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Behavior-Based Pricing and Signaling of Product Quality*

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Abstract

In a two-period model with repeat purchase, we compare the profit and social welfare effects of behavior-based pricing (BBP) and uniform pricing in a monopoly under quality uncertainty. We offer the novel insight that BBP increases the price elasticity of imitation demand and lowers the signaling cost relative to uniform pricing, and becomes a potentially profitable strategy even when the monopolist cannot commit to future prices. Moreover, the profitability of BBP does not arise at the expense of consumer surplus. Either upward or downward price distortion with use of BBP signals high quality, depending on the seller's commitment power. With more accurate tracking technology, the monopolist may forsake signaling for better consumer information.

Keywords: Behavior-Based Pricing; Uniform Pricing; Quality Uncertainty; Signaling

JEL classifications: D82, L11, L15, M31

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1 Introduction

Behavior-based pricing (BBP), the practice of conditioning prices on consumers' purchasing history, is a pricing strategy commonly observed in many markets, including retailing, data plan for smart phones, plane tickets, hotels, etc. The classic results, nevertheless, show that BBP is generally inferior to the simple uniform pricing for a monopolist. (See, e.g., Hart and Tirole, 1988; Taylor, 2004; Acquisti and Varian, 2005). We analyze a model with the novel feature that the monopolist holds private information about the quality of his product, and the prices that the seller posts signal product quality. We investigate how the option of BBP interacts with the signaling role of prices, and show that the option of price conditioning lowers the signaling cost and can be a profitable strategy even when the monopolist cannot commit to future prices.

When a seller has the option of using behavior-based pricing, consumers rationally adjust their initial purchase decisions, anticipating that their records will affect future prices when they make repeat purchase. This strategic interaction from the consumer side leads to a lower profit for the monopolist relative to uniform pricing when the consumers' valuations do not change over time. This far-reaching result has been obtained assuming perfect information about product quality.

The existence of imperfect information is, however, prevalent, in particular when a seller launches a new product or when consumers buy a product with which they are not familiar. (See, e.g., Gerstner, 1985; Yan and Sengupta, 2011.)[1] In such cases, consumers often rely on prices to make quality inferences. Classic wisdoms include "high prices signal high quality", or "high and declining prices signal product quality".[2] When a seller has the capacity of price conditioning, consumers' strategic adjustment of initial demand affects the effectiveness of signaling. On the other hand, a seller's signaling concern also affects his incentive to use conditional prices in the future. In particular, when a seller can commit to future prices, the use of BBP can itself be a part of the quality signal.

We set up a two-period model with repeat purchase, combining the idea that prices signal product quality with the theory of behavior-based pricing. The key features are that a monopolistic seller initially has private information about the quality of his product (type L or type H), meaning that the prices he posts convey valuable information about product quality, and that the seller can track consumers' purchasing records and charge them discriminatory prices in the sec-

[1]Imperfect information about product quality also has a profound impact on the efficiency of trade and may lead to a breakdown of the market. Sellers use quality signals, such as price, advertisement expenditure, brand name, etc., to convince consumers that their products are of high quality (see, e.g., Nelson, 1970; Milgrom and Roberts, 1986). Among various quality signals, price has been among the most important.

[2]See, e.g., Wolinsky, 1983; Riordan, 1986; Rogerson, 1988; Bagwell and Riordan, 1991; Bester, 1998; Janssen and Roy, 2010.

ond period, conditional on their purchasing history. We obtain novel insights on the profitability of behavior-based pricing and on the price trajectory as effective quality signals for a monopolist. First, behavior-based pricing has the benefit of lowering the signaling cost relative to *uniform pricing* (UP), and thus can be a profitable strategy for a monopolist, in sharp contrast to the classic result that BBP is detrimental and will not be used by a seller if he can avoid it. Moreover, the profitability of BBP does not come at the expense of the consumer surplus. Second, if the seller cannot commit to future prices, distorting the first-period price above the equilibrium level under public information signals product quality; if the seller can commit to future prices, a downward price distortion together with use of BBP signals product quality. In either case, the equilibrium price pattern exhibits widely observed first-purchase discounts, with the average price increasing over time. Third, if a seller can track consumers' behavior more accurately and has a high probability of obtaining precise information about their valuations, he may forsake signaling for better consumer information.

In the baseline model, we assume the seller has limited commitment power and thus can only post short-term prices. Moreover, we assume, in order to focus on type H's incentive to practice BBP, that all consumers have an identical valuation (v_L) of a type L product but have heterogeneous and private valuations (v_i) of a type H product. Absent quality uncertainty, we arrive at the familiar result that the option of BBP lowers type H's profits relative to uniform pricing. Foreseeing conditional prices, consumers strategically delay their initial consumption, pushing down the first-period price. As a result, the option of BBP increases the type H seller's second-period profit, thanks to the increased price for repeat purchasers and the expanded market coverage for first-time purchasers, but decreases the seller's first-period profit due to lower demand and a lower price. Under public information about product quality, the negative effect on the first-period profit dominates the positive effect on the second-period profit.

What happens when there is imperfect information about product quality in the first period? In this scenario, the type H seller's first-period price not only specifies the trading price, but also signals product quality. The consumer's first-period demand, however, is affected not only by the first-period price but also by the expected second-period prices, including whether or not BBP will be used. When BBP is not permitted and the seller is restricted to uniform pricing, due to the high production cost of a high-quality product, the type H seller posts a first-period price exceeding the equilibrium price under public information in order to prevent imitation by the type L seller in a separating equilibrium. This upward price distortion is the signaling cost which the type H seller has to bear in order to convince consumers of his product quality.

Relative to uniform pricing, BBP lowers the signaling cost, since it increases the price elasticity of the first-period demand. Under BBP, believing that the product is of high quality, a consumer makes a purchase in the first period if and only if her valuation is above $\hat{v}$, the valuation of the marginal consumer who is indifferent between buying and not buying in the first period. The marginal valuation $\hat{v}$ exceeds type H's first-period price, p_{1H}, and amplifies the demand reduction effect of a marginal price increase by the type H seller under BBP, relative to uniform pricing. This in turn lowers type L's imitation profit and relaxes his incentive compatibility constraint. As a result, the upward price distortion required to prevent type L's imitation is smaller under BBP, implying a lower signaling cost. This signaling cost effect affects the type H seller's profit positively, reinforcing the positive effect of BBP on the seller's second-period profit, and making BBP potentially profitable for the monopolist.

We show that an upward price distortion together with use of BBP signals product quality in the unique separating equilibrium that survives the intuitive criterion. The equilibrium price pattern exhibits flat prices for type L products and first-purchase discounts for type H products: consumers pay a lower price for their first unit than for their second if they consume in both periods, while in the second period, first-time purchasers pay a lower price than repeat purchasers. Compared with uniform pricing, BBP increases the type H seller's profit when the production cost of a high-quality product is sufficiently low, and decreases his profit when the production cost is high. Moreover, BBP benefits consumers because in equilibrium more consumers purchase the type H product at a lower average price. This benefit dominates the decrease in type H's profit when the production cost is high, and total welfare also increases relative to uniform pricing. Thus, a policy restricting consumer tracking, or prohibiting the use of behavior-based pricing, can actually harm sellers and consumers when there is imperfect information about product quality.

We then extend our analysis to a price-commitment regime where the seller posts prices for both periods at the beginning of the game. A key difference between this and the limited-commitment regime is that the prices of both periods serve as signaling instruments. With BBP, the type H seller is able to better leverage the first-period and the second-period prices to drive down type L's imitation profit in a separating equilibrium. As a result, the first-period price required to prevent imitation is further lowered, and the option of BBP allows type H to signal product quality via an equilibrium price lower than the public information outcome—meaning that a downward price distortion with use of BBP signals product quality. Consequently, when a seller can commit to future prices, the option of BBP always increases the type H seller's profit relative to uniform pricing.

Next, we explore the impact of more accurate tracking technology on the type H seller's signaling incentive in a setting where, after an initial purchase, the seller perfectly learns a consumer's willingness to pay with some probability, and can thus practice personalized pricing in the second period. While better information about a consumer's valuation increases the signaling cost, it also increases the type H seller's ability to extract consumer surplus from repeat purchasers. The type H seller faces a new tradeoff: while a high first-period price signals product quality, a low first-period price increases first-period demand and leads to better information about consumers. With low tracking accuracy, the lower signaling cost is still the dominant force that makes BBP more profitable than uniform pricing. When the tracking technology is sufficiently accurate, the type H seller gives up signaling and pools with the type L seller at a low price to extract more surplus from repeat purchasers.

Finally, we relax the assumption that consumers' valuations for a type L product are identical and consider an extension in which these valuations follow a distribution. As a result, the type L seller also uses conditional prices when BBP is permitted. Two forces affect the type L seller's imitation incentives in opposite directions under quality uncertainty. First, similar to the baseline model, the option of BBP amplifies the price elasticity of the first-period demand, thereby reducing the type H seller's signaling cost relative to uniform pricing. Second, the imitation of the type H seller's first-period price induces a higher valuation of the marginal consumer and thus a more profitable second-period market segmentation. This increases the type L seller's second-period imitation profit and adversely affects the type H seller's signaling cost. In aggregate, when the production cost of the type H product is sufficiently low, the first effect dominates and the option of BBP still lowers the signaling cost and increases the type H seller's profit relative to uniform pricing.

The remainder of this paper is organized as follows. The rest of this section relates our study to the existing literature. Section 2 presents the baseline model in which the seller has limited commitment and can only post short-term prices. Section 3 first derives the equilibrium under public information as a useful benchmark, and then analyzes the game with quality uncertainty under uniform pricing and BBP. In Sections 4–6 we analyse in sequence a price-commitment regime where the seller posts long-term prices, a setting with more accurate tracking technology, and an extension with heterogenous valuations for the type L product. Section 7 concludes. The proofs of Lemma 1 and 5, Remarks 1–2 and 4–5, Proposition 6 and Corollary 3 are substantiated in the main text, with all the remaining proofs found in the Appendix.

Related Literature

To our knowledge, this is the first paper to analyze the practice of BBP with the presence of imperfect information about product quality. We show that the prospect of future price conditioning has a nontrivial impact on the seller's signaling cost. We offer novel insights on the profitability of behavior-based pricing for a monopolist, and characterize the new price patterns that signal high product quality.

Our paper relates to the fast-growing literature on behavior-based pricing. Existing works (e.g., Taylor, 2004; Hart and Tirole, 1988; Acquisti and Varian, 2005; Laussel et al., 2020) highlight the far-reaching insight that the practice of conditioning price on consumers' purchasing history is typically inferior to simple uniform pricing for a monopolist, because consumers strategically delay their initial consumption or shield themselves by concealing their identities when they anticipate that higher prices will be charged to repeat purchasers.[3] This implies that a monopolist would choose not to track consumers in the first place, or would commit to not making use of consumers' purchasing information in subsequent interactions.[4] This implication, however, is called into question by the prevalence of behavior-based pricing in practice, even among sellers who enjoy a high level of market power.

The literature has also explored situations where behavior-based pricing is potentially profitable for a monopolist. Hart and Tirole (1988) argue that BBP can increase a seller's profit if consumers' valuations are revealed in the initial purchase, enabling the seller to extract all the surplus in the future.[5] Villas-Boas (2004) shows that monopoly pricing involves cycles in the prices offered to new consumers, and that if the seller is to a sufficient degree more forward-thinking than his consumers, customer recognition can be profitable for him. In Acquisti and Varian (2005), if consumers are myopic and do not anticipate future price changes based on their current behavior, or if the seller is able to provide repeat purchasers with value-added services, BBP can generate larger profits than uniform pricing. Jing (2011) shows that, for an experience good, BBP can generate a larger profit than uniform pricing when the mean consumer valuation is sufficiently high.[6]

Our work complements the existing literature by considering a setting where there is imperfect information and price signals product quality. We identify a new channel thus far absent in the literature through which BBP affects seller profits and consumer welfare: price conditioning

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[3]For comprehensive reviews of the BBP literature, see, e.g., Armstrong (2006), Fudenberg and Villas-Boas (2006) and Acquisti et al. (2016). Conitzer et al. (2012) and Lagerlöf (2023) analyze consumers' incentives to hide their purchase history when the seller can adopt BBP, and explore the welfare effects of anonymous shopping.

[4]The extensive literature on the economics of durable goods delivers the similar message that intertemporal price discrimination is never optimal for a monopolist who can commit to future prices due to the existence of ratchet effect. See, e.g., Stokey (1979) and Salant (1989). We consider a model of nondurable goods where consumers make repeat purchase in the second period.

[5]Laussel et al. (2020) also suggest that a monopolist can profit from BBP when he collects more detailed information on consumers.

[6]Studies on BBP in a competitive environment usually stress sellers' incentives for market segmentation and competitive customer poaching. See, e.g., Chen (1997), Fudenberg and Tirole (2000), Chen and Zhang (2009), Esteves (2009), Chen and Pearcy (2010), Li and Jain (2016), Jing (2017), Choe et al. (2018), Colombo (2018), Esteves et al. (2022), etc..

increases the price elasticity of imitation demand and lowers the signaling cost, thus making BBP a potentially profitable strategy for a monopolist. Our framework rationalizes the prevalence of BBP without sellers resorting to competitive poaching or relying on the irrationality of consumers. Moreover, the decrease in the signaling cost leads to an increase in consumer surplus because the average price decreases and overall demand increases. Thus, the benefits to the seller stem from improved efficiency of trade rather than being at the expense of consumer welfare.

Our paper also relates to the vast literature on prices as signals of product quality.[7] In a model where production cost increases with endogenous quality, Wolinsky (1983) proves the existence of a separating equilibrium in which each price signals a unique quality level. Rogerson (1988) shows that when firms compete both in price and in quality, price advertising is welfare improving because price can be a signal of the unobservable product quality. In a setting with exogenous quality, Bagwell and Riordan (1991) show that a high introductory price signals product quality, and that the price declines over time as information about the product quality diffuses among consumers. Thus, "high and declining prices signal product quality." Relaxing the assumption about a positive correlation between quality and production cost, Judd and Riordan (1994) prove that a high price can still signal high quality when consumers have some private information about the product quality. In Bester (1998), prices include a quality premium which keeps the sellers from producing low quality and leads to minimum horizontal product differentiation. Dana (2020) argues that not only is a high price an effective signal for product quality; it is also the only path a monopolist would choose among all actions that restrict sales. This literature has so far ignored the sellers' incentives to make use of the information that is revealed through the consumers' purchasing history. We make up this gap and consider how the option of behavior-based pricing affects the price trajectory when the price signals product quality. We show that both a high price and a low price with use of BBP can signal high quality, depending on whether the seller has commitment power.

2 The Model

A monopolist introduces to the market a new product which is of high or low quality, $q \in \{H, L\}$. The production cost of a type H product is constant and equals $c > 0$, while that of a type L product is normalized to 0. There is a continuum of consumers with total mass normalized to 1. Consumers have a common valuation $v_L > 0$ for a type L product,[8] but have heterogeneous

[7] See Kirmani and Rao (2000) for a critical review of the literature on quality signaling.

[8] In Section 6, we relax this assumption and analyze an extension in which consumers' valuations for the type L product also follow a distribution.

valuations for a type H product. In particular, consumer i's valuation for a type H product is v_i, which is a random draw following uniform distribution on the support $[v_L, v_L + 1]$, and the realization of v_i is the consumer's private information.

The seller and the consumers interact in two periods. Each consumer has a unit demand for the product in each period. At the beginning of the first period, nature draws the quality of the product and reveals the information privately to the seller; the consumers do not observe the realized quality of the product and they believe that the product is type H with probability ρ and type L with probability $1 - \rho$. However, consumers can infer some information about the product quality from the prices posted by the seller. At the beginning of the second period, the product quality becomes public information before consumers make their second-period purchasing decision. Both the seller and the consumers are risk neutral and there is no discounting.[9]

After the first-period interaction, the seller observes whether consumers have made a purchase or not, enabling him to base his second-period prices on consumers' first-period purchasing behavior. When *behavior-based pricing* (BBP) is permitted, the seller has the option of charging different prices to consumers with different purchase history. Let $p_q = \{p_{1q}, p_{2q}\} = \{p_{1q}, (p_{2q}^R, p_{2q}^N)\}$, where p_{1q} and p_{2q} denote type q's choice of first- and second-period prices under BBP, and p_{2q}^R and p_{2q}^N are the second-period prices for repeat and first-time purchasers respectively. When BBP is not permitted, the seller is obliged to charge the same price to all consumers in the second period, and thus is restricted to *uniform pricing* (UP). Let $s_q = \{s_{1q}, s_{2q}\}$ denote type q's choice of prices under UP. The seller has limited commitment power and posts one-period prices at the beginning of each period.[10] The timing of the game is as follows:

1. At $t = 1$, the seller privately learns the product quality $q \in \{H, L\}$ and posts the first-period price ℓ_{1q}, $\ell \in \{p, s\}$. A consumer learns privately her valuation v_i of the type H product. She then observes the posted price and updates her belief about the product quality, $\mu(\ell_{1q}) = \Pr\{H \mid \ell_{1q}\}$, which is the probability that the product is of type H given price ℓ_{1q}, and decides whether to purchase the product.

2. At $t = 2$, information about the product quality becomes common knowledge. The seller posts the second-period prices ℓ_{2q}, $\ell \in \{p, s\}$. Consumers make their second-period purchasing decisions.

At $t = 2$, knowing the product quality, a consumer will not buy a low-quality product at a price

[9]We discuss the case where only a fraction of consumers knows the product quality at $t = 2$ in Section 7. The assumption of no discounting is for clarity of exposition. The main results hold qualitatively when the seller and consumers discount future payoffs moderately.

[10]We analyze the price-commitment regime in Section 4.

exceeding v_L. Thus, whether BBP is permitted or not has no effect on type L's choice of a second-period price, and it is optimal for the seller to choose $s_{2L} = v_L$ and $p_{2L} = (v_L, v_L)$.[11] However, the option of BBP makes a difference to the type H seller because consumers have heterogenous valuations of a high-quality product.

At $t = 1$, the seller holds private information about q, while the first-period price, ℓ_{1q}, $\ell \in \{p, s\}$, conveys valuable information about the product quality. Although the second-period prices ℓ_{2q} do not directly serve as signals of product quality, they may affect consumers' first-period demand, because whether or not a consumer makes a first-period purchase will alter the prices she faces in the second period.[12]

To focus on the interesting cases, we make the following assumptions:[13]

$$v_L < 1; \qquad v_L < c < v_L + 1. \tag{1}$$

The solution concept of the game is a perfect Bayesian equilibrium (PBE) that satisfies: 1) the seller's choice of prices is optimal given his anticipation of consumers' beliefs and purchasing strategies; 2) consumers' purchasing decisions are rational given their updated beliefs about product quality; 3) consumers' beliefs about product quality are consistent with the seller's pricing strategies on the equilibrium path. Since signaling games have the disconcerting feature of multiple equilibria, we impose the widely-used intuitive criterion to rule out equilibria that are supported by implausible off-equilibrium path beliefs (See Cho and Kreps, 1987).

3 Analysis

In this section, we analyze the equilibria under uniform pricing and behavior-based pricing in sequence, and then compare the equilibria outcomes to show the effect of BBP on the signaling role of prices and to clarify the intuitions behind the profitability of price conditioning. In each case, we first characterize the public information outcome as a benchmark and then analyze the corresponding game under quality uncertainty. Since we are mainly interested in the interplay between the option of BBP and the signaling role of prices, we focus on separating equilibria in

[11] This would not be the case when consumers have heterogeneous valuations for the type L product. See the analysis in Section 6.

[12] The option of BBP differentiates our model from Bagwell and Riordan (1991), who assume that the seller can only charge uniform prices to all consumers in each period. On the other hand, with $\rho = 1$ our model becomes a continuous version of Acquisti and Varian (2005). This setup allows us to have a clean comparison with the results in both papers, in order to gain insights on how the option of BBP interacts with the signaling role of prices.

[13] Note that the type H seller never sells the product to anyone if $c \geq v_L + 1$. On the other hand, if $v_L \geq 1$, the type L seller has no incentive to imitate type H's price choice if the latter posts the equilibrium prices under public information about product quality, regardless of whether BBP is permitted or not. The analysis is qualitatively the same if $c \leq v_L$.

which type H and type L sellers post distinct first-period prices and consumers can infer product quality perfectly from these prices.

