n|\theta\right) \tag{36}$$

where the probability is measured with respect to the signal noise distribution $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$. An investor's expected payoff difference when observing signal θ_i and given a strategy profile $\{s_i\}_{i\in[0,1]}$ can, via the law of iterated expectation, be written as

$$H(\theta_i, \tilde{n}(\cdot)) = \frac{1}{2\varepsilon}\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon}\left(\int_0^{n_1} \upsilon(\theta, n)\, dF_\theta(n) + \cdots + \int_{n_k}^{1} \upsilon(\theta, n)\, dF_\theta(n)\right) d\theta \tag{37}$$

where $n_1, \ldots n_k$ are the jump points of $\upsilon(\theta, n)$ in the aggregate withdrawals n, and where the inner integrals of (37) are well-defined Lebesgue-Stieltjes integrals by assumption 3.1. The non-existence proof in Goldstein and Pauzner (2005) fully applies, because

Lemma 7.2. *$H(\theta_i, \tilde{n}(\cdot))$ is continuous in signal θ_i*

and because by the assumptions 2.1, and 3.2, the payoff difference function $\upsilon(n, \theta)$ satisfies single-crossing in n. Moreover, $\upsilon(n, \theta)$ is strictly decreasing in n whenever positive in the sense of assumption 3.2 and because $\upsilon(n, \theta)$ strictly increases in the state for state realizations in $[\underline{\theta}, \overline{\theta}]$. The proofs to Lemmata 7.1 and 7.2 can be found in the online appendix. □

7.2 Comparative statics under smooth intervention

Proof. [Proposition **??**] By Proposition **??**, for given $p > 0$ there exists a unique equilibrium trigger θ^* which is implicitly defined as the zero to

$$H(p,\theta^*) \equiv \int_0^1 \upsilon_p(n,\theta(n,\theta^*))\, dn = 0 \tag{38}$$

For sake of brevity, I suppress the dependence of θ^* on the policy p. The implicit function theorem delivers how θ^* changes as a function of p. By assumption, $\upsilon_p(n,\theta)$ is increasing in the state θ while $\theta(n,\theta^*)$ is strictly increasing in θ^*. Thus, $\frac{\partial H}{\partial \theta^*} > 0$. Next, since $\upsilon_p(n,\theta)$ is continuous in n, the boundary derivatives are zero, and we have $\frac{\partial H}{\partial p} = \int_0^1 \frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))\, dn = \int_{n\in\mathcal{N}(p)} \frac{\partial}{\partial p}\upsilon_p(n,\theta(n,\theta^*))\, dn$ which is positive under a prudent and negative under an adverse smooth policy. Altogether, $\frac{\partial \theta^*}{\partial p} = -(\frac{\partial H}{\partial p})/(\frac{\partial H}{\partial \theta^*}) < 0$ if and only if the policy is prudent. □

8 Supplementary Appendix

8.1 Additional Applications

8.1.1 Prudent smooth policy intervention via providing and raising partial Deposit Insurance (Guarantee)

The next regulatory policy I discuss is the provision of an increasing share of deposit insurance. I show, raising the partial deposit insurance provision constitutes prudent smooth intervention, thus raising bank stability ex ante by Proposition **??**. I consider partial insurance because if insurance is full there is no policy parameter to alter.[19] The following example revisits Schilling (2019) for the special case where there is no suspension of convertibility $a = 1$ (laissez-faire) but where the regulator provides partial deposit insurance, described by the share $\gamma \in (0,1)$. The resulting model is essentially the just-described Goldstein and Pauzner (2005) model, enriched by a partial deposit guarantee. The example nests the risk-neutral version of the Goldstein and Pauzner (2005) model when setting $\gamma = 0$.

Assume, deposits are insured up to the amount $\gamma \in [0,1)$, $\gamma \leq r_1$. Insurance alters the depositors' payoffs in the following way in comparison to the benchmark: In the case of a bank run $n \geq 1/r_1$, the depositors who roll over receive a positive payoff $\gamma \geq 0$, and the depositors who withdraw receive the face value r_1 with probability $\frac{1}{nr_1}$ (early in the queue) and receive the insured fraction γ with probability $1 - \frac{1}{nr_1}$ (late in the queue). Absent a run, if the asset does not pay off then the deposit insurance repays the depositors the insured share of their deposit.

