Myopic households on a stable path: the neoclassical growth model with rule-based expectations

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Myopic households on a stable path: the neoclassical growth model with rule-based expectations

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Abstract

The neoclassical growth model is extended to include limitations in the forecasting capability of a rational individual, who can predict the future state of the economy only for a short time horizon. Long-term predictions are formulated according to uninformed expectations, relying solely on myopic information about short-run dynamics, such as assuming a future persistent growth rate. Steady-state results are obtained in the case of iso-elastic utility and Cobb-Douglas technology. The model, characterized by forecasting errors and subsequent corrections, exhibits global stability and has relevant implications for welfare and policy. It is analyzed in comparison to the Solow–Swan model and the Ramsey model. Our approach, incorporating behavioral assumptions within a standard optimization rule, successfully yields explicit analytical solutions for the policy function in the neoclassical model. This strategy may also be extended to various modeling streams, including DSGE and HANK models.

Keywords: Expectations, Neoclassical growth, Bounded rationality, Myopic behavior, Dynamic optimization, Time inconsistency.

JEL Codes: C61, D83, D84, E21, E25, E71

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Introduction

The perfect foresight assumption of the neoclassical growth model (Ramsey, 1928, Koopmans, 1960, Cass, 1966) serves as the natural benchmark for evaluating alternative hypotheses on the formation of expectations. We posit that the household can accurately predict only the short-term impact of the saving rate on capital,[1] believing that this prediction will hold for a longer horizon. This myopic behavior reflects the individual's inability to see the future despite its rational intention to optimize inter-temporal utility over an infinite horizon. Among the many hypotheses on the way myopic households form expectations on the future state of the economy, it is reasonable to assume that they attempt to leverage the sole information at their disposal, namely, the short-term prediction on capital (or income) growth.[2] For instance, households may assume a constant growth rate in the future, extending their perceived short-run growth rate of capital to the long run. This assumption aligns with the primary objective of the paper, which is to explore a scenario in which households lack the ability to make reliable predictions about the future trajectory of the economy, being uninformed and thus forced to behave according to accessible strategies. We call "rule-based expectations" a predetermined notion of how the economy will evolve in the future, based on individuals' perception. The idea that short-term growth predictions will accurately hold for the long-term is, therefore, an example of myopic rule-based expectations. In this framework, the "perfect foresight" hypothesis can be considered a rule-based expectation, reflecting an individual's capacity to form consistent expectations with the underlying model, which implies knowledge of future interest rates. In all other cases of rule-based expectations, apart from perfect foresight, individuals will formulate incorrect predictions, but they will have the chance to adjust them at any point, taking into account updated information about the changed state of the world.

This process of expectation formation, where individuals rely on their current state (and different starting points lead to different expectations), has been coined "anchoring"[3] by Tversky and Kahneman, 1974, and typically results in forecast errors, as shown by Campbell and Sharpe, 2009 among others. The introduction of deviations from expected utility theory and/or perfect foresight behavior in the neoclassical growth model has been widely employed in the literature to provide a more comprehensive explanation of the empirical consumption smoothing stylized fact. For instance, Foellmi et al., 2011 employs prospect theory to introduce reference-based behavior (expectations) in the Ramsey model. This involves evaluating choices in relation to a certain reference point − the status quo − rather than in absolute terms.

The combination of myopia with rule-based expectations configures a dynamic sequence of static optimization problems that leads to the complete dynamic solution. Because the agent assumes that its current decision will last indefinitely, there is no need to estimate the feedback from the expected interest rate (which is dependent on future capital intensity) to consumption. Therefore, the myopic

[1] At date τ, the household can only predict the short-run evolution of capital stock as a function of saving rate $k_{\tau+1}(s_\tau)$ (discrete time) or $\dot{k}(s(\tau))$ (continuous time), but not the values of the following periods.

[2] Regardless of whether income or capital is selected as the variable to predict, the primary outcomes of the study remain unchanged. We will henceforth assume that households form expectations on future capital (see also footnote 6).

[3] For a complete survey on anchoring, see Furnham and Boo, 2011. The behavior of the individual in the current work is consistent with other observed cognitive anomalies, such as the "Reference point" effect and the "Status Quo/Endowment" effect. Further details on these anomalies can be found in McFadden, 1999.

problem-solving approach disregards the standard nexus that operates in the pure neoclassical growth model, leading to a breakdown of the inter-temporal relationship between consumption and capital intensity.

This paper is connected to several strands of literature, which will be discussed further, including Day's work on adaptive economizing agents (Day, 2000, Day, 1992), Barro's work on dynamic discount factor and time inconsistencies (Barro, 1999), and Evans and Honkapoya's work on learning and expectations (Evans and Honkapohja, 2001).

An important attempt to constrain the neoclassical growth model on a contingent plan (status quo) is the stream of literature dealing with the adaptive economizing agent (Day, 2000, Dawid, 2005). Agents use (linear) approximated representation of the production function and an adaptive expected interest rate. They are willing to leave to future generations (terminal condition) a constant level of capital.[4] All this provides a simplified future stream of expected utility value which leads to a static policy function for consumption, depending only on the current level of capital. Day's work demonstrates global stability and for some low impatience rates, limit cycles and chaos. In a similar way to the Day's economizing agents, the optimal solution for our myopic agent depends on a static policy function that is derived by assuming rule-based expectations. Therefore, the agents use a subjective solution of the model that is inconsistent with the rational expectation behavior that enables to solve the expected growth consistently with the structure of the economy. On this ground, we explore more general expectation rules, like exponential expectation or constant future capital levels, which represent opposite limiting cases of basic behavioral rules. We remark that the class of rule-based expectations that can be used within the model is flexible enough to incorporate realistic features that might descend from experiments or other empirical evidence. Our goal is to stress the properties in terms of welfare, dynamics and expenditure allocation of factor income, compared to the perfect foresight Ramsey model.

It's worth noting that, in our model, the evolving bias in expectations modifies the relative time preferences of the household, giving rise to a dynamic form of "effective impatience", resulting from the interplay of expectation errors and a fixed discount factor. This connects our work with the existing literature that addresses the issue of time inconsistency in optimal planning. Strotz, 1955 showed that, except in the case of exponential functions, varying time preferences lead to inconsistent plans that should be addressed with specific strategies. When agents apply time-varying discount factors, they can delay saving and increase current consumption, which may lead to sub-optimal outcomes without institutional settings that force agents to correct such behavior (via pre-commitment, for example). Pollak, 1968 and Goldman, 1980 demonstrated the condition for the existence of a consistent plan (equilibrium) under time-varying discount factors. Barro, 1999 extended the Ramsey model along these lines and found that the interplay between interest rates and variable rates of time preference generates dynamic effects similar to those resulting from differences in the rate of time preference in the standard model. However, in this literature, myopia stems from changes in individual preferences over time rather than from limited capabilities in the expectation formation process. In our work, instead, the sequential optimizing process creates time inconsistency, due to the interaction between rule-based expectations and impatience. Biased forecasts lead to time-varying utility plans, even when assuming constant time preferences, as they alter the perception of

[4] Another example of bounded rationality is found in Bellino, 2013, where it is assumed that agents project a constant income in the long-run horizon. Consumers set their consumption level sequentially, starting from the first period. This kind of adjustment process ensures stability.

discounted future income.

Barro's no-commitment setup leads to sub-optimal postponed saving, higher interest rate, lower capital accumulation and lower consumption and welfare levels, for every impatience level compared to the pure Ramsey case. In our work, expected constant growth leads to higher savings compared to Ramsey, due to a systematic overestimation of the effects of savings on future income, which translates into greater "effective" patience. The excess of savings becomes critical for very low values of impatience, leading rational and uninformed households to a dynamically inefficient region, resulting in a reduction in consumption and welfare. We identify an impatience threshold corresponding to higher capital accumulation compared to the Solow golden rule. This threshold generally falls within a range of realistic values for the interest rate, meaning that, in a world where agents are not able to perform accurate forecasting, sub-optimal long-run capital accumulation is a likely outcome and even a small time preference shock could lead to a significant reduction in welfare.

Our approach is also related to the extensive research on learning and expectations, expertly reviewed by Evans and Honkapohja, 2001. In the case of the Ramsey model, they demonstrate that the perfect-foresight saddle path is locally learnable, and agents can converge to the rational expectations equilibrium by employing adaptive rules. A mis-specified rational expectation equilibrium can arise from an incomplete representation of the saddle path. In such scenario, agents can only partially learn the intertemporal equilibrium. For example, Eusepi and Preston, 2011 enhanced the Real Business Cycle (RBC) model by introducing a learning process into the expectation formation mechanism. In this approach, beliefs function as a substitute for the precise relationship between exogenous variables, the aggregate economy, and market-clearing prices, leading to systematic forecast errors, even if the model's empirical fit is better than that of the original Kydland and Prescott, 1982.

Our contribution is similar in spirit, as the employment of myopic expectations results in a distortion of the relationship between expected interest (future capital productivity) and the saving decision, even though the agents exhibit forward-looking behavior. However, our approach is more parsimonious because it does not need to assume either the learning process of an equilibrium relation, or its mis-specification, as the analytical treatment allows for a direct comparison with the basic neoclassical model. Therefore, myopic agents with rule-based expectations tend to converge to an equilibrium different from the REE, even when making decisions based on an optimization process. Our behavioural assumption, along with the rolling optimization procedure and the particular rule-based expectation, even in a forward-looking behaviour, leads to a sub-optimal condition (Euler-equation) that lacks the forward-looking element (expected marginal productivity of capital). Expectation exuberance or under-exuberance is explicitly assumed, avoiding the need to impose super-rational behaviour to get a unique solution and stability.

The main contribution of this work is to introduce myopia alongside rule-based expectations into the neoclassical analytical model and to study analytically its properties and implications in terms of dynamic stability, welfare, and factor allocation. The model is compared with the classical Ramsey model and the Solow model in the case of golden rule savings, to highlight its characteristics more clearly. Compared to the Ramsey model, the main difference lies in the inability for agents to perform accurate long-horizon optimization and, therefore, to internalize the expected interest rates. This generally leads to a globally stable equilibrium rather than to saddle path (in)stability. Expectations of constant growth also lead to richer dynamics, including the possibility of accumulating too much capital, i.e., remaining in the region of dynamical inefficiency that is excluded in the Ramsey model. The excess savings, generated by the individual's bias on expectations, can lead to a reduction in the economy's welfare level. Therefore, compared to Ramsey, where agents' impatience always leads to a reduction in the saving rate, in the myopic case, there is also the reciprocal case where an excess of patience can lead the economy to excessive investment levels and a deterioration in welfare, compared to the optimum represented by Solow's golden rule. This can be also analyzed under a perspective of the interplay between the allocation of income to factors and factors' spending. In the Ramsey model, the high returns of capital always generate a capital income surplus that finances consumption spending, after the replacement of depreciated capital. On the other hand, the myopic model shows a more comprehensive behavior, as it can represent the case when part of the labor income must be used in steady state to finance the depreciation of the large accumulated capital. The Myopic household fears the effects of capital depreciation and is willing to contribute a share of labor income to maintain it, even at the cost of negative interest rates.