3.1 Uniform Pricing

Suppose the product quality is publicly observed at $t = 1$. A low-quality product will be sold at the same price to all consumers in both periods, $\tilde{s}_{1L} = \tilde{s}_{2L} = v_L$, leading to total profits $\tilde{\Pi}^u_L = 2v_L$ for the type L seller under uniform pricing.

For the type H seller, since the first-period price does not affect the demand which the seller faces at $t = 2$, the optimal prices for the two periods are independent. Thus, the type H seller chooses s_{tH} to maximize the monopoly profit in period t, $(v_L + 1 - s_{tH})(s_{tH} - c)$. It follows that the optimal prices $\tilde{s}_H = \{\tilde{s}_{1H}, \tilde{s}_{2H}\}$ and the type H seller's total profits are respectively

$$\tilde{s}_{1H} = \tilde{s}_{2H} = \frac{v_L + 1 + c}{2} \equiv \tilde{s}, \qquad \tilde{\Pi}^u_H = 2\frac{(v_L + 1 - c)^2}{4} \equiv 2\tilde{\pi}, \tag{2}$$

where $\tilde{s}$ represents the type H seller's static monopoly price and $\tilde{\pi}$ is his static monopoly profit. The next remark summarizes the equilibrium outcome under public information when BBP is not permitted.

Remark 1. *Suppose the product quality is publicly observed and the seller is restricted to uniform pricing. The type H seller's optimal price is $\tilde{s}_H = \{\tilde{s}, \tilde{s}\}$ which brings total profits $\tilde{\Pi}^u_H = 2\tilde{\pi}$, and the type L seller's optimal price is $\tilde{s}_L = \{v_L, v_L\}$ which brings total profits $\tilde{\Pi}^u_L = 2v_L$.*

Now consider the case of asymmetric information about product quality. The type H seller uses the first-period price to signal product quality. The equilibrium outcome in Remark 1 serves as a useful benchmark: if the type H seller can convince consumers of his product quality by setting $\tilde{s}_{1H} = \tilde{s}$, he will obviously do so; however, if $\tilde{s}_{1H}$ is insufficient to prevent type L from mimicking, type H needs to distort the first-period price from the equilibrium price under public information to signal high quality.

At $t = 2$, it is optimal for the two types to choose $s^*_{2H} = \tilde{s}$ and $s^*_{2L} = v_L$ respectively. At $t = 1$, consumers make their purchasing decisions solely on the basis of the first-period price, because the seller cannot charge conditional prices in the future. In a separating equilibrium with $s_{1H} \neq s_{1L}$, consumers' equilibrium path beliefs are given by $\mu(s_{1H}) = 1$ and $\mu(s_{1L}) = 0$. Suppose the off-equilibrium path belief is $\mu(s_1) = 0$ for $s_1 \neq s_{1H}$, and thus $s^*_{1L} = v_L$ must hold. Moreover, when the type H seller deviates from s_{1H}, his best choice is $s_1 > v_L$ leading to zero first-period profit.

When setting price s_{1H} and inducing consumer belief $\mu(s_{1H}) = 1$, the seller faces first-period demand given by

$$D_1^u(s_{1H}, 1) \equiv v_L + 1 - s_{1H}, \tag{3}$$

and the two types' incentive compatibility constraints are respectively:[14]

$$\Pi_L(s_{1H}, 1) = (v_L + 1 - s_{1H})s_{1H} + v_L \leq \Pi_L(v_L, 0) = 2v_L, \tag{4}$$

$$\Pi_H(s_{1H}, 1) = (v_L + 1 - s_{1H})(s_{1H} - c) + (v_L + 1 - \tilde{s})(\tilde{s} - c) \geq \max_{s_1 \neq s_{1H}} \Pi_H(s_1, 0) = \tilde{\pi}. \tag{5}$$

Thus, s_{1H}^* forms a separating equilibrium that survives the intuitive criterion *iff* it satisfies

$$s_{1H}^* \in \arg\max_{s_{1H}} \Pi_H(s_{1H}, 1) \quad \text{subject to (4) and (5).} \tag{6}$$

Note that type H's IC constraint (5) is always satisfied for $s_{1H} \geq c$, and that it is never optimal for the type H seller to set $s_{1H} < c$. Binding constraint (4) leads to two thresholds: $\bar{s} = 1$ and $\underline{s} = v_L$. Thus s_{1H} is supported in a separating equilibrium if and only if $s_{1H} \geq \bar{s} = 1$. The profit maximization program (6) implies that the price chosen by type H in a separating equilibrium is given by $s_{1H}^* = \max\{\tilde{s}_{1H}, \bar{s}\} = \max\{\tilde{s}, 1\}$, where $\tilde{s}_{1H}$ is the equilibrium first-period price under public information in Remark 1. Moreover, $\tilde{s} \gtreqless 1$ holds *iff* $c \gtreqless 1 - v_L$. We summarize the separating equilibrium under uniform pricing in the next proposition.

Proposition 1. *Under uniform pricing, there is a unique separating equilibrium that survives the intuitive criterion. (i) If $c \geq 1 - v_L$, the equilibrium outcome under public information stated in Remark 1 is supported. (ii) If $c < 1 - v_L$, the type L seller chooses $s_{1L}^* = s_{2L}^* = v_L$ and the type H seller chooses $s_{1H}^* = \bar{s} = 1$ and $s_{2H}^* = \tilde{s}$. The two types' expected profits are respectively $\Pi_L^u = 2v_L$ and $\Pi_H^u = v_L(1 - c) + \tilde{\pi}$.*

3.2 Behavior-Based Pricing

In this section, we first analyze the public information benchmark when BBP is permitted, and then proceed to the analysis under quality uncertainty.

3.2.1 Public Information Benchmark

Suppose product quality is publicly observed at $t = 1$. A type L product will be sold at a uniform price to all consumers in both periods, and thus $\tilde{p}_{1L} = \tilde{p}_{2L} = v_L$.[15] All consumers purchase the

[14] Throughout the paper, we use $\Pi_q(\ell, \mu)$ to denote type q's expected profits when he chooses price ℓ and is believed to be type H with probability μ.

[15] Here $\tilde{p}_{2L} = (v_L, v_L)$. We use the shorthand $p_{2L} = v_L$ where there is no confusion.

product in both periods and the total profits of the seller are $\tilde{\Pi}^b_L = 2v_L$.

Now consider the type H seller's price choices when BBP is permitted. At $t = 2$, with a price list $p_{2H} = (p^R_{2H}, p^N_{2H})$, repeat purchasers buy the product if and only if $v_i \geq p^R_{2H}$, and first-time purchasers buy the product if and only if $v_i \geq p^N_{2H}$. At $t = 1$, observing p_{1H} and anticipating p_{2H}, a consumer with value v_i purchases the product only if

$$(v_i - p_{1H}) + \max\{v_i - p^R_{2H}, 0\} \geq \max\{v_i - p^N_{2H}, 0\} \tag{7}$$

where the two terms on the LHS are respectively the consumer's utility from purchasing her initial unit at price p_{1H} at $t = 1$ and the option value of making a repeat purchase at price p^R_{2H} at $t = 2$, while the RHS is her option value of making a first-time purchase at price p^N_{2H} in the second period.

Define the marginal consumer as the one with valuation $\hat{v} \in [v_L, v_L + 1]$, and note that a consumer purchases in the first period *if and only if* $v_i \geq \hat{v}$. When a marginal consumer indeed exists, $\hat{v}$ divides consumers into a high-valuation segment where those with $v_i \geq \hat{v}$ face price p^R_{2H} and a low-valuation segment where consumers with $v_i < \hat{v}$ face price p^N_{2H} in the second period. It follows that the type H seller chooses $p^R_{2H} \geq \hat{v}$ to solve $\max_{p^R_{2H}} (v_L + 1 - p^R_{2H})(p^R_{2H} - c)$, leading to $p^R_{2H}(\hat{v}) = \max\{\tilde{s},\ \hat{v}\}$. Similarly, the type H seller chooses $p^N_{2H} \in [c, \hat{v}]$ to solve $\max_{p^N_{2H}} (\hat{v} - p^N_{2H})(p^N_{2H} - c)$, leading to $p^N_{2H}(\hat{v}) = \max\{\frac{\hat{v}+c}{2}, c\}$.

Making use of $p^R_{2H}(\hat{v})$ and $p^N_{2H}(\hat{v})$ and (7), the valuation of the marginal consumer $\hat{v}$ can be uniquely pinned down by

$$(\hat{v} - p_{1H}) + \max\left\{\hat{v} - \max\{\tilde{s},\ \hat{v}\}, 0\right\} = \max\left\{\hat{v} - \max\{\frac{\hat{v}+c}{2}, c\}, 0\right\},$$

which simplifies to

$$\hat{v} - p_{1H} = \max\{\frac{\hat{v} - c}{2}, 0\}.$$

It follows that if $\hat{v} \geq c$, $\hat{v} = 2p_{1H} - c$, and if $\hat{v} < c$, then $\hat{v} = p_{1H}$. Note that if $p_{1H} > \tilde{s}$, even a consumer with $v_i = v_L + 1$ will not purchase in the first period, and thus $\hat{v}$ does not exist. The next remark summarizes the relationship between $\hat{v}$ and type H's first-period price p_{1H}.

Remark 2. *A marginal consumer with $\hat{v}$ exists if and only if $p_{1H} \in [v_L, \tilde{s}]$. Moreover, if $p_{1H} \in [c, \tilde{s}]$, then $\hat{v} = 2p_{1H} - c$, and if $p_{1H} \in [v_L, c)$, then $\hat{v} = p_{1H}$.*

At $t = 1$, the type H seller chooses p_{1H} to maximize his expected profits from both periods

$$\Pi_H(p_{1H}, 1) = (v_L + 1 - \hat{v})(p_{1H} - c) + (\hat{v} - p^N_{2H})(p^N_{2H} - c) + (v_L + 1 - p^R_{2H})(p^R_{2H} - c), \tag{8}$$

where the first term on the RHS is the seller's profit from $t = 1$, and the second and third terms are his profits from the low- and high-valuation segments at $t = 2$, respectively. In the next remark we show that it is optimal for the type H seller to choose $p_{1H} \in [c, \tilde{s}]$.

Remark 3. *Under public information, type H's optimal price choice satisfies* $p_{1H} \in [c, \tilde{s}]$.

With $p_{1H} \in [c, \tilde{s}]$, $\hat{v} = 2p_{1H} - c$ follows from Remark 2, which in turn implies that $p_{2H}^N = p_{1H}$ and $p_{2H}^R = \max\{\tilde{s}, 2p_{1H} - c\}$. The type H seller's expected profit (8) can be rewritten as

$$\Pi_H(p_{1H}, 1) = (v_L + 1 - p_{1H})(p_{1H} - c) + (v_L + 1 - \max\{\tilde{s}, 2p_{1H} - c\})(\max\{\tilde{s}, 2p_{1H} - c\} - c) \tag{9}$$

where the first term on the RHS is the expected profit from consumers making their first-time purchase, either at $t = 1$ or $t = 2$, and the second term represents the profit from consumers making a repeat purchase at $t = 2$. Solving for p_{1H} that maximizes $\Pi_H(p_{1H}, 1)$ leads to the equilibrium outcome under public information when BBP is permitted.

Lemma 1. *Suppose product quality is publicly observed and BBP is permitted. The type L seller charges a flat price in both periods,* $\tilde{p}_L = \{v_L, v_L\}$. *The type H seller uses conditional prices* $\tilde{p}_H = \{\tilde{p}_{1H}, (\tilde{p}_{2H}^R, \tilde{p}_{2H}^N)\}$ *with*

$$\tilde{p}_{1H} = \tilde{p}_{2H}^N = \frac{3v_L + 7c + 3}{10}, \quad \tilde{p}_{2H}^R = \frac{3v_L + 2c + 3}{5}.$$

The two types' expected profits are, respectively, $\tilde{\Pi}_L^b = 2v_L$ *and* $\tilde{\Pi}_H^b = \frac{9}{20}(v_L + 1 - c)^2$. *The type H seller is worse off with the option of behavior-based pricing relative to uniform pricing.*

The last statement in Lemma 1 follows directly from comparing type H's profit with that under uniform pricing in Remark 1: $\tilde{\Pi}_H^b = \frac{9}{5}\tilde{\pi} < \tilde{\Pi}_H^u = 2\tilde{\pi}$. This confirms the insight in the existing literature that BBP is not an optimal strategy for a monopolist under public information about product quality. (See, e.g., Acquisti and Varian 2005.)

Without BBP (which is the case if discriminatory pricing is prohibited, or if the seller commits to not using price conditioning), the type H seller would post $\tilde{s}_H$, the equilibrium price under public information, and pocket $\tilde{\Pi}_H^u = 2\tilde{\pi}$ as stated in Remark 1. With the option of BBP, $\tilde{s}_H$ is no longer sustainable in equilibrium. BBP leads to two opposing effects on type H's profits relative to uniform pricing: a positive effect on the second-period profit and a negative effect on the first-period profit. (1) Given $p_{1H} \in [c, \tilde{s}]$, at $t = 2$ the type H seller finds it optimal to make use of the available information about consumers' purchasing records, and charges $p_{2H}^R = \max\{\tilde{s}, \hat{v}\} \geq \tilde{s}$ to repeat purchasers and $p_{2H}^N = p_{1H} < \tilde{s}$ to first-time purchasers. Thus, compared with uniform pricing, BBP increases the seller's second-period profit due to the increased price for repeat purchasers and the increased demand from first-time purchasers. (2) At $t = 1$, if there is no future price conditioning, consumers purchase as long as $v_i \geq p_{1H}$. However, anticipating BBP, consumers with $v_i \in [p_{1H}, \hat{v})$ delay their consumption until the second period, which forces type H to lower the first-period price below $\tilde{s}_{1H} = \tilde{s}$. Moreover, the first period demand falls despite the lower price, because consumers purchase if and only if $v_i \geq \hat{v} = 2p_{1H} - c > \tilde{s}$. As a result, the type H seller's first-period profit decreases due to lower demand and the lower first-period price. When the product quality is public information, the negative effect on type H's first-period profit dominates the positive effect on his second-period profit, and the option of BBP makes the type H seller worse off.

Note that Lemma 1 provides a useful benchmark for the subsequent analysis: when there is asymmetric information about product quality, the type H seller may distort the first-period price from the equilibrium price under public information ($\tilde{p}_{1H}$) to signal product quality, and this distortion has nontrivial effects on the seller's profits.

3.2.2 Separating Equilibria under Quality Uncertainty

In this subsection we analyze the game where the seller holds private information about the product quality at $t = 1$. Since the product quality is perfectly revealed at $t = 2$, the type L seller can only charge an unconditional price $p_{2L} = v_L$. Thus, type L's benefits from mimicking his high-quality counterpart derive solely from his first-period profit.

Consider a separating equilibrium where the two types post distinct prices, $p_{1H} \neq p_{1L}$. The equilibrium path beliefs are $\mu(p_{1H}) = 1$ and $\mu(p_{1L}) = 0$. Suppose the off-equilibrium path beliefs are such that $\mu(p_1) = 0$ for $p_1 \neq p_{1H}$. Following such beliefs, $p^*_{1L} = v_L$ must hold, and the type L seller has no incentive to deviate to any price different from p_{1H}. Moreover, if type H deviates to some $p_1 \neq p_{1H}$, he can only sell a positive quantity at price v_L in the first period, and all consumers buy at this price which leads to a loss $v_L - c$ for the seller, or he can set $p_1 > v_L$ and no consumer purchases at $t = 1$ leading to zero first-period profit; at $t = 2$, since all consumers have the same purchasing history, it is optimal for the seller to set price $\tilde{s}$ and receive $\tilde{\pi}$. Thus, if the type H seller deviates from the equilibrium candidate p_{1H}, the highest deviation payoff is given by $\max_{p_1 \neq p_{1H}} \Pi_H(p_1, 0) = \tilde{\pi}$.

Which p_{1H} can be supported in a separating equilibrium? For p_{1H} from different intervals, the valuation of the marginal consumer varies, and this in turn affects the demands of the high-valuation and low-valuation segments at $t = 2$. Note that, following belief $\mu(p_{1H}) = 1$, the relationship between $\hat{v}$ and p_{1H} is the same as that characterized in Remark 2.

Consider $p_{1H} \in [c, \tilde{s}]$, and it follows $\hat{v} = 2p_{1H} - c$. Believing $\mu(p_{1H}) = 1$, consumers with $v_i \geq \hat{v}$ purchase the product at price p_{1H} at $t = 1$, leading to the first-period demand

$$D_1^b(p_{1H}, 1) \equiv v_L + 1 - (2p_{1H} - c). \tag{10}$$

The two types' incentive compatibility constraints are given by

$$\Pi_L(p_{1H}, 1) = (v_L + 1 - 2p_{1H} + c)p_{1H} + v_L \leq \Pi_L(v_L, 0) = 2v_L \tag{11}$$

$$\Pi_H(p_{1H}, 1) \geq \max_{p_1 \neq p_{1H}} \Pi_H(p_1,\ 0) = \tilde{\pi} \tag{12}$$

where $\Pi_H(p_{1H}, 1)$ is given by (9), type H's profits under public information. Note that, for $p_{1H} \in [c, \tilde{s}]$, (12) is always satisfied. Thus, a separating equilibrium with $p_{1H} \in [c, \tilde{s}]$ exists if and only if (11) holds.

In the next lemma we show that $p_{1H} \in [v_L, c)$ is a dominated choice for the type H seller, and a candidate with $p_{1H} \in (\tilde{s}, v_L + 1]$ can be ruled out by invoking the intuitive criterion. Thus, a separating equilibrium that survives the intuitive criterion must satisfy $p_{1H} \in [c, \tilde{s}]$.

Lemma 2. *In a separating equilibrium that survives the intuitive criterion, $p_{1H} \in [c, \tilde{s}]$ and satisfies type L's incentive compatibility constraint* (11).

Note that binding constraint (11) leads to two thresholds $\underline{p}$ and $\bar{p}$ which are given by

$$\underline{p} = \frac{v_L + 1 + c - \sqrt{\Delta}}{4}, \qquad \bar{p} = \frac{v_L + 1 + c + \sqrt{\Delta}}{4}, \tag{13}$$

where $\Delta \equiv (v_L + 1 + c)^2 - 8v_L$. When $\Delta \geq 0$, type L's IC constraint (11) is satisfied if and only if $p_{1H} \geq \bar{p}$ or $p_{1H} \leq \underline{p}$. Since $\Pi_H(p_{1H}, 1)$ increases in p_{1H} for $p_{1H} \leq \tilde{p}_{1H}$ and achieves the maximal value at $\tilde{p}_{1H} > \frac{\underline{p}+\bar{p}}{2}$, $\Pi_H(\underline{p}, 1) < \Pi_H(\bar{p}, 1)$. As a result, $p_{1H} \geq \bar{p}$ must hold for a separating equilibrium to survive the intuitive criterion.[16]

When $\tilde{p}_{1H} \geq \bar{p}$, type L's IC constraint (11) is satisfied at $p_{1H} = \tilde{p}_{1H}$, and the type L seller has no incentive to mimic type H when the latter posts the equilibrium price under public information. Since $\tilde{p}_{1H}$ is the unique solution to type H's profit maximization problem, $p_{1H} = \tilde{p}_{1H}$ forms the unique separating equilibrium that survives the intuitive criterion in this case. When $\tilde{p}_{1H} < \bar{p}$, $p_{1H} = \tilde{p}_{1H}$ is insufficient to prevent type L's mimicking. In this case, type H needs to distort the price above $\tilde{p}_{1H}$ to send a convincing signal, and $p_{1H} = \bar{p}$ forms the unique separating

[16]Since $\Pi_H(\underline{p}, 1) < \Pi_H(\bar{p} + \epsilon, 1)$ for small positive ϵ and $\Pi_L(\bar{p} + \epsilon, 1) < \Pi_L(v_L, 0) = 2v_L$, the intuitive criterion requires $\mu(\bar{p} + \epsilon) = 1$. Thus $p_{1H} = \underline{p}$ cannot be optimal for the type H seller. Similar arguments can be used to rule out $p_{1H} < \underline{p}$.

equilibrium that survives the intuitive criterion. Thus, a high price together with the use of BBP signals product quality.

