MPRA
Munich Personal RePEc Archive

Effect of a cost channel on monetary policy transmission in a behavioral New Keynesian model

Ida, Daisuke and Kaminoyama, Kenichi

12 March 2024

Online at https://mpra.ub.uni-muenchen.de/120424/
MPRA Paper No. 120424, posted 23 Mar 2024 10:26 UTC

Effect of a cost channel on monetary policy transmission in a behavioral New Keynesian model*

Daisuke Ida†[a,b] and Kenichi Kaminoyama‡[a]

[a]Faculty of Economics, Momoyama Gakuin University
[b]Research Fellow, Graduate School of Economics, Kobe University

March 12, 2024

Abstract

This paper explores the impact of the cost channel on the monetary transmission mechanism in a behavioral New Keynesian model. In contrast to previous studies, we demonstrate that the degree of cognitive discounting significantly affects the determinacy condition in the model with a cost channel. Second, we show that the price puzzle arises only when a large value of the cost channel parameter, which is not empirically supported, is introduced with a high degree of cognitive discounting. Third, we find that the degree of cognitive discounting significantly impacts the effect of the cost channel on optimal monetary policy.

JEL codes: E52; E58;

Keywords: Cognitive discounting; New Keynesian model; Cost channel; Monetary policy rules; Price puzzle;

---

*Ida was supported by JSPS KAKENHI Grant Numbers JP20K01784 and JP23K01410. Kaminoyama was supported by JSPS KAKENHI Grant Number JP21K01454. All remaining errors are our own.

†Faculty of Economics, Momoyama Gakuin University, 1-1, Manabino, Izumi, Osaka 594-1198, Japan. Tel.: +81-725-92-6134. E-mail: ida-dai@andrew.ac.jp

‡Faculty of Economics, Momoyama Gakuin University, 1-1, Manabino, Izumi, Osaka 594-1198, Japan. E-mail: kaminoya@andrew.ac.jp

1 Introduction

A growing body of previous studies has addressed the important research topic of how monetary policy transmission works. The traditional monetary policy transmission indicates that contractionary monetary policy shocks lead to a decline in inflation and output gap. However, another channel for the supply-side effect of monetary policy has been explored empirically and theoretically (Abo-Zaid, 2022; Barth III and Ramey, 2001; Beaudry et al., 2024; Christiano et al., 2005; Henzel et al., 2009; Ida, 2023; Nie, 2023; Ravenna and Walsh, 2006; Tillmann, 2008), namely, the cost channel.[1] The cost channel focuses on the role of a firm's working capital, and its increase causes inflation through tightened marginal costs. If we allow for this channel, an increased nominal interest rate may provide a different consequence contrary to the traditional monetary policy transmission.[2] For instance, Ravenna and Walsh (2006) showed that the cost channel creates an endogenous policy trade-off between inflation and the output gap in terms of optimal monetary policy because the aggregate supply curve positively depends on the nominal interest rate. Llosa and Tuesta (2009) found that the cost channel restricts the central bank's policy reaction to fluctuations in inflation and the output gap to achieve the unique determinate equilibrium in the NK model.

The effectiveness of monetary policy depends on how the central bank successfully manipulates the expectations of the private sector. The expectations channel of monetary policy plays a significant role in the NK model (Woodford, 2003). Without the cost channel, if the central bank can correctly manipulate the expectations of the private sector, it can strengthen the demand channel of monetary policy. However, its presence may reduce the effectiveness of monetary policy through the demand channel through which rising nominal interest rates lead to a contraction in output. In addition, the policy prescription of the standard NK model is based on the assumption of rational expectations. Unfortunately, this assumption often yields a paradoxical policy prescription, such as the forward guidance puzzle (Del Negro et al., 2012).[3]

[1] Ida (2023) and Patel (2021) addressed the role of a cost channel in a two-country New Keynesian (NK) model.

[2] In contrast to previous studies that supported the cost channel hypothesis, Rabanal (2007) argued that the role of the cost channel is limited by implementing the Bayesian estimation of the dynamic stochastic general equilibrium model.

[3] The forward guidance puzzle indicates the inconsistent phenomenon in that a zero interest rate policy creates a substantial effect on inflation and the output gap, which is not observed in the data.

Therefore, several studies have considered alleviating the assumption of rational expectations in the standard NK model to prevent this unintended result (Airaudo, 2020; Gabaix, 2020). These studies have focused on the role of bounded rationality in the NK model. They showed that including bounded rationality can resolve some paradoxical phenomena observed in a standard NK model. Accordingly, this study casts the following question: Will departure from rational expectations significantly amplify or attenuate the cost channel of monetary policy? Although this important research question is simple, how the presence of bounded rationality delivers the effect of a cost channel on monetary policy transmission remains unclear.

This study aims to thoroughly explore the role of a cost channel in the NK model with bounded rationality, namely, the behavioral NK (BNK) model. Gabaix (2020) considered the role of cognitive discounting based on the presence of bounded rationality in agents in the NK model. We incorporate the firm's working capital channel considered by Ravenna and Walsh (2006) into the BNK model developed by Gabaix (2020) to consider the role of the cost channel.[4] More specifically, following Chowdhury et al. (2006), we employ incomplete pass-through from the interest rate to the loan rate to reveal the strength of the cost channel. In sum, our model can examine the interaction effect of cognitive discounting and cost channels on monetary policy transmission. Although this may be regarded as a simple model extension, we demonstrate that such a simple model can produce several interesting and important policy prescriptions, which are not addressed in previous studies.

The findings of this study are briefly summarized as follows. First, we derive the determinacy condition in the BNK model with a cost channel and show that, contrary to previous studies, the degree of cognitive discounting significantly affects the determinacy condition. Second, we demonstrate that the price puzzle occurs only when a large value of cost channel parameters, which is not empirically supported, is introduced with a high degree of cognitive discounting. Third, we find that the degree of cognitive discounting significantly impacts the effect of a cost channel on optimal monetary policy.

Our study contributes to previous studies as follows. First, we present a new insight into the determinacy problem in the NK model with a cost channel. In the standard NK model, the

[4] See Meggiorini and Milani (2021), Wagner et al. (2022), Benchimol and Bounader (2019), Ida (2024b), and Ilabaca et al. (2020) for a detailed discussion about the effect of introducing cognitive discounting on the effectiveness of monetary policy.

Taylor principle, which indicates that the central bank also one-for-one raises its interest rate in response to inflation, is required to guarantee the unique rational expectations equilibrium (REE) (Bullard and Mitra, 2002).[5] However, Llosa and Tuesta (2009) and Surico (2008) showed that the inclusion of the cost channel renders the determinacy condition necessary to achieve the unique REE highly complicated. In contrast to these studies, we analytically show how the presence of the cost channel affects the determinacy problem in the BNK model. Given that the introduction of bounded rationality mitigates the expectations channel of monetary policy, the presence of the cost channel does not alter the determinacy condition even with a high degree of cognitive discounting. Conversely, for a small degree of cognitive discounting, the cost channel leads to a severe determinacy condition to achieve the unique REE. These results are not addressed in previous studies.

Second, this study builds on the following literature by accounting for the price puzzle phenomenon observed in the empirical studies. In empirical studies using vector autoregressive (VAR) models, several studies have documented that monetary tightening shocks often cause positive rather than negative responses in the inflation rate (Florio, 2018; Hanson, 2004; Ida, 2014; Henzel et al., 2009; Ida, 2024a). Several studies have focused on the role of a cost channel to explain this paradoxical result (Chowdhury et al., 2006; Castelnuovo, 2007). Qureshi and Ahmad (2021) argued the role of trend inflation in explaining the effect of a monetary contraction on the inflation rate in the NK model with a cost channel. Ida (2024a) concentrated on the role of household heterogeneity to explain the price puzzle phenomenon in the NK model. Contrary to these studies, we show that a high value of a cost channel parameter causes the price puzzle once we allow for monetary policy shock persistence. Under such a model environment, a high degree of cognitive discounting should have a large value of the cost channel parameter to generate the price puzzle. However, given that such a large value is not empirically supported, we posit that the introduction of bounded rationality helps to prevent the occurrence of the price puzzle.

Third, our study is related to the literature on optimal monetary policy in economies with a cost channel. As mentioned earlier, Ravenna and Walsh (2006) demonstrated that the presence

[5] See Airaudo (2013), Ascari and Ropele (2009), Bullard and Mitra (2002), Bullard and Singh (2008), Ida (2023), Llosa and Tuesta (2009), and Surico (2008) for a detailed discussion about the determinacy problem in the NK model.

of a cost channel generates a policy trade-off between inflation and the output gap even without an exogenous supply shock such as a price mark-up shock. They showed that the discrepancy in policy responses between commitment and discretion becomes substantial if the cost channel is introduced in the NK model. Tillmann (2009) considered the effect of a cost channel on an optimal discretionary policy in the NK model with model uncertainty. Demirel (2013) examined whether the gain from a commitment policy is large in an NK model with a cost channel. Contrary to these studies, the present study focused on how the presence of a cost channel affects optimal monetary policy under bounded rationality. We highlight that a high degree of cognitive discounting makes the difference in policy response between commitment and discretion highly negligible.

In sum, our study underlines the importance of considering the presence of bounded rationality in agents in the NK model with a cost channel. Our simple model succeeds in accounting for the interaction effect of cognitive discounting the cost channel on monetary policy transmission. We show that introducing bounded rationality in agents can ease the indeterminacy problem stemming from the cost channel and it also helps account for the price puzzle phenomenon. To the best of our knowledge, no studies have attempted to explore the role of the cost channel in the NK model in which bounded rationality in agents matters.

This paper is further structured as follows. Section 2 briefly describes the model. Section 3 reports the analytical results of how the presence of cognitive discounting affects the determinacy properties in the NK model with a cost channel. This section also examines how the degree of cognitive discounting affects the response of inflation to a monetary contraction. Then, Section 4 considers the effect of a cost channel on optimal monetary policy in the BNK model. Finally, Section 5 concludes the study.

2 Model

We briefly describe the BNK model with a cost channel. We incorporate a simple cost channel into the BNK model developed by Gabaix (2020). Although the model is based on the standard forward-looking NK model, our assumption departs from the assumption of rational expectations. Gabaix (2020) presumed that the perceived actual law of motion regarding the current economic state may be less persistent than the actual law of motion. Thus, in this model, the behavioral agents imprecisely perceive the law of motion for the parameter $m \in [0, 1]$, representing the cognitive discount factor.

Next, we briefly explain the model's structure. The households maximize their utility function, comprising consumption and labor supply by following an intertemporal budget constraint. The firms that face monopolistically competitive environment determine their optimal prices under Calvo (1983)'s nominal price rigidity. In addition, firms borrow funds from financial intermediaries to pay employee wages. This assumption is the source of a cost channel in the model. Following Ravenna and Walsh (2006) and Chowdhury et al. (2006), financial intermediaries receive deposits from households and lend funds to firms. The central bank manipulates the nominal interest rate in accordance with an instrument rule or a targeting rule. Consequently, except for the monetary policy specification, the model consists of three equations: a dynamic IS equation, an NK Phillips curve (NKPC), and a monetary policy rule.

Finally, unless otherwise specified, hatted variables represent the logarithmic deviation from the steady state. We define $\hat{Z}_t = \log(Z_t/Z)$ as the deviation of Z_t from the steady state. Z represents the value of the steady state.

2.1 Brief explanation of cognitive discounting

Before providing a detailed model description, we briefly explain the definition of bounded rationality. Following Gabaix (2020), we assume that bounded rationality in agents perceive that the state vector (X_t) evolves as follows:

$$X_{t+1} = mf(X_t, \epsilon_{t+1}), \tag{1}$$

where ϵ_t denotes the vector of economic disturbances and m represents the degree of cognitive discounting, namely, the agents' myopia parameter on the state of the economy, for some equilibrium transition function $f(\cdot)$ and exogenous innovation with mean zero. Unlike Gabaix (2020), note that we simply apply cognitive discounting to all variables.[6] As noted earlier, following Gabaix (2020), this parameter value ranges from 0 to 1. As we will discuss, the case for $m = 1$ corresponds to the standard NK model with full information rational expectations (Woodford, 2003). Conversely, the case of $m \in (0, 1)$ indicates the model under the assumption of bounded rationality. Log linearization (1) leads to the following relationship between

[6] See Pfäuti and Seyrich (2022) for a detailed discussion about this issue.

subjective and objective expectations

$$E_t^{BR}\hat{X}_{t+k} = m^k E_t \hat{X}_{t+k}, \tag{2}$$

where E_t^{BR} represents the subjective behavioral expectations operator, namely, the degree of cognitive discounting. E_t denotes the rational expectations operator.

