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Abstract

Auction mechanisms commonly used in practice for awarding infrastructure concession contracts induce a bias towards the selection of concessionaires who are optimistic about demand, but are not necessarily cost-efficient. This helps to explain the frequent renegotiation of concessions observed in practice. This paper shows that the fixed-term nature of contracts is the key element for selection errors, and it proposes a better alternative mechanism based on flexible-term contracts. This new auction mechanism reduces the probability of selection errors and contract renegotiation, and it is simple enough to constitute a good option for concessions in sectors like transport and public utilities.

Keywords: concessions, auctions, renegotiation, infrastructure
JEL codes: D44, L91

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1. Introduction

The provision of basic infrastructure, like railways, bridges or seaports, has been traditionally a responsibility assumed by governments. However, since the late 1980's, there is a worldwide trend towards a more active participation of private investors in the construction and financing of large infrastructure projects. This is specially relevant for developing countries[1], but it is increasingly observed also in developed economies.

A popular model for the participation of private capital in infrastructure is the concession contract. This is an agreement between a government and a private firm –generally a consortium formed of several parties– for the construction or major rehabilitation of some infrastructure. Private investors assume all costs related to the project, and in return they obtain the right to operate the assets (power plants, airports, roads, and so forth) during some pre-specified period. Investment costs are thus recovered from charges or tolls on infrastructure's users. Governments retain ownership and control over the assets and may regulate prices and quality.

The mechanism for selecting concessionaires to implement infrastructure projects is usually a sealed-envelope type of auction. In the area of governments' contracts with private firms, the literature on auctions has devoted a great deal of attention to the analysis of competitive bidding for procurement contracts (Stark, 1971; Holt, 1980; Porter and Zona, 1993; Jofre-Bonet and Pesendorfer, 2000), and to study the problem of the winner's curse (for a general discussion, see Milgrom, 1989; for an application to the road sector, see Thiel, 1988; Levin and Smith, 1990).

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[1] A database on concessions for infrastructure projects signed during the 1990s in developing countries, compiled by the World Bank, reports the existence of 700 contracts. By sector, 45% of these contracts are transport projects, 25% water, 20% electricity, and 10% in telecommunications. By region, 60% are located in Latin America, 20% in Asia, and 20% in Eastern Europe.

Less attention has received the particular context of concession contracts, although there are many interesting questions about firms' selection and ex-post contract renegotiation[2]. However, this is an area where research on auctions is probably most valuable for governments, as stressed by Klemperer (1999). Infrastructure projects have long economic lives –generally above 25 years– and therefore are built under high uncertainty about costs and future levels of demand. Therefore, there are two dimensions to be considered: asymmetric information between governments and firms regarding costs; and different firms' beliefs about the future demand for the infrastructure. In the terminology of auction theory, governments are faced with a problem that combines elements both from a 'private-value' model (regarding costs), and a 'common-value' type of situation (related to the uncertainty of demand). Selection of the firm with lowest feasible cost, and minimisation of risks introduced by demand fluctuations, should be the main targets that an auction mechanism should pursue in this context. In particular, the risk of demand fluctuations is one of the main causes of difficulties for infrastructure projects. There are many international experiences about concessions entering into difficulties due to deviations between forecasted and actual levels of demand[3]. On the other hand, a reduction of the risk of demand may contribute to decrease concessionaires' costs, through lower risk premia associated to the cost of capital.

The frequent observed renegotiation of concession contracts and the need to bail out concessionaires indicate that selection mechanisms may be failing to achieve those objectives. Furthermore, there is evidence that the type of auction used to select a concessionaire might have an impact on the future performance of an infrastructure project. Using data from a large sample (see footnote 1),

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[2] Riordan and Sappington (1987), is one of the few contributions to the analysis of the problem of concessionaires' selection. Tirole (1986) presents a model of renegotiation between a government and a firm contracted for some procurement task, though it does not explicitly deal with the preceding stage of contract tendering.

[3] One of the more dramatic examples was a road program implemented in Mexico during the 1990s (52 concessions to build around 6,000 km of new highways). Actual levels of demand were on average 68 percent below expectations, and for 16 concessions traffic was below 50 percent of forecasted levels. In 1997, the Mexican government was forced to recover 25 of these concessions, assuming debts for US$ 7.7 billion. Besides, losses for private investors were estimated in US$ 3 billion (Fishbein and Babbar, 1996; Gómez-Ibáñez, 1997).

Guasch (2000) reports that 65 percent of all concession contracts are renegotiated. Interestingly, the probability of renegotiation is higher when the concessionaire is selected through an auction based on offers for minimum price (92 per cent of contracts renegotiated), than when a maximum payment is used as the bidding variable (29 per cent).

The objective of this paper is to show that auction mechanisms generally used in practice for awarding infrastructure concession contracts are far from being optimal, and to propose a better alternative mechanism. It is formally proved how traditional auctions based on bids for minimum prices or maximum payments do not generally select the most cost-efficient candidate. On the contrary, they introduce a bias towards the selection of concessionaires who are optimistic about future demand levels, which is likely to be one of the main reasons behind the frequent renegotiation of contracts. The fixed-term nature of traditional contracts is the basic element that induces selection errors, so the alternative proposed is to use a mechanism based on flexible-term contracts.

The structure of the paper is as follows. Section 2 presents the model used for the analysis of auctions. Section 3 uses a basic scenario with private values regarding construction costs, and presents the concept of flexible-term concession contracts. Section 4 introduces maintenance and operation costs, assumed to be equal across firms, to evaluate the impact of these costs on outcomes. Section 5 considers variability on construction and maintenance costs across firms, and discusses in detail the proposed new auction mechanism for the award of flexible-term concession contracts. Section 6 summarises the main results and concludes the paper.

2. A model to analyse auctions for infrastructure concessions

A simple framework is proposed to analyse auctions for the award of concession contracts. The number of users Q who will demand services from a new infrastructure to be built[4] is unknown at the moment of drafting the contract. All users are equal, and each of them demands one unit of service from the infrastructure. The only available information about demand is a range of feasible values [Q_{min} , Q_{max}.]. In order to represent a situation of maximum uncertainty about future demand, it is considered that Q is uniformly distributed over that range.. Once that uncertainty about the state of nature is resolved (the infrastructure is built and open to the public), demand is found to take a particular value Q^*. This level is considered to be constant over time, i.e. there will be Q^* users each year during the whole life of the concession.

There are N firms bidding for the contract, which are different in terms of cost efficiency. Two types of costs are considered: construction costs (I_i , $\underline{i}$ = 1,.., N), which represent all investments required before the infrastructure is operative; and maintenance and operation costs (M_i , i = 1,.., N), which are annual fixed expenses in which the concessionaire incurs for the provision of services (equipment and personnel) and to maintain the infrastructure (repairs, periodic revisions, and so forth). M_i is assumed to be constant over time, and does not vary with the level of demand. The reason is that only maintenance and operation costs of fixed nature are relevant for the analysis. All other variable costs that depend on Q are formally equivalent to a reduction in the price P paid by each user.

As indicated by notation, it is assumed that there are different firms' types regarding both construction costs (I_i) and maintenance and operation costs (M_i). This assumption is justified by the fact that firms' managers have some degree of control over firms' costs, because they might have different skills, or may exert more or less effort in reducing costs. In particular, it is assumed that I_i is drawn from a uniform distribution with support $[I_{min}, I_{max}]$, and M_i from another independent uniform distribution over $[M_{min}, M_{max}]$.

For fixed-term concession contracts, the total number of years during which the concessionaire operates the infrastructure is predetermined by the government, and denoted as T. The concessionaire knows that, once the level of demand Q^* is revealed, it will receive a total revenue equal to PQ^*T during the life of the concession. The price P may be set by the government or, in the case of an auction based on price offers, it will be the price bid for by the winner. It is considered that, apart from investment and maintenance costs, the concessionaire must make a lump-sum payment Z to the government. This payment is made at the beginning of the concession, and it could take negative values (a subsidy is demanded to operate the concession), or zero (no payment is required). In some cases, Z might be the bidding variable at the auction, while in others it is pre-specified by the government.

The project is assumed to be built in year t=0, when the concessionaire makes all investments I_i, and it starts receiving users in year t=1, when the firm initiates the collection of revenues through tolls or tariffs, and incurring into maintenance and operation costs. Without loss of generality, the discount rate is assumed to be equal to one, since considering a discount rate $\delta < 1$ does not alter the basic results of the paper. All firms are considered to be risk-neutral and to maximise expected profits for the whole life of the concession. For a given belief on future demand Q^e, expected profits for firm i are equal to $\Pi_i^e = (P\, Q^e - M_i)\, T - I_i - Z$.

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[4] Throughout the paper, all discussions focus on a case where the infrastructure to be built is new, but the same arguments apply to a project to enhance or rehabilitate existing assets (road enlargements, second runways at airports, improvement of power transmission lines, and so forth). Demand uncertainty, however, will be generally lower for

Several alternatives exist for the design of auctions to tender concession contracts. The most common are to invite offers for prices, awarding the contract to the lowest price; or offers for fee payments to the government, where the highest proposed fee wins. These are the two mechanisms analysed in this paper. Auctions for concessions are generally open only to those firms who pass a preliminary filtering process. Other auctions less frequently observed in practice are based on bids for investments, volumes of service, or duration of the contract. In some cases, several bidding variables can be used simultaneously, with some weighted criterion to select the auction's winner.

