Opportunity Cost of Capital, Marginal Cost of Funds and Numeraires in Benefit-Cost Analysis

Szekeres, Szabolcs

Independent researcher

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by Szabolcs Szekeres*

Abstract: The choice of social discount rate: social time preference rate (STPR) or social opportunity cost rate (SOCR), a long-standing dilemma, is related to the choice of BCA numeraire. A numerical example examines the consequences of discount rate choices and that of using two proposed methods of reconciling their differences. The role of the Marginal Cost of Funds and the Shadow Price of Capital are considered. While it is possible to reach analogous numerical results in any numeraire using conversion factors, the choice of numeraire should depend on the objective of the BCA performed.

Keywords: Social discount rate; STP discounting; SOC discounting; Descriptive discounting; Prescriptive discounting; Two-rate discounting; Shadow Price of Capital; Numeraires.

JEL classification: D61; H43

1. Introduction

Drèze and Stern (1990) wrote that the social discount rate (SDR) is the rate at which the social value of the numeraire falls over time. “We cannot, therefore, answer the question, ‘What should be the shadow discount rate?’ without being told, or without our choosing, what the numeraire is to be. And the apparent difference between the shadow discount rates proposed in alternative methods of cost-benefit analysis should not mislead us into thinking that the differences are necessarily real – alternative methods may simply involve different units of account.”

The dichotomy between Social Time Preference (STP) discounting and Social Opportunity Cost (SOC) discounting that has been unresolved for so long stems from differing numeraire choices and their consequences. In the consumption numeraire framework, the Social Time Preference Rate (STPR) is taken to correctly measure the rate of fall of the social value of consumption, whereas in the public-funds numeraire framework the Social Opportunity Cost Rate (SOCR) is taken to correctly measure the social cost of public funds. Because discounting both imputes capital costs and sets intertemporal preferences, discounting by the STPR results in undervalued capital costs while doing so with the SOCR results in undervalued future benefits – at least in the eyes of those making the opposite choice.

To explore the assertion of Drèze and Stern that the differences in discounting methods may be more apparent than real, this paper will test the consequences of numeraire choice on the results of analyzing a sample long-lived investment project and examine the performance of two discounting methods that propose to resolve the discount rate choice dilemma by using both the STPR and the SOCR simultaneously: Liu (2003) and Szekeres (2022, 2024).

Because Liu (2003) emphasizes the use of the Marginal Cost of Funds (MCF) correction and the method proposed in Szekeres (2022, 2024) is the equivalent of effecting a project specific Shadow Price of Capital (SPC) correction, the role of these two concepts will be illustrated.

* Independent researcher (https://orcid.org/0000-0003-3903-5377).
szsz@iid.hu
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2. The numerical example

The following illustrative project will be analyzed, with the assumption that STPR =2% and SOCR = 5%:

Table 1
SAMPLE PROJECT BENEFITS AND COSTS

Years	0	100
Direct benefits		15, 000
Indirect revenues		3,000
Operating costs		8,000
Investment	1	
Net flow	–1	10,000

The example is arbitrary, of course, as are all involved parameters. The composition of benefits and costs has been chosen so that all variables that Liu (2003) uses be present in the example. The fact that it has a single future benefit simplifies calculations and the fact that the benefit accrues in Year 100 makes for stark contrasts.

A significant part Liu (2003) is devoted to the estimation of the MCF, which it derives from a labor market model. This paper will not deal with that derivation, however, because we focus instead on the NPV calculation method proposed by Liu (2003), in which the MCF is just a parameter. We assume instead that the MCF is derived from the usual estimation of the dead-weight losses caused by taxation. Based on their survey of the opportunity cost of public funds literature, Massiani and Picco (2013) estimate that "every euro used by public authorities has a welfare cost of 1.20-1.30 euro." On this basis, we assume for our example that MCF = 1.25. This assumption requires the adjustment of the data in Table 1 to account for the effect of the MCF.

