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Abstract

The purpose of this paper is to incorporate a price-dependent utility function into the theory of price indices. The results presented in the paper provide justification for incorporating relative prices into a consumption aggregate for solving the household's optimization problem. Using a proposed CES aggregator, a simple model to approximate the conventional geometric price indices has been developed. The model provides the channels of convergence between the axiomatic, stochastic and economic approaches to index number theory.

Numerical results based on data published by the Office for National Statistics are presented to confirm the validity of the model.

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Consumption aggregator with price-dependent weights

Price index with price-dependent preferences

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I. Introduction

The model presented in this paper is inspired by the works of Feenstra (1994) and Redding and Weinstein (2020) in which they used the *weighted* generalized mean formula for constructing a consumption aggregate where weights play the role of structural residuals that ensure the model explains the observed data. These weights have been interpreted by Redding and Weinstein as preference ("demand") parameters for individual goods. In this paper, I suggest a different interpretation of structural residuals, under which the weights for each of products (goods or services) used for constructing the consumption aggregate are assumed to be inversely related to the differences between the price of each individual product and the overall price level. Such an approach enables the construction of a consumer price index under the condition that the change over time in squared differences between the price of each individual good and the overall price level is minimized. In other words, the model allows researchers to estimate what will the value of a price index be if the changes in squared deviations of relative prices from the mean are minimal. Secondary analysis of publicly available data produced by the Office for National Statistics (ONS) reveals the fact that the "hypothetical" price indices based on the above assumption are numerically close to the Jevons price indices, as shown in Table 1. This finding provides a broader view of the Jevons price index and its substantiation on the grounds of the economic approach to index number theory.

The inverse relationship between the above mentioned weights and the deviation of individual prices from the overall price level implies a price-dependent utility function. In simple words, it implies that there exist people, at least a small portion of the population, who feel uncomfortable with themselves while buying either very expensive or very cheap product.

Although price-dependent utility functions are not widely used in economic research, this type of utility function can be considered as a viable alternative for some economic models (see, e.g., Balasko, 2003; Pollak, 1977; Basmann, Molina, and Slottje, 1987; Barucci and Gazzola, 2014). According to Balasko (2003, p. 333), there exist circumstances under which individual preferences are affected by prices as, for example, when economic agents take relative prices as an indication of quality. In order to represent preferences that depend not only on direct consumption but also on relative prices, Balasko (2003, pp. 334-335) considers utility functions of the form $u_i(q, x_i)$, where q denote a price vector normalized by the simplex convention, and x_i denotes individual consumption.

The inverse relationship between relative prices and weights used for constructing a consumption aggregate can be justified on behavioural economics grounds, within which regret theory seems to provide a theoretical framework for explaining such an inverse relationship. It is widely accepted and empirically proven that for most products price and objective quality are positively correlated (see, e.g., Steenkamp, 1988; Gagnon-Bartsch and Rosato, 2022). However, most correlations are weak (Steenkamp, 1988, pp. 504-505). As Guo and Jiang (2016) noted, a firm may reduce its quality to a greater extent than it reduces its price. Analogously, a firm may increase the price of an item to a greater extent than it increases its quality. So, because of the low correlation between price and quality, there always exists a chance that retailers, whether intentionally or not, overestimate their product quality. Intuitively, the higher the difference between the price of a product and the overall price level, the higher the chance of inconsistency between its price and quality.

According to regret theory, one of the important factors affecting many people's choices is an individual's capacity to anticipate feelings of regret and rejoicing. If there exists a more desirable consequence than an individual has chosen, the individual may experience regret (Loomes and Sugden, 1982, pp. 822, 808). Regret theory postulates that the individual chooses between actions so as to maximise the mathematical expectation of modified utility that is defined as the sum of two functions, one of which is "a choiceless utility function" and the other is a regret-rejoice function, "which assigns a real-valued index to every possible increment or decrement of choiceless utility" (Loomes and Sugden, 1982, p. 808-809).