3. Scenario with no maintenance costs: fixed-term and flexible-term contracts

Using the proposed framework, it is possible to study the optimal strategies pursued by firms competing for a concession contract at an auction, and the outcomes that are obtained. We will consider initially a simple scenario with no maintenance costs ($M_i = 0$). The two most common types of auctions to award contracts (minimum price or maximum payment) are examined and compared, and then it is shown how an auction to award a flexible-term concession contract outperforms traditional auctions.

3.1 Minimum price auction

Consider an auction at which candidates must present sealed-envelopes with their bids for the price that they propose to charge users for the use of infrastructure (P_i). Envelopes are opened simultaneously and the winner is the firm with the lowest price. No possible modification or negotiation over the proposed price is permitted after the auction is resolved. The contract term T is set by the government and known in advance by bidders.

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rehabilitation than for greenfield projects.

Assume that all candidates share a common belief Q^e about the expected level of demand (this assumption will be later relaxed). This could be the case for example if the government provides results from demand studies in preparation for the auction, and participants do not invest to have other alternative estimates.

Consider the best strategy to be pursued by a firm with construction cost I_i. In general, bids will be based on firms' types (i.e. on their relative cost efficiency), so it is assumed that the price offered is a function of I_i. We then search for symmetric equilibria, where all firms use the same rule to calculate their bids. Thus, the price offered by firm i will be $P_i = P(I_i)$, with $P' > 0$. Using the known probability distribution of construction costs, it is possible for any firm i to compute the probability of winning the auction with a particular offer P_i. Comparing i's bid with that of any other firm j, firm i wins if $P_i < P_j$, or, applying the inverse function $P^{-1}(\cdot)$, if $P^{-1}(P_i) < P^{-1}(P_j) = I_j$. Therefore, by choosing carefully its bid, firm i knows that it would win the contract against any firm j with the same probability of the event $[I_j > P^{-1}(P_i)]$. Since there are N-1 rivals at the auction, the probability for firm i to win the contract with a bid P_i is equal to:

$$prob_i = \left(prob\left[I_j > P^{-1}(P_i)\right]\right)^{N-1} = \left(I_{max} - I_{min}\right)^{1-N}\left(I_{max} - P^{-1}(P_i)\right)^{N-1} \quad (1)$$

All firms are risk neutral, so each of them will calculate its optimal bid by maximising expected profits, given the probability of being selected as concessionaire as indicated by (1). When all firms share a common belief Q^e about the future level of demand, the problem that firm i solves to compute its optimal bid P_i can be expressed as:

$$Max_{P_i}\ \Pi_i^e = \left(P_i Q^e T - I_i - Z\right)\left(I_{max} - P^{-1}(P_i)\right)^{N-1} I_r^{1-N} \quad (2)$$

where $I_r = I_{max} - I_{min}$ is a constant that can be ignored. The first order condition of problem (2) is a differential equation in terms of the optimal function $P(\cdot)$:

$$P'(I_i) - \frac{N-1}{I_{max} - I_i} P(I_i) \;=\; -\frac{(N-1)\,(I_i + Z)}{Q^e T\;(I_{max} - I_i)} \tag{3}$$

Due to the uniform distribution assumed for construction costs I_i, the solution to equation (3) turns out to be a linear function in terms of I_i and other parameters:

$$P_i(I_i) = \frac{(I_i + Z)}{Q^e T} + \frac{I_{max} - I_i}{N\,Q^e T} \tag{4}$$

These optimal bids P_i exhibit some interesting properties. First, when all firms share the same expected demand Q^e, it is clear that the mechanism selects the most efficient firm, because the candidate with the lowest construction cost wins the contract. Second, the price offered allows the concessionaire to obtain some positive profits, given by the second term of the right-hand side of expression (4). Whether firm i to offer a price $P_i = (I_i + Z)/\,Q^e\,T$, its expected profit would be zero, therefore the term $(I_{max}\text{-}I_i)/N$ indicates the information rent that the winner extracts. This rent decreases with the number N of firms participating at the auction, it is zero for the least efficient firm with $I_i = I_{max}$, and it reaches a maximum for the most efficient firm.

3.2 Maximum payment auction

A second traditional model to award concession contracts is an auction at which firms make offers for a payment Z to the government, and the contract is won by the highest bid. These payments Z could even be negative, in which case candidates demand subsidies for the construction of the project, and the contract is awarded to the firm demanding the lowest subsidy (highest Z in that context). In this type of auction, the price P is set by the government, as well as the duration T of the contract.

As in the previous case, each firm will calculate its bid Z_i as a function of its type I_i, and again we search for a symmetric equilibrium in which all candidates use the same function $Z(\cdot)$, with $Z'(\cdot) <0$, to compute their bids Z_i. The probability of firm *i* winning the auction is now the probability of the event $Z_i > Z_j$, for any *j* other than *i*. In terms of construction costs, this can be expressed as:

$$prob_i = \left(prob\left[Z_i > Z_j \right] \right)^{N-1} = \left(prob\left[Z^{-1}(Z_i) < I_j \right] \right)^{N-1} = \left(I_{max} - Z^{-1}(Z_i) \right)^{N-1} I_r^{\,1-N} \qquad (5)$$

Optimal strategies are then derived from the maximisation of expected profits:

$$Max_{Z_i}\ \Pi_i^{\,e} = \left(P\,Q^e\,T - I_i - Z_i \right)\left(I_{max} - Z^{-1}(Z_i) \right)^{N-1} I_r^{\,1-N} \qquad (6)$$

The first order condition of problem (6) is a differential equation similar to (3), which yields:

$$Z_i\,(I_i) = (P\,Q^e T - I_i) - \frac{I_{max} - I_i}{N} \qquad (7)$$

Optimal strategies on payments Z_i share the same properties of P_i. Namely, if firms had the same beliefs about future demand, the mechanism would select the most efficient firm in terms of construction costs. The concessionaire is able to extract some informational rent, because the proposed payment to the goverment yields a positive expected profit equal to $(I_{max}$-$I_i)/N$. It is worth noticing that this rent is exactly the same that the winner of the price auction obtains, which is a revenue-equivalence result in the context of this model.[5]

This result has some practical lessons. It tells a government that the form used to auction a concession contract is irrelevant from the point of view of the firm, which can expect the same level of profits. However, the economic impact of each particular type of auction is different. When a price auction is used, users of infrastructure pay for the rent that the concessionaire obtains, through prices somewhat higher than the feasible minimum. Meanwhile, when using a payment auction, the

[5] A well known result from auction theory is the Revenue Equivalence Theorem (Vickrey, 1961; Myerson, 1981; Riley and Samuelson, 1981), which states that, if bidders are risk-neutral, the form used by a seller to auction a good (first- and second-price sealed bids, English and Dutch auctions) yields the same expected revenues. In our model, all auctions are first-price sealed bids, but results indicate that, in terms of expected revenues, it does not matter which bidding variable is used.

rent $(I_{max}-I_i)/N$ takes the form of a lower than optimal fee for the government, so taxpayers are in fact paying for that rent. Another lesson is the importance of having as many firms as possible competing for a concession contract. The number N of bidders reduces the information rent that a concesssionaire may extract. In the limit, if N tends to be sufficiently large, this rent would be zero, obtaining the known result of converge to real values (Wilson, 1977; Milgrom, 1979).

3.3 Problems suffered by traditional systems: selection errors and risk of renegotiation

As indicated when analysing optimal bidding rules given by (4) and (7), the traditional price and payment auctions will select the most efficient candidate (lowest I_i in this context), but only when all candidates share the same expected demand levels Q^e. This assumption will hardly hold in practice, given the difficulties associated with accurate demand forecasting. Therefore, it is more realistic to consider that Q^e will be a random variable drawn from some support $[Q_{min}, Q_{max}]$, and that each candidate may have a different belief Q_i^e. To represent a situation of maximum demand uncertainty, we will consider a uniform distribution $Q_i^e \rightarrow \mathrm{U}\ [Q_{min}, Q_{max}]$.

The impact of beliefs about demand on the bids submitted by candidates can be assessed by studying expressions (4) and (7). It can be observed that $\mathrm{d}P_i(I_i)\ /\ \mathrm{d}Q_i^e < 0$ and $\mathrm{d}Z_i(I_i)\ /\ \mathrm{d}Q_i^e > 0$, which means that those candidates with high expected demand levels will tend to submit better bids (lower prices and higher payments, respectively), than other candidates with similar costs but lower demand expectations. This creates two types of problems. First, the mechanisms do not longer guarantee the selection of the most efficient candidate, and the possibility of selection errors appears. Second, and related to the former, the selection of overoptimistic concessionaires with costs higher than optimal increases the risk that contract renegotiation[6] might be required to bail out bankrupted concessionaires, in situations of low demand.

(a) Selection error: Consider an auction at which candidates are ranked according to their construction costs, so that $I_1 < I_2 <...< I_N$. A selection error occurs if the auction mechanism selects a firm I_j other than I_1. The condition required for that event is simply that firm *j*'s bid turns out to be better than firm 1's (a lower price or a higher payment, respectively, at each type of auction). This can only occur when the belief of firm *j* about demand is sufficiently higher than firm 1's, i.e. $Q_j^e/Q_1^e = 1+\lambda$, with $\lambda > 0$ representing the degree of relative 'optimism' of firm *j*.

