



Munich Personal RePEc Archive

How Information Design Shapes Optimal Selling Mechanisms

Pham, Hien

Toulouse School of Economics

30 April 2023

Online at <https://mpra.ub.uni-muenchen.de/120462/>
MPRA Paper No. 120462, posted 19 Mar 2024 14:25 UTC

How Information Design Shapes Optimal Selling Mechanisms

Hien Pham*

March 17, 2024

Abstract

A monopolistic seller jointly designs allocation rules and (new) information about a pay-off relevant state to a buyer with private types. When the new information flips the ranking of willingness to pay across types, a *screening* menu of prices and threshold disclosures is optimal. Conversely, when its impact is marginal, *bunching* via a single posted price and threshold disclosure is (approximately) optimal. While information design expands the scope for random mechanisms to outperform their deterministic counterparts, its presence leads to an equivalence result regarding sequential versus. static screening.

1 Introduction

The evolution of informational technology has significantly broadened sellers' ways of selling their products. They can design not only *allocation rules* which specify how to allocate products and charge payments to buyers, but also *information policies* which control how much buyers learn about the products, thereby refining their willingness to pay. For instance, they may offer a posted price, associated with full information, to everyone. Alternatively, they could propose a rich menu of allocation rules and information policies.

As an example, many software such as McAfee and various (mobile) apps like Spotify provide users with a *single* free trial version, followed by a *single* subscription fee schedule. The trial version is, therefore, *merely* a learning opportunity for potential buyers to make well-informed purchasing decisions. An opposite example is travel agency platforms such as Priceline and Hotwire practice so-called "opaque pricing" by which, buyers either book hotels with detailed

*Toulouse School of Economics. Email: hien.pham@tse-fr.eu

information at standard prices or opt for limited details at discounted prices. Thus, these travel agencies *screen* their buyers via a menu of prices and information policies.

Price and information discrimination is also in the form of pre-order offers for buyers of not-yet-released products, as exemplified by Google's recent pre-order bonus for the Pixel 8. By contrast, well-known products are typically sold via a *single* posted price, coupled with a *single* timeframe for free return to all buyers.

What leads to these diverse selling strategies? In particular, when is a single posted price and disclosure policy optimal and conversely, when is it necessary to provide a screening menu of prices and information? In addition, is there any benefit from offering random mechanisms? Given that classical mechanism design results (Myerson (1981)) predict that a posted price is optimal when the informational environment is *fixed*, answering these questions explains how information design shapes optimal selling mechanisms. Regarding the timing, can the seller's revenue be improved by contracting with the buyer at the "*interim*" stage where he knows his type but before the seller's information disclosure? Or equivalently, should she allow the buyer to walk away at the "*posterior*" stage where he observes both his type and the information provided? Answering this question helps understand the impact of consumer protection regulations that grant the consumer a withdrawal right such as the European directive 2011/83/EU.¹ Finally, if the buyer privately observes the information disclosed by the seller, can the buyer enjoy any rent induced from such an *endogenously private* information?

This paper aims to answer these questions. The model, as formally described in Section 2, features a seller (she) who sells an object to a buyer (he) with a privately known initial valuation (*initial type*). The seller controls how much the buyer learns about an *additional component* in his valuation. For example, this additional component represents what the buyer learns via product trials. The seller designs a menu of *information policies* for different types of the buyer, and *allocation rules* for different types and signals. Therefore, she solves a joint mechanism and information design problem in which information plays a dual role. First, it allows the seller to screen the buyer's type through discriminatory disclosure policies. Second, disclosed information serves as input for designing allocation rules. We focus on the case where the buyer privately observes the new information (private signals) and investigate the case with public signals as a benchmark.

¹For a detailed discussion on such policies, see Krämer and Strausz (2015b).

1.1 Summary of results

First, we establish a revenue-equivalence result regarding sequential vs. static screening. Specifically, we show that for any *feasible* and *deterministic* mechanism, there exists a mechanism that generates the same revenue for the seller and non-negative payoff for the buyer at any type and signal realization. As a consequence, there is no revenue loss if contracting at the posterior stage when the buyer knows both his type and signal. This result counters the well-established idea in sequential screening suggesting that the seller's revenue is strictly higher if contracting with the buyer before, rather than after, he learns additional information.² The basic intuition is that the seller's ability to flexibly design information can crowd out the advantages of sequential over static screening. A practical implication is that afore-mentioned consumer protections do not necessarily harm the seller, rationalizing the prevalence of free information in many markets.

Second, we investigate the (ir)relevance of signal privacy. In the benchmark problem with public signals, only *expected* allocations and payments (over signals) matter. Hence, this benchmark admits multiple solutions, including \mathbf{M}^* , a *screening* menu of threshold disclosures π^* and prices paid conditional on trade.³ We provide a simple way to verify the (ir)relevance of signal privacy, which is to check if, under \mathbf{M}^* , the highest type pays the lowest price. If this is true, privacy of signals is irrelevant and \mathbf{M}^* solves the seller's original problem. We find that this is not always the case and consequently, not observing signals generally hurts the seller. Moreover, *per-signal* allocations and payments matter, which significantly complicates the characterization of optimal mechanisms. In particular, it is not *a priori* clear how many signals are needed and which incentive compatibility (IC) constraints are relevant. The seller must also handle double deviations when the buyer lies about both his type and observed signal. Leveraging techniques for mechanisms with non-convex type spaces, we make it *always* possible for the buyer to "correct his lie," facilitating the characterization of optimal double deviations and thereby, optimal mechanisms.

Our main result characterizes optimal mechanisms, starting with binary types. The seller faces a trade-off between maximizing virtual surplus and minimizing the *posterior* rent. A threshold disclosure rule, under which signal realization is either "*good news*" if the state is above some cutoff or "*bad news*" otherwise, is optimal in both targets.⁴ Under the optimal mechanism, the seller either *screens* the buyer's types (via a menu of threshold disclosures and posted prices) or *bunches* them (via a single posted price and threshold disclosure), depending on whether the threshold disclosure π^* induces a *threshold flip* of type order: the high type's value after "*bad*

²See Courty and Li (2000) and Krämer and Strausz (2015b).