Proposition 2. *Under BBP, there is a unique separating equilibrium that survives the intuitive criterion. (i) If $\tilde{p}_{1H} \geq \bar{p}$, the equilibrium outcome under public information in Lemma 1 is supported. (ii) If $\tilde{p}_{1H} < \bar{p}$, the type L seller chooses $p_L^* = \{v_L, v_L\}$ and the type H seller charges $p_H^* = \{p_{1H}^*, (p_{2H}^{*R}, p_{2H}^{*N})\} = \{\bar{p}, (2\bar{p} - c, \bar{p})\}$. The two types' expected profits are respectively*

$$\Pi_L^b = 2v_L; \qquad \Pi_H^b = \frac{(1 - 3c + v_L + \sqrt{\Delta})(7 + 3c + 7v_L - 5\sqrt{\Delta})}{16}.$$

Note that $\tilde{p}_{1H} < \bar{p}$ is equivalent to $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 < 0$ which implies $\Delta \geq 0$. Fixing v_L, part (i) of Proposition 2 applies if c is sufficiently large, while part (ii) applies if c is small. Note that $\Pi_L(p_{1H}, 1)$ in (11) is a downward-facing parabola in p_{1H} and $\frac{\partial \Pi_L(p_{1H},1)}{\partial p_{1H}} = v_L + 1 + c - 4p_{1H}$. Starting from $p_{1H} = \frac{v_L+1+c}{4}$, when c is large, $\Pi_L(p_{1H}, 1)$ decreases quickly with p_{1H} and drops below $\Pi_L(v_L, 0) = 2v_L$ before p_{1H} reaches $\tilde{p}_{1H}$, and thus the public information outcome is sufficient to deter imitation by type L. If c is small, $\Pi_L(p_{1H}, 1)$ decreases at a low speed with p_{1H}, and $p_{1H} > \tilde{p}_{1H}$ is needed for $\Pi_L(p_{1H}, 1)$ to drop below $\Pi_L(v_L, 0)$. In both cases, the equilibrium price patterns exhibit a flat price for a type L product and BBP for a type H product, with first-time purchasers paying a lower price than repeat purchasers.

3.3 Benefits of Behavior-Based Pricing

Comparing Propositions 1 and 2, we are able to get a clear picture of the benefits and costs of BBP when asymmetric information exists about product quality.[17] Recalling the discussions in Section 3.2.1, there is no need for price signaling under public information. The negative effect of BBP on the type H seller's first-period profit dominates its positive effect on his second-period profit, and the option of BBP makes the type H seller worse off relative to uniform pricing. The same holds true when part (i)s of the two Propositions apply under quality uncertainty. In these cases, the corresponding public information outcome is supported in equilibrium under both BBP and uniform pricing, and there is no need for the seller to distort the price to signal product quality. The option of BBP hurts the type H seller when a signaling consideration is unimportant.

However, when part (ii)s of Propositions 1 and 2 apply, the corresponding public information outcome cannot be supported in equilibrium, and the type H seller needs to distort the first-period price upward to signal product quality; that is, $p_{1H}^* = \bar{p} > \tilde{p}_{1H}$ under BBP and $s_{1H}^* = \bar{s} > \tilde{s}$

[17]Note that the seller's profit, the consumer surplus and the social welfare associated with a type L product do not change when the price scheme changes, with or without quality uncertainty. Thus, in the analysis of the welfare effect, we focus on the welfare levels associated with a type H product.

under uniform pricing. Recall the demand functions (3) and (10) when the type L seller imitates the price choice of type H. Note that the price-elasticity of the imitation demand at every price k exhibits

$$\eta^b(k) \equiv -\frac{(dD_1^b(k,1)/dk)k}{D_1^b(k,1)} = \frac{2k}{v_L + 1 - (2k-c)} > \eta^u(k) \equiv -\frac{(dD_1^u(k,1)/dk)k}{D_1^u(k,1)} = \frac{k}{v_L + 1 - k}.$$

This implies that a marginal increase in the first-period price by the type H seller leads to a larger reduction of type L's imitation demand under BBP than under uniform pricing. This in turn lowers the type L seller's imitation profit, relaxes IC constraint (11), and effectively lowers the threshold of the first-period price which type H needs to set to send a convincing signal. As a result, the equilibrium price is closer to the corresponding public information outcome under BBP than under UP; that is, $\bar{p} - \tilde{p}_{1H} < \bar{s} - \tilde{s}$, implying a lower signaling cost for the type H seller under BBP. The lower signaling cost affects the type H seller's profit positively and may turn BBP into a profitable strategy for the seller. In the next corollary, we provide sufficient conditions under which BBP is more profitable than uniform pricing for the type H seller.

Corollary 1. *For given v_L with $v_L \in [0, \frac{1}{5})$, there exist thresholds c_1 and c_2 with $c_1, c_2 \in (v_L, v_L + 1)$ and $c_1 \leq c_2$, such that BBP increases the type H seller's profit by comparison with uniform pricing if $c \leq c_1$, and lowers the type H seller's profit by comparison with uniform pricing if $c \geq c_2$.*

In summary, BBP affects the type H seller's profits in two opposite directions. Without signaling considerations, the negative effect on the first-period profit dominates the positive effect on the second-period profit, and the option of BBP reduces the seller's profit relative to simple uniform pricing. When the signaling consideration is important, BBP has the additional benefit of lowering the signaling cost of the type H seller, which reinforces the positive effect that BBP has on the second-period profit. For given v_L, when c is small, the price elasticity of the imitation demand under BBP is large and the signaling cost effect is prominent, making BBP more profitable for the type H seller relative to uniform pricing.

Consumer Surplus and Social Welfare When behavior-based pricing is profitable for the seller, one naturally wonders whether such profitability comes at the expense of consumer surplus and social welfare. The next corollary shows that this is not the case under quality uncertainty.

Corollary 2. *When there is asymmetric information about product quality, BBP increases consumer surplus and total welfare by comparison with uniform pricing.*

When the price regime moves from uniform pricing to BBP, the consumer surplus increases because the average price decreases and the total demand for type H products increases. For illustration, consider the case where part (ii)s of Propositions 1 and 2 apply. Under uniform pricing, type H products are sold at price $s_{1H}^* = \bar{s} = 1$ with demand $v_L + 1 - \bar{s}$ in the first period, and sold at price $s_{2H}^* = \tilde{s}$ with demand $v_L + 1 - \tilde{s}$ in the second period. Under BBP, repeat purchasers pay the price $p_{2H}^{*R} = 2\bar{p} - c$ with demand $v_L + 1 - (2\bar{p} - c)$, and first-time purchasers pay the price $p_{1H}^* = p_{2H}^{*N} = \bar{p}$ for their first unit, while the total quantity is $v_L + 1 - \bar{p}$. Note that $\bar{p} < \tilde{s}$ and $2\bar{p} - c < \bar{s} = 1$, meaning that all consumers who purchase a type H product under uniform pricing pay less with BBP, leading to an increase in consumer surplus. Moreover, total demand under BBP is also larger than that under uniform pricing. Thus, BBP increases consumer surplus relative to uniform pricing.

Moreover, social welfare increases even when the type H seller is worse off when the price regime moves from uniform pricing to BBP. To see this, note that consumers with $v_i \in [\bar{s}, v_L + 1]$ purchase two units while consumers with $v_i \in [\tilde{s}, \bar{s})$ purchase one unit under uniform pricing. These consumers will also consume under BBP, and from these purchases the increase in consumer surplus precisely offsets the decrease in the type H seller's profits when the price regime changes. However, consumers with $v_i \in [\bar{p}, \tilde{s}]$, who do not purchase under uniform pricing, will purchase one unit under BBP at price $\bar{p} > c$, while consumers with $v_i \in [2\bar{p} - c, \bar{s}]$, who purchase one unit under uniform pricing, will purchase two units under BBP, both of these leading to an increase in total welfare.

Numerical Example We close this section with a numerical example illustrating the benefits of BBP relative to uniform pricing. Suppose $v_L = 0.1$ and $c \in (0.1, 1.1)$. Under uniform pricing, by Proposition 1 we have $s_{1H}^* = s_{2H}^* = \tilde{s} = \frac{1.1+c}{2}$ if $c \geq 0.9$, and $s_{1H}^* = 1$ and $s_{2H}^* = \frac{1.1+c}{2}$ if $c < 0.9$. Under BBP, by Proposition 2, when $c \geq 0.824$, $p_{1H}^* = p_{2H}^{*N} = \frac{3.3+7c}{10}$, and $p_{2H}^{*R} = \frac{3.3+2c}{5}$; when $c < 0.824$, $p_{1H}^* = p_{2H}^{*N} = \frac{1.1+c+\sqrt{c^2+2.2c+0.41}}{4}$, and $p_{2H}^{*R} = \frac{1.1-c+\sqrt{c^2+2.2c+0.41}}{2}$.

In equilibrium, $\Pi_H^b > \Pi_H^u$ if and only if $c < 0.696$, but the consumer surplus and the social welfare are higher for all c under BBP.[18] Figure 1 illustrates the incremental changes in type H's profits and in the consumer surplus when the pricing scheme moves from uniform pricing to

[18] In this example, $c < 0.696$ is a sufficient and necessary condition for $\Pi_H^b > \Pi_H^u$. This does not contradict the statement in Corollary 1, because $c \leq c_1$ and $c \geq c_2$ are sufficient but not necessary conditions for the profitability of BBP.

behavior-based pricing.

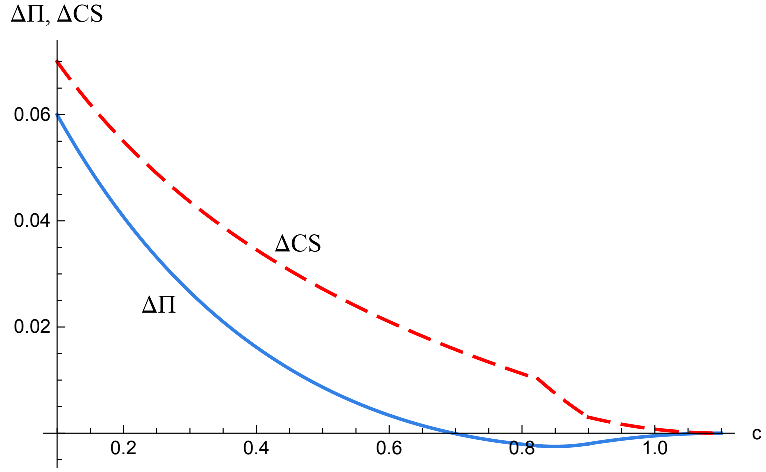

Figure 1: The incremental change in type H's profits ($\Delta\Pi = \Pi^b_H - \Pi^u_H$) and in the consumer surplus ($\Delta CS = CS^b_H - CS^u_H$) when the pricing scheme moves from uniform pricing to BBP for $v_L = 0.1$.

4 Price Commitment

So far, we have assumed that the seller has limited commitment and posts short-term prices at the beginning of each period. In some cases, the seller may have the ability to commit to future prices, post price scheme $\ell_q = \{\ell_{1q}, \ell_{2q}\}$ for both periods at $t = 1$, and honor the price offers at $t = 2$. Recall that, under public information, the option of BBP reduces type H's profit relative to uniform pricing in the limited-commitment regime. It follows that in the price-commitment regime, since uniform pricing is a degenerate form of BBP, the seller will ignore the information about consumers' purchasing records and choose the static monopoly price in each period under public information. We summarize this observation in the next remark.[19]

Remark 4. *Under public information about product quality, if the seller can commit to future prices, the type H seller posts the same price scheme under BBP and uniform pricing,* $\tilde{s}^c_H = \{\tilde{s}, \tilde{s}\}$, $\tilde{p}^c_H = \{\tilde{s}, (\tilde{s}, \tilde{s})\}$, *and obtains profit* $\tilde{\Pi}^c_H = 2\tilde{\pi}$*; the type L seller posts* $\tilde{p}^c_L = \tilde{s}^c_L = \{v_L, v_L\}$ *and obtains* $\tilde{\Pi}^c_L = 2v_L$. *It is not optimal for the type H seller to condition second-period prices on consumers' purchasing history.*

Under quality uncertainty, the type H seller has more instruments to signal in the price-commitment regime, because the second-period prices ℓ_{2q} (including with BBP being used or not), posted together with ℓ_{1q}, also convey information about product quality.[20] It is natural to

[19] We use superscript c to denote the profit functions and equilibrium outcomes in the price-commitment regime.

[20] Recall that in the limited-commitment regime, ℓ_{2q} is posted at $t = 2$ and cannot be used as a quality signal.

wonder how the direct signaling role of the second-period prices affects the seller's profits and consumer surplus, and whether the seller has an incentive to use BBP in contrast to the public information outcome.

4.1 Uniform Pricing under Quality Uncertainty

In the price-commitment regime, a seller of type q posts the prices for the two periods, $s_q = \{s_{1q}, s_{2q}\}$, at $t = 1$. In a separating equilibrium with $s_H \neq s_L$, consumers' equilibrium path beliefs satisfy $\mu(s_H) = 1$ and $\mu(s_L) = 0$. Given such beliefs, the equilibrium prices of the type L seller are uniquely given by $s_L^c = \{v_L, v_L\}$. Moreover, $s_{2H} > v_L$ must hold in equilibrium because $s_{2H} \leq v_L$ is dominated for the type H seller. Therefore, type L receives zero second-period profit by choosing s_H. On the other hand, making use of the off-equilibrium path belief $\mu(s) = 0$ for $s \neq s_H$, the type H seller's best deviation choice is $s^d = \{s_{1H}^d, \tilde{s}\}$ with $s_{1H}^d > v_L$, leading to an expected profit $\tilde{\pi}$ for the seller. Thus, the two types' incentive compatibility constraints can be written as

$$\Pi_L^c(s_H, 1) = (v_L + 1 - s_{1H})s_{1H} + 0 \leq \Pi_L^c(s_L^c, 0) = 2v_L, \tag{14}$$

$$\Pi_H^c(s_H, 1) = (v_L + 1 - s_{1H})(s_{1H} - c) + (v_L + 1 - s_{2H})(s_{2H} - c) \geq \max_{s^d \neq s_H} \Pi_H^c(s^d, 0) = \tilde{\pi}. \tag{15}$$

A separating equilibrium that survives the intuitive criterion is given by

$$s_H^c = \arg\max_{s_H} \Pi_H^c(s_H, 1) \qquad \text{subject to} \quad (14) \text{ and } (15). \tag{16}$$

In the next proposition, we summarize the equilibrium outcome in the price-commitment regime when the seller is restricted to uniform pricing.

Proposition 3. *Suppose the seller can commit to future prices and BBP is not permitted. Under asymmetric information about product quality, there is a unique separating equilibrium that survives the intuitive criterion.*

(i) If $c^2 \geq (v_L + 1)^2 - 8v_L$, the equilibrium outcome under public information stated in Remark 4 is supported.

(ii) If $c^2 < (v_L + 1)^2 - 8v_L$, the type L seller chooses $s_L^c = \{v_L, v_L\}$ and the type H seller sets $s_H^c = \{s_{1H}^c, s_{2H}^c\}$ with $s_{1H}^c = \frac{v_L + 1 + \sqrt{(v_L+1)^2 - 8v_L}}{2}$ and $s_{2H}^c = \tilde{s}$. The equilibrium profits of the two types are $\Pi_L^{c,u} = 2v_L$ and $\Pi_H^{c,u} = (v_L + 1 - s_{1H}^c)(s_{1H}^c - c) + \tilde{\pi}$.

Part (ii) of Proposition 3 applies when the equilibrium outcome under public information is insufficient to prevent mimicking, and the type H seller needs to distort prices from $\tilde{s}_H^c = \{\tilde{s}, \tilde{s}\}$

to signal product quality. Note that $s^c_{1H} < \bar{s}$, where $\bar{s}$ is the price that type H needs to set in the limited-commitment regime to deter mimicking. Commitment power leads to a smaller price distortion because the second-period price, s_{2q}, also conveys information about product quality. If type L imitates type H's price choice, posting $s^c_{2H} = \tilde{s}$ leads to zero second-period profit, since no consumer will purchase a low-quality product at a price exceeding v_L. This lowers type L's imitation incentive relative to the limited-commitment regime, where type L can always ensure himself a positive profit by charging v_L at $t = 2$.

4.2 Behavior-Based Pricing under Quality Uncertainty

In a separating equilibrium with $p_H \neq p_L$, consumers' equilibrium path beliefs satisfy $\mu(p_H) = 1$ and $\mu(p_L) = 0$. Suppose the off-equilibrium path belief is $\mu(p) = 0$ for $p \neq p_H$. Then $p^c_L = \{v_L, v_L\}$ must hold in equilibrium, and the type L seller's equilibrium profit is $\Pi^{c,b}_L = 2v_L$.

With the option of BBP, the type H seller chooses $p_H = \{p_{1H}, p_{2H}\} = \{p_{1H}, (p^R_{2H}, p^N_{2H})\}$ at the beginning of $t = 1$. He has a large set of price schemes to consider because the three prices, p_{1H}, p^R_{2H} and p^N_{2H}, can all be different. In the next lemma we show that, in the analysis of separating equilibria that survive the intuitive criterion, it is without loss of generality to focus on price schemes with identical prices to first-time purchasers; that is, $p_{1H} = p^N_{2H}$. This observation helps us to simplify the subsequent analysis substantially.

Lemma 3. *Suppose the seller can commit to future prices and BBP is permitted. In searching for equilibrium prices of the type H seller in a separating equilibrium that survives the intuitive criterion, it suffices to consider price schemes in the form of* $p_H = \{p_{1H}, (p^R_{2H}, p^N_{2H})\} = \{\tau, (\beta\tau, \tau)\}$ *with* $\beta > 0$.

The logic behind the proof of Lemma 3 is that when the seller can commit to future prices, for any price scheme $p_H = \{p_{1H}, (p^R_{2H}, p^N_{2H})\}$ with $p_{1H} \neq p^N_{2H}$, an alternative scheme $\hat{p}_H$ exists with $\hat{p}_{1H} = \hat{p}^N_{2H}$ that brings the type H seller (weakly) larger profits while keeping type L's incentive compatibility constraint satisfied in a separating equilibrium. Since price schemes with $p_{1H} = p^N_{2H}$ can be conveniently represented by $p_H = \{\tau, (\beta\tau, \tau)\}$ with $\beta > 0$, type H's equilibrium price choice is thus transformed into one of finding an equilibrium combination of τ and β.

Given price scheme $p_H = \{\tau, (\beta\tau, \tau)\}$ and belief $\mu(p_H) = 1$, a consumer purchases at $t = 1$ if and only if $v_i \geq \beta\tau$ when $\beta \geq 1$, and if and only if $v_i \geq \frac{1+\beta}{2}\tau$ when $\beta < 1$. Thus, a seller faces the following first-period demand when choosing p_H and inducing consumer belief $\mu(p_H) = 1$:

$$D^{c,b}_1(p_H, 1) = \begin{cases} v_L + 1 - \beta\tau & \text{if } \beta \geq 1 \\ v_L + 1 - \frac{\beta+1}{2}\tau & \text{if } \beta < 1 \end{cases}.$$

Since $c > v_L$, p_H satisfies $\tau > v_L$ and $\beta\tau > v_L$ in equilibrium, type L receives zero second-period profit by imitating type H's price choice. Thus, type L's incentive compatibility constraint can be written as

$$\Pi_L^c(p_H, 1) = D_1^{c,b}(p_H, 1)\tau + 0 \leq \Pi_L^c(p_L^c, 0) = 2v_L. \tag{17}$$

For the type H seller, when $\beta \geq 1$, consumers with $v_i \geq \beta\tau$ purchase their second unit at price $\beta\tau$, and consumers with $v_i \in [\tau, \beta\tau)$ purchase their first unit at price τ in the second period; when $\beta < 1$, consumers with $v_i \geq \frac{\beta+1}{2}$ purchase their second unit at price $\beta\tau$ in the second period. Thus, the type H seller's expected profit from choosing p_H is

$$\Pi_H^c(p_H, 1) = \begin{cases} (v_L + 1 - \beta\tau)(\tau - c) + (\beta\tau - \tau)(\tau - c) + (v_L + 1 - \beta\tau)(\beta\tau - c) & \text{if } \beta \geq 1 \\ (v_L + 1 - \frac{\beta+1}{2}\tau)(\tau - c) + (v_L + 1 - \frac{\beta+1}{2}\tau)(\beta\tau - c) & \text{if } \beta < 1 \end{cases}. \tag{18}$$

Note that, when deviating to an alternative price, type H's maximal deviation profit is $\tilde{\pi}$. Making use of (18), type H's incentive compatibility constraint is given by

$$\Pi_H^c(p_H, 1) \geq \max_{p^d \neq p_H} \Pi_H^c(p^d, 0) = \tilde{\pi}. \tag{19}$$

It follows that, in a separating equilibrium surviving the intuitive criterion, p_H^c solves the following maximization problem:

$$p_H^c = \arg\max_{p_H} \Pi_H^c(p_H, 1) \quad \text{subject to (17) and (19)}. \tag{20}$$

When $(v_L + 1)^2 - 8v_L \leq c^2$ holds, the equilibrium price under public information, $\tilde{p}_H^c = \{\tilde{s}, (\tilde{s}, \tilde{s})\}$, satisfies type L's incentive compatibility constraint (17) and forms the unique equilibrium that survives the intuitive criterion. When $(v_L + 1)^2 - 8v_L > c^2$ holds instead, the public information outcome is insufficient to deter imitation by type L, so the type H seller needs to distort his prices to convince consumers of his product quality. We show in the next proposition that the type H seller indeed uses BBP as a signaling instrument, and the equilibrium price scheme exhibits first-purchase discount ($\beta > 1$).