Using expressions (4) and (7) for optimal bids, it is possible to determine the required size of λ for a firm other than 1 to win each type of auction and thus to be selected as concessionaire :

Price auction:
$$\lambda > \frac{N-1}{N} \frac{I_j - I_1}{I_1 + Z + \dfrac{I_{max} - I_i}{N}} \tag{8}$$

Payment auction:
$$\lambda > \frac{N-1}{N} \frac{I_j - I_1}{P\, Q_1^e\, T} \tag{9}$$

The basic factors that affect the possibility of making selection errors are reflected in expressions (8) and (9). The cost gap between firms (I_j-I_1) and λ are directly related, therefore a wider range of feasible construction costs makes errors to be less likely (because, on average, the difference between I_1 and I_j will tend to be larger). A large number *N* of bidders reduces the probability of selection errors for both types of auctions. It is noticeable that the contract term *T* does not have an effect on the probability of error for the auction price, while it does on the payment auction, increasing the possibility of selection errors for longer contracts.

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[6] Throughout the paper, we assume that renegotiation is only 'honestly' sought by firms, i.e. there is no strategic bidding anticipating the possibility of contract renegotiation. Firms compute their bids based on real costs and demand

It is possible to compute explicit expressions for the probability of selection errors, based on the assumed uniform distributions for demand beliefs $Q_i^e \rightarrow U[Q_{min}, Q_{max}]$ and construction costs $I_i \rightarrow U[I_{min}, I_{max}]$. The demand belief of any firm *j*, relative to that of the most efficient firm in terms of costs, is by definition a random variable $(1+\lambda) = Q_j^e / Q_1^e$, with probability distribution function:

$$F(1+\lambda) = \begin{cases} \dfrac{[Q_{max}(1+\lambda) - Q_{min}]^2}{2(1+\lambda)(Q_{max} - Q_{min})^2} & ; \ if \quad \dfrac{Q_{min}}{Q_{max}} \leq (1+\lambda) \leq 1 \\ 1 - \dfrac{[Q_{max} - Q_{min}(1+\lambda)]^2}{2(1+\lambda)(Q_{max} - Q_{min})^2} & ; \ if \quad 1 < (1+\lambda) \leq \dfrac{Q_{max}}{Q_{min}} \end{cases} \tag{10}$$

At an auction with *N* bidders, the expected values for costs I_i of each of them can be calculated for any given value of the most efficient firm I_1. Because the cost of the remaining *N*-1 bidders must be uniformly distributed between I_1 and I_{max}, any firm at position *j* in the ranking of costs will have an expected construction cost equal to $I_j = I_1 + j\,(I_{max}-I_1)/N$. Thus, it is feasible to compute, for any firm *j*, the required relative demand belief to overcome the gap I_j-I_1, necessary for firm *j* to submit an offer better than the one from the most efficient firm 1.

For the price auction, and disregarding situations in which a firm with cost $\{I_3,...,I_N\}$ could outbid the most efficient firm[7], the probability of selection errors, *prob* (s_P), is equal to:

$$prob\,(s_P) = \int_{I_{min}}^{I_{max}} \frac{(I_{max} - I_1)^{N-1}}{(I_{max} - I_1)^N} \, \frac{[Q_{max} - \theta(I_1)\,Q_{min}]^2}{2\theta(I_1)\,(Q_{max} - Q_{min})^2} \, dI_1 \tag{11}$$

where $\theta(I_1) = \dfrac{(N-1)^2\, I_1 + (2N-1)\, I_{max} + N^2\, Z}{N\,(N-1)\, I_1 + N\, I_{max} + N^2\, Z}$.

expectations. Therefore, renegotiation only takes place when demand turns out to be low and the concessionaire faces a situation with $\Pi_i^* < 0$. For a model of renegotiation and strategic bidding, see Wang (2000).

The probability of selection error for the payment auction, *prob* (s_Z), can be similarly calculated. Its expression is slightly more complex, because in this case the condition to be satisfied for firm 2 to be selected instead of firm 1 varies with Q_1^e.

$$prob\,(s_Z) = \int_{I_{min}}^{I_{max}} \int_{Q_{min}}^{Q_{max}} \frac{(I_{max} - I_1)^{N-1}}{(I_{max} - I_1)^N} \, \frac{\left[Q_{max} - \theta'(I_1, Q_1^e)\,Q_{min}\right]^2}{2\theta'(I_1, Q_1^e)\,(Q_{max} - Q_{min})^2} \, dQ_1^e \, dI_1 \qquad (12)$$

with $\theta'(I_1, Q_1^e) = 1 + \dfrac{(N-1)(I_{max} - I_1)}{N^2 \, P Q_1^e \, T}$.

(b) Contract renegotiation: The conditions for a concession contract to enter into financial problems and require to be renegotiated are similar in nature to those of selection errors. The event $\Pi_i^* < 0$ occurs when the actual demand for the infrastructure Q^* is low, and falls below expectations. Revenues obtained from users might then result insufficient to cover for infrastructure costs, so the concessionaire is forced to go bankrupt unless the government accepts to introduce changes in the contract terms, like rising the price, providing subsidies or extending the life of the concession. As described above, this is a practice too frequently observed for real concessions.

Consider the random variable $\mu = Q^* / Q_1^e$, representing the deviation of actual demand Q^* with respect to the forecasted value used by the winner when computing its bid. The condition for a concession contract awarded through a price auction to be renegotiated is:

$$\mu \; < \; \frac{I_1 + Z}{I_1 + Z + \dfrac{I_{max} - I_i}{N}} \qquad (13)$$

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[7] Total probability associated to an event in which a firm $j \in \{3...N\}$ outbids firms 1 and 2 is of second order compared to the probability of firm 2 beating firm 1 at the auction. An event with a firm $j > 2$ winning the auction is only likely to take place when firm 1 has a cost I_1 close to I_{max}, and all firms have costs higher than I_1, which is a situation with extremely low probability.

Using the fact that μ is uniformly distributed on the support $[Q_{min}/Q_1^e, Q_{max}/Q_1^e]$, the probability of contract renegotiation, *prob* (r_P) will be equal to:

$$prob\ \ (r_P) = \frac{I_1 + Z}{I_2 + Z}\,\frac{Q_1^e - Q_{min}}{Q_{max} - Q_{min}} - \frac{Q_{min}\ (I_{max} - I_{min})}{(N+1)\ (I_2 + Z)\ (Q_{max} - Q_{min})} \tag{14}$$

where $I_1 = N/(N+1)\ I_{min} + 1/(N+1)\ I_{max}$, and $I_2 = (N-1)/(N+1)\ I_{min} + 2/(N+1)\ I_{max}$ are the expected values for the construction costs of the first and second best firms in the cost ranking of bidders.

A similar condition to (13) is derived for the case of the payment auction, from which the corresponding probability of contract renegotiation *prob* (r_Z) is calculated:

$$prob\ \ (r_Z) = \frac{Q_1^e - Q_{min}}{Q_{max} - Q_{min}} - \frac{I_{max} - I_{min}}{(N+1)\ P\ T\ (Q_{max} - Q_{min})} \tag{15}$$

An examination of expressions (11) and (14) (and of (12) and (15), respectively), reveals that factors that affect the probabilities of selection errors and contract renegotiation are very similar. Thus, for example, the contract term T does not affect the probability of renegotiation for a price auction *prob* (r_P), while it does for the payment auction. Longer periods increase the probability of renegotiation, because an optimistic firm relies on obtaining substantial revenues from the concession when submitting its bid, and this assumption is reinforced by the duration of the contract.

It is not surprising to observe that *prob* (s_P) and *prob* (r_P) are highly correlated because both effects are linked (the same applies to *prob* (s_Z) and *prob* (r_Z)). An auction mechanism is more likely to make selection errors when the winner has a belief for high expected demand, which in turn rises the probability of contract renegotiation when demand falls below expectations. This is the form that the “winners’ curse” takes in the context of this model. Even though the probabilities of selection error and renegotiation generally vary in the same direction, however, the probabilities of renegotiation are more affected by demand uncertainty. It can be proved that the range ($Q_{max} - Q_{min}$) of feasible demand has a larger effect over *prob* ($r_{\{P,Z\}}$) than over *prob* ($s_{\{P,Z\}}$).

Expressions (14) and (15) for the probabilities of renegotiation of contracts can be used to try to explain the observed fact about *prob* (r_P) being much higher in practice than *prob* (r_Z) (Guasch, 2000). The condition for *prob* (r_P) > *prob* (r_Z) is:

$$P\,T \;<\; \frac{I_2 + Z}{Q_{min} + (N+1)\,(Q_1^e - Q_{min})\,\dfrac{I_2 - I_1}{I_{max} - I_{min}}} \tag{16}$$

Condition (16) allows to compare price and payment auctions. The conclusions derived from this condition are quite revealing. Low prices (i.e., charges to users), short contract terms, and high fee payments to governments are all factors that make contract renegotiation to be a more likely event for the minimum price auction than for the maximum payment auction. These three targets appear frequently in many infrastructure concession programs announced by governments. Therefore, it is likely that many of the frequently observed concession contracts' renegotiations could be partly caused by the type of auction mechanism used for the selection of concessionaires and the values chosen for *P*, *Z* and *T*.

3.4 Flexible-term concession contracts

Problems suffered by traditional auction mechanisms stem from a basic common feature: uncertainty about future demand levels introduces uncertainty about total revenue expected by firms. This induces bids submitted by firms to be 'contaminated' by their beliefs on demand, and therefore not to represent accurately their underlying construction costs. Formally, this can be observed by the presence of demand expectations (Q_i^e) on expressions (4) and (7), which indicate that firms include their beliefs in their bids for prices and payments.