The paper is organized as follows: section 1 presents the model in the case of exponential growth expectations, section 2 discusses its properties with respect to the Ramsey–Cass–Koopmans (RKC) and Solow models, section 3 extends the model to include a different rule-based expectation mechanism and discusses the dynamic properties of the model. The last section concludes.

1 The model

We assume a representative household as in the standard neoclassical model. To simplify the analysis, we do not consider technological progress or population growth. The expected utility at date τ can be defined as,

$$U(\tau) = \int_{\tau}^{\infty} e^{-\rho(t-\tau)} u\big[E_{\tau}\{c(t)\}\big]\, dt \tag{1}$$

where $u[\cdot]$ is utility with the usual properties, $\rho > 0$ is the constant rate of time preference, and $E_{\tau}\{c(t)\}$ indicates the forecast[5] at date τ for the future value of the variable c at time t. Assuming perfect foresight, $E_{\tau}\{c(t)\}$ becomes simply $c(t)$, leading to the conventional Ramsey model.

We call a household "myopic", if it makes decisions at time τ, according to its current information set and forecasting capability, which is perfect only for a short time ahead. The household is still rational and willing to optimize its intertemporal utility on an infinite horizon, however, it needs to use some rule-based criteria to predict the future state of the economy. In particular, the household will make use of its "short time ahead" information in order to predict the values of future variables.

If we consider the saving rate as the decision variable, and income as the variable to predict, expected utility in τ should be maximized according to:

$$\max_{s(\tau)} \quad \int_{\tau}^{\infty} e^{-\rho(t-\tau)} u\big[\big(1-s(\tau)\big)E_{\tau}\{y(t)\}\big] dt \tag{2}$$

The problem for the household is to decide $s(\tau)$ in order to optimize $U(\tau)$, given current expectations on future income $E_{\tau}\{y(t)\}$, which depend on the instantaneous capital growth rate $\frac{\dot{k}(\tau)}{k(\tau)}$. Then, at each following date ($> \tau$), the

[5] $E_{\tau}\{x(t)\}$ represents the predicted value of the deterministic variable $x(t)$ that an individual does not know, due to either insufficient information or the inability to utilize the available information. This should not be confused with the role of expectations in stochastic models.

household makes a new decision. In other words, the household is not capable to choose dynamically an infinite stream of future saving rates but it myopically chooses a static one ($s(t) = s(\tau)\ \forall\ t > \tau$), which optimizes inter-temporal utility, given its forecast about the evolution of future income. Then it repeats the process on each subsequent date.

One simple assumption on the myopic expectation formation process at date τ is that the household believes that capital[6] will grow in the future at the observed short-run rate $\dot{k}(\tau)/k(\tau)$, thus anchoring long-term growth expectations to short-term ones.

If we consider a Cobb Douglas production function $y(t) = k(t)^\alpha$, where $y(t)$ is output per worker and $k(t)$ is capital per worker, the expected future income becomes,[7]

$$E_\tau\{y(t)\} = E_\tau\{k(t)^\alpha\} = \left[k(\tau)\left(1+\frac{\dot{k}(\tau)}{k(\tau)}\right)^{t-\tau}\right]^\alpha. \tag{3}$$

If utility takes the iso-elastic form,

$$u(c) = \frac{c^{1-\theta}-1}{1-\theta}, \tag{4}$$

and we substitute 3 into 2, the consumer problem takes the form,

$$\max_{s(\tau)} \quad U(\tau) = \int_\tau^\infty e^{-\rho(t-\tau)} \frac{\left[\left(1-s(\tau)\right)k(\tau)^\alpha\left(1+\frac{\dot{k}(\tau)}{k(\tau)}\right)^{\alpha(t-\tau)}\right]^{1-\theta}-1}{1-\theta}\, dt \tag{5}$$

subject to $\dot{k}(\tau) = s(\tau)k(\tau)^\alpha - \delta k(\tau)$.

We report here some key steps of the solution, provided in appendix A, focusing for the moment on the special case of logarithmic utility $u(c) = \log(c)$. The first step is finding the optimal saving rate as a function of current capital, $s(\tau) = s(k(\tau))$,

$$s(\tau) = \frac{\alpha - \rho(1-\delta)k(\tau)^{1-\alpha}}{\alpha+\rho}, \tag{6}$$

which allows us to derive the dynamics of capital in the myopic model,

$$\frac{\dot{k}}{k} = \frac{\alpha}{\rho+\alpha}k^{\alpha-1} - \frac{\rho+\alpha\delta}{\rho+\alpha}. \tag{7}$$

Equations 6 and 7 are obtained by equations 21 and 22 in appendix A in the special case of logarithmic utility ($\theta = 1$). The dynamic optimization problem becomes a static problem, where the household finds the best $s(\tau)$, which in turn determines the new capital endowment. Then, the household will make a new decision on savings, based on the new level of capital and new expectations of capital growth, which takes into account information acquired in the last step. The process continues until the steady state is reached:

[6]If the household expects a constant growth of income, rather than capital, i.e., $E_\tau\{y(t)\} = y(\tau)\left(1+\frac{\dot{y}(\tau)}{y(\tau)}\right)^{t-\tau} = k(\tau)^\alpha\left(1+\alpha\frac{\dot{k}(\tau)}{k(\tau)}\right)^{t-\tau}$, it can be shown that the steady state of the economy is not affected. Equations 8 and 9 still hold.

[7]Eq. 3 is a first-order Taylor expansion of exponential expectations, which may be more intuitive to compute by the myopic households. In any case, even for exponential expectations, i.e., $E_\tau\{k(t)\} = k(\tau)e^{\frac{\dot{k}(\tau)}{k(\tau)}(t-\tau)}$, the same steady state of eqs. 8 and 9 still holds.

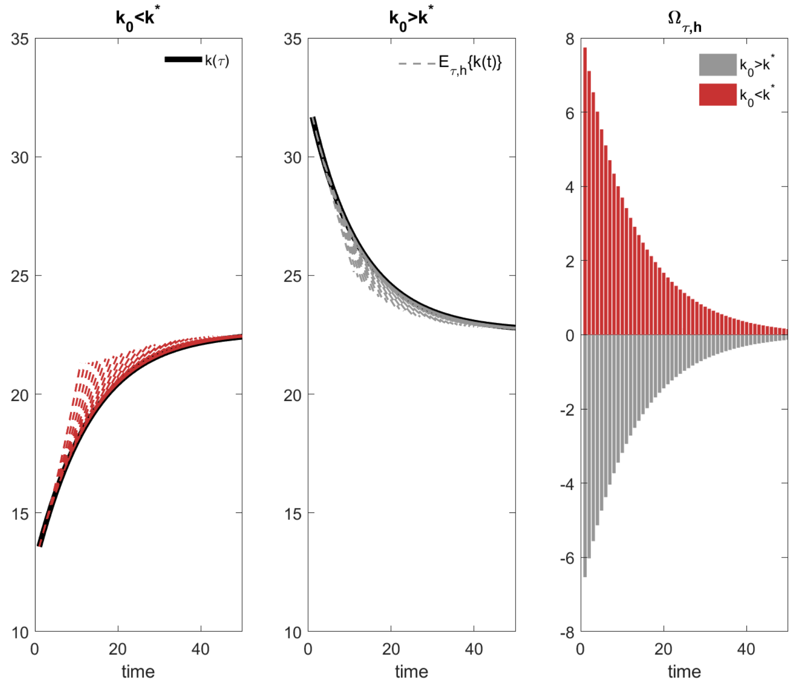

Figure 1: Myopic model solution (black solid) for capital vs 10 periods ahead rolling expectation (dotted). Negative shock from equilibrium capital stock (red, left). Positive shock (grey, center). Percentage error (right) (eq. (10)).

$$k^* = \left(\frac{\alpha}{\rho+\alpha\delta}\right)^{\frac{1}{1-\alpha}} \tag{8}$$

$$s^* = \frac{\alpha\delta}{\rho+\alpha\delta} \tag{9}$$

The household makes a prediction error Ω when forecasting future capital, which depends on the current date τ and the prediction horizon h.

$$\Omega(\tau,h,k) = E_{\tau,h}\{k(t)\} - k(\tau+h) = k(\tau)\left(1+\frac{\dot{k}(\tau)}{k(\tau)}\right)^h - k(\tau+h), \tag{10}$$

This prediction error, at any given date τ, is positively related to the prediction horizon h, but decreases as the economy approaches its steady state. Equation 10 demonstrates that, for a finite horizon, the prediction error is zero as τ tends to infinity, because $\dot{k}(\tau)$ approaches zero and $k(\tau)$ tends to k^* (see figure 1 as an example). Additionally, as τ approaches infinity, the prediction error also converges to zero for an infinite horizon (see appendix B.1 for the proof). This indicates that the household does not make any systematic error and correctly forecasts the future state of the economy as it approaches equilibrium.

The dynamic properties of the error are simulated and shown in figure 1 for the logarithmic utility case. Capital per worker converges to the steady state, after a shock of $\pm 40\%$ from the equilibrium capital stock. The figure includes dotted lines representing different forecast vintages, each starting on a different date and considering a forecast horizon of $h = 10$ periods. As previously mentioned, the perceived law of motion (based on exponential rules of expectation) and the actual law of motion interact. Even if agents anticipate a positive or negative exponential law of motion, the periodic revision of expectations leads to a stabilizing process.

