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ABSTRACT

This paper presents a model to calculate daily returns and corresponding NAV changes of hedge funds. In the past, the values of hedge funds were typically available on monthly basis. The model link daily hedge fund performance with the returns on indices selected to provide a comprehensive spectrum of possible market exposures. The model gives an estimate of the daily returns of hedge funds based on the daily values of a list of market indices. The daily return of each hedge fund is estimated as a linear combination of daily market index returns. The coefficients of this linear combination are obtained through linear regression of monthly index returns against monthly hedge fund returns.

Key words: hedge fund, daily return, cash flow, market index, linear regression.

JEL Classification: E44, G21, G12, G24, G32, G33, G18, G28

Hedge fund returns come from trading in various sectors of the market, such as equities, foreign exchange, fixed income, etc., and index data capturing market fluctuations in these areas is available on a daily basis.

Moreover, fund managers generally rely on strategies based on hedging long positions in one sector with long positions in another (for example, playing currencies against each other, or taking offsetting positions in large and small-cap equities).

Therefore, it appears reasonable to attempt to describe daily hedge fund performance on the basis of "portfolios" (mathematically speaking, *linear combinations*) of returns on indices selected to provide a comprehensive spectrum of possible market exposures. Indices are designed to capture the general direction and size of market movements and are generally accepted as good proxies for overall market behavior of a given sector.

These "portfolios" should be allowed to have positive or negative weights, depending on the style of trading of each fund. The problem therefore revolves around finding suitable weights (*coefficients*). We have taken an approach that relies on the (reasonable) assumption that if a fund has a certain exposure to some combination of market factors on a monthly basis, that will also be the case for daily returns, and that daily exposures will reflect monthly ones. Note that we use the term "portfolios" only in a figurative sense here, since from the mathematical point of view there is no requirement that the weights add up to 100%.

Harvey et al (2016) introduce a new multiple testing framework and provides historical cutoffs from the first empirical test in 1967 to 2016. Dimmock and Gerken (2016) and Honigsberg (2019) show that various measures of misreporting decline after increases in regulation, and this could worsen observed performance. If fund managers smooth returns less intensively, then reported volatility would increase and Sharpe ratios would decrease.

Aragon and Nanda (2017) study the timeliness of hedge fund monthly performance disclosures and conclude that timely disclosure is an important consideration for hedge fund managers and investors. Barth et al (2021) estimate that the worldwide net assets under hedge fund management is larger than the most generous estimate and also show that the total returns earned by funds that report to the public databases are significantly lower than the returns of funds that report only on regulatory filings.

Jackwerth and Slavutskaya (2016) assess the addition of alternative assets to pension fund portfolios in terms of the total benefit derived from diversification, addition of positive skewness, and the elimination of left tails of returns. Joenväärä et al (2019) re-examine the fundamental questions regarding hedge fund performance and find a significant association between fund-characteristics related to share restrictions as well as compensation structure and risk-adjusted returns.

Jorion and Schwarz (2019) show that truncation largely preserves backfilled returns and document that either of these backfill treatments can lead to biased empirical findings, including cross-sectional results. McLean and Pontiff (2016) study the out-of-sample and post-publication return predictability of variables shown to predict cross-sectional stock returns.

This paper proposes a method to calculate the daily returns of hedge funds when only *monthly* data for the funds is available. In the first version of the model an attempt was made to use "all available indices" simultaneously. Empirical testing showed that this approach leads to deterioration of the quality of estimation of daily returns. In addition, when the large number of indices is present in the model, the regression R-Squared becomes high regardless of the predictive power of the model and cannot be used as an accurate measure of the accuracy of the estimation.

It became clear that in order to improve accuracy of estimation it is necessary: a) To develop a method to identify a set of indices that is relevant for each particular hedge fund and to use only these indices to estimate daily returns and b) To develop a measure of accuracy of the estimation that does not depend on the number of indices used in the model.

In the subsequent versions of the model, in order to identify a minimum set of indices that should be used in exposure profile, we used a combination of two methods – forward regression and backward elimination.

The approach consists of two stages. The first stage (forward regression) starts with a single index selected based on maximum correlation with the returns of the hedge fund.

Other indexes are sequentially added to the model using maximum partial correlation as the criterion for inclusion in the model.

After the number of indices included in the model reaches certain predetermined number, the reverse process (backward elimination) starts – indices are sequentially removed from the model using the value of t-statistic as the criterion for removing an index. The process stops when only one index is left.

