Labor markets, wage inequality, and hiring selection

Pizzo, Alessandra and Villena-Roldán, Benjamin

Université de Paris VIII, Universidad Andres Bello

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Labor Markets, Wage Inequality, and Hiring Selection*

Alessandra Pizzo
Université Paris 8, LED

Benjamín Villena-Roldán
Universidad Andres Bello & LM^{2}C^2 & MIPP

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Abstract

Employers hire more selectively between heterogeneous productivity workers when applicants' queues are longer. Consistently, CPS data reveal a positive and concave relation between unemployment rates and wage inequality. We rationalize intuition and evidence altogether using a nonsequential search model in which selective hiring stretches out the right tail of the wage distribution and compresses the left one. Using GMM-estimated parameters, we show that mean worker productivity distribution shifts are consistent with the evidence. Welfare analysis suggests that regressive taxation may enhance efficiency because expected good matches stimulate vacancies, creating a positive externality for other job seekers.

Keywords: Nonsequential search, Hiring, Inequality, Unemployment, Worker Flows, Efficiency.

1 Introduction

The most qualified applicants often get hired and workers with the poorest fit are fired. Employers usually poach the most talented workers of other firms. That ability drives labor market transitions is a commonsensical premise. Labor markets reward the most talented workers through high wages and beneficial transitions, and the opposite occurs for the most unproductive ones. Since "all the good things come together," a common focus of the literature

*Email: ale.pizzo@gmail.com and benjamin@benjaminvillena.com. We are grateful to colleagues at the 2024 SAET; 2023 SED; 2022 LACEA-LAMES; 2022 EEA; Le Mans University; Catholic University of Chile; Alberto Hurtado University; 2019 Annual Search and Matching workshop; University of Bristol; and Central Bank of Chile for useful comments and discussions at different stages of this project. Villena-Roldán is grateful for financial support from FONDECYT project 1191888; ANID Milenio Labor Market Mismatch – Causes and Consequences, LM^{2}C^2 (NCS2022_045); and the Institute for Research in Market Imperfections and Public Policy, MIPP (ICS13_002 ANID). All remaining errors are ours.

on earnings of the employed probably understates the true role of individual ability in shaping (labor) income inequality. In particular, the role of labor market frictions in amplifying or dampening inequality has been scarcely researched empirically or theoretically.

To do so, in this paper we first document a number of empirical facts about the US labor market. There is a large positive and concave cross-sectional and temporal relation between wage inequality and the state-time unemployment rate using merged CPS monthly and CPS-ORG data. More detailed analysis shows that high inequality is often related to low probabilities of finding a job and to job-to-job transitions, suggesting that a part of the phenomenon may be explained by delving deeper into the hiring mechanism of firms.

To explain this evidence we construct a general equilibrium model of nonsequential employer search with hiring selectivity and heterogeneous workers, and characterize its equilibrium. The model departs from the standard search model by allowing firms to simultaneously meet several applicants and choose the best. Since the seminal work of Stigler (1961), there have been scattered empirical and theoretical pieces of research that have been pointing out the importance of nonsequential search in modeling behavior (Stern 1989; van Ours and Ridder 1992; Villena-Roldan 2010a; van Ommeren and Russo 2013; Wolthoff 2017; Davis and Samaniego de la Parra 2017). Indeed, nonsequential search seems a critical ingredient to explain how individual productivity jointly affects wages and transitions in the labor market.

A nonsequential search model provides an endogenous matching process in which the probabilities of finding a job and of making a job-to-job transition increase in worker productivity. Under selective hiring, lifetime inequality increases relative to a sequential search benchmark because low productivity workers go through longer spells of unemployment and have lower chances of job-to-job transitions. In general equilibrium, more productive workers, more often employed and potentially engaged in on-the-job search, compete for positions with the unemployed, as in Eeckhout and Lindenlaub (2019). In equilibrium, employers receive a pool of both employed and unemployed applicants, learn the true productivity type by exerting some effort, and offer the position to the most productive candidate. In the competitive equilibrium solution, employers do not take into account that they worsen the average composition of applicants when hiring the top candidates and dismissing the less attractive ones. We also derive a result that establishes that the labor market in our model amplifies inequality at the right tail of the wage distribution and dampens it at the left tail, in comparison to a Walrasian framework.

We take the model to the data by calibrating the average unemployment rate for the US, as well as the frequencies of job-to-job transitions and of separations, and the wage distribution during 1994–2019. Since we do not try to match the observed correlations between inequality and worker flow measures per se, we investigate —for instance— what type of shocks can replicate a positive comovement between the unemployment rate and different measures of wage inequality. We find that an increase in the mean of productivities replicates the aforementioned empirical fact. The first intuition goes as in standard search and matching models: an increase in the average productivity of workers spurs more creation of vacancies, making it easier to find jobs, and reducing the unemployment rate. As the number of applicants per vacancy (average queue length) declines, screening becomes less selective, and thus the composition of the pool of the unemployed *improves* its average productivity. The employed pool, on the other hand, *worsens*. Hence, the pool of applicants, a mixture of employed and unemployed workers, becomes more similar to the population's distribution of productivities. Therefore, hired applicants tend to be more similar to each other, reducing inequality. An additional general equilibrium effect is the reduction of variance in the distribution of applicants which leads to a lower likelihood of hiring top applicants. This offsets the increased average productivity and deters, to some extent, the posting of vacancies. In the end, the general equilibrium adjustment partially undoes the negative initial impact of higher productivity on both unemployment and inequality.

Finally, we take a normative point of view and characterize a social planner's problem. We find the optimal allocation, which takes into account that making the most productive workers employed requires substantial effort in screening applicants. Under the baseline estimation/calibration, we show that the optimal unemployment rate for non-college workers is slightly below its competitive equilibrium steady-state level. In contrast, for college workers, the optimal unemployment rate is slightly higher than the market solution. Next, we consider the profit tax schedule under a balanced budget that implements the social planner's solution. This takes into consideration a coincidence ranking equilibrium constraint, i.e., the after-tax profit must be strictly increasing in the type of worker so that employers do not change their preferences for workers in an application pool due to taxes. This exercise yields an apparently paradoxical regressive taxation scheme for non-college types, which is explained by the large externality created by high types in the distribution, providing large ex ante incentives to employers to open vacancies. Ex post, however, many employers cannot hire stars, so that non-top applicants obtain jobs more easily. Implementing an optimal social allocation must pay the top types for the positive externalities they generate. For college workers, instead, the tax and transfer scheme is slightly progressive, since in this case the social planner targets a marginally lower unemployment level, and therefore it is not necessary to provide incentives to employers to open more vacancies.

Our model is also related to theoretical and empirical approaches explaining rising income or labor income inequality in recent years. While increased education, skill-biased technical change, globalization, and taxation are often regarded as the principal drivers of these trends, our theory complements them by explaining how labor markets amplify or attenuate secular trends in the distribution of productivity.[1] In that sense, we do not have a theory of why the mean productivity or its dispersion has seemingly increased over time. Instead, our purpose is to theoretically understand how the selectivity prevailing in hiring and poaching workers in the labor market can interact with the pre-market productivity inequality, whatever the underlying cause is. We also try to provide an explanation simultaneously accounting for the empirical correlations between inequality and worker flows, and a sensible measurement of the effect of the selective activity of employers into inequality.

Literature review Our paper is linked to several strands of the literature.

First, considering the nonsequential matching process, there is a long-standing tradition of modeling the matching function as an urn-ball process, starting from Butters (1977). Blanchard and Diamond (1994) analyse a framework in which firms can receive more than one application and rank candidates according to the duration of their unemployment, so that the equilibrium exit rate from unemployment negatively depends on the duration of the unemployment, as well as on the measure of labor market tightness. Moen (1999) builds on Blanchard and Diamond (1994) and extends the analysis by considering that workers' rankings for firms depends on their productivity level, and productivity is linked to education, which can be acquired through a monetary cost. He thus focuses on the optimal level of education, considering that, on one hand education improves their prospects, but on the other, introduces a so-called "rat-race" among the workers, who acquire education just to move up in the ranking.

Some have considered a ranking mechanism for hiring in a directed search approach, as for example Shimer (2005), Shi (2006) and more recently Fernández-Blanco and Preugschat (2018), who endogeneize the mechanism highlighted by Blanchard and Diamond (1994), Wolthoff (2017), and Cai, Gautier, and Wolthoff (2021).

Wolthoff (2017) considers a framework in which *ex ante* homogeneous workers can send multiple applications to firms who decide on the level of wage to post, the hiring standard (a minimum threshold level of productivity) and recruiting intensity. He uses this model to study the effects of aggregate productivity shocks on the decisions of firms regarding screening and hiring standards. Some aspects of this paper, such as ex ante worker heterogene-

[1] There is a large literature examining these issues. An incomplete enumeration of papers studying the merits and demerits of these hypotheses includes Krusell, Ohanian, Ríos-Rull, and Violante (2000), Card and DiNardo (2002), Moore and Ranjan (2005), Goldin and Katz (2008), Helpman (2016), and Ravallion (2018).

ity and nonsequential search, are similar to Villena-Roldan (2010a), Villena-Roldan (2010b). However, these papers do not take into account on-the-job search, nor do they analytically characterize the competitive equilibrium and the efficiency of the economy. Cai, Gautier, and Wolthoff (2021) develop a directed search model with two-sided heterogeneity to study the conditions for sorting in applications as well as matches.

Secondly, there are a number of recent papers which have focused on the importance of the composition of the pool of employed and unemployed agents in relation with the presence of on-the-job search. Eeckhout and Lindenlaub (2019) develop a framework with random search, endogenous on-the-job search, endogenous creation of vacancies, and sorting, and show that the presence of a complementarity between the creation of vacancies and on-the-job search affects the cyclical composition of the searchers and unemployed agents, thus creating self-fulfilling states of booms and recessions.

Engbom (2021) considers a similar setting in which firms pay a cost to screen job applicants, who decide to apply based on a noisy signal on the productivity of the match. He shows that, unlike Eeckhout and Lindenlaub (2019), it is not the procyclical search behavior of workers that amplifies shocks to the labor market, but rather the countercyclical recruiting costs for firms: when unemployment is high in fact firms receive more applications from unemployed workers who are more likely to be a bad fit. Similarly to Engbom (2021), Bradley (2020) considers a model with on-the-job search where firms screen applicants. The search frequency of the unemployed and employed is different and so is the number of available vacancies they can see. In consequence, the unemployed search more intensively but are exposed to fewer possibilities, which leads to a situation in which workers, in practice, sample job opportunities from different wage distributions.

The last three cited papers consider workers as *ex ante* homogenous and focus on match-specific shocks. Merkl and van Rens (2019) develop a framework where there are no search frictions but workers are heterogeneous in terms of training costs, and consider a polar case in which this heterogeneity is permanent or individual-specific. This case gives rise to what they define as selective hiring, where the pool of unemployed is essentially composed of "lemons". They then use their framework to study the welfare costs of unemployment and the trade-offs a benevolent social planner faces in providing unemployment insurance.

2 Empirics

In this section, we document significant relations between worker flows and inequality. We are *not* claiming any causal effects. Our preferred interpretation is that worker flows cause changes in wage inequality and we will provide a model that provides a plausible mechanism that explains the facts. In this section, however, our purpose is to document correlations and partial correlations that may be interpreted later by using a structural model.

Using CPS monthly files made available by IPUMS-CPS (Flood et al. 2022), we construct unemployment rates (U) and unemployment-to-employment transitions (UE) for the period 1994–2019 by state and year.[2] We also construct job-to-job (JJ) transition rates using the standard method in Fallick and Fleischman (2004) by state, year, and college status. We use hourly wages from the Outgoing Rotation Group (ORG), also available in IPUMS-CPS and deflate them using national CPI.[3] By state, year, and college cells, we compute measures of wage inequality such as the 90–10 percentile gap for log hourly real wages (G9010), as well as other gaps (75–25, 90–50, 50–10, etc.). Our measures have time and geographical cross-sectional variation for two levels of skill or education of the worker. Our idea is to group workers who compete for similar jobs in the same market (state) in a specific year. While workers may have also being grouped by occupation, there is substantial occupational mobility in the CPS even for 1-digit codes: Moscarini and Thomsson (2007) report a 3.2% monthly occupational transition rate at the 3-digit level and approximately 2.2% at the 1-digit level for 2004–2005 (4.2% and 3.7% for 1983–1986). Kambourov and Manovskii (2009) obtain comparable results using the PSID. Our split according to college status intends to generate coarse categories in which job transitions between them is likely to be rare. Suggestive evidence for the CPS shows that for a majority of non-college occupations, less than 1% have a college degree (Gottschalk and Hansen 2003).

We mainly present the results for the log of the 90–10 wage percentile ratio, although we do not belittle the relevance of other measures: we will in fact show that different parts of the distribution can be affected differently by changes in the unemployment rate.

Figures 1–4 show the fitted values of a quadratic regression between the 90–10 log hourly real wage percentile gap (G9010) by state and year on a number of flow or stock variables, one at a time. The figures also depict the 95% confidence intervals for the fitted values.

The relation between G9010 and the unemployment rate (U) is mostly increasing but flattens out near 7% for non-college workers in the panels of Figure 1. The pattern seems quite similar when controlling for state fixed effects, suggesting that time variability is driving the pattern to a large extent. For college workers, the relation between G9010 and the unemployment rate is also positive, but mostly in a linear way, although the range of unemployment rates covered is narrower.

[2] We also compute transitions from 1976-2019 and the results do not change qualitatively.

[3] All items in U.S. city average, all urban consumers, not seasonally adjusted, series CUUR0000SA0. Of course, for inequality measures based on log wages, this adjustment is irrelevant.

Figures 2–4 try to dig deeper into the previous relation, as flows in and out of unemployment could shape the unemployment rate as in a standard two-state search and matching model (Pissarides 2000). The job finding frequency (UE)[4] is mostly negatively correlated with wage dispersion, showing that wage inequality tends to increase in times of long unemployment duration, which remains true after controlling for state fixed effects. The sensitivity of wage inequality has a similar magnitude for both college and non-college workers, although for the latter there is greater concavity.

In the case of the separation frequency (EU flow), we observe mostly positive correlations. The sensitivity remains if we control for state fixed effects. Both job finding and separation frequencies are associated to wage dispersion in a way that is consistent with the correlation between wage dispersion and unemployment. Taken together, these pieces suggests a clear linkage between worker flows and inequality.

As occurs with the job finding frequency (UE), for college workers there is a negative relation between job-to-job frequency transitions and inequality, although the slope of the relation is flatter than the one for the UE flow. For non-college workers, the slope is, in contrast, mainly positive.

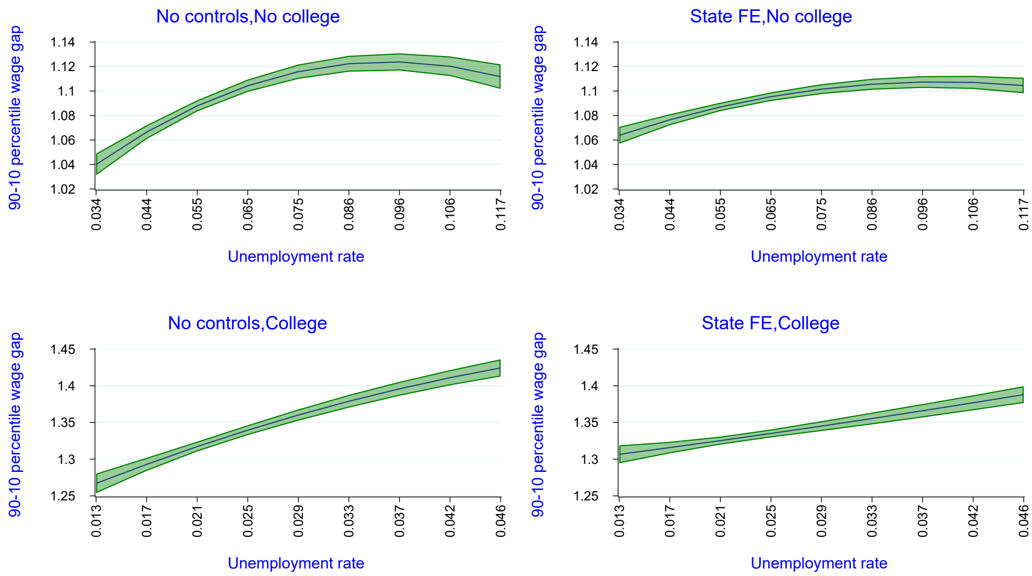
Our preferred explanation, among the many possible, for the relation between wage inequality and unemployment assumes that employers hire selectively, i.e., they compare applicants and offer the job to the most appropriate one. When the unemployment rate increases, there are larger queues of workers applying for jobs. In this situation, employers hire better applicants on average, all else constant. Therefore, jobs are allocated to the most productive workers or best matches, on average, so that the composition of employed workers improves, widening the gap with previously employed workers. However, as the unemployment rate keeps increasing, the composition of the employed becomes better but less heterogeneous, driving down inequality. This would explain why the increase of inequality associated with unemployment becomes smaller in higher unemployment markets, showing a concave relation. For a sufficiently high unemployment rate, selectivity is so strong that hired workers may be homogeneous enough to make wage inequality to decrease.

We perform a number of robustness checks to be sure these relations are meaningful and unrelated to mechanical biases. We refer the reader to the relevant figures in the Appendix. In particular, we checked that

- The 1994 CPS sampling redesign does not substantially change the observed patterns

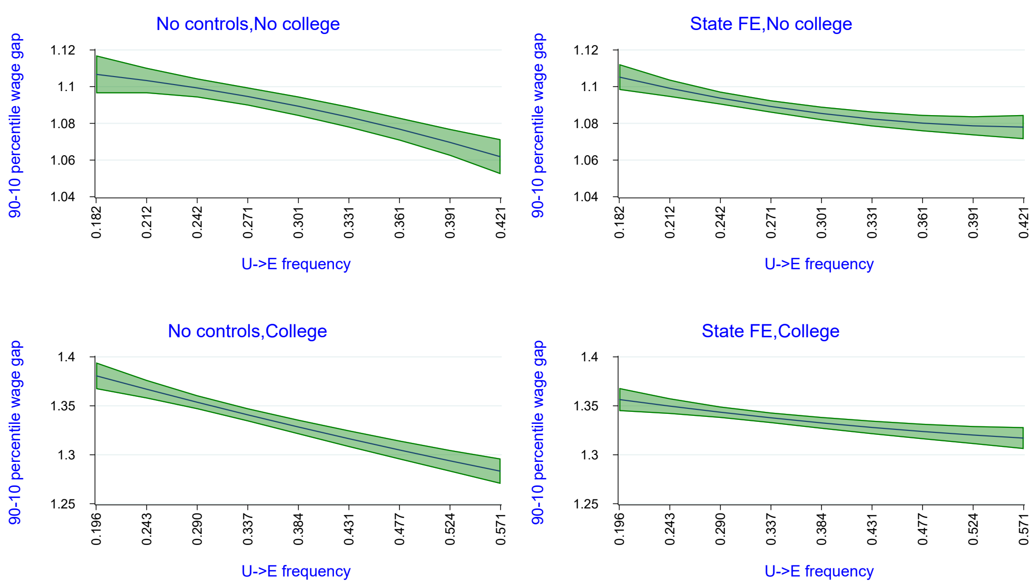
[4]Worker flow transitions cannot be computed for individuals whose month-in-sample is 1 or 5 in the CPS sampling design, which contrasts with the computation of the unemployment rate, which covers all individuals in the sample.