Overall, the tension between rule-based expectations and bounded rationality, brought about by periodic re-optimization, constrains the parametric space available to achieve equilibrium determinacy. We can demonstrate (as shown in appendix A.1) that such a dynamic process has a parametric restriction for the stability condition that needs to be satisfied in the case of a CRRA utility function:

$$\rho > \alpha(1-\theta)ln(1+\gamma_k), \tag{11}$$

where $\gamma_k = sk(\tau)^{\alpha-1} - \delta$ represent the constant capital growth rate estimated instantaneously. Similarly to the Ramsey model,[8] there are different forces at play: the constant rate of time preference ρ, the household's willingness to shift consumption between periods θ, and the myopic expectation bias $\alpha ln(1+\gamma_k)$. The myopic bias leads to an overestimation of the future consequences of today's savings on income growth, resulting in an excessive shift of consumption towards the future. This delay in consumption can cause the optimization problem to diverge, which can be prevented by a sufficiently high impatience (sufficiently large ρ) or a strong willingness to smooth consumption (θ close to 1). In the case of logarithmic utility ($\theta = 1$), convergence can be achieved with a positive discount rate ($\rho > 0$), as in the Ramsey model with no trend in technology or population.

The equilibrium in equations 8-9 is a globally stable fixed point with speed of convergence λ, derived in appendix B.3,

$$\lambda = \frac{(\rho + \alpha\delta)(1-\alpha)}{\alpha + \rho} > 0. \tag{12}$$

The analogous problem in discrete time yields the same steady state values of equations 8 and 9, provided that the standard discount rate transformation $\beta = 1/(1+\rho)$ is applied, as demonstrated in appendix C.

2 Comparison with Ramsey and Solow models

In this section, we compare the properties of the myopic model to those of the neoclassical models with non-optimizing and optimizing agents, namely the Solow-Swan at the golden rule (GR henceforth) saving rate and the Ramsey (R) models, respectively. As shown in the previous section, the myopic model differs from the Ramsey model because the household is unable to predict correctly the future evolution of the economy. As a result, it makes an error that gradually decreases when approaching the equilibrium. However, the accumulation of errors committed by the household affects the dynamics of the model and generates a steady-state that is different from the one of the Ramsey model. Table 1 displays the values of several key variables at steady-state for the different models under consideration.

2.1 Steady state

It may be useful to remember that sharing the same level of consumption across different generations, as happens in the Solow model with the golden rule, implies a null relative price between future and current consumption, that is, a

[8] In the Ramsey model the transversality condition imposes a similar parameter restriction:

$$\rho > (1-\theta)g,$$

where g is the growth rate for technology. The condition imposes that discounting is a stronger force than the growth rate of capital to have finite intertemporal utility objective function, and collapses to $\rho > 0$ for stable population and productivity. The condition resembles equation (11), with the exception that the exogenous growth g is replaced by the household's expectation of growth, which determines their choices.

Variable	Golden rule	Ramsey	Myopic
Interest rate r^*	0	ρ	$\rho-\delta(1-\alpha)$
Capital stock k^*	$\left(\frac{\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}$	$\left(\frac{\alpha}{\rho+\delta}\right)^{\frac{1}{1-\alpha}}$	$\left(\frac{\alpha}{\rho+\alpha\delta}\right)^{\frac{1}{1-\alpha}}$
Saving rate s^*	α	$\frac{\alpha\delta}{\rho+\delta}$	$\frac{\alpha\delta}{\rho+\alpha\delta}$
Consumption rate $1-s^*$	$1-\alpha$	$\frac{\rho-\delta(1-\alpha)}{\rho+\delta}$	$\frac{\rho}{\rho+\alpha\delta}$

Table 1: Steady state values for Solow, Ramsey and Myopic models

null interest rate. In general, at equilibrium, the capital steady state is the ratio between the investment share s and the dismissed capital rate δ, which at the GR level corresponds to α/δ, modified by the elasticity of substitution between labor and capital $1/(1-\alpha)$.[9]

Introducing consumer optimization as in the Ramsey model means assuming inter-temporal selfishness. In this case utility is not discounted uniformly over time and generations, due to a positive impatience rate ρ. Such impatience implies to anticipate consumption and to introduce a positive interest rate at the equilibrium, $r^* = f'(k^*) - \delta = \rho$. This anticipation erodes optimal savings and therefore reduces the capital steady state level, as analytically ρ enters in the numerator and introduces a perpetual higher discounting $\alpha/(\rho+\delta)$. In the case of the myopic behavior, the steady state level has the same form of the Ramsey model, except for a lower impact of capital depreciation $\alpha/(\rho+\alpha\delta)$, originating from the overestimation of the capital accumulation process, which translates in a diminished perception of capital depreciation. The internalization of the technology in the myopic expectation process alters the neoclassical equilibrium. Different technology assumptions would lead to different deviations from the Ramsey model steady state.

$>$	k^*_{gr}	k^*_r	k^*_m
k^*_{gr}	-	$\rho \geq 0$	$\rho > \delta(1-\alpha)$
k^*_r	$\rho < 0$	-	$\alpha \geq 1$
k^*_m	$\rho < \delta(1-\alpha)$	$\alpha < 1$	-

Table 2: Ranking of capital steady state values as a function of parameters

Table 2 displays the relationship between the steady-state capital levels of the three models. For standard parametrization ($\rho \geq 0,\ \alpha < 1$), the Ramsey model always has a lower steady-state capital level compared to both the Solow model, i.e., $k^*_r < k^*_{gr}$, and the Myopic model, i.e., $k^*_r < k^*_m$. The first inequality depends on the household's impatience. The second, instead, depends on the bias in the expectations of the myopic agent, who tends to overestimate future income changes. When starting from a low capitalization, the household tends to overestimate the effect of savings on future capital growth, investing more

[9] If we consider the equation $\dot{x} = \phi x^z - \psi x^y$, where ϕ and ψ are the birth and death rates, and z and y are returns to scale parameters, the implied steady state is

$$x = \left(\frac{\phi}{\psi}\right)^{\frac{1}{y-z}}.$$

This shows the Solow model as a special case of a birth and death process.

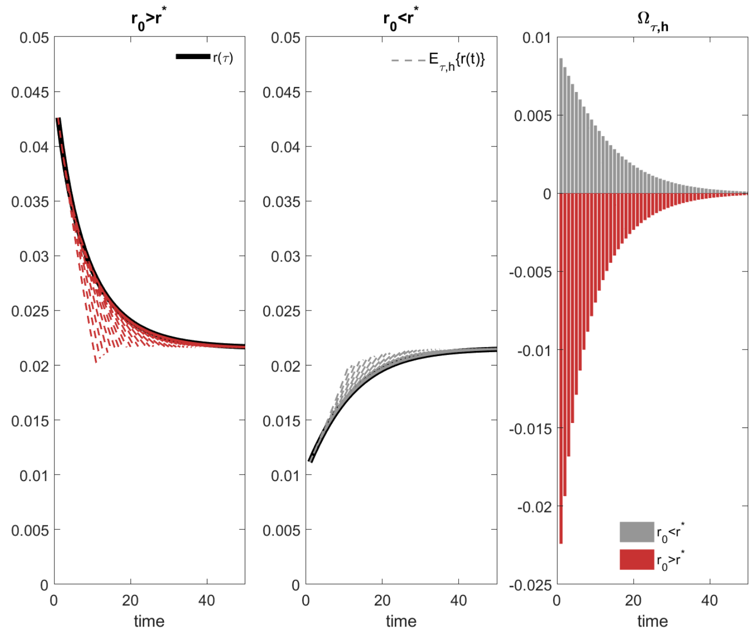

Figure 2: Myopic model solution (black solid) for interest rate vs 10 periods ahead rolling expectation (dotted). Negative shock from equilibrium interest rate (red, left). Positive shock (grey, center). Percentage error (right).

than in the Ramsey case and accumulating more capital. In the case of excess capitalization, the myopic household still tends to overestimate the negative effect of a reduction in savings on future income and therefore chooses to maintain a higher saving rate ($s_r^* < s_m^*$), resulting again in a steady-state capital higher than in Ramsey. We will discuss on the consumption level ranking in section 2.2.

Symmetrically, the equilibrium interest rate in the myopic case is lower than Ramsey ($r_r^* > r_m^*$), which is always positive in the presence of impatience.

The inability to accurately predict the future leads households to make an error in calculating the expected interest rate. The process is similar to what has already been observed in the case of capital, and a graphical representation of the error, for horizon $h = 10$, is given in figure 2. Specifically, we define the expected future interest rate at time t, as perceived at time τ, as:

$$E_\tau\{r(t)\} = E_\tau\{f'(k(t)) - \delta\} \tag{13}$$

In the case of Cobb-Douglas technology, given the expectations about the future value of capital $E_\tau\{k(\tau)\}$ from equation 3, we obtain:

$$E_\tau\{\alpha k(t)^{\alpha-1}\} - \delta = \alpha k(\tau)^{\alpha-1}\left(1+\frac{\dot{k}(\tau)}{k(\tau)}\right)^{(t-\tau)(\alpha-1)} - \delta \tag{14}$$

We can therefore define the error made in predicting the interest rate with horizon h similarly to equation 10, as $\Omega(\tau,h,r) = E_{\tau,h}\{r(t)\} - r(\tau+h)$. Appendix B.3 demonstrates that the interest rate at the steady state is $r^* = \rho - \delta(1-\alpha)$ and that $\lim_{\tau\to\infty}\Omega(\tau,h,r) = 0$. As observed for capital, the error $\Omega(\tau,h,r)$ tends to zero for sufficiently large times, indicating that the agent in the steady state is able to accurately predict future interest rates.