In the end, this approach produces several sets of indices that are reasonable candidates for the final model. The choice among these sets represents a complex task because standard statistics, such as F-statistic, t-statistic and R-Squared are biased when a selection process is used to choose indices from a large set.

Several approaches have been tested on a representative set of hedge funds leading to the conclusion that the choice of indices based of R-Squared adjusted for selection bias yields best results.

We assume that the all the ticker symbols are unique, and that for a given symbol and valid date there is at most one record. We also assume that all records in the input holding data of the appropriate type are valid. Records containing one or more blank or invalid fields are taken to be invalid and discarded.

Calculation of the daily index returns is based on the difference in value between two consecutive valid dates. If index data is missing for the current valid date, the return is calculated as 0 (no change in the index value with respect to the latest previous valid date). If data for a given fund is available for the current date, but missing or unavailable for the latest valid previous date, the current return is calculated by searching backwards from the list of valid dates until a value is found and calculating a daily return by interpolation.

Calculation of monthly returns for indices is based on the difference between the last piece of data available for each index on two consecutive months. It is assumed that the data. It is assumed that there will be no gaps in the data stream, that is, after inception of any given fund, at least one daily value *must* be available on any given month.

Calculation of the monthly index return for the current month (which is typically not used in the process, since hedge funds report their monthly returns after the end of the month) is done on a month-to-date basis.

The rest of this paper is organized as follows: The model is described in Section 1; Section 2 elaborates index selection. Practical discussions are presented in Section 3; the conclusions are given in Section 4.

1. Model

Hedge fund returns come from trading in different sectors of the market, such as domestic and international equities, fixed income, currencies, etc. Each return can be partitioned into two components – systematic and specific components of the return.

The systematic return component can be represented as a linear combination (a weighted sum) of returns of market indices and factors, such as US equity indices, fixed income indices, international equity indices, etc.

The specific return component is determined by the factors specific to the individual securities the fund has a position in and is not related to the general economic and market indices. It follows, that for each period the return of a hedge fund can be written as

$$R_F = \sum_j b_j R_j + S_F \qquad (1)$$

Where:

R_F - return of a hedge fund during the time period

R_j – return of an index j during the time period

b_j – coefficients describing the sensitivity of the fund to the changes in the index

S_F – the specific component of the hedge fund's return

The expression (1) describes the relationship between returns of the fund and returns on the market indices and factors. The values of the coefficients b_j can be estimated based on the history of the returns of the hedge fund and returns on the corresponding indices using multiple linear regression.

Mathematically, then, we say that if the monthly returns of a fund f can be approximately expressed as

$$MR_f \approx \sum_j c_j MR_{j_j} \qquad (2)$$

in terms of regression monthly index returns MR_j and coefficients c_j, then

$$DR_f \approx \sum_j c_j DR_j \qquad (3)$$

approximately expresses the daily fund returns in terms of the same coefficients and daily index returns.

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$$R_F = \sum_j b_j R_j + S_F \qquad (4)$$

Where:

R_F - return of a hedge fund during the time period

R_j – return of an index *j* during the time period

b_j – coefficients describing the sensitivity of the fund to the changes in the index

S_F – the specific component of the hedge fund's return

The expression describes the relationship between returns of the fund and returns on the market indices and factors. The values of the coefficients b_j can be estimated based on the history of the returns of the hedge fund and returns on the corresponding indices using multiple linear regression.

The key to the functionality of the program lies in the use of *linear regression* to produce a suitable set of coefficients. Linear regression is a time-tested methodology to find the set of coefficients (weights) that would result in the linear combination of a set of variables providing the best approximation to another variable.

2. Index Selection

The "naïve" approach to the problem of selecting independent variable (factors and indices) for the model (see https://finpricing.com/lib/EqWarrant.html) is to include all available indices and factors in the model. This approach does not work for three reasons:

First, the number of indices cannot exceed the number of dates for which returns are available – this is the mathematical requirement of the linear regression model used to estimate the exposure profile.

Second, the accuracy of forecast cannot increase (and usually decreases) when indices that are not relevant for the particular fund are included in the model.

Finally, inclusion of highly correlated indices in the model leads to large errors in estimation of *b*-coefficients and unstable forecasts.

These lead to the necessity to develop an algorithm to choose a relatively small number of highly relevant indices out of thousands of indices available from online data sources.