To pin down payoffs, for a given state realization $\theta \in [0,\overline{\theta})$[20], and in the case where the bank

[19] Considering partial insurance is reasonable, because from different models we know that full insurance does not lead to efficient allocations due to moral hazard because depositors stop monitoring the bank (Cooper and Ross, 2002) or because of inefficient continuation of investment because depositors liquidate the bank too seldom (Schilling, 2019, 2023). The literature that analyzes the economics of deposit insurance is large, and the example here serves to provide one example where deposit insurance acts smoothly on payoffs. For a different analysis of partial insurance, see Dávila and Goldstein (2016) who analyze optimal insurance provision in the case of asymmetric deposits and lump-sum deposit insurance in a Diamond and Dybvig (1983) model.

[20] For states in $[\overline{\theta},1]$ all depositors roll over because this is the dominant action, see (Goldstein and Pauzner, 2005). We therefore exclude these states from the analysis here.

remains liquid (L) in $t = 1$, $n < 1/r_1$, the payoff difference between roll-over and withdrawal equals

$$\upsilon_L(n,\gamma) = \underbrace{\left(p(\theta)\max\left(\frac{R(1-nr_1)}{1-n},\gamma\right) + (1-p(\theta))\times\gamma\right)}_{u_2(n,\theta)} - \underbrace{r_1}_{u_1} \tag{39}$$

In the case where the bank becomes illiquid (Ill), $n \geq 1/r_1$, the payoff difference becomes

$$\upsilon_{Ill}(n,\gamma) = \underbrace{\gamma}_{u_2} - \underbrace{\left(\frac{1}{nr_1}\times r_1 + \left(1-\frac{1}{nr_1}\right)\times\gamma\right) = \frac{1}{nr_1}(\gamma - r_1)}_{u_1(n)} \tag{40}$$

The payoff difference function is continuous in n for every insurance choice $\gamma \in [0,1)$. Thus, the provision of partial deposit insurance constitutes smooth policy intervention. Further, increasing the share of deposit insurance provision γ constitutes prudent smooth intervention:

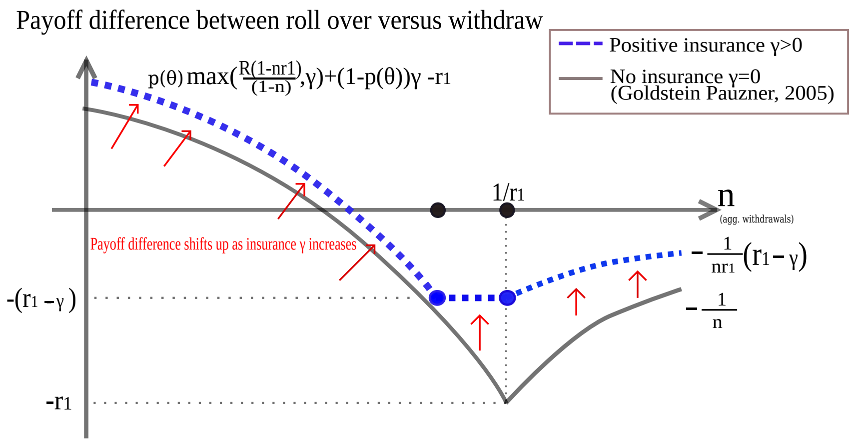

Figure 5: The payoff difference function $\upsilon(n)$ shifts up the more insurance coverage γ is provided.

In the liquid case, $n < 1/r_1$, it holds $\frac{\partial}{\partial\gamma}\upsilon_L(n,\gamma) > 0$. Similarly, in the illiquid case, $n \geq 1/r_1$, $\frac{\partial}{\partial\gamma}\upsilon_{Ill}(n,\gamma) > 0$. Allover, $\frac{\partial}{\partial\gamma}\upsilon(n,\gamma) > 0$ for all $n \in [0,1]$, and the intervention interval is given as $\mathcal{N}_\gamma = [0,1]$. As a Corollary of Proposition **??**(i), I obtain:

Corollary 8.1 (Raising partial deposit insurance is prudent smooth policy)
An increase of partial deposit insurance $\gamma \in [0,1)$ constitutes prudent smooth policy intervention. In the unique equilibrium, ex ante bank stability improves in the guaranteed share $\gamma \in (0,1)$.