Figure 1: 90–10 log wage percentile gap vs. unemployment rate by state & year



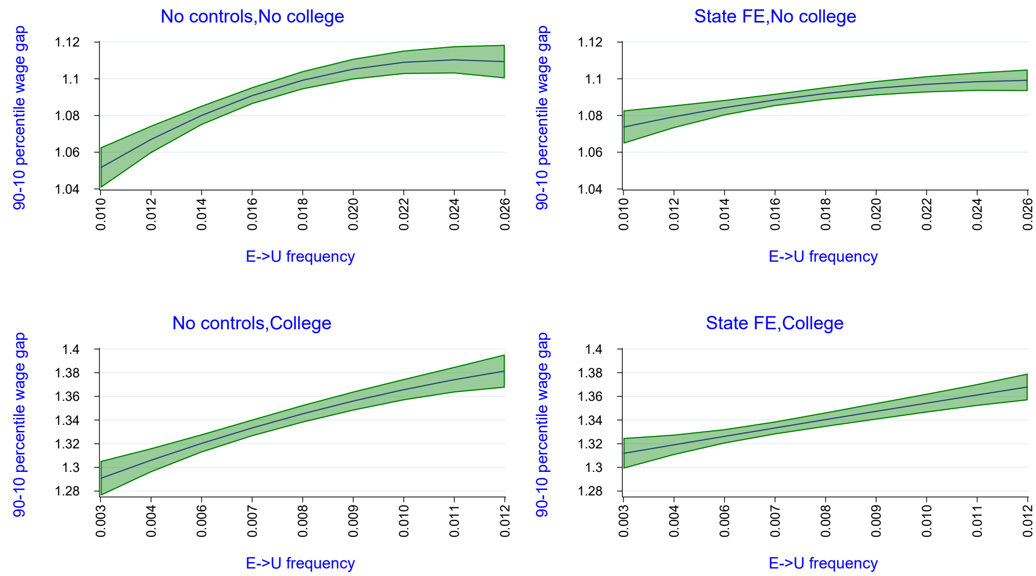
Sample 1994-2019. US real log hourly wages.

Figure 2: 90–10 log wage percentile gap vs. job finding freq. (UE) by state & year



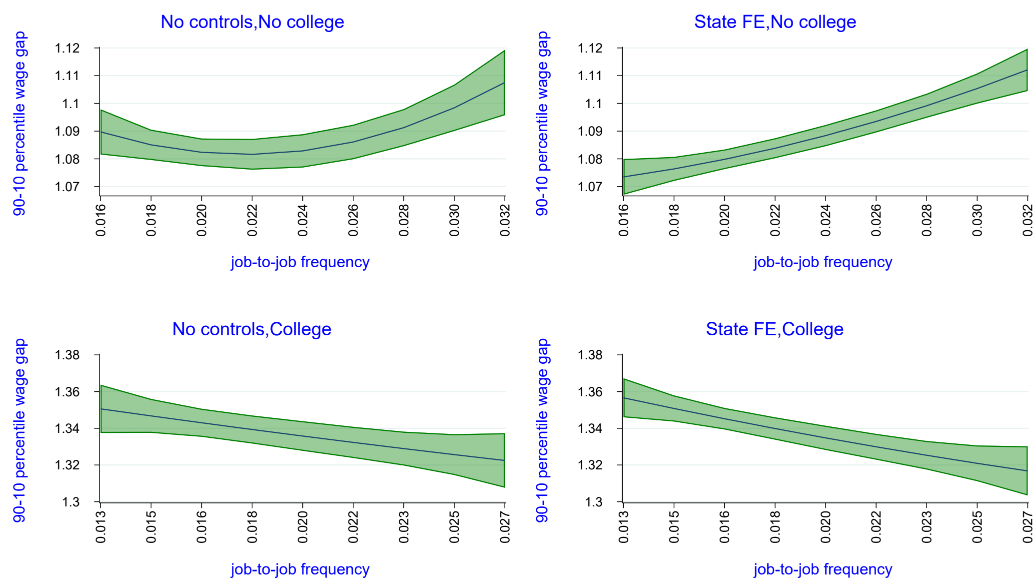
Sample 1994-2019. US real log hourly wages.

Figure 3: 90–10 log wage percentile gap vs. separation freq. (EU) by state & year



Sample 1994-2019. US real log hourly wages.

Figure 4: 90–10 log wage percentile gap vs. job-to-job freq. (JJ) by state & year



Sample 1994-2019. US real log hourly wages.

in the relation between the log wage dispersion and the unemployment rate, nor the relation between the wage dispersion and the EU and UE flow frequencies. These can be seen in Figures 15, 16, and 17.

- The findings generally remain unaltered for alternative measures of wage dispersion, such as the 75–25 percentile wage gaps and the standard deviation of log wage, as can be seen in Figures 18–25. While the former is less sensitive to outliers, the latter is more commonly used.

- We introduced controls for year fixed effects only in Figures 26, 27, 28, and 29, obtaining qualitatively similar results. This shows that the underlying mechanism generating the shapes in the previous evidence is pertinent to explaining the cross-sectional diversity of inequality across different states.

While we uncover these empirical facts in a somewhat novel fashion, other papers have investigated the relation between inequality and unemployment. There is a relatively old literature which assessed the existence of a relation between income inequality and unemployment. For example Jäntti (1994)[5], using US data from 1948 to 1989, shows that there is a non-linear relation between unemployment and the quintile shares in family income: in particular, while the first three quantiles lose from an increase in unemployment, the opposite is true for the top quintile. Mocan (1999), using a different specification, confirms those findings with respect to unemployment, and concludes that "an increase in structural unemployment increases the income share of the highest quintile, and decreases the shares of the bottom sixty percent of the population."

In an effort to shed light on the mechanisms that could link unemployment and inequality, Cysne and Turchick (2012), building on Cysne (2009), focus on the Gini index as a measure of income inequality, and analytically show that a standard model with search frictions, on-the-job search, and wage posting, such as Burdett and Mortensen (1998), can generate a positive correlation between unemployment and the Gini index (whenever unemployment remains under some reasonable value). We differ from the approach in Cysne and Turchick (2012), since we do not focus on the Gini index and our mechanisms will be generated by a search and matching frictions model (and not a wage posting model).

Papers such as Heathcote, Perri, and Violante (2010), Heathcote, Perri, and Violante (2020) and Heathcote, Perri, Violante, and Zhang (2023) have approached the complex and vast question of documenting the inequality's evolution over the past decades. When focusing on wage inequality, they distinguish between inequality at the bottom of the distribution and

[5] For a list of earlier papers, see his literature review.

that at the top, identifying different evolutions and drivers. However, our empirical exercise differs from theirs, as we do not consider a time analysis, or distinguish between trend and cycle.

3 The Model

In the model, time is discrete. There is a continuum of homogeneous risk-neutral firms or employers that can post ex ante identical job vacancies. There is also a fixed mass of size 1 of workers, who are characterized by a time-invariant productivity θ according to an exogenous distribution with density $g(\theta)$. Workers can either be employed or unemployed. Employed workers apply to a position with a fixed exogenous probability λ. The mass of applicants $\mathcal{A}$ is given by the sum of the unemployed and the proportion of employed who apply for a job: $\mathcal{A} = \mathcal{U} + \lambda(1 - \mathcal{U})$. Jobs are destroyed with exogenous probability η. Workers cannot borrow or save.

The general state of the economy is a tuple $\mathcal{X} \equiv (\mathcal{A}, \mathcal{V}, G_A(\theta))$, where $\mathcal{A}$ represents the mass of applicants in the economy, $\mathcal{V}$ is the mass of aggregate vacancies posted, and $G_A(\theta)$ is the endogenous joint distribution of types of the applicants, including unemployed and employed jobseekers. While the setting could be extended to a dynamic framework, in this paper we solely focus on the steady-state symmetric equilibrium of the economy.

3.1 Matching and Job Finding Rate

Since all jobs are ex ante identical, workers randomly apply to vacancies. Thus, the probability that a particular worker applies for a given vacancy is $1/\mathcal{V}$ and the number of applications for a vacancy, K, follows a binomial distribution:

$$Prob(K = k) = \binom{\mathcal{A}}{k} (1/\mathcal{V})^k (1 - 1/\mathcal{V})^{\mathcal{U}-k}$$

As both $\mathcal{A}, \mathcal{V} \to \infty$ with its ratio $q = \mathcal{A}/\mathcal{V}$ constant, the number of applicants per vacancy K converges to a Poisson distribution with mean q, what from now on we refer to as the queue length.

A worker is hired whenever they generate a profit for the employer which is greater than that of the other applicants for the same vacancy. To ease the exposition, we assume for now that the ranking of productivity of the applicants portrayed by the cumulative distribution function of the applicants, $G_A(\theta)$ is the same as the ranking of profits generated in the population of applicants, i.e., a Coincidence Ranking Equilibrium (CRE) holds. This is not guaranteed to actually be the case in any equilibrium, since wage determination may give a high weight to the worker's outside option, making high productivity types less attractive to employers. However, under our simple wage-setting mechanism, to be explained later, the applicant with the highest productivity yields the highest profit in equilibrium.

Under these assumptions, if an employer screens s applicants, the top candidate gets the offer with probability $(\phi G_A(\theta))^{s-1}$, where ϕ is the probability of interviewing an applicant, ex ante decided by the employers in a symmetric equilibrium.

The total number of screened applicants follows a binomial distribution. If k applicants apply for a job, the probability that a worker of type θ will be hired is

$$Prob(\theta \text{ hired}|k \text{ total applicants}) = \sum_{s=1}^{k} \binom{k}{s} (\phi G_A(\theta))^{s-1}(1-\phi)^{k-s+1} = (\phi G_A(\theta) + 1 - \phi)^{k-1}$$

Nevertheless, when workers apply, they ignore how many applicants are competing for the same job they applied to. Assuming that the number of applicants follow a Poisson distribution, the probability of being hired $p(G_A(\theta), q)$ is a Poisson–Binomial mixture.

$$\tilde{p}(G_A(\theta), q) = \sum_{k=1}^{\infty} \frac{e^{-q} q^{k-1}}{(k-1)!} (\phi G_A(\theta) + 1 - \phi)^{k-1} = e^{-\phi q (1 - G_A(\theta))} \tag{1}$$

Henceforth, we take $\phi = 1$ to ease our exposition and defer the explanation of the irrelevance of this assumption. Intuitively, the expected number of interviews ϕq is the relevant variable for both employers and applicants, regardless of the specific values ϕ and q take. Then, we can write the probability of being hired as a function of q and the applicant's ranking $x = G_A(\theta)$:

$$p(x) = e^{-q(1-x)} \tag{2}$$

The average probability of an applicant's being hired, $\mathbb{E}[\tilde{p}(\theta, q)|q]$ is therefore given by

$$\mathbb{E}[p(G_A(\theta), q)|q] = \overline{p}_A = \int \sum_{k=1}^{\infty} \frac{e^{-q} q^{k-1}}{(k-1)!} (\phi G_A(\theta) + 1 - \phi)^{k-1} \, dG_A(\theta) = \frac{1 - e^{-q}}{q} \tag{3}$$

3.2 Distributions

The recruiting selection process affects the distribution of unemployed and employed workers. In this section we show how this distribution is endogenously determined in steady state.

First, the exogenous density of types is a weighted average of the densities of the unemployed g_U and of the employed g_E, as follows.

$$g(\theta) = \mathcal{U} g_U(\theta) + (1 - \mathcal{U}) g_E(\theta)$$

In steady state, for a given type θ and queue length q, the flows in and out of unemployment must be equal, i.e.,

$$\tilde{p}(G_A(\theta), q) g_U(\theta) = \eta^* g_E(\theta)$$

where $\eta^* \equiv \eta(1-\lambda)$ is the effective separation rate once on-the-job searchers skip the separation shock as we assume that an on-the-job application occurs first than a separation shock within a period.

Combining both equations, we obtain that the population density of the type θ is weighted by its steady-state probability of unemployment, and scaled by the mass of the unemployed to ensure the expression integrates to 1.

$$g_U(\theta) = \frac{g(\theta)}{\mathcal{U}} \frac{\eta^*}{\eta^* + \tilde{p}(G_A(\theta), q)} \tag{4}$$

In a similar fashion, we can obtain the density of employed workers as

$$g_E(\theta) = \frac{g(\theta)}{1-\mathcal{U}} \frac{\tilde{p}(G_A(\theta), q)}{\eta^* + \tilde{p}(G_A(\theta), q)} \tag{5}$$

Since a fraction λ of the employed workers apply for jobs just as the unemployed do, the density of the applicants is

$$g_A(\theta) = \frac{g_U(\theta)\mathcal{U} + g_E(\theta)\lambda(1-\mathcal{U})}{\mathcal{U} + \lambda(1-\mathcal{U})}$$

which turns out to be a separable differential equation

$$\frac{dG_A(\theta)}{d\theta} = \frac{\eta^* + \lambda\tilde{p}(G_A(\theta), q)}{\eta^* + \tilde{p}(G_A(\theta), q)} \frac{1}{\mathcal{U} + \lambda(1-\mathcal{U})} g(\theta) \tag{6}$$

We make a change of variable and define the quantile of the distribution of applicants as $G_A(\theta) = x$. Since $G_A(\theta)$ is a cumulative distribution function, the boundary conditions are $G_A(\infty) = G(\infty) = 1$ and $G_A(0) = G(0) = 0$.

Using the boundary conditions, we can determine the value of the constant of integration and therefore the expression for the mass of the applicants, $\mathcal{A}$, and the unemployment rate, $\mathcal{U}$.[6]

$$\mathcal{A} = \frac{1}{1 + \frac{1-\lambda}{\lambda q} \log\left(\frac{\eta^* + \lambda}{\eta^* + \lambda e^{-q}}\right)} \tag{7}$$

[6] Using L'Hôpital's rule, we can show that the mass of the applicants converges to a well-known formula when there is no on-the-job search, i.e., when $\lambda \to 0$

$$\lim_{\lambda \to 0} \mathcal{U} = \frac{\eta^*}{\eta + \frac{1-e^{-q}}{q}}$$

where $\frac{1-e^{-q}}{q}$ is the average probability of being hired when there is no on-the-job search.

$$\mathcal{U} = \frac{\mathcal{A} - \lambda}{1 - \lambda} = 1 - \frac{1}{q} \log \left(\frac{\eta^* + \lambda}{\eta^* + \lambda e^{-q}} \right) \tag{8}$$

Equation (6) shows that there is a closed-form mapping between quantiles of the applicants distribution and the quantiles of the original population, given an equilibrium queue length q.

Using the transformation $x = G_A(\theta) \to G_A^{-1}(x) = \theta$, we therefore obtain

$$G^{-1}(M(x, q)) = G_A^{-1}(x) = \theta \tag{9}$$

with

$$M(x, q) \equiv \frac{m(x, q) - m(0, q)}{m(1, q) - m(0, q)} \text{ with } m(x, q) = x + \left(\frac{1 - \lambda}{\lambda q} \right) \log \left(\eta + \lambda e^{-q(1-x)} \right) \tag{10}$$

The result in Equation (9) is key to expressing the equilibrium conditions in a way that they do not depend on the unknown distribution $G_A(\theta)$, but rather on the distribution of the population's productivity $G(\theta)$, which is a primitive of the model.

We can also write down the cumulative distribution functions of the unemployed and the employed by realizing that the population is a weighted average of these two groups

$$G(\theta) = \mathcal{U} G_U(\theta) + (1 - \mathcal{U}) G_E(\theta)$$

In addition, the distribution of applicants equals

$$G_A(\theta) = \frac{\mathcal{U} G_U(\theta) + \lambda(1 - \mathcal{U}) G_E(\theta)}{\mathcal{A}}$$

Combining these two conditions and the quantile mapping in Equation 9, we obtain

$$G_U(\theta) = \frac{G_A(\theta) - \lambda G(\theta)}{(1 - \lambda)\mathcal{U}} = \frac{\mathcal{A}x - \lambda M(x, q)}{(1 - \lambda)\mathcal{U}} \tag{11}$$

$$G_E(\theta) = \frac{G(\theta) - \mathcal{A} G_A(\theta)}{(1 - \lambda)(1 - \mathcal{U})} = \frac{M(x, q) - \mathcal{A}x}{(1 - \lambda)(1 - \mathcal{U})} \tag{12}$$

We can also determine the average job finding probabilities for applicants, the unemployed and the on-the-job seekers. The last two provide a link between the model and empirical measurements. Thus, the average probability of an applicant's being hired is

$$\overline{p}_A = \int e^{-q(1 - G_A(\theta))} dG_A(\theta) = \int_0^1 e^{-q(1-x)} dx = \frac{1 - e^{-q}}{q}$$

By integrating over the equilibrium distribution of the unemployed, one can also compute the average probability for an unemployed person to be hired, that is, a UE transition[7]

$$\overline{p}_U = \int e^{-q(1-G_A(\theta))} dG_U(\theta) = \frac{\mathcal{A}\overline{p}_A - \lambda \int_0^1 e^{-q(1-x)} dM(x,q)}{(1-\lambda)\mathcal{U}}$$

The probability for an employed person to be hired is

$$\overline{p}_E = \int e^{-q(1-G_A(\theta))} dG_E(\theta) = \frac{\int_0^1 e^{-q(1-x)} dM(x,q) - \mathcal{A}\overline{p}_A}{(1-\lambda)(1-\mathcal{U})}$$

Therefore, the average job-to-job transition probability is $\lambda \overline{p}_E$.

Some algebra then shows that

$$\overline{p}_A = \frac{\mathcal{U}\overline{p}_U + \lambda(1-\mathcal{U})\overline{p}_E}{\mathcal{A}}.$$

Moreover, the unemployment rate can also be expressed as

$$\mathcal{U} = \frac{\eta}{\eta + \overline{p}_U}.$$

3.3 Workers

All unemployed workers receive an exogenous income $\rho\theta$ with $0 < \rho < 1$. An applicant randomly chooses a vacancy and faces an *equilibrium* job finding probability $\tilde{p}(\theta, q)$ which depends on the applicant's type and the queue length, as derived above. In case the applicant obtains the job, the worker gets the value of being employed $W(\cdot)$ earning a wage $w(\theta)$. If the applicant comes from unemployment and receives no offers, the applicant remains in this state and applies again the next period.

As previously noted, an employed worker applies to another job with an exogenous probability λ, actually switching with overall probability $\lambda\tilde{p}(\theta, q)$. Employed workers who do not move remain in their current job until the next period. If the workers do not apply to another job, they become unemployed with exogenous probability η. Hence, the effective separation probability is $\eta^* = (1-\lambda)\eta$.

Workers have linear preferences over consumption and have a constant discount factor $\beta \in (0, 1)$. Hence, an unemployed worker's lifetime utility is

$$U(\theta) = \rho\theta + \beta[\tilde{p}(\theta, q)W(\theta) + (1 - \tilde{p}(\theta, q))U(\theta)] \tag{13}$$

[7] The integral $\int_0^1 e^{-q(1-x)} dM(x,q)$ equals

$$\mathcal{A}\int_0^1 \frac{\eta^* e^{-q(1-x)} + e^{-2q(1-x)}}{\eta^* + \lambda e^{-q(1-x)}} = \mathcal{A}\left(\frac{1-e^{-q}}{\lambda q} - \frac{\eta^*(1-\lambda)}{q\lambda^2}\left(\log\left(\eta^* + \lambda\right) - \log\left(\eta^* + \lambda e^{-q}\right)\right)\right)$$

while for the employed agent, the value of being employed W depends on the potential value of a new job $\widetilde{W}$.

$$W(\theta) = w(\theta) + \beta[\lambda\left(\tilde{p}(\theta,q)\widetilde{W}(\theta) + (1-\tilde{p}(\theta,q))W(\theta)\right) + (1-\lambda)\left((1-\eta)W(\theta) + \eta U(\theta)\right)$$

3.4 Firms

A job filled with a worker of productivity θ generates a value $J(\theta)$ and a profit flow $\theta - w(\theta)$. After production, matched workers apply to another job with probability λ, in which case they are hired by another employer with probability $\tilde{p}(\theta,q)$. In this case, the original employer obtains the value of posting a vacancy again as described later, V.