The idea of using concession contracts with flexible terms, proposed for the road sector by Engel et al (1997) (for a detailed discussion, see Tirole, 1997, 1999), aims at breaking this link between demand uncertainty and revenue uncertainty. A flexible-term contract works as follows: firms submit bids for the total revenue that they want to obtain from the concession (R_i). An auction based on this type of contract awards the concession to the lowest bid $R_1 < R_2 <...<R_N$. The winner then is ensured to obtain a revenue equal to R_1 by extending the life of the concession as necessary. If actual demand is Q^*, the contract lasts until $P\ Q^*\ T(Q^*) = R_1$, at which point the concession ends.

Before uncertainty about demand is resolved, firms have total expected revenues equal to $R = PQ^eT$. Traditional auctions based on a fixed-term T thus force expected revenues to be a function of the uncertain demand, $R = R(Q^e)$, and bids are computed on the basis of these uncertain revenues. Meanwhile, if the term T were flexible and endogenously determined by demand, total revenue could be regarded as constant by each firm. No matter what its belief about demand might be, submitting a bid for revenue R_i would yield that amount if the firm is selected as concessionaire. If a firm expects a demand level Q_i^e, and submits a bid R_i, it can expect the concession to last for a period $T(Q_i^e) = R_i/(P\ Q_i^e)$. Therefore, its expected revenue would be $R = PQ_i^eT(Q_i^e) = R_i$. This induces bids that are not affected by beliefs about future demand.

This type of flexible-term concession contracts has already been applied in practice, although the number of examples around the world is very reduced[8]. In a context with discount factors δ smaller than one, firms participating at auctions for flexible-term contracts must submit bids for discounted revenues, $R_i = \sum_{t=1}^{T_i^e} \delta^t\ P\ Q_i^e$. This is the reason why this type of auction has been named as '*least-present-value of revenue*' (LPVR), because the concession is awarded to the candidate with the lowest bid for discounted revenue R_i.

[8] To our knowledge, the first experience with flexible-term concessions was a UK road project (Dartford bridge). A well

In the context of our model, with no discount rate, a firm participating at an LPVR auction will compute its bid according to its type, following some rule $R(I_i)$, with $R'(\cdot) > 0$. As in the cases of price and payment auctions, we search for a symmetric equilibrium with all firms using the same rule $R(\cdot)$ to compute their bids. The probability of winning the auction with an offer R_i has an expression similar to (1) and (6). The expected contract-term is $T_i^e = R_i/(PQ_i^e)$, so the problem solved by each firm is now:

$$Max_{R_i}\ \Pi_i^{\ e} = \left(PQ_i^e \left(\frac{R_i}{PQ_i^e} \right) - I_i - Z \right) \left(I_{max} - R^{-1}(R_i) \right)^{N-1} I_r^{\ 1-N} \qquad (17)$$

As in previous cases, the first-order condition of this maximisation problem is a differential equation on $R(I_i)$, which yields the linear solution:

$$R(I_i) = I_i + Z + \frac{I_{max} - I_i}{N} \qquad (18)$$

Expression (18) reveals the enormous advantages of awarding a flexible-term concession contract over the traditional fixed-term contracts. First, no demand belief Q_i^e enters expression (18), which indicates that bids are free from the effect of demand uncertainty. Thus, any possibility of having selection errors is eliminated. No matter the distribution of beliefs across candidates, this mechanism will always select the most efficient one in terms of costs, which will be the firm submitting the offer for lowest revenue. Second, renegotiation never takes place, because the firm receives exactly the amount requested as total revenue. In the terminology of auction theory, this mechanism allows the selection of concessionaire to be transformed from a private- and common-value problem, to a much simpler private-value situation, in which firms only differ according to their construction costs.

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documented experience is the concession of a Chilean highway (Santiago-Valparaíso-Viña del Mar), awarded through an auction based on bids for total discounted revenues (Gómez-Lobo and Hinojosa, 2000).

Further examination of optimal offers from (18) indicates that expected profits are positive for the winner. Bids for revenue cover for all costs paid by the firm ($I_i + Z$), and allow for some extra rent, which is equal to the information rents that firms expected to extract from the traditional fixed-term auction systems. This revenue equivalence result can be expressed as:

$$\Pi_i^e\Big|_P = \Pi_i^e\Big|_Z = \Pi_i^e\Big|_{LPVR} = \frac{I_{max} - I_i}{N} \tag{19}$$

Although the three analysed auction mechanisms are ex-ante equivalent for firms, in terms of expected revenues, they perform very differently ex-post. As it has been already examined, price and payment auctions have positive probabilities of making selection errors, and it is also likely that the concession contract could be required to be renegotiated in the event of low demand for the infrastructure. Meanwhile, the LPVR type of auction yields optimal results. No selection error is ever made, because firms' bids are completely free from beliefs about demand. More interestingly, a flexible-term concession is never renegotiated. In situations of low demand, the only effect is that the duration of the contract is automatically extended to allow the firm to collect revenues from charges during a longer period, and thus obtain the requested total amount R_i.

Flexible-term concession contracts, however, suffer at least from four drawbacks. First, because the firm is ensured to receive its claimed revenue, it has no incentives at all to promote the use of infrastructure, as pointed out by Tirole (1997). This means that users may end up receiving a poor service from the infrastructure operator. Second, and related to the former, the concessionaire may not maintain adequate quality and safety standards, an aspect which should be strictly supervised by governments. Third, for large projects the amount of required investments is usually high, and the period needed to recover them will be typically long. Although at the end of the concession the firm will have obtained its total requested revenue, it might experience some cash-flow shortages during intermediate periods, which might affect its ability to repay debts. A necessary condition for a smooth performance of flexible-term concessions is the existence of well-developed capital markets, which allow concessionaires to obtain 'bridge-loans' for such periods.

There is a fourth limitation of flexible-term contracts, which turns out to be quite serious, and it arises due to the existence of maintenance and operation costs. In the scenario considered in this section, in which firms only incur into construction costs, bidders are indifferent to the duration of the concession. However, when the concessionaire faces annual expenses to operate an infrastructure, longer periods imply higher costs that might affect its profitability and, in extreme cases, may even require contract renegotiation.

This latter problem is extensively analysed in the next sections, where a modified LPVR mechanism is proposed to solve the problems posed by maintenance costs M_i. Before turning to the study of the case with maintenance and operation costs, some figures are provided to assess the magnitude of the problems discussed.

3.5 Simulations of price, payment and LPVR auctions

In order to evaluate probabilities of selection errors and renegotiation of concession contracts, some simulations have been performed. Parameters are chosen to represent a typical case for infrastructure concessions, in terms of the relative size of variables involved. Using a particular benchmark of reference, it is possible to assess the impact of each factor by introducing changes in parameters and studying the impact on outcomes.

The benchmark case corresponds to an auction with N=5 potential candidates. Firms' construction costs are randomly drawn from a uniform distribution $I_i \rightarrow$U [100; 1,000], and demand beliefs from an independent distribution $Q_i^e \rightarrow$U [10; 30]. The contract-term is set at T=30 years for the fixed-term auctions. Price is fixed at P=0.75 for the case of maximum payment auction; and payment at Z=100, for the price auction. With these values, the expected range of total revenue for firms is [225, 675]. For the average type of firm, with cost I_i = 550, this implies that profits may vary between [-325, 125], so the project is quite risky and only attractive to firms with low costs.

Table 1 shows the outcomes of each type of auction after performing 5,000 simulations, at each of which five bidders were randomly chosen regarding both their cost types and beliefs on future demand levels. Bids from all firms are computed and evaluated to determine the auction's winner. The auction's outcome is then compared with the actual ranking of firms' costs to calculate the probability of selection error.

[INSERT TABLE 1]

For the benchmark case considered, the probabilities of selection errors and contract renegotiation are considerably high. Figures indicate that price and payment auctions are selecting on average concessionaires with costs above the optimal level ($I_1 = (5\ I_{\min}+I_{\max})/6$). The basic reason explaining selection problems is the bias towards optimistic firms. This bias can be assessed by comparing the average expected demand levels by auction winners with the unbiased expected demand (Q^e=20). Meanwhile, an auction based on the LPVR mechanism yields optimal outcomes: there is no bias in the selection of concessionaire, which implies optimal selection, and the probability of contract renegotiation is equal to zero.

In contrast to the situation described in the real world, with a higher probability of renegotiation for the price auction than for the payment auction, results show similar values for both types in this case. This is simply due to the size of parameters chosen for the benchmark case: it is possible to obtain easily situations in which *prob* (r_P) is larger (or lower) than prob (r_Z) by changing the values of P, Z or T.

4. Scenario with equal maintenance and operation costs

As a first step towards the analysis of the more general case in which firms differ both on construction and maintenance costs, consider initially a scenario in which we introduce some fixed annual expenses on maintenance and operation of infrastructure assets, but these costs are known for all parties and are equal across firms, $M_i = M$.

For fixed-term auctions, the existence of these costs is irrelevant. It only changes slightly the functions $P_i(I_i)$ and $Z(I_i)$ used by firms to compute their bids (see expressions (4) and (7)). Firms simply accommodate for the higher costs and obtain the same expected profits $(I_{max}-I_i)/N$. In the case of the price auction, bids P_i are increased by a factor of M/Q_i^e, which means that each user is required to pay a share of the annual maintenance costs. Meanwhile, for the payment auction, bids Z_i are reduced by an amount MT, which is equal to total maintenance and operation costs during the complete life of the concession. In this latter case, the winner is simply subtracting those costs from the payment proposed to be made to the government, so maintenance costs are in fact transferred to taxpayers.