Table 2
ADJUSTED SAMPLE PROJECT BENEFITS AND COSTS

Years	0	100
Direct benefits		15,000
Indirect revenues (× MCF)		3,750
Operating costs (× MCF)		10,000
Investment	1	
MCF effect on investments (MCF–1)	0.25	
Net flow	–1.25	8,750

Because Liu (2003) assumes that indirect revenues accrue to the public sector, these are multiplied by the MCF. Likewise, investment and operating costs are multiplied by the MCF, as Liu (2003) assumes these to be public sector expenditures. The adjustment of the investment by the MCF is shown separately from the amount of investment because the implicit welfare cost is incurred when funds are raised, but it is not an expenditure that needs financing.

3. The two roles of discounting

Analyzing this project in the public funds numeraire context, we discount the project net flow of Table 2 by the SOCR to obtain NPV = $-1.25 + 8{,}750 / (1.05)^{100} = 65.289$

Doing so in the consumption numeraire context requires discounting by the STPR to obtain a significantly higher result: NPV = – 1.25 + 8,750 / $(1.02)^{100}$ = 1,206.538.

The difference is due to the fact that the SOCR implies a higher cost of capital, and also a lower present value of the after-capital-costs net benefits. Table 3 separates these two effects. It shows the net flow of the project in Year 100, the opportunity cost of capital in the same year, and the net benefits after capital costs. With the STPR the opportunity cost of capital is 1.02^{100} = 7.245, while with the SOCR it is 1.05^{100} = 131.501.

Table 3
FUTURE VALUES AND PRESENT VALUES
AS A FUNCTION OF THE DISCOUNT RATE

	STPR	SOCR
Net Flow in Year 100	8,750.000	8,750.000
Opportunity cost of capital in Year 100	7.245	131.501
Net benefit after costs of capital in Year 100	8,742.755	8,618.499
Present value of net benefits	1,206.788	65.539
NPV (–0.25, the Year 0 MCF adjustment of the above)	1,206.538	65.289

By subtracting the opportunity costs of capital from the net flows (already net of operating costs) we obtain net benefits after costs of capital. Discounting these values at the corresponding discount rates we get the present value of future net benefits. Subtracting from that the MCF effect accruing in Year 0 we obtain the NPVs of the project, which are the same as the values obtained by simply discounting the net flow at the alternative rates.

It is instructive to note that the difference in opportunity costs of capital derives not from the investment amount of \$1, which is the same in both cases, but from the cost of keeping a stock of capital of \$1 immobilized for 100 years. In the first case it costs 2% per year, in the second it costs 5%. Table 3 illustrates that discounting always attributes capital costs corresponding to the discount rate used and that it subsequently discounts the remaining net benefits with the intertemporal weight implicit in the same rate.

Spackman (2020) reported that several authors sustain that “taxes on consumption impose no significant further social cost,” which implies that once the cost of taxation has been reflected by the MCF, it is no longer necessary to consider the SOCR, which is the shadow price at which capital is costed. But as argued in Szekeres (2024) and evident from the presented numerical example, the MCF is a correction to flows, whereas the SOCR is used to quantify the cost of capital stock being immobilized, so the two measure different things that are in no way overlapping. They couldn’t possibly be, on account of their different dimensionalities. The MCF is a multiplicative factor to be applied to flows, whereas the SOCR is a rate over time to be applied to each year’s stock of capital.

Even though the opportunity cost of capital computed at the SOCR is about 18 times as high as when computed with the STPR, it still only takes about 2% out of the project’s net flow, because this sample project is not capital intensive. The large difference in NPVs between the two cases lies not in the cost of capital, therefore, but in the difference that the choice of discount rate makes in terms of intertemporal weighing.