Quiggin (1994) noted that the original version of regret theory might equally be called rejoicing theory; he suggested a model which is strictly a theory of regret. According to Quiggin (1994), a particularly tractable functional form arises when the function ϕ representing utility is multiplicatively separable, so that

$$\phi(x, M) = u(x)\psi(M) , \quad (1)$$

where *x* is the chosen outcome and *M* the best outcome available (see also Goossens, 2022). Thus, "the effect of regret is simply to attach different weights to the states of the world" (Quiggin, 1994).

In the model presented below, the functional form of the utility function is in accord with equation (1).

II. Incorporating relative prices into the consumption aggregate for solving the household's optimization problem

In line with economic approach to index number theory, it is assumed that households regard the observed price data as given, while the quantity data are regarded as solutions to economic optimization problem (Diewert, 2004, p. 313; Feenstra and Reinsdorf, 2000).

Assume there is a basket of products selected randomly or purposefully from the entire universe of products. Let there be *n* products in the basket with prices $p_{i,t}$, where *t* denotes time.

Denote by $\Psi_{i,t}$ the relative price of product *i* in period *t*, that is, $\Psi_{i,t} = \frac{p_{i,t}}{P_t}$, where P_t is the average of current prices across the entire spectrum of products produced in the economy.

As assumed above, the chance of inconsistency between the price of i-th product and its quality increases when $|\Psi_i - 1|$ increases. So, it is reasonable to assume that higher value of $|\Psi_i - 1|$ leads to stronger regret associated with a higher risk of buying a product that is overpriced compared to its quality. For the sake of concreteness, suppose regret associated with buying a product whose relative price is $\Psi_{i,t}$ equals $\frac{\mathrm{e}^{-b\left(\Psi_{i,t}-1\right)^2}}{\sum_{i=1}^{n} \mathrm{e}^{-b\left(\Psi_{i,t}-1\right)^2}}$, where *b* is some constant.

Let the aggregate consumption in period *t*, C_t, be defined using the following CES aggregator:

$$C_t = \left[\sum_{i=1}^{n} c_{i,t}^{(\eta-1)/\eta} \, w_{C\ i,t}\right]^{\eta/(\eta-1)}, \quad (2)$$

where $c_{i,t}$ denotes the consumption of product *i* in period *t* (*i* = 1, 2,..., *n*), η is the elasticity of substitution between any two products, and the weight attached to product *i* is

$$w_{C\ i,t} = \frac{\mathrm{e}^{-b\left(\Psi_{i,t}-1\right)^2}}{\sum_{i=1}^{n} \mathrm{e}^{-b\left(\Psi_{i,t}-1\right)^2}}. \quad (3)$$

Here, $w_{C\ i,t}$ represents the effect of regret which attaches different weights to $c_{i,t}$s. So, C_t is the weighted generalized mean of $c_{i,t}$s. Intuitively, the effect of regret should be rather small, that is, the value of *b* should be small, and $w_{C\ i,t}$ is close to 1/*n*.

The function represented by equation (2) can be referred to as a CES consumption aggregator with price-dependent weights.

The most widely accepted approach to solving the household's optimization problem (Dixit and Stiglitz, 1977; Galí, 2008, pp.41-43; Woodford, 2003, pp.143-147), which is to maximize C_t for the given expenditure level $S_t = \sum_{i=1}^{n} p_{i,t} c_{i,t}$, yields the set of equations

$$\frac{p_{i,t}}{p_{j,t}} = \frac{\mathrm{e}^{-b\left(\Psi_{i,t}-1\right)^2}}{\mathrm{e}^{-b\left(\Psi_{j,t}-1\right)^2}} \left(\frac{c_{i,t}}{c_{j,t}}\right)^{-1/\eta} . \quad (4)$$

Let $c_{j,t}$ be chosen equal to the average consumption $\bar{C}_t$ such that total consumption, C_t, is equal to $n\bar{C}_t$, where n is the number of products. Then the total monetary price of all products is $P_t n\bar{C}_t$, hence $P_t = \frac{\text{total monetary price of all products}}{n\bar{C}_t}$ is an average price level.