³See Definition 4 for a formal description of \mathbf{M}^* .

⁴See Definition 1 for our formal definition of a threshold disclosure.

news" is lower than the low type's after "good news." Specifically, screening is optimal when this flip of type order occurs, and bunching otherwise.

To grasp the intuition, note that such a flip of type order occurs when the variation of valuations is mainly driven by the unknown component, leaving some room for the threshold disclosure π^* to reverse the ranking of valuation. Information (about the unknown component) matters, serving as a screening tool. Conversely, if the buyer's type is the main driver, which prevents π^* from flipping the type order, information is not crucial and screening disappears. The optimal mechanism echoes its counterpart in standard mechanism design where the buyer's valuation is his type: a posted price (but associated with threshold disclosure) is optimal.

The significance of this bunching vs. screening result is two-fold. First, it implies that in the above-mentioned scenarios, eliciting signals and random mechanisms are worthless. Second, it rationalizes observed mechanisms in practice. For coming-soon items, the unknown component's impact on the variation of valuations is large and a screening menu is employed. By contrast, its impact is marginal for well-known products where bunching comes into play. The significance of the unknown component also varies across different industries. In the realm of hotels, it matters much more than in software or mobile apps, leading to screening for the former and bunching for the latter.

Having characterized the optimal mechanism for the binary-type setting, we consider larger type spaces. With more than two types, there are also cases where an information policy reverses the ranking of valuations within a group of types but fails to do so for another. Consequently, not only information but also trading probabilities are needed to screen the buyer, leading to a *random* solution. However, the two scenarios of bunching/screening extend to the case with finitely many types, under stronger notions of flip (no flip) of type order. Specifically, a screening menu is optimal under a *partition* flip by π^* of type order - which generalizes the threshold flip of type order by π^* , taking into account medium types and their associated cut-off states. Instead, bunching via a fixed price and threshold disclosure maximizes the seller's revenue when there is *uniformly* no threshold flip of type order under which, the type order is to be preserved between *any* pair of types and after *any* threshold disclosure. This strong requirement of type order preservation helps deal with the challenge of determining the lowest type being served in a rich type space.

As binding (IC) constraints can involve local, global, and upward ones, characterizing optimal random mechanisms becomes difficult. We thus focus on shedding light on how random mechanisms outperform their deterministic counterparts.⁵ We first establish the "*no randomization*

⁵In the Online Appendix, we solve for the optimal random mechanism in several examples.

at the top" result, extending the well-known "no distortion at the top" to a setting with information design: the highest type receives an efficient (and hence, deterministic) allocation. In turn, this implies an optimal contract for this type, featuring a posted price and no disclosure. While randomization is not needed for the highest type, it can be helpful for the lower types, leading to a better balance of the efficiency vs. rent trade-off.⁶ We analyze, by examples, how random mechanisms facilitate screening distant types as well as screening signals.

Finally, we consider a setting with a continuum of types. In this case, the optimality of a screening menu of posted prices and threshold disclosures under a partition flip of type order extends readily. Particularly, in a "continuous" model when valuation shifts smoothly across types and states, this notion corresponds to the ranking of valuations at the zero-virtual-value states by types being reversed. On the other hand, the fact that there are always types whose valuations are close to others' makes it impossible to flip the ranking of willingness to pay across *all* types. We show that when the type order is *almost* preserved, bunching via a fixed price-threshold disclosure bundle is *approximately* optimal. If there is only two states, we establish the "exact" optimality of bunching within the class of deterministic mechanisms..

1.2 Related literature

We contribute to the literature on joint mechanism and information design, comprising two main strands. The first, more related, strand endows the buyer with a private type, initiated by Eső and Szentes (2007) who focus on full disclosure. Most other papers focus on *posted-price* mechanisms,⁷ which in turn, makes it without loss of generality to focus on *binary-signal* information structures (Li and Shi (2017), Guo et al. (2022), Wei and Green (2023), Smolin (2023)).⁸ Our findings imply that these restrictions are not innocuous in general.

Our model builds on Eső and Szentes (2007) who focus on full disclosure and an environment with (i) the above-mentioned "continuous" model and (ii) certain assumptions on the valuation function. Under such an environment, they show that the upper bound of revenue with public signals can be achieved via full disclosure, associated with a screening menu of prices (for the

⁶While it is natural to expect the two-dimensionality feature of the buyer's valuation to lead to random mechanisms, the seller has another tool for randomization: the distribution of signals, which potentially makes random mechanisms redundant. However, signal misreporting off-path shuts off this additional instrument. Thus, random mechanisms arise to deter double deviations, minimizing the *posterior* rent.

⁷In posted-price mechanisms, each type receives a posted price for the good and in some cases, a posted fee for information.

⁸Exceptions include Zhu (2023) and Kräbmer (2020) who establish full surplus extraction results when the seller can correlate information disclosed to multiple buyers, and when randomizing over information structures is allowed and the buyer's type correlates with the unknown component, respectively.

good) and information fees. However, their optimal mechanism is not incentive compatible and moreover, privacy of signals generally matters outside their environment.⁹ Not only do we allow for general information structures, we also characterize a *joint* design of information and allocation rules in a more general environment of type space and valuation functions. This allows us to uncover how information design reshapes the optimal selling mechanism which features not just screening, but also bunching and a random mechanism. At the same time, we strengthen Eső and Szentes (2007)'s finding by showing that the irrelevance of signals extends to other (but not all) environments, with appropriate information design.