8.2 Proofs of Lemmata

Proof. [Proof Lemma 7.1] Consider two triggers θ_x^* and θ_y^*. Without loss of generality, $\theta_x^* < \theta_y^*$, and I can write $\theta_y^* = \theta_x^* + d$, $d > 0$. I want to show: $\lim_{d\to 0} H(\theta_y^*, n(\cdot, \theta_y^*)) = H(\theta_x^*, n(\cdot, \theta_x^*))$. As the state θ increases in $[\theta^* - \varepsilon, \theta^* + \varepsilon]$, the function $n(\theta, \theta^*)$ crosses all jump points $n_1, \ldots n_k$. The according jump states, however, depend on the trigger θ^*: By $\theta_x^* < \theta_y^*$, we have $n(\theta, \theta_x^*) \leq n(\theta, \theta_y^*)$. Because we require for all jump points $j = 1, \ldots k$

$$n(\theta_j^x, \theta_x^*) = n_j = n(\theta_j^y, \theta_x^*), \tag{41}$$

and because $n(\theta, \theta_x^*)$ is increasing in the trigger but decreasing in the state it follows $\theta_j^x < \theta_j^y$ for all $j = 1, \ldots k$. Further, note that $n(\theta_j^x, \theta_x^*) = n(\theta_j^y, \theta_x^*)$ implies that $\theta_x^* - \theta_y^* = \theta_j^x - \theta_j^y$ for all j. That is, $\theta_y^* = \theta_x^* + d$ implies $\theta_j^y = \theta_j^x + d$. Therefore,

$$\begin{aligned}
2\varepsilon\, H(\theta_y^*, n(\cdot, \theta_y^*)) &= \int_{\theta_y^*-\varepsilon}^{\theta_{j_1}^y} \upsilon(\theta, n(\theta, \theta_y^*))d\theta + \cdots + \int_{\theta_{j_k}^y}^{\theta_y^*+\varepsilon} \upsilon(\theta, n(\theta, \theta_y^*))d\theta && (42)\\
&= \int_{\theta_x^*+d-\varepsilon}^{\theta_{j_1}^x+d} \upsilon(\theta, n(\theta, \theta_y^*))d\theta + \cdots + \int_{\theta_{j_k}^x+d}^{\theta_x^*+d+\varepsilon} \upsilon(\theta, n(\theta, \theta_y^*))d\theta && (43)\\
&= \int_{\theta_x^*-\varepsilon}^{\theta_{j_1}^x} \upsilon(\theta+d, n(\theta+d, \theta_y^*))d\theta + \cdots + \int_{\theta_{j_k}^x}^{\theta_x^*+\varepsilon} \upsilon(\theta+d, n(\theta+d, \theta_y^*))d\theta && (44)\\
&= \int_{\theta_x^*-\varepsilon}^{\theta_{j_1}^x} \upsilon(\theta+d, n(\theta, \theta_x^*))d\theta + \cdots + \int_{\theta_{j_k}^x}^{\theta_x^*+\varepsilon} \upsilon(\theta+d, n(\theta, \theta_x^*))d\theta && (45)
\end{aligned}$$

where the last step follows from $n(\theta + d, \theta_y^*) = n(\theta + d, \theta_x^* + d) = n(\theta, \theta_x^*)$. Therefore,

$$\begin{aligned}
&|H(\theta_x^*, n(\cdot, \theta_x^*)) - H(\theta_y^*, n(\cdot, \theta_y^*))| && (46)\\
&= \frac{1}{2\varepsilon} \Big| \int_{\theta_x^*-\varepsilon}^{\theta_{j_1}^x} (\upsilon(\theta, n(\theta, \theta_x^*)) - \upsilon(\theta+d, n(\theta, \theta_x^*)))\, d\theta && (47)\\
&+ \cdots + \int_{\theta_{j_k}^x}^{\theta_x^*+\varepsilon} (\upsilon(\theta, n(\theta, \theta_x^*)) - \upsilon(\theta+d, n(\theta, \theta_x^*)))\, d\theta \Big| && (48)\\
&\leq \frac{1}{2\varepsilon} \Big(\int_{\theta_x^*-\varepsilon}^{\theta_{j_1}^x} |\upsilon(\theta, n(\theta, \theta_x^*)) - \upsilon(\theta+d, n(\theta, \theta_x^*))|\, d\theta && (49)\\
&+ \cdots + \int_{\theta_{j_k}^x}^{\theta_x^*+\varepsilon} |\upsilon(\theta, n(\theta, \theta_x^*)) - \upsilon(\theta+d, n(\theta, \theta_x^*))|\, d\theta \Big) && (50)
\end{aligned}$$