Under the above assumption, equation (4) can be rewritten as

$$\frac{p_{i,t}}{P_t} = \mathrm{e}^{-b\left(\Psi_{i,t}-1\right)^2} \left(\frac{c_{i,t}}{\bar{C}_t}\right)^{-1/\eta} \quad (5)$$

or, equivalently,

$$\frac{c_{i,t}}{\bar{C}_t} = \mathrm{e}^{-\eta b\left(\Psi_{i,t}-1\right)^2} \left(\frac{p_{i,t}}{P_t}\right)^{-\eta} . \quad (6)$$

Multiplying both sides of equation (5) by $\left(\frac{p_{i,t}}{P_t}\frac{1}{n}\right)^{-1/\eta}$ and solving for P_t yield

$$P_t = p_{i,t} \left(\frac{1}{n}\right)^{-\frac{1}{\eta-1}} \mathrm{e}^{\frac{\eta}{\eta-1} b\left(\Psi_{i,t}-1\right)^2} w_{i,t}{}^{\frac{1}{\eta-1}} , \quad (7)$$

where $w_{i,t} = \frac{p_{i,t}c_{i,t}}{P_t n\bar{C}_t} = \frac{p_{i,t}c_{i,t}}{S_t}$ is the share of product *i* in the total household spending in periods *t*.

Assume that $n_t = n_{t-1} = n$, and $\eta_t = \eta_{t-1} = \eta$, then it follows from equation (7) that

$$\frac{P_t}{P_{t-1}} = \frac{p_{i,t}}{p_{i,t-1}} \left[\frac{\mathrm{e}^{b\left(\Psi_{i,t}-1\right)^2}}{\mathrm{e}^{b\left(\Psi_{i,t-1}-1\right)^2}}\right]^{\frac{\eta}{\eta-1}} \left(\frac{w_{i,t}}{w_{i,t-1}}\right)^{\frac{1}{\eta-1}} , \quad (8)$$

where $w_{i,t}$ and $w_{i,t-1}$ are the shares of product *i* in the total household spending in periods *t* and *t*-1, respectively.

If $\mathrm{e}^{-b(\Psi_{i,t}-1)^2}$ is ignored then equations (6) come down to the set of demand equations in the Basic New Keynesian Model (Galí, 2008, p. 42; Woodford, 2003, p. 147):

$$\frac{c_{i,t}}{\bar{C}_t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\eta} . \quad (9)$$

Differentiating both sides of equations (9) and (6) with respect to $\Psi_{i,t}$ gives the following equations, respectively:

$$\frac{\mathrm{d}}{\mathrm{d}\Psi_{i,t}}\left(\frac{c_{i,t}}{\bar{C}_t}\right) = -\eta {\Psi_{i,t}}^{-\eta-1} , \quad (10)$$

$$\frac{\mathrm{d}}{\mathrm{d}\Psi_{i,t}}\left(\frac{c_{i,t}}{\bar{C}_t}\right) = -\eta \mathrm{e}^{-\eta b(\Psi_{i,t}-1)^2} {\Psi_{i,t}}^{-\eta-1}\left\{2b\Psi_{i,t}\left(\Psi_{i,t}-1\right)+1\right\} . \quad (11)$$

Figure 1 below provides the graphical comparison of equations (10) and (11).