Bergemann and Wambach (2015) and Wei and Green (2023) revisit Eső and Szentes (2007)'s continuous model, showing that the latter's *optimal* allocation can be implemented under stronger participation constraints. We show that with deterministic allocations (including Eső and Szentes (2007)'s), this is true for any *feasible* allocations, not just optimal. In turn, this provides an alternative proof for Wei and Green (2023).¹⁰

In the second, less related, strand of this literature, the buyer's valuation is the unknown component itself. See, for example, Lewis and Sappington (1994), Bergemann and Pesendorfer (2007), Bergemann et al. (2022). Without the buyer's private types, information cannot serve as a screening tool. Moreover, the buyer's private information (about his valuation) arrives only once, making the seller's problem static.¹¹

We also contribute to the literature on dynamic mechanism design in which handling off-path misreporting is a notable issue. Eső and Szentes (2007) explicitly characterize an agent's optimal double deviation, which is to "correct the lie". However, such a lie correction is feasible only if the agent's payoff shares a *common support* across types, which is rather restrictive. We show that by leveraging mechanism design techniques for a non-convex type space, lie correction is feasible even with non-common supports. Moreover, the existing literature (for instance, Battaglini (2005), Eső and Szentes (2007), Pavan et al. (2014)) extensively relies on the first-order approach considering only local incentive compatibility constraints.¹² Instead, we characterize different scenarios of binding constraints, showing that global deviations (associated with double deviation off-path) lead to bunching and random solutions.¹³

⁹See Krähmer and Strausz (2015a) for a detailed discussion

¹⁰Wei and Green (2023) also shows that information disclosure triggers reverse price discrimination. We show that this can also be derived from the properties of Eső and Szentes (2007)'s optimal mechanism.

¹¹If the buyer in our model has no private type, the seller fully extracts the surplus by offering no disclosure and a posted price for the good, which is equal to the expected valuation.

¹²The validity of this approach usually requires certain regularity conditions, which are not easy to satisfy, see Battaglini and Lamba (2019).

¹³Even with full disclosure, which makes our problem become a standard dynamic screening problem, random

Finally, we contribute to the recent literature on Bayesian persuasion following Kamenica and Gentzkow (2011), where a sender designs *only* information disclosure to affect a receiver's action. When the latter has a private type, Kolotilin et al. (2017) show that with binary actions and linear valuation functions, non-discriminatory disclosure is optimal. In our *joint* design problem, the buyer's action space (which is the menu of allocations and payments) is endogenous and can consist of more than two options. We show that the optimality of non-discriminatory disclosure, while not being true in general, holds in some environments even if the seller also designs allocation rules and the valuation function is non-linear.

2 Model

2.1 Environment

The principal, a seller (she) sells an object to an agent, the buyer (he). The buyer's valuation for the object, $v(\theta, x) \in \mathbb{R}_+$, depends on two components: (i) the buyer's type $\theta \in \Theta \subset \mathbb{R}$ and (ii) an unknown state $x \in X \subset \mathbb{R}$. There are a finite number of possible types and states, *i.e.*, $|\Theta| < \infty$ and $|X| < \infty$.¹⁴ Random variables θ and x are independent. Let $f(\theta)$ be the probability of each type θ and $\mu(x)$ of each state x . Without loss of generality, assume $f(\theta) > 0$ and $\mu(x) > 0$ for all θ and x .

The realization of $\theta \in \Theta$ is privately known by the buyer. Neither the seller nor the buyer knows the state $x \in X$. The seller commits to a policy of information disclosure about the state, formally defined in Section 2.2.

To define payoffs, let $q \in [0, 1]$ be the trading probability and $p \in \mathbb{R}$ the expected transfer from the buyer to the seller. The seller's *ex post* payoff is then p and the buyer's is $v(\theta, x)q - p$.

For expositional clarity, we use the following notations $\bar{\theta} \equiv \max \Theta$, $\underline{\theta} \equiv \min \Theta$, $\theta^+ \equiv \min_{\theta' \in \Theta} \{\theta' \mid \theta' > \theta\}$ for $\theta < \bar{\theta}$, $\theta^- \equiv \max_{\theta' \in \Theta} \{\theta' \mid \theta' < \theta\}$ for $\theta > \underline{\theta}$, and $\bar{\theta}^+ = \bar{\theta}$, $\underline{\theta}^- = \underline{\theta}$. We define \bar{x} and \underline{x} similarly. Let

$$\phi(\theta, x) \equiv v(\theta, x) - [v(\theta^+, x) - v(\theta, x)] \frac{\sum_{\theta' > \theta} f(\theta')}{f(\theta)}$$

denote the buyer's virtual value. Throughout, assume that both the valuation and virtual valuation increase in the buyer's type and the state.

Assumption 1 (Monotone value). $v(\theta, x)$ increases in θ and x .

mechanisms can outperform their deterministic counterparts. See Example 3(b).

¹⁴We study the infinite type and state spaces in Section 7.

Assumption 2 (Monotone virtual value). $\phi(\theta, x)$ increases in θ and x .

2.2 Selling mechanism

The seller designs, and *ex ante* commits to a *grand* mechanism or a menu of (i) information policies for different types of the buyer and (ii) allocation rules for different types and information received by the buyer.

Information policies: We model information policies as information structures (experiments) $\Pi \equiv (S, \pi)$, which consists of a countable set of signals $S \subset \mathbb{R}$,¹⁵ and a mapping π , which associates to each state θ a distribution over signals $\pi(\cdot | x) \in \Delta(S)$. Given a mapping π and a signal realization $s \in S$, the corresponding posterior belief $\Psi(\cdot | s) \in \Delta(X)$ is obtained by Bayes' rule whenever possible, and is given by

$$\mu_{s, \pi}(x) = \frac{\mu(x)\pi(s | x)}{\sum_{x' \in X} g(x')\pi(s | x')}$$

An example of information structures is the threshold rule, defined as follows.

Definition 1 (Threshold disclosure). *If the information policy follows a threshold rule, each signal realization is classified as either "good news" or "bad news". Moreover,*

$$\pi(\text{"good news"}, x) = \begin{cases} 1 & \text{if } x > \hat{x}, \\ \lambda & \text{if } x < \hat{x}, \\ \lambda & \text{if } x = \hat{x}, \end{cases} \quad \text{for some } \hat{x} \in X \text{ and } \lambda \in [0, 1].$$

Thus, a threshold disclosure is represented by a pair (\hat{x}, λ) where \hat{x} is the cut-off state and λ the probability with which "good news" is sent at the cut-off state. It informs the buyer whether the state is (weakly) higher or lower than \hat{x} . To simplify notations, throughout the paper, we use " s^g " to represent "good news" and " s^b " for "bad news".