Proposition 4. *Suppose the seller can commit to future prices and BBP is permitted. Under asymmetric information about product quality, a separating equilibrium exists that satisfies the intuitive criterion.*

(i) If $c^2 \geq (v_L + 1)^2 - 8v_L$, the public information outcome stated in Remark 4 is supported as the unique separating equilibrium.

(ii) If $c^2 < (v_L + 1)^2 - 8v_L$, the type H seller uses BBP and offers price scheme $p_H^c = \{\tau^c, (\beta^c \tau^c, \tau^c)\}$ in which $\tau^c \in (\frac{v_L+1-\sqrt{(v_L+1)^2-8v_L}}{2}, \tilde{s})$ and $\beta^c = \frac{1}{\tau^c}(v_L + 1 - \frac{2v_L}{\tau^c}) > 1$, and the type L seller chooses $p_L^c = \{v_L, v_L\}$. The equilibrium profits of the two types are

$$\Pi_L^{c,b} = 2v_L, \qquad \Pi_H^{c,b} = \frac{2v_L}{\tau^c}\left(v_L + 1 - c - \frac{2v_L}{\tau^c}\right) + (v_L + 1 - \tau^c)(\tau^c - c).$$

Part (ii) of Proposition 4 contrasts sharply with the result in Remark 4. When $c^2 < (v_L + 1)^2 - 8v_L$ holds, absent quality uncertainty, the seller does not use BBP even though it is an option; with quality uncertainty, BBP becomes a profitable strategy and occurs as an equilibrium choice of the type H seller. The existence of quality uncertainty reverses the result in the existing literature (e.g., Hart and Tirole, 1988; Acquisiti and Varian, 2005) that price conditioning is not optimal for a monopolist that can commit to future prices.

Given $c^2 < (v_L + 1)^2 - 8v_L$, the type H seller adopts BBP as a part of his quality signals, and the equilibrium price scheme has the feature of a first-purchase discount: $\beta^c > 1$ and first-time purchasers pay a lower price than repeat purchasers. Note that consumers' first-period demand, $D_1^{c,b}(p_H, 1)$, is more sensitive to a first-period price change when $\beta > 1$ than when $\beta = 1$ or $\beta < 1$. Compared with no price conditioning ($\beta = 1$) or a repeat-purchaser discount ($\beta < 1$), a marginal first-period price increase ($\Delta\tau$) when $\beta > 1$ leads to a larger reduction in first-period demand when the type L seller imitates the price choice of the type H seller, and this in turn leads to a larger decrease in his imitation profit $\Pi_L(p_H, 1)$. Moreover, price commitment pushes type L's second-period imitation profit down to zero. These two forces work in the same direction to lower type L's imitation profit, relax constraint (17), and make BBP a profitable strategy for the type H seller.

In classic works on price signaling, e.g., Bagwell and Riordan (1991), the type H seller needs to post a first-period price higher than the public information outcome in order to signal high product quality. This is not, however, necessarily true when the seller has the option of BBP and can commit to future prices. When part (ii) of Proposition 4 applies, the first-period price is lower than the equilibrium price under public information; that is, $p_{1H}^c = \tau^c < \tilde{s}$. This downward price distortion, together with conditional second-period prices, serves as a convincing signal of high product quality. We record this result in the next corollary.

Corollary 3. *Suppose the seller can commit to future prices and BBP is permitted. When $c^2 < (v_L + 1)^2 - 8v_L$ holds, the type H seller uses a first-period price that is lower than the equilib-*

rium price under public information and second-period prices that are conditional on consumers' purchasing history, to signal product quality.

4.3 Benefits of Behavior-Based Pricing

Comparison of respective part (ii)s in Propositions 3 and 4 shows how the option of BBP affects the type H seller's price signaling choice relative to uniform pricing. When $c^2 < (v_L + 1)^2 - 8v_L$ holds, the public information outcome is not supported under either pricing scheme. Under uniform pricing, $s^c_{1H} > s^c_{2H} = \tilde{s}$, and the type H seller makes an upward first-period price distortion to signal product quality. However, when BBP is permitted, the type H seller uses second-period price conditioning as part of his quality signals, and distorts the first-period price downward to signal product quality.

When the seller can commit to future prices, the signaling cost reduction effect of BBP relative to uniform pricing is even more significant, and, as a result, the type H seller is always better off using BBP. On the one hand, the first-period demand reduction effect of BBP is still present, similar to that in the limited-commitment regime. On the other hand, commitment to future prices drives type L's second-period imitation profit down to zero. This effect exists under both uniform pricing and BBP. With the option of BBP, type H can better leverage first- and second-period prices by pushing down the first-period price while keeping the price high for repeat purchasers, further lowering the imitation incentive of type L.

Note that both first-time and repeat purchasers pay lower prices under BBP, and total demand is higher under BBP than under uniform pricing. Thus, the option of BBP also increases consumer surplus and social welfare in the price-commitment regime.

Corollary 4. *When the seller can commit to future prices and there is asymmetric information about product quality, BBP increases the type H seller's profit, consumer surplus, and social welfare relative to uniform pricing.*

Numerical Example. We close this section by revisiting the example in Section 3.3 in which $v_L = 0.1$. Let $c = 0.5$. Then $c^2 < (v_L + 1)^2 - 8v_L$ holds and $\tilde{s} = 0.8$. Under uniform pricing, applying Proposition 3, the equilibrium prices for the type H seller are $\{s^c_{1H}, s^c_{2H}\} = \{0.870, 0.8\}$. Seller profit, consumer surplus, and social welfare associated with type H products are respectively $\Pi^{c,u}_H \approx 0.175$, $CS^{c,u}_H \approx 0.071$, and $TS^{c,u}_H \approx 0.246$.

Under BBP, by Proposition 4, numerical calculation shows that the equilibrium prices for the type H seller are $\{p^c_{1H}, p^c_{2H}\} = \{\tau^c, (\beta^c\tau^c, \tau^c)\} = \{0.785, (0.845, 0.785)\}$ and $\beta^c \approx 1.07$. Both first-time and repeat purchasers pay lower prices under BBP than under uniform pricing. Seller profit, consumer surplus, and social welfare associated with type H products are respectively $\Pi_H^{c,b} \approx 0.178$, $CS_H^{c,b} \approx 0.082$, and $TS_H^{c,b} \approx 0.260$. BBP increases the type H seller's profit by 1.48%, consumer surplus by 14.7%, and total welfare by 5.31%.

5 Tracking Technology

In the baseline model, the monopolist can only condition the second-period prices on a consumer's purchasing history. With the development of modern technology, he may be able to acquire more accurate information, following an initial purchase, about a consumer's willingness to pay. In an extreme case, a seller may learn perfectly the consumer's valuation of the product and practice personalized pricing accordingly. To investigate the effect of more accurate information on the seller's incentive to practice BBP and price signaling, we modify the baseline model in Section 2 in the following way: after a consumer makes a purchase at $t = 1$, the seller learns her valuation v_i with probability $\lambda \in [0, 1]$ and can charge her a personalized price equal to v_i at $t = 2$, while with probability $1 - \lambda$, the seller only recognizes the consumer's purchasing history as in the baseline model. Thus, the seller's information about a consumer becomes more accurate as λ increases. With $\lambda \to 0$, the model degenerates to the baseline model.[21]

Public Information First, consider the benchmark that product quality is publicly observed. A marginal consumer with $\hat{v}$ divides the second-period market into a high-valuation segment and a low-valuation segment. For the high-valuation segment, with probability λ, the type H seller learns his consumers' valuations and charges a personalized price v_i to each consumer with $v_i \geq c$, while with probability $1 - \lambda$ the seller knows only his consumers' purchasing history and chooses $p_{2H}^R \geq \hat{v}$ to solve $\max_{p_{2H}^R} (v_L + 1 - p_{2H}^R)(p_{2H}^R - c)$, leading to $p_{2H}^R(\hat{v}) = \max\{\tilde{s},\ \hat{v}\}$. For the low-valuation segment, the type H seller chooses $p_{2H}^N \in [c, \hat{v}]$ to solve $\max_{p_{2H}^N} (\hat{v} - p_{2H}^N)(p_{2H}^N - c)$, leading to $p_{2H}^N(\hat{v}) = \max\{\frac{\hat{v}+c}{2}, c\}$.

A consumer's expected valuation of a repeat purchase at $t = 2$ is given by $\lambda \cdot 0 + (1 - \lambda)(\max\{v_i - p_{2H}^R, 0\})$. Thus, the marginal consumer's valuation $\hat{v}$ is determined by

$$\hat{v} - p_{1H} + (1 - \lambda) \max\{\hat{v} - \max\{\tilde{s}, \hat{v}\}, 0\} = \max\{\hat{v} - \max\{\frac{\hat{v} + c}{2}, c\}, 0\},$$

which simplifies to

$$\hat{v} - p_{1H} = \max\{\frac{\hat{v} - c}{2}, 0\}.$$

Following similar arguments made in Section 3.2, $\hat{v} = 2p_{1H} - c$ holds in equilibrium, which in

[21] We use superscript t to denote the profit functions and equilibrium outcomes in this section.

turn implies $p_{2H}^R = \max\{2p_{1H} - c, \tilde{s}\}$ and $p_{2H}^N = p_{1H}$. The type H seller's profit as a function of p_{1H} under public information can be written as

$$\begin{aligned}\Pi_H^t(p_{1H}, 1) =&(v_L + 1 - \hat{v})(p_{1H} - c) + (\hat{v} - p_{2H}^N)(p_{2H}^N - c)\\&+ \lambda \int_{\hat{v}}^{v_L+1} (v_i - c)\mathrm{d}v_i + (1-\lambda)(v_L + 1 - p_{2H}^R)(p_{2H}^R - c)\\=&(v_L + 1 - p_{1H})(p_{1H} - c) + \lambda \int_{2p_{1H}-c}^{v_L+1} (v_i - c)\mathrm{d}v_i\\&+ (1-\lambda)(v_L + 1 - \max\{2p_{1H} - c, \tilde{s}\})(\max\{2p_{1H} - c, \tilde{s}\} - c).\end{aligned} \tag{21}$$

In contrast to the baseline model, where $2p_{1H} - c > \tilde{s}$ and $p_{2H}^R = 2p_{1H} - c$ always hold, it is possible to have $\tilde{s} > 2p_{1H} - c$ and $p_{2H}^R = \tilde{s}$ in equilibrium when λ is sufficiently large. Solving for the p_{1H} that maximizes $\Pi_H^t(p_{1H}, 1)$ in (21) gives us the equilibrium outcome under public information, which we summarize in the next lemma.

Lemma 4. *Suppose product quality is publicly observed and behavior-based pricing is permitted. The type L seller charges a flat price v_L in both periods, $\tilde{p}_L^t = \{v_L, v_L\}$; the type H seller charges*

$$\tilde{p}_{1H}^t = \begin{cases} \frac{(3-2\lambda)(v_L+1)+(7-2\lambda)c}{10-4\lambda} & \text{if } \lambda \leq \frac{1}{2} \\ \frac{v_L+1+(4\lambda+1)c}{4\lambda+2} & \text{if } \lambda > \frac{1}{2} \end{cases}; \quad \tilde{p}_{2H}^{t,N} = \tilde{p}_{1H}^t$$

and $\tilde{p}_{2H}^{t,R} = \max\{v_i, c\}$ if he learns his consumers' valuations, and $\tilde{p}_{2H}^{t,R} = \max\{\frac{(3-2\lambda)(v_L+1)+2c}{5-2\lambda}, \tilde{s}\}$ if he learns only their purchasing history. Relative to uniform pricing, BBP decreases the type H seller's profits if $\lambda < \frac{1}{2}$ and increases his profits if $\lambda > \frac{1}{2}$.

Note that the equilibrium price charged to first-time purchasers, $\tilde{p}_{1H}^t = \tilde{p}_{2H}^{t,N}$, and the price charged to repeat purchasers, $\tilde{p}_{2H}^{t,R}$, decrease as λ increases, but the type H seller's profit $\tilde{\Pi}_H^t$ in (39) increases with λ. When the seller's information becomes more accurate, consumers anticipate a larger probability of being charged a personalized price and of receiving zero surplus at $t = 2$, and they thus have a stronger incentive to delay their initial consumption. This strategic response from consumers pushes the first-period price down further, leading to a larger first-period profit loss for the type H seller as λ increases. However, as λ increases, the seller also extracts more surplus from repeat purchasers, and this benefit increases more quickly than his first-period profit decreases. As a result, relative to uniform pricing, the type H seller is better off with the option of behavior-based pricing when $\lambda > \frac{1}{2}$ and worse off when $\lambda < \frac{1}{2}$.

Quality Uncertainty Since the seller cannot commit to future prices, only the first-period price signals quality, as in the baseline model. Under uniform pricing, the unique separating equilibrium that survives the intuitive criterion remains the same as that characterized in Proposition 1. Under behavior-based pricing, following a similar logic to the baseline model, in a separating equilibrium that survives the intuitive criterion, $p_{1H}^t \in \arg\max_{p_{1H}} \Pi_H^t(p_{1H}, 1)$ where p_{1H} and $p_{1L}^t = v_L$ satisfy the two types' incentive compatibility constraints. To prevent the type L seller from mimicking, p_{1H} needs to satisfy

$$\Pi_L^t(p_{1H}, 1) = (v_L + 1 - 2p_{1H} + c)p_{1H} + v_L \leq \Pi_L^t(v_L, 0) = 2v_L, \tag{22}$$

which is the same as (11). Thus $p_{1H} \geq \bar{p}$ needs to hold in equilibrium. Recall that $\tilde{p}_{1H}^t$ is the equilibrium price that uniquely maximizes the type H seller's profits under public information (see Lemma 4). It follows that $p_{1H}^t = \max\{\tilde{p}_{1H}^t, \bar{p}\}$ holds in equilibrium.

Next, consider type H's IC constraint. The type H seller has two options of deviation: $p^d > v_L$ and $p^d = v_L$. If $p^d > v_L$, $\mu(p^d) = 0$ and no consumers make a purchase at $t = 1$. The type H seller's profit would be $\Pi_H^t(p^d > v_L, 0) = \tilde{\pi}$. If $p^d = v_L$, $\mu(p^d) = 0$, and all consumers purchase the product in the first period. Then, in the second period, with probability λ the seller charges a personalized price v_i to consumers with $v_i \geq c$, and with probability $1 - \lambda$ he posts the uniform price $\tilde{s}$ to all consumers. The seller's profit in this case is

$$\Pi_H^t(v_L, 0) = v_L - c + \lambda \int_c^{v_L+1} (v - c)dv + (1 - \lambda)\tilde{\pi} = v_L - c + (1 + \lambda)\tilde{\pi}. \tag{23}$$

Thus, the type H seller's IC constraint can be written as

$$\Pi_H^t(p_{1H}, 1) \geq \max\{\Pi_H^t(p^d > v_L, 0), \Pi_H^t(v_L, 0)\} = \max\{\tilde{\pi}, v_L - c + (1 + \lambda)\tilde{\pi}\}, \tag{24}$$

where $\Pi_H^t(p_{1H}, 1)$ is given in (21). Note that if $\lambda > \frac{c - v_L}{\tilde{\pi}}$, constraint (24) becomes $\Pi_H^t(p_{1H}, 1) \geq v_L - c + (1 + \lambda)\tilde{\pi}$, which differs from (12) in the baseline model and becomes crucial for the existence of a separating equilibrium.

If λ is sufficiently large, $\Pi_H^t(\bar{p}, 1) < v_L - c + (1 + \lambda)\tilde{\pi}$ holds, and the type H seller has an incentive to pool with the type L seller at price v_L, destroying the separating equilibrium with $p_{1H} = \bar{p}$. When pooling with type L at price v_L, the type H seller suffers a loss in the first period by selling below his marginal cost. However, in doing so he gains the benefit of learning all his consumers' valuations and extracting the entire consumer surplus in the second period. By contrast, by posting $\bar{p}$ to signal product quality, the type H seller makes a positive profit in the first period, but extracts less consumer surplus in the second period. Thus, when choosing the first-period price, the seller faces a tradeoff between making a positive first-period profit through price signaling and forsaking signaling to acquire better information about consumers' valuations. In the next proposition we characterize the equilibrium under quality uncertainty when the seller has the option of behavior-based pricing.

Proposition 5. *Suppose there is asymmetric information about product quality and the seller can use behavior-based pricing.*

(i) If $\tilde{p}_{1H}^t \geq \bar{p}$, the public information outcome stated in Lemma 4 is supported as the unique separating equilibrium that survives the intuitive criterion.

(ii) There exists a threshold value $\bar{\lambda}$ such that if $\tilde{p}_{1H}^t < \bar{p}$ and $\lambda \in [0, \bar{\lambda}]$, a unique separating equilibrium exists that survives the intuitive criterion, where $p_L^t = \{v_L, v_L\}$, $p_{1H}^t = p_{2H}^{t,N} = \bar{p}$, and $p_{2H}^{t,R} = \max\{v_i, c\}$ with probability λ, and $p_{2H}^{t,R} = 2\bar{p} - c$ with probability $1 - \lambda$.

(iii) If $\tilde{p}_{1H}^t < \bar{p}$ and $\lambda \in (\bar{\lambda}, 1]$, no separating equilibrium exists that survives the intuitive criterion. However, there exists a pooling equilibrium surviving the intuitive criterion, in which the two types post the same price $p_{1q}^t = v_L$ in the first period. At $t = 2$, the type L seller sets $p_{2L}^t = v_L$; with probability λ the type H seller charges $p_{2H}^{t,R} = \max\{v_i, c\}$ and with complementary probability he charges $p_{2H}^{t,R} = \tilde{s}$.

When $\tilde{p}_{1H}^t < \bar{p}$, the type H seller needs to distort the first-period price above the equilibrium price under public information to establish a signal of high quality. The extent of price distortion $\bar{p} - \tilde{p}_{1H}^t$ measures the signaling cost. Lemma 4 implies that behavior-based pricing is less profitable relative to uniform pricing when $\lambda < \frac{1}{2}$ under public information. When quality uncertainty is present, behavior-based pricing lowers the signaling cost relative to uniform pricing in a separating equilibrium. Similar to the results in Corollary 1, for given small v_L, there exist thresholds c_1 and c_2 such that behavior-based pricing increases the type H seller's profit relative to uniform pricing if $c \leq c_1$, and lowers the type H seller's profit relative to uniform pricing if $c \geq c_2$.[22]

What happens if λ gets larger? While $\bar{p}$ is invariant with the accuracy parameter λ, $\tilde{p}_{1H}^t$ decreases with λ. Thus, as λ gets larger, the signaling cost increases, affecting type H's first-period profit adversely. Meanwhile, the type H seller has more accurate information about his consumers' valuations and receives a higher second-period profit from practicing personalized prices. The increased signaling cost has a second-order effect, but the increased capacity for surplus extraction has a first-order effect on type H's equilibrium profit. When λ is sufficiently large, the latter effect dominates the former despite the monopolist's profit loss to sell the product

[22] For the numerical example in Section 3.3 with $v_L = 0.1$, let $\lambda = 0.2$. Behavior-based pricing brings the type H seller higher profit relative to uniform pricing if $c < 0.753$.

below margin cost in the first period, and the equilibrium shifts from the separating to the pooling one. Overall, the type H seller's equilibrium profit from BBP increases with λ.