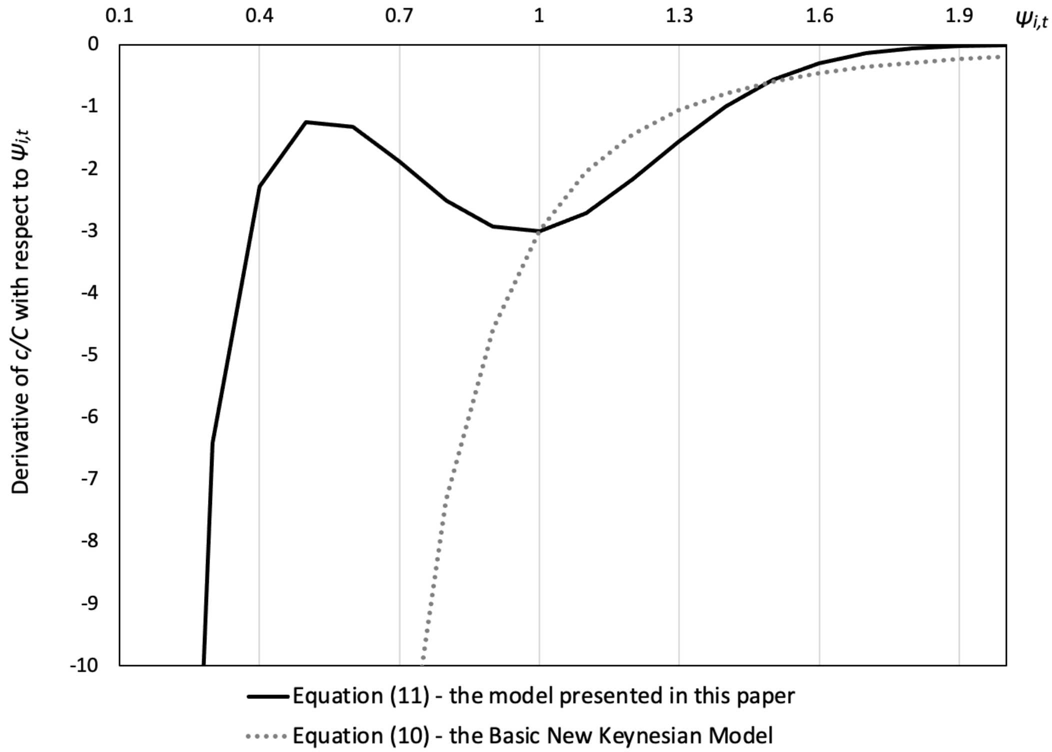

FIGURE 1. Graphical Representation of Equations (10) and (11) (*η*=3; *b*=1.8)

Consider two scenarios: in the first, the relative price for a product increases from $1 - \Delta\Psi_{i,t}$ to 1; in second, the relative price for the same product increases from 1 to $1 + \Delta\Psi_{i,t}$ (where $\Delta\Psi_{i,t}$ is very small). Intuitively, the change in $\frac{c_{i,t}}{\bar{C}_t}$, $\Delta\frac{c_{i,t}}{\bar{C}_t}$, should be the same for the two scenarios. Obviously, the equality between these two changes in $\frac{c_{i,t}}{\bar{C}_t}$ is inconsistent with equation (9). However, in the case of equation (6) the difference in $\Delta\frac{c_{i,t}}{\bar{C}_t}$ between the first and second scenarios is smaller as compared to that obtained from equation (9), as can be seen in Figure 1. This provides some justification for the correctness of equations (2) and (3).

III. Estimating the consumer price index under the assumption that the change over time in squared deviations of relative prices from 1 is minimized

Taking the logs of both sides of equation (8) and rearranging yields

$$b\left(\Psi_{i,t}-1\right)^2 - b\left(\Psi_{i,t-1}-1\right)^2 = \ln\frac{P_t}{P_{t-1}} - \ln\frac{p_{i,t}}{p_{i,t-1}} - \frac{1}{\eta}\left(\ln\frac{P_t}{P_{t-1}} - \ln\frac{p_{i,t}}{p_{i,t-1}} + \ln\frac{w_{i,t}}{w_{i,t-1}}\right). \quad (12)$$

The value of the expression shown below can serve as a summary measure of change from one time period to another in squared deviations of relative prices from 1:

$$D = \sum_{i=1}^{n}\left[b\left(\Psi_{i,t}-1\right)^2 - b\left(\Psi_{i,t-1}-1\right)^2\right]^2. \quad (13)$$

D is equal to the sum of the squares of the right side of equation (12), it is a function of η. The value of η that minimizes *D* is the hypothetical value of elasticity which, had it been the actual numerical value of elasticity, would have ensured minimal year-over-year changes in the distances between Ψ_is and 1. Let η^* denote this hypothetical value of elasticity.