A menu of experiments is a set $\{\pi_\theta\}_{\theta \in \Theta}$. The paper focuses on the case in which the buyer privately observes the signal. The benchmark case with public signals is examined in Section 3.3.

Without loss of generality, assume that signals are ordered such that upon observing a higher signal, the buyer's posterior valuation is higher, as follows.

Assumption 3 (Ranking of signals).

$$s > s' \Leftrightarrow \sum_x v(\theta, x)\mu_{s, \pi_\theta}(x) \geq \sum_x v(\theta, x)\mu_{s', \pi_\theta}(x)$$

¹⁵Assuming S is a countable set of \mathbb{R} is without loss.

Allocation rules: An allocation rule specifies the trading probability, q , and the expected transfer from the buyer to the seller, p . Given the information structure, by the revelation principle (see, for example, Myerson (1986)), we focus on direct allocation rules $\{q(\theta, s), p(\theta, s)\}_{\theta, s}$.

Thus, a selling mechanism is a tuple $\mathbf{M} \equiv \left\{ \pi_{\theta}, (q(\theta, s), p(\theta, s)) \right\}_{\theta, s}$. The formal definitions of a deterministic mechanism and its random counterpart are as follows.

Definition 2. *An mechanism \mathbf{M} is deterministic if under \mathbf{M} , $q(\theta, s) \in \{0, 1\}$ for all $\theta \in \Theta$ and $s \in S$. \mathbf{M} is random otherwise.*

Timing: The timing of interactions is as follows:

1. The seller offers a selling mechanism \mathbf{M} .
2. The buyer learns his type θ and decides to accept or reject the offer. In case of acceptance, he reports a type $\hat{\theta}$ to receive information generated from $\pi_{\hat{\theta}}$.
3. The buyer *privately* observes a signal s and reports a signal \hat{s} .
4. The allocation $(q(\hat{\theta}, \hat{s}), p(\hat{\theta}, \hat{s}))$ is implemented.

According to this timing, the buyer's participation is decided at the *interim* state, as commonly assumed in the mechanism design literature. See our discussion on the timing structure in Section 3.2.

2.3 Seller's problem

An optimal mechanism refers to a revenue-maximizing mechanism. By the revelation principle, it is without loss of generality to focus on direct mechanisms such that the buyer finds it optimal to (i) participate in the mechanism, (ii) truthfully report his type, and (iii) truthfully report his signal conditional on being truthful about his type. Let

$$u(\theta, \theta', s, s') \equiv \sum_x [v(\theta, x)q(\theta', s') - p(\theta', s')] \Psi_{\theta}(x|s)$$

denote the *ex post* payoff for type- θ buyer, who reports θ' , observes s , and reports s' . Note that if the buyer lies about his type, he may want to lie again about the signal. In other words, double deviations from truth-telling may be attractive. Let

$$s^*(\theta, \theta', s) \in \underset{s'}{\operatorname{argmax}} u(\theta, \theta', s, s')$$

be the optimal signal reporting of type- θ buyer who reports θ' and observes signal s .¹⁶ The *ex ante* payoff for type- θ buyer, who reports θ' and then $s^*(\theta, \theta', s)$, is then given by

$$U(\theta, \theta') \equiv \sum_x \sum_s u(\theta, \theta', s, s^*(\theta, \theta', s)) \pi(s|x).$$

With abuse of notation, let $u(\theta, s) \equiv u(\theta, \theta, s, s)$, $u(\theta, s, s') \equiv u(\theta, \theta, s, s')$, and $U(\theta) \equiv U(\theta, \theta)$. For the buyer to truthfully report his signal on the equilibrium path (conditional on reporting his type truthfully), it must be that for all θ and s ,

$$u(\theta, s) \geq u(\theta, s, s'). \quad (\text{IC-signal})$$

For the buyer to truthfully report his type, it must be that for all θ and θ' ,

$$U(\theta) \geq U(\theta, \theta'). \quad (\text{IC-type})$$

Finally, the buyer participates in the mechanism if and only if

$$U(\theta) \geq 0. \quad (\text{IR})$$

Definition 3. *A mechanism is feasible if it satisfies all constraints (IR), (IC-type), and (IC-signal),*

Formally, the seller's maximization problem is given by

$$\begin{aligned} & \sup_{\{\pi_\theta, q(\theta, s), p(\theta, s)\}_{s, \theta}} \sum_\theta \sum_x \sum_s p(\theta, s) \pi(s|x) \mu(x) f(\theta) \\ & \text{s.t.} \quad (\text{IR}), (\text{IC-type}), (\text{IC-signal}). \end{aligned}$$

3 Preliminary results

3.1 No distortion at the top and no rent at the bottom

First, we establish that the solution to the seller's joint design problem bears commonly known features: the highest type receives an efficient allocation while the lowest is fully extracted.

Lemma 1. *Under any optimal mechanism,*

(a) *the lowest type gets a zero payoff: $U(\underline{\theta}) = 0$, and*

¹⁶In case the buyer is indifferent between signals off the equilibrium path, fix arbitrarily one of the seller-preferred signals.

(b) the highest type an efficient allocation: $q(\bar{\theta}, x) = \begin{cases} 1 & \text{if } v(\bar{\theta}, x) > 0, \\ \in [0, 1] & \text{if } v(\bar{\theta}, x) = 0. \end{cases}$

To prove Part (a) or the "no rent at the bottom" feature, we first show that the buyer's rent $U(\theta)$ increases in θ under any incentive-compatible mechanism. For this purpose, consider the buyer of type $\theta > \underline{\theta}$. He reveals his type only if his payoff from truth-telling is at least that obtained by mimicking some type $\theta' < \theta$ and reporting signals truthfully. Formally,

$$U(\theta) \geq \sum_x [\nu(\theta, x)q(\theta', s) - p(\theta', s)] \pi_{\theta'}(s|x)g(x) \quad (1)$$

As $\nu(\theta, x) \geq \nu(\theta', x)$ for all x , (1) implies $U(\theta) \geq \sum_x [\nu(\theta', x)q(\theta', s) - p(\theta', s)] \pi_{\theta'}(s|x)g(x) = U(\theta')$. Therefore, $U(\cdot)$ is an increasing function. Armed with this result, we now show that $U(\underline{\theta}) = 0$ under optimal mechanisms. By contradiction, suppose $U(\underline{\theta}) = \varepsilon > 0$ under an optimal mechanism. Then, if increasing $p(\theta, s)$ by ε for all θ and s , the seller strictly increases her revenue while not violating any IC and IR conditions. A contradiction. Thus, $U(\underline{\theta}) = 0$ at optimum.