Replacing the perfect foresight assumption with exponential rule-based expectations generates a deviation of $\delta(1-\alpha)$ from the Ramsey model in the equilibrium interest rate. This deviation, which we label as "myopic bias", is due to the fact that the household is unable to properly internalize the net benefit of holding a higher marginal unit of capital. We can define,

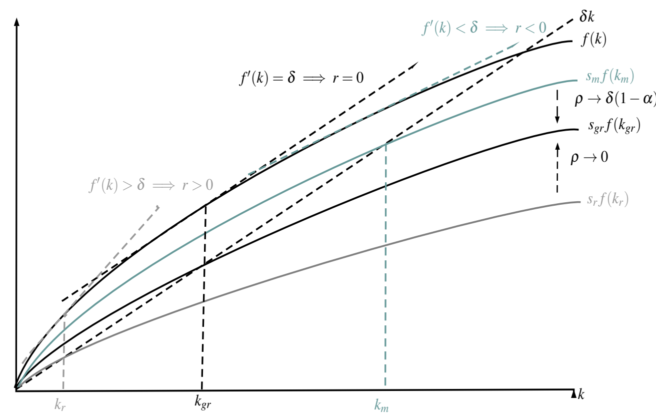

Figure 3: Ramsey (grey), Solow (black) and Myopic (green) steady state capital. If $\rho \to \delta(1-\alpha)$ the myopic investment $s_m f(k_m) \to s_{gr} f(k_{gr})$ and $k_m \to f(k_{gr})$. $\rho \to 0$ then $s_r f(k_r) \to s_{gr} f(k_{gr})$ and $k_r \to f(k_{gr})$.

$$\rho_m = \rho - \delta(1-\alpha) = r_m^* < r_r^* = \rho, \tag{15}$$

where ρ_m represents a sort of "equivalent impatience", partly generated by the individual's time preference and partly by their imperfect expectations. When the value of impatience equals the myopic bias, i.e., $\rho = \delta(1-\alpha)$, two conditions are met: the steady-state capital of the myopic model equals the golden rule capital ($k_m^* = k_{gr}^*$, see table 2), and the equilibrium interest rate r_m^* is zero. Otherwise[10], if $\rho > \delta(1-\alpha)$, then $k_m^* < k_{gr}^*$ and $r_m^* > 0$; and if $\rho < \delta(1-\alpha)$, then $k_m^* > k_{gr}^*$ and $r_m^* < 0$. The last condition states that when the household has a high level of patience, the steady-state capital accumulation is larger than the Golden rule level and, therefore, interest rate is negative. This specific scenario is depicted in Figure 3, which shows the standard textbook representation of the steady state for the three economic models under consideration.

It is well-known that in the Ramsey model, capital cannot be to the right of the golden rule capital because the solution would be dynamically inefficient. In other words, the individual could lower their savings rate to improve their utility in both the short and long run. However, the steady-state of the myopic model can remain in this situation of excess savings, as a decrease in the saving rate would lead to a significant decrease in future expected income, according to the myopic household's perspective. This would discourage such a reduction in savings.

2.2 Welfare and policy

The focus of this section is to examine the discrepancy between the maximum steady-state consumption, which represents a benchmark achievable under the Solow golden rule saving level ($s = \alpha$), and consumption levels attained in the Ramsey (r), Barro (b) and Myopic (m) models. To quantify this difference, we calculate the percentage deviation of c_i from c_{gr}, where $i = \{m, r, b\}$. Figure 4 presents this deviation for various values of $\alpha = [0.2, 0.3, 0.4]$ and $\delta = 0.03$.

The use of rule-based expectations introduces time-inconsistent plans that result from the biased expected growth, which alters the perceived discounted stream

[10] In the limiting case of full patience ($\rho = 0$), it is optimal for the agent to save the entire output ($s_m = 1$). This leads the agent to reduce consumption to zero.

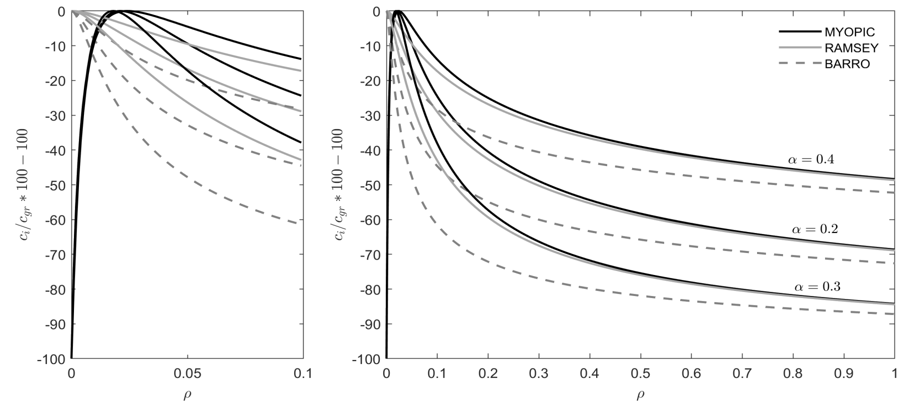

Figure 4: Percentage Deviation of Steady State Optimal Consumption for different values of $0 < \rho < 1$. Golden Rule vs Myopic, Ramsey, and Barro (1999) no commitment Model. $\Delta_c(\rho)_i = \frac{c(\rho)_i^*}{c(\rho)_{gr}^*}100 - 100$.

of utility. At each time step, a new plan arises, leading to modifications in consumption allocation. This heterogeneity ultimately leads to higher consumption given a higher capital accumulation. Specifically, when the time preference parameter falls within the range of $\delta(1-\alpha) < \rho < 1$, the Myopic model yields greater welfare than the Ramsey model.

We emphasize that comparing welfare should be perceived merely as an ex-post evaluation of consumption levels between two distinct economies. It is evident that any path derived from myopic behavior and assessed within the discounted utility function of a standard Ramsey-type optimization plan would yield lower expected utility than the optimal solution advocated by pure neoclassical behavior.

In contrast to the Ramsey model, the impact of time preference ρ on welfare in the Myopic model is non-monotonic. This is due to the fact that expectation-biased growth drives capital allocation towards the Golden Rule level within the range of admissible values of impatience, i.e, $\rho = \delta(1-\alpha)$. The maximum welfare attained by the Myopic model is, however, fragile, as it is vulnerable to potential positive shifts in patience that can lead to a substantial drop in consumption.

Indeed, for $\rho \to 0$, the myopic patient household tends to save their entire income ($s_m \to 1$) in anticipation of a very strong future growth that makes saving today convenient. More generally, for values of ρ lower than $\delta(1-\alpha)$, myopia leads to excess capital accumulation, rewarding the choice of over-saving at the expense of economic welfare. As seen in the previous section, the dynamically inefficient zone for the Ramsey agent is instead a zone where the myopic agent can remain, by virtue of their optimization process which involves an overestimation of the future income response to variations in the saving rate.

This property of the myopic model has interesting implications for economic policy, as it predicts the possibility that households without perfect foresight may rationally choose to save too much, with negative consequences for economic welfare. This occurs particularly when households are inclined to overestimate future economic growth, or rather the effect of savings on future growth of income. For instance, if there is a prevalent tendency in economic and financial expectations to overestimate financial investments returns, with a consequent incentive for households to save excessively, then there is room for policy to disincentive excess of savings.

If this were the case, it would suggest the need for policy intervention aimed at mitigating the potentially negative effects of the mechanism previously described, namely the risk of reduced welfare due to excessive savings. Policy interventions such as stimulating consumption or increasing public spending, among other possibilities, could be considered. For example, a policymaker, internalizing the inefficiency measured as biased welfare with respect to the full-commitment rational case, could incentivize consumption plan adjustments by using fiscal policy. It is worth noting that myopic optimization, tied to the anchored expectation hypothesis, leads to a different form of time inconsistency in our model compared to Barro's. In Barro's scenario without commitment, the time-inconsistent household can not fully commit over the infinite optimization horizon, resulting in higher effective time preference, higher interest rate, reduced capital, lower consumption, and welfare. In our case, the incentive to deviate from the fully time-consistent committed plan (Ramsey solution) arises due to sequential time windows splitting the optimization plan. Myopic agents don't vary in time preference but in expectations due to updated information altering the expected capital growth rate. In contrast, for high impatient rate, the interest rate is lower leading to higher savings and capital accumulation. Consequently, the final optimization yields a much higher welfare levels in equilibrium compared to both the full-commitment case and Barro's no-commitment case except for very patient individuals. The myopic bias also affects the expenditure allocation of factor income. Two metrics, the relative excess of wage over consumption $(1-c/w)$ and the relative excess of capital income (π) over investments $(1-i/\pi)$, allow us to make some considerations about expenditure allocation of income factors, despite the simplified structure of the representative agent economy. They measure the percentage of wages that is not spent on consumption and the percentage of capital income that is not allocated to investment.

$$\begin{cases} 1-\frac{c}{w}=\frac{s-\alpha}{1-\alpha} \\ 1-\frac{i}{\pi}=\frac{\alpha-s}{\alpha} \end{cases} \tag{16}$$

It is known that at the maximum level of consumption, reached in the case of the golden rule, the flow of income that remunerates capital is equal to investments $(\pi_{gr}=i_{gr})$, and the flow that remunerates labor is equal to consumption $(w_{gr}=c_{gr})$. In the Ramsey model, it holds that $\pi_r > i_r$ and $w_r < c_r$, meaning that part of the capital income is always used to purchase consumption. The equilibrium interest rate is sufficiently high to ensure a surplus to finance consumption, after the replacement of depreciated capital. On the other hand, wage is obviously not enough to buy consumption.

In Figure 5, we display the metrics defined in Equation 16 as a function of natural logarithm of the impatience rate ρ, for both the Myopic and the Ramsey model. As impatience increases, the proportion of consumption purchased by capital income also increases. In this pursuit of immediate consumption, the capitalist sacrifices future growth by using capital income to buy more goods today. The Myopic household behaves like the Ramsey household under normal conditions, i.e., as long as $\rho > \delta(1-\alpha)$. However, once this threshold is crossed, part of the labor income must be used in steady state to finance the depreciation of the large accumulated capital. The Myopic household fears the effects of capital depreciation and is willing to contribute a share of labor income to maintain it, even at the cost of negative interest rates. However, the slope of the curve shown in Figure 4 in this region is very steep, and the risk of welfare loss due to potential fluctuations in time preferences, expectations, or other shocks can be significant.

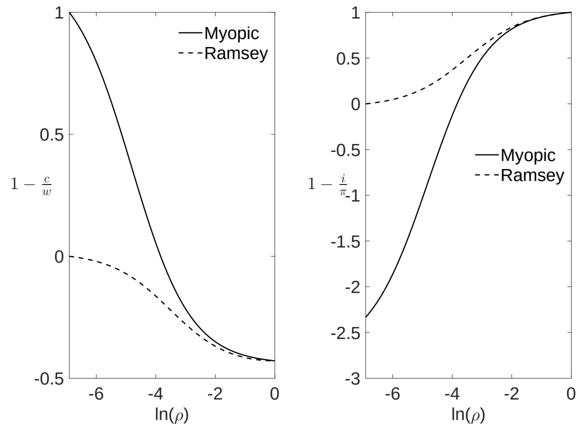

Figure 5: Excess wage over consumption and excess capital income over investments

2.3 The dynamics

The myopic nature of expectations, independently from their specific form assumed like the exponential proposed until now (see section 3 for a generalization), would lead to an essential difference in the nature of the solution obtained in the neoclassical model. Indeed, the standard Euler equation relates not the level but the consumption growth rate to the interest rate deviation from the time preference. The primary difference is due to the inability for the myopic agent to foresee the exact evolution of the economy and to incorporate it in the expectations. Agents use a certain belief of the future, given the local and partial knowledge of the economy. This process translates into solving a static intertemporal optimization problem. In this way, the agent gets the optimal (biased) policy function explicitly. In the Ramsey model, instead, the dynamic optimal condition implies solving for the control variable's initial condition (consumption) to start on the top of the saddle path and converge to the long-run equilibrium, i.e. to impose optimality via the transversality condition. Moreover, the general policy function is unknown, and in general, a numerical solution has to be adopted to obtain an equivalent form as in our optimal analytical condition (6). Such condition dictates myopic households to set the level of the saving rate $s(\tau)$ anchoring it at the observed current level of capital. Such property would have two significant consequences: the global stability and the empirical relevance of the model.