Figure 2 below illustrates parts (ii) and (iii) of Proposition 5. In the left panel, $v_L = 0.1$ and $c = 0.2$, $\tilde{p}_{1H}^t < \bar{p}$ and $\bar{\lambda} = 1$. The separating equilibrium exists for all $\lambda \in [0, 1]$. The type H seller's profit is given by $\Pi_H^t(\bar{p}, 1)$ in the unique separating equilibrium. In the right panel, $v_L = 0.1$ and $c = 0.12$, $\tilde{p}_{1H}^t < \bar{p}$ holds, and $\bar{\lambda} = 0.740$. When $\lambda \leq 0.740$, the prevailing equilibrium is the separating one in which the type H seller's profit is given by $\Pi_H^t(\bar{p}, 1)$. When $\lambda > 0.740$, the prevailing equilibrium is the pooling one in which the type H seller's profit is equal to $\Pi_H^t(v_L, 0)$.[23]

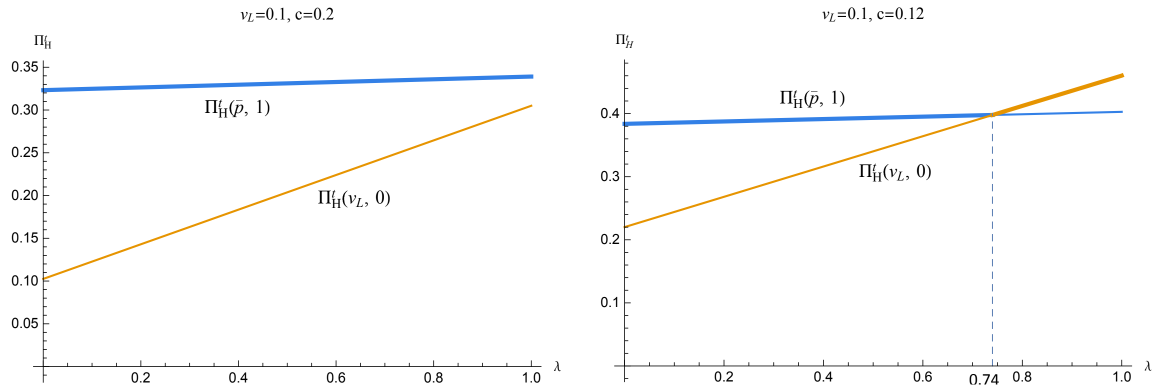

Figure 2: Separating versus pooling equilibrium. Type H seller's equilibrium profit is illustrated by the thick curve in each panel.

6 Heterogeneous Valuations for the Type L Product

So far we have assumed that consumers have identical valuation v_L for a type L product and thus the type L seller charges the same price v_L under different pricing schemes in equilibrium. We now relax this restriction and explore the effect of heterogenous valuations for a type L product.[24] For this purpose, we assume that a consumer's valuation for a type L product is given by θv_i with $\theta \in (0, 0.5)$,[25] where v_i follows uniform distribution on $[v_L, v_L + 1]$ as in Section 2. The specifications of the model are otherwise the same as those in Section 2. In particular, a consumer's valuation for the type H product is still given by v_i. Thus at $t = 1$, when a consumer privately learns her v_i, she learns her valuations for both types of products.

Heterogeneous valuations for the type L product lead to the consequence that the type L seller also employs conditional prices in equilibrium when BBP is permitted. We will demonstrate that

[23] When the two types pool at price v_L, the induced equilibrium-path belief is $\mu(v_L) = \rho$. All the consumers purchase the product at $t = 1$. The type H seller's profit is $\Pi_H^t(v_L, \rho) = \Pi_H^t(v_L, 0)$.

[24] We thank an anonymous referee for suggesting this extension.

[25] The existence of a separating equilibrium requires consumers' valuations for the type L product to be sufficiently lower than those for the type H product. See also the discussion in Footnote 26.

the option of BBP can still reduce signaling cost of the type H product, akin to the insights obtained in the baseline model.

To focus on the interesting case that the type H seller's main concern is to prevent mimicking from the type L seller, we replace assumption (1) with the following condition in this section[26]

$$v_L < 1, \quad \max\{v_L, \frac{\theta+1}{3}(v_L+1)\} < c < v_L + 1. \tag{25}$$

First consider the case of public information when BBP is not permitted. The type H seller's equilibrium choice remains the same as that in Remark 1. Given a uniform price s, a consumer buys a type L product if and only if $\theta v_i \leq s$ which is equivalent to $v_i \leq \frac{s}{\theta}$. Thus, the type L seller chooses s_{tL}, $t = \{1, 2\}$, to maximize his static monopoly profit, $(v_L + 1 - \frac{s_{tL}}{\theta})s_{tL}$, leading to the equilibrium outcome summarised in the next Remark.[27]

Remark 5. *Suppose the product quality is publicly observed and the seller is restricted to uniform pricing. The type L seller's optimal price is* $\tilde{s}_L^h = \{\frac{\theta(v_L+1)}{2}, \frac{\theta(v_L+1)}{2}\}$, *which brings total profits* $\tilde{\Pi}_L^{h,u} = \frac{\theta(v_L+1)^2}{2} \equiv 2\tilde{\pi}_L$. *The type H seller's optimal price is* $\tilde{s}_H^h = \{\frac{v_L+1+c}{2}, \frac{v_L+1+c}{2}\} = \{\tilde{s}, \tilde{s}\}$, *which brings total profits* $\tilde{\Pi}_H^{h,u} = \frac{(v_L+1-c)^2}{2} = 2\tilde{\pi}$.

In the case of asymmetric information under uniform pricing, since product quality becomes public information in the second period, the two types receive their static monopoly profits, $\tilde{\pi}_L$ and $\tilde{\pi}$, at $t = 2$. In a separating equilibrium, given the first-period prices, s_{1H} and s_{1L}, and consumers' belief $\mu(s_{1H}) = 1$ and $\mu(s_1) = 0$ for $s_1 \neq s_{1H}$, the two types' IC constraints are respectively

$$\Pi_L^h(s_{1H}, 1) = (v_L + 1 - s_{1H})s_{1H} + \tilde{\pi}_L \leq \Pi_L^h(s_{1L}, 0) = \tilde{\Pi}_L^{h,u}; \tag{26}$$

$$\Pi_H^h(s_{1H}, 1) = (v_L + 1 - s_{1H})(s_{1H} - c) + \tilde{\pi} \geq \max_{s_1 \neq s_{1H}} \Pi_H^h(s_1, 0) = \tilde{\pi}. \tag{27}$$

Constraint (27) holds because the highest deviation profit for the type H seller is given by $\max_{s_1 \neq s_{1H}} \Pi_H^h(s_1, 0) = \tilde{\pi}$. To see this, note that $\theta \in (0, 0.5)$ and assumption (25) imply $c > \theta(v_L + 1)$. Then a deviation with $s_1 < \theta(v_L + 1)$ results in a negative first-period profit for the type H seller, and a deviation with $s_1 \geq \theta(v_L + 1)$ leads to zero demand and zero profit in the first period.

Since (27) always holds for $s_{1H} \geq c$, s_{1H} is supported in a separating equilibrium if and only

[26] As in the baseline model, $v_L < 1$ and $c \in (v_L, v_L + 1)$ ensure that the static monopoly prices for both types are interior. Moreover, $c > \frac{\theta+1}{3}(v_L + 1)$, together with $\theta < 0.5$, ensures that the type H seller has no incentive to mimic the type L seller under uniform pricing and under BBP. These restrictions on the parameters simplify the exposition without losing the main insight.

[27] We use superscript h to denote the profit functions and equilibrium outcomes in this section.

if (26) holds, which is equivalent to $s_{1H} \geq \bar{s}^h \equiv \frac{1+\sqrt{1-\theta}}{2}(v_L+1)$ or $s_{1H} \leq \underline{s}^h \equiv \frac{1-\sqrt{1-\theta}}{2}(v_L+1)$. Following similar arguments as in the proof of Proposition 1, we can show that the type H seller's optimal first-period price is $s_{1H}^{h,*} = \max\{\tilde{s}, \bar{s}^h\}$, where $\tilde{s} = \frac{v_L+1+c}{2}$ as defined in (2).

Proposition 6. *Under uniform pricing, there is a unique separating equilibrium that survives the intuitive criterion. (i) If $c \geq \sqrt{1-\theta}(v_L+1)$, the equilibrium outcome under public information stated in Remark 5 is supported. (ii) If $c < \sqrt{1-\theta}(v_L+1)$, the type L seller chooses $s_{1L}^{h,*} = s_{2L}^{h,*} = \frac{\theta(v_L+1)}{2}$ and the type H seller chooses $s_{1H}^{h,*} = \bar{s}^h$ and $s_{2H}^{h,*} = \tilde{s}$. The two types' expected profits are respectively $\Pi_L^{h,u} = \frac{\theta(v_L+1)^2}{2}$ and $\Pi_H^{h,u} = \frac{(v_L+1-c)^2}{4} + \frac{\theta(v_L+1)^2}{4} - \frac{1-\sqrt{1-\theta}}{2}(v_L+1)c$.*

6.1 Behavior-Based Pricing

Suppose BBP is permitted and product quality is publicly observed. The type H seller's equilibrium choice is the same as that in Lemma 1. Following similar analysis as in Section 3.2.1, we can show that given p_{1L} and the value of the marginal consumer $\hat{v}_L$, it is optimal for the type L seller to choose $p_{2L}^R = \max\{\frac{\theta(v_L+1)}{2}, \theta\hat{v}_L\}$ and $p_{2L}^N = \frac{\theta\hat{v}_L}{2}$ at $t=2$. Solving the game backward, $\hat{v}_L = \frac{2p_{1L}}{\theta}$ holds.

In the first period, consumers with $v_i \geq \hat{v}_L = \frac{2p_{1L}}{\theta}$ buy the product at price p_{1L}; in the second period, consumers with $v_i \in [\frac{p_{2L}^N}{\theta}, \hat{v}_L) = [\frac{p_{1L}}{\theta}, \hat{v}_L)$ buy the product at price $p_{2L}^N = p_{1L}$ and consumers with $v_i \in [\hat{v}_L, v_L+1]$ buy the product at price $p_{2L}^R = 2p_{1L}$. Thus, the optimal p_{1L} solves

$$\max_{p_{1L}} \Pi_L^h(p_{1L}, 0) = (v_L + 1 - \frac{p_{1L}}{\theta})p_{1L} + \big(v_L + 1 - \frac{2p_{1L}}{\theta}\big)2p_{1L}.$$

We summarise the equilibrium outcome under public information when BBP is permitted in the next Lemma.

Lemma 5. *Suppose product quality is publicly observed and BBP is permitted. The equilibrium prices of the two types are*

$$\tilde{p}_{1L}^h = \tilde{p}_{2L}^{h,N} = \frac{3\theta(v_L+1)}{10}, \quad \tilde{p}_{2L}^{h,R} = \frac{3\theta(v_L+1)}{5};$$
$$\tilde{p}_{1H}^h = \tilde{p}_{2H}^{h,N} = \frac{3v_L+7c+3}{10}, \quad \tilde{p}_{2H}^{h,R} = \frac{3v_L+2c+3}{5}.$$

The two types' expected profits are, respectively, $\tilde{\Pi}_L^{h,b} = \frac{9}{20}\theta(v_L+1)^2$ and $\tilde{\Pi}_H^{h,b} = \frac{9}{20}(v_L+1-c)^2$. Without information asymmetry, both types are worse off with the option of BBP.

When consumers' valuations for the type L product follow a distribution, the optimal prices for the type L seller are also conditional prices when BBP is permitted, differing from the flat prices in Lemma 1 when these valuations are identical. A comparison of Lemma 5 with Remark 5 shows that, under public information, the type L seller is worse off with the option of BBP. The driving force of this outcome is the strategic delay of consumptions by the consumers.

We now consider the case of quality uncertainty under BBP. Given consumers' belief $\mu(p_{1H}) = 1$ and $\mu(p_1) = 0$ for $p_1 \neq p_{1H}$, $p_{1H} \geq c$ and $p_{1L} = \tilde{p}^h_{1L}$ can be supported in a separating equilibrium if and only if

$$\Pi^h_L(p_{1H}, 1) \leq \Pi^h_L(p_{1L}, 0) = \tilde{\Pi}^{h,b}_L; \quad \Pi^h_H(p_{1H}, 1) \geq \max_{p_1 \neq p_{1H}} \Pi^h_H(p_1, 0) \tag{28}$$

with

$$\Pi^h_H(p_{1H}, 1) = (v_L + 1 - p_{1H})(p_{1H} - c) + (v_L + 1 - 2p_{1H} + c)(2p_{1H} - 2c), \tag{29}$$

$$\Pi^h_L(p_{1H}, 1) = \underbrace{(v_L + 1 - 2p_{1H} + c)p_{1H}}_{\text{imitation profit at } t=1} + \underbrace{\frac{\theta(2p_{1H} - c)^2}{4} + (v_L + 1 - 2p_{1H} + c)\theta(2p_{1H} - c)}_{\text{imitation profit at } t=2} \tag{30}$$

where (29) follows directly from (8). In (30), when the type L seller imitates type H by setting p_{1H}, consumers with valuation $v_i \geq 2p_{1H} - c \equiv \hat{v}_H$ purchase at price p_{1H} in the first period, leading to the imitation profit at $t = 1$. Then it is optimal for the type L seller to charge $p^N_{2L} = \frac{\theta(2p_{1H}-c)}{2}$ and $p^R_{2L} = \theta(2p_{1H} - c)$ in the second period. Thus, the type L seller obtains $\frac{\theta(2p_{1H}-c)^2}{4}$ from consumers with $v_i < \hat{v}_H$ and $(v_L + 1 - 2p_{1H} + c)\theta(2p_{1H} - c)$ from consumers with $v_i \geq \hat{v}_H$, leading to the imitation profit at $t = 2$ in (30).

Note that the type L seller's imitation profit (30) differs from its counterpart in (11) in the imitation profit at $t = 2$. This is exactly because the type L seller also uses conditional prices. Imitating the type H seller's first-period price induces a marginal consumer with $v_i = \hat{v}_H$, and this in turn affects the market segmentation and type L's profit at $t = 2$.

Let $\bar{p}^h$ be the larger root of equation $\Pi^h_L(p_{1H}, 1) = \tilde{\Pi}^{h,b}_L$. We arrive at the following equilibrium outcome.

Proposition 7. *Under BBP, there exists a unique separating equilibrium that survives the intuitive criterion. In this equilibrium, the type L seller chooses $\tilde{p}^h_{1L}$ and $(\tilde{p}^{h,N}_{2L}, \tilde{p}^{h,R}_{2L})$ as specified in Lemma 5 under public information.*

(i) If $c \geq \hat{c} \equiv \frac{4+\theta+\sqrt{5}\sqrt{20-8\theta-7\theta^2}}{14+6\theta}(v_L + 1)$, the equilibrium outcome under public information is supported, and the type H seller chooses $\tilde{p}^h_{1H}$ and $(\tilde{p}^{h,N}_{2H}, \tilde{p}^{h,R}_{2H})$ as specified in Lemma 5.

(ii) If $c < \hat{c}$, the type H seller chooses $p^{h,*}_{1H} = p^{h,N}_{2H} = \bar{p}^h$, and $p^{h,R}_{2H} = 2\bar{p}^h - c$.

Since the type L seller's equilibrium prices are the same as those under public information, it follows immediately from Lemma 5 and Remark 5 that under quality uncertainty, the type L seller is worse off with the option of BBP relative to uniform pricing.

Similar to Proposition 2 in the baseline model, the type H seller may need to raise the first-period price above the equilibrium price under public information to signal product quality. However, the welfare implication of BBP for the type H seller is more subtle when the type L seller also uses conditional prices. A comparison of the imitation profits of the type L seller under uniform pricing and BBP, (26) and (30), leads to the observation that the option of BBP affects the type L seller's imitation profits, thus type H's signaling cost, in two opposite directions. First, the price-elasticity of the imitation demand at $t = 1$ is higher under BBP. Therefore, similar to the baseline model, a marginal increase in the first-period price leads to a larger reduction in the imitation demand, and thus a lower first-period imitation profit for the type L seller. This reduces the type H seller's signaling cost under BBP relative to uniform pricing. Second, because the type L seller can also condition prices on consumers' purchasing history, mimicking the type H seller induces a higher valuation of marginal consumer ($\hat{v}_H = 2p_{1H} - c$ instead of $\hat{v}_L = \frac{2p_{1L}}{\theta}$) which implies a more profitable second-period market segmentation. This increases type L's second-period imitation profit and adversely affects the type H seller's signaling cost. When production cost c is sufficiently low, the first effect dominates, and BBP reduces the signaling cost and increases the type H seller's profit relative to uniform pricing, as in the baseline model.

In the next corollary, we state a sufficient condition under which the type H seller is better off with BBP for small c due to reduced signaling cost of product quality.

Corollary 5. *When $\theta < 0.165$, there exist $\frac{\theta+1}{3}(v_L + 1) < c_1^h \leq c_2^h < \sqrt{1-\theta}(v_L + 1)$ such that the type H seller is better off for $c < c_1^h$ and worse off for $c > c_2^h$ with the option of BBP by comparison with uniform pricing.*

Numerical Example. We close this section with a numerical example. Suppose $v_L = 0.1$, $\theta = 0.1$ and $c \in (0.404, 1.1)$. Under uniform pricing, by Proposition 6 we have $s_{1L}^{h,*} = s_{2L}^{h,*} = 0.055$, $s_{1H}^{h,*} = s_{2H}^{h,*} = \frac{1.1+c}{2}$ when $c \geq 1.043$, and $s_{1H}^{h,*} = 1.071$, $s_{2H}^{h,*} = \frac{1.1+c}{2}$ when $c < 1.043$. Under BBP, by Proposition 7, $p_{1H}^{h,*} = p_{2H}^{h,N} = \frac{3.3+7c}{10}$ and $p_{2H}^{h,R} = \frac{3.3+2c}{5}$ when $c \geq 1.045$; when $c < 1.045$, $p_{1H}^{h,*} = p_{2H}^{h,N} = 0.0972203\sqrt{5.c^2 + 12.1c + 6.2073} + 0.282609c + 0.286957$, and $p_{2H}^{h,R} = 0.194441\sqrt{5.c^2 + 12.1c + 6.2073} - 0.434782c + 0.573914$.

As a result, $\Pi_H^{h,b} > \Pi_H^{h,u}$ if and only if $c < 0.514$ while $\Pi_L^{h,b} < \Pi_L^{h,u}$ for all c. The consumer surplus and the social welfare are both higher under BBP. Figure 3 illustrates the welfare change when the pricing scheme moves from uniform pricing to BBP.

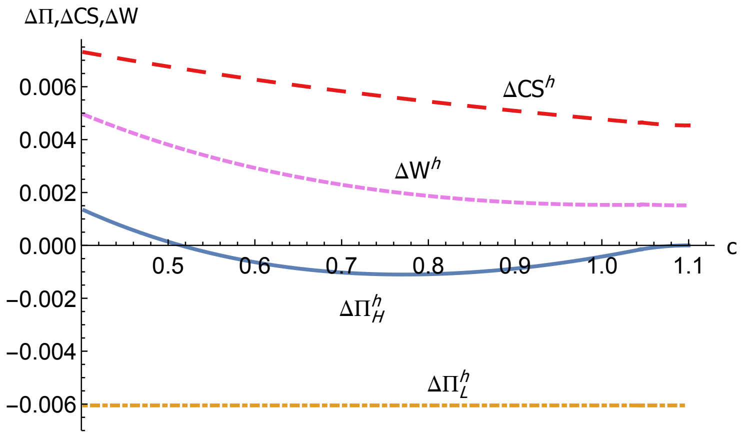

Figure 3: The two types' incremental profits, $\Delta\Pi_H^h$ and $\Delta\Pi_L^h$, incremental consumer surplus (ΔCS^h) and incremental total welfare (ΔW^h) when the pricing scheme moves from uniform pricing to BBP for $v_L = 0.1$, $\theta = 0.1$, and $\rho = 0.5$.

7 Conclusions

We analyzed a two-period model with the key features that price signals product quality in the first period and the seller charges discriminatory prices to consumers in the second period conditional on their purchasing history. Contrary to the classic result that BBP reduces monopoly profit (and thus will not be chosen in equilibrium), we identify a novel channel through which BBP potentially benefits the monopolist: BBP increases the price elasticity of imitation demand and lowers the signaling cost if consumers have imperfect information about product quality, and can be an equilibrium choice for a monopolist.