The payoff difference function $\upsilon(\theta, n(\theta, \theta_x^*))$ is continuous between the jump points, implying $\lim_{d\to 0} |\upsilon(\theta, n(\theta, \theta_x^*)) - \upsilon(\theta + d, n(\theta, \theta_x^*))| = 0$. Moreover, the payoff difference function is bounded by assumption 3.1. Thus, $|H(\theta_x^*, n(\cdot, \theta_x^*)) - H(\theta_y^*, n(\cdot, \theta_y^*))| \to 0$ as $d \to 0$ by Lebesgue's dominated convergence theorem. □

Proof. [Proof Lemma 7.2] To show continuity of $H(\theta_i, \tilde{n}(\cdot))$ in signal θ_i, I show that for $h > 0$, $\lim_{h\to 0} |H(\theta_i + h, \tilde{n}(\cdot)) - H(\theta_i, \tilde{n}(\cdot))| = 0$. Observe that for a small $h > 0$, the intervals

$[\theta_i - \varepsilon, \theta_i + \varepsilon]$ and $[\theta_i + h - \varepsilon, \theta_i + h + \varepsilon]$ overlap. Therefore,

$$
\begin{aligned}
&H(\theta_i + h, \tilde{n}(\cdot)) - H(\theta_i, \tilde{n}(\cdot)) \qquad (51)\\
&= \frac{1}{2\varepsilon}\int_0^1 (\mathbf{1}_{\{\theta\in[\theta_i+h-\varepsilon,\theta_i+h+\varepsilon]\}} - \mathbf{1}_{\{\theta\in[\theta_i-\varepsilon,\theta_i+\varepsilon]\}}) \left(\int_0^{n_1} \upsilon(\theta,n)\,dF_\theta(n) + \cdots + \int_{n_k}^1 \upsilon(\theta,n)\,dF_\theta(n)\right) d\theta\\
&= \frac{1}{2\varepsilon}\int_0^1 (\mathbf{1}_{\{\theta\in[\theta_i+\varepsilon,\theta_i+h+\varepsilon]\}} - \mathbf{1}_{\{\theta\in[\theta_i-\varepsilon,\theta_i+h-\varepsilon]\}}) \left(\int_0^{n_1} \upsilon(\theta,n)\,dF_\theta(n) + \cdots + \int_{n_k}^1 \upsilon(\theta,n)\,dF_\theta(n)\right) d\theta
\end{aligned}
$$

where I have used that on $[\theta_i - \varepsilon, \theta_i + \varepsilon] \cap [\theta_i + h - \varepsilon, \theta_i + h + \varepsilon]$ the indicator functions cancel out to zero. For every state θ, the Lebesgue-Stieltjes integrals $\left(\int_0^{n_1} \upsilon(\theta,n)\,dF_\theta(n) + \cdots + \int_{n_k}^1 \upsilon(\theta,n)\,dF_\theta(n)\right)$ exist, that is, are bounded by assumption 3.2. Further, as $h \to 0$, it holds $\mathbf{1}_{\{\theta\in[\theta_i+\varepsilon,\theta_i+h+\varepsilon]\}} \to \mathbf{1}_{\{\theta\in[\theta_i+\varepsilon,\theta_i+\varepsilon]\}} = 0$ almost everywhere. Likewise, for $h \to 0$, $\mathbf{1}_{\{\theta\in[\theta_i-\varepsilon,\theta_i+h-\varepsilon]\}} \to \mathbf{1}_{\{\theta\in[\theta_i-\varepsilon,\theta_i-\varepsilon]\}} = 0$ almost everywhere. Thus, $|\mathbf{1}_{\{\theta\in[\theta_i+\varepsilon,\theta_i+h+\varepsilon]\}} - \mathbf{1}_{\{\theta\in[\theta_i-\varepsilon,\theta_i+h-\varepsilon]\}}| \to 0$, and $|\mathbf{1}_{\{\theta\in[\theta_i+\varepsilon,\theta_i+h+\varepsilon]\}} - \mathbf{1}_{\{\theta\in[\theta_i-\varepsilon,\theta_i+h-\varepsilon]\}}| \leq 1$. Thus, by Lebesgue's dominated convergence theorem, $\lim_{h\to 0} |H(\theta_i + h, \tilde{n}(\cdot)) - H(\theta_i, \tilde{n}(\cdot))| = 0$. □