We can appreciate the stability properties by re-examining the saving rate equation (6). If we rearrange that equation, we will have the following:

$$i = sy = \frac{\alpha y}{\alpha+\rho} - \frac{\rho(1-\delta)k}{\alpha+\rho}.$$

The agent will set investment (sy) as the difference between the capital share of income (αy) and the discounted value of capital ($\rho(1-\delta)k$). Both elements are discounted by the factor $(1/(\alpha+\rho))$, which is the result of constant discounting $(1/\rho)$ and the exponential over discounting $(1/\alpha)$.

The resulting dynamics is represented in figure 6 where we can identify the optimal path converging to the myopic equilibrium given an initial condition on capital, i.e., simulating the system of equations (6) and the capital law of motion $\dot{k}(\tau) = s(\tau)k(\tau)^{\alpha} - \delta k(\tau)$, with parameters $\rho = 0.05, \alpha = 0.3, \delta = 0.1$. As shown, in contrast with the neoclassical model, the dynamics is not saddle-path unstable

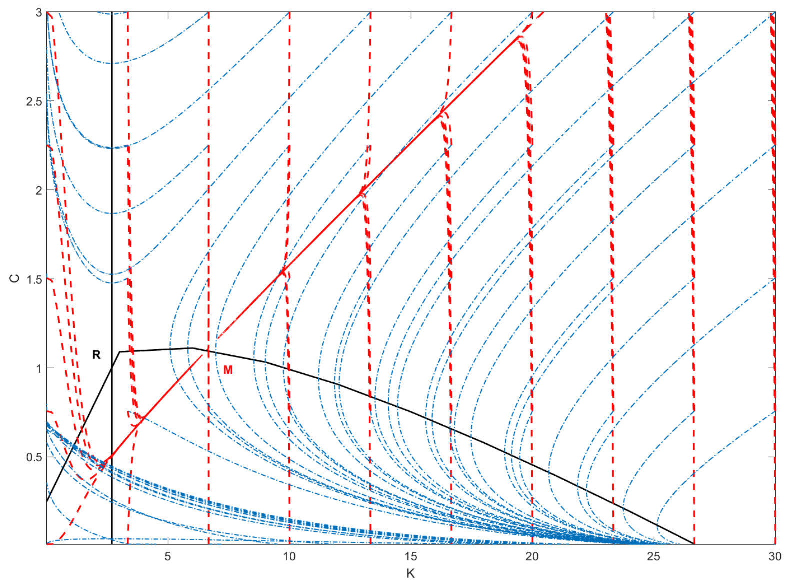

Figure 6: Myopic and Ramsey model phase spaces and gridded initial conditions. Dashed blue and dashed red lines are respectively the unstable Ramsey and the stable Myopic model trajectories. Myopic equilibrium (M) is globally stable, while Ramsey equilibrium (R) is saddle path un-stable.

but globally stable as, given any combination of $(s(\tau), k(\tau))$, the model reaches the myopic stable arm approaching the steady state.

Linearizing the myopic model around the steady state and calculating the speed of convergence as in equation (12) determines the model's global stability in the neighborhood of the steady state, that is granted for any admissible parameter values $(0 \leq \delta \leq 1,\ 0 \leq \alpha \leq 1,\ 0 \leq \rho \leq 1)$. Moreover, the myopic behavior implies a lower speed of convergence compared to Ramsey.[11] Indeed, in figure 7, we compare the non-linear speed-of-convergence, showing that the myopic model has a slower convergence for any initial conditions.

In general, a slower speed of convergence grounded on limited information and behavioural bases implies empirically more relevant properties as it better explains the convergence process between advanced and emerging countries. The literature often argues that the time required for emerging countries to catch up to advanced ones is longer, highlighting a limitation of the Ramsey model. King and Rebelo, 1993 initially raised this concern, later discussed in Barro and Sala-i-Martin, 2003. They proposed augmenting the capital share by encompassing human capital in an expanded definition, suggesting a parameter $\alpha = 0.75$, significantly higher than empirical observations. In our model, the myopic bias helps to fix the empirical relevance of the transitional dynamics. For example, for $\alpha = 0.3$ as shown in figure 8, the Ramsey model has the counterfactual implication that the initial value of the speed of convergence γ_y is implausibly large (17%), while in the myopic model,

[11] The neoclassical model convergence is given by:

$$\lambda = \frac{1}{2}\left(\left[\rho^2 + 4(1-\alpha)(\rho+\delta)\left(\frac{\rho+\delta}{\alpha} - \delta\right)\right]^{\frac{1}{2}} - \rho\right)$$

with constant population and technology, as provided in Barro and Sala-i-Martin, 2003.

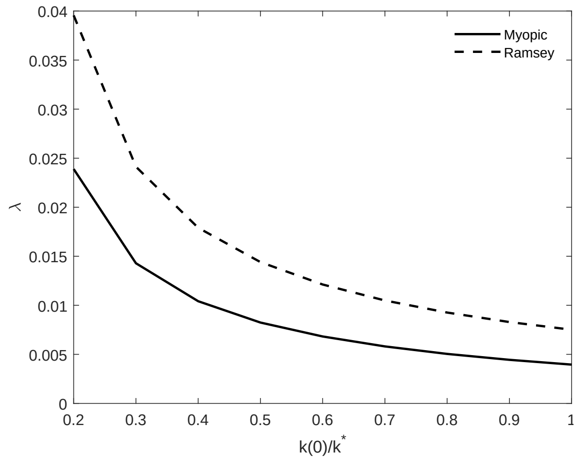

Figure 7: Non-linear speed of convergence versus initial condition level. Ceteris paribus, the myopic model has a lower speed of convergence.

the initial value is lower. Another important stylized fact in transitional dynamic is the lower interest rate $r(0)$ implied by the myopic behavior is 20% against the 40% implied by the neoclassical model.

In essence, the myopic model conciliates more with the Euler-puzzle (Kremer et al., 2019), which posits the empirical observation that high interest rates and high consumption growth rates are inconceivable together as implied by the neoclassical model. All else being equal, not only are interest rates lower in the early stages of growth convergence but also consumption growth rates are lower in the myopic model.

Once again we stress the fact that the limited-information and computational ability of the agent - embedded in the expectation formation process leading to extrapolate long term properties of the economy using the observed relationship between the control variable (saving rate) and the rate of growth of the capital and income - would lead to a more plausible dynamic behavior of the economy. This notion is supported by analyses advocating for the integration of behavioral elements in contemporary versions of macroeconomic models that retain the underlying neoclassical structure. For instance, Jang and Sacht, 2022 substitute the rational expectation assumption with forecast heuristics in a New-Keynesian DSGE model, thereby enhancing the model's capacity to explicate the persistence of consumption fluctuations.

3 Generalizing rule-based expectations

The entire approach described in the preceding sections depends on the assumption that has been made about how economic agents with limited rationality form their expectations about the future. As soon as one deviates from the idea that individuals have perfect foresight, one falls into an inevitable variety of alternative assumptions: there is one way to be rational and thousands of ways not to be. Even if we stick to the idea that individuals anchor their expectations to some value, the selected variable that serves as an anchor may change, thus affecting the structure of the expectations.

The hypothesis of the paper was that the myopic individual were able to calculate the short-term capital level for each possible saving rate choice and therefore

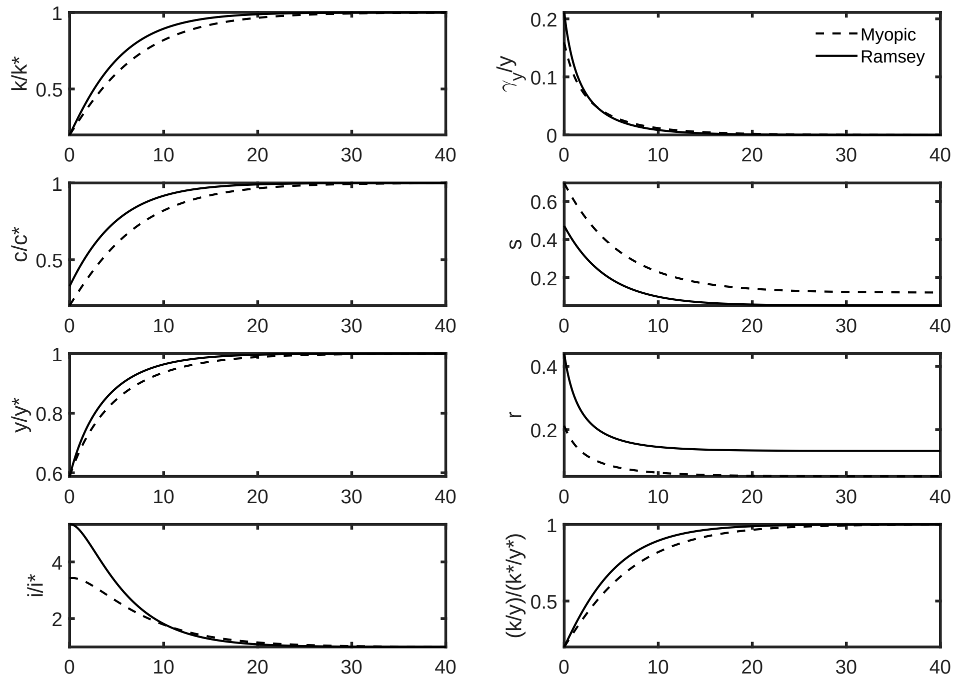

Figure 8: Ramsey vs Myopic transition from an initial condition on capital of 20% its steady state value. On the left-hand panels are displayed the capital, consumption, production and investment ratio to their steady state values. On the right-hand side panels: income percentage speed of convergence, saving rate, interest rate, and capital-to-output ratio.

to derive a short-term growth rate. The anchoring assumption was that the individual expected a constant growth rate for the future, equal to the short-term rate, as in equation 3. This assumption is essential to the narrative because it determines the constant overvaluation of future output variations, which is the primary reason for all the results obtained so far.