The analysis suggests that when promoting a new product whose quality is unknown to consumers, behavior-based pricing can be a profitable strategy for managerial practice. Moreover, the profitability of BBP does not come at the expense of consumer surplus because it reduces the average price and increases total demand. Therefore, a policy restricting consumer tracking or prohibiting discriminatory prices conditional on consumers' prior purchases may actually backfire when there is asymmetric information about product quality.

We examined a simple information structure in which all consumers hold the same prior belief, and information about product quality is publicly revealed in the second period. An obvious direction is to consider an alternative setting in which informed consumers with perfect knowledge about product quality coexist with uninformed consumers who know only the prior distribution, with the fraction of informed consumers increasing as time elapses. In this setting, the type H seller uses price to signal product quality to uninformed consumers, and the purchasing decisions of informed consumers also affect the type L seller's imitation incentive. Since the channel through which the option of BBP interacts with the signaling role of prices remains unchanged, our main insights on the profitability of BBP and the equilibrium price patterns that signal product quality extend to this more general information structure. One issue that arises with consumer tracking is that consumers with privacy concerns may take costly measures to maintain anonymity in their interactions with sellers. The incentive for consumers to remain anonymous may reduce the significance of the effect of BBP on price signaling; however, the main insights still hold if the cost of remaining anonymous is sufficiently large or if the portion of consumers remaining anonymous is not too large.

In an oligopolistic market with multiple firms, sellers may engage in competitive price signaling in the first period and in consumer poaching in the second period. In this case, BBP can be beneficial for sellers due to lower signaling costs, but at the same time can be harmful to sellers due to endogenous market segmentation and intensified competition in the second period. In such an environment, BBP's overall impact on sellers' profits and consumer surplus becomes more subtle and merits further research in the future.

Appendix

This appendix contains the proofs of Lemmas 2–4, Propositions 1–5 and 7, Remark 3 and Remark 6 which is a preparation for the proof of Proposition 4, and Corollaries 1, 2, 4 and 5.

Proof of Proposition 1. (i) When $c \geq 1 - v_L$, $\tilde{s} = \frac{v_L+1+c}{2} \geq \bar{s} = 1$ which means type L's incentive compatibility constraint (4) is satisfied when type H chooses the equilibrium price under public information, $\tilde{s}_{1H} = \tilde{s}$. So $s_{1H} = \tilde{s}_{1H}$ is supported in a separating equilibrium. Furthermore, any price with $s_{1H} > \tilde{s}$ cannot survive the intuitive criterion because there exists $\epsilon > 0$ such that $s^d = s_{1H} - \epsilon > \tilde{s}$ and

$$\Pi_H(s_{1H}, 1) = (v_L + 1 - s_{1H})(s_{1H} - c) + \tilde{\pi} < \Pi_H(s^d, 1) = (v_L + 1 - s^d)(s^d - c) + \tilde{\pi},$$
$$\Pi_L(s^*_{1L}, 0) = 2v_L > \Pi_L(s^d, 1) = (v_L + 1 - s^d)s^d + v_L.$$

Therefore, intuitive criterion requires $\mu(s^d) = 1$ and given such belief, the type H seller is better off choosing s^d instead of s_{1H}. Thus, $s_{1H} > \tilde{s}$ cannot survive the intuitive criterion. Using similar arguments, we can also rule out $s_{1H} < \tilde{s}$. We conclude that $s^*_{1H} = \tilde{s}_{1H}$ constitutes the unique separating equilibrium that survives the intuitive criterion.

(ii) When $c < 1 - v_L$, $s_{1H} = \bar{s}$ is the lowest price that satisfies type L's IC constraint (4). Thus, $s_{1H} = 1$ is supported in a separating equilibrium. Using similar arguments as in (i), we can show that no $s_{1H} > \bar{s}$ can survive the intuitive criterion. Thus $s^*_{1H} = \bar{s}$ constitutes the unique separating equilibrium that survives the intuitive criterion. The equilibrium profits are obtained by plugging the equilibrium price $s^*_{1H} = \bar{s} = 1$ into $\Pi_H(s_{1H}, 1)$ in (5). □

Proof of Remark 3. First, price $p_{1H} \in (\tilde{s}, v_L + 1]$ is strictly dominated for the type H seller. Suppose $p_{1H} \in (\tilde{s}, v_L + 1]$ holds in equilibrium. Then no consumer makes a purchase at $t = 1$, and following such price it is optimal for the seller to choose $p^R_{2H} = p^N_{2H} = \tilde{s}$ at $t = 2$, and the seller's total profit is $\Pi_H(p_{1H}, 1) = \tilde{\pi}$. However, by setting some price $p'_{1H} \in [c, \tilde{s}]$, the type H seller earns a strictly positive profit from $t = 1$ and then at least $\tilde{\pi}$ from $t = 2$, and the total expected profit $\Pi_H(p'_{1H}, 1) > \tilde{\pi} = \Pi_H(p_{1H}, 1)$.

Second, $p_{1H} \in [v_L,\ c)$ cannot be optimal for the type H seller either. Suppose $p_{1H} \in [v_L,\ c)$ holds in equilibrium. By Remark 2, $\hat{v} = p_{1H}$ and all consumers with $v_i \geq p_{1H}$ purchase the product at p_{1H} at $t = 1$, and it is optimal for the seller to charge $p^R_{2H} = \tilde{s}$ and $p^N_{2H} = c$ at $t = 2$. With price p_{1H}, the seller suffers a loss in the first period. Then the seller can increase his profit by choosing $p'_{1H} = c - \epsilon$ with small positive ϵ instead because the seller's first-period profit loss is smaller while the second-period profit remains the same. □

Proof of Lemma 2. First, note that $p^*_{1L} = v_L$ follows directly from the equilibrium path belief $\mu(p_{1L}) = 0$. Moreover, given the belief system $\mu(p_1) = 0$ for $p_1 \neq p_{1H}$, constraints (11) and (12) ensure that the two types will not mimic each other's price choice, thus $p^*_{1L} = v_L$ and p_{1H} are indeed supported in a separating equilibrium.

Second, $p_{1H} \in [v_L,\ c)$ cannot be supported in a separating equilibrium. Suppose $p_{1H} \in [v_L,\ c)$ holds in equilibrium, then by Remark 2, $\hat{v} = p_{1H}$ and all consumers with $v_i \geq p_{1H}$ purchase the product at p_{1H}

at $t = 1$, and it is optimal for the seller to charge $p_{2H}^R = \tilde{s}$ and $p_{2H}^N = c$ at $t = 2$. Type H is willing to post $p_{1H} \in [v_L,\ c)$ if and only if

$$\Pi_H(p_{1H}, 1) = (v_L + 1 - p_{1H})(p_{1H} - c) + \tilde{\pi} \geq \max_{p_1 \neq p_{1H}} \Pi_H(p_1, 0) = \tilde{\pi}$$

which cannot be satisfied given $p_{1H} < c$.

Third, a separating equilibrium with $p_{1H} > \tilde{s}$ and $p_{1L}^* = v_L$ cannot survive the intuitive criterion. Suppose $p_{1H} > \tilde{s}$ holds in equilibrium. No consumers make a purchase at $t = 1$ given their belief $\mu(p_{1H}) = 1$, and the optimal second-period price for the type H seller is $p_{2H}^R = p_{2H}^N = \tilde{s}$. Then $\Pi_H(p_{1H}, 1) = \tilde{\pi}$ and $\Pi_L(p_{1L}^*, 0) = 2v_L$. Consider deviation $p^d = \tilde{s} - \epsilon$ where ϵ is an infinitely small positive number. With first-period price p^d, $\hat{v} = 2p^d - c > \tilde{s}$. Then we have

$$\begin{aligned}
\Pi_H(p^d, 1) &= (v_L + 1 - p^d)(p^d - c) + (v_L + 1 - \max\{\tilde{s}, 2p^d - c\})(\max\{\tilde{s}, 2p^d - c\} - c) \\
&= (p^d - c)(3v_L - 5p^d + 2c + 3) = 4\epsilon(\tilde{s} - c) - 5\epsilon^2 + \tilde{\pi} > \Pi_H(p_{1H}, 1) = \tilde{\pi}; \\
\Pi_L(p^d, 1) &= (v_L + 1 - 2p^d + c)p^d + v_L = 2\epsilon(\tilde{s} - \epsilon) + v_L < \Pi_L(p_{1L}^*, 0) = 2v_L.
\end{aligned}$$

Thus, intuitive criterion requires $\mu(p^d) = 1$. Given such belief, it is better for the type H seller to choose p^d instead of p_{1H}, contradicting the assumption that $p_{1H} > \tilde{s}$ forms a separating equilibrium.

Therefore, we conclude that $p_{1H} \in [c,\ \tilde{s}]$ must hold in a separating equilibrium that survives the intuitive criterion. □

Proof of Proposition 2. Recall that $\tilde{p}_{1H} = \frac{3v_L + 7c + 3}{10} \in [c, \tilde{s}]$ from Lemma 1 and $\bar{p} = \frac{v_L + 1 + c + \sqrt{\Delta}}{4}$ from (13).

(i) When $\tilde{p}_{1H} \geq \bar{p}$, constraint (11) is satisfied if $p_{1H} = \tilde{p}_{1H}$. Thus, $p_{1H} = \tilde{p}_{1H}$ and p_{1L}^* are indeed supported in a separating equilibrium. Furthermore, since $\tilde{p}_{1H}$ is the unique global maximizer of $\Pi_H(p_{1H}, 1)$, any $p_{1H} = p^d \neq \tilde{p}_{1H}$ cannot survive the intuitive criterion. To see this, suppose $p_{1H} = p^d$ and $p_{1L} = v_L$ indeed form a separating equilibrium, since $\Pi_H(p^d, 1) < \Pi_H(\tilde{p}_{1H}, 1)$, there exists small positive ϵ such that $\Pi_H(p^d, 1) < \Pi_H(\tilde{p}_{1H} + \epsilon, 1)$ and

$$\Pi_L(\tilde{p}_{1H} + \epsilon, 1) = [v_L + 1 - 2(\tilde{p}_{1H} + \epsilon) + c]\,(\tilde{p}_{1H} + \epsilon) + v_L < \Pi_L(v_L, 0) = 2v_L,$$

where the inequality holds because $\tilde{p}_{1H} \geq \bar{p}$ implies $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 \geq 0$. Thus, intuitive criterion requires $\mu(\tilde{p}_{1H} + \epsilon) = 1$, and the type H seller prefers $\tilde{p}_{1H} + \epsilon$ over p^d which is a contradiction. Thus $p_{1H}^* = \tilde{p}_{1H}$ and $p_{1L}^* = v_L$ form the unique separating equilibrium that survives the intuitive criterion.

(ii) If $\tilde{p}_{1H} < \bar{p}$, we show that $p_{1H}^* = \bar{p}$ is the unique equilibrium that survives the intuitive criterion. Note that $\bar{p}$ is the local maximizer of $\Pi_H(p_{1H}, 1)$ for $p_{1H} \in [\bar{p}, \tilde{s}]$, any $p_{1H} \neq \bar{p}$ that satisfies (11) cannot survive the intuitive criterion. To see this, suppose $p_{1H} = p^d > \bar{p}$ and $p_{1L}^* = v_L$ indeed form a separating equilibrium, then there exists a small positive ϵ such that $p^d - \epsilon > \bar{p}$ and

$$\Pi_H(p^d, 1) < \Pi_H(p^d - \epsilon, 1), \quad \text{and} \quad \Pi_L(p^d - \epsilon, 1) < \Pi_L(p_{1L}^*, 0) = 2v_L.$$

Intuitive criterion requires $\mu(p^d - \epsilon) = 1$, and the type H seller prefers $p^d - \epsilon$ to p^d, which leads to a contradiction. Using similar logic we can also rule out $p_{1H} \leq \underline{p}$ to be a separating equilibrium. Thus $p^*_{1H} = \bar{p}$ and $p^*_{1L} = v_L$ form the unique separating equilibrium that survives the intuitive criterion. Plugging p^*_{1H} into $\Pi_H(p_{1H}, 1)$ in (12) gives us the claimed equilibrium profits of the type H seller. $\square$

Proof of Corollary 1. First consider $c \geq 1 - v_L$ which implies $\tilde{p}_{1H} \geq \bar{p}$. Part (i)s of Propositions 1 and 2 apply and the respective public information outcome forms the unique separating equilibrium under uniform pricing and behavior-based pricing. It follows that

$$\Pi^u_H = \tilde{\Pi}^u_H = 2\tilde{\pi} > \Pi^b_H = \tilde{\Pi}^b_H = \frac{9(v_L + 1 - c)^2}{20}.$$

Let $c_2 \equiv 1 - v_L$. Thus when $c \geq c_2$, BBP lowers type H seller's profit relative to uniform pricing.

Next consider $\tilde{p}_{1H} < \bar{p}$ which is equivalent to $7c^2 - 4(v_L + 1)c + 25v_L - 3(v_L + 1)^2 < 0$. This implies $c < \frac{1}{7}\left(2 + 2v_L + 5\sqrt{1 - 5v_L + v_L^2}\right)$ with $v_L < \frac{1}{5}$ and $c < 1 - v_L$. From Proposition 1 and equation (8), type H seller's equilibrium profits under uniform pricing and BBP are respectively

$$\Pi^u_H = v_L(1 - c) + \frac{(v_L + 1 - c)^2}{4}, \quad \Pi^b_H = (\bar{p} - c)(3v_L - 5\bar{p} + 2c + 3).$$

Note that at $c \to v_L$, $\bar{p} = \frac{1}{2}$ and

$$\Pi^b_H(c \to v_L) = (\frac{1}{2} - v_L)(3v_L - \frac{5}{2} + 2v_L + 3) = \frac{1}{4} + 2v_L - 5v_L^2 > \Pi^u_H(c \to v_L).$$

Moreover note that $\Pi^b_H < \frac{9(v_L+1-c)^2}{20}$. It follows that

$$\Pi^b_H(c = 1 - \frac{3 + \sqrt{5}}{2}v_L) \leq \Pi^u_H(c = 1 - \frac{3 + \sqrt{5}}{2}v_L).$$

By continuity there must exist a threshold value $\hat{c} \in (v_L, 1 - \frac{3+\sqrt{5}}{2}v_L]$ such that $\Pi^b_H \geq \Pi^u_H$ if $c \leq \hat{c}$. Let $c_1 \equiv \min\{\hat{c}, \frac{1}{7}\left(2 + 2v_L + 5\sqrt{1 - 5v_L + v_L^2}\right)\}$. BBP increases type H seller's profits in comparison to uniform pricing when $c \leq c_1$.

$\square$

Proof of Corollary 2. First consider $c \geq 1 - v_L$. Part (i) of Proposition 1 applies under uniform pricing. The consumer surplus and total welfare when the product is type H are given by

$$CS^u_H = 2\int_{\tilde{s}}^{v_L+1} (x - \tilde{s})dx = \frac{(v_L + 1 - c)^2}{4}, \quad TS^u_H = \tilde{\Pi}^u_H + CS^u_H = \frac{3(v_L + 1 - c)^2}{4}.$$

Under BBP, part (i) of Proposition 2 applies. Consumers with $v_i \geq \hat{v} = 2\tilde{p}_{1H} - c = \frac{3v_L+2c+3}{5}$ purchase at $t = 1$; consumers with $v_i \geq \tilde{p}^R_{2H} = \frac{3v_L+2c+3}{5}$ purchase a second unit at $t = 2$, and consumers with $v_i \in [\tilde{p}^N_{2H}, \hat{v}]$, where $\tilde{p}^N_{2H} = \frac{3v_L+7c+3}{10}$, purchase their first unit at $t = 2$. Thus, the consumer surplus and total welfare when the product is type H are

$$CS_H^b = \int_{\hat{v}}^{v_L+1}(x-\tilde{p}_{1H})dx + \int_{\tilde{p}_{2H}^N}^{\hat{v}}(x-\tilde{p}_{2H}^N)dx + \int_{\tilde{p}_{2H}^R}^{v_L+1}(x-\tilde{p}_{2H}^R)dx$$
$$= \frac{(v_L+1-\tilde{p}_{1H})^2}{2} + \frac{(v_L+1-\tilde{p}_2^R)^2}{2} = \frac{13(v_L+1-c)^2}{40} > CS_H^u, \tag{31}$$
$$TS_H^b = \tilde{\Pi}_H^b + CS_H^b = \frac{31(v_L+1-c)^2}{40} > TS_H^u. \tag{32}$$

Next consider $c < 1-v_L$. Part (ii) of Proposition 1 applies under uniform pricing and the equilibrium prices are given by $s_{1L}^* = s_{2L}^* = v_L$ and $s_{1H}^* = 1$ and $s_{2H}^* = \tilde{s}$. Following this, the consumer surplus and total welfare when the product is type H are respectively:

$$CS_H^u = \int_1^{v_L+1}(x-1)dx + \int_{\tilde{s}}^{v_L+1}(x-\tilde{s})dx = \frac{v_L^2}{2} + \frac{(v_L+1-c)^2}{8},$$
$$TS_H^u = \Pi_H^u + CS^u = v_L(1-c) + \frac{v_L^2}{2} + \frac{3(v_L+1-c)^2}{8}.$$

Under BBP, we differentiate two cases following Proposition 2:

(i) If $\tilde{p}_{1H} \geq \bar{p}$, part (i) of Proposition 2 applies and the consumer surplus and total welfare related to type H products are given by (31) and (32). It follows that $CS_H^b > CS_H^u$ and $TS_H^b > TS_H^u$.

(ii) If $\tilde{p}_{1H} < \bar{p}$ which is equivalent to $7c^2 - 4(v_L+1)c + 25v_L - 3(v_L+1)^2 < 0$, part (ii) of Proposition 2 applies. The consumer surplus and total welfare related to type H products are:

$$CS_H^b = \int_{\bar{p}}^{v_L+1}(x-\bar{p})dx + \int_{2\bar{p}-c}^{v_L+1}(x-2\bar{p}+c)dx = \frac{(v_L+1-\bar{p})^2}{2} + \frac{(v_L+1-2\bar{p}+c)^2}{2},$$
$$TS_H^b = (\bar{p}-c)(3v_L - 5\bar{p} + 2c + 3) + \frac{(v_L+1-\bar{p})^2}{2} + \frac{(v_L+1-2\bar{p}+c)^2}{2}.$$

To show $CS_H^b > CS_H^u$, it suffices to show the equilibrium prices are lower in both periods under BBP than those under uniform pricing, that is, $\bar{p} \leq \tilde{s}$ and $2\bar{p} - c \leq 1$. While $\bar{p} \leq \tilde{s}$ is obvious, $2\bar{p} - c \leq 1$ holds because

$$\bar{p}(v_L \to 0) = \frac{1+c+\sqrt{(1+c)^2}}{4} = \frac{1+c}{2}, \quad \frac{\partial \bar{p}}{\partial v_L} = \frac{1}{4}[1 + \frac{(v_L+1+c)-4}{\sqrt{(v_L+1+c)^2 - 8v_L}}] < 0$$

where the inequality holds because $c < 1-v_L$. Since the equilibrium prices under BBP and uniform pricing exceed the production cost c, lower prices for all purchasers lead to larger social welfare.

□

Proof of Proposition 3. In a separating equilibrium, for s_H^c to satisfy (16), $s_{2H}^c = \tilde{s}$. (i) When $(v_L+1)^2 - 8v_L \leq c^2$ holds, $s_{1H} = \tilde{s}$, together with $s_L^c = \{v_L, v_L\}$, satisfies type L's incentive compatibility constraint (14), and constitutes a separating equilibrium. Since $s_H^c = \{s_{1H}^c, s_{2H}^c\} = \{\tilde{s}, \tilde{s}\}$ is the unique global maximizer to $\Pi_H^c(s_H, 1)$ in (15), it constitutes the unique separating equilibrium that survives the intuitive criteria.