2.2 Households

Households solve the standard intertemporal utility maximization problem. A representative household's intertemporal utility is as follows:

$$E_0^{BR}\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\gamma}}{1-\gamma}-\frac{N_t^{1+\eta}}{1+\eta}\right]. \tag{3}$$

where N_t denotes the household's labor hour. The parameters γ and η denote the Constant Relative Risk Aversion (CRRA) parameter and the inverse of the elasticity of the labor supply, respectively. The aggregate consumption C_t is defined as follows:

$$C_t = \left[\int_0^1 c_t(j)^{\frac{\xi-1}{\xi}} dj\right]^{\frac{\xi}{\xi-1}}. \tag{4}$$

$c_t(j)$ denotes the demand for goods j and ξ is the elasticity of substitution for individual goods. From the cost-minimization problem of households, we obtain the following demand for good j:

$$c_t(j) = \left(\frac{p_t(j)}{P_t}\right)^{-\xi} C_t, \tag{5}$$

where $p_t(j)$ represents the prices for goods j and the aggregate price index P_t is given as follows:

$$P_t = \left[\int_0^1 p_t(j)^{1-\xi} dj\right]^{\frac{1}{1-\xi}}. \tag{6}$$

The representative household faces the following budget constraints:

$$P_tC_t + M_{t+1} + B_{t+1} + D_t = (1+i_t)B_t + (1+i_t^d)D_t + M_t + W_tN_t + \Pi_t^f + \Pi_t^b + T_t, \tag{7}$$

where B_t represents nominal bonds held for one period and M_t denotes money stock, and i_t is the nominal interest rate. In addition, Π_t^f and Π_t^b are dividends earned from intermediate goods firms and financial intermediaries, respectively. W_t and T_t denote nominal wages and lump-sum transfers, respectively. D_t denotes the deposit service from financial intermediaries and i_t^d is the nominal interest rate on deposits.[7] The representative households also face the following cash-in-advance constraint:

$$P_t C_t \leq M_t - D_t + W_t N_t + T_t. \tag{8}$$

By solving the household's problem and using the goods market clearing condition, we obtain the following IS:

$$x_t = M E_t x_{t+1} - \sigma(\hat{i}_t - E_t \pi_{t+1} - r_t^n), \tag{9}$$

where $M = m$ and $\sigma = 1/\bar{r}\gamma$, and $\bar{r}$ denotes the real interest rate at the steady state. $E_t \pi_{t+1}$ is the expected inflation rate defined as $E_t \log(P_{t+1}/P_t)$, and the output gap is defined as $x_t = \hat{Y}_t - \hat{Y}_t^n$. Here, $\hat{Y}_t$ is the current output and $\hat{Y}_t^n$ denotes the natural rate of output, which is given by the following:

$$\hat{Y}_t^n = \frac{1+\eta}{\gamma+\eta} A_t,$$

where A_t denotes the productivity shock disturbance.[8] Finally, r_t^n is the natural level of the real interest rate, which is given by the following:

$$r_t^n = \sigma^{-1}(M E_t \hat{Y}_{t+1}^n - \hat{Y}_t^n).$$

Contrary to the standard NK model, the cognitive discounting parameter m affects the DIS curve. This case indicates that the presence of the cognitive discounting factor mitigates future endogenous variables on the current inflation and output gap. That is, iterating Eq. (9) forwardly leads to

$$x_t = -\sigma E_t \sum_{k=0}^{\infty} M^j (\hat{i}_{t+k} - \pi_{t+k+1} - r_{t+k}^n). \tag{10}$$

As argued by Gabaix (2020), introducing cognitive discounting helps to ease the several puzzles that emerged in the standard NK model, such as the forward guidance puzzle. The presence of M in a DIS indicates that it discounts the effect of the future real interest rate on the current output gap.

[7] The household's optimal condition indicates that the nominal interest rate equals the interest rate on deposits.

[8] Productivity shock disturbance is explained in the next subsection.

2.3 Financial intermediaries

Following Chowdhury et al. (2006), we consider the role of financial intermediaries.[9] Financial intermediaries provide deposit services to households. If households deposit the amount of D_t in period t, then they will receive the amount of $(1 + i_t)D_t$ at the end of the period. In turn, financial intermediaries receive deposits from households and lend these funds to firms. The lending rate is given by i_t^L. In addition, financial intermediaries incur a monitoring cost, $\Gamma(i_t)$ when lending funds to goods producers. We assume that this monitoring cost is differentiable and satisfies the following properties: $\Gamma^{'}(i_t) \geq 0$ and $\Gamma^{''}(i_t) \geq 0$.

Financial intermediaries face the following profit maximization problem:

$$\Pi_t^b = (1 + i_t^L)[1 - \Gamma(i_t)]L_t - (1 + i_t)D_t - kL_t, \tag{11}$$

subject to $L_t = D_t$, where L_t denotes loans to a firm. The parameter k represents the management cost, which is constant. The equilibrium for the lending market is $D_t = W_t N_t^d$, where N_t^d denotes the demand for labor.

After calculating the financial intermediaries' profit maximization problem, the log-linearized lending rate is given as follows:

$$\hat{i}_t^L = \psi_i \hat{i}_t, \tag{12}$$

where ψ_i represents the degree of the cost channel strength.[10] As pointed out by Chowdhury et al. (2006) and Castelnuovo (2007), when ψ_i exceeds unity, the cost channel amplifies the loan rate, thereby increasing the inflation rate through a rise in the real marginal costs.

2.4 Firms

There is a continuum of firms that face monopolistic competition and Calvo (1983)'s nominal price rigidity. The firms, similar to households, have problems with future cognition. In other words, they cannot accurately recognize how the macroeconomic state will be realized in the future.

---

[9] Castelnuovo (2007) and Tillmann (2009) considered the similar introduction of financial intermediaries into the standard NK model. Ida (2023) introduced the idea of Chowdhury et al. (2006) in modeling financial intermediaries into a two-country NK model.

[10] See Chowdhury et al. (2006) for a detailed derivation of this equation.

Firm j's production function is given as follows:

$$Y_t(j) = A_t N_t(j), \tag{13}$$

where $N_t(j)$ denotes the labor supply employed in firm j, and A_t denotes an aggregate productivity disturbance, following an autoregressive (AR)(1) process given by $\log A_t = \rho_a \log A_{t-1} + \epsilon_t^a$ with $0 \leq \rho_a < 1$, where ϵ_t^a is an independent and identically distributed (i.i.d.) shock with constant variance σ_a^2.

We explain the derivation of real marginal costs. Following Chowdhury et al. (2006) and Ravenna and Walsh (2006), we assume that the firm must borrow an amount $W_t N_t$ from intermediaries at the gross nominal interest rate $1 + i_t^L$. The real marginal cost for all firms is as follows:

$$mc_t = \frac{(1 + i_t^L) w_t}{A_t}, \tag{14}$$

where w_t is the real wage. Thus, contrary to the textbook NK model, the real marginal cost depends on a change in the loan rate in the presence of the cost channel. At this point, we can obtain the real marginal cost as the expression using the output gap as follows:

$$\widehat{mc}_t = (\gamma + \eta) x_t + \psi_i \hat{\hat{i}}_t. \tag{15}$$

Following Calvo (1983), we assume that firms face price rigidity in each period. More precisely, a fraction $1 - \theta$ of all firms adjusts their price, whereas the remaining fraction of firms θ does not. When revising their prices, these firms contemplate uncertainty concerning when they can adjust prices again. The firm's optimization problem is given by the following:

$$E_t^{BR} \sum_{k=0}^{\infty} \theta^j Q_{t,t+k} Y_{t+k}(i)(P_t^* - P_{t+k} mc_{t+k}). \tag{16}$$

subject to the following demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\xi} Y_t, \tag{17}$$

where $Q_{t,t+j}$ is the stochastic discount factor, $Y_{t+k|t}$ is the output in period $t + k$ expected by the firm in period t, and P_t^* is the optimal price. By log-linearizing the FOC of the firm's optimal condition (16) around the steady state, we obtain the following:

$$\hat{P}_t^* - \hat{P}_{t-1} = (1 - \beta\theta) E_t^{BR} \sum_{t=0}^{\infty} (\beta\theta)^k [\widehat{mc}_{t+k} + \hat{P}_{t+k} - \hat{P}_{t-1}]. \tag{18}$$

After some manipulations and using Eq. (15), we obtain the following new Keynesian Phillips curve (NKPC) under bounded rationality:

$$\pi_t = M_f \beta E_t \pi_{t+1} + \delta[(\gamma + \eta)x_t + \psi_i \hat{i}_t], \tag{19}$$

where $\pi_t = \hat{P}_t - \hat{P}_{t-1}$ and $M_f = m[\theta + (1-\theta)(1-\theta\beta)/(1-m\theta\beta)]$ and $\delta = (1-\theta)(1-\theta\beta)/\theta$.[11] In contrast to Gabaix (2020), the nominal interest rate is augmented in the NKPC owing to the presence of a cost channel.

Compared with the standard NK model with rational expectations, the NKPC is also discounted in the BNK model as follows:

$$\pi_t = \delta E_t \sum_{k=0}^{\infty} (\beta M_f)^k [(\gamma + \eta)x_{t+k} + \psi_i \hat{i}_{t+k}]. \tag{20}$$

Thus, cognitive discounting attenuates the effect of future output gaps on current inflation.[12] Surico (2008) documented that in the absence of a cost channel, the discounted value expression of the NKPC indicates that monetary tightening can generate a negative response of inflation by successfully creating a negative response of the discounted value of the expected output gap. However, the cost channel generates a wedge into this relationship, as it may prevent monetary contraction from generating a negative response of inflation.[13] Thus, cognitive discounting helps dampen the positive effect of the future nominal interest rate on current inflation.

2.5 Calibration

Before further analysis, we briefly describe the calibrated values mainly used in the study. We set the deep parameters based on Gabaix (2020), except for several parameter values. The discount factor β is set to 0.99. We set the degree of nominal price stickiness, namely, the Calvo parameter, to 0.75 as a benchmark case. Then, we set the CRRA coefficient to 2.0 and select the value of the Frisch elasticity of labor supply as 1.0. The elasticity of substitution is set to 7.88.

Next, we briefly explain the value of cognitive discounting parameter m. As noted earlier, $m = 1$ corresponds to the standard NK with full information rational expectations.Following

[11] See Gabaix (2020) for a detailed derivation of the NKPC under bounded rationality.

[12] Benchimol and Bounader (2019) focused on which values of M and M_f influence the properties of optimal monetary policy in a BNK model.

[13] Ida (2023) also pointed out this mechanism in a two-country NK model.

Gabaix (2020), we set the degree of cognitive discounting m to 0.85 as a benchmark calibration.[14] We consider the range of m from 0 to 1 to explore how the degree of cognitive discounting impacts the properties of optimal monetary policy rules. Although the calibrated value of a small value of m may not be empirically supported, such a parameterization is useful to obtain the intuitive mechanism of our model.

Finally, we provide the calibrated value on the cost channel parameter. Several studies have reported the degree of a cost channel parameter, ψ_i. For instance, Ravenna and Walsh (2006) reported that the value of the parameter ψ_i is 1.276. Chowdhury et al. (2006) estimated a value of ψ_i of 1.32 for the United States. Patel (2021) estimated the cost channel parameter in a two-country NK model using a Bayesian technique and reported that the estimated values of the cost channel parameter ranged from 1.8 to 2.3.[15] This study therefore uses an empirically plausible value of $\psi_i \in [0, 1.8]$ to account for the severity of the cost channel.[16]

3 Role of a cost channel in the BNK model

This section provides the role of a cost channel in the NK model in which bounded rationality in agents is present. In this section, we assume that the central bank adopts a simple rule when conducting monetary policy. In Section 3.1, we analytically examine the effect of a cost channel on the determinacy condition under bounded rationality. In this exercise, we adopt current-looking and forward-looking Taylor rules to derive the determinacy condition. In Section 3.2, we explore how the degree of cost channel strength changes the dynamic properties to calculate the impulse response function. In Section 3.3, we investigate whether the assumption of the cost channel hypothesis creates the price puzzle phenomenon in our model.

---

[14]See Meggiorini and Milani (2021) for a detailed discussion about the empirical studies on the degree of cognitive discounting. Moreover, Hirose et al. (2023) estimated the degree of cognitive discounting in the NK model with zero lower bounds on nominal interest rates.

[15]Patel (2021) regarded the cost channel parameter as the trade finance parameter. Strictly speaking, the trade finance parameter assumes the pass-through of the loan rate between two countries. See Patel (2021) for a detailed description of the trade finance parameter.

[16]The degree of the cost channel ψ_i becomes below unity if the pass-through from the interest rate to the loan rate takes a negative value. We do not focus on this case because the main results of our study remain unaffected by such a parameterization.

3.1 Determinacy properties

First, we explore the determinacy analysis in our model. Bullard and Mitra (2002) showed that the Taylor principle is necessary to attain the unique REE in the standard NK model. However, Llosa and Tuesta (2008) found that introducing the cost channel makes the determinacy condition highly complicated in the NK model.[17] Conversely, Gabaix (2020) demonstrated that an increased degree of cognitive discounting is likely to attain unique REE, contrary to the rational expectations model. However, how the interaction effect of cognitive discounting and the cost channel affects the determinacy properties remains unclear. Hence, we employ two simple monetary policy rules, namely, the current-looking and forward-looking policy rules, to answer this question.

3.1.1 Current-looking Taylor rule

First, as a benchmark case, we consider that the central bank responds to the current inflation rate and the current output gap (Taylor, 1993). More specifically, we adopt the following monetary policy rule:

$$i_t = \phi_\pi \pi_t + \phi_x x_t, \tag{21}$$

where ϕ_π and ϕ_x are the degree of responsiveness of the policy interest rate to inflation and output gap, respectively. Our main question in this section is how the departure from the rational expectations model affects the equilibrium determinacy in the presence of the cost channel. More precisely, we examine the interaction effect of cognitive discounting and the cost channel on the uniqueness of REE. Particularly, as shown in Gabaix (2020), a small value of the inflation stabilization coefficient achieves unique REE. In what follows, we examine whether the findings of Gabaix (2020) are significantly affected by introducing the cost channel.