If the worker does not apply to another job, the match is destroyed with exogenous probability η, in which case the employer obtains the value of posting a vacancy V described below. In case the on-the-job application and the separation shocks do not take place, the match goes on and the employer obtains the discounted profit flow next period. Hence, the value $J(\theta)$ is

$$J(\theta) = \theta - w(\theta) + \beta\left(\lambda\tilde{p}(\theta,q)V + (1-\lambda)\left(\eta V + (1-\eta)J(\theta)\right)\right) \tag{14}$$

Employers observe the aggregate state of the labor market and pay a fixed entrance cost χ when they open a vacancy if it is profitable to do so. This cost is related to securing financial funds, obtaining capital goods, or access to product markets in environments modeling aspects of other markets (Petrosky-Nadeau and Wasmer 2015; Petrosky-Nadeau and Wasmer 2017). After this, employers optimally create vacancies by paying a fixed flow cost κ. Vacancies simultaneously receive K applications drawn from the distribution of applicants $G_A(\theta)$, which will be defined below.This simultaneous hiring is a key departure from most of sequential search and matching models.

In addition, after receiving applications, the employer attaches a probability ϕ of interviewing or screening each applicant with marginal cost $\xi \geq 0$.[8]

After each costly interview, the employer perfectly learns the applicant's type θ. Due to this assumption, we focus on selection issues, leaving aside informational effects. The employer offers the position to the most profitable worker or posts the vacancy again. Thus, the value

[8]This decision could be contingent on the realized number of arrived applicants k, in which case an optimal hiring policy would set a cap on the number of interviewed candidates. This would assuage the hiring advantage high productivity applicants have, as does the ex ante hiring probability ϕ, at the cost of substantially decreasing tractability. Therefore, for the sake of simplicity, we only consider the case in which ϕ is constant and set before learning the realized number of applicants.

of posting a vacancy is

$$V = \max\left\{\max_{\phi}\left\{-\kappa + \beta(Prob(K>0)H(k) + Prob(K=0)V)\right\}, V\right\}$$

where $H(k)$ is the maximum profit obtained from a pool of $k > 0$ effectively screened applicants.

$$H(k) = \mathbb{E}_K\left[-\xi k + \max_j \left\{J(\theta_j)\right\}_{j=1}^{k} | k > 0\right]$$

3.5 Solving the Competitive Equilibrium

3.5.1 Solving the hiring problem

To construct the solution we assume (and show later) a Coincidence Ranking Equilibrium (CRE) in which the productivity and profitability rankings coincide, i.e., employers always prefer more productive types. Under CRE, we characterize the problem of (3.4) given a distribution of applicants $G_A(\theta)$ which is exogenous from the viewpoint of an individual employer. We note that employers never choose to reject all applicants and repost a vacancy since no applicant whose value to the firm is lower than V will ever submit a marginally costly application.

In Appendix B we show that the entry condition can be written as

$$\kappa + \beta\xi\phi q + \chi(1 - \beta e^{-\phi q}) = \beta\phi q\left(\int_0^1 J(G^{-1}(x,q))e^{-\phi q(1-x)}dx\right) \tag{15}$$

as long as the expected value of hiring an average worker is at least as high as the costs of interviewing and entering the market, i.e., $E[J(\theta)] > \xi + \chi$.

In general equilibrium, the queue length must satisfy the entry condition $V = \chi$, that is, an employer posts vacancies up to the point where the expected value of doing so exactly compensates for the opportunity cost of entry.

3.5.2 The choice of the hiring probability ϕ

The information we can retrieve from the data is the average number of interviews per vacancy $\tilde{q}$. This can be thought of as resulting from a choice of the firm, which decides to interview each applicant with probability ϕ: $\tilde{q} \equiv \phi q$, where q is the number of applications received per vacancy.

When we consider the optimal choice of vacancy opening, we realize that in fact ϕ and q always enter in a multiplicative way in the model, as we can see in Equation 16, where, for the reader's convenience, we show the expressions for $p(G_A(\theta))$ and $J(\theta)$.

$$V = \max_{\phi} -\kappa - \beta\xi\phi q(1 - e^{-\phi q}) + \beta \int_0^\infty J(\theta)\phi q p(G_A(\theta))d(G_A(\theta)) \tag{16}$$

$$J(\theta) = \frac{\theta(1-\rho) + \beta[\lambda p(G_A(\theta)) + \eta^*]\chi}{1 - \beta[1 - \eta^* - \lambda p(G_A(\theta))]} \tag{17}$$

$$p(G_A(\theta)) = e^{-\phi q(1-G_A(\theta))} \tag{18}$$

Since they always enter as a product in the model, we cannot distinguish between ϕq and $\tilde{q}$. Indeed, employers can set an average number of interviews per vacancy by either adjusting vacancy-posting or choosing the interview probability ϕ to achieve the desired level $\tilde{q} = \phi q$. For this reason, without loss of generality we fixe $\phi = 1$ henceforth, so that $\tilde{q} = q$.

3.6 Wages

Since we consider job-to-job transitions, the traditional Nash bargaining between the worker and the firm would pose some theoretical problems of non-uniqueness of the equilibrium, as discussed by Shimer (2006). Gottfries (2017) recently proposed a bargaining protocol, with infrequent renegotiation, to address the problems raised by Shimer (2006)[9], However, in order to keep the model as simple as possible, we decide to be conservative and adopt the assumption, widely used in the literature, that the firm has full bargaining power: to maximize profits, wages are set to make the employee indifferent between taking the job or their outside option.

Bargained wages are never turned down in equilibrium because: (1) the value of the match is solely determined by θ; (2) all employers are identical and therefore pay the same wages, and (3) workers only send one application per period. The Nash axiomatic solution solves the following problem.

$$\max_{w} J(w,\theta) - V$$
$$\text{s. t. } W(\theta) > U$$

Substituting Equations (3.3), (13) and (14), and using the free entry condition $V = \chi$ for an interior solution, the problem can be expressed as

[9] Gottfries (2017) shows that when wages can be continuously renegotiated, then the solution corresponds to the Nash equilibrium, as in Mortensen and Pissarides (1994), while the framework described in Shimer (2006) corresponds to the case in which wages cannot be renegotiated at all.

$$\max_{w} \frac{\theta - w(\theta) + \beta\chi[\lambda p(\theta) + \eta^*]}{1 - \beta[1 - \eta^* - \lambda p(\theta)]}$$
$$\text{s. t. } \frac{w - (1-\beta)U}{1 - \beta(1 - \eta^*)} \geq 0$$

and therefore by substituting Equation (13), the solution is simply

$$w(\theta) = \rho\theta \tag{19}$$

3.7 Inequality measures

Remarkably, in our framework we obtain a quasi-closed form for the equilibrium distribution, making the model amenable to studying comparative statics for inequality statistics. In particular, we construct a ratio to compare the outcome of the model in terms of the ranking gap for the employed distribution with the correspondent measure characterizing the population distribution: we thus compare an indicator of *ex post* inequality with an indicator of *ex ante* inequality. If a Walrasian market prevails ex ante inequality would reflect one-to-one in wage inequality. Therefore, this ratio would be interpreted as the amplification or compression of the wage distribution due to search and selection frictions.

We consider two levels of productivity ($\underline{\theta}$ and $\overline{\theta}$) that correspond to two percentiles of the applicants' distribution ($\underline{x}$ and $\overline{x}$), for example the $\overline{x} = 90^{th}$ and the $\underline{x} = 10^{th}$ percentiles: we therefore have that $\overline{\theta} = G_A^{-1}(\overline{x}) = G^{-1}(M(\overline{x}; q))$ and $\underline{\theta} = G_A^{-1}(\underline{x}) = G^{-1}(M(\underline{x}; q))$. We then compute the ratio between the gap of the c.d.f. evaluated at these levels for the employed agents with respect to the gap evaluated for the entire population:

$$\frac{G_E(\overline{\theta}) - G_E(\underline{\theta})}{G(\overline{\theta}) - G(\underline{\theta})} = \frac{G_E(G^{-1}(M(\overline{x}; q))) - G_E(G^{-1}(M(\underline{x}; q))}{M(\overline{x}; q) - M(\underline{x}; q)}$$

.

If this ratio is bigger than one, then, for the chosen percentiles, and for an equilibrium queue length, the hiring process in the labor market magnifies the pre-existing level of inequality observed in the population expressed as the ranking gap, and attenuates it if the ratio is smaller than one. To characterize this ratio, we apply the Cauchy Mean Value Theorem and obtain the following proposition.

Proposition 1 ***Quantile amplification:*** *The quantiles of the employed distribution and the population distribution are related through*

$$\frac{G_E(\overline{\theta}) - G_E(\underline{\theta})}{G(\overline{\theta}) - G(\underline{\theta})} = \frac{s_E(x_c; q)}{s_E(x^*; q)} \tag{20}$$

in which $s_E(x;q)$ denotes the probability of employment of the type with ranking x given a certain queue length q, i.e.,

$$s_E(x;q) = \frac{p(x;q)}{\eta^* + p(x;q)};$$

x_c denotes the applicants' ranking associated to the Cauchy Mean Value (CMV) of productivity θ_c, i.e., $x_c = G_A(\theta_c)$, with $\theta_c \in (\underline{\theta}, \overline{\theta})$. Finally, x^* denotes the ranking of the type that defines the average applicants' ratio, i.e., $x^* = G_A(\theta^*)$ is such that

$$\mathcal{A}(q) = \frac{\eta^* + \lambda p(x^*;q)}{\eta^* + p(x^*;q)}$$

Proof. By the Cauchy Mean Value Theorem, we have

$$\frac{G_E(\overline{\theta}) - G_E(\underline{\theta})}{G(\overline{\theta}) - G(\underline{\theta})} = \frac{g_E(\theta_c)}{g(\theta_c)} \text{ with } \theta_c \in (\underline{\theta}, \overline{\theta})$$

in which θ_c is referred to as the Cauchy Mean Value (CMV).

Using the definitions of the density of the employed, the previous expression can be written as

$$\frac{g_E(\theta)}{g(\theta)} = \frac{p(G_A(\theta_c);q)}{\eta^* + p(G_A(\theta_c);q)} \frac{1}{1 - \mathcal{U}(q)} \text{ with } \theta_c \in (\underline{\theta}, \overline{\theta})$$

where $x_c = G_A(\theta_c)$ is the ranking in the applicants' distribution of CMV type θ_c.

On the other hand, the aggregate level of employment $1 - \mathcal{U}$ can be written in terms of the mass of applicants $\mathcal{A}$ as

$$1 - \mathcal{U}(q) = \frac{1 - \mathcal{A}(q)}{1 - \lambda}$$

We can express the density function of the applicants' distribution as the sum of the unemployed distribution and a share λ of the employed. Hence,

$$g_A(\theta) = \frac{g(\theta)}{\mathcal{U}(q) + \lambda(1 - \mathcal{U}(q))} \frac{\eta^* + \lambda p(G_A(\theta);q)}{\eta^* + p(G_A(\theta);q)}$$

where we define $s_A(G_A(\theta)) \equiv \frac{\eta^* + \lambda p(G_A(\theta);q)}{\eta^* + p(G_A(\theta);q)}$.

By integrating the density of the applicants, we obtain

$$\mathcal{A}(q) = \int_0^1 s_A(G_A(\theta);q) dG(\theta)$$

By the mean value theorem for integrals, there exists a value $\theta^* \in (0, \infty)$ such that

$$\mathcal{A}(q) = s_A(G_A(\theta^*))$$

Remembering that $x \equiv G_A(\theta)$, we can therefore write

$$\mathcal{A}(q) = s_A(G_A(\theta^*)) = s_A(x^*)$$

where x^* is the average applicant type.

We then substitute the expression for the applicants' mass in the definition of the employment rate:

$$\begin{aligned} 1 - \mathcal{U}(q) &= \frac{1 - \mathcal{A}(q)}{1 - \lambda} \\ &= \frac{1 - s_A(G_A(\theta^*))}{1 - \lambda} = \frac{1 - \frac{\eta^* + \lambda p(G_A(\theta^*);q)}{\eta^* + p(G_A(\theta^*);q)}}{1 - \lambda} = \frac{p(G_A(\theta^*);q)}{\eta^* + p(G_A(\theta^*);q)} = s_E(x^*;q) \end{aligned}$$

Using the latter, we obtain the result stated in Equation (20). ■

Proposition 1 states that the ranking gap for types $(\underline{\theta}, \overline{\theta})$ is amplified by the functioning of the labor market if the average applicant type θ^* is less than θ_c, which obviously can also be stated in terms of the applicants' rankings $x_c > x^*$; in the opposite case, the ranking gap is compressed. In other words, when comparing two types, the hiring selection in the labor market amplifies the ranking gap for individuals whose productivity is higher than the average applicant, and does the opposite for those whose productivity is below that point. Loosely speaking, the hiring selectivity generates an employed distribution with a thickened left tail and a stretched right tail in comparison to the population distribution. Thus, selectivity generates positive skewness, as often occurs in empirical wage distributions.

In terms of the consequences for inequality of the hiring process in the labor market, the model is thus able to generate heterogeneous effects, according to the productivity level. This conclusion is valid for a given queue length and therefore unemployment rate. Any changes in the equilibrium value of q imply a change in the value of the ranking gap ratio, for any chosen percentiles. In Section 4.4 we will provide some numerical illustration of the heterogenous consequences in terms of inequality for different percentiles, considering the steady state equilibrium. In Section 5, we will show numerically how the ranking gap is affected when the unemployment rate changes, as a consequence of some exogenous parameter shift.

4 Calibration and Results

Once we solved the model, we bring it to the data to estimate some of the parameters, and test its empirical performance. Our strategy consists on targeting monthly CPS and CPS-ORG statistics such as the unemployment rate, the job-to-job transition probability (EE), the separation rate (EU), and some of the moments of the wage distribution. We aggregate the data that at the state-year level as in Section 2 and assume that the parameters are time-invariant. Moreover, in our model, agents with different levels of productivity compete for the same type of jobs. As also explained in Section 2, we consider two separate labor markets, one for college and the other for non-college workers. We also assume that the population productivity θ is lognormal, i.e. $\theta \sim$ lognormal (μ, σ^2).

Once we obtain the estimated values, we use the equilibrium entry condition (Equation 15) to calibrate some extra parameters. As will be shown in Section 4.3, we calibrate the parameters β, the discount factor; ξ, the screening cost per applicant; and κ, which we interpret as the cost of posting a vacancy (online). With the calibrated and estimated values of the parameters, we obtain the fixed entry cost, χ, from the equilibrium entry condition.

4.1 Estimation procedure

We use a Generalized Method of Moments (GMM) approach: we look for those values of the parameters that minimize the difference between the moments of the data and those implied by the model. It is important to note that for our steady state calibration, we also need to estimate the queue length, even if this is not a parameter but an endogenous variable of the model. Once we obtain the estimated values of the parameters of interest, we perform some counter-factual experiments: in these cases, the value of q changes endogenously.

The set of parameters we need to estimate is given by $\Psi = \{\eta, \lambda, \mu, \sigma, \rho, q\}$ where η is the exogenous match destruction rate (remember that the overall separation rate is given by $\eta^* = \eta(1 - \lambda)$), λ is the probability of applying for a job while working, while μ and σ are the parameters characterizing the lognormal productivity distribution of the population, and ρ is the proportion of the productivity that the worker receives when unemployed, and q is the length of the queue length in steady state equilibrium. Since in our setting we do not explicitly model labor market institutions such as unemployment benefits, the coefficient ρ is an overall measure of the outside option of the workers, including income support such as unemployment insurance.

For what regards the wage distributions, we compare the predictions of the model with the data in terms of the level of the wage for a selection of percentiles, in particular the 10th, the 25th, the 50th, the 75th and the 90th. Since we solve the model in terms of the applicants' cumulative distribution function, the predictions of the model are therefore made in terms of the applicant's percentiles. We therefore transform the empirical wage distribution to obtain the percentile of the applicants' distribution that corresponds to the observed wage distribution.

Using the definition of the density of employed, as we did in (12), we can write

$$G_E(G^{-1}(M(x;q,\eta,\lambda))) = \frac{M(x;q,\eta,\lambda) - \mathcal{A}x}{(1-\lambda)(1-\mathcal{U})}$$

Let us write $\tilde{G}_w(.)$ for the empirical cumulative distribution function of observed wages. The cumulative distribution function of the employed $G_E(.)$ is unknown, but we can approximate it using the empirical wage c.d.f.: $G_E(G^{-1}(M(x;q,\eta,\lambda))) \approx \hat{G}_w(\hat{w}(x)) = \frac{M(x;q)-\mathcal{A}x}{(1-\lambda)(1-\mathcal{U})}$. Thus if we want to obtain the level of wages implied by the observed wage distribution for a certain percentile x of the applicants' distribution, we just need to take the inverse function of the expression for $\hat{G}_w$:

$$\hat{w}(x) = \widehat{G}_w^{-1}\left(\frac{M(x;q,\eta,\lambda) - \mathcal{A}x}{(1-\lambda)(1-\mathcal{U})}\right) \tag{21}$$

In our model, the wage is simply the reservation productivity of the worker, i.e., $w(\theta) = \rho\theta = \rho G^{-1}(M(x;q,\eta,\lambda))$. To find the parameters' values, we thus solve the following minimization problem:

$$\min_{\eta,\lambda,\mu,\sigma,\rho,q} Q = \Bigg\{\varphi_1 \sum_{i=1}^{N} \frac{1}{N}(\mathcal{U}(q,\eta,\lambda) - \mathcal{U}_i)^2 + \varphi_2 \sum_{i=1}^{N} \frac{1}{N}(\lambda\overline{p}_E(q,\eta,\lambda) - JJ_i)^2 + \varphi_3 \sum_{i=1}^{N} \frac{1}{N}(\eta(1-\lambda) - EU_i)^2$$
$$+ \varphi_4 \sum_{x\in N_x} \frac{1}{5}\left(\rho G^{-1}(M(x;q,\eta,\lambda)) - \hat{w}(x)\right)^2 \Bigg\}$$

where N represents the number of observations by state and by year in the CPS data[10], while $N_x = \{0.1, 0.25, 0.5, 0.75, 0.9\}$. The parameters ϕ_1, ϕ_2 and ϕ_3 are all set to one, while the parameter ϕ_4 is set to 1% to take into account the different scale of the variable (hourly wages are expressed in dollars, while the first three variables are rates).

4.2 Results

Table 1 shows the results of our estimation of the benchmark model. The results in terms of the exogenous separation rate η and the poaching probability λ are in line with standard values in the literature. The estimated values for the parameters μ and σ imply a higher mean productivity for college workers, as expected, together with a higher variance. The parameter ρ is a rough measure that includes the value of home production and unemployment benefits, and it is also quite in line with standard calibration values.

Table 2 compares the informative moments from the data with the ones obtained by simulating the model. Figure 5 shows the actual and simulated cumulative distribution functions for log wages.