4. Szekeres two-rate discounting

The insight that the two functions of discounting can be performed separately (Szekeres, 2020, 2024) largely reconciles the STP and SOC discounting approaches by (1) using the SOCR to explicitly calculate the opportunity cost of capital of projects and then (2) discounting the after-capital-costs net flows by the STPR. In this way the chief requirements of both discounting approaches are simultaneously fulfilled, because the opportunity cost of capital is computed as in the public funds numeraire but the remaining net project benefits are discounted as in the consumption numeraire.

Table 4
SZEKERES TWO-RATE DISCOUNTING
OF THE SAMPLE PROJECT

Values in Year 100, except for present values	SOCR, STPR
Net Flow	8,750.000
Opportunity cost of capital (@ SOCR)	131.501
Net benefit after costs of capital	8,618.499
Present value of net benefits	1,189.637
NPV (after the Year 0 MCF adjustment of the above)	1,189.387

An important consequence of this is that the SOCR becomes the project feasibility hurdle rate (if the opportunity cost of capital were higher than net benefits, the NPV would be negative), so there will be no dispute concerning which projects are economically feasible. However, differences will remain because their ranking will depend on intertemporal weighing.

Szekeres two-rate discounting is equivalent to conventional discounting after capital has been appropriately shadow priced. The present value at the STPR of the opportunity cost of capital is the shadow priced value of capital, in this case $131.501 / 1.02^{100} = 18.152$. Shadow pricing capital (SPC) is achieved through replacing the invested amount by the shadow priced value of capital. As explained in Szekeres (2024), this adjustment must be project specific, to reflect the actual capital usage of projects. With this adjustment, the net flow in Table 2 becomes the following.

Table 5
SAMPLE PROJECT NET FLOW
AFTER SPC CORRECTION

Years	0	100
Net flow	–18.402	8,750

The Year 0 value is the negative of the PV of the opportunity cost of capital ($131.501/1.02^{100} = 18.152$) less the MCF adjustment of 0.25 (–18.402–0.25 = –18.402). The conventional NPV of this flow is $-18.402 + 8{,}750 / 1.02^{100} = 1{,}189.387$, the same as with Szekeres two-rate discounting.

It should be clear that conventional STPR discounting without appropriate SPC correction overstates the NPV of projects.

5. Liu two-rate discounting

In discussing how to deal with the discount rate choice Liu (2003) wrote: “We should use both market rates in a more direct way. The gross rate representing the opportunity cost of government revenues [assumed to equal the SOCR in this paper] should be used for the discounting of project costs while the net rate representing the valuation of consumers [assumed to equal the STPR in this paper] should be used for the discounting of consumer benefit.”

Liu's proposed discounting method is as follows: "the MCF approach to multi-period project evaluation consists of the following components. (i) A project should be represented as a stream of direct investments [interpreted by Liu (2003) to include all government outlays, that is investments and operating costs], a stream of direct benefits measured as contemporaneous willingness to pay and a stream of indirect revenue benefits; (ii) future project direct benefits should be discounted at the net rate of return while future project costs, including indirect revenue benefits as negative costs, should be discounted at the gross rate of return; (iii) the present value of net costs should be multiplied by the marginal cost of funds before being compared to the present value of the direct benefits."

The above quote describes the following project feasibility criterion proposed by Liu (2003, expression 15):

$$\sum_{t=0}^{\infty}\frac{B_t}{(1+r_n)^t} - MCF\sum_{t=0}^{\infty}\frac{\Delta I_t - \Delta R_t}{(1+r_g)^t} > 0 \tag{1}$$

where B_t stands for benefits, ΔI_t for both investments and operating costs and ΔR_t for revenues.