Using Eqs. (9), (19), and (21), we obtain the following dynamic system for $z_t = [\pi_t, x_t]'$ and r_t^n:

$$z_t = \Omega E_t z_{t+1} + \Psi r_t^n, \tag{22}$$

[17] Surico (2008) examined the effect of the cost channel on the equilibrium determinacy by using a monetary policy rule with interest rate smoothing.

where coefficient matrices are

$$\Omega = \omega \begin{bmatrix} (\sigma+\phi_x)M_f\beta + \delta[(\sigma+\eta)+\psi_i\phi_x]\beta & \sigma\delta[(\sigma+\eta)+\psi_i\phi_x]M \\ 1 - M_f\beta\phi_\pi - \delta\psi_i\phi_\pi & \sigma(1-\delta\psi_i\phi_\pi)M \end{bmatrix}, \tag{23}$$

where $\omega = (\sigma+\phi_x-\sigma\delta\psi_i\phi_\pi+\delta(\sigma+\eta)\phi_\pi)^{-1}$. The determinacy conditions of the current Taylor type are summarized as follows:[18]

Proposition 1 *Under a simple Taylor type in which the central bank reacts to current price inflation and the current output gap the necessary and sufficient conditions for determinacy are*

$$\phi_\pi + \varpi\phi_x + \frac{(1-M)(1-M_f\beta-\delta\psi_i)}{\delta[\sigma(\gamma+\eta)-(1-M)\psi_i]} > 1, \tag{24}$$

$$\sigma(1+M_f\beta+\delta\psi_i)\phi_x + \delta[\sigma(\gamma+\eta)-(1+M)\psi_i]\phi_\pi + (1+M)(1+M_f\beta)$$
$$+\sigma(\gamma+\eta)\delta > 0, \tag{25}$$

where $\varpi = \sigma(1-M_f\beta-\delta\psi_i)/\delta[\sigma(\gamma+\eta)-(1-M)\psi_i]$.

Proof. See Appendix A. ■

These conditions are generalized versions of Gabaix (2020) and Llosa and Tuesta (2009). Llosa and Tuesta (2009) showed the determinacy condition under the cost channel in the NK model with rational expectations. The condition (24) states the generalized Taylor principle, which contains the third term of the left-hand side in contrast to their study. Intuitively, in the long run, the condition (24) indicates that each percentage point of permanently high inflation reflects a permanent reduction in the output gap of ϖ percentage point. When $\partial\varpi/\partial\psi_i < 0$ holds, a rise in the output gap is less responsive to an increase in the inflation rate in the presence of a severe cost channel. This term makes the REE less determinate in the case of a weak response of ϕ_π and ϕ_x. In our model, that term is affected by the presence of the cost channel. The condition (25) is also a generalized version of Llosa and Tuesta (2009) in a model with cognitive discounting. Notably, the condition (25) is trivially satisfied in the case of no cost channel.[19]

---

[18] See LaSalle (1986) and Woodford (2003) for a detailed explanation of the determinacy condition in a discrete-time macroeconomic model.

[19] See Bullard and Mitra (2002) for a detailed discussion of this point.

Remarkably, unlike Llosa and Tuesta (2009), we address the role of the output gap response ϕ_x in accounting for the determinacy condition in our model. If we consider the value of m, which is slightly smaller than unity, then the value of the third term of the left-hand side is less than unity. In this case, for a given the value of ϕ_π and ϕ_x, a large value of the cost channel parameter may violate the condition (24). When the value of m is slightly smaller than unity, we can easily confirm $\partial\varpi/\partial\psi_i < 0$. Therefore, for small coefficients of ϕ_π and ϕ_x, the determinacy condition (24) does not guarantee the unique REE when the cost channel parameter takes a large value. Put differently, when the severe cost channel exists, for a small value of ϕ_π, the central bank adopts a high weight on the output gap coefficient to attain the unique REE.

Llosa and Tuesta (2009) showed that for a high value of the cost channel parameter, the determinacy condition requires a high ϕ_π in the case of $m = M = 1$. However, as noted by Gabaix (2020), the presence of bounded rationality in agents enlarges the determinacy regions. We underline that when a small cognitive discounting is introduced into the model, the cost channel effect partially deprives this enlarging effect of the determinacy region associated with the presence of cognitive discounting. This effect is captured by $(1-M)\psi_i$ in the coefficient ϖ. Therefore, our finding indicates that if the central bank employs a weak inflation stabilization coefficient, it must put a heavy weight on the output gap stabilization in the presence of the cost channel. From another aspect, a high value of ϕ_x is likely to achieve the unique REE, even for a smaller value of ϕ_π. This channel is in contrast to the result obtained by Llosa and Tuesta (2009).

If the central bank ignores fluctuations in inflation and the output gap, namely, $\phi_\pi = \phi_x = 0$, then we have a unique REE if and only if the following conditions hold:

$$(1-M)(1-M_f\beta) > \delta\sigma(\gamma+\eta). \tag{26}$$

This equation is similar to that derived by Gabaix (2020), and thus, we show that the condition is unaffected by the presence of the cost channel. Intuitively, although the cost channel influences the private sector's expectations, it is attenuated by a small value of the cognitive discounting parameter. Thus, the cost channel does not matter when the cognitive discounting parameter takes a predominately small value. Previous studies do not also underline this result.

Finally, we numerically confirm the above results. Figure 1 depicts how the cost channel affects the determinacy region under several parameterizations of the degree of cognitive discounting. In the case of the rational expectations assumption, as shown by Llosa and Tuesta (2009), including the cost channel in the model requires an aggressive inflation reaction coefficient to attain the unique REE, considering the output gap response. However, when a benchmark model slightly departs from the assumption of the rational expectations (i.e., $m = 0.85$), a small value of ϕ_π makes the REE determinate for the large value of ϕ_x. We address that this result applies to the case where bounded rationality in agents is allowed in the model. If the private sector's expectations are highly discounted (for instance, $m = 0.25$ as an extreme case), the determinacy region remains unaffected by the presence of the cost channel.

[Figure 1 around here]

3.1.2 Forward-looking Taylor rule

Next, we consider the case where the central bank responds to the expected inflation rate and the expected output gap. Contrary to the current-looking policy rule, as noted by Bullard and Mitra (2002) and Llosa and Tuesta (2009), the adoption of the forward-looking monetary policy rule significantly affects determinacy properties. This section aims to explore how the presence of the cost channel changes the determinacy condition in the BNK model in which the central bank employs the forward-looking monetary policy rule. Hence, we consider the following monetary policy rule:

$$\begin{aligned} i_t &= \phi_\pi E_t^{BR}\pi_{t+1} + \phi_x E_t^{BR} x_{t+1}, \\ &= \phi'_\pi E_t \pi_{t+1} + \phi'_x E_t x_{t+1}, \end{aligned} \tag{27}$$

where $\phi'_\pi = m\phi_\pi$ and $\phi'_x = m\phi_x$ are the degrees of responsiveness of the policy interest rate to inflation and the output gap, respectively. Unlike Bullard and Mitra (2002) and Llosa and Tuesta (2009), our forward-looking rule does not rely on the assumption of rational expectations but on the expectations under bounded rationality. Therefore, given the values of ϕ_π and ϕ_x, a high degree of cognitive discounting makes the response of the nominal interest rate less responsive to fluctuations in inflation and the output gap. This monetary policy rule can also be interpreted as nesting the forward-looking one and the interest rate peg. On the one hand, for the extreme case, that is, $m = 0$, our forward-looking policy rule corresponds to the interest rate peg because of $\phi'_\pi = 0$ and $\phi'_x = 0$. On the other hand, if we adopt the assumption of rational expectations, namely, $m = 1$, the rule (27) corresponds to the forward-looking one with rational expectations considered by Bullard and Mitra (2002) and Llosa and Tuesta (2009).

As in the current looking case, using (27), (9) and (19), we can obtain the dynamic system for the endogenous vector $z_t = (\pi_t, x_t)'$. The coefficient matrices of the reduced system Ω are defined by as follows:

$$\Omega = \sigma^{-1} \begin{bmatrix} \sigma M_f \beta - \delta(\sigma+\eta)(\phi'_\pi - 1) + \delta\phi'_\pi \sigma\psi_i & \delta(\sigma+\eta)(\sigma M - \phi'_x) + \sigma\delta\psi_i\phi'_x \\ -\phi'_\pi + 1 & \sigma M - \phi'_\pi \end{bmatrix}. \tag{28}$$

The determinacy properties of the forward-looking expectations in the Taylor rule are summarized as follows:

Proposition 2 *Under interest rate rules with forward expectations, the necessary and sufficient conditions for determinacy are given as follows:*

$$\phi'_\pi + \frac{\sigma(1 - M_f\beta - \delta\psi_i)}{\delta[\sigma(\gamma+\eta) - (1-M)\psi_i]}\phi'_x + \frac{(1-M)(1-M_f\beta - \delta\psi_i)}{\delta[\sigma(\gamma+\eta) - (1-M)\psi_i]} > 1, \tag{29}$$

$$\sigma(M_f\beta + \delta\psi_i)\phi'_x - \delta\psi_i M\phi'_\pi < 1 + MM_f\beta, \tag{30}$$

$$-\sigma(M_f\beta + \delta\psi_i)\phi'_x + \delta\psi_i M\phi'_\pi < 1 - MM_f\beta, \tag{31}$$

$$\begin{aligned} &\sigma(1 + M_f\beta + \delta\psi_i)\phi'_x + \delta[\sigma(\gamma+\eta) - (1+M)\psi_i]\phi'_\pi \\ &< (1+M)(1+M_f\beta) + \sigma(\gamma+\eta)\delta. \end{aligned} \tag{32}$$

Proof. See Appendix A. ■

The case for $m = 1$ corresponds to the determinacy conditions obtained by Llosa and Tuesta (2009). Their determinacy conditions are modified when cognitive discounting and cost channels interact. First, the condition (29) represents the generalized Taylor principle, which holds even in the case of a current-looking rule. Second, the conditions (30) and (31) imply the upper bound on the coefficient for inflation and the output gap, respectively. Third, the inequality (32) requires that an upper bound restricts the total response to inflation and the output gap. Furthermore, our result reveals that even in the case of a severe cost channel, a small value of the cognitive discounting parameter can easily satisfy conditions (30) and (31). As mentioned earlier, a large value of the cost channel parameter delivers the amplifying effect of the expectations channel, whereas a small value of m captures its dampening effect. Our result indicates that the central bank can easily attain the unique REE when the latter effect dominates the former one. In an extreme case of $\phi_\pi = \phi_x = 0$, the condition required to attain the unique REE corresponds to Eq. (26). Put differently, this study shows that despite the current-looking or forward-looking policy rule specification, if all agents form their expectations bounded rationally, the satisfaction of the condition (26) suffices to achieve the unique REE under an interest rate peg. This result is not addressed by Gabaix (2020) and Llosa and Tuesta (2009).

Figure 2 illustrates the determinacy region under a forward-looking monetary policy rule. In the case of $m = 1$, Llosa and Tuesta (2009) found that introducing a cost channel narrows the determinacy region compared with the standard NK model. Thus, unlike the model without the cost channel, a strong reaction to inflation is necessary to retain the unique REE with a small response to the output gap. Next, in the case of bounded rationality for slightly discounted expectations ($m = 0.85$), the presence of bounded rationality in agents expands the determinacy region in a model with and without the cost channel. Thus, the central bank can achieve the unique REE even when $\phi_\pi < 1$, with a small output gap response. As shown in Figure 2, determinacy regions are narrower in the NK model with the cost channel than without. Furthermore, this figure depicts that a high degree of cognitive discounting leads to a further expansion of the determinacy region despite introducing the cost channel. Accordingly, our result indicates that for a small value of m, the determinacy region remains unaffected by the cost channel.

[Figure 2 around here]

3.2 Model dynamics

In this section, we examine the dynamic properties of our model. This section focuses on the impulse response of inflation to contractionary monetary policy shocks to address the role of the cost channel in the BNK model. More precisely, we analytically calculate the equilibrium response of the inflation rate to monetary policy shocks. In this section, we consider the current-looking monetary policy rule with exogenous monetary policy shocks, which follows an AR(1) process. Concretely, we adopt the following monetary policy rule specification:

$$i_t = \phi_\pi \pi_t + \phi_x x_t + \nu_t, \tag{33}$$

where $\nu_t = \rho \nu_{t-1} + \varepsilon_t$ and ε_t is i.i.d. shock $(0, \sigma_\varepsilon^2)$.

We employ the undetermined coefficient method to derive the equilibrium response to monetary policy shocks. More precisely, we follow the solution method that McCallum (1983) offered as the minimum state variable (MSV) solution to obtain the analytical solution in the model. More concretely, we obtain the following reduced form for the inflation rate:

$$\begin{aligned}\pi_t &= -\left\{\frac{\delta[(\gamma+\eta)\sigma-\psi_i(1-\rho M)]}{(1-\rho M+\sigma\phi_x)[(1-\beta\rho M_f)-\delta\psi_i\phi_\pi]+\sigma\delta[(\gamma+\eta)+\psi_i\phi_x](\phi_\pi-\rho)}\right\}\nu_t,\\ &= -\Psi\nu_t. \end{aligned} \tag{34}$$

Unlike Castelnuovo (2007), our solution allows for a persistent monetary policy shock. The solution with $\rho = 0$ corresponds to that derived by Castelnuovo (2007). We emphasize the role of monetary policy shock persistence in preventing the price puzzle phenomenon and the assumption of bounded rationality. Unlike the standard NK model with rational expectations, this MSV solution is affected by the degree of the cognitive discounting parameter through the DIS curve and the NKPC. Under a plausible calibration, the denominator never becomes a positive value. However, a high value of the cost channel parameter may lead to a positive value of the numerator. Thus, a monetary tightening shock increases the inflation rate because the cost channel dominates the demand channel (Castelnuovo, 2007; Chowdhury et al., 2006; Ida, 2014). The next subsection discusses how the price puzzle emerges in our model.