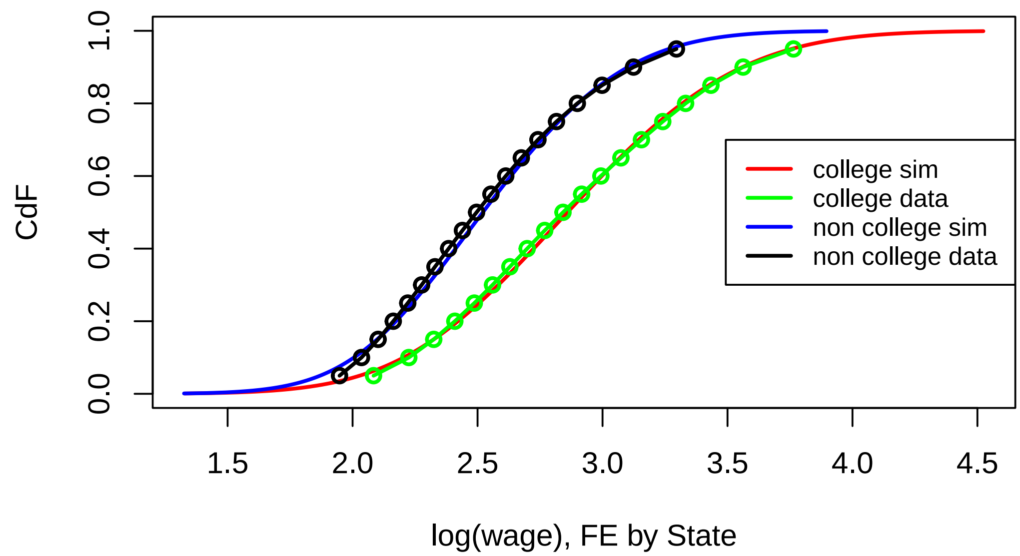
Table 1: Parameters' baseline estimation

Parameter	College	Non-College
η	0.008	0.018
λ	0.044	0.052
q	2.386	2.751
μ	3.749	3.369
σ	0.512	0.410
ρ	0.468	0.508
Min fun	0.310	0.774

Table 2: Data vs. Model generated moments, baseline estimation

Statistic	College	Non-College
Unempl. rate data (%)	2.599	6.412
Unempl. rate model (%)	2.607	6.417
Job-to-job trans. data (%)	1.923	2.319
Job-to-job trans. model (%)	1.916	2.310
Separation rates data (%)	0.742	1.718
Separation rates model (%)	0.734	1.710
Median wage data ($)	17.148	12.135
Median wage model ($)	17.458	12.442
Mean wage data ($)	19.572	13.495
Mean wage model ($)	20.305	13.880
90th/10th log wage data	1.337	1.088
90th/10th log wage model	1.355	1.096

Figure 5: Cdf of baseline log wages



In order to assess the goodness of the fit, we present the mean square errors (MSE) for the four types of moments we used in the estimation. For each variable x, let us define x_i to be the observations by state and year, $\bar{x}$ the simple average, and $\hat{x}$ the simulated value according to the model using the estimated values for the parameters.

For the unemployment rate, separation rates and job-to-job transitions, we compute the MSE in the standard way:

$\bar{Q}_1 = \Sigma_i(\mathcal{U}_i - \overline{\mathcal{U}})^2; \bar{Q}_2 = \Sigma_i(JJ_i - \overline{JJ})^2; \bar{Q}_3 = \Sigma_i(EU_i - \overline{EU})^2$

$\hat{Q}_1 = \Sigma_i(\mathcal{U}_i - \widehat{\mathcal{U}})^2; \hat{Q}_2 = \Sigma_i(JJ_i - \widehat{JJ})^2; \hat{Q}_3 = \Sigma_i(EU_i - \widehat{EU})^2$

For the moments of the wage distribution, it is important to remark that the data are not "model independent," since we used the transformation of equation 21.

4.3 Closing the model

In the previous section, we estimated the values of the parameters of interest and of the queue length, distinguishing between the two types with different levels of education, $j = \{\text{college, non-college}\}$. We now proceed to recover the fixed entry cost χ, by using the general equilibrium condition given by Equation 15. In order to do so, we need additional evidence

[10] N=1308 for college and N=1326 for non-college.

Table 3: Mean Square Errors, baseline estimation

MSE	College	Non-College
$\bar{Q}_1$ (data)	0.139	0.627
$\hat{Q}_1$ (model)	0.140	0.627
$\bar{Q}_2$ (data)	0.024	0.031
$\hat{Q}_2$ (model)	0.024	0.031
$\bar{Q}_3$ (data)	0.009	0.023
$\hat{Q}_3$ (model)	0.009	0.023
$\bar{Q}_4$ (data)	0.134	0.086
$\hat{Q}_4$ (model)	0.137	0.093

on the screening costs ξ and the vacancy posting costs κ.

We follow Villena-Roldan (2010a) and consider the National Employer Survey 1997 (NES97) to compute the average monetary cost incurred in that specific year for recruiting activities. To do this, we use the following questions of the survey:

Q29: What percent of total labor costs is spent annually on the recruitment and selection of employees?

Q3: What was the total labor cost used in the production of your 1996 sales?

Q30A: How many people have you hired in the past two years?

Q41: How many candidates do you interview for each [JOB TITLE] opening?

Therefore, using these responses, the annual recruitment and selection cost (RSC) in 1996–1997 is computed as $RSC = Q29/100 \times Q3$. The total number of interviews over the last two years is $NIN = Q30A/2 \times Q41$, assuming each position is filled after interviewing $Q41$ applicants on average. Hence, an estimate for the average recruiting cost is $\xi = RSC/NIN$. Indeed, RSC could be interpreted as the total amount spent including fixed costs. Nevertheless, there is a substantial non-response rate in the survey. Only 48.2% (1486 out of 3081 respondants) answered the four questions we need to compute ξ. We then regress the observed ξ on a set of categorical variables for size, industry and multi-establishment firm, using the weights of the survey. Since these variables are observed for all surveyed employers, we predict the value non-respondents would have declared. The actual distribution of ξ and the one with imputed values are highly skewed. The results are presented in Table 4.

Given the influence that outliers and measurement error may have on the averages, we may consider medians as more reliable measures for ξ in 1997.

Table 4: Measurement of screening cost ξ (NES 1997)

	Mean (USD 1997)	Median (USD 1997)
Actual ξ	1172	80
ξ with imputations	1337	193

How can we compute values for ξ for other years? We adapt the idea of Landais, Michaillat, and Saez (2018), who use some NES information reported by Villena-Roldan (2010a). Instead of measuring the share of workers, we use the average hourly wage of workers in areas related to Human Resources Departments.

To compute the recruiting cost ξ, we assume there is a proportionality, given by an adjustment factor ζ, between screening costs and recruiters' wages at any moment in time, such that $\xi_{1997} = \zeta \times \text{wage recruiters}_{1997}$. The recruiters' wage in CPS-ORG data is 13.57 USD per hour, using the median of the NES97 screening cost adjusted by non-response, we compute an adjustment factor $\zeta = 14.2$.

The vacancy posting cost κ can be considered as the cost of keeping an online job post for one month[11] current costs oscillate between 200 USD and 400 USD, so we consider an average value of 300 USD.[12]

Considering the mean of screening costs ξ over the sample period, and the vacancy posting costs κ, we then use the free entry condition to obtain the value of the fixed entry cost χ:

$$\kappa + \beta\xi\phi\hat{q} + \chi(1 - e^{-\phi\hat{q}}) = \beta \int_0^1 J(G^{-1}(M(x; \hat{q}, \hat{\eta}, \hat{\lambda}, \chi)))\phi\hat{q}e^{-\phi\hat{q}(1-x)}dx$$

The values for χ we obtain for our benchmark case for college and non-college are, respectively, 256,970 USD and 147,832 USD. The amount of fixed cost are quantitatively important, considering that the average hourly wage for college and non-college workers imply respectively an average annual income (supposing full time work) of approximately 36,000 USD and 25,000 USD.

4.4 The ranking gap

Once the baseline steady state of the model has been characterized, in this section we provide a numerical illustration of the ranking gap, i.e., of the amplification/contraction effect of the

[11] Since our data start from 1994, we can make the hypothesis that job posting is mainly done through the Internet.

[12] For example, https://www.glassdoor.com/employers/blog/how-much-it-costs-to-post-a-job-online/

functioning of the model on inequality that was illustrated in Section 3.7.

The ranking gap can be expressed as

$$\frac{G_E(\overline{\theta}) - G_E(\underline{\theta})}{G(\overline{\theta}) - G(\underline{\theta})} = \frac{s_E(x_c; q)}{s_E(x^*; q)}$$

Tables 5 and 6 present the values for the amplification/contraction factor, as well as the corresponding percentiles of the Cauchy Mean Value (x_c) and the percentile corresponding to the average applicant (x^*).

Considering college workers, for applicants' types above the percentile 0.458, the ranking gap is amplified with respect to the population, while for non-college workers, the amplification happens for applicants' types above the percentile 0.495.

Table 5: Ranking gap, baseline steady state, College

	$\frac{G_E(\overline{\theta})-G_E(\underline{\theta})}{G(\overline{\theta})-G(\underline{\theta})}$	x_c	x^*
$\overline{x} = 0.9, \underline{x} = 0.1$	1.00048	0.46560	0.45785
$\overline{x} = 0.9, \underline{x} = 0.5$	1.01061	0.67388	0.45785
$\overline{x} = 0.5, \underline{x} = 0.1$	0.98647	0.28069	0.45785

Table 6: Ranking gap, baseline steady state, Non-College

	$\frac{G_E(\overline{\theta})-G_E(\underline{\theta})}{G(\overline{\theta})-G(\underline{\theta})}$	x_c	x^*
$\overline{x} = 0.9, \underline{x} = 0.1$	0.99915	0.49054	0.49532
$\overline{x} = 0.9, \underline{x} = 0.5$	1.02605	0.67832	0.49532
$\overline{x} = 0.5, \underline{x} = 0.1$	0.95288	0.28769	0.49532

5 Counterfactual experiments

In this section, we perform some counterfactual experiments to explore the mechanisms of the model and to explain the empirical findings of Section 2. We consider three exogenous changes to the model's parameters: *i)* an increase in the mean of the exogenous distribution of productivity that preserves the spread, and *ii)* an increase in the variance of the exogenous distribution of productivity that preserves the mean. Finally, *iii)* is a combination of both cases.

While individual wages are, in our model, just proportional to the level of productivity of the worker, an exogenous change in the population distribution of productivity also affects the distribution of wages (or the distribution of types for employed workers) through the functioning of the labor market. In particular, the hiring process determines the endogenous distribution of productivity of the employed workers.

In Appendix D, we present the results of additional experiments, such as an increase in the exogenous probability λ of poaching and an increase in the screening costs ξ. For each case, we keep all other parameters fixed and compute the equilibrium queue length (q) that satisfies the entry condition (Equation 15).

5.1 The effects of changes in average productivity

We start by focusing on the effects of shifts in the productivity distribution that leave its variance unchanged. Assuming that productivity follows a log-normal distribution, i.e., $\theta \sim$ lognormal (μ, σ^2), we consider an exogenous change in μ and compute the value of σ that keeps the variance constant.[13]

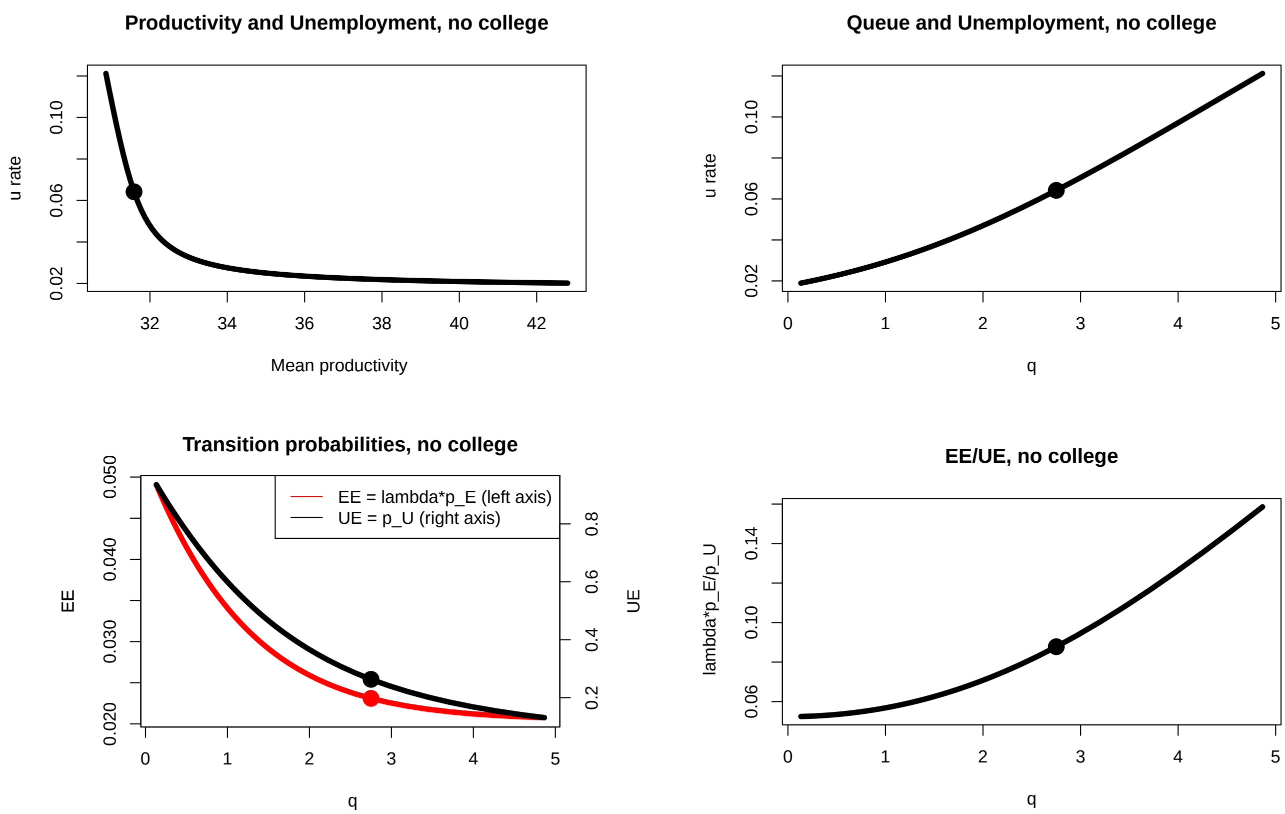
The increase in average productivity is beneficial for the firm, which therefore opens more vacancies. The length of the queue and therefore the unemployment rate decrease monotonically as average productivity increases, as we can see in the top panels of Figure 6, where the dots represent the benchmark equilibrium values. For brevity, here we show the results of the counterfactual experiment for non-college workers. The results are qualitatively very similar for college workers, too, as shown in Appendix E.

As productivity increases (and therefore as the queue length decreases), both the job-to-job (EE) and the unemployment-to-employment (UE) transition rates increase, as can be seen in the bottom-left panel of Figure 6. Moreover, the bottom right panel of Figure 6 shows that when the queue length decreases, the unemployed agents gain relatively more in terms of transitions (the ratio EE/UE decreases as the length of the queue decreases, i.e., with the reduction in unemployment).

Figure 7 shows the consequences for wages of a spread-preserving increase in the mean of the productivity distribution. The left panel of Figure 7 shows that, in general, mean productivity is positively related to the mean log wage. However, if one starts from a very low level of mean productivity, the first effect of a positive shift in average productivity is a decrease in the average wage. By looking at the right panel of Figure 7, this initial reduction in wages is related to high unemployment rates in the right portion of the right panel. As we move

[13] The mean of a variable following a lognormal distribution of parameters (μ, σ^2) is given by $\exp(\mu + \frac{\sigma^2}{2})$ and the variance is $[\exp(\sigma^2) - 1] \exp(2\mu + \sigma^2)$.

Figure 6: The effects of an increase in average productivity on unemployment and transition probabilities

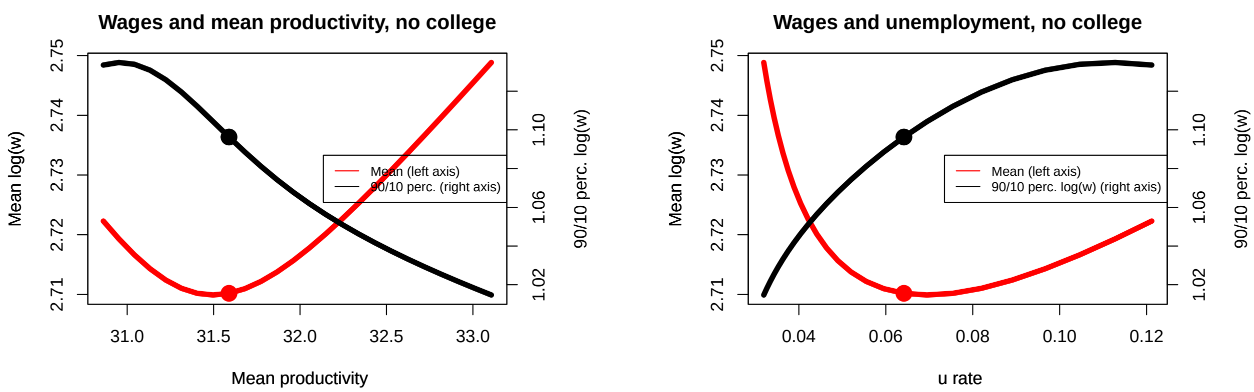


The dots represent the values for the benchmark calibration and estimation exercise.

from right to left, unemployment decreases, and so do average wages. In our numerical simulation, the steady-state equilibrium unemployment is almost at the level associated with the minimum level of the average wage. Starting from the initial steady state level (the red dot), further decreases in unemployment rates, driven by the increase in mean productivity, produce an increase in the average wage.

How to explain these results? The change in average wage is driven by a composition effect: individual wages in our model just reflect the productivity of the employed agents, so the average wage reflects changes in the distribution of the employed agents as well as changes in the wage function. When the unemployment rate is very high (because the average productivity is low), the relatively few employed people are concentrated among the most productive. This occurs because employers receive a large number of applicants and are able to hire very selectively. The left panel of Figure 8 shows that the densities of employed and unemployed workers are substantially different when the unemployment rate is high (solid lines).

Figure 7: The effects of an increase in average productivity on wages

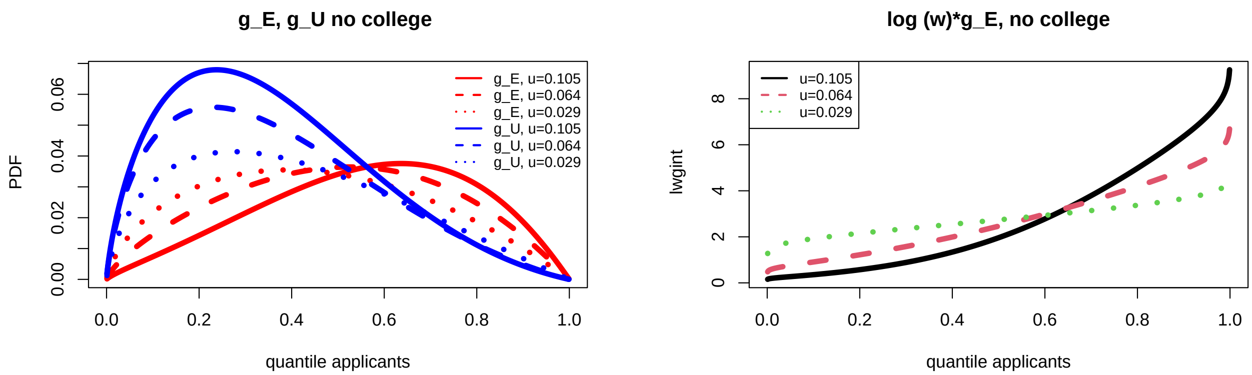


The dots represent the values for the benchmark calibration and estimation exercise.

When average productivity increases, vacancy postings increase. Because the unemployed are both numerous and of low productivity, most new hires come from this pool. As a consequence, the unemployment rate decreases and the average wage of the employed decreases, as shown in the right part of the right panel of Figure 7. In line with this, the density functions of the employed and unemployed become more similar, as can be seen in the left panel of Figure 8. Starting from low levels of productivity, the composition effect dominates the improvement in average productivity, driving the average wage down. If we start with a more moderate unemployment rate, the increase in productivity effect dominates, driving the average wage up because the unemployed are not that abundant and their productivity distribution is not that far from that of the employed. The economy is not that selective in this case, attenuating the composition effect. The right panel of Figure 8 conveys this intuition in a different way: the weighted log wage for high-productivity workers is much higher when the economy becomes very selective with a high unemployment rate and large queue lengths.