Taking the first order condition for minimizing *D* with respect to $1/\eta$ gives the following:

$$\frac{1}{\eta^*} = \sum_{i=1}^{n}\frac{\ln\frac{P_t}{P_{t-1}} - \ln\frac{p_{i,t}}{p_{i,t-1}}}{\ln\frac{P_t}{P_{t-1}} - \ln\frac{p_{i,t}}{p_{i,t-1}} + \ln\frac{w_{i,t}}{w_{i,t-1}}} W_{\eta\, i,t}\,, \quad (14)$$

where $W_{\eta\, i,t} = \frac{\left(\ln\frac{P_t}{P_{t-1}} - \ln\frac{p_{i,t}}{p_{i,t-1}} + \ln\frac{w_{i,t}}{w_{i,t-1}}\right)^2}{\sum_{i=1}^{n}\left(\ln\frac{P_t}{P_{t-1}} - \ln\frac{p_{i,t}}{p_{i,t-1}} + \ln\frac{w_{i,t}}{w_{i,t-1}}\right)^2}$

It follows from equation (8) that if both $e^{b\left(\Psi_{i,t}-1\right)^2}$ and $e^{b\left(\Psi_{i,t-1}-1\right)^2}$ are ignored then

$$\frac{1}{\eta_{(\text{structural residuals ignored})}} = \frac{\ln\frac{P_t}{P_{t-1}} - \ln\frac{p_{i,t}}{p_{i,t-1}}}{\ln\frac{P_t}{P_{t-1}} - \ln\frac{p_{i,t}}{p_{i,t-1}} + \ln\frac{w_{i,t}}{w_{i,t-1}}}. \quad (15)$$

Thus, according to (14) the reciprocal of the hypothetical value of elasticity is the weighted average of the set of inverse elasticity values calculated using equation (15) with weights $W_{\eta\ i,t}$.

Summing both sides of (12) over all products, dividing by n, and rearranging yields

$$\frac{\sum_{i=1}^{n}\left[b\left(\Psi_{i,t}-1\right)^{2}-b\left(\Psi_{i,t-1}-1\right)^{2}\right]}{n}=\frac{\eta-1}{\eta}\ln\frac{P_t}{P_{t-1}}-\frac{1}{n\eta}\sum_{i=1}^{n}\left\{(\eta-1)\ln\frac{p_{i,t}}{p_{i,t-1}}+\ln\frac{w_{i,t}}{w_{i,t-1}}\right\}. \quad (16)$$

or, equivalently,

$$\ln\frac{P_t}{P_{t-1}}=b\frac{\eta}{\eta-1}\left[Var\left(\Psi_{i,t}\right)-Var\left(\Psi_{i,t-1}\right)\right]+\frac{1}{n}\sum_{i=1}^{n}\ln\left[\frac{p_{i,t}}{p_{i,t-1}}\left(\frac{w_{i,t}}{w_{i,t-1}}\right)^{\frac{1}{\eta-1}}\right]. \quad (17)$$

Firms keep their relative prices close to 1 unless their marginal cost is so high that it is not profitable to sell (Klenow and Willis, 2016, p. 463), so Ψ_i is usually small. The value of b is presumably small as well. Therefore, the left-hand side of equation (16) does not significantly differ from zero, and

$$\ln\frac{P_t}{P_{t-1}}\approx\frac{1}{n}\sum_{i=1}^{n}\ln\left[\frac{p_{i,t}}{p_{i,t-1}}\left(\frac{w_{i,t}}{w_{i,t-1}}\right)^{\frac{1}{\eta-1}}\right], \quad (18)$$

or, equivalently,

$$\frac{P_t}{P_{t-1}}\approx\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\left(\frac{\widetilde{w}_t}{\widetilde{w}_{t-1}}\right)^{\frac{1}{\eta-1}}, \quad (19)$$

where tilde above a variable is used to denote a geometric mean across products.