We leave the proof of Part (b) or the "no distortion at the bottom" result in Appendix A.1. The idea is that whenever this type does not trade with probability 1 (at some state), it is possible to improve the seller's revenue by letting him always trade under no disclosure and a posted price being equal to his original expected payment, adding the new surplus.

It is worth noting that by Lemma 1(b), random allocations are not needed for the highest type. This is not necessarily true for the lower types to which, offering efficient allocations is generally sub-optimal. See Section 6.1 for a detailed discussion.

3.2 Sequential vs. static screening

This section establishes an irrelevance result regarding the timing structure of interactions. We show that contracting at the *posterior* stage (after the buyer observes both his type and signal) does not necessarily hurt the seller. Specifically, within the class of deterministic mechanisms, there is no revenue loss if the buyer can walk away after information disclosure.

Proposition 1. *For any deterministic and feasible mechanism, there exists a mechanism which generates the same revenue for the seller and a non-negative ex post pay-off for the buyer.*

Let $\mathbf{M}^d \equiv \{q(\theta, s), p(\theta, s), \pi_\theta\}$ be an arbitrary deterministic and feasible mechanism. Hence, under \mathbf{M}^d , $q(\theta, s) \in \{0, 1\}$ for any θ and s . Fix $\theta \in \Theta$. Let $S_\theta^g \equiv \{s \mid q(\theta, s) = 1\}$ and $S_\theta^b \equiv \{s \mid q(\theta, s) = 0\}$. To induce signal truth-telling by θ , $p(\theta, s) = p(\theta, s') = \underline{p}(\theta)$ if $s \in S_\theta^g$; and $p(\theta, s) = p(\theta, s') \equiv \bar{p}(\theta)$

and if $s \in S_\theta^b$. Let

$$\mathcal{Q}(\theta) \equiv \sum_x \sum_{s \in S_\theta^g} \pi_\theta(s|x) g(x)$$

represent type θ 's trade probability. Consider the following two cases:

Case 1: $\underline{p}(\theta) < 0$. If $\exists s$ such that $\pi(\theta, s) < 0$, then type θ who observes s (strictly) prefers to misreport signal s' with $q(\theta, s') = 0$ to receive a negative transfer without buying the good. This contradicts with \mathbf{M}^d being feasible. Hence, $\pi(\theta, s) \geq 0$ for all s in this case.

Case 2: $\bar{p}(\theta) \geq 0$. Revise \mathbf{M}^d as follows. For any $s \in S_\theta^g$, replace s with signal " s^g "; and for any $s \in S_\theta^b$, replace it with signal " s^b ". In addition, each type θ now receives a posted price

$$\tilde{p}(\theta) = \bar{p}(\theta) + \underline{p}(\theta) \frac{1 - \mathcal{Q}(\theta)}{\mathcal{Q}(\theta)}, \quad (2)$$

As θ pays only if he decides to buy the good, his payoff is non-negative at any signal realization. In Appendix A.2 we show that the seller's revenue remains unchanged under this revision, which completes the proof for Proposition 1.

Proposition 1 has two implications. First, if only deterministic mechanisms are allowed, there is no loss for the seller to contract after the buyer observes both type and signal realizations. Therefore, despite the sequential arrival of his private information, sequential screening the buyer is not beneficial, unless random mechanisms are necessary.

Second, by the "no rent at the bottom", the lowest type earns a zero payoff under optimal mechanisms. Therefore, if there exists θ such that $\underline{p}(\theta) < 0$, the lowest type mimics θ and not buying the good to enjoy a positive payoff. Consequently, if \mathbf{M}^d is optimal, $\underline{p}(\theta) \geq 0 \forall \theta$. Then, as argued above, \mathbf{M}^d do no better than "posted-price" mechanisms, which are *signal-independent*. As we will show, such posted-price mechanisms are optimal in several, but not all, environments.

3.3 (Ir)relevance of signal privacy

As the buyer privately observes signals only after the contract is signed, one might expect that the privacy of signals does not hurt the seller's revenue. To investigate this conjecture, we first consider the benchmark problem with public signals. There, the buyer's payoff and the seller's revenue depends only on *expected* payments and *expected* allocations over signals, defined as

$$\mathbb{Q}(\theta, x) \equiv \sum_x \sum_s q(\theta, s) \pi_\theta(s|x) \mu(x), \quad \mathbb{P}(\theta) \equiv \sum_x \sum_s p(\theta, s) \pi_\theta(s|x) \mu(x).$$

As a result, the seller's problem reduces to

$$\begin{aligned}
(\overline{\mathcal{P}}) \quad & \sup_{\mathbb{Q}, \mathbb{P}} \sum_{\theta} \mathbb{P}(\theta) f(\theta) \\
s.t. \quad & \sum_x v(\theta, x) \mathbb{Q}(\theta, x) \mu(x) - \mathbb{P}(\theta) \geq \sum_x v(\theta, x) \mathbb{Q}(\theta', x) \mu(x) - \mathbb{P}(\theta'), \quad (\overline{IC}\text{-type}) \\
& \sum_x v(\theta, x) \mathbb{Q}(\theta, x) \mu(x) - \mathbb{P}(\theta) \geq 0. \quad (\overline{IR})
\end{aligned}$$