Wage inequality, measured by the 90–10 percentile gap for log hourly real wages, also has a non-monotonic relation with average mean productivity. If the mean productivity is quite low, an increase in it implies a reduction in the unemployment rate and an increase in wage inequality. However, beyond some point, as mean productivity increases, inequality decreases. The overall result in terms of the relation between the unemployment rate and wage inequality is thus a concave function, as can be seen in the right panel of Figure 7.

Figure 8: The effects of an increase in average productivity on wages and mass functions



When the unemployment rate is high, hiring is very selective, and the relatively few workers are concentrated among the most productive. As average productivity increases and unemployment starts to decrease, the newly hired from the unemployment pool are not only relatively less productive (the average wage initially decreases), but therefore also relatively more diverse in terms of productivity, implying a larger degree of inequality in wages. As the effect of the positive shift in productivity distribution gains, not only does the average wage increase but also wage inequality decreases. The pool of applicants becomes more homogeneous because the economy becomes less selective, as is clear from comparing the employed and the unemployed productivity distributions at different levels of the unemployment rate in the left panel of Figure 8.

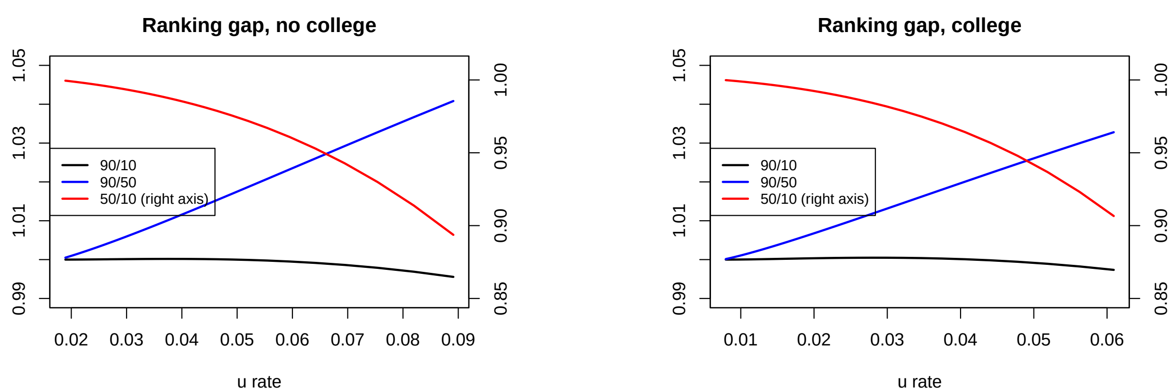
As we saw in Section 3.7, the model predicts a heterogeneous effect of the functioning of the labor market in terms of amplifying or compressing the *ex ante* ranking gap among different percentiles of the distribution. The ranking gap, using equations (9) and (12), is expressed as

$$\frac{G_E(\overline{\theta}) - G_E(\underline{\theta})}{G(\overline{\theta}) - G(\underline{\theta})} = \frac{M(\overline{x};q) - M(\underline{x};q) - \mathcal{A}(\overline{x} - \underline{x})}{(1-\lambda)(1-\mathcal{U})(M(\overline{x};q) - M(\underline{x};q))}$$

can therefore be higher or lower than one. In particular, it is lower than one in the left tail of the distribution and higher than one in the right one, as was illustrated in Section 4.4. Here we plot the values for the ranking gaps obtained for three different sets of percentiles of the applicants' distribution (90–10, 90–50 and 50–10), to show the general equilibrium effects of a change in the unemployment rate, driven by an increase in the mean of the population distribution.

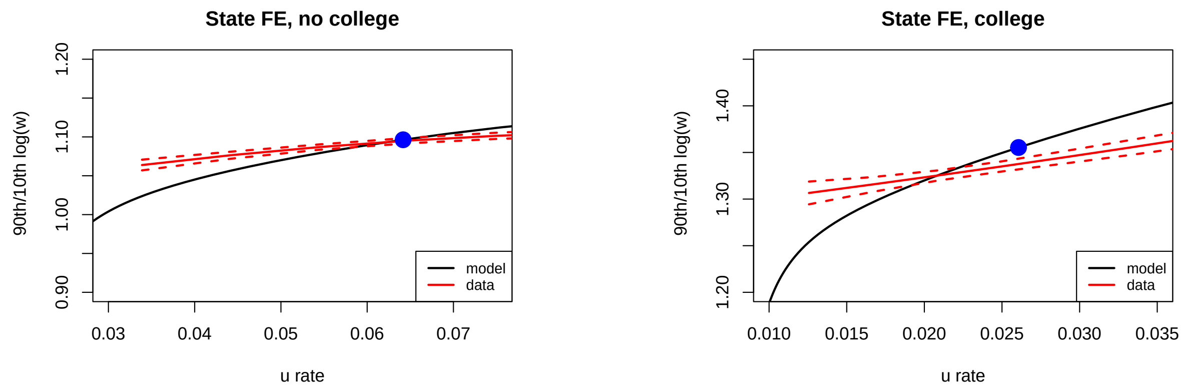
Figure 9 shows the heterogenous effects of the unemployment rate on the ranking gap according to the chosen portion of the distribution: the ranking gap, already less than one, further decreases in the left tail of the distribution (i.e., for the 50–10 percentiles), while it increases with the unemployment rate for the right tail of the distribution (the 90–50 percentiles). In other words, as unemployment increases, the left tail of the employed distribution is even more compressed than the population distribution, while the opposite happens in the right tail. The overall quite flat effect for the combination of the 90th and 10th percentiles is thus the result of very different effects on the two tails of the distribution.

Figure 9: The effects of an increase in average productivity on the ranking gap



Finally, Figure 10 compares the results of the model in terms of wage inequality with the data. The model is calibrated to quantitatively match the blue dot with its counterpart in the data. The variability in the data is therefore quite well captured by the changes in steady-state combinations of unemployment rate and wage inequality if the exogenous driver is considered a shift in the productivity distribution of the population. For college workers, the model clearly overstates the reaction of wage inequality to changes in mean productivity.

Figure 10: The effects of an increase in average productivity on unemployment and wage inequality: Model and data



The dots represent the values for the benchmark calibration and estimation exercise.

5.2 The effects of a mean-preserving spread in productivity

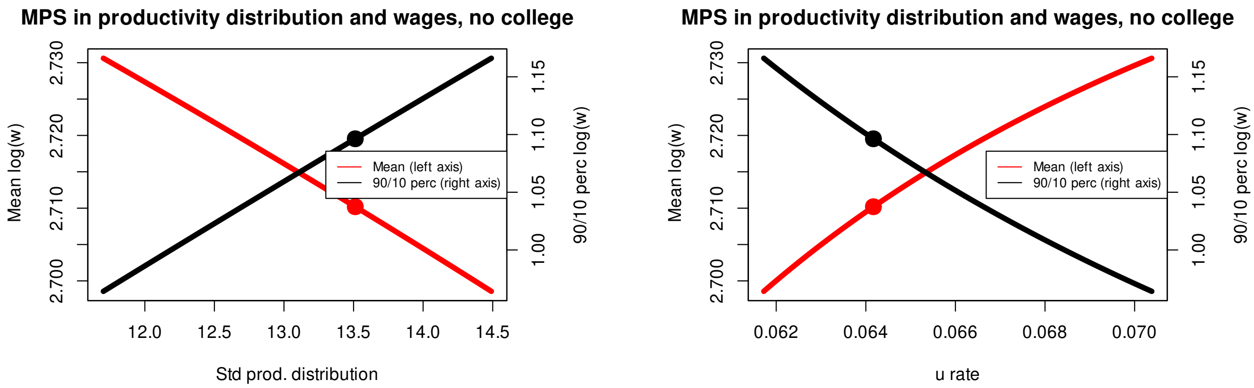
In this section, we show the effects of a mean-preserving spread in the exogenous distribution of productivity. We consider exogenous changes in parameters μ and σ that imply a change in variance, while keeping constant the mean of the population distribution. As expected, a higher variance in the distribution of productivity is reflected, in equilibrium, in a higher variance in terms of wages, as can be seen in the left panel of Figure 11.

In the same graph, we see that our model also generates an inverse relation with the average log wage, even though quantitatively the impact is quite small. With an increasing standard deviation of the productivity distribution, the unemployment rate decreases too, but only very slightly. The overall relation between the unemployment rate and wage inequality (right panel of Figure 11), driven by an exogenous mean-preserving spread in the distribution of productivity, is thus a decreasing one. Through the lens of the model, this kind of shock cannot generate the relation between unemployment and inequality seen in the data.

5.3 The effects of changes in both mean and variance of productivity distribution

The counterfactual exercises on the effects of changes in only the mean or only the variance of the population productivity distribution showed that: *(i)* an increase in the mean implies a U-shape relation between the average productivity and the average wage, as well as a concave relation between the unemployment rate and wage inequality; *(ii)* an increase in the standard deviation of the population productivity distribution implies a slight reduction in the average wage and an overall decreasing relation between unemployment and wage inequality.

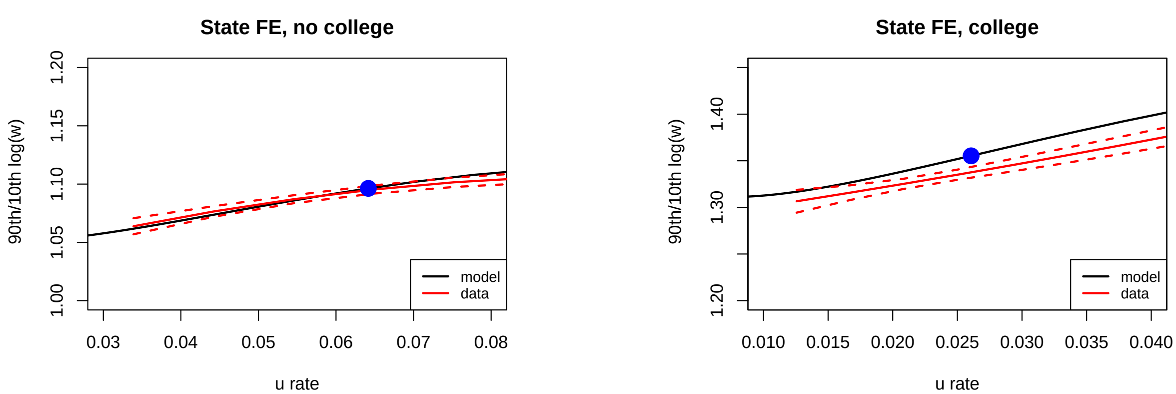
Figure 11: The effects of a mean-preserving spread in productivity on wages



The dots represent the values for the benchmark calibration and estimation exercise.

Our model, driven by an increase in average productivity, can reproduce the relation between the unemployment rate and wage inequality, but it also overstates it, as was shown in Figure 10. We do not claim that only changes in the average production distribution drive the relation between unemployment and wage inequality, but we are rather interested in proposing potential mechanisms. We therefore considered at the same time changes in the mean and in the standard deviation of the productivity distribution. In this counterfactual exercise, we vary the value of the parameter μ of the lognormal productivity distribution; an increase in μ thus implies at the same time an increase in the average wage and in the variance of the distribution. The results are shown in Figure 12: the combination of both shocks is better suited to explain the overall trend observed in the data. In the obtained counterfactual result, the increase in average productivity prevails, but the associated increase in the variance of the population distribution attenuates the main effect.

Figure 12: The effects of changes in both average and spread of productivity



6 Efficiency Analysis

As is common in matching models, there are many Pareto-constrained allocations. Indeed, it is not possible to switch a given employed worker with an unemployed one without hurting the former. We focus on allocations that maximize aggregate welfare or production since workers are risk neutral, following the tradition of Hosios (1990). The Social Planner (SP) faces the same constraints as private agents, in particular the same recruiting and screening technologies. The SP may instruct firms to post vacancies and workers to apply. The control of screening effort is embedded into the decision about the optimal q, because the planner can affect the effective queue length by changing the probability of screening ϕ or by posting vacancies and changing q, i.e., $q = \phi\tilde{q}$.

The matching process faced by the firms instructed by the SP is still characterized by the same risk of the decentralized economy: it is possible that the firm does not receive any applications at all. Since workers are sending their applications to firms, the number of applications received K still follows a Poisson distribution with a mean of $q = \mathcal{A}/\mathcal{V}$. Therefore, the probability of matching at least one applicant is $1 - e^{-q}$. Thus, the Social Planner's objective is

$$Y = \max_{q,\mathcal{V}} \left\{ \beta(1-\mathcal{U}) \int_0^\infty \theta g_E(\theta) d\theta + \beta\mathcal{U} \int_0^\infty \rho\theta g_U(\theta) d\theta - \underbrace{[\kappa\mathcal{V} + (1 - e^{-q})\chi\mathcal{V} + \beta\xi\mathcal{A}]}_{\text{recruiting costs}} \right\}$$

where $g_E(\theta) \equiv \frac{g(\theta)p(\theta)}{(1-\mathcal{U})(\eta+p(\theta))}$ is the density of productivities of employed workers and $g_U(\theta) \equiv \frac{g(\theta)\eta}{\mathcal{U}(\eta+p(\theta))}$ stands for that of the unemployed. In addition to the costs of posting $\kappa\mathcal{V}$ for all vacancies, the SP has to pay the present value of the screening costs ($\beta\xi\mathcal{A}$) and the entry cost in the market, χ. By the law of large numbers, a fraction e^{-q} of all vacancies posted by the SP are not filled due to the lack of applicants, in which case no entry cost needs to be paid. Since the economy operates in a steady state, only a fraction $1 - e^{-q}$ of vacancies posted in each period has to pay the entry cost, χ.

Using the definition of the distribution of applicants $g_A(\theta)$ and the fact that the population distribution always equals $g(\theta) = \mathcal{U}g_U(\theta) + (1-\mathcal{U})g_E(\theta)$, we realize that the distribution of the unemployed is

$$g_U(\theta) = \frac{\mathcal{A}g_A(\theta) - \lambda g(\theta)}{\mathcal{U}(1-\lambda)},$$

while the distribution of the employed can be written as

$$g_E(\theta) = \frac{g(\theta) - \mathcal{A}g_A(\theta)}{(1-\mathcal{U})(1-\lambda)}.$$

With some algebra and using the quantile mapping, we can rewrite the SP problem as

$$\max_q Y(q) = \frac{\beta(1-\rho\lambda)}{(1-\lambda)}\mathbb{E}[\theta] - \mathcal{A}(q)\left[\frac{\kappa}{q} + (1-e^{-q})\frac{\chi}{q} + \beta\xi + \beta\frac{(1-\rho)}{1-\lambda}\int_0^1 G^{-1}(M(x;q))dx\right] \tag{22}$$

The first term of Equation (22) is proportional to the output obtained in a frictionless environment, $\overline{Y} = \mathbb{E}[\theta]$. The proportionality factor $\frac{1-\rho\lambda}{1-\lambda}$ indicates there is an obvious loss because unemployed workers only generate a fraction ρ of their productivity. However, the probability of on-the-job search, λ attenuates this effect because a fraction of new hirings do not originate from jobless workers.

The first two terms in the square brackets indicate the recruiting and screening costs. The number of posted vacancies $\mathcal{V}$ is managed by the Social Planner to control the extensive margin of hiring at the firm level, whereas q controls the screening activity. The last term in the square brackets refers to the loss due to the negative impact that recruiting activities generate on the quality of the pool of applicants.

The more selective the recruiting process is through a higher q, the lower the quality of the average pool of applicants. Although more frequent on-the-job search offsets this effect to some extent, the overall impact is a decrease in the productivity of the average hiring of firms in the economy. If the Social Planner is allowed to mandate vacancy-posting and screening, it is clearly constrained by the identity $q = \mathcal{A}(q)/\mathcal{V}$.

Imposing first-order conditions on the Social Planner's program, we obtain the following expression:

$$\begin{aligned}&\frac{\beta(1-\rho)}{1-\lambda}\left[\widetilde{\mathcal{A}}(q)\int_0^1 G^{-1}(M(x,q))dx + q\int_0^1 \frac{M_q(x;q)}{g(G^{-1}(M(x,q)))}dx\right] = \\ &\frac{\kappa}{q}\left[1-\widetilde{\mathcal{A}}(q)\right] - \widetilde{\mathcal{A}}(q)\left[\beta\xi + (1-e^{-q})\frac{\chi}{q}\right] - \chi\left(qe^{-q} - \frac{1-e^{-q}}{q}\right)\end{aligned} \tag{23}$$

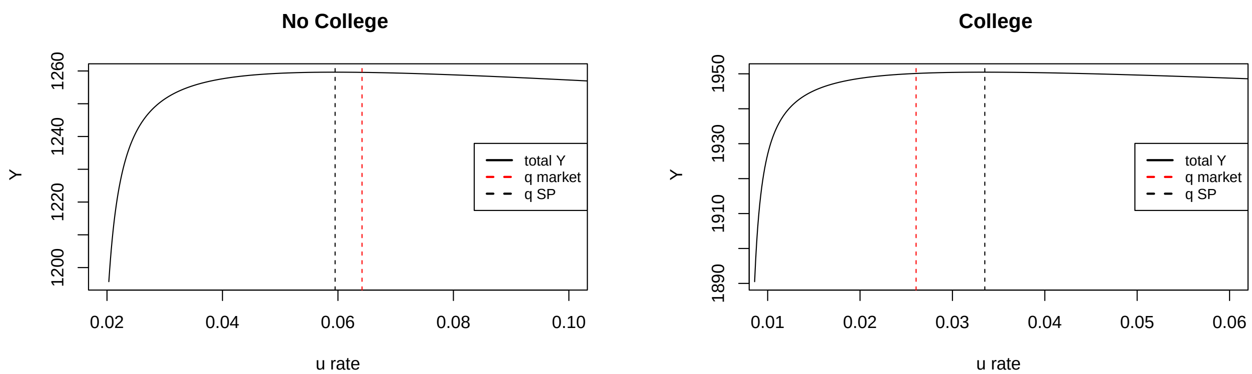
where $\widetilde{\mathcal{A}}(q) = \frac{\partial \log \mathcal{A}(q)}{\partial \log q}$ is the elasticity of $\mathcal{A}$ with respect to q.

The term $\frac{\beta(1-\rho)}{1-\lambda}\int_0^1 G^{-1}(M(x,q))dx$ is the expected loss due to an unmatched applicant given the prevailing equilibrium composition of applicants reflected by $G_A^{-1}(x,q) = G^{-1}(M(x,q))$ and the queue length q. In addition to this loss, the SP also considers the recruiting cost associated per applicant, including the interview cost, ξ, an the flow vacancy cost per applicant, κ/q.

Therefore, the optimality condition just says that the increase of applicants that the SP generates when choosing a higher queue length must equate the whole expected cost increase of that decision, including more selectivity through the $M_q(x,q)$ term, larger screening costs, and lower average fixed costs, through the term κ/q.

Figure 13 shows that for the calibrated and estimated values of the parameters, the optimal unemployment rate would be slightly lower than the market solution for non-college (5.95% instead of 6.42%) and slightly higher for college workers (3.35% instead of 2.61%).

Figure 13: **Queue length and unemployment: Market vs. social planner**



In a competitive equilibrium, q is determined by the entry condition (Equation 15). We can therefore look for a tax/subsidy schedule that allows obtaining the queue length that the Social Planner would choose as the outcome of a decentralized market equilibrium.