Equation (19) is identical to equation (9) in Redding and Weinstein's 2020 article. Namely, equation (9) in Redding and Weinstein's 2020 article (p. 512) states that the exact price index is equal to $\frac{\tilde{p}_t}{\tilde{p}_{t-1}}\left(\frac{\tilde{s}_t}{\tilde{s}_{t-1}}\right)^{\frac{1}{\eta-1}}$ where s is the expenditure share of any individual product i.

The price index defined by equation (18) is a function of price changes $\frac{p_{i,t}}{p_{i,t-1}}$, but it is not their weighted (or unweighted) average of any kind. Because of this, price index (18) differs from the most commonly used price indices as well as from the less frequently used the quadratic mean price index (Diewert, 1978, p. 888) and the Lloyd index (Lloyd, 1975, p. 305).

If

$$\eta = 1 + \frac{\sum_{i=1}^{n} w_{i,t}^{*}\left(\ln\frac{w_{i,t}}{w_{i,t-1}} - \ln\frac{\tilde{w}_t}{\tilde{w}_{t-1}}\right)}{\sum_{i=1}^{n} w_{i,t}^{*}\left(\ln\frac{\tilde{p}}{\tilde{p}_{t-1}} - \ln\frac{p_{i,t}}{p_{i,t-1}}\right)}, \quad (20)$$

where $w_{i,t}^{*}$ are the Sato-Vartia weights, then the price index calculated according to equation (19) is the Sato-Vartia price index (see online appendix to Redding and Weinstein's 2020 article).

If $\eta = \eta^{*}$ then (19) is the formula for the consumer price index that takes place if

(1) the aggregate consumption is defined using CES aggregator (2),

(2) the variance of Ψ remains the same for two consecutive time periods,

(3) the dispersion of relative price changes between two time periods is minimized.

The value of η^{*} and the corresponding value of $\frac{P_t}{P_{t-1}}$ can be found by solving the system of equations (18) and (14). The numerical solution to the above system can be obtained iteratively. Before the first iteration, η^{*} should be set to some initial starting value, say η^{*}=2. At the first iteration, this value of η^{*} should be substituted into equation (18) to find the first iteration value of $\ln\frac{P_t}{P_{t-1}}$, which in turn can be substituted into equation (14) to find the first iteration value of η^{*}. The difference between the first iteration value of η^{*} and the initial value of η^{*} should be calculated. If the difference is close to zero, then the iterative process can be stopped, otherwise the next iteration is performed.

At the second iteration, the first iteration value of η^{*} is substituted into equation (18), which is solved to find the second iteration value of $\ln\frac{P_t}{P_{t-1}}$, which in turn is substituted into equation (14) to find the second iteration value of η^{*}, and so on until the difference in η^{*} between two consecutive iterations is close to zero. At last, η^* and $\frac{P_t}{P_{t-1}}$ obtained at the final iteration are the hypothetical value of elasticity and the consumer price index that are the target of the computational procedure just presented in this paper.

Solving the system of equations (18) and (14) yields the consumer price index that takes place if the change over time in squared deviations of relative prices from 1 is minimized. So, the consumer price index obtained as a result of the above iterative process can be referred to as a price index with price-dependent preferences and the minimal dispersion of relative price changes, let it be denoted by $I_{p\ proposed\ model}$.

Once the numerical value of price index (18) is calculated, it can be compared to the most commonly used conventional price indices. The empirical results demonstrate that the consumer price indices calculated according to equations (18) and (14) are close enough to the Jevons price indices.

Feenstra (1994, p. 175) and Redding and Weinstein (2020, pp. 533-534, 511-512) used a statistical technique for estimating the elasticity of substitution that permits correlation between the taste parameters, prices, and quantities. Then, the consumer price index is derived analytically from the equation containing observable variables and the estimated elasticity of substitution.