In what follows, we numerically explore how the presence of cognitive discounting affects the dynamic response of inflation rates to monetary tightening in the NK model with a cost channel. Figure 3 shows the impulse response of inflation rates to a contractionary monetary policy shock. Figure 3(i) plots the case without the cost channel because it helps to understand how the presence of cognitive discounting impacts the dynamic response of inflation. This figure shows that introducing bounded rationality in agents renders the response of inflation to monetary contraction highly attenuated. As noted earlier, intuitively, given that the presence of bounded rationality in agents weakens the central bank's ability to manipulate the expectations of the private sector, equilibrium inflation is less responsive to monetary contraction.

[Figure 3 around here]

Figure 3(ii) illustrates the inflation response of rising nominal interest rates under a benchmark cost channel model. Unlike models without the cost channel, introducing a cost channel further attenuates the response of inflation to monetary contraction. This figure also shows the widening gap between rational expectations and bounded rationality. As noted earlier, the reason is that an increased degree of cognitive discounting weakens the demand channel of monetary policy relative to the cost channel. However, even in this case, a rise in nominal interest rates yields a negative inflation response because the demand channel still dominates the cost channel. Finally, as shown in Figure 3 (iii), we consider the case where a strong cost channel parameter is introduced. The strong cost channel leads to a further dampening effect on monetary contraction. Moreover, a significant difference exists between rational expectations and bounded rationality models.

Intuitively, the presence of bounded rationality in agents affects the shape of the DIS curve and NKPC. As shown in Eq.(10), the effect of the future real interest rate on the current output gap is counteracted by a high degree of cognitive discounting. Thus, the presence of bounded rationality in agents weakens the demand channel of monetary policy. In addition, from Eq.(20), the effect of future interest rates on current inflation is attenuated by cognitive discounting. Therefore, when the degree of cognitive discounting is high, whether the central bank can induce a negative response of inflation to monetary tightening depends on the dominance of the demand channel over the cost channel in the initial period.

We calculate the impact response of the inflation rate to monetary contraction under several parameterizations of m and ψ_i to grasp further insights into the interaction effect between cognitive discounting and cost channels. Figure 4 depicts the impact response of the inflation rate to contractionary monetary policy shock. Figure 4 also shows how the degree of cognitive discounting leads to the impact of an increase in the nominal interest rate on the initial response of inflation, given a cost channel parameter. Even with a high value of ψ_i, monetary contraction causes a decline in the inflation rate. A small value of the cognitive discounting parameter mitigates the impact of monetary tightening on inflation. The price puzzle occurs when ψ_i takes a high value and a high degree of cognitive discounting. Given that empirical studies do not support such a cost channel parameter, we note that the presence of bounded rationality in agents helps to prevent the occurrence of the price puzzle even if we incorporate the cost channel into the model.

[Figure 4 around here]

The presence of cognitive discounting makes the expectations channel of monetary policy less effective. This case indicates that the central bank has difficulty fully manipulating the expectations of the private sector through demand and supply channels of monetary policy. However, even if we allow for the role of bounded rationality in agents, the model still captures the impact response of inflation to an increase in the nominal interest rate through a rise in the real marginal costs. As long as the demand channel dominates the cost channel, a contractionary monetary policy shock decreases the inflation rate even if cognitive discounting weakens the demand channel through the expectations channel of the private sector. However, monetary contraction yields a positive response to the inflation rate if the cognitive discounting parameter m magnifies the cost channel relative to the demand channel. This counterintuitive response of inflation to monetary contraction is referred to as the price puzzle (Sims, 1992).

3.3 Price puzzle phenomenon

In this subsection, we examine the interaction effect of cognitive discounting and cost channels to understand the price puzzle phenomenon. Many empirical studies have argued that a contractionary monetary policy shock results in a rise in inflation when the VAR model is used to assess monetary policy transmission. Several studies have focused on whether introducing the cost channel helps to account for price puzzle phenomenon (Castelnuovo, 2007; Chowdhury et al., 2006; Rabanal, 2007; Tillmann, 2008).[20] This section aims to theoretically reconsider the problem of the price puzzle in a BNK model.[21]

In the following discussion, without loss of generality, we assume that $\phi_x = 0$.[22] First, we demonstrate how the cost channel parameter contributes to the occurrence of the price puzzle.

Proposition 3 *The price puzzle occurs if the cost channel parameter exceeds the following threshold:*

$$\psi_i^* = \frac{\sigma(\gamma + \eta)}{1 - \rho M}. \tag{35}$$

[20] Some studies have found that misspecification in the VAR model leads to the price puzzle (Hanson, 2004; Florio, 2018; Sims, 1992).

[21] We consider the effect of a monetary tightening shock on the inflation rate in the short run. Several studies have explored the increase in the transitory and persistent interest rate on the inflation rate to assess the neo-Fisherian effect in the NK model (Garín et al., 2018; Uribe, 2022). Considering that the occurrence of the neo-Fisherian effect in our model is beyond the scope of the study, we do not discuss this issue.

[22] The same discussion is applicable when we allow for the case of $\phi_x > 0$.

Proof. The price puzzle requires the condition $\partial \pi_t / \partial \nu_t > 0$. Considering that the denominator in Eq.(34) never becomes a negative value, the above condition holds if the denominator takes a negative value. Thus, $\psi_i(1-\rho M) > (\gamma+\eta)\sigma$. Moreover, the threshold that the response of inflation to monetary tightening becomes a positive value is given by $\psi_i^* = \sigma(\gamma+\eta)/(1-\rho M)$. The price puzzle emerges when ψ_i exceeds this threshold. ■

This proposition states that the threshold of the cost channel parameter depends on the value of the CRRA coefficient, the inverse elasticity of labor supply, monetary policy shock persistence, and the degree of cognitive discounting. Castelnuovo (2007) pointed out the first two parameters, but he derived the threshold of the cost channel parameter in the case of no monetary policy shock persistence. Unlike his study, we underline the role of monetary policy shock persistence. A large value of ρ can prevent the price puzzle phenomenon because it requires a large threshold value of ψ_i. Intuitively, a large value of ρ strengthens the demand channel of monetary policy. Conversely, given the degree of the parameter ρ, a high degree of cognitive discounting reduces the threshold of ψ_i. The reason is that a small value of m makes the central bank's ability to manage the expectations of the private sector highly mitigated. This result is in contrast to existing literature.

Next, we examine how this threshold is affected by the deep parameters. In this regard, we focus on the effect of a change in the parameters, ρ, m, and γ. We obtain the following results:

Proposition 4 *The threshold of ψ_i is characterized by the following properties:*

$$\frac{\partial \psi_i^*}{\partial \rho} > 0; \ \frac{\partial \psi_i^*}{\partial m} > 0; \ \frac{\partial \psi_i^*}{\partial \gamma} < 0.$$

Proof. We easily confirm the above conditions by differentiating the threshold of ψ_i with respect to the parameters, ρ, m, and γ, respectively. ■

The first condition indicates that a high degree of monetary policy shock persistence prevents the occurrence of the price puzzle by bolstering the demand channel of monetary policy. Thus, the high value of ρ needs a high value of ψ_i to induce the positive response of inflation to monetary tightening.[23] The second condition implies that a high degree of cognitive discounting—a

[23] Ida (2024a) also noted out this point in a two-agent NK (TANK) model to explain the occurrence of the price puzzle.

small value of m—lowers the threshold of ψ_i, which easily produces the price puzzle by lowering the central bank's ability to manage the expectations of the private sector. As argued by Chowdhury et al. (2006) and Castelnuovo (2007), the third condition indicates that a high value of the CRRA coefficient can easily create the price puzzle phenomenon to attenuate the demand channel of monetary policy in the DIS curve.

In what follows, we focus on the effect of ρ and γ on the cost channel parameter thresholds in accounting for the price puzzle in a BNK model. Figure 5 depicts how a change in the degree of monetary policy shock persistence affects the threshold of ψ_i. A high degree of monetary policy shock persistence significantly increases the threshold value of ψ_i when $m = 1$, namely, the case for rational expectations. As mentioned repeatedly, a high value of ρ indicates that the reinforced demand channel of monetary policy avoids the occurrence of the price puzzle. The cost channel parameter that exceeds roughly 15 generates the price puzzle in the case of $\rho = 0.9$. Of course, such a parameterization is not empirically supported by previous studies. Conversely, increased cognitive discounting decreases the threshold value of ψ_i. This result indicates that even with a high degree of monetary policy shock persistence, a low value of a cognitive discounting parameter may induce the price puzzle. For instance, in the case of $\rho = 0.9$, the cost channel parameter below 2.0 results in the price puzzle when the private sector's expectations are predominately discounted, namely, a small value of m.

[Figure 5 around here]

Finally, we examine whether the degree of a CRRA coefficient affects the price puzzle phenomenon in our model. Castelnuovo (2007) pointed out that given the value of the inverse elasticity of labor supply η, a large value of a CRRA coefficient is likely to produce the price puzzle because it dampens the demand channel of monetary policy through a change in the real interest rate. However, whether a large value of a CRRA coefficient causes the positive response of inflation to the monetary contraction in a BNK model is unclear. Gali et al. (2007) argued that the estimated value of a CRRA parameter ranges from 1.0 to 10.0. Therefore, we also adopt their parameter ranges to assess the effect of a CRRA coefficient on the threshold of ψ_i by considering the degree of cognitive discounting.

Figure 6 reports how a change in a CRRA coefficient impacts the threshold of ψ_i. Considering the case of the rational expectations model, namely, $m = 1$, a large value of γ decreases the threshold of ψ_i. This result is consistent with that shown by Castelnuovo (2007). However, as shown in this figure, the empirically plausible value of ψ_i that is generally less than 1.5 does not lead to the price puzzle phenomenon. As the cognitive discounting parameter takes a small value, a large value of a CRRA coefficient creates the threshold value of ψ_i that is less than 1.5. The value of ψ_i, which is less than 1.5, may be supported by previous studies (Castelnuovo, 2007). For instance, although the parameter γ is set to 6, which is roughly calibrated by Woodford (2003), such a parameterization causes the price puzzle when the expectations of the private sector are extremely discounted. When the cost channel parameter takes more than 1.5 in the case of $\gamma = 6.0$ and $m = 0.5$, such a parameter value exceeds the threshold, resulting in the occurrence of the price puzzle. When we consider the large value of γ, the small value of ψ_i is likely to yield the price puzzle for a small value of m.

[Figure 6 around here]

In sum, the cost channel plays a significant role in the BNK model regarding equilibrium determinacy and model dynamics. Although a high degree of cognitive discounting is likely to generate the price puzzle, its occurrence still requires a high value of the cost channel parameter, which empirical studies may not support. Thus, this study demonstrated that even if we permit the presence of bounded rationality in agents, an empirically plausible cost channel parameter does not always cause the price puzzle phenomenon. Therefore, the result of this study indicates that the departure from the rational expectations model presents one solution for the price puzzle problem when the cost channel matters.

4 Cost channel and optimal monetary policy in the BNK model

In the previous section, we considered that the central bank conducts its monetary policy by following the simple instrument rule. This section explores the effect of a cost channel on optimal monetary policy in the BNK model. Section 4.1 briefly discusses the loss function of the central bank in our model. Section 4.2 examines how the cost channel affects the properties of optimal monetary policy in the BNK model, compared with the case for the assumption of rational expectations.

4.1 Central bank's loss function

In this section, we examine the properties of optimal monetary policy in the BNK model with a cost channel. Following Ravenna and Walsh (2006) and Gabaix (2020), we adopt the loss function of the central bank, which contains the traditional stabilization terms of inflation and the output gap.[24] More concretely, the loss function is given by the following:

$$\mathcal{L}_t = E_t \sum_{k=0}^{\infty} \beta^k \left\{ \pi_{t+k}^2 + \lambda x_{t+k}^2 \right\}, \tag{36}$$

where λ denotes the output gap stabilization relative to inflation stabilization, which is defined as follows:

$$\lambda = \frac{\delta(\gamma + \eta)}{\xi}.$$

More precisely, the term for inflation stabilization stems from the presence of staggered nominal prices. Gabaix (2020) derived the loss function of the central bank in the BNK model and showed that the traditional objectives with stabilization terms for inflation and the output gap still hold in the case where bounded rationality in agents is allowed. In addition, Ravenna and Walsh (2006) derived the loss function of the central bank in the case of a cost channel and demonstrated that the loss function consists of the standard policy objectives even in the case of a cost channel. In sum, except for the presence of cognitive discounting, we can use the standard loss function, which corresponds to the second-order approximation of the household's utility function in a model with a cost channel.

4.2 Properties of optimal monetary policy

We evaluate the policy performance of commitment and discretionary policies using the impulse response analysis. According to the standard NK model, a commitment policy generally outperforms a discretionary policy (Woodford, 2003). The reason is that central banks can commit to future monetary policy stances in the current period under a commitment policy. In contrast to discretionary policies, policy inertia allows central banks to influence private sector expectations because central banks that commit to an optimal policy can appropriately

[24] These studies delivered the central bank's loss function by calculating the second-order approximation of the household's utility function and showed that the loss function includes the stabilization terms for inflation and the output gap.

introduce policy inertia into the economy. Ravenna and Walsh (2006) also considered the role of a cost channel in accounting for the properties of optimal monetary policy. In this study, we focus on whether the presence of a cost channel affects the performance of optimal monetary policy in a BNK model.

We consider the role of the demand and supply shock when examining the impact of a cost channel on optimal monetary policy in the BNK model. In the absence of the cost channel, Gabaix (2020) showed that the central bank can exclude the constraint regarding the DIS curve from its optimization problem.[25] However, the central bank's constraints consist of DIS and NKPC curves in the case of a cost channel. In what follows, we numerically examine the properties of optimal monetary policy in our model. Appendix B presents some analytical expressions of the properties of optimal monetary policy in our model.