The only modification worth to be mentioned for fixed-term auctions, with respect to the analysis of the previous section, is that the concession term T now has an impact on the probabilities of selection error and renegotiation for the case of the price auction. In particular, condition (8) for the selection of a firm I_j other than the best firm I_1 is transformed so that parameter T now affects the condition. The effect of T on probabilities, however, remains being more important for the case of the payment auction. Another relevant feature is that probabilities of selection errors and renegotiation for the payment auction are not affected by the size of maintenance cost M, while probabilities for the price auction do change according to M.

4.1 Effect of maintenance costs over LPVR auctions

The introduction of maintenance costs M has a dramatic effect on the flexible-term contract. To realise this, consider the problem that a firm with maintenance cost M participating at an LPVR auction must solve now. As before, when submitting a bid for revenue R_i, the candidate is determining its expected term for the contract, which will be equal to $T_i^e = R_i/PQ_i^e$. Taking into account the impact of R_i on the probability of winning, firm i calculates its optimal bid by solving:

$$Max_{R_i}\ \Pi_i^{\ e} = \left((PQ_i^e - M)\left(\frac{R_i}{PQ_i^e} \right) - I_i - Z \right)\left(I_{max} - R^{-1}(R_i) \right)^{N-1} I_r^{\ 1-N} \qquad (20)$$

Although in principle this maximisation problem seems to be almost identical to (17), the presence of M in the expected profits has a strong impact on the solution:

$$R(I_i) = \frac{PQ_i^e}{PQ_i^e - M}\left[I_i + Z + \frac{I_{max} - I_i}{N} \right] \qquad (21)$$

As it can be observed from expression (21), in the presence of maintenance and operation costs, bids submitted under LPVR auctions suffer exactly from the same problems detected for the traditional fixed-term auctions. Beliefs about future demand (Q_i^e) are used when computing bids for revenue R_i, implying the possibility of distorted information for the selection of concessionaire. Again, it is possible that an overoptimistic firm may outbid the most efficient firm in terms of costs, inducing the mechanism to make selection errors. Even more serious is the effect that M has on the probability of contract renegotiation, which now is different from zero. Although under a flexible term contract a concessionaire is guaranteed to obtained its claimed R_i revenue, it is no longer immediate that this amount of revenue will always be sufficiently large to cover for all costs. In the event of a low demand situation ($Q^* \rightarrow Q_{min}$), the contract term must be extended, which causes total maintenance costs for the life of the concession to rise. Renegotiation might then be required in cases where $I_i + Z + M\,T^*(Q^*) > R_i$.

4.2 A new proposed mechanism: least-present-value of net revenue (LPVNR)

The impact of maintenance costs and their associated problems can be avoided by designing a new type of auction, also based on flexible-term concession contracts. As the aim of the auction mechanism is to extract information from firms regarding their true construction costs (I_i), the objective should be to ensure that the bidding variable is as close as possible to I_i. This can be done by ensuring firms that their annual maintenance cost M will be covered in any case.

The mechanism is similar in nature to LPVR, but yields better results. Firms submit bids B_i for the total amount of money they demand to cover for investment costs. From revenues collected from users, each year a part M is considered to be destined to cover for maintenance costs, while the rest is accounted for as 'net income perceived'. When total net income perceived is equal to the bid B_i, the concession ends. Formally expressed, firm *i* with a bid B_i can therefore expect to hold the concession until $P\,Q_i^e\,T = B_i + M\,T$, and the expected term is then equal to $T_i^e = B_i / (P\,Q_i^e\text{-}M)$. This new auction mechanism will render exactly the same optimal results in a context with discount rates $\delta < 1$, therefore it is named as 'least-present-value of *net* revenue' (LPVNR), because the contract is awarded to the candidate with the lowest bid B_i

Bidders participating at an LPVNR auction will compute their bids B_i according to some function $B(I_i)$ which is assumed to be shared by all participants. Each firm then solves:

$$Max_{B_i}\ \Pi_i^{\,e} = \left((P\,Q_i^e - M)\left(\frac{B_i}{P\,Q_i^e - M} \right) - I_i - Z \right)\left(I_{max} - B^{-1}(B_i)\right)^{N-1}\ I_r^{\;1-N} \qquad (22)$$

Solving for B_i shows that this maximisation problem (22) is exactly the same that the firm was confronted with at an LPVR auction when there are no maintenance and operation costs (see expression (17)). The solution to (22), therefore, restores the good properties about selection and no renegotiation that were derived in that case. Optimal bids are equal to:

$$B_i = B(I_i) = I_i + Z + \frac{I_{max} - I_i}{N} \tag{23}$$

Expression (23) indicates why a LPVNR auction outperforms any other auction mechanism in this context of equal maintenance and operation costs. Submitted bids B_i are completely independent from beliefs about future demand, and $B'(I_i) > 0$, therefore the mechanism guarantees the selection of the most efficient candidate. The auction's winner extracts some information rent from the concession, which takes exactly the same expected value as in the other auctions.

$$\Pi_i^e\Big|_P = \Pi_i^e\Big|_Z = \Pi_i^e\Big|_{LPVR} = \Pi_i^e\Big|_{LPVNR} = \frac{I_{max} - I_i}{N} \tag{24}$$

Results from simulations performed for this scenario with constant maintenance costs across firms are presented in table 2. It can be observed that the LPVR auction mechanism suffers exactly from the same problems as auctions based on fixed-term concessions, although its probabilities for selection errors and contract renegotiation are smaller. Meanwhile, the LPVNR auction always selects the candidate with lowest cost, and it has zero probability of contract renegotiation.

[INSERT TABLE 2]

5. Scenario with different maintenance and operation costs

The most relevant case for the analysis of auctions for infrastructure concessions is a context in which firms may differ both on their construction costs (I_i) and also on their maintenance and operation costs (M_i). The logic behind the assumption of different M_i's across firms is the same that justifies the existence of differences in construction costs. Firms can vary according to their efficiency levels, for example because managers exert more or less effort in controlling costs.

The existence of different maintenance and operation costs poses some difficulties for the formal analysis of auctions' expected outcomes. In particular, firm's types are no longer uniformly distributed over some range of construction costs, because the variability of maintenance costs introduces some changes in the probabilities of each type of firm. A second issue is the definition of cost efficiency. Total costs expected by each firm are now equal to $C_i = I_i + M_i T$, therefore the definition of the lowest-cost firm now depends on the contract-term T. For flexible-term concessions is difficult to determine *ex-ante* which firm has the lowest cost, due to the fact that actual duration of the contract (T^*) may change according to demand. Some rule is required to calculate probabilities of selection errors, and to evaluate bids if it is decided to use some form of bi-dimensional auction.

5.1 Probability distribution of firms' types

Using the assumption of uniform independent distributions for construction costs, $I_i \rightarrow U\,[I_{min}; I_{max}]$, and for maintenance costs, $M_i \rightarrow U\,[M_{min}; M_{max}]$, it is possible to derive the probability distribution of firms' types. A type now is defined as the value of total costs, $C_i = I_i + M_i T$. As discussed above, these types may vary according to the contract-term T predefined by the government. For a given value of T, the distribution function of C_i is equal to:

$$F(C_i) = \begin{cases} \dfrac{1}{2\, I_r\, M_r\, T}\,(C_i - C_{min})^2 & ; \; if \quad C_{min} \le C_i \le C_a \\[2ex] F(C_a) + \dfrac{1}{I_r}\,(C_i - C_a) & ; \; if \quad C_a < C_i \le C_b \\[2ex] F(C_b) + \dfrac{1}{2\, I_r\, M_r\, T}\left[C_b^{\,2} - C_i^{\,2} + 2\, C_{\max}\,(C_i - C_b)\right] & ; \; if \quad C_b < C_i \le C_{\max} \end{cases} \tag{25}$$

where $I_r = I_{max} - I_{min}$; $C_a = I_{min} + M_{max}T$; $M_r = M_{max} - M_{min}$; $C_b = I_{max} + M_{min}T$; $C_{min} = I_{min} + M_{min}T$; $C_{max} = I_{max} + M_{max}T$.

Because construction costs are generally much larger than maintenance and operation costs for most types of infrastructure, it is assumed that $C_a < C_b$ (the other alternative case does not imply any other significant change than some modifications in the expressions for $F(C_i)$). Expression (25) shows that the probability distribution of C_i is not uniform. It exhibits a linear part in the range $C_i \in [C_a, C_b]$, but the function is convex on $[C_{min}, C_a]$, and concave on $[C_b, C_{max}]$. This reflects the fact that extreme types with very low or very high total costs have small probability masses.

Given this three-part probability distribution, the calculation of optimal bids for the different types of auctions becomes in general more complex. Strategies must now be a function of full costs C_i, and therefore the probability of winning with each possible bid of the form $f(C_i)$ has to be calculated from $F(C_i)$, where $f(\cdot)$ will be the corresponding function $P(\cdot)$, $Z(\cdot)$ or $R(\cdot)$, for each type of auction.

5.2 Impact of maintenance and operation costs M_i on fixed-term concessions

The maximisation problems for computing optimal bids to participate at a minimum price or maximum payment auction are not basically altered from those cases analysed in previous sections. The only difference is the change in the definition of firms' types. In this scenario, firms' bids are functions of real full costs $C_i = I_i + M_i T$.