Because in Table 2 we have already applied the MCF adjustment to obtain an adjusted net flow, we can rewrite expression (1) to recognize this fact. Thus $I_t = \Delta I_t \times MCF$ and $R_t = \Delta R_t \times MCF$. To accommodate expression (1) to the categories in Table 2 we decompose I_t into K_t and O_t, capital and operating costs, respectively, to obtain the following expression that already includes the MCF adjustment:

$$\text{NPV} = \sum_{t=0}^{\infty}\frac{B_t}{(1+r_n)^t} - \sum_{t=0}^{\infty}\frac{(K_t+O_t)-R_t}{(1+r_g)^t} \tag{2}$$

To calculate the Liu NPV we can eliminate the summation signs, given that we only have two time periods in our example. Recognizing that values in the present are not discounted and taking r_n = 2% and r_g = 5%, we have the following result:

$$NPV = \frac{B}{1.02^{100}} - K - \frac{O-R}{1.05^{100}} = \frac{15{,}000}{1.02^{100}} - 1.25 - \frac{10{,}000-3{,}750}{1.05^{100}} = 2{,}021.716 \tag{3}$$

This result is higher than that obtained by any of the other methods considered so far. To explore the source of the differences in computed NPVs between the Liu method and the rest, Table 6 shows the PVs and FVs of the adjusted items in Table 2.

Table 6

PRESENT AND FUTURE VALUES ACCORDING TO THE FOUR DISCOUNTING METHODS

	SOC		STP	
	PV	FV	PV	FV
Direct benefits	114.067	15,000.000	2,070.495	15,000.000
Indirect revenues (× MCF)	28.517	3,750.000	517.624	3,750.000
Operating costs (× MCF)	76.045	10,000.000	1,380.330	10,000.000
Investment	1.000	131.501	1.000	7.245
Net benefits after all cots	65.539	8,618.499	1,206.788	8,742.755
NPV after Year 0 MCF adjustment	65.289	5.0%	1,206.538	2.0%

	Szekeres		Liu	
	PV	FV	PV	FV
Direct benefits	2,070.495	15,000.000	2,070.495	15,000.000

Indirect revenues (× MCF)	517.624	3,750.000	28.517	3,750.000
Operating costs (× MCF)	1,380.330	10,000.000	76.045	10,000.000
Investment	18.152	131.501	1.000	131.501
Net benefits after all cots	1,189.637	8,618.499	2,021.966	8,618.499
NPV after Year 0 MCF adjustment	1,189.387	2.0%	2,021.716	1.5%

In all cases shown in Table 6, the FV of direct benefits, indirect revenues and operating costs are simply the adjusted Year 100 values from Table 2. For the FV of investment, we have the opportunity cost of capital as specified by each method: $(1+\text{STPR})^{100}$ in the case of STP discounting and $(1+\text{SOCR})^{100}$ in all other cases. The PV of these items varies by method. SOC discounting discounts all items at the SOCR, STP discounting and Szekeres two-rate discounting do so at the STPR and Liu two-rate discounting discounts direct benefits at the STPR and all else at the SOCR.

Table 7 shows the proportion of direct benefits that opportunity cost of capital and operating costs represent, in present value terms, for each of the discounting methods, as computed from the data in Table 6. Also shown in this table is the effective discount rate that each method uses, computed as the IRR between the FV of net benefits after all costs and the NPV computed by each method.

Table 7

COST ITEMS IN PROPORTION OF DIRECT BENEFITS
AND EFFECTIVE DISCOUNT RATE

	SOC	STP	Szekeres	Liu
Operating costs	53%	53%	53%	3.6%
Opportunity cost of capital	0.88%	0.05%	0.88%	0.05%
Effective discount rate	5%	2.0%	2.0%	1.5%

We can see from the above Tables that the Liu method seriously understates all cost items. The reason is that it uses two numeraires simultaneously, which leads to consequences that Liu (2003) probably did not intend. What Liu two-rate discounting does in effect is to capitalize operating costs using the SOCR and then uses the STPR to discount the resulting net flow. This method overstates the project's net operating benefits by understating operating costs relative to benefits, and it implicitly equates the project's opportunity cost of capital to the STPR, contrary to its stated intention to let the SOCR define it. More specifically:

- Discounting operating costs at a rate that differs from that used for benefits is problematic conceptually, because operating costs are a measure of foregone consumption, and therefore should be valued on the same basis as consumption. The fact that in this case the operating costs are public expenses and therefore should be corrected by the MCF factor does not require them to be discounted at a different rate. By discounting costs at a higher rate than benefits are discounted at, the Liu two-rate method overestimates net operating benefits, which is one of the reasons why it yields the highest computed NPV of all the reviewed discounting methods.
- Liu (2003) stated that "future project costs [which includes investments], including indirect revenue benefits as negative costs, should be discounted at the gross rate of return." This reveals the intention of using the SOCR to measure the opportunity cost of capital. However, discounting benefits at the STPR effectively results in discounting the initial investment of Year 0 at the STPR, not the SOCR. This is confirmed in Table 7, which shows that the opportunity cost of capital in Liu two-rate discounting is the same as in STP discounting. It is easy to see why this is so directly. Take a project with only benefits and capital costs. Its Liu two-rate NPV would equal $B/(1+\text{STPR})^{100}-K$, which means that the discount rate being used is the STPR, not the SOCR. It is benefits that need to be discounted at the cost of capital, not the value of the investments, to impute the correct

capital costs and thereby arrive at the correct value of net benefits after capital costs. In Liu two-rate discounting, as far as the investment in Year 0 is concerned, the discount rate used is the STPR, not the SOCR. This is the second reason why this method yields the highest computed NPV.

The combined effect of these two underestimations is that the effective discount rate that the Liu two-rate method uses is 1.5% (See Table 7), whereas it is 5% for SOC discounting and 2% for STP discounting as well as for the Szekeres two-rate method. Changing the value of the MCF does not solve the problem. If MCF = 2.5, operating costs continue to be significantly undervalued and the effective discount rate becomes an even lower 0.18%.

For these reasons, the use of Liu (2003) two-rate discounting method is not recommendable for use in BCA. Reducing the direct benefits of the sample project to 354 would result in a Liu NPV of 0.086 despite net benefits becoming –6,027.501 in Year 100. All the other methods analyzed would show the modified project to be unfeasible by computing negative NPVs.

This serious underestimation of project costs results from the direct addition of present values computed using different discount rates, that is, from using two numeraires at once without adequate conversion. Liu's (2003) statement that "the present value of net costs should be multiplied by the marginal cost of funds before being compared to the present value of the direct benefits" is insufficient to effect a conversion between numeraires, for it only corrects the relative dimensions of the unadjusted values through the MCF adjustment. Multiplication by the MCF is simply the addition of a welfare cost of (MCF-1) for each $1 of affected cost[1], but it does nothing to address the differences in numeraire, a subject to which we turn in the next section.

6. Changing numeraires

Because different numeraires have different rates of fall, converting a value from one numeraire to another requires compensating for the difference in discount rates if the objective is to preserve the PV of the affected value.

The after-capital-cost benefits of the STP discounting case can be seen in the third line of the first column of Table 3. They equal 8,742.755. If we want to convert this value to the public funds numeraire and preserve its present value after discounting by the SOCR, we must multiply it by a conversion factor equal to $((1+\text{SOCR})/(1+\text{STPR}))^{100}$, which with our values is $(1.05/1.02)^{100} = 18.152$. Multiplying remaining benefits of 8,742.755 by this factor equals 158,694.201, which discounted at the SOCR of 5% and correcting for the Year 0 MCF effect gives an NPV = 1,206.538, the same as we obtained by STP discounting, see the last line of Table 3.

The correction also works in the opposite direction. Should we believe that the correct value of future benefits is that which SOC discounting attributes to it, we can translate this value to the consumption numeraire. If we divide the after-capital-cost benefits of the SOC discounting case, 8,618.499, from the third line of the last column of Table 3, by the same correction factor we get 474.809, which discounted at the STPR of 2% and correcting for the Year 0 MCF effect gives NPV = 65.289, the same as that obtained by SOC discounting, see the last line of Table 3.