In the model presented in this paper, both the hypothetical value of elasticity, η^*, and the corresponding price index are derived analytically.

IV. Empirical results

The here presented results were produced in accordance with equations (18) and (14), and are based on data published by the Office for National Statistics (ONS)[1].

In the preprocessing stage, the consumer price indices for lower-level aggregates (with 2015=100) published by the ONS have been unchained to get year-over-year CPIs with previous year=1.

After completing the fourth iteration for solving the system of equations (18) and (14), the condition $\eta^*_{current\ iteration} - \eta^*_{previous\ iteration} \approx 0$ has been met. The Sato-Vartia price indices and the Jevons price indices have also been calculated in order to compare them with the indices obtained from equations (18) and (14). The results obtained have been tabulated as follows.

TABLE 1

The values of price indices obtained from the model, previous year = 100

	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019
Price index with price-dependent preferences and the minimal dispersion of relative price changes. ($I_{p\ proposed\ model}$)	102.1	103.7	104.0	102.3	101.8	101.0	99.3	100.2	102.8	102.4	101.6
Jevons price index ($I_{p\ J}$)	102.1	103.3	104.1	102.1	101.9	101.0	99.3	100.2	102.8	102.3	101.6
Price index with price-dependent preferences and the Sato-Vartia weights ($I_{p\ proposed\ model\ with\ SV\ weights}$)	101.5	103.6	104.2	102.9	102.5	101.4	100.2	100.6	102.4	102.3	101.8
Sato-Vartia price index ($I_{p\ SV}$)	101.5	103.2	104.2	102.7	102.5	101.4	100.2	100.6	102.3	102.2	101.8

[1] Office for National Statistics (22 June 2022) 'Consumer price inflation, UK: June 2022', Table 23. Available at: https://www.ons.gov.uk/economy/inflationandpriceindices/bulletins/consumerpriceinflation/june2022 (Accessed 25 August 2022).
Office for National Statistics (June 2023) 'Consumer trends April to June 2023', Data tables 'Consumer trends: current price, not seasonally adjusted'. Available at: https://www.ons.gov.uk/economy/nationalaccounts/satelliteaccounts/datasets/consumertrendscurrentpricenotseasonallyadjusted (Accessed 19 October 2023).

Within the stochastic approach framework the Jevons price index is derived from the regression model of the form

$$\ln\frac{p_{i,t}}{p_{i,t-1}} = \ln\frac{P_t}{P_{t-1}} + \ln\xi_i\ , \qquad (21)$$

where $\xi_i = 1 + \varepsilon_i$, and ε_i are random variables with mean 0.

The best linear estimator for $\ln\frac{P_t}{P_{t-1}}$ is

$$\widehat{\ln\frac{P_t}{P_{t-1}}} = \ln\left[\prod_{i=1}^{n}\left(\frac{p_{i,t}}{p_{i,t-1}}\right)^{\frac{1}{n}}\right]. \qquad (22)$$

It is obtained by solving the quadratic minimization problem

$$\min_{\ln\frac{P_t}{P_{t-1}}} \sum_{i=1}^{n}\left(\ln\frac{p_{i,t}}{p_{i,t-1}} - \ln\frac{P_t}{P_{t-1}}\right)^2 = \min_{\ln\frac{P_t}{P_{t-1}}} \sum_{i=1}^{n}\left(\ln\frac{p_{i,t}}{P_t} - \ln\frac{p_{i,t-1}}{P_{t-1}}\right)^2. \qquad (23)$$

In this paper, as can be seen from equations (14) and (18), the consumer price index $I_{p\ proposed\ model}$ is derived by minimizing (13) with respect to η. Minimization problems (23) and $\min_{\eta} D$ may not be mathematically identical, however, they are closely related from an economic point of view. Therefore, the values of $I_{p\ proposed\ model}$ and $I_{p\ J}$ are close to each other, with the exception of those compiled for calendar year 2010 when $I_{p\ proposed\ model} = 103.7$ and $I_{p\ J} = 103.3$. The most probable explanation for this difference is that equation (2) might not be fully compatible with the real-life consumer behavior in the year of 2010. However, for 10 years shown in Table 1 (2009 and 2011-2019) the difference between $I_{p\ proposed\ model}$ and $I_{p\ J}$ is small, thus providing justification for equation (2).