The probability of a firm *i* winning an auction with a bid $P_i=P(C_i)$, or $Z_i=Z(C_i)$, now varies according to the range where the bid falls. In general, for any bid P_i or Z_i there will be three possible cases, according to the possibilities of having either $P^{-1}(P_i) < C_a$; $C_a \leq P^{-1}(P_i) \leq C_b$; or $C_b < P^{-1}(P_i)$. The two extreme cases yield non-linear solutions, while the intermediate case is linear, due to the fact that types C_i are uniformly distributed between C_a and C_b. (A complete derivation of the optimal strategy, taking into account the whole distribution of C_i, for the case of a price auction is presented in detail in the appendix)

For infrastructure projects with large construction costs, the extreme cases can be disregarded, because when M_i is small relative to I_i, $C_a \to C_{min}$ and $C_b \to C_{max}$. Considering that all bids submitted by candidates lie within the range [C_a, C_b], optimal bids for price auctions and payment auctions will have linear forms, similar to (4) and (7), respectively.

$$P_i = P(C_i) = \frac{1}{Q_i^e T}\left[C_i + Z + \frac{1}{N}\left(I_r - \frac{M_r T}{2} - (C_i - C_a)\right)\right] \tag{26}$$

$$Z_i = Z(C_i) = P Q_i^e T - C_i - \frac{1}{N}\left(I_r - \frac{M_r T}{2} - (C_i - C_a)\right) \tag{27}$$

A simple examination of expressions (26) and (27) indicates that these solutions share the same features of those analysed on previous sections. Both $P(C_i)$ and $Z(C_i)$ and functions that depend on beliefs Q_i^e, therefore outcomes will again be plagued from the same problems of selection errors and possible contract renegotiation discussed above.

5.3 Impact of maintenance and operation costs M_i on LPVR and LPVNR concessions

As it is the case for fixed-term concessions, optimal bidding strategies to follow at an LPVR auction are not fundamentally altered. The only major change is that bids for revenue R_i are calculated as a function of total costs C_i. Assuming that all bids lie within the range [C_a, C_b], yields the solution:

$$R_i = R(C_i) = \frac{P Q_i^e}{P Q_i^e - M_i}\left[I_i + Z + \frac{1}{N}\left(I_r - \frac{M_r T}{2} - (C_i - C_a)\right)\right] \tag{28}$$

A comparison of expressions (28) and (21) shows that results obtained for the case with equal maintenance costs M_i=M are not altered for LPVR auctions when variability on M_i is introduced.

The bid for revenue R_i is slightly more complex, to take into account the true value of maintenance and operation costs for each firm *i*, but its main features remain unaltered, included the effect of beliefs on future demand on bids R_i.

For the LPVNR auction, the existence of different values for M_i across firms introduces changes. Because the government now does not have accurate information about the size of these costs, a first alternative could be to use the expected average value $M_{av} = (M_{min}+M_{max})/2$, to determine the part of revenues collected from charges that will be assigned each year to cover for maintenance costs. Firms are again invited to submit bids on the revenue, net of maintenance and operation costs, taht they demand to recover their investment costs, and the contract is awarded to the lowest bid. Thereafter, when the actual level of demand Q^* is known, the duration of the contract wil be equal to $T^* = B_i /(P\, Q^* - M_{av})$

Considering that firms will compute their bids B_i based on their real full costs[9], $B_i=B(C_i)$, the problem that they solve is:

$$Max_{B_i}\ \Pi_i^e = \left(\frac{PQ_i^e - M_i}{PQ_i^e - M_{av}} B_i - I_i - Z \right)\left(1 - \frac{M_r T}{2I_r} - \frac{1}{I_r}\left(B^{-1}(B_i) - C_a\right)\right)^{N-1} \qquad (29)$$

Optimal strategies are then:

$$B_i = B(C_i) = \frac{PQ_i^e - M_{av}}{PQ_i^e - M_i}\left[I_i + Z + \frac{1}{N}\left(I_r - \frac{M_r T}{2} - (C_i - C_a) \right)\right] \qquad (30)$$

Expression (30) shows how, in the presence of different maintenance costs across firms, the mechanism LPVNR does not have the property of eliminating completely the presence of demand

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[9] Another possible alternative could be to consider that firms only take into account construction costs to calculate bids, i.e. $B_i = B(I_i)$. In that case, it is found that the optimal strategy is $B_i = (PQ_i^e-M_{av})\ [I_i+Z+(I_{max}-I_i)/N]\ /\ (PQ_i^e-M_{av})$, which is similar to (30) and yields on average the same profits.

beliefs in bids B_i. Therefore, it is possible that this auction mechanism could also make selection errors. A careful examination of expressions (28) and (30) indicates that, although they appear to be similar, the effect of Q_i^e on R_i and B_i is different. This effect can be checked by taking the corresponding derivatives:

$$\frac{\partial R_i}{\partial Q_i^e} = \frac{-P M_i}{(P Q_i^e - M_i)^2} \left[I_i + Z + \frac{1}{N}\left(I_r - \frac{M_r T}{2} - (C_i - C_a) \right) \right] < 0 \tag{31}$$

$$\frac{\partial B_i}{\partial Q_i^e} = \frac{P (M_{av} - M_i)}{(P Q_i^e - M_i)^2} \left[I_i + Z + \frac{1}{N}\left(I_r - \frac{M_r T}{2} - (C_i - C_a) \right) \right] \lessgtr 0 \tag{32}$$

The negative sign of derivative $\partial R_i/\partial Q_i^e$ indicates the bias towards selecting optimistic candidates for the LPVR auction. High expected values Q_i^e result in lower bids, thus increasing the probability of selection errors. This is the same effect that had been already detected in the case of equal maintenance costs.

Meanwhile, distortions introduced by demand beliefs Q_i^e on bids for net revenues at a LPVNR auction are smaller (observe that $\left| \partial R_i / \partial Q_i^e \right| > \left| \partial B_i / \partial Q_i^e \right|$). Moreover, the effect of Q_i^e on the bid may vary according to the type of firms. For those candidates with maintenance costs $M_i > M_{av}$ the sign of $\partial B_i/\partial Q_i^e$ is negative, and the effect is the same as observed for $\partial R_i/\partial Q_i^e$. But, for candidates with costs M_i smaller than M_{av}, the effect is exactly the opposite. In this latter case, more optimistic firms tend to rise their bids, thus decreasing the probability of winning the auction. The combination of both these effects is likely to result in better selection outcomes for LPVNR auctions than for the other mechanisms.

A comparison of all outcomes derived in the context of this scenario with different M_i costs for the four auctions examined again provides a revenue equivalence result: ex-ante profits expected by the winner of any of the auctions are again equal. The existence of differences across firms allow those firms with lower maintenance costs to obtain higher profits than those obtained in the case of constant costs, $M_i=M$ (compare the result with expression (24) above).

$$\Pi_i^e\Big|_P = \Pi_i^e\Big|_Z = \Pi_i^e\Big|_{LPVR} = \Pi_i^e\Big|_{LPVNR} = \frac{I_{max} - I_i}{N} + \frac{(M_{av} - M_i)\,T}{N} \tag{33}$$

However, this expected revenue-equivalence does not imply that the ex-post performance of all types of mechanisms is the same. This can be proved by numerical simulations for this scenario, which reveal that flexible-term concessions perform generally better than the traditional mechanisms. Table 3 presents the results obtained from these simulations, based again on the benchmark case, and assuming that $M_i \to U[3,7]$.

[INSERT TABLE 3]

As expected, the LPVNR auction now presents positive probabilities of selection errors and renegotiation. However, it is interesting to observe how this mechanism yields better results than the alternative LPVR, and also outperforms the traditional auctions based on fixed-term contracts.

5.4 LPVNR auction based on bids with two dimensions

This final section aims to complete the analysis of feasible auctions to award concession contracts for infrastructure projects by analysing a possible refinement of the LPVNR auction. This new proposed mechanism has been proved to yield optimal outcomes in simple scenarios with constant maintenance costs, clearly outperforming both traditional auction models and the LPVR type of auction (which does not take into account explicitly the existence of maintenance costs in the auction design). However, in a real context of strong asymmetries of information, it is possible that a government could be completely uninformed about the size of maintenance costs M_i, so it will be unable to use an LPVNR auction as described in the previous section (which implies the use of some known average maintenance cost M_{av}). The refinement proposed is then to invite firms to submit bids with two dimensions: (a) total revenues to be obtained from the concession, net of maintenance costs (B_i); and (b) annual average expenses on maintenance and operation costs (E_i).

The analysis of this proposed variation of LPVNR is interesting both from a practical point of view (because it could be an alternative for governments to auction concession projects), and also from a theoretical perspective. There are not many examples in the auction literature dealing with situations in which bidders submit bids in more than one dimension (Che, 1993; Branco, 1997). However, as pointed out by McAfee and McMillan (1987), it is frequent that when awarding contracts, governments are interested in more than on dimension of information (e.g. quality, prices, investment levels, and so forth). It is then interesting to try to compute equilibrium strategies when firms are requested to submit this type of bi-dimensional bids.