If we consider that one value is correctly estimated under one numeraire and the other value is correct under the other numeraire, then with the appropriate conversion the desired result can be reached under either numeraire. Szekeres two-rate discounting combines the cost of capital of the public funds numeraire

[1] The added welfare cost is not monetary, so if Liu's intention was to segregate items that affect the government budget, the welfare losses measured by the MCF should have been added the consumption numeraire part of his analysis, rather than to the public funds one, because they are neither paid for nor received by the government.

with the intertemporal valuation of the consumption numeraire. Notice that in the case of this example the numeraire conversion factor and the shadow price of capital factor are the same, so the shadow price of capital adjustment can be viewed as an instance of value preserving numeraire conversion of the opportunity cost of capital.

We can use the calculated numeraire conversion factors to eliminate the heterogeneity of Liu two-rate discounting. This is done in Table 8.

Table 8

CORRECTION OF LIU TWO-RATE NPV BY CONVERSION TO A SINGLE NUMERAIRE

	NPV of benefits	Undiscounted values	NPV of all else	SUM	Note
Liu NPVs	2,070.495	–1.25	–47.528	2,021.716	From expression (3)
Conversion to consumption numeraire	2,070.495	–1.25	–862.706	**1,206.538**	Result in consumption numeraire
Conversion to public funds numeraire	114.067	–1.25	–47.528	**65.289**	Result in public funds numeraire

In line 1 of Table 8 we have the Liu two-rate NPV components, as calculated by expression (3). In the first column we have the NPV of benefits. In the second we have the MCF adjusted investment, which is not discounted because it is in the present. In the third column we have the NPV of indirect revenues and operating costs. In the fourth column we have the sum of the values to the left. In the case of the first line, we see the result obtained by expression (3), the heterogeneous NPV that Liu two-rate discounting computes.

In line 2 we have the conversion of the Liu results to the consumption numeraire. The value in column 1 remains unchanged, as it is already in the target numeraire, but the value in column 3 must be converted by multiplication of the conversion factor. That is: –47.528 × 18.152 = –862.706. As now all numbers in line 2 are in the consumption numeraire, they can be added safely. The result is 1,206.538, identical to the conventional STP discounting value of the adjusted net flow of Table 2.

In row 3 we have the conversion of the Liu results to the public funds numeraire. In this case it is the value in the first column that gets converted: 2,070.495 / 18.152 = 114.067 while the remaining ones remain unchanged. The sum in this case is 65.289, which is the value that was obtained by conventional SOC discounting of the adjusted net flow of Table 2. This is a correspondence that Burgess (2013) also remarked upon, albeit through a different line of reasoning.

The preceding shows that Liu two-rate discounting, when corrected, is not really a two-rate discounting method because it becomes either STP or SOC single-rate discounting, depending on the direction of the correction.

7. Choice of discount rate

The numerical example used highlights the stunning difference in the valuation of the future that the choice of numeraire entails. Benefits in Year 100 are valued about 18 times higher in the consumption numeraire than in the public funds numeraire, a fact that partly explains the controversy that surrounds the choice of discount rate.

As Szekeres (2024) explains, once STP discounting is corrected through Szekeres two-rate discounting, the pool of feasible projects will be the same independently of which numeraire is used in their analysis, because either way the hurdle rate of return is the SOCR. The only difference that the choice of numeraire will then make is in the ranking of projects. As all feasible projects cover their capital costs in full, no choice would cause a direct welfare loss, and the selection between them will only be a function of intertemporal preferences.

The choice of discount rate should depend on the objective of the BCA, therefore.

- SOC discounting should be used if the objective is to maximize the present value of future benefits in allocating public funds.
- STP discounting (appropriately corrected) should be used if the objective is to measure the welfare impact of projects as defined by the adopted social intertemporal preference rate.