We adapt a general tax schedule that has been studied at length in the literature on optimal income taxation and apply it to our framework, in which we consider the firm to be the subject of taxation. A widely used tax schedule in public finance, as described by Heathcote, Storesletten, and Violante (2017), defines the total tax revenues ($T(y)$) for the level of income y with the two-parameter functional form $T(y) = y - \tau_0 y^{1-\tau_1}$, where the parameter τ_1 is an index of the progressivity of the tax system.[14]

A tax schedule is considered progressive if the Coefficient of Residual Income Progression (CRIP) is less than one and regressive if larger than one; in the case of a flat tax, the CRIP is equal to one. One advantage of this measure of tax progressivity is that it is well defined even when the average tax rate is zero. The CRIP is related to the marginal and average tax rates as follows:

$$\text{CRIP}(y) = \frac{\partial y^d}{\partial y}\frac{y}{y^d} = \frac{1 - T'(y)}{1 - \overline{T}(y)} \tag{24}$$

where $\overline{T}(y)$ is the average and $T'(y)$ is the marginal tax rate.

With the adopted functional form for the tax and transfer schedule, the expression for the CRIP is given by $1 - \tau_1$.

[14] We briefly recall that the Coefficient of Residual Income Progression (CRIP) is one of the most used measures of progressivity: it represents the elasticity of post-tax income to pre-tax income.

In our framework, we assume that the tax schedule applies to the profit function of the firm, which in itself depends on the level of productivity of the hired worker. The net value of a filled job, $J(\theta)$, is thus given by $(1-\tilde{t}(\theta))J(\theta)$, where $\tilde{t}(\theta)$ is the average tax rate paid for productivity level θ. Therefore, by using the tax schedule as in Heathcote, Storesletten, and Violante (2017), $\tilde{t}(\theta) = 1 - \tau_0\theta^{-\tau_1}$. By applying the same change of variables as in Equation (9), the tax schedule can be rewritten as $\bar{t}(x) = 1 - \tau_0(G^{-1}(M(x;q)))^{-\tau_1}$.

The entry condition including taxes is thus

$$\kappa+\beta\xi\phi q(1-e^{-q})+\chi(1-\beta e^{-q}) = \beta\int_0^1 \frac{(1-\rho)G^{-1}(M(x;q)) + \beta\chi[\eta^* + \lambda e^{-q(1-x)}]}{1-\beta[1-\eta^*-\lambda e^{-q(1-x)}]} e^{-q(1-x)}(1-\bar{t}(x))dx \tag{25}$$

In addition to the entry condition, we impose a balanced government budget: the net tax revenues that are levied on the work force of the firm have to be null. In fact, the general tax and transfer schedule allows for positive as well as negative taxes, i.e., for subsidies.

The balanced budget condition for the government is thus given by

$$(1-\mathcal{U}(q))\int_0^\infty \tilde{t}(\theta)\theta dG_E(\theta) = 0 \tag{26}$$

By applying the usual change of variables and re-writing the integral accordingly, after some algebra we obtain the condition

$$\int_0^1 \bar{t}(G^{-1}(M(x;q))M_x(x;q)dx = \mathcal{A}(q)\int_0^1 \bar{t}(G^{-1}(M(x;q))dx \tag{27}$$

Finally, to implement the SP solution as a competitive equilibrium, the tax schedule must also preserve the coincidence ranking equilibrium: the firm has to continue to strictly prefer hiring higher productivity types, even after paying the corresponding tax. We therefore check *ex post* that the coincidence ranking equilibrium is satisfied.

The values for the parameters of the tax schedule for non-college types are found to be $\tau_0 = 0.96$ and $\tau_1 = -0.035$, while for college $\tau_0 = 1.01$ and $\tau_1 = 0.008$. Therefore the tax and transfer schedule is regressive for non-college types and slightly progressive for college ones: the coefficients of residual income progression $(1-\tau_1)$ are respectively 1.035 and 0.992.

Figure 14 shows that the optimal tax and transfer schedule would imply that for non-college workers characterized by low levels of productivity the firm is taxed, while non-college hires with high levels of productivity are subsidized through negative average tax rates. For college workers, instead, high productivity types are taxed and low productivity types are subsidized.

The Social Planner wishes to obtain a slightly smaller queue length for non-college workers and a marginally larger one for college workers. The lower the target level for the queue length and, therefore, the unemployment rate, the greater the need to give incentives to the firm to open vacancies. This is achieved for non-college workers through a "regressive" tax and transfer schedule that provides more motivation for employers to hire high-productivity workers. For college workers, instead, the opposite holds true.

This curious result can be understood by examining the incentives for employers. Top matches chiefly drive firms' interest in the labor market because they can avoid mediocre matches by hiring selectively. An extra incentive for hiring at the top of the distribution is a great incentive to post vacancies. However, good types are scarce, and many employers end up hiring not-so-good applicants, resulting in a trickle-down of job opportunities. Hence, high types generate a positive externality for their lower productivity counterparts by driving more open vacancies.

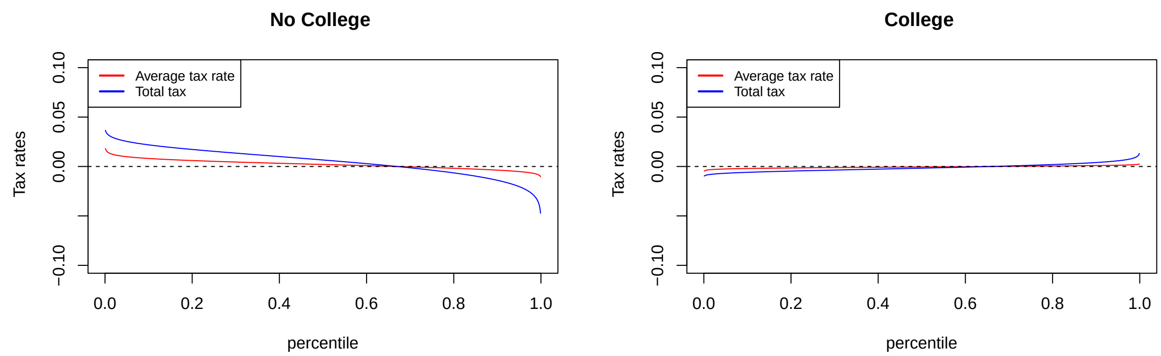
In addition, there is another composition externality of high types. As their attractiveness spurs more vacancies and shortens queues, subsidies for them reduce hiring selectivity, improve the composition of the unemployed, and increase the share of on-the-job seekers. As employers expect to hire better workers under this scenario, more vacancy posting is reinforced, which translates into higher chances of being hired for less productive workers. Last but not least, the tax schedule we obtain clearly preserves a coincidence-ranking equilibrium in which employers prioritize hiring top applicants. This condition resembles an incentive-compatibility constraint: the SP cannot prescribe employers to choose candidates who do not maximize their profits.

With these concepts in mind, we can understand the transfer scheme as a simple application of the Pigouvian principle that aligns private costs and benefits with social costs and benefits to enhance efficiency. The tax schedule that implements the SP allocation for non-college types subsidizes hiring high types because of the positive externalities they generate for lower types in an economy with search frictions and hiring selectivity; the opposite is true for college workers. As we impose a balanced budget condition, those resources need to be provided by individuals benefitting from positive externalities.

7 Conclusions

We developed a theoretical model that offers insights into the consequences of selectivity in the hiring process for labor market performance and wage inequality. Within our framework, workers exhibit *ex ante* heterogeneity in their productivity levels. This theoretical model en-

Figure 14: **Tax and transfer schedule**



ables us to understand how variations in *ex ante* inequality translate into *ex post* wage inequality, primarily through the endogenous composition of the pool of employed and unemployed workers.

Our analysis began by examining empirical evidence that reveals a positive correlation between the unemployment rate and inequality, specifically as measured by the 90–10 percentile gap in log hourly wages. We also observed a positive correlation when considering job separation probabilities and a negative correlation between job finding probabilities and our measure of inequality. Building on these insights, we constructed a nonsequential search model, where firms can incur screening costs to perfectly discern the productivity type of a worker. Workers can apply for jobs while already employed, resulting in a mix of employed and unemployed job seekers. Labor market transitions significantly impact the composition of the employed workforce, making the productivity distribution of the employed an endogenous variable.

We take our model to the data by estimating a set of parameters using the generalized method of moments and performed various counterfactual experiments. These experiments revealed that the observed correlation between unemployment and wage inequality in the data is consistent with shifts in the distribution of productivity, influencing both its mean and variance.

Furthermore, our counterfactual analyses unveiled non-linearities in the relation between labor market variables and inequality of wages. Notably, our measure of the ranking gap showed that, within a given steady state equilibrium, the selective hiring process in the labor market results in a productivity distribution of the employed with a skewed left tail and a stretched right tail. This implies that ex ante inequalities in terms of productivity are accentuated for highly productive individuals, while the opposite holds for individuals with relatively lower productivity levels. Moreover, when exogenous parameters change, such as shifts in the distribution of productivities that alter its mean, this has different effects on different parts of the wage distribution. An increase in average productivity, while keeping the variance constant, leads to a reduction in the ranking gap at the left tail of the distribution (the 50/10 percentiles ratio) but an increase at the right tail (the 90/50 percentiles ratio). Consequently, an overall nearly neutral effect on measures like the 90/10 percentiles ratio masks distinct effects at different locations of the distribution.

Finally, we conducted an efficiency assessment. We considered the scenario of a Social Planner subject to the same technological constraints as the individual agents. Our findings suggest that achieving the optimal allocation within our baseline steady-state economy may require implementing either a regressive or a progressive tax and transfer schedule, depending on the objective of attaining a lower or higher unemployment rate. Specifically, if the optimal unemployment rate is lower than the competitive equilibrium market rate, a regressive tax and transfer schedule would be necessary. In such a scenario, the tax and transfer schedule, applied to firms and contingent upon workers' productivity levels (i.e., wages), entails providing subsidies to more productive workers while taxing less productive individuals. The motivation behind this approach lies in maintaining incentives for firms to hire their preferred candidates and the positive externalities derived from encouraging firms to create job opportunities for the most productive individuals.

Taking stock, our model offers a framework to understand how ex ante productivity differences in the labor market map into realized differences in the wage distribution. We highlight hiring selectivity mechanisms that link worker flows, composition, and inequality. By also taking the model to the data and using it for normative economic analysis, we hope to have made a stride in understanding the central role of labor markets in shaping inequality.

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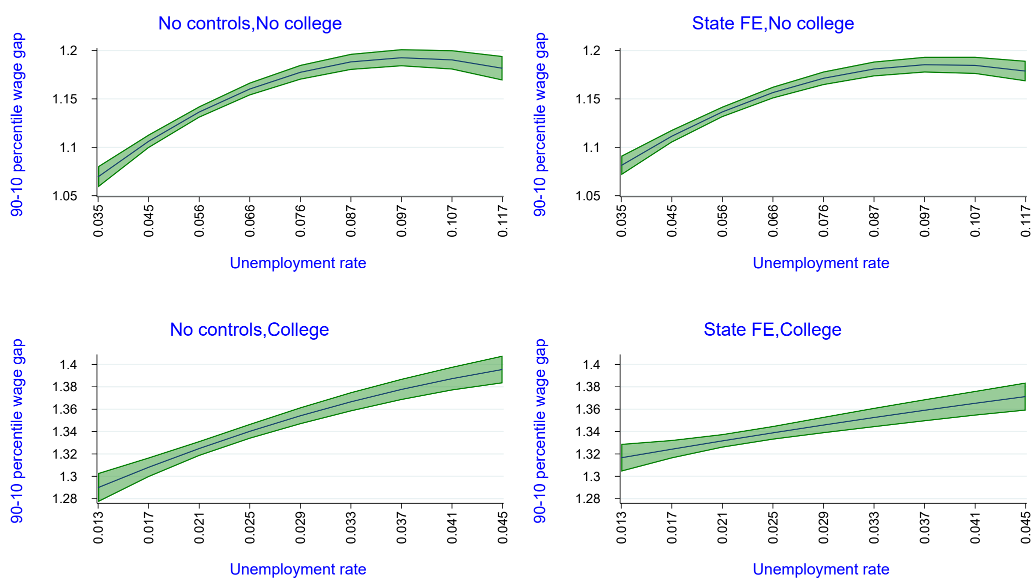
Appendix

Appendix A Robustness of empirical facts

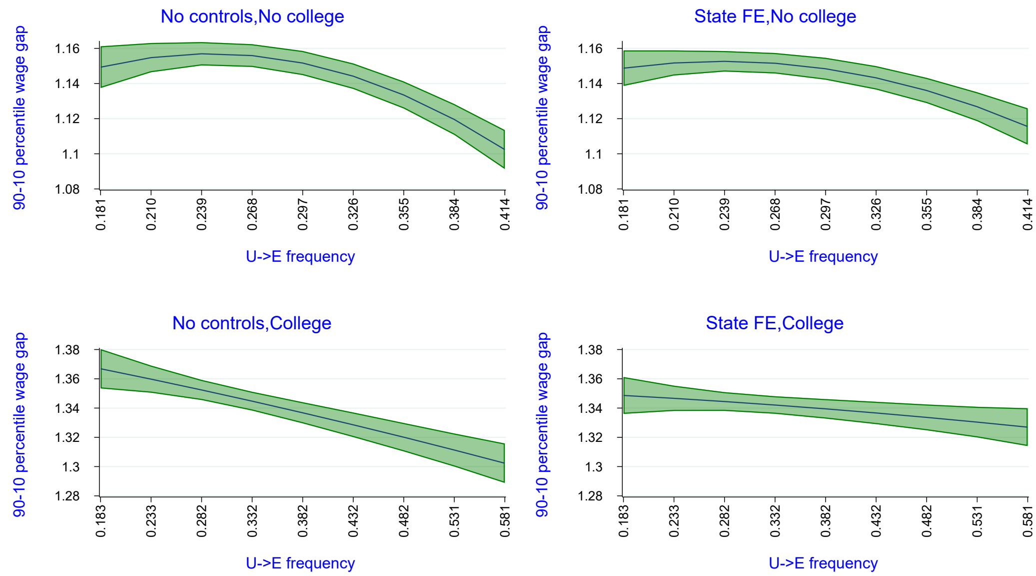
A Robustness to 1994 CPS sampling redesign

Figure 15: 90–10 log wage percentile gap vs. unemployment rate by state & year



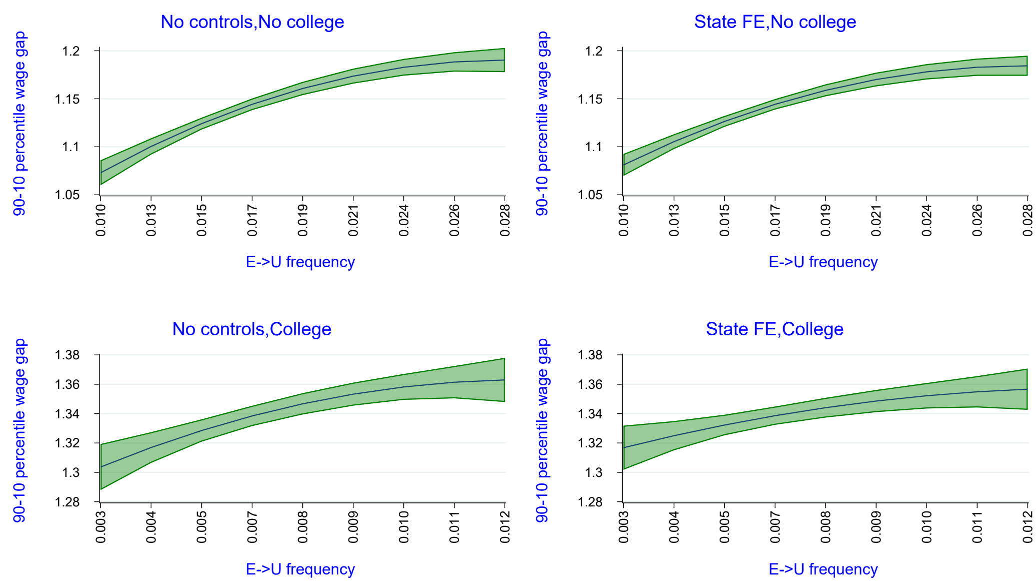
Sample 1976-2019. US real log hourly wages.

Figure 16: 90–10 log wage percentile gap vs. UE frequency by state & year



Sample 1976-2019. US real log hourly wages.

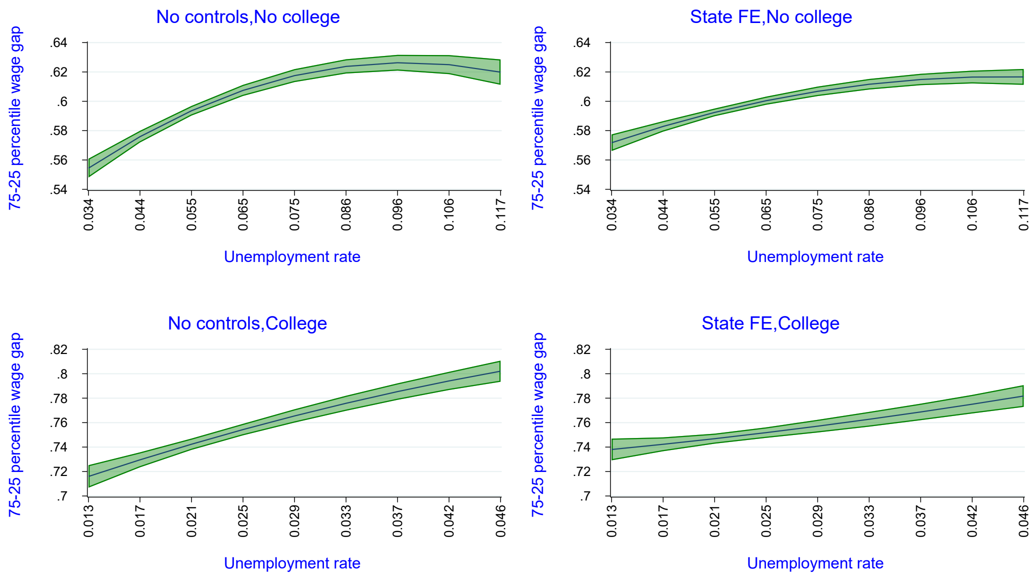
Figure 17: 90–10 log wage percentile gap vs. EU frequency by state & year



Sample 1976-2019. US real log hourly wages.