Under Assumption 1 and 2, only local IC constraints bind under $(\overline{\mathcal{P}})$. By standard arguments (omitted), this problem reduces to point-wise maximization w.r.t \mathbb{Q} only:

$$\sup_{\mathbb{Q}} \sum_{\theta} \sum_x \phi(\theta, x) \mathbb{Q}(\theta, x) \mu(x) f(\theta). \quad (\star)$$

A solution to (\star) exists and is generically unique:¹⁷ $\mathbb{Q}(\theta, x) = \mathbb{1}_{\phi(\theta, x) \geq 0}$. *Expected* payment (over signals) is pinned down by $(\overline{IC}_{\theta^+ \rightarrow \theta})$ and (\overline{IR}_{θ}) . For any $\theta \geq \underline{\theta}$, let

$$x_{\theta} \equiv \begin{cases} \min\{x \mid \phi(\theta, x) \geq 0\} & \text{if } \phi(\theta, \bar{x}) \geq 0, \\ +\infty & \text{if } \phi(\theta, \bar{x}) < 0 \end{cases} \quad (3)$$

denote the lowest state at which type θ 's virtual value is non-negative. Note that x_{θ}^* decreases in θ by Assumption 2.

Lemma 2 (Benchmark problem). *With public signals, the optimal mechanism is generically unique, given by*

$$\mathbb{Q}(\theta, x) = \mathbb{1}_{\phi(\theta, x) \geq 0}, \quad (4)$$

$$\mathbb{P}(\theta^+) = \mathbb{P}(\theta) + \sum_{x_{\theta^+} \leq x < x_{\theta}} v(\theta^+, x) \mu(x) \quad \forall \theta \geq \underline{\theta}. \quad (5)$$

The seller retains a certain level of freedom in designing disclosure and *per-signal* allocation rules as long as (i) upon observing any signal, one knows whether the state is above or below the cut-off x_{θ} and (ii) expected terms are given by equations in Lemma 2. This leads to a multiplicity of solutions to $(\overline{\mathcal{P}})$, including the following menu of threshold disclosures and prices (paid conditional on trade), under which each type of the buyer (i) knows whether his virtual value is positive or not and (ii) pays only if trade happens. Formally:

Definition 4. *Under $M^* \equiv \{p^*(\theta, s), q^*(\theta, s), \pi_{\theta}^*\}_{\theta \in \Theta, s \in \{s^g, s^b\}}$ is a menu of threshold disclosures and prices, in which*

¹⁷When $\phi(\theta, x) = 0$, any $\mathbb{Q}(\theta, x) \in [0, 1]$ is optimal.

1. $\pi_\theta^*(s^g|x) = \mathbb{1}_{x \geq x_\theta}$, where x_θ is given by equation (3).

$$2. (q^*, (\theta, s), p^*(\theta)) = \begin{cases} (1, \frac{\mathbb{P}(\theta)}{\sum_{x \geq x_\theta} \mu(x)}) & \text{if } s = s^g, \\ (0, 0) & \text{if } s = s^b, \end{cases} \text{ where } \mathbb{P}(\theta) \text{ is given by equation (5).}$$

Let $V(\overline{\mathcal{P}})$ represent the value of problem $(\overline{\mathcal{P}})$. Then, $V(\overline{\mathcal{P}})$ is an upper bound on the seller's revenue with private signals. Under a mild condition, Proposition 2(a) below shows that if this upper bound is achieved via some mechanism, it is via \mathbf{M}^* . The basic intuition is that relative to other solutions to $(\overline{\mathcal{P}})$, \mathbf{M}^* provides less information (just enough to know the sign of virtual values) and a higher price for the good (payments are paid only when trade happens). Hence, if there exists a solution that induces truth-telling with private signals, so does \mathbf{M}^* . This is the case, by Proposition 2(b), if and only if the highest type pays the lowest price under \mathbf{M}^* .

To formally state Proposition 2, let $R_{\mathbf{M}}$ represent the revenue level obtained with private signals from an arbitrary mechanism \mathbf{M} .

Proposition 2.

a) Suppose $\phi(\theta, x_\theta) > 0 \forall \theta$. If there exists \mathbf{M} such that $R_{\mathbf{M}} = V(\overline{\mathcal{P}})$, then $R_{\mathbf{M}^*} = V(\overline{\mathcal{P}})$.

b) $R_{\mathbf{M}^*} = V(\overline{\mathcal{P}})$ if and only if $p^*(\bar{\theta}, s^g) = \min_{\theta} \{p^*(\theta, s^g)\}$.

It seems counter-intuitive that the highest type pays the lowest price (conditional on buying the good). However, it is worth noting that information disclosure can flip the ranking of (*posterior*) willingness to pay across types, leading to non-monotone price discrimination.¹⁸ As will be shown formally in later sections, this occurs in some, but not all environments.

4 A restatement of the seller's problem

Without loss of generality, assume that each signal induces a single (on-path) posterior valuation. Therefore, each signal s observed by type- θ buyer corresponds to his on-path posterior value after observing such a signal, given by

$$\omega^{\pi_\theta}(\theta, s) \equiv \sum_x v(\theta, x) \mu_{s, \pi_\theta}(x)$$

¹⁸That information disclosure can lead to non-monotone price discrimination has been observed in Bang and Kim (2013) and Wei and Green (2023) where prices decrease in types. Throughout our paper, several examples are presented where under \mathbf{M}^* , prices can be decreasing, increasing and even concave in types (see Example 6).