Equation (18) is the formula for calculating an unweighted price index. After introducing weights, equation (18) becomes

$$\ln \frac{P_t}{P_{t-1}} \approx \sum_{i=1}^{n} \left[w_{i,t}^* \ln \frac{p_{i,t}}{p_{i,t-1}} + \frac{1}{n} \ln \left(\frac{w_{i,t}}{w_{i,t-1}} \right)^{\frac{1}{\eta^*-1}} \right], \quad (24)$$

where $w_{i,t}^*$ denotes the weight attached to $\frac{p_{i,t}}{p_{i,t-1}}$, $\sum_{i=1}^{n} w_{i,t}^* = 1$.

Equation (24) should be used only at the final iteration of the iterative procedure described in section III, after a prior iteration determines the final value of η*.

After compiling price index (24) with $w_{i,t}^* =$ Sato-Vartia weights, it becomes clear that the resulting index, $I_{p\ proposed\ model\ with\ SV\ weights}$, well approximates the Sato-Vartia price index (with the exception of indices compiled for calendar year 2010). This also serves as a justification for equation (2).

The empirical research results presented in Table 1 are based on partially aggregated data (low-level aggregates), not product-level data. However, for the purpose of this paper that is to incorporate a price-dependent utility function into the theory of price indices, the use of partially aggregate data seems to be sufficient.

If the consumer price index calculated according to the model proposed in this paper accurately reflects the change in the overall price level, then it is easy to find out whether the price for any specific product *i* approaches the level of average prices or moves away from it. The expression $\frac{p_{i,t}}{p_{i,t-1}} / \frac{P_t}{P_{t-1}} - 1$ gives the relative change in the relative price for product *i*, $\Psi_i = \frac{p_i}{P}$, from period *t-1* to *t*. If the value of the expression $\frac{\mathrm{e}^{-b\left(\Psi_{i,t}-1\right)^2}}{\mathrm{e}^{-b\left(\Psi_{i,t-1}-1\right)^2}}$ that can be obtained from equation (8) is greater than 1, then the relative price for product *i*, Ψ_i, moves towards 1 from the left or the right (under the plausible assumption that the distribution of products' relative prices is normal with mean 1). So, for instance, if $\frac{p_{i,t}}{p_{i,t-1}} / \frac{P_t}{P_{t-1}} < 1$ and $\frac{e^{-b(\Psi_{i,t}-1)^2}}{e^{-b(\Psi_{i,t-1}-1)^2}} > 1$, then Ψ_i is moving towards 1 from the right.

V. Conclusion

The results of the study presented here show that, in most of the years during the time period under consideration, the unweighted price index constructed under the assumptions of price dependent utility function well approximates the Jevons price index. Such a good approximation, in turn, serves as a justification for using the price-dependent utility function.

The only exception is the year 2010 for which the model presented in this paper does not provide a good approximation of the conventional geometric price indices. This fact suggests that, in 2010, there was a temporary shift in the functional form of the utility function. At least, utility function (2) cannot serve as a starting point for finding a numerical approximation to the conventional geometric price indices for the year 2010.

Thus, there are two main findings in this paper. First, this research shows how the price-dependent utility function can be used, within the scope of the economic approach to index number theory, for the purpose to approximate the conventional geometric price indices. Second, the model allows researchers to identify the year in which a temporary shift in the functional form of the utility function was likely to occur. It is very likely to happen when the price index calculated according to the model does not provide a good approximation to the Jevons price index. However, the temporary shift in the functional form of the utility function may not be the only possible cause of an inaccurate approximation. So, in the case of an inconsistency between the two above-mentioned price indices, additional research is required to clarify this issue.

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