(ii) When $(v_L+1)^2-8v_L > c^2$, $s_{1H}^c = \frac{v_L+1+\sqrt{(v_L+1)^2-8v_L}}{2}$ is the unique first-period price that maximizes $\Pi_H^c(s_H,1)$ subject to binding constraint (14). Thus s_{1H}^c, together with $s_{2H}^c = \tilde{s}$ and $s_L^c = \{v_L, v_L\}$, constitutes the unique separating equilibrium that survives the intuitive criterion. The equilibrium profit of the type H seller is obtained by plugging the equilibrium prices into (15). □

Proof of Lemma 3. We show that in a separating equilibrium with the option of BBP, for any price scheme $p_H = \{p_{1H}, (p_{2H}^R, p_{2H}^N)\}$, there exists an alternative price scheme $\hat{p}_H = \{\hat{p}_{1H}, (\hat{p}_{2H}^R, \hat{p}_{1H})\}$ that brings the type H seller weakly (sometimes strictly) higher profits than p_H while keeping type L's incentive compatibility constraint satisfied. We will prove this claim for price scheme with $p_{1H} < p_{2H}^N$ first and then for $p_{1H} > p_{2H}^N$.

Given price scheme p_H, a consumer has four options: purchasing in both periods at price p_{1H} and p_{2H}^R, purchasing in the first period only at price p_{1H}, purchasing in the second period only at price p_{2H}^N, and not purchasing in any period. Recall that in a separating equilibrium with $p_H \neq p_L$, equilibrium path belief is given by $\mu(p_H) = 1$ and $\mu(p_L) = 0$, and type L's equilibrium choice is uniquely given by $p_L^c = \{v_L, v_L\}$.

1. Consider a price scheme p_H with $p_{1H} < p_{2H}^N$. No consumers purchase only in the second period. By lowering p_{2H}^N to p_{1H} while keeping p_{2H}^R unchanged, the demands in the first and second period do not change, and thus price schemes p_H and $p'_H = \{p_{1H}, (p_{2H}^R, p_{1H})\}$ bring the type H seller the same expected profit. Moreover, a price scheme $\hat{p}_H = \{p_{1H}, (\hat{p}_{2H}^R, p_{1H})\}$ with optimally chosen $\hat{p}_{2H}^R$ brings the type H seller a (weakly) larger profit than p'_H, thus dominates p_H for the type H seller.

   If $p_{1H} \geq p_{2H}^R$, a seller faces the same first-period demand with price scheme p_H and p'_H. Thus type L seller's imitation profit is the same under price scheme p_H and p'_H. If $p_{1H} < p_{2H}^R$, consumers make a purchase in the first period under p_H if $v_i \geq p_{1H}$, while under p'_H consumers purchase if $v_i \geq p_{2H}^R$. Thus, p'_H decreases first-period demand and strictly lowers type L's imitation payoff relative to p_H. Therefore, if type L's IC constraint is satisfied with price scheme p_H, it is also satisfied with price scheme p'_H and $\hat{p}_H$.

2. Consider a price scheme p_H with $p_{1H} > p_{2H}^N$. We differentiate two cases:

   (i) $p_{1H} + p_{2H}^R \leq 2p_{2H}^N$. Consumers with $v_i \geq \frac{p_{1H}+p_{2H}^R}{2}$ purchase in both periods at prices p_{1H} and p_{2H}^R, and those with $v_i < \frac{p_{1H}+p_{2H}^R}{2}$ do not make a purchase. Note that lowering p_{2H}^N while keeping $p_{1H} + p_{2H}^R \leq 2p_{2H}^N$ satisfied does not change the first-period and second-period demand. Thus the same logic as in the case $p_{1H} < p_{2H}^N$ applies, and a price scheme $\hat{p}_H = \{p_{1H}, (\hat{p}_{2H}^R, p_{1H})\}$ with optimally chosen $\hat{p}_{2H}^R$ dominates p_H for the type H seller.

   (ii) $p_{1H} + p_{2H}^R > 2p_{2H}^N$. Consumers with $v_i \geq p_{1H} + p_{2H}^R - p_{2H}^N$ purchase in both periods at prices p_{1H} and p_{2H}^R, and consumers with $v_i \in [p_{2H}^N, p_{1H} + p_{2H}^R - p_{2H}^N]$ purchase only in the second period at price p_{2H}^N.[28] Type H seller's profits from the two periods are

$$\Pi_H^c(p_H,1) = \left[v_L+1-(p_{1H}+p_{2H}^R-p_{2H}^N)\right](p_{1H}-c+p_{2H}^R-c)+(p_{1H}+p_{2H}^R-p_{2H}^N-p_{2H}^N)(p_{2H}^N-c)$$

[28]If $p_{1H} + p_{2H}^R - p_{2H}^N > v_L + 1$, consumers with $v_i \geq p_{2H}^N$ purchase only at $t = 2$. Then, compared with p_H, $\hat{p}_H = \{p_{2H}^N, (\hat{p}_{2H}^R, p_{2H}^N)\}$ with $\hat{p}_{2H}^R = v_L + 1$ neither changes type H's profits nor type L's imitation incentive.

where the first term is type H's total profits from the high valuation segment and the second term is his total profits from the low valuation segment.

By posting a price scheme $p'_H = \{p^N_{2H}, (p'^R_{2H}, p^N_{2H})\}$ with $p'^R_{2H} = p_{1H} + p^R_{2H} - p^N_{2H} > p^R_{2H}$ instead, the type H seller earns exactly the same profits both from the high valuation segment and the low valuation segment:

$$\begin{aligned}\Pi^c_H(p'_H, 1) =&[v_L + 1 - (p^N_{2H} + p'^R_{2H} - p^N_{2H})](p^N_{2H} - c + p'^R_{2H} - c) + (p^N_{2H} + p'^R_{2H} - p^N_{2H} - p^N_{2H})(p^N_{2H} - c)\\ =&[v_L + 1 - (p_{1H} + p^R_{2H} - p^N_{2H})](p_{1H} - c + p^R_{2H} - c) + (p_{1H} + p^R_{2H} - p^N_{2H} - p^N_{2H})(p^N_{2H} - c)\\ =&\Pi^c_H(p_H, 1).\end{aligned}$$

Furthermore, under price scheme p'_H, $p'_{1H} = p^N_{2H} < p_{1H}$, the type L seller has strictly less incentive under p'_H than under p_H to imitate the price choice of type H:

$$\begin{aligned}\Pi^c_L(p'_H, 1) = &\left(v_L + 1 - (p^N_{2H} + p'^R_{2H} - p^N_{2H})\right)(p^N_{2H} - c) = (v_L + 1 - [p_{1H} + p^R_{2H} - p^N_{2H}])(p'_{1H} - c)\\ &< (v_L + 1 - [p_{1H} + p^R_{2H} - p^N_{2H}])(p_{1H} - c) = \Pi^c_L(p_H, 1) \le \Pi^c_L(p^*_L, 0) = 2v_L.\end{aligned}$$

It follows that a price scheme $\hat{p}_H = \{\hat{p}_{1H}, (\hat{p}^R_{2H}, \hat{p}^N_{2H})\}$ with optimally chosen $\hat{p}_{1H} = \hat{p}^N_{2H}$ and $\hat{p}^R_{2H}$ brings the type H seller larger profits than $\Pi^c_H(p'_H, 1)$, and larger resulting profits than $\Pi^c_H(p_H, 1)$.

Finally, to see that $\Pi^c_H(\hat{p}_H, 1)$ can be strictly larger than $\Pi^c_H(p'_H, 1)$, note that $p'_{1H} = p'^N_{2H}$ and p'^R_{2H} cannot be both equal to $\tilde{s}$. ($p'_{1H} = p'^N_{2H} = p'^R_{2H} = \tilde{s}$ implies $(p_{1H} + p^R_{2H})/2 = p^N_{2H} = \tilde{s}$, violating the assumption $p_{1H} + p^R_{2H} > 2p^N_{2H}$.) Suppose $p'^R_{2H} < \tilde{s}$, then there exists small $\epsilon > 0$ such that $\hat{p}^R_{2H} = p'^R_{2H} + \epsilon$ is closer to $\tilde{s}$ than p'^R_{2H}, and $\hat{p}_H = \{p'_{1H}, (\hat{p}^R_{2H}, p'^N_{2H})\}$ brings the type H seller strictly larger profit than p'_H:

$$\begin{aligned}\Pi^c_H(\hat{p}_H, 1) =&(v_L + 1 - \hat{p}^R_{2H})(\hat{p}^R_{2H} - c) + (v_L + 1 - p'_{1H})(p'_{1H} - c)\\ >&(v_L + 1 - p'^R_{2H})(p'^R_{2H} - c) + (v_L + 1 - p'_{1H})(p'_{1H} - c) = \Pi^c_H(p'_H, 1).\end{aligned}$$

And type L has no incentive to mimic the price choice of type H given that ϵ is sufficiently small and $\Pi_L(p'_H, 1) < 2v_L$ because

$$\Pi^c_L(\hat{p}_H, 1) = [v_L + 1 - (p'^R_{2H} + \epsilon)](p'_{1H} - c) = \Pi^c_L(p'_H, 1) - \epsilon(p'_{1H} - c) < 2v_L.$$

When $p'^R_{2H} > \tilde{s}$, following similar logic, the price scheme $\hat{p}_H = \{p'_{1H}, (\hat{p}^R_{2H}, p'^N_{2H})\}$ with $\hat{p}^R_{2H} = p'^R_{2H} - \epsilon$ brings the type H seller strictly larger profit than p'_H, while satisfying type L's incentive compatibility constraint. When $p'^R_{2H} = \tilde{s}$, it follows that $p'_{1H} = p'^N_{2H} < \tilde{s}$, and then price scheme $\hat{p}_H = \{\hat{p}_{1H}, (p'^R_{2H}, \hat{p}^N_{2H})\}$, where $\hat{p}_{1H} = \hat{p}^N_{2H} = p'_{1H} + \epsilon$, brings the type H seller strictly larger profit than p'_H while satisfying type L seller's incentive compatibility constraint.

□

We state and prove Remark 6 below to prepare for the proof of Proposition 4.

Remark 6. *In the price-commitment regime, there exists no separating equilibrium with* $\beta \leq 1$ *that survives the intuitive criterion when* $c^2 < (v_L+1)^2 - 8v_L$.

Proof of Remark 6. Suppose $c^2 < (v_L+1)^2 - 8v_L$ holds and we show in sequence that an equilibrium candidate p_H with $\beta < 1$ or $\beta = 1$ violates the intuitive criterion.

Consider an equilibrium candidate $p_H^c = \{\tau^c, (\beta^c\tau^c, \tau^c)\}$ with $\beta^c < 1$. Recall that $p_L^c = \{v_L, v_L\}$ in a separating equilibrium. Making use of (17), the equilibrium β^c must satisfy

$$\Pi_L^c(p_H^c, 1) = (v_L + 1 - \frac{1+\beta^c}{2}\tau^c)\tau^c + 0 \leq \Pi_L^c(p_L^c, 0) = 2v_L.$$

Then consider the price scheme $\hat{p}_H = (\hat{\tau}, (\hat{\beta}\hat{\tau}, \hat{\tau}))$ with $\hat{\tau} = \frac{1+\beta^c}{2}\tau^c$ and $\hat{\beta} = 1$. We have

$$\begin{aligned}\Pi_L^c(\hat{p}_H, 1) &= (v_L + 1 - \hat{\tau})\hat{\tau} + 0 = (v_L + 1 - \frac{1+\beta^c}{2}\tau^c)\frac{1+\beta^c}{2}\tau^c \\ &< \Pi_L^c(p_H^c, 1) \leq \Pi_L^c(p_L^c, 0).\end{aligned}$$

Thus under $\hat{p}_H$ the type L seller has no incentive to imitate the high quality counterpart. Furthermore, note that type H seller's profits under p_H^c and $\hat{p}_H$ are the same:

$$\begin{aligned}\Pi_H^c(p_H^c, 1) &= (v_L + 1 - \frac{1+\beta^c}{2}\tau^c)(\tau^c - c) + (v_L + 1 - \frac{1+\beta^c}{2}\tau^c)(\beta^c\tau^c - c) \\ &= 2(v_L + 1 - \frac{1+\beta^c}{2}\tau^c)(\frac{1+\beta^c}{2}\tau^c - c), \\ \Pi_H^c(\hat{p}_H, 1) &= (v_L + 1 - \hat{\tau})(\hat{\tau} - c) + (v_L + 1 - \hat{\tau})(\hat{\tau} - c) \\ &= 2(v_L + 1 - \frac{1+\beta^c}{2}\tau^c)(\frac{1+\beta^c}{2}\tau^c - c) = \Pi_H^c(p_H^c, 1).\end{aligned}$$

Moreover, since $c^2 < (v_L+1)^2 - 8v_L$, the equilibrium outcome under public information, $\tilde{p}_H^c = \{\tilde{s}, (\tilde{s}, \tilde{s})\}$, cannot be supported in a separating equilibrium and we have $\Pi_L^c(\tilde{p}_H^c, 1) > \Pi_L^c(p_L^c, 0) = 2v_L$, thus $\hat{\tau} \neq \tilde{s}$ must hold. Then for $\hat{\tau} \lesseqgtr \tilde{s}$ there exists $\check{\tau}_H$ with $\check{\tau} = \hat{\tau} \pm \epsilon$ and $\check{\beta} = 1$ such that for sufficiently small positive ϵ we have

$$\begin{aligned}\Pi_L^c(\check{p}_H, 1) &= (v_L + 1 - \check{\tau})\check{\tau} + 0 = (v_L + 1 - (\hat{\tau} \pm \epsilon))(\hat{\tau} \pm \epsilon) < \Pi_L^c(p_L^c, 0), \\ \Pi_H^c(\check{p}_H, 1) &= 2(v_L + 1 - \check{\tau})(\check{\tau} - c) > \Pi_H(\hat{p}_H, 1) = \Pi_H^c(p_H^c, 1).\end{aligned}$$

Thus the intuitive criterion requires $\mu(\check{p}_H) = 1$ and the type H seller prefers price scheme $\check{p}_H$ to p_H^c, which violates the optimality of p_H^c. Therefore, an equilibrium candidate p_H^c with $\beta^c < 1$ cannot survive the intuitive criterion.

Next we show that p_H^c with $\beta^c = 1$ cannot survive the intuitive criterion either. Suppose there exists a separating equilibrium with $p_H^c = \{\tau^c, (\tau^c, \tau^c)\}$. The optimal $\tau^c > \tilde{s}$[29] for the type H seller must satisfy

$$\Pi_L^c(p_H^c, 1) = (v_L + 1 - \tau^c)\tau^c + 0 \leq \Pi_L(p_L^c, 0) = 2v_L. \tag{33}$$

Then consider the price scheme $\hat{p}_H = \{\tilde{s}, (\hat{\beta}\tilde{s}, \tilde{s})\}$ with $\hat{\beta}\tilde{s} = \tau^c$ where $\hat{\beta} > 1$. We have

$$\Pi_L^c(\hat{p}_H, 1) = (v_L + 1 - \hat{\beta}\tilde{s})\tilde{s} + 0 = (v_L + 1 - \tau^c)\tilde{s} < \Pi_L^c(p_H^c, 1) \leq \Pi_L^c(p_L^c, 0).$$

Thus under $\hat{p}_H$ the type L seller will also not imitate the price choice of the type H seller. Moreover, type H seller's profit under $\hat{p}_H$ is higher than that under p_H^c:

$$\begin{aligned}\Pi_H^c(\hat{p}_H, 1) &= (v_L + 1 - \tau^c)(\tau^c - c) + (v_L + 1 - \tilde{s})(\tilde{s} - c)\\ &> (v_L + 1 - \tau^c)(\tau^c - c) + (v_L + 1 - \tau^c)(\tau^c - c) = \Pi_H^c(p_H^c, 1).\end{aligned}$$

Thus both $\Pi_H^c(\hat{p}_H, 1) > \Pi_H^c(p_H^c, 1)$ and $\Pi_L^c(\hat{p}_H, 1) < \Pi_L^c(p_L^c, 0)$ hold, intuitive criterion requires $\mu(\hat{p}_H) = 1$ and as a result the type H seller prefers $\hat{p}_H$ over p_H^c which violates the optimality of p_H^c. Therefore, p_H^c with $\beta^c = 1$ cannot be supported in a separating equilibrium that survives the intuitive criterion. □

Proof of Proposition 4. Part (i) of the statement follows directly from the discussions in the text before Proposition 4. In the following we prove part (ii) of the proposition.

In Remark 6 we show that a price scheme $p_H = \{\tau, (\beta\tau, \tau)\}$ with $\beta \leq 1$ and $p_L = \{v_L, v_L\}$ cannot be supported in a separating equilibrium that survives the intuitive criterion. Next we prove that if $c^2 < (v_L + 1)^2 - 8v_L$, which implies $v_L < \frac{1}{6}$ and $c < \sqrt{v_L^2 - 6v_L + 1}$, there exists a separating equilibrium with $\beta > 1$ that survives the intuitive criterion. That is, there exists a combination of $\beta^c > 1$ and τ^c such that $p_H^c = \{\tau^c, (\beta^c\tau^c, \tau^c)\}$ and $p_L^c = \{v_L, v_L\}$ solve the maximization problem (20). We will solve for β^c and τ^c assuming that type H's incentive compatibility constraint (19) is not binding, and then confirm that the derived p_H^c indeed satisfies (19).

With $\beta > 1$, type H seller's equilibrium path profit (18) and type L's IC constraint (17) are respectively

$$\Pi_H^c(p_H, 1) = (v_L + 1 - \beta\tau)(\tau - c) + (v_L + 1 - \beta\tau)(\beta\tau - c) + (\beta\tau - \tau)(\tau - c) \tag{34}$$

$$\Pi_L^c(p_H, 1) = (v_L + 1 - \beta\tau)\tau \leq \Pi_L(p_L^c, 0) = 2v_L. \tag{35}$$

The lagrangian function of the maximization problem in (20) is written as follow:

$$\mathcal{L}(\beta, \tau) = (v_L + 1 - \beta\tau)[(1 + \beta)\tau - 2c)] + (\beta - 1)\tau(\tau - c) + \eta[2v_L - (v_L + 1 - \beta\tau)\tau]$$

[29] Type L's binding IC constraint (33) gives two candidates for τ^c with midpoint $\frac{v_L+1}{2}$. The optimal τ^c takes the higher value which is closer to $\tilde{s}$.

in which η is the lagrangian multiplier. The first-order conditions with respect to τ and β are respectively

$$(1+\beta)(v_L+1-\beta\tau)-\beta[(1+\beta)\tau-2c)]+(\beta-1)(2\tau-c)-\eta(v_L+1-2\beta\tau)=0, \tag{36}$$
$$-\tau[(1+\beta)\tau-2c)]+\tau(v_L+1-\beta\tau)+\tau(\tau-c)+\eta\tau^2=0. \tag{37}$$

Suppose $\eta=0$, (37) implies that $\beta\tau=\tilde{s}$ in equilibrium. Plugging this back into (36), we get $\beta=1$, which leads to $\tau=\beta\tau=\tilde{s}$ as the equilibrium choice of the type H seller. This is a contradiction because under $(v_L+1)^2-8v_L>c^2$, the public information outcome, $\tilde{p}_H^c=\{\tilde{s},(\tilde{s},\tilde{s})\}$, cannot be supported in a separating equilibrium. Therefore, in a separating equilibrium, $\eta>0$ must hold and type L's IC constraint (35) must be binding, which implies

$$(v_L+1-\beta\tau)\tau=2v_L \Leftrightarrow \beta(\tau)=\frac{v_L+1}{\tau}-\frac{2v_L}{\tau^2}.$$

Thus, type H seller's profit maximization problem in (20) simplifies to:

$$\max_{\tau}\ \Pi_H^c(p_H,1)=\frac{2v_L}{\tau}(\tau+v_L+1-\frac{2v_L}{\tau}-2c)+(v_L+1-\frac{2v_L}{\tau}-\tau)(\tau-c). \tag{38}$$

Then the derivative with respect to τ is

$$\frac{\partial\Pi_H^c(p_H,1)}{\partial\tau}=-\frac{2v_L(v_L+1-c)}{\tau^2}+\frac{8v_L^2}{\tau^3}+v_L+1-2\tau+c.$$

Note that for $\tau\le\frac{(v_L+1)-\sqrt{(v_L+1)^2-8v_L}}{2}\in[v_L,\tilde{s}]$ in which $\tilde{s}=\frac{v_L+1+c}{2}$, we have

$$\begin{aligned}\frac{\partial\Pi_H^c(p_H,1)}{\partial\tau}&=\frac{v_L}{\tau^3}[8v_L-2\tau(v_L+1-c)]+v_L+1-2\tau+c\\&\ge\frac{v_L}{\tau^3}[8v_L-(v_L+1-\sqrt{(v_L+1)^2-8v_L})(v_L+1-c)]+c+\sqrt{(v_L+1)^2-8v_L}\\&=\frac{v_L}{\tau^3}(v_L+1-\sqrt{(v_L+1)^2-8v_L})(\sqrt{(v_L+1)^2-8v_L}+c)+c+\sqrt{(v_L+1)^2-8v_L}\\&>2c>0.\end{aligned}$$

For $\tau\ge\tilde{s}$, we have

$$\begin{aligned}\frac{\partial\Pi_H^c(p_H,1)}{\partial\tau}&=-\frac{2v_L(v_L+1-c)\tau-8v^2}{\tau^3}+v_L+1+c-2\tau\\&\le-\frac{v_L[(v_L+1-c)(v_L+1+c)-8v_L]}{\tau^3}+v_L+1+c-(v_L+1+c)<0.\end{aligned}$$

Since (38) is differentiable for $\tau\in[v_L,v_L+1]$, there exists a $\tau^c\in(\frac{(v_L+1)-\sqrt{(v_L+1)^2-8v_L}}{2},\tilde{s})$ that maximizes the objective function (38). Furthermore, since $(v_L+1-\tau)\tau>2v_L$ for all $\tau\in(\frac{(v_L+1)-\sqrt{(v_L+1)^2-8v_L}}{2},\tilde{s})$, $\beta^c=\frac{1}{\tau^c}(v_L+1-\frac{2v_L}{\tau^c})>1$.