However, it is possible to assume that the myopic individual believes the future level of capital to be constant rather than growing, as assumed in Dawid, 2005 and Day, 2000. This leads generally to an underestimation of future changes in capital and income, resulting in different outcomes compared to the previous case of expected constant growth rate.

In the following, we assume that the household forms expectations at time τ, assuming future capital to be constant at a given level, which depends on the effects of the low of motion for the next step $(\tau+1)$, plus an inertial component $\omega(k_{\tau+1}-k_\tau)$ that adjusts the capital in the same direction of the first $\tau \to \tau+1$ movement. This inertial component encompasses all expected variations in capital beyond the initial period, in which the household is capable of making an accurate prediction. If $\omega = 0$, the household expects capital to be constant at $k_{\tau+1}$.

Hence, the equation for expected future capital is:

$$E_\tau\{k_{\tau+h}\} = k_{\tau+1} + \omega(k_{\tau+1} - k_\tau) \qquad \forall h \geq 1 \tag{17}$$

Discrete time is employed as it offers a more intuitive interpretation of the equation. The forecast for future capital occurs at time τ. The anticipated value of capital in $\tau+h$ equals the short-term predicted value of capital $k_{\tau+1}$, adjusted

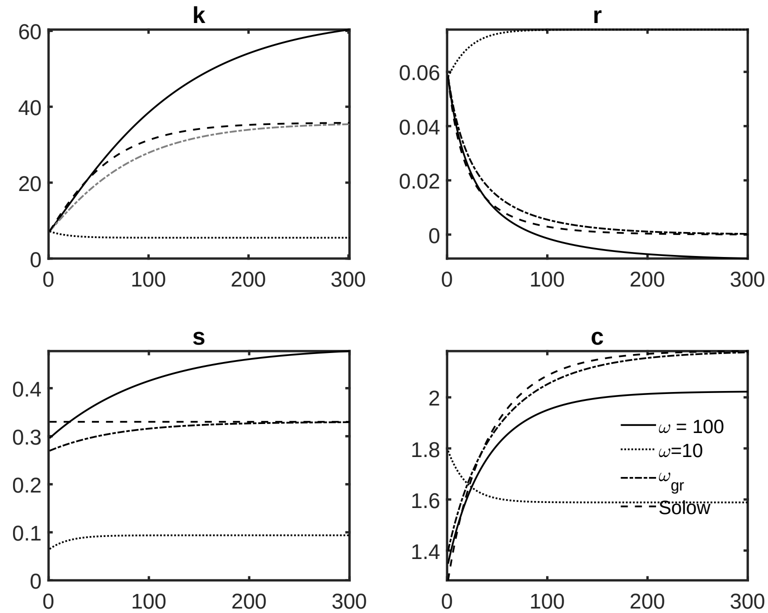

Figure 9: Solow model and different calibrations for the ω- model. The ω- model exactly replicates the Solow model steady state for ω_{gr}.

by the adjustment factor ω, which denotes the degree to which the individual expects short-term growth to impact the constant long-term capital level.

In this case, writing $U(\tau)$ and solving the optimization problem leads to:

$$k^* = \left[\frac{\alpha\beta(1+\omega)}{1+\alpha\beta\delta(1+\omega)}\right]^{\frac{1}{1-\alpha}} \tag{18}$$

The details of this derivation can be found in appendix D.

The parameter ω, which represents the degree of persistence of growth in an individual's conception, enhances the versatility of the results obtained using exponential expectations. Depending on the value of ω, the model can stabilize at different levels of capital. It is always possible to identify a value of $\omega = \omega_{gr}$ at which the steady-state capital is equal to that of the Solow Golden Rule ($k^* = k^*_{gr}$), or another value, ω_r, at which it is equal to that of Ramsey[12] ($k^* = k^*_r$), assuming the same values of parameters α, β, and δ.

In particular, if $\omega < \omega_r$, capital per worker accumulated in the economy is lower than Ramsey's; if $\omega \in (\omega_r, \omega_{gr})$ it is between Ramsey and Solow; and if $\omega > \omega_{gr}$ it will be higher than Solow. In figures 9 and 10 we show a comparison of different calibration for the parameter $\omega = [100, 10, \omega_r, \omega_{gr}]$. As shown, calibrating opportunely the parameter the same steady state is approached, compared to the Golden Rule or the Ramsey model. However the dynamics is different given the behavioral nature of the expectations. In particular the transition is approximated by the myopic model. However, the ω expectation model displays a slower transition speed compared to the full neoclassical model for consumption, savings, capital and output, similarly to the case of exponential expectation presented in the previous sections.

[12] With logarithmic utility: $\omega_{gr} = \frac{\beta(\delta(1-\alpha)-1)}{(\beta-1)-\delta\beta(1-\alpha)}$ and $\omega_r = \frac{1}{\beta\delta(1-\alpha)} - 1$

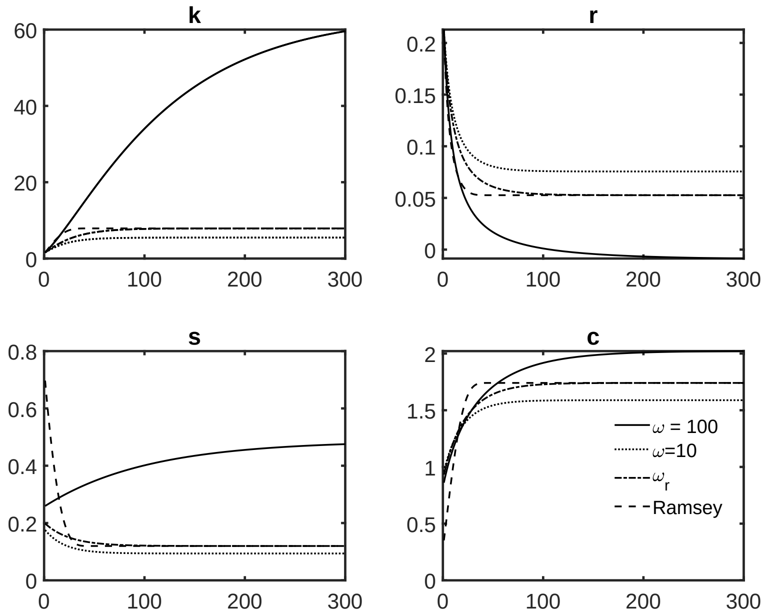

Figure 10: Ramsey model and different calibrations for the ω- model. The ω-model exactly replicates the Ramsey model steady state for ω_r.

Conclusions

The neoclassical growth model is extended to incorporate limitations in the forecasting capability of rational individuals. Specifically, the introduction of myopic behavior with rule-based expectations results in a framework where individuals can only predict short-term impacts on the economy, lacking accurate long-term predictions. This approach, distinct from perfect foresight assumptions, acknowledges the influence of cognitive anomalies, such as anchoring, reference point effects, and status quo biases.

The interaction between myopia and rule-based expectations shapes a dynamic sequence of static optimization problems, fundamentally altering the intertemporal relationship between consumption and capital intensity. Unlike the pure neoclassical growth model, this myopic approach disregards the traditional nexus between these variables, generating forecasting errors and subsequent adjustments.

This paper connects to various strands of literature exploring adaptive economizing agents, dynamic discount factors, learning, and expectations. Unlike previous works emphasizing time inconsistency arising from changes in individual preferences, our approach highlights time inconsistency emerging from the interaction between myopic rule-based expectations and impatience. Biased forecasts underpin time-varying utility plans, impacting perception of discounted future income, despite assuming constant time preferences.

We demonstrate how myopic agents, relying on rule-based expectations, deviate from rational expectation behavior. While rational agents in perfect foresight models converge to the long-run equilibrium à la Ramsey, our myopic model leads to a distinct equilibrium with higher saving rate and capital accumulation. This discrepancy emerges even though the myopic model is based on forward-looking behavior and optimization process.

Moreover, our study delves into the effects of myopic behavior on welfare, dynamics, and expenditure allocation of factor income. Notably, the myopic assumption results in a modified dynamic form of "effective impatience", influencing time preferences of households, leading to sub-optimal long-run capital accumulation. This outcome persists even with small deviations in time preferences, potentially reducing welfare significantly compared to the prediction of neoclassical models.

Our study highlights the need for policymakers to consider the intrinsic human behavioral nature of expectation formation mechanisms. It underscores potential risks associated with the belief that agents are entirely rational in the RKC sense, as this can lead to adverse long-run consequences. In particular, under standard model parametrization and given an impatience level that matches observed interest rates, our findings suggest that policy intervention may be needed to reduce excessive savings.

Our model might help reconcile with some recent developments on a global scale. The world economy has witnessed a simultaneous increase in global saving and investment rates, along with a prolonged period of declining interest rates. Various explanatory factors have been proposed, including demographic and technological evolution in both advanced and emerging countries. In our analysis, we incorporate the myopic rule-based expectation growth model as one such contributing factor. The ascent of developing countries in terms of population and GDP share could potentially contribute to an increase in over-saving behavior. Emerging and advanced countries exhibit distinct characteristics in growth expectations and commitment technologies within financial markets. Saving rates, especially in emerging countries, operate within a framework of myopic expectations, introducing a macro myopic bias at the global level. This bias may lead to an excess in capital accumulation, compression of consumption, and diminished welfare. These trends have manifested in a global reduction of interest rates, even reaching negative territory. The myopic bias may offer an explanation for such phenomena.

Our behavioral assumption, integrated into a standard optimization rule, has demonstrated the potential to derive analytical and explicit solutions for the policy function, describing the optimal behavior of the economic agent within the standard neoclassical model. As it lies at the core of neoclassical development in macroeconomic modeling, we believe our strategy could help in designing explicit analytical models that incorporate behavioral rules. This extends to descendant streams of modeling, including dynamic stochastic general equilibrium models (DSGEs), heterogeneous agents models (HANKs), and dynamic games models. Our upcoming research will focus on further exploration of this avenue.