Figures 7 and 8 show the impulse response of inflation and the output gap to a positive demand shock for the cases of commitment and discretion. The central bank can fully stabilize the inflation rate if the cost channel does not exist. Moreover, the output gap in that the central bank can simultaneously achieve zero inflation and zero output gap (Woodford, 2003). However, in the presence of the cost channel, the central bank faces the trade-off between inflation and the output gap. An increase in the inflation rate accompanies a decline in the output gap to meet the optimal targeting criteria for both the cases of commitment and discretion. Thus, the demand shock increases the inflation rate but leads to a decline in the output gap. When the cognitive discounting parameter is close to unity, the central bank can ease this policy trade-off by manipulating the expectations of the private sector: see the bottom panels of Figures 7 and 8. However, as the degree of cognitive discounting increases—a small value of m—the central bank no longer introduces policy inertia into the economy. In particular, for $m = 0.25$, the impulse response of the inflation rate under commitment is almost identical to that under discretion. Conversely, the output gap response seems to be characterized by inertial behavior.

[Figure 7 around here]

[Figure 8 around here]

Figures 9 and 10 depict the impulse response of inflation and the output gap to a price mark-up shock for the cases of commitment and discretion. A price mark-up shock causes a

[25] See Walsh (2017) for a detailed discussion about this issue.

policy trade-off between inflation and the output gap (Woodford, 2003). First, considering the case of $m = 1$, as shown in both figures, the responses of inflation and the output gap to a cost-push shock are almost unaffected by introducing the cost channel. In addition, the commitment solution can impart policy inertia into the economy. Thus, as argued by Woodford (2003), the commitment policy can alleviate the policy trade-off associated with a price mark-up shock by correctly managing the expectations of the private sector. However, Figures 9 and 10 show that a high degree of cognitive discounting, namely, a small value of m, renders the optimal response of inflation and the output gap less responsive to a cost-push shock. When $m = 0.25$, the response of inflation under commitment is the same as that under discretion despite the presence of the cost channel.

[Figure 9 around here]

[Figure 10 around here]

5 Concluding remarks

Considerable studies have argued how monetary policy transmission works. The traditional transmission mechanism of monetary policy indicates that monetary contraction induces a decline in inflation and the output gap. However, an additional channel, namely the cost channel, is empirically and theoretically addressed regarding the supply-side effect of monetary policy. If we permit this channel, an increase in the nominal interest rate yields a different consequence in contrast to the traditional monetary policy transmission. In addition, notably, the effectiveness of monetary policy relies on how the central bank successfully manipulates the expectations of the private sector. The expectations channel of monetary policy plays a significant role in the NK model. However, the policy prescription of the standard NK model, which relies on the assumption of rational expectations, may provide a paradoxical policy prescription. We explore how the departure from rational expectations significantly magnifies or attenuates the cost channel.

We examined the impact of a cost channel on monetary policy transmission in the BNK model to answer this question. First, we analytically derived the determinacy condition in the BNK model with a cost channel. We found that, unlike previous studies, the degree of cognitive discounting significantly affects the determinacy condition. Second, we demonstrated the price puzzle phenomenon in our model. We found that the price puzzle occurs only when a large value of the cost channel parameter, which is not empirically supported, is introduced with a high degree of cognitive discounting. Third, we documented that the degree of cognitive discounting significantly impacts the effect of a cost channel on optimal monetary policy.

A Appendix A: Proof of proposition

A.1 Proof of Proposition 1

The characteristic polynomial of Ω is $P(\mu) = \mu^2 + A_1\mu + A_0$, where

$$A_0 = \omega\sigma M M_f \beta,$$
$$A_1 = -\omega[M_f\beta(\sigma + \phi_x)\delta(\gamma + \eta) + \delta\psi_i\phi_x + \sigma(1 - \delta\psi_i\phi_\pi)].$$

The determinacy condition requires that both eigenvalues of Ω must be inside the unit circle. The necessary and sufficient conditions for determinacy are given as follows:

$$|A_0| < 1, \tag{A.1}$$

$$|A_1| < 1 + A_0. \tag{A.2}$$

For the case of the contemporaneous rule, condition (A.1) holds automatically from the range of each parameter, whereas condition (A.2) indicates (24) and (25).

A.2 Proof of Proposition 2

In the case of the current-looking Taylor rule, the characteristic polynomial of Ω is $P(\mu) = \mu^2 + A_1\mu + A_0$ where

$$A_0 = \sigma^{-1}[\beta M_f(\sigma M - \phi'_x) + \delta\psi_i(M\sigma\phi'_\pi - \phi'_x)],$$
$$A_1 = \sigma^{-1}[\phi'_x + \delta(\gamma + \eta)(\phi'_\pi - 1) - \sigma(M_f\beta + M) - \delta\psi_i\sigma\phi'_\pi].$$

As in the previous case, the determinacy condition requires that eigenvalues of Ω must be inside the unit circle. The determinacy properties are characterized as follows:

$$|A_0| < 1, \tag{A.3}$$

$$|A_1| < 1 + A_0. \tag{A.4}$$

For the case of the current-looking Taylor rule, condition (A.3) indicates (30) and (31), whereas condition (A.4) indicates (29) and (32).

B Appendix B: Derivation of optimal monetary policy

In this section, we briefly provide the central bank's optimization problem. The central bank minimizes a policy objective function, considering the Lagrange multipliers associated with DIS and NKPC. In this Appendix, we focus on the case for a cost-push shock.

B.1 Commitment

Specifically, the Lagrangian of the central bank's problem under commitment is as follows:

$$\Lambda = E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \pi_t^2 + \lambda x_t^2 - 2\phi_{1,t}[M x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^n) - x_t]$$
$$- 2\phi_{2,t}(M_f \beta \pi_{t+1} + \delta(\gamma + \eta) x_t + \delta \psi_i i_t - \pi_t) \Big\}.$$

The first order conditions with respect to π_t, x_t, and i_t are given as follows:

$$\pi_t - \sigma\beta^{-1}\phi_{1,t-1} + \phi_{2,t} - M_f \phi_{2,t-1} = 0,$$
$$\lambda x_t + \phi_{1,t} - M\beta^{-1}\phi_{1,t-1} - \delta(\gamma+\eta)\phi_{2,t} = 0,$$
$$\sigma\phi_{1,t} - \delta\psi_i \phi_{2,t} = 0,$$

where $\phi_{1,t}$ and $\phi_{2,t}$ denote the Lagrange multipliers associated with DIS and NKPC, respectively. Using these conditions, we obtain the following first-order difference equation system:

$$\phi_{2,t} = (M_f + \beta^{-1}\delta\psi_i)\phi_{2,t-1} - \pi_t,$$
$$x_t = \lambda[\delta(\gamma+\eta) - \sigma^{-1}\psi_i \delta]\phi_{2,t} + \lambda\beta^{-1}\sigma^{-1}\delta\psi_i\phi_{2,t-1}.$$

After several manipulations, we obtain the following reduced system under a commitment policy:

$$y_t = \Omega E_t y_{t+1} + \Phi y_{t-1} + u_t,$$

where u_t denotes the term for an exogenous cost-push shock disturbance, $y_t = [\pi_t, \phi_{2,t}]'$, and

$$\Omega = \vartheta \begin{pmatrix} \beta\lambda(M_f\beta + \delta\psi_i) & \sigma^{-1}\beta\delta\psi_i M\kappa_0 \\ -\beta\lambda(M_f\beta + \delta\psi_i) & -\sigma^{-1}\beta\delta\psi_i M\kappa_0 \end{pmatrix},$$
$$\Phi = \vartheta \begin{pmatrix} 0 & \sigma^{-1}\kappa_0\delta\psi_i + (\beta\kappa_0^2 + \sigma^{-2}\delta^2\psi_i^2 M)(M_f + \beta^{-1}\delta\psi_i) \\ 0 & \beta\lambda(M_f + \beta^{-1}\delta\psi_i) - \sigma^{-1}\kappa_0\delta\psi_i \end{pmatrix}.$$

and $\vartheta = (\beta\lambda + \beta\kappa_0^2 + \delta^2\psi_i^2 M\sigma^{-2})^{-1}$, $\kappa_0 = \delta[(\gamma+\eta) - \sigma^{-1}\psi_i)]$.

B.2 Discretion

Under a discretionary policy, we obtain the following targeting rule:

$$\pi_t = -\frac{\lambda}{\kappa_0} x_t,$$

where $\kappa_0 = \delta(\gamma + \eta) - \delta\psi_i\sigma^{-1}$.

After several manipulations, we obtain the reduced form under discretion:

$$\pi_t = \omega_0 E_t \pi_{t+1} + \omega_1 u_t,$$

where

$$\omega_0 = \frac{\lambda(M_f\beta + \delta\psi_i) - \delta\psi_i\sigma^{-1}M\kappa_0}{\lambda + \kappa_0^2}.$$

References

Abo-Zaid, S., 2022. The government spending multiplier in a model with the cost channel. Macroeconomic Dynamics 26, 72–101.

Airaudo, M., 2013. Monetary policy and stock price dynamics with limited asset market participation. Journal of Macroeconomics 36, 1–22.

Airaudo, M., 2020. Temptation and forward-guidance. Journal of Economic Theory 186, 104989.

Ascari, G., Ropele, T., 2009. Trend inflation, Taylor principle, and indeterminacy. Journal of Money, Credit and Banking 41, 1557–1584.

Barth III, M.J., Ramey, V.A., 2001. The cost channel of monetary transmission. NBER macroeconomics annual 16, 199–240.

Beaudry, P., Hou, C., Portier, F., 2024. Monetary policy when the Phillips curve is quite flat. American Economic Journal: Macroeconomics 16, 1–28.

Benchimol, J., Bounader, L., 2019. Optimal monetary policy under bounded rationality. International Monetary Fund.

Bullard, J., Mitra, K., 2002. Learning about monetary policy rules. Journal of Monetary Economics 49, 1105–1129.

Bullard, J., Singh, A., 2008. Worldwide macroeconomic stability and monetary policy rules. Journal of Monetary Economics 55, S34–S47.

Calvo, G.A., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics 12, 383–398.

Castelnuovo, E., 2007. Cost channel and the price puzzle: The role of interest rate smoothing. Unpublished manuscript, University of Padua .

Chowdhury, I., Hoffmann, M., Schabert, A., 2006. Inflation dynamics and the cost channel of monetary transmission. European Economic Review 50, 995–1016.

Christiano, L.J., Eichenbaum, M., Evans, C.L., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113, 1–45.

Del Negro, M., Giannoni, M.P., Patterson, C., 2012. The forward guidance puzzle. FRB of New York Staff Report .

Demirel, U.D., 2013. Gains from commitment in monetary policy: Implications of the cost channel. Journal of Macroeconomics 38, 218–226.

Florio, A., 2018. Nominal anchors and the price puzzle. Journal of Macroeconomics 58, 224–237.

Gabaix, X., 2020. A behavioral New Keynesian model. American Economic Review 110, 2271–2327.

Gali, J., Gertler, M., Lopez-Salido, J.D., 2007. Markups, gaps, and the welfare costs of business fluctuations. The Review of Economics and Statistics 89, 44–59.

Garín, J., Lester, R., Sims, E., 2018. Raise rates to raise inflation? Neo-Fisherianism in the New Keynesian model. Journal of Money, Credit and Banking 50, 243–259.

Hanson, M.S., 2004. The "price puzzle" reconsidered. Journal of Monetary Economics 51, 1385–1413.

Henzel, S., Hülsewig, O., Mayer, E., Wollmershäuser, T., 2009. The price puzzle revisited: Can the cost channel explain a rise in inflation after a monetary policy shock? Journal of Macroeconomics 31, 268–289.

Hirose, Y., Iiboshi, H., Shintani, M., Ueda, K., 2023. Estimating a behavioral new keynesian model with the zero lower bound. Journal of Money, Credit and Banking .

Ida, D., 2014. Role of financial systems in a sticky price model. Journal of Economics and Business 72, 44–57.

Ida, D., 2023. Cost channel, determinacy, and monetary policy in a two-country new Keynesian model. Economic Modelling 119, 106112.

Ida, D., 2024a. Household heterogeneity and the price puzzle in a new Keynesian model. Journal of Macroeconomics 79, 103587.

Ida, D., 2024b. Optimal monetary policy rules in a behavioral new Keynesian model. Manuscript .

Ilabaca, F., Meggiorini, G., Milani, F., 2020. Bounded rationality, monetary policy, and macroeconomic stability. Economics Letters 186, 108522.

LaSalle, J., 1986. The Stability and Control of Discrete Processes. Applied Mathematical Sciences .

Llosa, L.G., Tuesta, V., 2008. Determinacy and learnability of monetary policy rules in small open economies. Journal of Money, Credit and Banking 40, 1033–1063.

Llosa, L.G., Tuesta, V., 2009. Learning about monetary policy rules when the cost-channel matters. Journal of Economic Dynamics and Control 33, 1880–1896.

McCallum, B.T., 1983. On non-uniqueness in rational expectations models: An attempt at perspective. Journal of Monetary Economics 11, 139–168.

Meggiorini, G., Milani, F., 2021. Behavioral New Keynesian Models: Learning vs. Cognitive Discounting. CESifo Working Paper .

Nie, H., 2023. Government spending multipliers with the real cost channel. Macroeconomic Dynamics , 1–22.

Patel, N., 2021. International trade finance and the cost channel of monetary policy in open economies. International Journal of Central Banking 17, 117–178.

Pfäuti, O., Seyrich, F., 2022. A behavioral heterogeneous agent New Keynesian model. DIW Berlin Discussion Paper .

Qureshi, I.A., Ahmad, G., 2021. The cost-channel of monetary transmission under positive trend inflation. Economics Letters 201, 109802.