The first step is to calculate the probability for a firm i to win the auction with a bid (B_i ,E_i). In order to do that, we need first to establish how the government will evaluate offers. Due to the fact that, for a flexible-term concession, the contract term is ex-ante undetermined, there is not a unique option to evaluate which offer is the best in terms of total expected costs. One alternative is to set some arbitrary period of reference T^{ev}, and calculate expected total costs as $C_i^e = B_i + E_i\, T^{ev}$, awarding the contract to the lowest offer. Other options are to announce a range of periods [T_{min}, T_{max}], and compute average expected costs over that range, or to choose the firm with lowest costs during more periods within that range. It can be proved that when firms know the range of periods used for evaluation, all these criteria are equivalent to use the intermediate point of the range as a period of reference, i.e. $T^{ev} = (T_{min}+T_{max})/2$. Therefore, for a simpler exposition, we will work with a single period T^{ev} for the evaluation of bids.

It is no longer feasible in this context to use general strategies of the form $B_i = B(C_i)$, $E_i = E(C_i)$, because evaluation of bids would then imply the use of a joint-inverse function $[B(C_i)+ E(C_i)\, T^{ev}]^{-1}$, from which it is impossible to retrieve individual functions for $B(\cdot)$, $E(\cdot)$. Instead, a simpler way to calculate probabilities of winning the auction is to assume some particular forms for the functions $B(\cdot)$ and $E(\cdot)$ that can be related to some random variable with known probability distribution. Natural candidates are linear forms for B_i and E_i, in terms of construction and maintenance costs, respectively. It is then assumed that firms choose their bids for net revenue as a function of construction costs, $B_i = \alpha + \beta\, I_i$, and bids for reimbursement of maintenance costs as a function of their true costs, $E_i = \gamma\, M_i$.[10]

Using these rules, the probability of firm i winning the auction is equal to the probability of the event $B_i + E_i\, T^{ev} < B_j + E_j\, T^{ev}$, for any possible j, which can be expressed as:

$$prob_i = \left(prob\,\left[B_i + E_i\, T^{ev} < \alpha + \beta\, I_j + \gamma\, M_j\, T^{ev} \right] \right)^{N-1} \tag{34}$$

The probability distribution of $\alpha + \beta\, I_j + \gamma\, M_j\, \mathrm{T}^{ev}$ can be calculated from the uniform distributions of I_j and M_j (it has a form very similar to that of C_i, see expression (25)). Therefore, it is possible to derive an explicit expression for $prob_i$, which can be used to maximise expected profits.

Assuming, as in previous sections, that only the linear part of the probability distribution is relevant (which is equivalent to say that all bids $C'_i = B_i + E_i\, T^{ev}$ fall within a given range), the probability of firm i winning the auction with a bid C_i' is equal to:

$$prob_i = \left(1 - \frac{\gamma\, M_r\, T}{2\beta\, I_r} + \frac{C_a'}{\beta\, I_r} - \frac{C_i'}{\beta\, I_r} \right)^{N-1} \tag{35}$$

where $C_a' = \alpha + \beta\, I_{min} + \gamma\, M_{max}\, T^{ev}$.

[10] Another possible option is to make both B_i and E_i dependent on full cost C_i.(e.g. $B_i = \alpha + \beta C_i$, $E_i = \alpha' + \beta' C_i$). However, this is equivalent to the forms adopted, implying only a reinterpretation of coefficients α, β and γ. The same argument applies to the choice of introducing a constant term α only in the function of B_i and not on E_i.

Considering that the concession will last for an expected term $T_i^e = B_i/(PQ_i^e - E_i)$, discarding some constants terms, and denoting by $\theta = \alpha + \beta\, I_{max} + \gamma\, T^{ev}\, (M_{min}+M_{max})/2$, the problem that firm i solves to determine its optimal bid (B_i, E_i) can be expressed as:

$$Max_{B_i,E_i}\ \Pi_i^{\,e} = \left((PQ_i^e - M_i)\frac{B_i}{PQ_i^e - E_i} - I_i - Z \right)\left(\theta - B_i - E_i\, T^{ev}\right)^{N-1} \qquad (36)$$

The two first-order conditions of this problem constitute a non-linear system of equations in terms of B_i and E_i:

$$(PQ_i^e - M_i)\left(\theta - N\,B_i - E_i\, T^{ev}\right) - (N-1)\,(I_i + Z)(PQ_i^e - M_i) = 0 \qquad (37)$$

$$\frac{(PQ_i^e - M_i)\,B_i}{(PQ_i^e - E_i)^2}\left(\theta - B_i - E_i\, T^{ev}\right) - (N-1)\,T^{ev}\left(\frac{PQ_i^e - M_i}{PQ_i^e - E_i}\,B_i - I_i - Z\right) = 0 \qquad (38)$$

There exist two solutions, (B_i', E_i') and (B_i'', E_i''), for this system of equations, and for each of them it is possible to obtain explicit expressions for parameters α, β and γ, using the assumed forms for the optimal strategies. Both of them have some economic interpretation. The first solution (B_i', E_i') is a bid that yields expected profits equal to zero, so any firm *i* will only submit this bid in absence of any other better option.

The second solution (B_i'', E_i'') yields an expected profit equal to $\Pi_i^{e''} = P\, Q_i^e\, T^{ev} - (I_i + M_i\, T^{ev} + Z)$, i.e., the firm tries to extract from the concession all the existent surplus expected from the project, computed on the basis of a contract-term equal to the evaluation period T^{ev}. This second solution is in fact a whole family of feasible bids, with $E_i'' = E(I_i, M_i, Q_i^e)$, and $B_i'' = T^{ev}\,(P\, Q_i^e - E_i'')$, all of which yield the same expected level of profit ($\Pi_i^{e''}$). Firms that have negative expected values for this profit level –a case that might occur for high-cost firms with low expected demands– will opt for the other alternative bid (B_i', E_i'), or will drop out from the auction.

Optimal strategies for firms derived for the context of this bi-dimensional auction can be easily interpreted. Due to the fact that bids (B_i, E_i) must be eventually reduced to a single-dimension score for evaluation purposes, candidates know that they can try to extract rents from the concession, either by rising bids B_i over real construction costs (I_i), or rising the other dimension E_i over real maintenance costs (M_i). Tailoring the strategy in such a way that the total bid $Ci' = B_i + E_i\, T^{ev}$ is kept constant does not affect the probability of winning, but the effects of moving B_i or E_i can be very different on the expected profit levels. When a firm considers that the concession will last for a longer period than the one used for evaluation, it will be interested in increasing the gap E_i-M_i, because each additional year as operator of the infrastructure provides that difference as extra profits. On the contrary, if a candidate expects the concession to be short, it will be more rewarding to try to extract rents from the difference between real and declared construction costs, (B_i-I_i).

The main conclusion for this proposed refinement of the LPVNR mechanism is that beliefs Q_i^e still enter the calculation of optimal strategies. Therefore, once again it is concluded that the mechanism is susceptible to make selection errors, as it is the case with the other auction systems studied in this paper. Nevertheless, empirical simulations (performed within the framework of the benchmark case analysed above) indicate that this variant of LPVNR yields reasonable good outcomes. This makes it a good candidate to be used in practice by governments that want to implement auctions based on the idea of flexible-term contracts, but are completely uninformed about the possible range of values for maintenance and operation costs of an infrastructure.

[INSERT TABLE 4]

One interesting finding from the empirical analysis is that this type of auction seems to favour low cost-firms, as revealed from the average size I_i of winners, which is much lower than for other auctions. This results in higher expected profits for concessionaires (as a result of larger informational rents), but it has a good impact on the probability of renegotiation, which is very low compared to that of other auction systems. Meanwhile, the probability of selection errors is relatively high for this case (18.9%), but nevertheless this is far better than those found for the traditional auction mechanisms, and also for the LPVR type of auction (see table 3). The relatively high probability of selection errors of LPVNR auctions with bi-dimensional bids is originated by a bias towards optimistic firms, as it can be observed from the average expected demand levels of winners (24.8 against an unbiased average of 20). This effect is shared with all the other auction systems analysed, with the exception of the LPVNR auction based on the use of average maintenance and operation costs.

The bi-dimensional auction proposed also has some other features that can make it very attractive for practical purposes. A frequent event during the life of infrastructure concessions is that governments may at some point want to modify an on-going project (e.g. power generation capacity enlargements, modifications of road layouts, and the like). Renegotiation is then required in those cases, not due to financial problems of the concessionaire, but because the new framework may require additional investments or significant changes in financial arrangements. These renegotiatons are usually complex, because firms exploit their advantages of information, and may try to extract additional profits from the concession. Transaction costs can also be an element of importance in this process.

If a contract is awarded through a LPVNR auction, based on bi-dimensional bids, renegotiation problems are completely eliminated. This is achieved because the concessionaire reveals at the auction all the required information to the government. In case that a government decides to modify an infrastructure project, or if there is a conflict between parties that makes it recommendable to terminate the contract, the concessionaire can be automatically compensated for its costs and the contract ended.

The bid for net revenue B_i can be easily used to calculate the amount of money by which a firm needs to be compensated for the income that it ceases to receive if forced to abandon a concession (a matter which in many occasions must be decided by courts when renegotiated concessions are of fixed-term nature). On the other hand, the bid for annual maintenance costs E_i demanded by the own concessionaire is a tremendously useful value of reference to estimate actual expenses for that concept. In real renegotiation situations, when a firm bargains with a government about compensation payments, it benefits from the asymmetry of information regarding those costs. Renegotiation of LPVNR contracts has no difficulties at all on the point of maintenance and operation costs: the company can be compensated exactly by the amount that it claimed in its bid, allowing governments to reject any other demands.