Appendices

A First order conditions in continuous time

Under the assumption of iso-elastic utility, equation 5 becomes:

$$U(\tau) = \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \frac{\{(1-s(\tau))\left[k(\tau)\left(1+\frac{\dot{k}(\tau)}{k(\tau)}\right)^{t-\tau}\right]^{\alpha}\}^{1-\theta}}{1-\theta} dt - \frac{1}{(1-\theta)\rho} \tag{19}$$

By substituting the capital law of motion, $\dot{k}(\tau) = s(\tau)k(\tau)^{\alpha} - \delta k(\tau)$, into 19 and imposing $\frac{\partial U(\tau)}{\partial s(\tau)} = 0$, one obtains, after some algebraic manipulation:

$$k(\tau)^{\alpha(1-\theta)}(1-s(\tau))^{-\theta}\int_{\tau}^{\infty} e^{-\rho(t-\tau)}(1+\gamma_k)^{a(t-\tau)}dt+$$
$$+\frac{k(\tau)^{a}\alpha k(\tau)^{\alpha-1}(1-s(\tau))^{1-\theta}}{1+\gamma_k}\int_{\tau}^{\infty} e^{-\rho(t-\tau)}(1+\gamma_k)^{a(t-\tau)}(t-\tau)dt = 0,$$

where $1+\gamma_k = \frac{k(\tau)(1-\delta)+s(\tau)k^{\alpha}(\tau)}{k(\tau)}$ and $a = \alpha(1-\theta)$. $(1+\gamma_k)$ can be interpreted as the growth factor of capital $k(\tau)$. Computing the definite integrals we get:

$$\frac{-k(\tau)^{a}(1-s(\tau))^{-\theta}}{\rho - a\ln(1+\gamma_k)} \cdot \Big| -(1+\gamma_k)^{a(t-\tau)}e^{-\rho(t-\tau)}\Big|_{\tau}^{\infty}+$$
$$+\frac{k(\tau)^{a+\alpha+1}\alpha(1-s(\tau))^{1-\theta}}{(1+\gamma_k)(\rho-a\ln(1+\gamma_k))^2}\cdot\Big|-(1+\gamma_k)^{a(t-\tau)}e^{-\rho(t-\tau)}\left(\rho - a\ln(1+\gamma_k)(t-\tau)+1\right)\Big|_{\tau}^{\infty} = 0.$$

The convergence of the integral is subordinated to the following transversality condition, that will be discussed below in appendix A.1:

$$\lim_{t\to+\infty} -(1+\gamma_k)^{a(t-\tau)}e^{-\rho(t-\tau)} = 0. \tag{20}$$

Given equation 20, we can derive the optimal saving rate $s(\tau)$ as:

$$s(\tau) = 1 - \frac{(1+\gamma_k)(\rho - a\ln(1+\gamma_k))}{\alpha}k(\tau)^{1-\alpha}, \tag{21}$$

which simplifies to eq. (6) in the case of log-utility.

By substituting $s(\tau)$ from 21 into the capital law of motion we get:

$$1 - \frac{\dot{k}(\tau)+\delta k(\tau)}{k(\tau)^{\alpha}} = \frac{(1+\gamma_k)(\rho - a\ln(1+\gamma_k))}{\alpha k(\tau)^{\alpha-1}}, \tag{22}$$

which allows us to derive the steady state capital and saving rate reported in eqs. 8 and 9:

$$k^* = \left(\frac{\alpha}{\rho+\delta\alpha}\right)^{\frac{1}{1-\alpha}},$$
$$s^* = \delta k^{*1-\alpha} = \frac{\delta\alpha}{\rho+\alpha\delta}.$$

It should be noticed that neither the steady state capital k^* nor the saving rate s^* depend on θ.

A.1 Integral convergence condition

The convergence condition for integral 19 is contingent upon satisfying equation 20, which can be expressed as:

$$\lim_{t\to+\infty} -(1+\gamma_k)^{\alpha(1-\theta)(t-\tau)} e^{-\rho(t-\tau)} = 0.$$

By performing a change of base, this equation transforms into:

$$\lim_{t\to+\infty} -e^{(t-\tau)[\alpha(1-\theta)\ln(1+\gamma_k)-\rho]} = 0.$$

Since $t-\tau > 0$ for all t, the convergence condition of the integral depends on the following constraint:

$$\rho > \alpha(1-\theta)ln(1+\gamma_k),.$$

B Stability and errors

In this section we collect some results concerning convergence, stability and errors in the expectation formation process.

B.1 Convergence to the equilibrium

Under the convergence condition presented in appendix A.1, $\lim_{\tau\to\infty} k(\tau) = k^*$. The first order Taylor approximation of $\dot{k}(\tau)$ is:

$$\dot{k}(\tau) = \left.\frac{\partial \dot{k}(\tau)}{\partial k(\tau)}\right|_{k^*} \Big[k(\tau) - k^*\Big].$$

Differentiating equation (7) for $k(\tau)$ and using the expression of the steady state of capital in eq. 8, leads to:

$$\left.\frac{\partial \dot{k}(\tau)}{\partial k(\tau)}\right|_{k^*} = \frac{(\rho+\alpha\delta)(\alpha-1)}{\alpha+\rho} < 0.$$

Therefore, the first-order Taylor expansion around the steady state equals:

$$k(t) = k^* + e^{-\lambda t}\Big[k(0) - k^*\Big] \tag{23}$$

with

$$\lambda = \frac{(\rho+\alpha\delta)(1-\alpha)}{\alpha+\rho} > 0.$$

B.2 Error in future capital expectations

To compute the asymptotic behaviour of $\Omega(\tau,h,k)$ for both $\tau\to\infty$ and $h\to\infty$, without much loss of generality we assume that the prediction horizon h is a multiple of τ: $h = m\tau$. This implies that $h\to\infty$ when $\tau\to\infty$, and we can write eq. 10 as:

$$\lim_{\tau\to\infty}\Omega(\tau,h,k) = \lim_{\tau\to\infty} k(\tau)\left(1+\frac{\dot{k}(\tau)}{k(\tau)}\right)^{m\tau} - k(\tau(1+m)) \quad = k^*\left[\lim_{\tau\to\infty}\left(1+\frac{\dot{k}(\tau)}{k(\tau)}\right)^{m\tau} - 1\right].$$

Let's define the percentage deviation of capital from the steady state, as:

$$\widetilde{k(\tau)} = \frac{k(\tau)-k^*}{k^*}$$

Deriving $k(\tau)$ in eq. 23, we get:

$$\dot{k}(\tau) = -\lambda e^{-\lambda\tau}\Big[k(0) - k^*\Big]$$

Given the two relationships above, finally:

$$\begin{aligned} 1 + \frac{\dot{k}(\tau)}{k(\tau)} &= \frac{k^* + e^{-\lambda\tau}\Big[k(0) - k^*\Big](1+\lambda)}{k^* + \Big[k(0) - k^*\Big]e^{-\lambda\tau}} \\ &= \frac{1 + e^{-\lambda\tau}\widetilde{k(\tau)}\,(1+\lambda)}{1 + e^{-\lambda\tau}\widetilde{k(\tau)}}. \end{aligned}$$

Then we can write:

$$\begin{aligned} \lim_{\tau\to\infty}\left(1 + \frac{\dot{k}(\tau)}{k(\tau)}\right)^{m\tau} &= \lim_{\tau\to\infty}\left[\frac{1 + e^{-\lambda\tau}\widetilde{k(\tau)}\,(1+\lambda)}{1 + e^{-\lambda\tau}\widetilde{k(\tau)}}\right]^{m\tau} \\ &= \frac{\lim_{\tau\to\infty}\left(1 + ae^{-\lambda\tau}\right)^{m\tau}}{\lim_{\tau\to\infty}\left(1 + be^{-\lambda\tau}\right)^{m\tau}} \end{aligned}$$

where $a = (1+\lambda)\widetilde{k(\tau)}$ and $b = \widetilde{k(\tau)}$ are finite values $\forall\tau$. It can be demonstrated that the limit above converges to 1:

$$\begin{aligned} \lim_{\tau\to\infty}\left(1 + ae^{-\lambda t}\right)^{m\tau} &= \lim_{\tau\to\infty}\left[\left(1 + \frac{a}{e^{\lambda\tau}}\right)^{e^{\lambda\tau}}\right]^{\frac{m\tau}{e^{\lambda\tau}}} \\ &= \lim_{x\to\infty}\left[\left(1 + \frac{a}{x}\right)^{x}\right]^{\frac{m\ln(x)}{\lambda x}} \end{aligned}$$

where: $x = e^{\lambda\tau}, \tau = \frac{\ln(x)}{\lambda}$. This results in:

$$\lim_{x\to\infty}\left(e^{a}\right)^{\frac{m\ln(x)}{\lambda x}} = 1.$$

Therefore:

$$\lim_{\tau\to\infty}\left(1 + \frac{\dot{k}(\tau)}{k(\tau)}\right)^{\tau} = 1 \tag{24}$$

and finally the asymptotic behaviour of the prediction error Ω made by the representative agent becomes:

$$\lim_{\tau\to\infty}\Omega(\tau, h, k) = 0$$

B.3 Interest rate bias

Considering a generic capital law of motion $\frac{\dot{k}}{k} = ak^{\alpha-1} - b$, and differentiating saving rate as in equation 6 with respect to time, we get:

$$\dot{s} = \eta k^{1-\alpha}\left(ak^{\alpha-1} - b\right).$$

where $\eta = \frac{\rho(1-\delta)(1-\alpha)}{\alpha+\rho}$. From equation 7, then we have:

$$\dot{s} = -\frac{\eta}{\rho+\alpha}\left[(\rho + \alpha\delta)k^{1-\alpha} - \alpha\right],$$

leading to

$$\dot{s} = -\frac{\eta k^{1-\alpha}}{\rho+\alpha}\left[\alpha k^{\alpha-1} - \delta + \delta(1-\alpha) - \rho\right],$$

and

$$\dot{s} = -\frac{\eta k^{1-\alpha}}{\rho+\alpha}\left[r + \delta(1-\alpha) - \rho\right],$$

where $r = \alpha k^{\alpha-1} - \delta$. The equation shows that the interest rate at the steady state is

$$r^* = \rho - \delta(1-\alpha). \tag{25}$$

The excess of capital productivity $r = \alpha k^{\alpha-1} - \delta$ with respect to the 'equivalent impatience' $\rho_m = \rho - \delta(1-\alpha)$ leads to a decrease in saving rate (see section 2.1), diminishing capital accumulation. The saving rate stabilizes when the interest rate is equal to ρ_m.