Rabanal, P., 2007. Does inflation increase after a monetary policy tightening? answers based on an estimated dsge model. Journal of Economic Dynamics and Control 31, 906–937.

Ravenna, F., Walsh, C.E., 2006. Optimal monetary policy with the cost channel. Journal of Monetary Economics 53, 199–216.

Sims, C.A., 1992. Interpreting the macroeconomic time series facts: The effects of monetary policy. European Economic Review 36, 975–1000.

Surico, P., 2008. The cost channel of monetary policy and indeterminacy. Macroeconomic Dynamics 12, 724.

Taylor, J.B., 1993. Discretion versus policy rules in practice, in: Carnegie-Rochester Conference Series on Public Policy, Elsevier. pp. 195–214.

Tillmann, P., 2008. Do interest rates drive inflation dynamics? an analysis of the cost channel of monetary transmission. Journal of Economic Dynamics and Control 32, 2723–2744.

Tillmann, P., 2009. Robust monetary policy with the cost channel. Economica 76, 486–504.

Uribe, M., 2022. The neo-Fisher effect: Econometric evidence from empirical and optimizing models. American Economic Journal: Macroeconomics 14, 133–62.

Wagner, J., Schlanger, T., Zhang, Y., 2022. A horse race of alternative monetary policy regimes under bounded rationality. Technical Report. Bank of Canada.

Walsh, C., 2017. Monetary Theory and Policy. fourth ed., MIT Press, Cambridge.

Woodford, M., 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton Princeton University Press, Princeton.

Figure 1: Equilibrium determinacy in a BNK with a contemporaneous Taylor rule

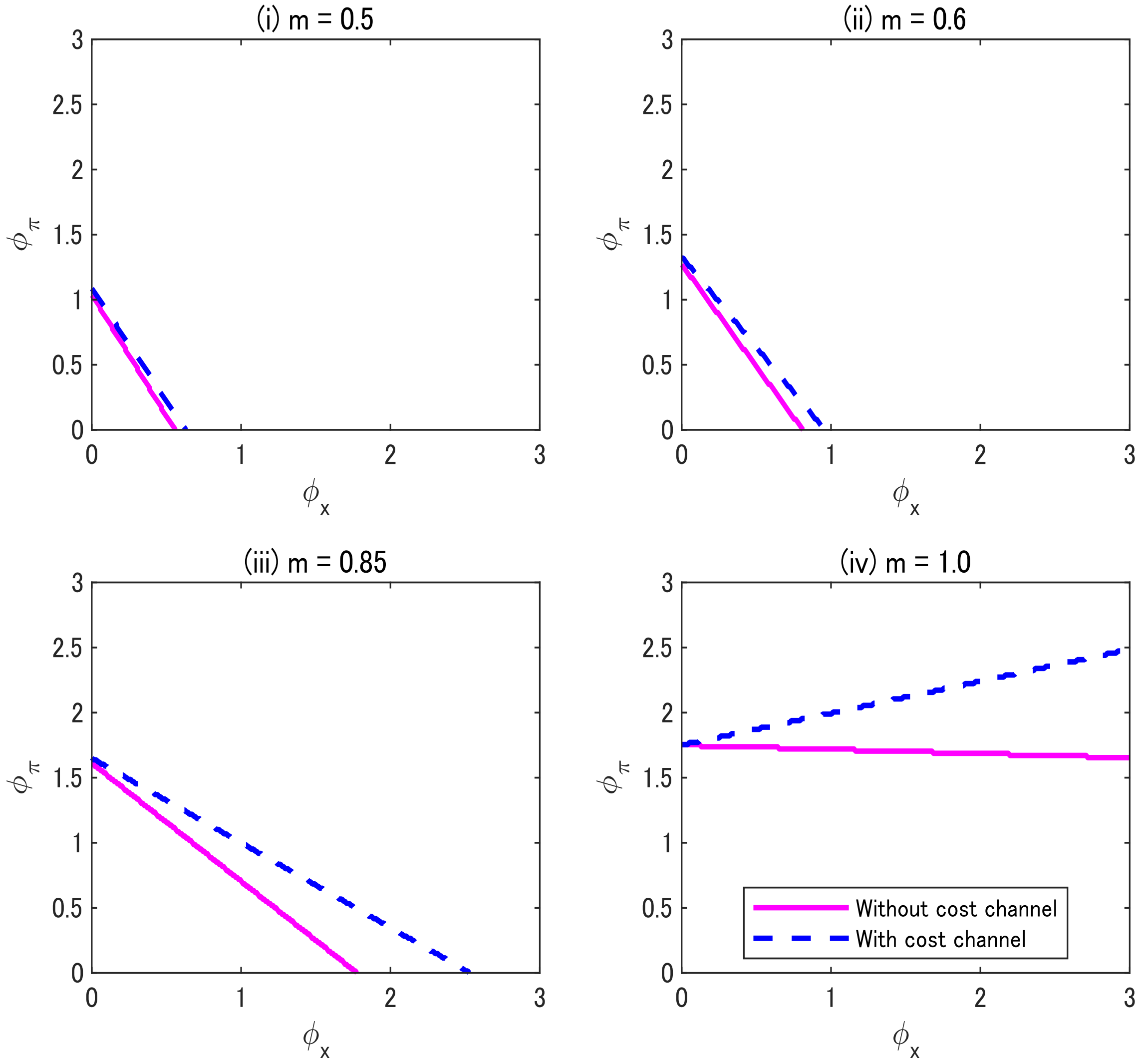


Figure 2: Equilibrium determinacy under a forward-looking Taylor rule

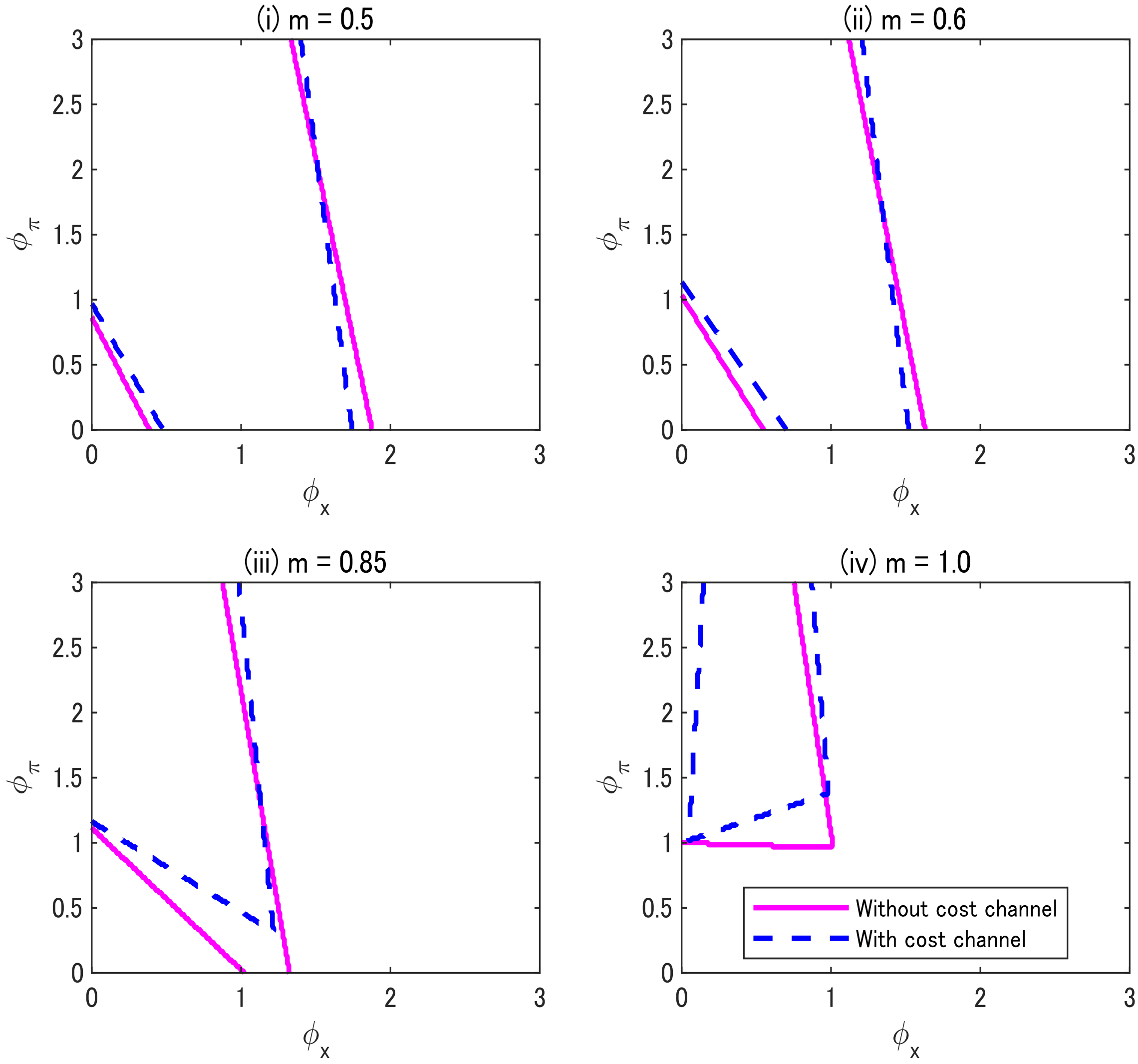


Figure 3: Impulse response of inflation to a monetary tightening shock

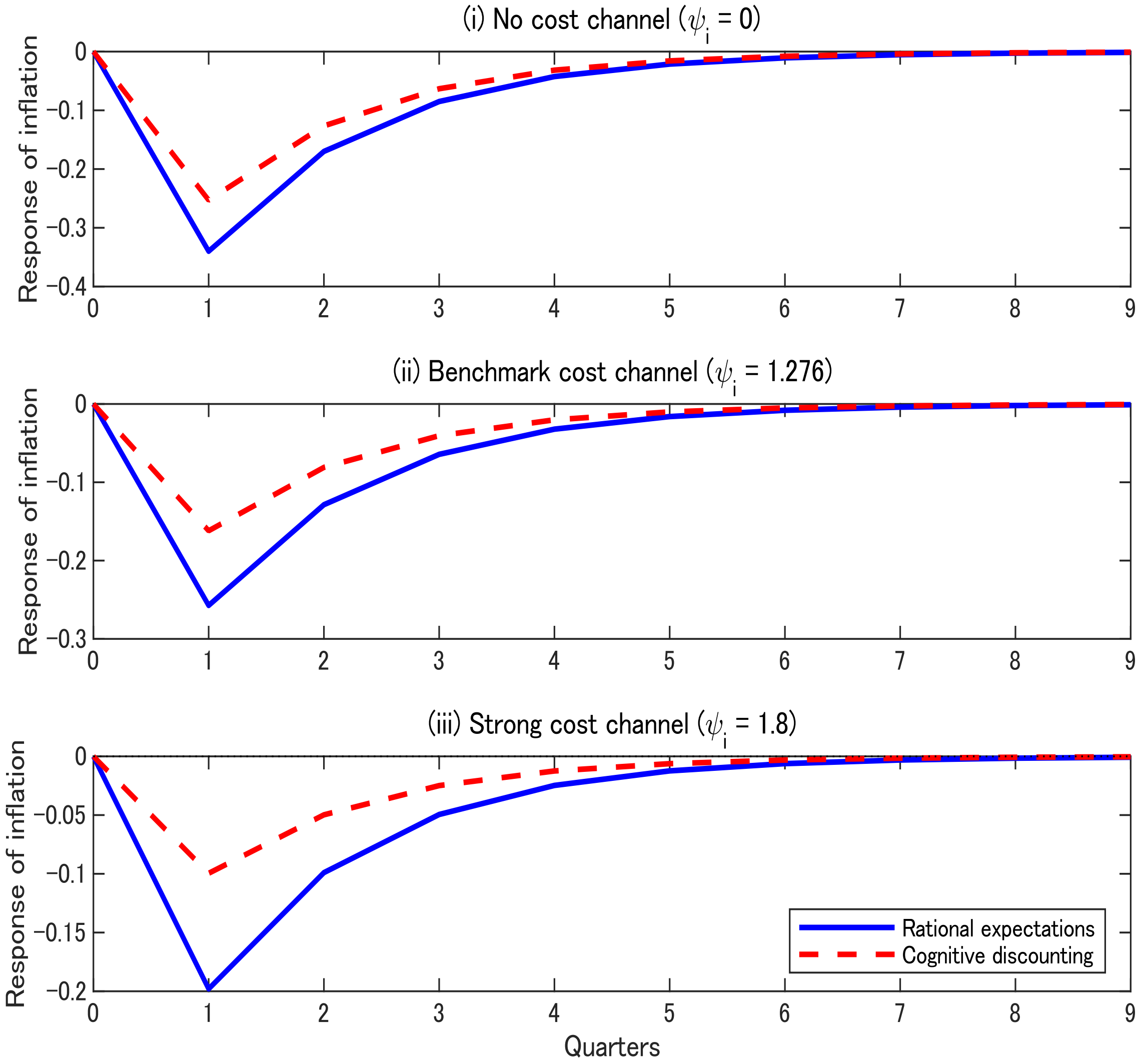


Figure 4: Impact response of inflation to a monetary tightening shock: The role of cognitive discounting