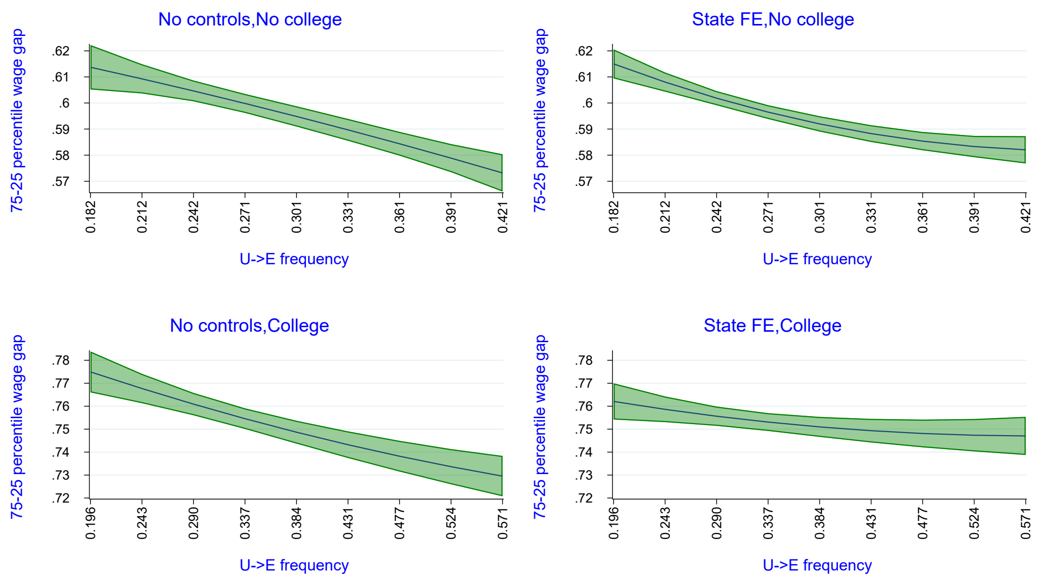
B Alternative measures of dispersion

Figure 18: 75–25 percentile log wage gap vs. unemployment rate by state & year



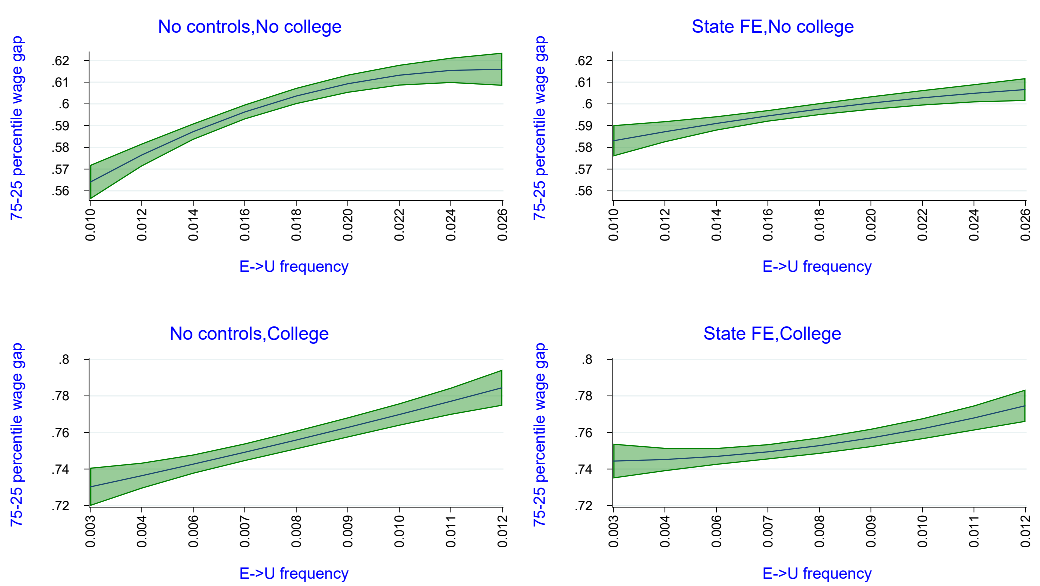
Sample 1994-2019. US real log hourly wages.

Figure 19: 75–25 percentile log wage gap vs. UE frequency by state & year



Sample 1994-2019. US real log hourly wages.

Figure 20: 75–25 percentile log wage gap vs. EU frequency by state & year

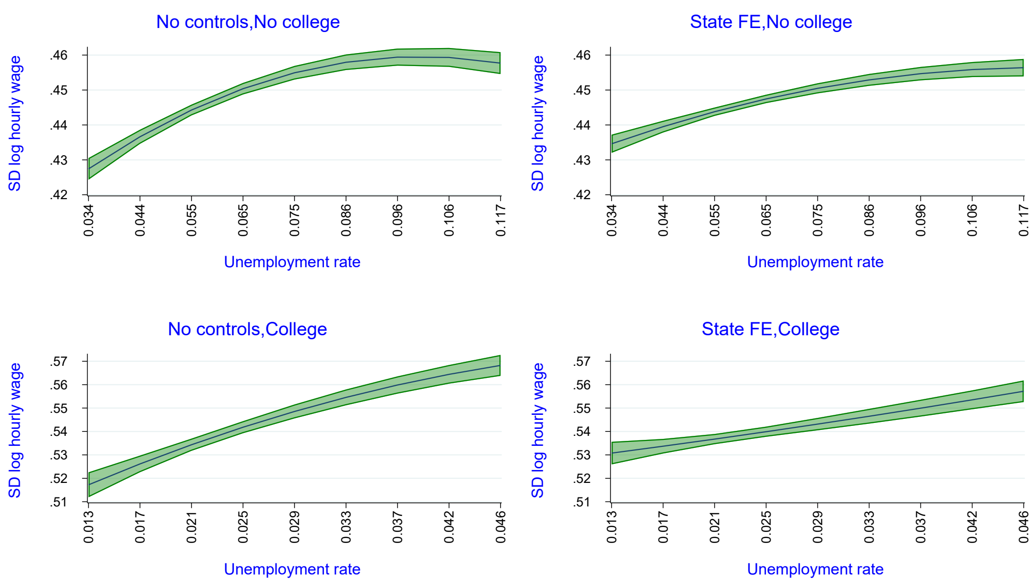


Sample 1994-2019. US real log hourly wages.

Figure 21: 75–25 percentile log wage gap vs. JJ frequency by state & year

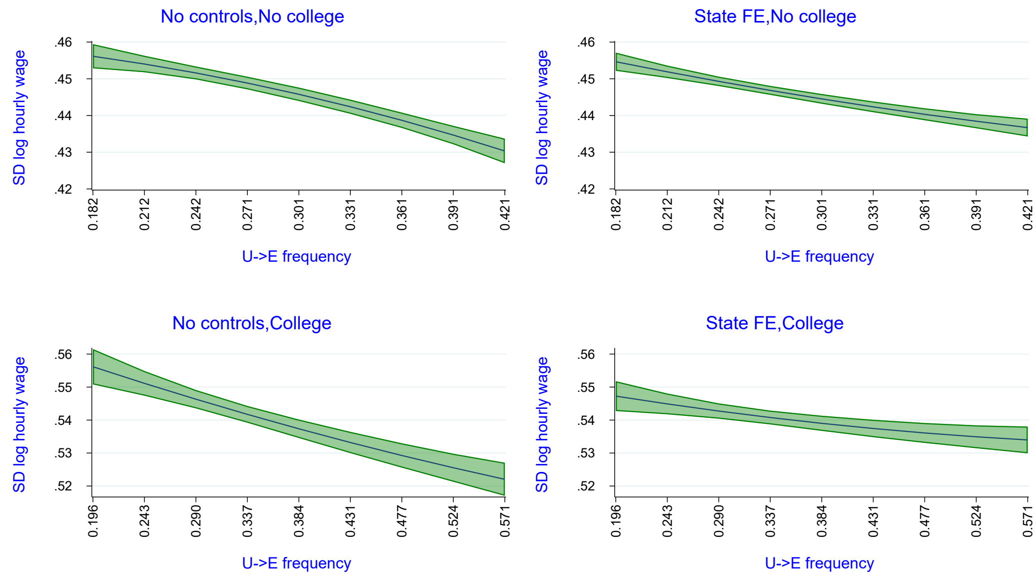
Sample 1994-2019. US real log hourly wages.

Figure 22: Standard deviation log wage vs. unemployment rate by state & year



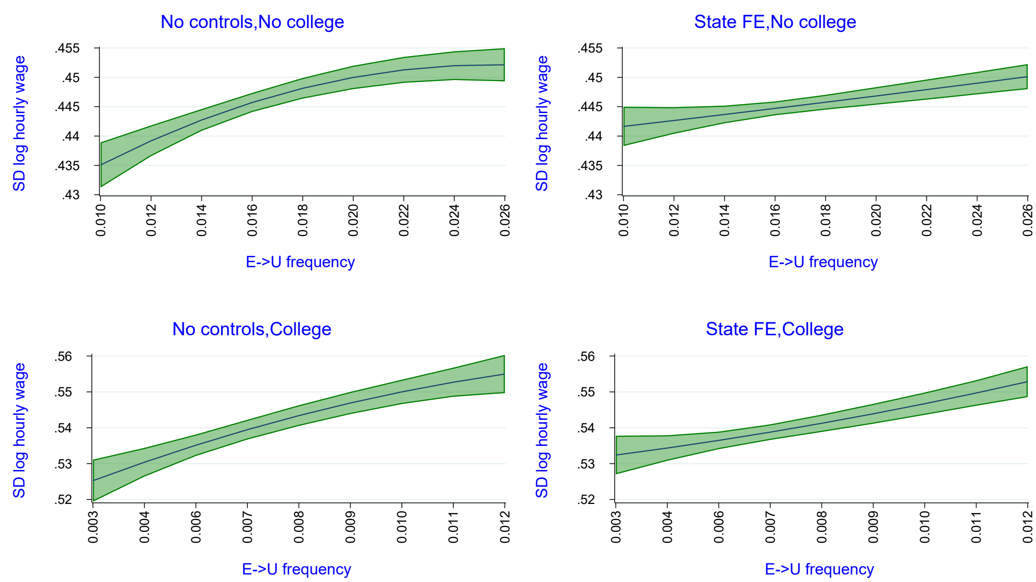
Sample 1994-2019. US real log hourly wages.

Figure 23: Standard deviation log wage vs. UE frequency by state & year



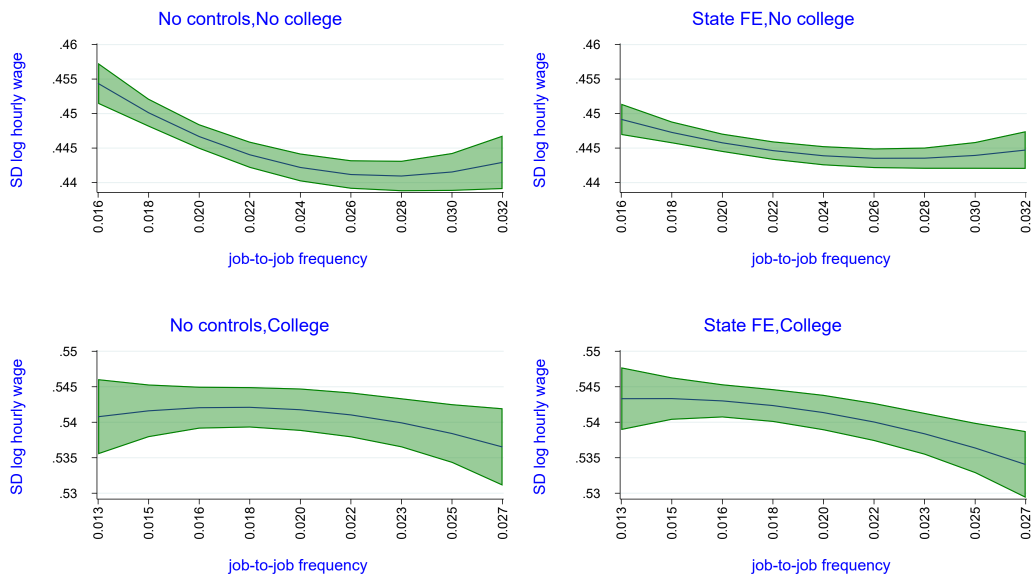
Sample 1994-2019. US real log hourly wages.

Figure 24: Standard deviation log wage vs. EU frequency by state & year



Sample 1994-2019. US real log hourly wages.

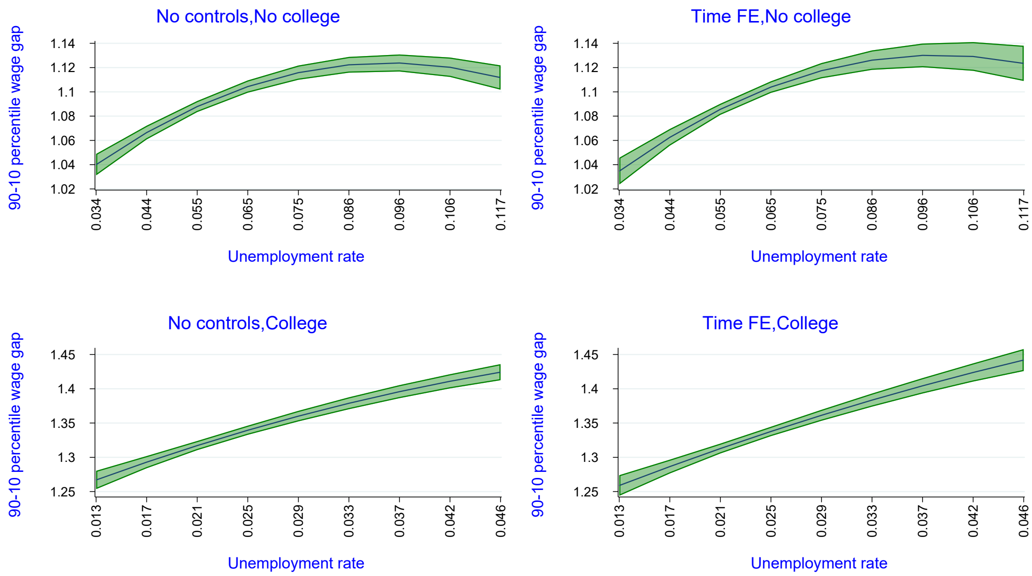
Figure 25: Standard deviation log wage vs. JJ frequency by state & year



Sample 1994-2019. US real log hourly wages.

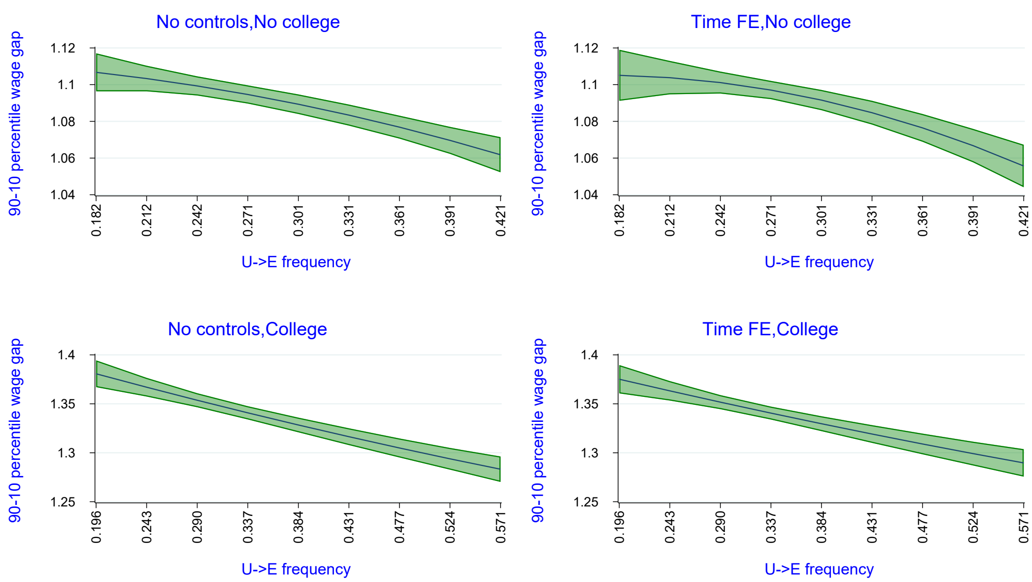
C Time fixed effects

Figure 26: 90–10 percentile wage gap vs. unemployment rate by state & year



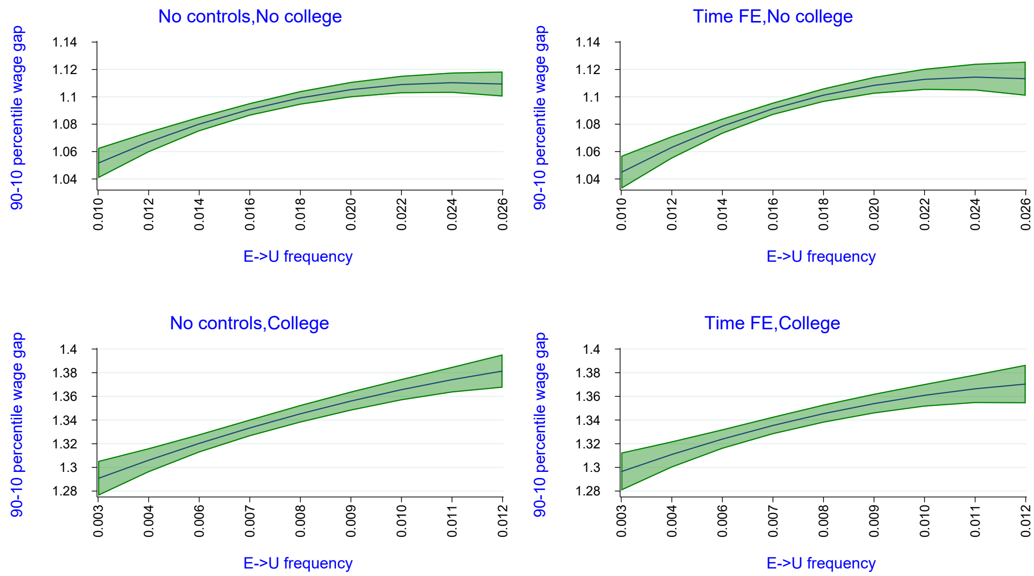
Sample 1994-2019. US real log hourly wages.

Figure 27: 90–10 percentile wage gap vs. UE frequency by state & year



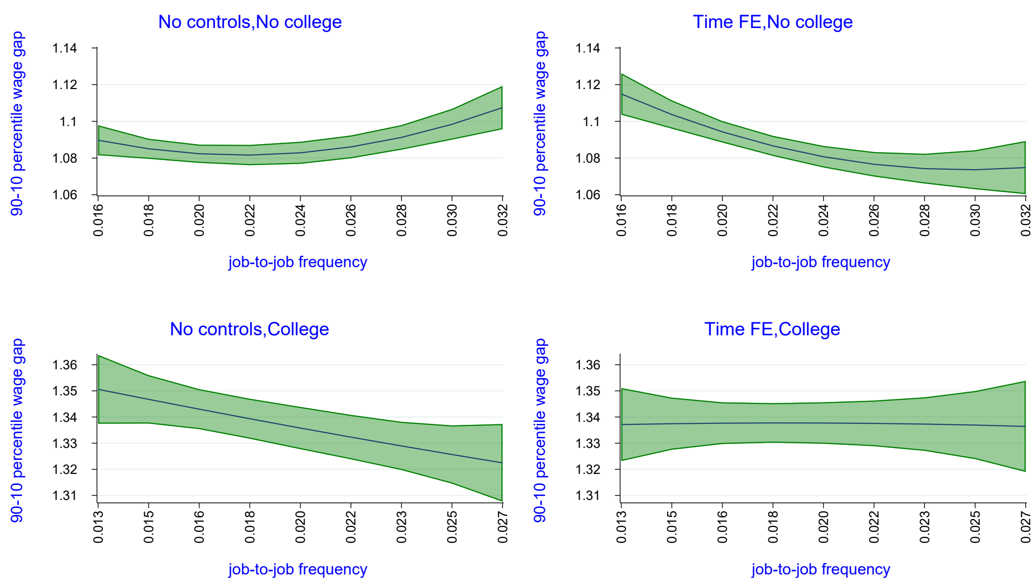
Sample 1994-2019. US real log hourly wages.

Figure 28: 90–10 percentile wage gap vs. EU frequency by state & year



Sample 1994-2019. US real log hourly wages.

Figure 29: 90–10 percentile wage gap vs. JJ frequency by state & year



Sample 1994-2019. US real log hourly wages.

Appendix B Deriving Entry Condition (15)

The entry condition can be written as

$$-\kappa+\beta\left\{\sum_{k=1}^{\infty}\frac{e^{-\phi q}(\phi q)^k}{k!}\left[-\xi k+\int_0^{\infty}J(v)k(\phi G_A(v)+(1-\phi))^{k-1}g_A(v)dv\right]+e^{-\phi q}V\right\}=V \tag{28}$$

To arrive at condition (28), we use that:

- The queue length q follows a Poisson distribution.
- The hired applicant follows the distribution of the maximum productivity applicant over k applicants
- There is an ex ante determined probability of interviewing each applicant, ϕ.
- If the vacancy posted is not filled, the employer receives the value of posting the vacancy again.

In equilibrium, $V=\chi$, the entry cost. We also assume that the value of hiring the first worker surpasses the entry cost plus the interviewing cost, e.g., $E[J(\theta)]=\int J(\theta)dG_A(\theta)>\chi+\xi$. This condition ensures that the employer expects to hire a worker rather than discarding the pool of applicants received in one period, even if the employer gets only one applicant.