Those opting for the second choice should be aware of two facts:

- Only in the case of SOC discounting is it true that the computed NPV is the amount of money that will compound back to the equivalents of project benefits at the SOCR, which is the cost of funds of the public sector[2].
- The fact that NPVs computed in the consumption numeraire are higher than those computed in the public funds numeraire means that there are cheaper ways of matching project benefits through investments in the capital market.

8. Conclusions

Drèze and Stern (1990) were correct in stating that it is possible to select arbitrarily the numeraire of the analysis, but the conversion from one to the other via conversion factors cannot be performed mechanically because discounting imputes the capital costs implicit in the rate of fall of the numeraire and these differ. While it is possible to use any numeraire and reach conclusions deemed appropriate using conversion factors, the choice of numeraire is not immaterial, and its discount rate is in fact the main reason why a numeraire is chosen. The choice should correspond to the objectives of the BCA being performed: SOC discounting for a budget allocation that maximizes the present value of attainable benefits, and STP discounting to measure perceived welfare impact.

Liu two-rate discounting, as defined by his computational expression (1), computes an invalid mix of heterogeneous NPVs and is therefore not useful in BCA. If its heterogeneity is corrected, then it becomes either STP discounting or SOC discounting, depending on the direction of the correction, and is then no longer two-rate discounting.

By making the cost of capital explicit, Szekeres two-rate discounting allows treating capital like any other project input, to be valued by its shadow price, the SOCR. After having accounted for capital costs explicitly, discounting no longer computes capital costs, it just defines intertemporal weighting. Consequently, Szekeres two-rate discounting will work even if the opportunity cost of capital happened to be different from the rate of fall of any of the plausible numeraires that might be used.

REFERENCES

Burgess, David (2012) “Reconciling alternative views about the appropriate social discount rate.” *Journal of Public Economics* 97 (2013) 9–17 http://dx.doi.org/10.1016/j.jpubeco.2012.08.009

[2] While the SOCR measures the welfare cost of using capital, it is also equal to the financial cost to national treasuries. This was demonstrated with a numerical example by Harberger (2007), asserted by Burguess and Zerbe (2013) and experimentally demonstrated in a capital market model by Szekeres (2022).

Burgess, David, and Richard Zerbe. (2013) “The Most Appropriate Discount Rate.” *Journal of Benefit-Cost Analysis*, 4(3): 391–400. https://doi.org/10.2202/2152-2812.1065

Drèze, Jean and Nicholas Stern (1990) “Policy reform, shadow prices, and market prices,” *Journal of Public Economics*, Volume 42, Issue 1, 1990, Pages 1-45,ISSN 0047-2727, https://doi.org/10.1016/0047-2727(90)90042-G.

Harberger, A. C. (2007) “Suggested New Steps Toward the Practical Implementation of Cost-Benefit Analysis.” Conference on Cost-Benefit Analysis, University of Washington, Seattle.

Liu, Liqun (2003) A Marginal Cost of Funds Approach to Multi-Period Public Project Evaluation: Implications for the Social Discount Rate, *Journal of Public Economics*, Volume 87, Issues 7–8, 2003,Pages 1707-1718, ISSN 0047-2727, https://doi.org/10.1016/S0047-2727(01)00179-7

Massiani J., Picco, G., (2013), “The opportunity costs of public funding: concept and issues,” *Public Budgeting & Finance,* 33(3), pagg.96–114. https://doi.org/10.1111/j.1540-5850.2013.12016.x

Spackman, Michael. (2020) “Social Discounting and the Cost of Public Funds: A Practitioner’s Perspective.” *Journal of Benefit Cost Analysis*; 11(2):244–271. https://doi:10.1017/bca.2020.5

Szekeres, Szabolcs (2022) “Answering the Social Discount Rate Question,” MPRA Paper No. 115848. https://mpra.ub.uni-muenchen.de/117718/

Szekeres, Szabolcs (2024) “Resolving the Discounting Dilemma,” MPRA Paper No. 120014. https://mpra.ub.uni-muenchen.de/120014/