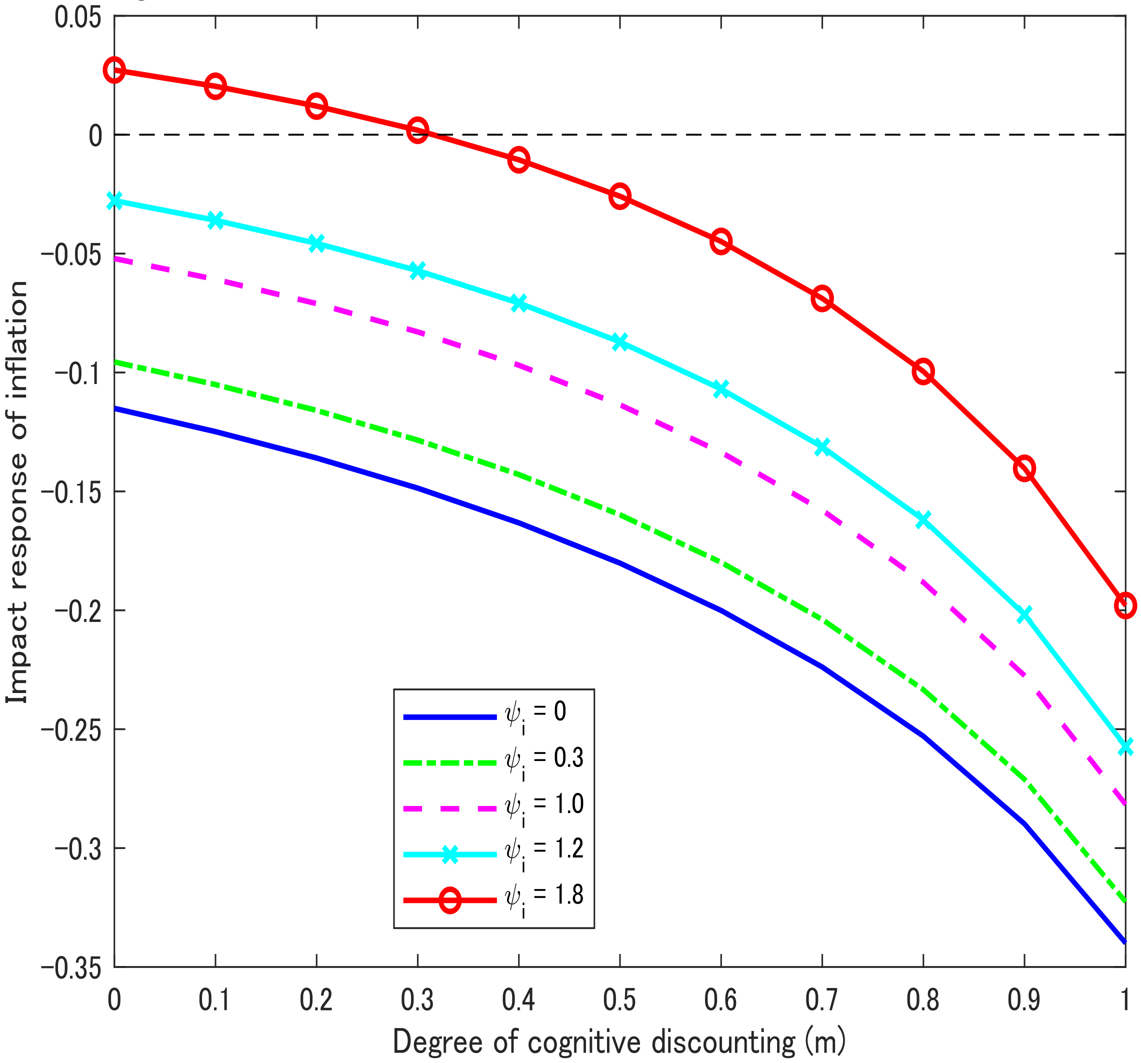


Figure 5: Effect of the cost channel parameter on the price puzzle: The role of policy shock persistence

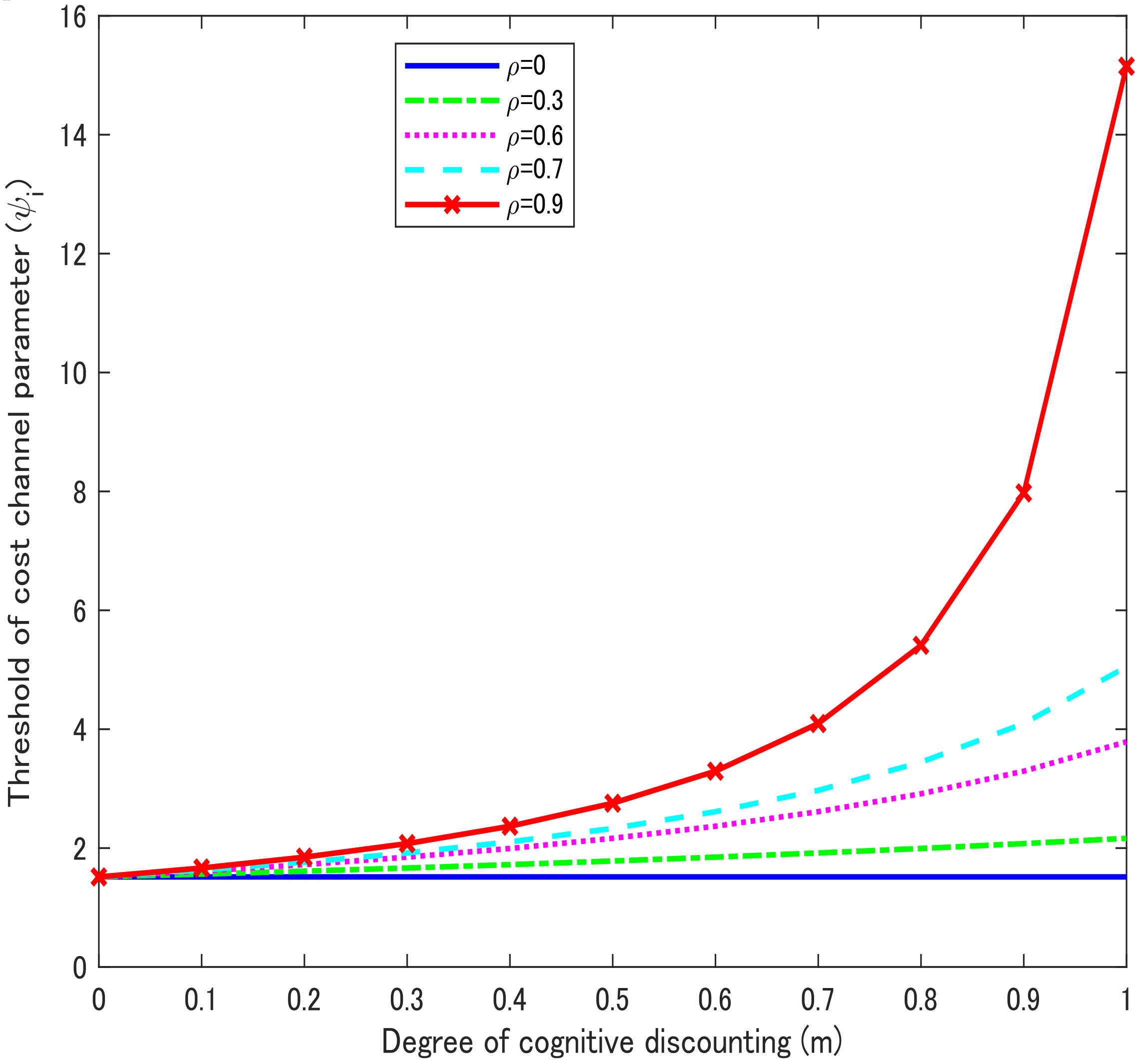


Figure 6: Effect of the cost channel parameter on the price puzzle: The role of CRRA coefficient

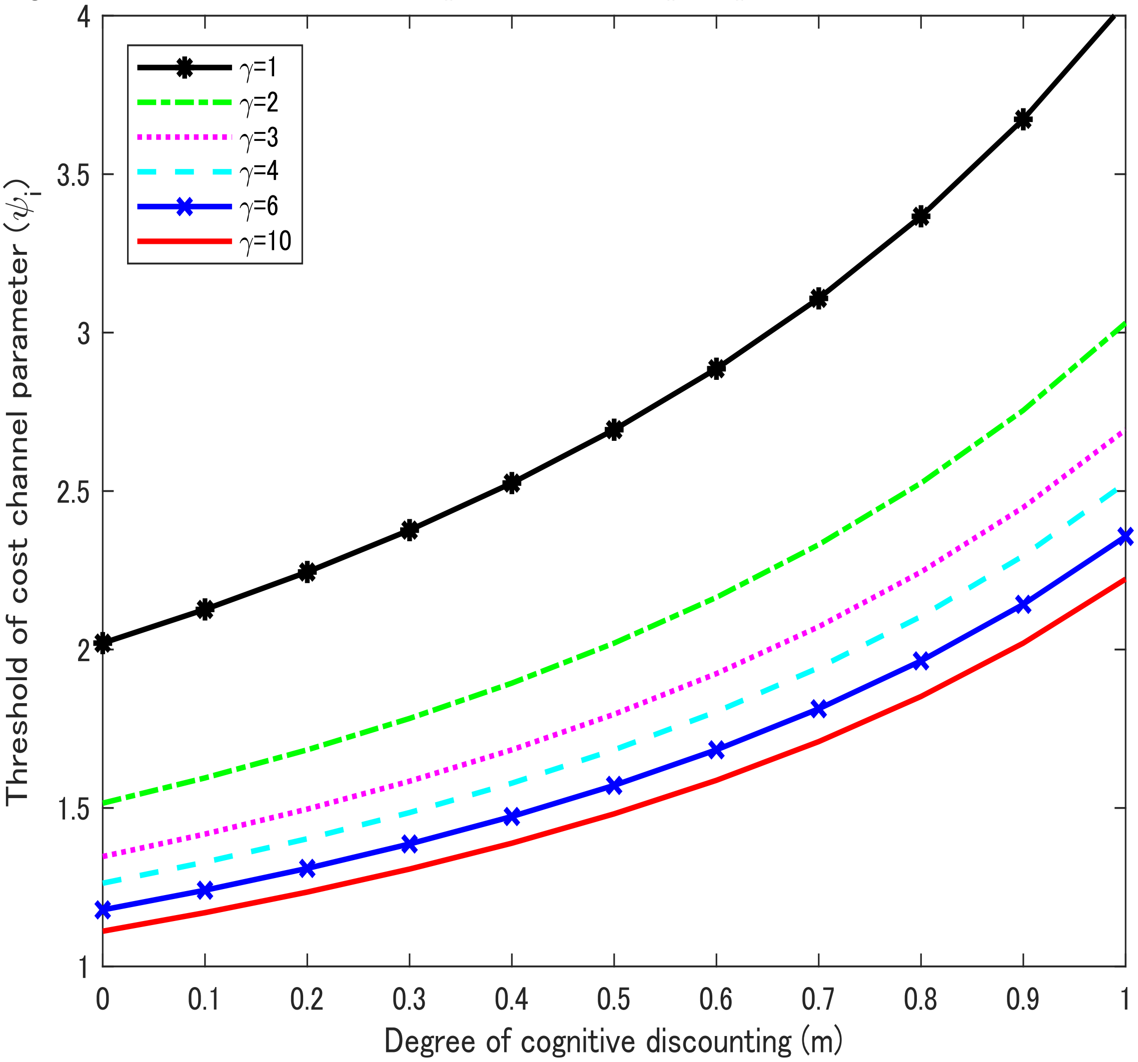
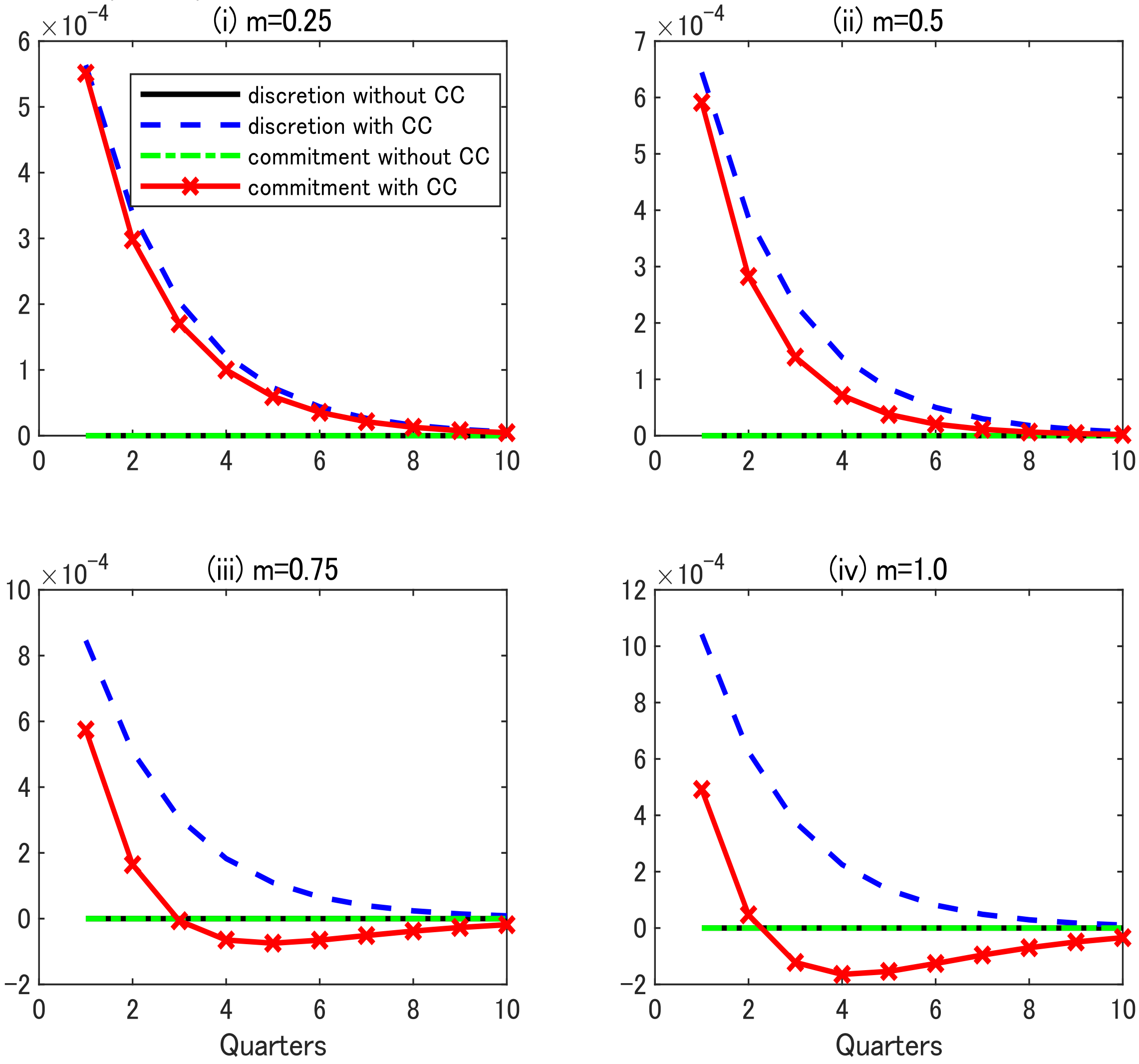
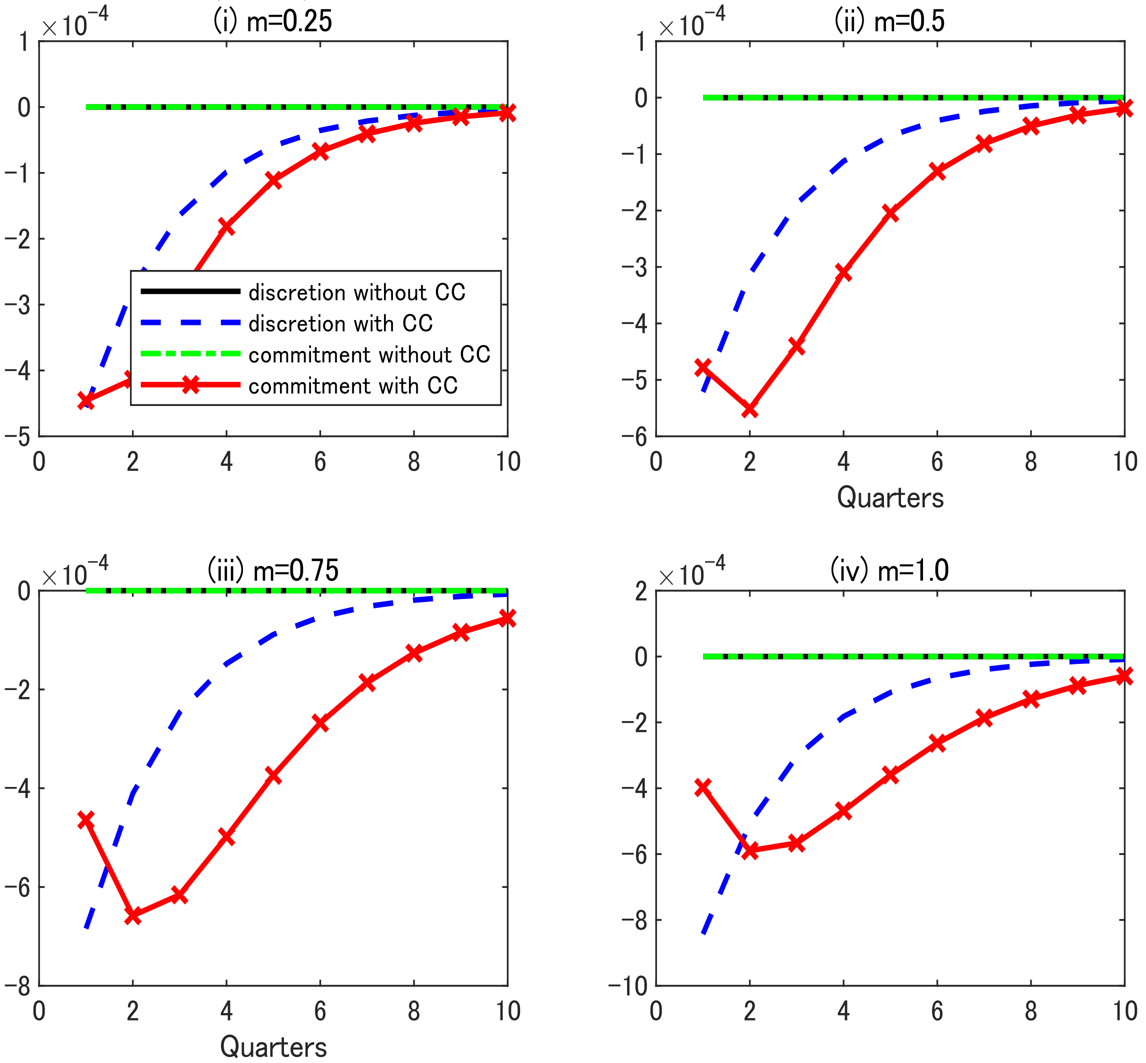