Doing some algebra helps us show that

$$\kappa+\beta\xi\phi q+\chi(1-\beta e^{-\phi q})=\beta\left(\sum_{k=1}^{\infty}\frac{e^{-\phi q}(\phi q)^k}{k!}\int_0^{\infty}J(v)k(\phi G_A(v)+(1-\phi))^{k-1}g_A(v)dv\right)$$

$$\kappa+\beta\xi\phi q+\chi(1-\beta e^{-\phi q})=\beta\left(\sum_{k=1}^{\infty}\frac{e^{-\phi q}(\phi q)^{k-1}}{(k-1)!}\int_0^{\infty}J(v)\phi q(\phi G_A(v)+(1-\phi))^{k-1}g_A(v)dv\right)$$

$$\kappa+\beta\xi\phi q+\chi(1-\beta e^{-\phi q})=\beta\left(\int_0^{\infty}J(v)\phi q e^{-\phi q(1-G_A(v))}dG_A(v)\right)$$

$$\kappa+\beta\xi\phi q+\chi(1-\beta e^{-\phi q})=\beta\phi q\left(\int_0^{1}J(G^{-1}(x;q))e^{-\phi q(1-x)}dx\right)$$

For the last step, we use the inverse quantile mapping in (9) to replace the unknown distribution of applicants G_A by the population distribution G, a primitive of the model.

Appendix C Calibrating screening costs

The occupation codes used to compute the adjustment factor are:

Occupation codes 1992–2002:

- 8: Personnel and labor relations workers.
- 27: Personal, training and labor relation specialists.

Occupation codes 2003–2010:

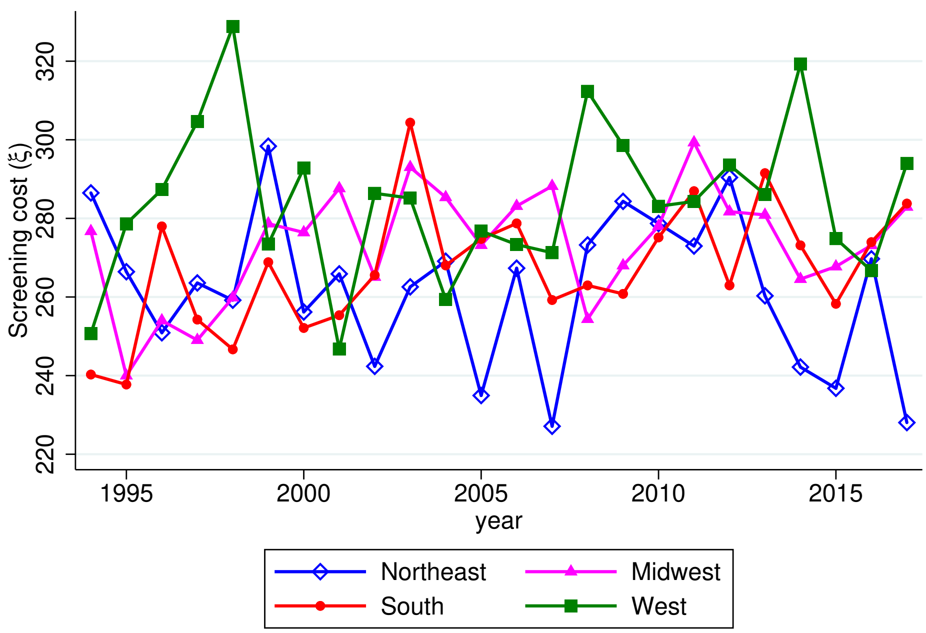
- 130: Human Resources Managers.
- 620: Human resources, training, and labor relations specialists.
- 5360: Human resources assistants, except payroll and timekeeping.

Occupation codes since 2011:

- 136: Human Resources Managers.
- 630: Human resource workers.
- 5360: Human resources assistants, except payroll and timekeeping.

The series of reconstructed recruiting costs is shown in Table 30.

Figure 30: Real recruiting cost wages CPS-ORG since 1994



Magnitudes in 2010 USD, deflated by regional CPI.

Estimated recruiting cost is based on the median of the NES97 recruiting cost with imputations due to non-response.

Appendix D Counterfactual experiments

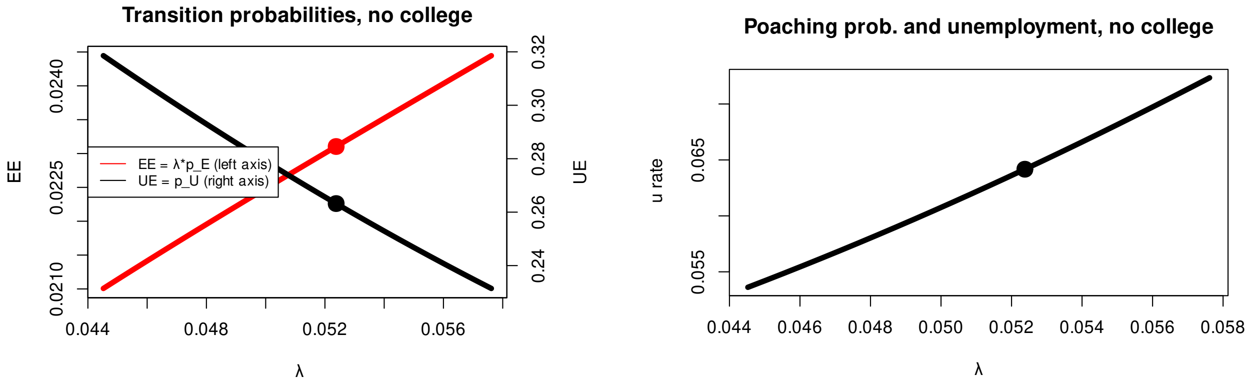
A The effects of an increase in poaching probability λ

If the probability of applying to a job while working (represented by the parameter λ) exogenously increases, there is more competition for jobs, since more employed agents enter the pool of applicants. The average job finding probability of the unemployed (in our notation, $\bar{p}_U$) decreases, while the job-to-job transition probability, $\lambda\bar{p}_E$, increases, as can be seen in the left panel of Figure 31. The average length of the queue, and therefore the unemployment rate, increases with λ, as is shown in the right panel of Figure 31.

In terms of the wage distribution, this additional competition and therefore selectivity implies that there are fewer employed (a higher unemployment rate), but those who have a job earn more, because they are the most productive: the average log-wage increases as the parameter λ increases, as we can see in Figure 32.

The predictions of the model in terms of the consequences for wage inequality of an increase in the poaching probability depend on the chosen measure of inequality: while the model predicts a slight decrease of the 50/10 percentiles of log wages as λ increases, the con-

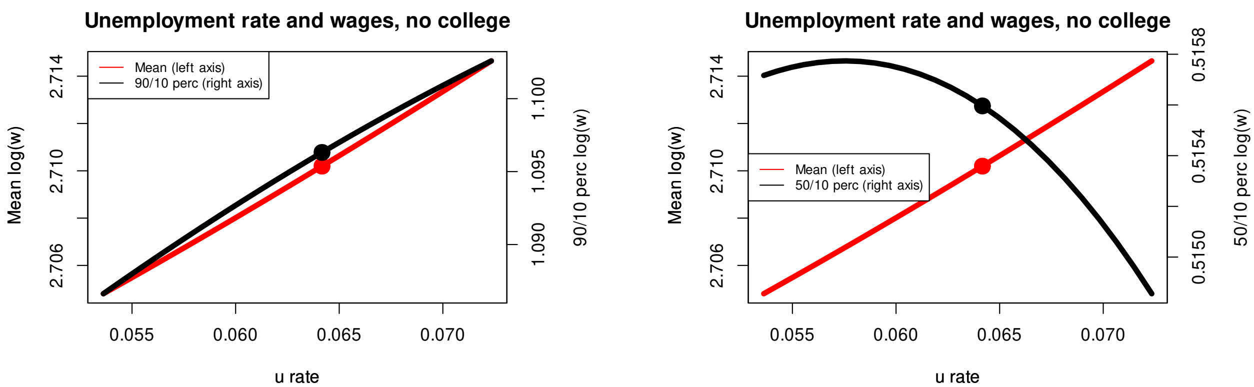
Figure 31: The effects of poaching probability on transitions and average wage



The dots represent the values for the benchmark calibration and estimation exercise.

sequences in terms of the ratio of the 90th over 10th percentile are the opposite, as we can see in Figure 32.

Figure 32: The effects of poaching probability on wages



The dots represent the values for the benchmark calibration and estimation exercise.

B The effects of an increase in screening costs ξ

Increasing the screening costs ξ implies a decrease in job vacancies and therefore a decrease in job finding probabilities and an increase in the length of the queue and the unemployment rate, as can be seen in Figure 33.

Similarly to the results of the counterfactual experiments on the poaching probability λ, an increase in the unemployment rate is associated with an increase in the average wage; the effects of screening costs on wage inequality depend on the chosen measure of inequality, as we can see in Figure 34.

Figure 33: The effects of screening costs ξ on the labor market

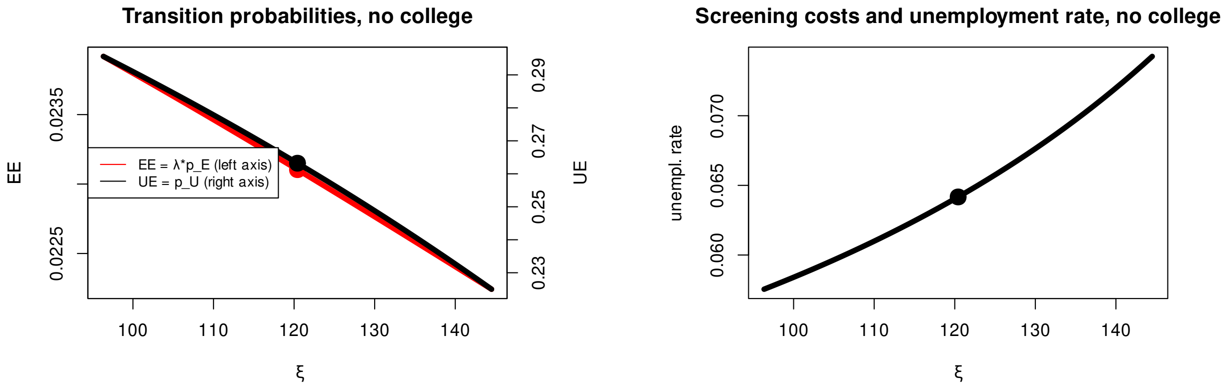
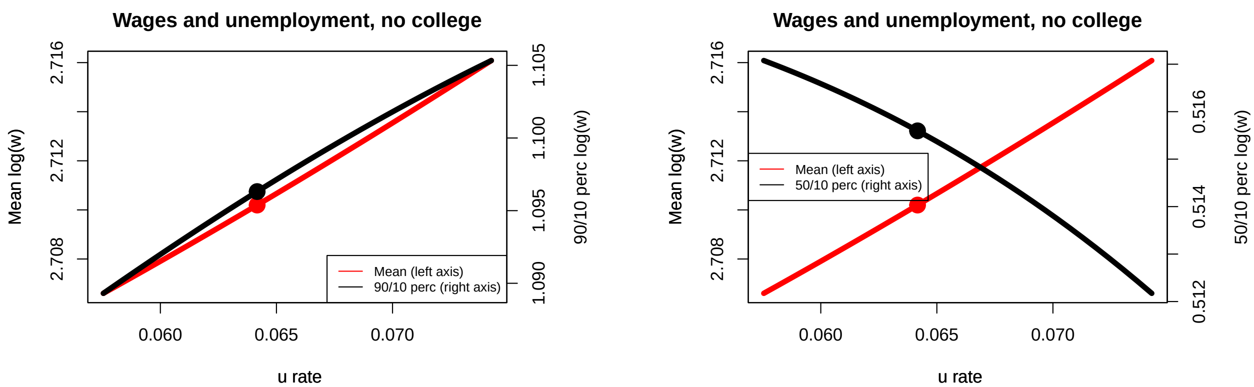


Figure 34: The effects of screening costs ξ on wages



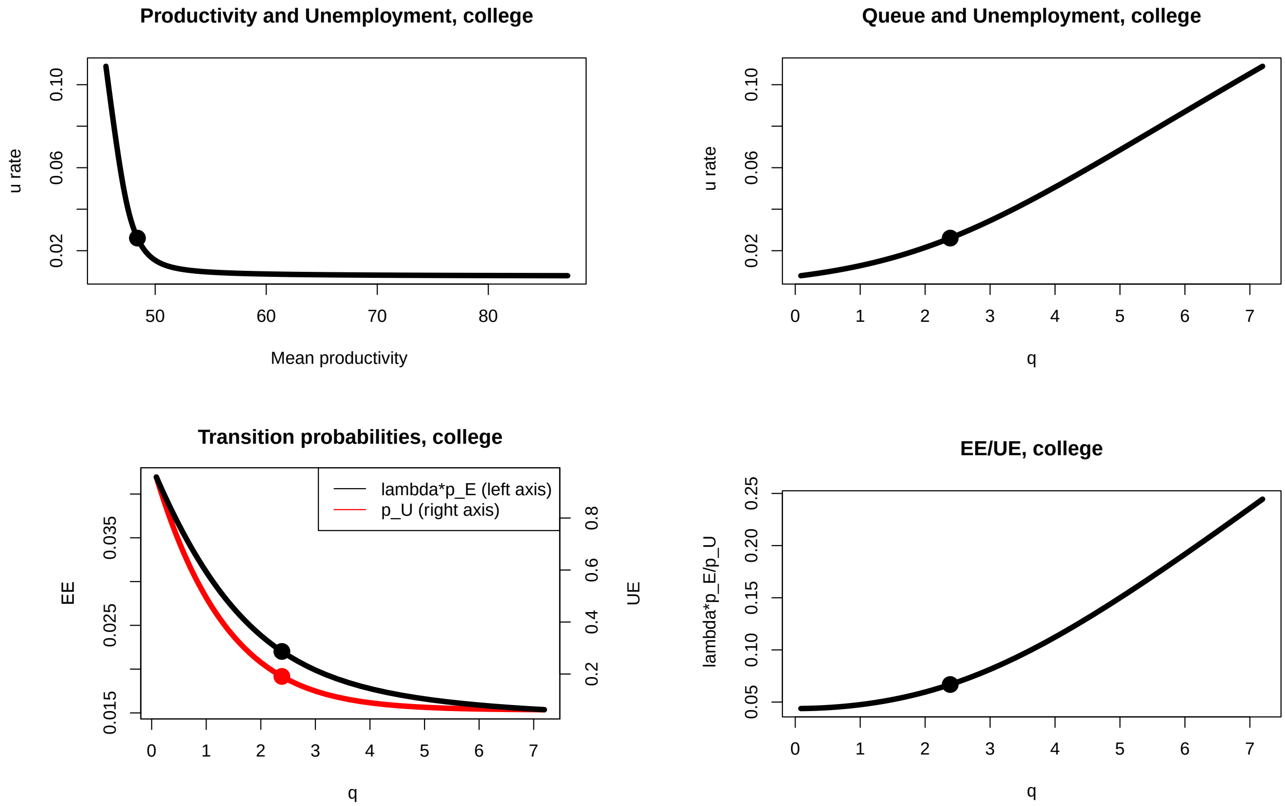
Appendix E Additional counterfactuals: The effects of an increase in mean productivity for college workers

In this section we present the results of a counterfactual experiment consisting in increasing the mean of the productivity ditribution while keeping constant the variance for college workers.

Appendix F Proofs omitted in the main text

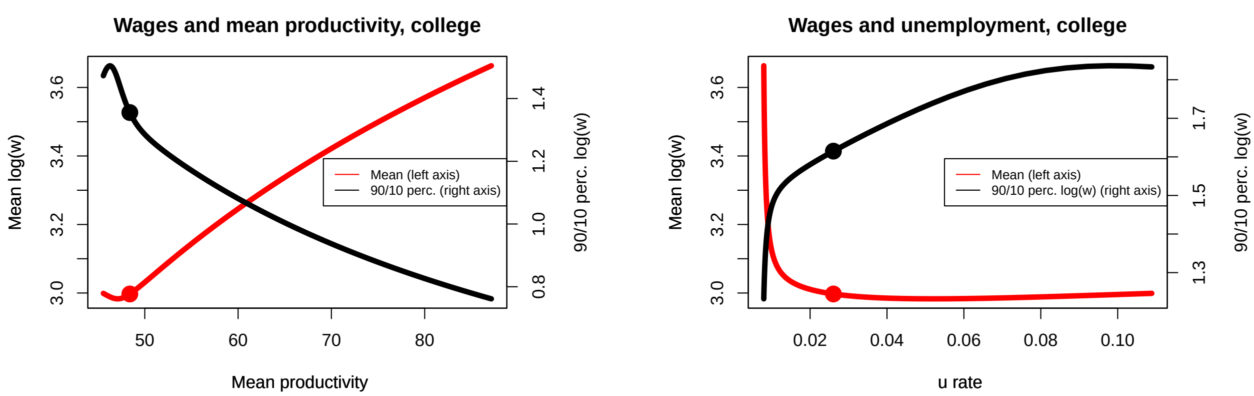
Proposition 2 ***Increasing unemployment rate*** *The unemployment rate is increasing in* q.

Figure 35: The effects of an increase in average productivity on unemployment and transition probabilities (college)



The dots represent the values for the benchmark calibration and estimation exercise.

Figure 36: The effects of an increase in average productivity on wages (college)



The dots represent the values for the benchmark calibration and estimation exercise.

Proof.

$$\mathcal{A} = \frac{1}{m(1,q) - m(0,q)}$$

with

$$m(x,q) \equiv x + \frac{1-\lambda}{\lambda q} \log\left(\eta + \lambda e^{-q(1-x)}\right)$$

We have to compute the derivative

$$\frac{\partial(1/\mathcal{A})}{\partial q} = -\frac{1-\lambda}{\lambda q^2} \log\left(\frac{\eta+\lambda}{\eta+\lambda e^{-q}}\right) + \frac{1-\lambda}{\lambda q}\frac{\lambda e^{-q}}{\eta+\lambda e^{-q}}$$

Using the Mean Value Theorem we realize that the term with a logarithm in the previous expression is a difference evaluated at 1 and 0, so it can be written as

$$\log(\eta + \lambda e^{-q(1-1)}) - \log(\eta + \lambda e^{-q(1-0)}) = \frac{q\lambda e^{-q(1-\bar{x})}}{\eta + \lambda e^{-q(1-\bar{x})}}(1-0) \text{ with } \bar{x} \in (0,1)$$

Hence, the original derivative can be expressed as

$$-\frac{1-\lambda}{\lambda q}\left(\frac{\lambda e^{-q}}{\eta+\lambda e^{-q}} - \frac{\lambda e^{-q(1-\bar{x})}}{\eta+\lambda e^{-q(1-\bar{x})}}\right)$$

$$-\frac{1-\lambda}{q}\left(\frac{1}{\eta e^{q}+\lambda} - \frac{1}{\eta e^{q(1-\bar{x})}+\lambda}\right) < 0$$

where the last expression holds because $\frac{1}{\eta e^q + \lambda}$ is strictly decreasing in q.

Since $\frac{\partial(1/\mathcal{A})}{\partial q} < 0$, it follows that $\frac{\partial \mathcal{A}}{\partial q} > 0$ and also $\frac{\partial \mathcal{U}}{\partial q} > 0$ because $\mathcal{U} = \frac{\mathcal{A}-\lambda}{1-\lambda}$.

■

If on-the-job search is prevalent in the labor market, i.e., λ is high, hiring selectivity q has less of an effect on the unemployment rate. In the extreme scenario of $\lambda = 1$, i.e., all workers apply, selectivity does not matter as it does not affect the composition of the unemployed pool.