This considerations allow us to compute the error committed by predicting the future interest rate with horizon $h = t - \tau$, given by eq. 14. The error $\Omega(\tau, h, r) = E_{\tau,h}\{r(t)\} - r(\tau+h)$ becomes:

$$\Omega(\tau, h, r) = \alpha k(\tau)^{\alpha-1}\left(1 + \frac{\dot{k}(\tau)}{k(\tau)}\right)^{h(\alpha-1)} - \delta - r(\tau+h). \tag{26}$$

By replicating the considerations outlined in appendix B.2, we can demonstrate that $\lim_{\tau\to\infty}\Omega(\tau, h, r) = 0$. This result can be obtained by taking the limit of eq. 26 and substituting eqs. 24, 8 and 25 into it.

C Discrete time myopic model

Utility maximization in discrete time can be written as:

$$\max_{s_\tau, k_{\tau+1}} = E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \ln\left\{(1-s_\tau)\left[k_\tau\left(\frac{k_{\tau+1}}{k_\tau}\right)^{t-\tau}\right]^{\alpha}\right\}$$

$$s.t.$$

$$k_{\tau+1} = s_\tau k_\tau^{\alpha} + (1-\delta)k_\tau,$$

where $k_{\tau+1}/k_\tau$ denotes the capital growth rate contingent upon the capital stock one period ahead, a variable influenced by the corresponding saving decision.

C.1 First order conditions

We write the Lagrangian formulation:

$$\mathscr{L} = E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} \ln\left\{(1-s_\tau)\left[k_\tau\left(\frac{k_{\tau+1}}{k_\tau}\right)^{t-\tau}\right]^{\alpha}\right\} + E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau}\lambda_\tau(s_\tau k_\tau^{\alpha} + (1-\delta)k_\tau - k_{\tau+1})$$

Before deriving the first order conditions, we solve the setup for the infinite periods such as:[13]

$$\mathscr{L} = \frac{1}{1-\beta}\ln[(1-s_\tau)k_\tau^{\alpha}] + \alpha \ln\left[\frac{k_{\tau+1}}{k_\tau}\right]\frac{\beta}{(1-\beta)^2} + \frac{1}{1-\beta}\lambda_\tau(s_\tau k_\tau^{\alpha} + (1-\delta)k_\tau - k_{\tau+1})$$

[13]The following properties of infinite series has been used:

$$\sum_{t=0}^{\infty} \alpha^t x = \frac{1}{1-\alpha}x,$$
$$\sum_{t=0}^{\infty} \alpha^t t x = \frac{1}{(1-\alpha)^2}x.$$

The household takes their decisions at period τ, considering the whole time horizon and taking a new decision at time $\tau+1$.

The first order conditions are:

$$\frac{\partial \mathscr{L}}{\partial s_\tau} = 0 \Leftrightarrow \frac{1}{k_\tau^\alpha(1-s_\tau)(1-\beta)} = \lambda_\tau$$

$$\frac{\partial \mathscr{L}}{\partial k_{\tau+1}} = 0 \Leftrightarrow \frac{\alpha\beta}{k_{\tau+1}(1-\beta)^2} = \lambda_\tau$$

$$\frac{\partial \mathscr{L}}{\partial \lambda_\tau} = 0 \Leftrightarrow k_\tau = s_\tau k_\tau^\alpha + (1-\delta)k_\tau$$

It is worth noting that, in the second equation, the myopic behavior implies that the households do not internalize the expected effect of a marginal unit of capital in the next period $\alpha s_{\tau+1} k_{\tau+1}^{\alpha-1} - (1-\delta)$. Combining the F.o.c.s for optimal saving and capital we get the condition:

$$k_{\tau+1} = (1-s_\tau)\frac{\alpha\beta}{1-\beta}k_\tau^\alpha.$$

Substituting the law of motion of capital $k_{\tau+1} = s_\tau k_\tau^\alpha + (1-\delta)k_\tau$, after some manipulations, we obtain the optimal saving rate in period τ:

$$s_\tau = \frac{\alpha\beta - (1-\delta)(1-\beta)k_\tau^{1-\alpha}}{\alpha\beta + 1 - \beta}$$

Finally, substituting s_τ in $k_{\tau+1}$, we define the optimal law of motion of capital:

$$k_{\tau+1} = \frac{\alpha\beta}{\alpha\beta + 1 - \beta}\left(k_\tau^\alpha + (1-\delta)k_\tau\right).$$

From this last equation it is possible to derive the steady state level of capital and saving rate, which correspond to eqs. 8 and 9.

D Constant capital expectations

The representative agent in time τ assumes that future capital will be constant at level $k_{\tau+h}$, which derives from applying the low of motion for one step, plus an inertial component $\omega(k_{\tau+1} - k_\tau)$ that adjusts the capital in the same direction of the first step. This inertial component represents all expected variations in capital beyond the initial period in which it is capable of making an accurate prediction. Therefore the agent's perceived law of motion can be describes as follows:

$$E_\tau\{k_{\tau+h}\} = k_{\tau+1} + \omega(k_{\tau+1} - k_\tau) \qquad \forall h \geq 1$$

If $\omega = 0$, the household expects capital to be constant at $k_{\tau+1}$. Given such relationship, the agent's discounted utility at time $t = \tau$ is:

$$U_\tau = \ln\left[(1-s_\tau)k_\tau^\alpha\right] + \sum_{t=\tau+1}^{\infty} \beta^{t-\tau} \ln\left\{(1-s_\tau)\left[k_{\tau+1} + \omega(k_{\tau+1} - k_\tau)\right]^\alpha\right\} + \lambda\left[k_{\tau+1} - (1-\delta)k_\tau - s_\tau k_\tau^\alpha\right]$$

In order to find the optimal saving rate s_τ it is necessary to optimize the above function U_τ,

$$\max \quad U_\tau(s_\tau, k_{\tau+1})$$

subject to the constrain

$$g(k_\tau, s_\tau) = k_{\tau+1} - (1-\delta)k_\tau - s_\tau k_\tau^\alpha$$

Then:

$$\mathscr{L} = U_\tau - \lambda g(k_\tau, s_\tau)$$

$$\text{w.r.t. } s_\tau : \qquad \frac{\partial U_\tau}{\partial s_\tau} = -\frac{1}{1-s_\tau} + \frac{\beta}{1-\beta}\left(-\frac{1}{1-s_\tau}\right) - \lambda k_\tau^\alpha = 0$$

$$\text{w.r.t. } k_{\tau+1} : \qquad \frac{\partial U_\tau}{\partial_{k_{\tau+1}}} = \alpha \frac{\beta}{1-\beta} \frac{1+\omega}{k_{\tau+1} + \omega\left(k_{\tau+1} - k_\tau\right)} - \lambda = 0$$

By equating the λs of the two F.O.Cs:

$$\frac{k_\tau^{-\alpha}}{(1-s_\tau)(1-\beta)} = \frac{\alpha\beta(1+\omega)}{(1-\beta)\left[k_{\tau+1} + \omega(k_{\tau+1} - k_\tau)\right]}$$

$$k_\tau^{-\alpha}\left[k_{\tau+1} + \omega(k_{\tau+1} - k_\tau)\right] = \alpha\beta(1+\omega)(1-s_\tau)$$

By substituting $k_{\tau+1}$ inside the equation with its law of motion:

$$k_\tau^{-\alpha}\left\{\left[k_\tau(1-\delta) + s_\tau k_\tau^\alpha\right](1+\omega) - \omega k_\tau\right\} = \alpha\beta(1+\omega)(1-s_\tau)$$

$$k_\tau^{1-\alpha}(1-\delta)(1+\omega) + s_\tau(1+\omega) - \omega k_\tau^{1-\alpha} = \alpha\beta(1+\omega)(1-s_\tau)$$

Finally the optimal saving rate s_τ equals to:

$$s_\tau = \frac{\alpha\beta(1+\omega) - k_\tau^{1-\alpha}\left[(1-\delta)(1+\omega) - \omega\right]}{(1+\omega)(1+\alpha\beta)}$$

and by plugging s_τ into the law of motion of capital, one gets the result:

$$k_{\tau+1} = \frac{\alpha\beta}{1+\alpha\beta}\left[k_\tau(1-\delta) + k_\tau^\alpha\right] - \frac{\omega}{(1+\omega)(1+\alpha\beta)}k_\tau$$

The steady state is be reached when $k_\tau = k_{\tau+1}$:

$$k_\tau = \frac{\alpha\beta}{1+\alpha\beta}\left[k_\tau(1-\delta) + k_\tau^\alpha\right] - \frac{\omega}{(1+\omega)(1+\alpha\beta)}k_\tau;$$

$$k_\tau\left[\omega + (1+\omega)(1+\alpha\beta)\right] = \alpha\beta(1+\omega)(1-\delta)k_\tau + \alpha\beta(1+\omega)k_\tau^\alpha;$$

$$\alpha\beta(1+\omega)k_\tau^{\alpha-1} = 1 + 2\omega + \alpha\beta\delta(1+\omega).$$

From one finally gets:

$$k^* = \left[\frac{\alpha\beta(1+\omega)}{1+2\omega+\alpha\beta\delta(1+\omega)}\right]^{\frac{1}{1-\alpha}}$$

If one were willing to make a comparison with the Ramsey model steady state, the equality between the two would be reached for the following value of ω:

$$\frac{\alpha\beta}{1-\beta(1-\delta)} = \frac{\alpha\beta(1+\omega)}{1+2\omega+\alpha\beta\delta(1+\omega)}$$

$$1 + 2\omega + \alpha\beta\delta(1+\omega) = 1 + \omega - \beta(1+\omega)(1-\delta)$$

$$\omega + \alpha\beta\delta(1+\omega) + \beta(1+\omega)(1-\delta) = 0$$

Equality to Ramsey–Cass–Koopmans would hold for:

$$\omega_r = \frac{\beta(\delta - \alpha\delta - 1)}{1 + \alpha\beta\delta + \beta - \delta\beta}$$

Finally, the same comparison with Solow-Swan's model would yield the ω value of:

$$\omega_{gr} = \frac{\beta\delta(1-\alpha) - 1}{2 + \beta\delta(1-\alpha)} < 0$$

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