Moreover, that the buyer reveals the realized signal is equivalent to him reporting his posterior valuation. For any type θ , let

$$\Omega_\theta \equiv \{\omega \mid \omega = \omega^{\pi_\theta}(\theta, s) \text{ for some } s \in S\}$$

be the set of all possible on-path posterior values for type θ . Then, requiring signal truth-telling on-path is equivalent to ensuring truth-telling about on-path posterior values, or

$$\omega q(\theta, \omega) - p(\theta, \omega) \geq \omega q(\theta, \omega') - p(\theta, \omega') \quad \forall \theta, \forall \omega, \omega' \in \Omega_\theta$$

As mentioned, the buyer may want to coordinate lies about the realized type and signal. Given that the signal space is endogenous, this significantly complicates the characterization of truth-telling conditions. To facilitate characterizing the buyer's optimal double deviation, we extend the allocation rule to be defined on the set of all possible on-path and off-path posterior valuations, denoted by

$$\Omega \equiv [v(\underline{\theta}, \underline{x}), v(\bar{\theta}, \bar{x})].$$

Moreover, it is without loss of generality to require truthful signal reporting on this set Ω , rather than in only $\{\Omega_\theta\}_\theta$,¹⁹ i.e.,

$$\omega q(\theta, \omega) - p(\theta, \omega) \geq \omega q(\theta, \omega') - p(\theta, \omega') \quad \forall \theta, \forall \omega, \omega' \in \Omega \quad (\text{IC-value})$$

The characterization of (IC-value) is standard.

Lemma 3 (Myerson, 1981). *An allocation rule $(q, p) : \Theta \times \Omega \rightarrow [0, 1] \times \mathbb{R}$ satisfies (IC-value) if and only if*

1. $\omega q(\theta, \omega) - p(\theta, \omega) = \hat{\omega} q(\theta, \hat{\omega}) - p(\theta, \hat{\omega}) + \int_{\hat{\omega}}^{\omega} q(\theta, z) dz$,
2. $q(\theta, \omega)$ increases in ω .

It then follows from Lemma 3 that the buyer, after having lied about his type, reveals his true (off-path) posterior valuation.

Lemma 4 (Optimal double deviations). *Under any allocation rule $(q, p) : \Theta \times \Omega \rightarrow [0, 1] \times \mathbb{R}$ that satisfies (IC-value), it is optimal for type θ who mimics θ' and observe signal s to report his off-path posterior valuation, given by*

$$\omega^{\pi_\theta}(\theta', s) \equiv \sum_x v(\theta, x) \mu_{s, \pi_{\theta'}}(x)$$

¹⁹See, for example, Skreta (2006), for mechanism design with non-convex type spaces.

The proof (omitted) is similar to what is called "correcting the lie" in the dynamic mechanism design literature. Often, this lie correction is made feasible by assuming that the agent's (new) private information shares a common support across types.²⁰ This is not applicable in our model as the buyer's new private information, which is his posterior valuation, is endogenous. By extending the allocation rule to be defined in the extended signal space Ω , we make it possible for the buyer to "correct his lie."²¹

Consider $\theta, \theta' \in \Theta$ with $\theta > \theta'$. Then,

$$\begin{aligned} U(\theta, \theta') &\equiv \sum_x \sum_s [\omega^{\pi_{\theta'}}(\theta, s) q(\theta', \omega^{\pi_{\theta'}}(\theta, s)) - p(\theta', \omega^{\pi_{\theta'}}(\theta, s))] \pi_{\theta'}(s|x) \mu(x) \\ &= \sum_x \sum_s \left[[\omega^{\pi_{\theta'}}(\theta', s) q(\theta', \omega^{\pi_{\theta'}}(\theta', s)) - p(\theta', \omega^{\pi_{\theta'}}(\theta', s))] + \sum_s \int_{\omega^{\pi_{\theta'}}(\theta', s)}^{\omega^{\pi_{\theta'}}(\theta, s)} q(\theta', z) dz \right] \pi_{\theta'}(s|x) \mu(x) \\ &= U(\theta') + \sum_x \sum_s \int_{\omega^{\pi_{\theta'}}(\theta', s)}^{\omega^{\pi_{\theta'}}(\theta, s)} q(\theta', z) dz \pi_{\theta'}(s|x) \mu(x). \end{aligned}$$

Thus, θ does not benefit from misreporting θ' if and only if

$$U(\theta) - U(\theta') \geq \sum_x \sum_s \int_{\omega^{\pi_{\theta'}}(\theta', s)}^{\omega^{\pi_{\theta'}}(\theta, s)} q(\theta', z) dz \pi_{\theta'}(s|x) \mu(x).$$

By similar arguments, θ' does not benefit from misreporting θ if and only if

$$U(\theta) - U(\theta') \leq \sum_x \sum_s \int_{\omega^{\pi_{\theta}}(\theta', s)}^{\omega^{\pi_{\theta}}(\theta, s)} q(\theta, z) dz \pi_{\theta}(s|x) \mu(x).$$

To sum up, the seller's problem can be expressed as follows.

$$\begin{aligned} (\mathcal{P}) \quad &\max_{(\pi, q, U)} \sum_{\theta} f(\theta) \left[\sum_x \sum_s v(\theta, x) q(\theta, \omega^{\pi_{\theta}}(\theta, s)) \pi_{\theta}(s|x) \mu(x) - U(\theta) \right] \\ \text{s.t.} \quad &\forall \theta, \quad U(\theta) - U(\theta') \geq \sum_x \sum_s \int_{\omega^{\pi_{\theta'}}(\theta', s)}^{\omega^{\pi_{\theta'}}(\theta, s)} q(\theta', z) dz \pi_{\theta'}(s|x) \mu(x) \quad \forall \theta' < \theta \quad (\text{dwIC-type}) \\ &U(\theta) - U(\theta') \leq \sum_x \sum_s \int_{\omega^{\pi_{\theta}}(\theta, s)}^{\omega^{\pi_{\theta}}(\theta', s)} q(\theta, z) dz \pi_{\theta}(s|x) \mu(x) \quad \forall \theta' > \theta \quad (\text{uwIC-type}) \\ &U(\theta) \geq 0 \quad (\text{IR}) \\ &q(\theta, \omega) \text{ increases in } \omega. \quad (\text{MON}) \end{aligned}$$

5 Optimal mechanism for $|\Theta| = 2$

In this section, we characterize the optimal mechanism for binary types. We derive two findings. First, screening is optimal if and only if the ranking of willingness to pay is flipped under a

²⁰See Esó and Szentés (2007) and Krämer and Strausz (2015b) for example.