6. Conclusions

Results obtained in this paper make a good case for the revision of traditional models used to select concessionaires for infrastructure projects. Auctions based on bids for minimum prices or maximum payments exhibit a poor performance in terms of selecting the best candidate among a group of bidders. It has been shown that, by design, this type of auctions introduce a bias towards selecting firms that are optimistic about expected demand, but are not necessarily the most efficient in terms of costs. Infrastructure projects are then likely to be implemented at costs higher than optimal, which results in a social misuse of resources. In situations of weak demand, high-cost concessionaires are more likely to experience financial difficulties and claim for contract renegotiation. This is a form of "winner's curse" in the context of concession contracts. Mechanisms used to select firms may be in part responsible for the high rate of renegotiation of concessions observed in practice.

The use of flexible-term contracts could greatly improve firms' selection. Contracts with flexible duration eliminate the need to use demand forecasts to compute bids, and the bias suffered by traditional mechanisms. It has been shown that a form of revenue-equivalence theorem applies in this context. All mechanisms to award concession contracts allow auction winners to extract exactly the same informational rents, regardless of the system employed. Therefore, flexible-term concessions improve the selection of concessionaires at zero cost: information rents extracted by winners are the same as those obtained under price or payment auctions. On the other hand, the reduction of risks associated to demand fluctuations allows concessionaires to obtain resources from capital markets at lower costs, because risk premia on borrowing rates will tend to be smaller.

In situations in which a government is completely uninformed about maintenance and operation costs, a new type of auction mechanism is proposed. Firms are invited to submit bi-dimensional bids on the following variables: (a) total revenues, net from maintenance and operation costs, to obtain from the concession; and (b) average annual amount of maintenance and operation expenses. Although this new mechanism is not completely error-free, outcomes from simulations are reasonably good in terms of selection of concessionaires and low probability of renegotiation.

The analysis of auctions for concession contracts must be extended in several directions. First, we have not considered the possibility of risk aversion. Bidders' attitudes with respect to risk are known to have effects on auctions' outcomes, (Holt, 1980; Maskin and Riley, 1984), so it would be interesting to examine what is the impact of considering different types of bidders. Asymmetries among bidders regarding costs and information is also another area which is of great importance for infrastructure projects. The issue of collusion should also be studied, as pointed out by Laffont (1997), because usually the number of firms participating in auctions for concessions is reduced, and the same agents participate repeatedly at auctions for projects. A final open question is the possible existence of a mechanism, simple enough to be used in practice, and that could be able to completely isolate firms' bids from their expectations about future demand levels.

Appendix

Derivation of bidding rule $P(I_i)$ for a minimum price auction considering the full range of feasible values for cost C_i

Consider that each bidder uses the same function $P(C_i)$ to calculate its optimal bid P_i, where $C_i = I_i + M_i\, T$ is the real full cost of the firm. The probability of firm i winning the auction against N-1 rivals ($prob_i$) is then equal to $(prob\ [P_i < P_j]\ )^{N-1}$, which, applying the inverse function $P^{-1}(\cdot)$, can be expressed as $(prob\ [P^{-1}(P_i) < C_j]\ )^{N-1}$. Using the cumulative distribution function defined in (27), $prob_i\ (P^{-1}\ [P_i])$ takes the form:

$$prob_i\,(\cdot)= \begin{cases} 1-\dfrac{1}{2\,I_r\,M_r\,T}\left(P^{-1}(P_i)-C_{min}\right)^2 & ;\ \ if \quad C_{min} \le P^{-1}(P_i) \le C_a \\[2ex] 1-\dfrac{M_r\,T}{2\,I_r}-\dfrac{1}{I_r}\left(P^{-1}(P_i)-C_a\right) & ;\ \ if \quad C_a < P^{-1}(P_i) \le C_b \\[2ex] \dfrac{M_r\,T}{2\,I_r}+\left[P^{-1}(P_i)\right]^2-2\,C_{\max}\,P^{-1}(P_i)-C_b^{\,2}+2\,C_{\max}\,C_b & ;\ \ if \quad C_b < P^{-1}(P_i) \le C_{\max} \end{cases} \tag{A.1}$$

where $I_r = I_{max} - I_{min}$; $M_r = M_{max} - M_{min}$; $C_a = I_{min} + M_{max}T$; $C_b = I_{max} + M_{min}T$; $C_{min} = I_{min} + M_{min}T$; $C_{max} = I_{max} + M_{max}T$. If firm i is participating at a price auction, in order to choose its optimal bidding rule $P(C_i)$, it solves:

$$Max_{P_i(C_i)}\ \Pi_i^{\,e} = \left(P_i(C_i)\,Q_i^{\,e}\,T - I_i - M_i\,T - Z\right)\quad prob_i\left(P^{-1}\left[P_i(C_i)\right]\right) \tag{A.2}$$

Solution to problem (A.2) has three different parts, according to the size of firm i's full cost C_i compared to the reference values C_a and C_b. These three parts stem from solving three separate differential equations obtained from the first-order condition of the maximisation problem (A.2):

(a) If C_{min} ∟C_i ∟C_a, the foc in this case is:

$$P'(C_i)-\frac{2\,(N-1)\,(C_i-C_{min})}{2\,I_r\,M_r\,T-(C_i-C_{min})^2}\ P(C_i)+\frac{2\,(N-1)\,(C_i-C_{min})\,(C_i+Z)}{Q_i^{\,e}\,T\left(2\,I_r\,M_r\,T-(C_i-C_{min})^2\right)}=0 \tag{A.3}$$

yielding the solution:

$$P_i = P(C_i) = \frac{1}{Q_i^e T}\left[C_i + Z - \frac{\int \left[2\, I_r\, M_r\, T - (C_i - C_{min})^2\right]^{N-1} dC_i}{\left[2\, I_r\, M_r\, T - (C_i - C_{min})^2\right]^{N-1}} \right] \qquad \text{(A.4)}$$

(b) If $C_a < C_i$ ∟C_b, the foc takes the form:

$$P'(C_i) - \frac{(N-1)}{I_r - \dfrac{M_r\, T}{2} - (C_i - C_a)} P(C_i) + \frac{(N-1)\,(C_i + Z)}{Q_i^e\, T \left(I_r - \dfrac{M_r\, T}{2} - (C_i - C_a) \right)} = 0 \qquad \text{(A.5)}$$

Solution for $P(C_i)$: see expression (28) in the text.

(c) If $C_b < C_i$ ∟C_{max}, the foc is:

$$P'(C_i) - \frac{2\,(N-1)\,(C_{max} - C_i)}{C_i^2 - 2\, C_{max}\, C_i + \dfrac{M_r\, T}{2\, I_r} - C_b^2 + 2\, C_{max}\, C_b} P(C_i) + \frac{2\,(N-1)\,(C_{max} - C_i)\,(C_i + Z)}{Q_i^e\, T \left(C_i^2 - 2\, C_{max}\, C_i + \dfrac{M_r\, T}{2\, I_r} - C_b^2 + 2\, C_{max}\, C_b \right)} = 0$$

(A.6)

and its corresponding solution:

$$P_i = P(C_i) = \frac{1}{Q_i^e T}\left[C_i + Z - \frac{\int \left[C_i^2 - 2\, C_{max}\, C_i + \dfrac{M_r\, T}{2\, I_r} - C_b^2 + 2\, C_{max}\, C_b \right]^{N-1} dC_i}{\left[C_i^2 - 2\, C_{max}\, C_i + \dfrac{M_r\, T}{2\, I_r} - C_b^2 + 2\, C_{max}\, C_b \right]^{N-1}} \right] \qquad \text{(A.7)}$$

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Table 1. Average outcomes from auctions (Case with no maintenance costs)

	Winner's real construction cost	Winner's expected profits	Winner's expected demand	Probability of selection error	Probability of contract renegotiation
Price auction	314.9	137.0	24.2	41.2%	38.5%
Payment auction	291.6	141.7	23.5	35.5%	37.9%
LPVR auction	248.3	150.3	19.9	0	0
Optimal expected values	**250**	**0**	**20**	**0**	**0**

Table 2: Average outcomes from auctions. Case with equal maintenance costs (*M*=5)

	Winner's real construction cost	Winner's expected profits	Winner's expected demand	Probability of selection error	Probability of contract renegotiation
Price auction	342.5	131.5	24.8	47.9%	49.9%
Payment auction	295.3	140.9	23.6	36.4%	38.3%
LPVR auction	282.5	143.5	22.9	28.9%	18.3%
LPVNR auction	251.5	149.7	20.0	0	0
Optimal expected values	**250**	**0**	**20**	**0**	**0**

Table 3: Average outcomes from auctions.
Case with different maintenance costs; $M_i \rightarrow U[3,7]$

	Winner's real construction cost	Winner's expected profits	Winner's expected demand	Probability of selection error	Probability of contract renegotiation
Price auction	343.0	132.1	24.8	47.5%	49.5%
Payment auction	298.4	141.2	23.6	36.6%	38.5%
LPVR auction	289.4	144.2	22.7	27.5%	16.2%
LPVNR auction (using M_{av})	281.7	143.6	19.7	18.8%	0.04%
Optimal expected values	**250**	**0**	**20**	**0**	**0**

Table 4: Average outcomes from an LPVNR auction with bi-dimensional bids

	Winner's real construction cost	Winner's expected profits	Winner's expected demand	Probability of selection error	Probability of contract renegotiation
LPVNR auction (B_i, E_i)	200.4	238.4	24.8	18.9%	0.02%
Optimal expected values	**250**	**0**	**20**	**0**	**0**