Finally, we confirm that $p_H^c=\{\tau^c,(\beta^c\tau^c,\tau^c)\}$ and $p_L^c=\{v_L,v_L\}$ indeed constitute a separating equilibrium by showing that type H's IC constraint (19) is satisfied. Consider $\hat{p}_H=\{\tilde{s},(\hat{\beta}\tilde{s},\tilde{s})\}$ with

$\hat{\beta} = \frac{1}{\tilde{s}}(v_L + 1 - \frac{2v_L}{\tilde{s}})$. Then making use of (34), we have

$$\Pi_H^c(\hat{p}_H, 1) = (v_L + 1 - \tilde{s})(\tilde{s} - c) + (v_L + 1 - \hat{\beta}\tilde{s})(\hat{\beta}\tilde{s} - c) = \tilde{\pi} + \frac{2v_L}{\tilde{s}}\left(v_L + 1 - \frac{2v_L}{\tilde{s}} - c\right) > \tilde{\pi}$$

because $c^2 < (v_L + c)^2 - 8v_L$ implies $v_L + 1 - \frac{2v_L}{\tilde{s}} - c > 0$. Since p_H^c and p_L^c maximizes $\Pi_H^c(p_H, 1)$ subject to constraint (35), making use of the consumer's off-equilibrium belief $\mu(p^d) = 0$ for $p^d \neq p_H^c$, we have

$$\Pi_H^c(p_H^c, 1) \geq \Pi_H^c(\hat{p}_H, 1) > \max_{p^d \neq p_H^c} \Pi_H^c(p^d, 0) = \tilde{\pi}.$$

Thus the type H seller has no incentive to deviate from p_H^c. □

Proof of Corollary 4. From part (i)s of Proposition 3 and 4, if $c^2 \geq (v_L + 1)^2 - 8v_L$ holds, the public information outcome in Remark 4 is supported as the unique separating equilibrium under both BBP and uniform pricing, and type H seller's profits under the two pricing regimes are the same: $\Pi_H^{c,b} = \tilde{\Pi}_H^c = 2\tilde{\pi}$.

If $c^2 < (v_L + 1)^2 - 8v_L$ holds, the proof of Lemma 3 implies that under BBP a price scheme $p_H = \{s_{1H}^c, (s_{2H}^c, s_{2H}^c)\}$, where $s_{1H}^c > s_{2H}^c$, is (weakly) dominated by some price scheme $\hat{p}_H = \{s_{2H}^c, (s_{1H}^c, s_{2H}^c)\}$. The analysis in Proposition 4 suggests that when the price scheme takes the form $p_H = \{\tau, (\beta\tau, \tau)\}$ the equilibrium prices are $p_H^c = \{\tau^c, (\beta^c\tau^c, \tau^c)\}$, with $\beta^c > 1$ specified in Proposition 4. Thus type H seller's profit with BBP must be larger than that under uniform pricing in the price-commitment regime.

To show that consumer surplus is higher under BBP than under uniform pricing, note that under BBP consumers with $v_i \geq \beta^c\tau^c$ purchase at price τ^c in the first period, consumers with $v_i \in [\tau^c, \beta^c\tau^c)$ purchase their first unit in the second period at price τ^c, and consumers with $v_i \in [\beta^c\tau^c, v_L + 1]$ purchase their second unit at price $\beta^c\tau^c$, while under uniform pricing consumers with $v_i \geq s_{1H}^c$ purchase in the first period at price s_{1H}^c and consumers with $v_i \geq \tilde{s}$ purchase in the second period at price $\tilde{s}$. Thus the consumer surplus associated with type H products under BBP and uniform price are respectively

$$\begin{aligned} CS_H^{c,b} &= \int_{\beta^c\tau^c}^{v_L+1}(x - \tau^c)dx + \int_{\tau^c}^{\beta^c\tau^c}(x - \tau^c)dx + \int_{\beta^c\tau^c}^{v_L+1}(x - \beta^c\tau^c)dx \\ &= \int_{\tau^c}^{v_L+1}(x - \tau^c)dx + \int_{\beta^c\tau^c}^{v_L+1}(x - \beta^c\tau^c)dx. \\ CS_H^{c,u} &= \int_{s_{1H}^c}^{v_L+1}(x - s_{1H}^c)dx + \int_{\tilde{s}}^{v_L+1}(x - \tilde{s})dx. \end{aligned}$$

In Proposition 4 we have shown that $\tau^c < \tilde{s}$. Moreover,

$$\beta^c\tau^c = v_L + 1 - \frac{2v_L}{\tau^c} < v_L + 1 - \frac{2v_L}{s_{1H}^c} = s_{1H}^c.$$

Thus $CS_H^{c,u} < CS_H^{c,b}$ holds. Since both consumer surplus and type H seller's profits are higher, total surplus is also higher under BBP than under uniform pricing. □

Proof of Lemma 4. Solving for the p_{1H} that maximizes $\Pi_H^t(p_{1H}, 1)$ in (21) gives us the equilibrium first-period price $\tilde{p}_{1H}^t$. Making use of $p_{2H}^R = \max\{2p_{1H} - c, \tilde{s}\}$ gives us the equilibrium price for repeat purchasers when the seller only learns the consumers' purchasing history.

Plugging the equilibrium prices back into (21), we obtain the type H seller's equilibrium profits with the option of discriminatory prices under public information

$$\tilde{\Pi}_H^t = \begin{cases} \frac{9-2\lambda}{20-8\lambda}(v_L+1-c)^2 & \text{if } \lambda \le \frac{1}{2} \\ \left(\frac{9-2\lambda}{20-8\lambda} + \frac{(2\lambda-1)^2(1-\lambda)}{4(2\lambda+1)^2}\right)(v_L+1-c)^2 & \text{if } \lambda > \frac{1}{2} \end{cases}. \tag{39}$$

Comparing $\tilde{\Pi}_H^t$ with $\tilde{\Pi}_H^u$ shows that $\tilde{\Pi}_H^t \ge \tilde{\Pi}_H^u$ if and only if $\lambda \le \frac{1}{2}$. □

Proof of Proposition 5. Note that when $\tilde{p}_{1H}^t \ge \bar{p}$, $p_{1H} = \tilde{p}_{1H}^t$ satisfies (22) and (24), thus can be supported in a separating equilibrium. Moreover, since $\tilde{p}_{1H}^t$ is the first-period price that uniquely maximizes type H seller's profit given belief $\mu(\tilde{p}_{1H}^t) = 1$, $p_{1H}^t = \tilde{p}_{1H}^t$ forms the unique separating equilibrium that survives the intuitive criterion. This proves part (i) of the proposition.

We now prove part (ii) and (iii) of the proposition. If $\tilde{p}_{1H}^t < \bar{p}$, in a separating equilibrium that survives the intuitive criterion, the type L seller chooses $p_L^t = \{v_L, v_L\}$ and the type H seller chooses $p_H^t = \{\bar{p}, (2\bar{p} - c, \bar{p})\}$, which maximizes type H's profit given type L's IC constraint (22). Using (21), type H seller's profit by setting $p_{1H} = \bar{p}$ is

$$\Pi_H^t(\bar{p}, 1) = \frac{(1 - 3c + v_L + \sqrt{\Delta})\left[(7-4\lambda)(v_L+1) + (3-4\lambda)c - (5-4\lambda)\sqrt{\Delta}\right]}{16} + \lambda\frac{(v_L + 1 + c - \sqrt{\Delta})(3v_L + 3 - 5c + \sqrt{\Delta})}{8}.$$

Let $\bar{\lambda} \equiv \min\{\hat{\lambda}, 1\}$ where $\hat{\lambda}$ is given by

$$\Pi_H^t(\bar{p}, 1) = v_L - c + (1+\lambda)\tilde{\pi}$$
$$\Leftrightarrow \lambda = \frac{\frac{1}{16}(1 - 3c + v_L + \sqrt{\Delta})(7v_L + 7 + 3c - 5\sqrt{\Delta}) + c - v_L - \frac{1}{4}(v_L + 1 - c)^2}{\frac{1}{4}(v_L+1-c)^2 - \frac{1}{8}(v_L + 1 + c - \sqrt{\Delta})^2} \equiv \hat{\lambda}.$$

Suppose $\Pi_H^t(\bar{p}, 1) \ge \Pi_H^t(v_L, 0)$ if and only if $\lambda \le \bar{\lambda}$. It follows that if $\lambda \le \bar{\lambda}$, $p_{1H} = \bar{p}$ satisfies type H seller's IC constraint (24), and the separating equilibrium with $p_{1H} = \bar{p}$ exists and survives the intuitive criterion. On the other hand, if $\lambda > \bar{\lambda}$, $p_{1H} = \bar{p}$ violates type H's IC constraint (24). Then there exists no separating equilibrium that survives the intuitive criterion. It is straightforward to see that if $\lambda > \bar{\lambda}$ both types choosing $p_{1H} = p_{1L} = v_L$ constitute a pooling equilibrium with the belief system $\mu(p_1) = \rho$ for $p_1 = v_L$ and $\mu(p_1) = 0$ for $p_1 \neq v_L$. The two types' equilibrium profits are respectively $\Pi_H^t(v_L, \rho) = \Pi_H^t(v_L, 0)$ and $\Pi_L^t(v_L, \rho) = \Pi_L^t(v_L, 0) = 2v_L$. Moreover, this equilibrium survives the intuitive criterion because $\Pi_L^t(p_1^d, 1) < \Pi_L^t(v_L, \rho) = 2v_L$ requires that $p_1^d > \bar{p}$. However, with such p_1^d and belief $\mu(p_1^d) = 1$, $\Pi_H^t(p_1^d, 1) \le \Pi_H^t(\bar{p}, 1) < \Pi_H^t(v_L, \rho)$. Thus there exists no price under which if the seller is believed to be of type H, type H's profit increases compared with his equilibrium profit $\Pi_H^t(v_L, \rho)$ and type L's profit decreases compared with $\Pi_L^t(v_L, \rho)$. Thus part (ii) and (iii) of the proposition hold under the condition that $\Pi_H^t(\bar{p}, 1) \ge \Pi_H^t(v_L, 0)$ if and only if $\lambda \le \bar{\lambda}$.

We now show that $\Pi_H^t(\bar{p}, 1) \gtreqless \Pi_H^t(v_L, 0)$ implies $\lambda \lesseqgtr \bar{\lambda}$. Note that given v_L and c, $\Pi_H^t(\bar{p}, 1)$ strictly increases in λ since $\frac{d\Pi_H^t(\bar{p},1)}{d\lambda} = \frac{(v_L+1+c-\sqrt{\Delta})^2}{8} > 0$. $\Pi_H^t(v_L,0)$ in (23) is also strictly increasing in λ and $\frac{d\Pi_H^t(v_L,0)}{d\lambda} = \frac{(v_L+1-c)^2}{4} > 0$. Since $\Pi_H^t(\bar{p},1) > \tilde{\pi} > \Pi_H^t(v_L,0)$ at $\lambda = 0$, if

$$\frac{d\Pi_H^t(\bar{p},1)}{d\lambda} = \frac{(v_L+1+c-\sqrt{\Delta})^2}{8} < \frac{d\Pi_H^t(v_L,0)}{d\lambda} = \frac{(v_L+1-c)^2}{4} \tag{40}$$

holds, then either $\Pi_H^t(\bar{p},1) > \Pi_H^t(v_L,0)$ holds for all $\lambda \in [0,1]$ in which case $\hat{\lambda} > 1$ and $\bar{\lambda} = 1$, or $\Pi_H^t(\bar{p},1)$ and $\Pi_H^t(v_L,0)$ cross only once at $\hat{\lambda} \in (0,1)$ such that $\Pi_H^t(\bar{p},1) \geq \Pi_H^t(v_L,0)$ if and only if $\lambda \leq \bar{\lambda}$. Thus, the separating equilibrium exists if and only if $\lambda \leq \bar{\lambda}$.

It remains to prove (40). Note that $\tilde{p}_{1H}^t < \bar{p}$ is equivalent to

$$c < \frac{1}{7-2\lambda}\left(2 + 2v_L + (5-2\lambda)\sqrt{1-(5-2\lambda)v_L + v_L^2}\right)$$

for $v_L \leq \frac{1}{2}(5 - 2\lambda - \sqrt{21 - 20\lambda + 4\lambda^2})$ and $v_L < \frac{1}{5-2\lambda}$. It follows $c < 1 - v_L$. Thus, to prove (40), it suffices to show that $G(v_L,c) \equiv \sqrt{2}(v_L+1-c) - (v_L+1+c-\sqrt{\Delta}) > 0$ for all $v_L < \frac{1}{5-2\lambda} \leq \frac{1}{3}$ and $c \in (v_L, 1-v_L)$. First, consider $v_L \leq \frac{\sqrt{2}-1}{2}$. $G(v_L,c)$ strictly decreases in $c \in (v_L, 1-v_L)$ and $G(v_L, c \to 1-v_L) = 2\sqrt{2}v_L - 2 + 2\sqrt{1-2v_L} > 0$. Second, consider $v_L \in (\frac{\sqrt{2}-1}{2}, 1/3]$. $G(v_L,c)$ increases in c for $c \in (v_L, 2\sqrt{(1+\sqrt{2})v_L} - (v_L+1)]$ and decreases in c for $c \in (2\sqrt{(1+\sqrt{2})v_L} - (v_L+1), 1-v_L]$. And $G(v_L, c \to v_L) = \sqrt{2} - 4v_L > 0$, $G(v_L, c \to 1-v_L) > 0$. Combining the two cases shows that $G(v_L,c) > 0$ holds and this completes the proof of part (ii) and (iii). □

Proof of Proposition 7. Recall (28), (29), and (30). Given belief $\mu(p_1) = 0$ for $p_1 \neq p_{1H}$, when the type H seller deviates to $p_1 \geq \frac{\theta(v_L+1)}{2}$, $\hat{v}_L \geq (v_L+1)$. Both the first-period demand and the first-period profit are zero, and the type H seller makes the maximal profit $\tilde{\pi}$ by setting the static monopoly price in the second period.

When deviating to $p_1 < \frac{\theta(v_L+1)}{2}$, $\hat{v}_L = \frac{2p_1}{\theta}$, and the first-period profit for the type H seller is negative because $p_1 < c$. Following this, the optimal second-period prices are $p_{2H}^N = \frac{\frac{2p_1}{\theta}+c}{2}$, $p_{2H}^R = \frac{2p_1}{\theta}$. The type H seller's deviation profit in this case is given by

$$\Pi_H^h(p_1,0) = (v_L+1-\frac{2p_1}{\theta})(p_1-c) + \frac{(\frac{2p_1}{\theta}-c)^2}{4} + (v_L+1-\frac{2p_1}{\theta})(\frac{2p_1}{\theta}-c).$$

Making use of the condition $c \geq \frac{\theta+1}{3}(v_L+1)$ in (25), we have

$$\begin{aligned}\frac{\partial}{\partial p_1}\Pi_H^h(p_1,0) =& (1+\frac{2}{\theta})(v_L+1) - \frac{4\theta+6}{\theta^2}p_1 + \frac{3}{\theta}c \\ \geq& (1+\frac{2}{\theta})(v_L+1) - \frac{4\theta+6}{\theta^2}p_1 + \frac{\theta+1}{\theta}(v_L+1) \\ =& \frac{2\theta+3}{\theta}(v_L+1-\frac{2}{\theta}p_1) > 0.\end{aligned}$$

It follows that $\Pi_H^h(p_1,0)\mid_{p_1<\frac{\theta(v_L+1)}{2}} < \Pi_H^h(p_1,0)\mid_{p_1=\frac{\theta(v_L+1)}{2}}$. Thus, the type H seller's best deviating price is $p_1 \geq \frac{\theta(v_L+1)}{2}$ which brings a maximal profit equal to $\tilde{\pi}$. Therefore, $\max_{p_1\neq p_{1H}} \Pi_H^h(p_1,0) = \tilde{\pi}$ holds, and the type H seller's IC constraint in (28) is transformed into

$$\Pi_H^h(p_{1H},1) \geq \tilde{\pi}$$

which always holds for $p_{1H} \geq c$.

We now turn to type L's IC constraint in (28). Solving $\Pi_L^h(p_{1H},1) = \tilde{\Pi}_L^{h,b}$ leads to

$$\bar{p}^h = \frac{(5+10\theta)(v_L+1) + 5(1+3\theta)c + \sqrt{5}\sqrt{(5+2\theta-7\theta^2)(v_L+1)^2 + 10c(1+\theta)(v_L+1) + 5c^2}}{20+30\theta}.$$

Note that when $c \geq \hat{c} \equiv \frac{4+\theta+\sqrt{5}\sqrt{20-8\theta-7\theta^2}}{14+6\theta}(v_L+1)$, $\tilde{p}_{1H}^h = \frac{3(v_L+1)+7c}{10} \geq \bar{p}^h$ is sufficient to prevent mimicking by the type L seller and thus constitutes a separating equilibrium. Since $\tilde{p}_{1H}^h$ maximizes the type H seller's profit globally, this equilibrium also survives the intuitive criterion. When $c < \hat{c}$, the type H seller chooses $p_{1H} = \bar{p}^h > \tilde{p}_{1H}^h$ to prevent mimicking by the type L seller, which is also the unique separating equilibrium that survives the intuitive criterion. □

Proof of Corollary 5. Notice that $\hat{c} > \sqrt{1-\theta}(v_L+1)$ for all $\theta \in (0, 0.5)$. Consider $c < \sqrt{1-\theta}(v_L+1)$ where part (ii)s of Proposition 6 and 7 apply. The type H seller distorts first-period prices upward to signal product quality, $s_{1H}^{h,*} = \bar{s}^h > \tilde{s}$ under uniform pricing and $p_{1H}^{h,*} = \bar{p}^h > \tilde{p}_{1H}^h$ under BBP. From Proposition 6, the type H seller's equilibrium profit under uniform pricing is

$$\Pi_H^{h,u} = \frac{(v_L+1-c)^2}{4} + \frac{\theta(v_L+1)^2}{4} - \frac{1-\sqrt{1-\theta}}{2}(v_L+1)c.$$

Making use of (29), the type H seller's equilibrium profit under BBP is

$$\Pi_H^{h,b} = \Pi_H^h(\bar{p}^h,1) = \big(3(v_L+1) - 5\bar{p}^h + 2c\big)(\bar{p}^h - c)$$

where the expression for $\bar{p}^h$ is given in the proof of Proposition 7.

For $\theta < 0.165$, $\Pi_H^{h,b} > \Pi_H^{h,u}$ when $c \to \frac{\theta+1}{3}(v_L+1)$, and $\Pi_H^{h,b} < \Pi_H^{h,u}$ when $c \to \sqrt{1-\theta}(v_L+1)$. Since both $\Pi_H^{h,b}$ and $\Pi_H^{h,u}$ are continuous in c, there exist c_1^h and c_2^h with $\frac{\theta+1}{3}(v_L+1) < c_1^h \leq c_2^h < \sqrt{1-\theta}(v_L+1)$ such that BBP increases the type H seller's profit by comparison with uniform pricing if $c < c_1^h$ and decreases the profit if $c > c_2^h$.

□

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