The choice of the best set of indices is made based on comparison of the actual value of the regression R-Squared to the value of the regression R-Squared expected under the hypothesis that there is no relationship between the independent and dependent variables This latter value is called Selection R-Squared.

The algorithm to calculate the value of selection R-Square consists of three steps:

First considering a random variable Y that under a null hypothesis H_0 has distribution $F(y|H_0)$. Given an outcome y of a random experiment e we reject the null hypothesis H_0 when the probability of the outcome y is less than P-Value

$$F(y|H_0) < \text{P-Value}. \qquad (4)$$

Then it can be shown that if we measure the maximum outcome y' = MAX(y) of n experiments, we should reject the H_0 if $F(y'|H_0)$ < AdjP-Value, where

$$\text{Adjusted P-Value} = 1\text{-}\exp^{\ln((1\text{-P-Value})/n)} \qquad (5)$$

Now let's assume that we selected k "best" indices out of K available indices. This can be viewed as selecting the maximum observation out of n= C^k_K random experiments, where C^k_K denotes the number of combinations from K by k.

Secondly, in our case H_0 means that there is no relationship between the index returns and returns of the hedge fund. Then under H_0 the value of regression R^2 is distributed as Beta (R^2, a,b), where

a = k/2

b = (p-k-1)/2

k – the number of indices

p – number of observations

Third, given the index selection procedure, the null hypothesis H_0 should be rejected if

$$R^2 > Beta^{-1}(\text{Adjusted P-Value, a,b}) \qquad (6)$$

The threshold value

$$R^2_s = Beta^{-1}(\text{Adjusted P-Value, a,b}) \qquad (7)$$

is called selection R Squared.

Fourth, when the value of regression R^2 is less or equal to the value of selection R^2, the model has no predictive value and the predictive R^2_p is 0.

When $R^2 > R^2_s$, the value of predicted R^2_p is estimated as

$$R^2_p = (R^2-R^2_s)/(1-R^2_s) \qquad (8)$$

The obtained value of the R^2_p is further adjusted to take into account p-levels of the individual regression coefficients, i.e., the probability that the "least reliable" of the regression coefficient is zero. This adjustment is accomplished by the following formula

$$R^2_p = R^2_p\,(1-PV), \qquad (9)$$

where PV is the maximum p-value across all independent variables in the model.

Empirical testing shows that the values of R^2_p obtained by the process described above tend to underestimate the actual predictive power of the model. This fact is not important in the index selection process, because the actual goal is to rank index set.

The section above describes the process of selecting the best combination of indices. After the best combination is determined, the predictive power of the model is further adjusted based on the results of the Jackknife simulation according to the formula

$$R^2_p = ((R^2 + R^2_j) - R^2_s)/(2 - R^2_s)$$

The value of R^2_j represents the outcome of the Jackknife simulation of the prediction errors. It is computed as a squared correlation of the actual returns and returns predicted during the Jackknife process.

This adjustment is desirable in order to compensate for the consistent underestimation of predictive power of the model caused, among other factors, by the lack of independence among indices.

3. Practical Discussion

The main problem in the practical application of this approach lies in the fact that on one hand the number of indices that can be used to build a model is very large and, on the other hand, the particular combination of indices relevant to a particular fund is usually not known.

The predictive power of the model is estimated based on the average of the values of Jackknife R-Squared and regression R-Squared, adjusted by the number of indices available in the index selection process.

The value of the R-Squared is computed by the linear regression module, while the value of the of Jackknife R-Squared statistic is computed by systematically excluding dates from the model and comparing predicted returns to the actual returns for each excluded date.

Even given the best model, the predictive power may be low for a substantial proportion of hedge funds – specifically those that hedge their exposure to the market factors. To forecast daily returns for these “low R-Squared” funds, Version 3.2 implements a proxy-based approach; the hedge funds with low R-Square are combined in a fund of funds, and the index selection procedure is applied to this fund of funds.

This approach leads to the determination of the exposure profile of the group of hedge funds, as opposed to the individual funds. This exposure profile represents the “best proxy” in the sense that when daily returns of the low R-Squared funds are estimated based on this proxy, the sum of estimated daily returns of individual hedge funds provides the best estimate for the returns of the total portfolio.

In particular, we apply this methodology to the returns on monthly market indices to come up with a suitable linear combination of index returns, approximating the monthly performance of each hedge fund. Once these coefficients are obtained, capturing the dependence of hedge fund returns on the underlying market variables, they are used to calculate a linear combination of the current daily index returns, thus providing the desired daily estimates.