Figure 7: Interaction effect of cognitive discounting and cost channel on inflation under optimal monetary policy: Impulse response function to a demand shock



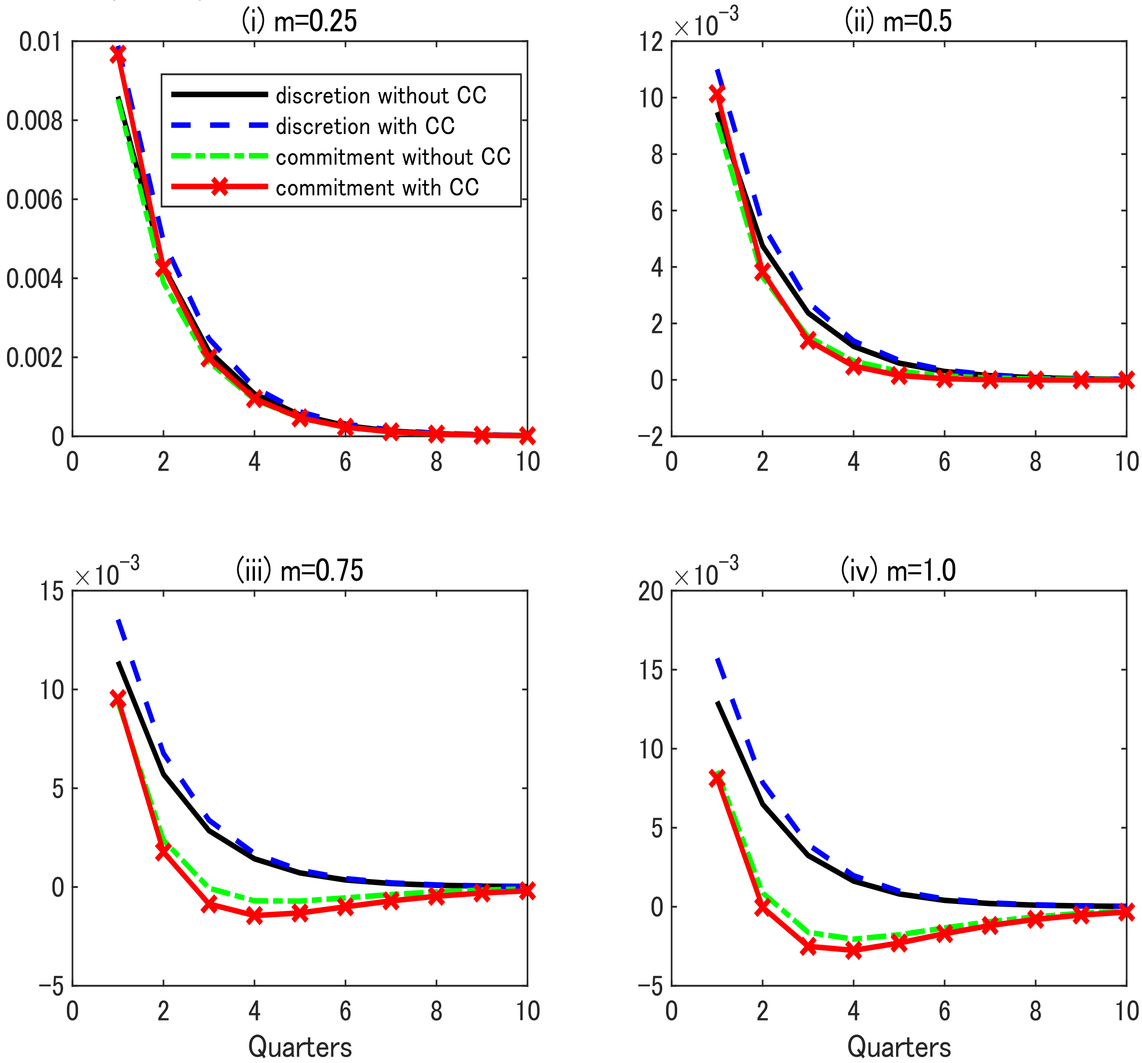
(Note) CC denotes the cost channel.

Figure 8: Interaction effect of cognitive discounting and cost channel on the output gap under optimal monetary policy: Impulse response function to a demand shock



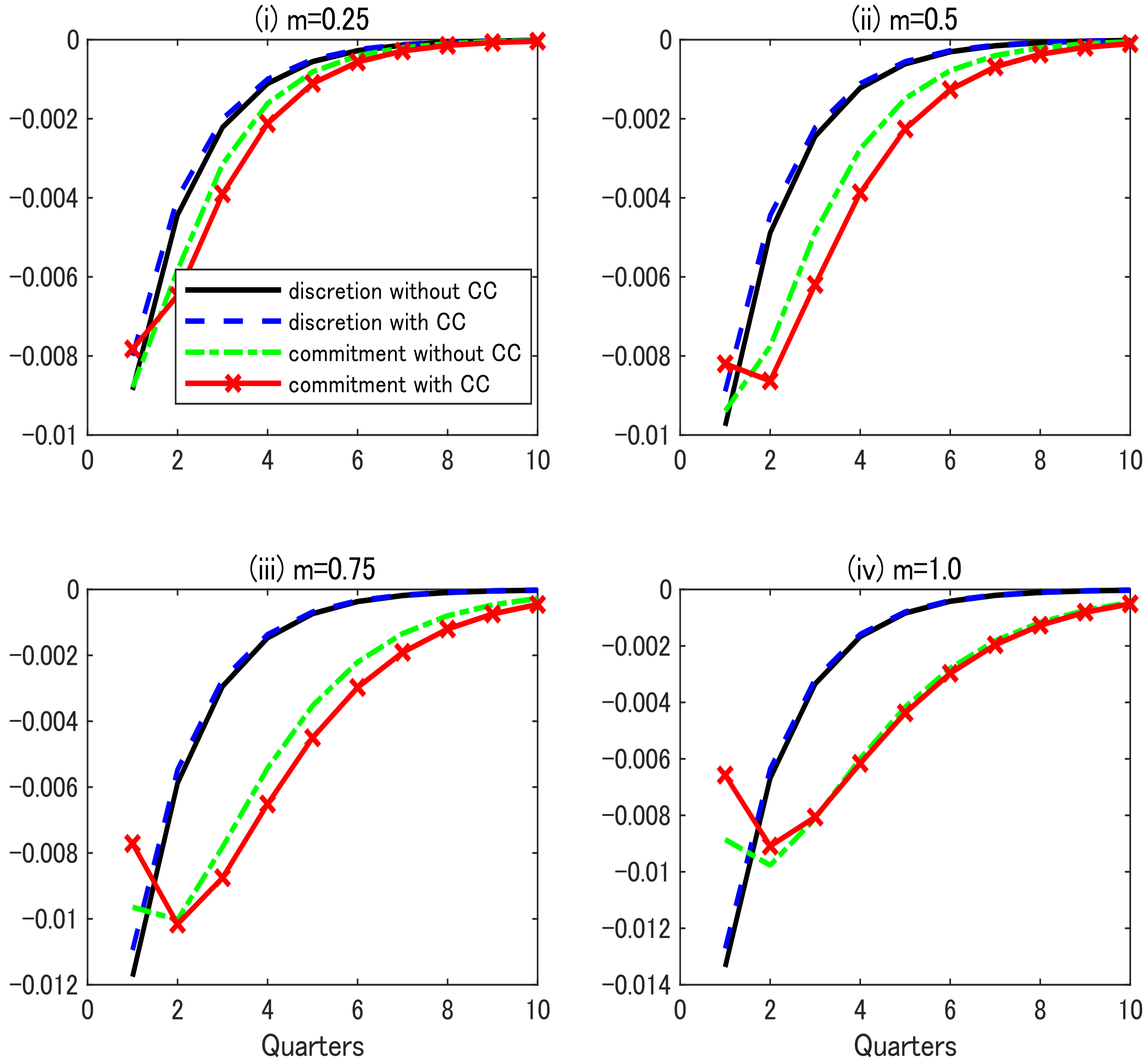
(Note) CC denotes the cost channel.

Figure 9: Interaction effect of cognitive discounting and cost channel on inflation under optimal monetary policy: Impulse response function to a cost-push shock



(Note) CC denotes the cost channel.

Figure 10: Interaction effect of cognitive discounting and cost channel on the output gap under the optimal monetary policy: Impulse response function to a cost-push shock



(Note) CC denotes the cost channel.