We conducted an extensive research project to search for the market indices that would be most representative of hedge fund returns. The results of that research were key to the selection of the market indices selected for this daily NAV return calculator.

What was found is that a good proportion of hedge funds returns exhibit some degree of dependence to linear combination of the following indices:

- SPX, NASDAQ return and/or historical Implied return
- Market volatilities (historical, implied, equity, bonds). VIX indices.
- SML historical implied return
- Currency exchange rates
- Barra and Russel indices.
- SSB U.S. , Asia and Latin America indices.
- Lehman Mortgage Backed Index, and High Yield Credit Bond Index
- Government bond returns, yield and price basis (act/act); swap returns, yield and price basis (act/365 fixed)
- Curve exposure and Shape exposure (PV01 weighted).
- HFRI Indice

Volatility and volatility indices tend to be good trackers for momentum traders, short term traders and counter-trend traders.

Spreads between Russel indices, for instance, is a good proxy for long-short funds that trade sector or cap spreads in the equity markets.

Fixed income traders are the hardest to track, but bond, swap and mortgage indices provide partial return proxy information.

Many hedge funds obtain returns trading credit qualities of different instruments, and the credit index captures the contributions to the hedge fund returns coming from these trades.

In what follows, the term *valid date* will mean a date for which at least one of the index values is available. Examples of invalid dates include weekends and holidays. The term *inception* will refer to the beginning of a data stream.

The interval over which the regression is calculated is user-selected, but typically expected to be a 30-month period. The choice of a suitable length of time for this interval has to balance the facts that regression operates better as more data is available, and on the other hand, that as the data goes further back in time, it may be less relevant to current conditions. The bare minimum number of data points that regression demands equals the number of regression variables (currently 12); therefore 30 months constitutes a sensible compromise.

The regression and all the subsequent calculations are calculated against the list of indices appearing on the input data. If for any reason a smaller set of indices is desired for the procedure, it is enough to change that list accordingly, without any modifications to the input, or any impact on the behavior of the program.

First we calculate the monthly returns against which the monthly fund data will be regressed, and the current daily return for each index.

The second step takes the list of hedge funds and executes a number of sub-steps for each fund. The first one is the extraction and completion of the data for the given fund. If the fund has an inception date later than the beginning of the regression period, the initial section of the data stream is completed with the data from a proxy index. We are currently using the HFR Fund of Funds Index, under the symbol HFRIFOFI, but other choices are possible.

After extraction and completion of the data, it is possible that gaps occurring after the fund's inception still remain. If the number of such gaps is below a user-defined threshold (our current default for a 30-month period is 5), the issue is ignored, and the next steps continue on the basis of the available data.

Next the fund is regressed against the list of monthly indices, thus obtaining the regression coefficients, which are finally used to compute the linear combination of daily index returns stored in the *Output* sheet.

4. Conclusion

This article presents an approach for the purpose of modeling daily returns and corresponding net asset value (NAV) changes of individual hedge funds. NAV values of hedge funds are typically available on monthly basis. The approach to estimate the daily NAV for a hedge fund is based on modeling daily returns of the hedge fund as a weighted sum of returns of a combination of several market indices and factors.

For each hedge fund, the Model is calibrated based on historical monthly returns of the fund and the market indices and factors, resulting in an “exposure profile”. Exposure profile is a vector of index weights representing the “best” estimate of the systematic return of the fund in the past.

The approach consists of two stages. The first stage (forward regression) starts with a single index selected based on maximum correlation with the returns of the hedge fund. Other indexes are sequentially added to the model using maximum partial correlation as the criterion for inclusion in the model.

After the number of indices included in the model reaches certain predetermined number, the reverse process (backward elimination) starts – indices are sequentially removed from the model using the value of t-statistic as the criterion for removing an index. The process stops when only one index is left.

In order to eliminate irrelevant indices implemented an index selection process. Variables were added and deleted from the model based on the maximum and minimum value of the corresponding t-statistic. The process stopped when the maximum value of t-statistic (across all variables in the model) was less than the value of a dynamically computed threshold. The threshold value took into account the number of choices (independent variables) available at each step of the selection process.

The choice of the best model is based on the maximum value of the Predicted R-Squared for the model. The value of the predicted R-Squared was calculated as a minimum variance estimate of the Jackknife R-Squared (adjusted for the number of choices available for each model) and *a priori* estimate of the multiple correlation coefficient.

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