²¹This trick can also be helpful in other dynamic mechanism design problems where the agent(s)' private information does not share common support across types.

certain threshold disclosure and bunching is optimal otherwise. Second, eliciting signals and random mechanisms are worthless. Formally, $\Theta = \{h, l\}$ and hence, the seller's problem reduces to (\mathcal{P}_b) , given by

$$\begin{aligned}
(\mathcal{P}_b) \quad & \max_{(\pi, q, U)} \sum_{\theta} f(\theta) \left[\sum_x \sum_s v(\theta, x) q(\theta, \omega^{\pi_{\theta}}(\theta, s)) \pi_{\theta}(s|x) \mu(x) - U(\theta) \right] \\
s.t.: \quad & U(h) - U(l) \geq \sum_x \sum_s \int_{\omega^{\pi_l(l, s)}}^{\omega^{\pi_l(h, s)}} q(l, z) dz \pi_l(s|x) \mu(x) & (IC_{hl}) \\
& U(h) - U(l) \leq \sum_x \sum_s \int_{\omega^{\pi_h(l, s)}}^{\omega^{\pi_h(h, s)}} q(h, z) dz \pi_h(s|x) \mu(x) & (IC_{lh}) \\
& U(h) \geq 0 & (IR_h) \\
& U(l) \geq 0 & (IR_l) \\
& q(\theta, \omega) \text{ increases in } \omega.
\end{aligned}$$

To state the main result of this section, we introduce the following notion of type order flip, which shapes the optimal mechanism. Recall that π^* is the threshold disclosure associated with \mathbf{M}^* formally defined in Definition 4, with $\pi_l^*(s^g|x) = \mathbb{1}_{x \geq x_l}$.

Definition 5 (Threshold flip of type order by π_l^*).

If π_l^* induces the threshold flip of type order, $\mathbb{E}[v(h, x) | x < x_l] \leq \mathbb{E}[v(l, x) | x \geq x_l]$.

By Definition 5, π_l^* induces the threshold flip of type order when $\omega^{\pi_l^*}(h, s^b) \leq \omega^{\pi_l^*}(h, s^g)$. In words, this threshold disclosure overturns the ranking of willingness to pay with h 's valuation after "bad news" being lower than l 's after "good news". Intuitively, this is the case when the unknown component x causes significant variations of valuations, creating room for π_l^* to flip the type order. By contrast, it does not happen in, for example, an extreme case in which valuation is constant with respect to this component (i.e., $g(\cdot)$ is a degenerate distribution with $\bar{x} = \underline{x}$), as in standard mechanism design problems.

We are now ready to state the main result of this section, assuming that type l 's virtual value is either strictly positive or negative, i.e., $\phi(l, x_l) > 0$. Accordingly, the benchmark allocation is unique, given by $Q(l, x) = \mathbb{1}_{x \geq x_l}$.

Theorem 1 (Binary types). Fix $\Theta = \{h, l\}$. There exists some $\lambda \in [0, 1]$ and $\hat{x}_l \in \cdot$, such that in the unique optimal mechanism, the allocation is given by

$$q(h, x) = 1 \quad \forall x, \quad q(l, x) = \begin{cases} 1 & \text{if } x > \hat{x}_l, \\ 0 & \text{if } x < \hat{x}_l, \\ \lambda & \text{if } x = \hat{x}_l. \end{cases}$$

Moreover,

- (a) If π_l^* induces the threshold flip of type order, $(\hat{x}_l, \lambda) = (x_l, 1)$. A menu of posted prices and threshold disclosures is optimal.
- (b) If π_l^* does not induce the threshold flip of type order, $(\hat{x}_l, \lambda) \neq (x_l, 1)$. A posted price, associated with a uniform threshold disclosure, is optimal.

In short, Theorem 1 states that the optimal mechanism features *screening* whenever π_l^* leads to the threshold flip of type order and *bunching* otherwise. Intuitively, when the unknown component x dominates the buyer's private type θ in triggering the variation of the buyer's valuation (to induce the threshold flip of type order), the new information matters and helps screen the buyer. Conversely, when the ranking of willingness to pay mainly depends on the buyer's type, screening disappears. Then, the optimal mechanism closely resembles its counterpart in standard mechanism design where the state is known: a posted price (but associated with threshold disclosure) is optimal. The optimal mechanism in each case is explicitly characterized in the remainder of this section. To illustrate Theorem 1, consider the following examples.

Example 1 (Binary types and states). $\Theta = \{l, h\}$ and $X = \{b, g\}$. Types and states are equally likely. Assume that $\phi(\theta_1, x_1) < 0 < \phi(\theta_1, x_2)$ to make the problem non-trivial.

In this simple binary-type, binary-state setting, there are two scenarios of optimal mechanisms. If $v(\theta_3, x_1) \geq v(\theta_1, x_2)$, then π^* does not induce the threshold flip of type order. By Theorem 1(a), a fixed price and threshold disclosure is optimal. On the other hand, if $v(\theta_3, x_1) < v(\theta_1, x_2)$, then π^* leads to the threshold flip of type order. By Theorem 1(b), a menu of prices and threshold disclosures is optimal.

Example 2. $\Theta = \{l, h\}$ and X is a finite subset of \mathbb{N} . Types and states are equally likely. Valuations are given by: $v(\theta, x) = \theta + x$.

Let

$$\begin{aligned}\Delta_\theta &\equiv v(h, x) - v(l, x) = h - l \quad \forall x, \\ \Delta_x &\equiv v(\theta, \bar{x}) - v(\theta, \underline{x}) = \bar{x} - \underline{x} \quad \forall \theta.\end{aligned}$$

Then, Δ_θ represents the variation of valuation due to the buyer's type, whereas Δ_x due to the state x . For any state $\hat{x} \in \Omega$,

$$\mathbb{E}[v(h, x) \mid x < \hat{x}] - \mathbb{E}[v(l, x) \mid x \geq \hat{x}] = \left(h + \frac{\hat{x} - 1 + \underline{x}}{2}\right) - \left(l + \frac{\hat{x} + \bar{x}}{2}\right) = \Delta_\theta - \frac{\Delta_x + 1}{2},$$

Thus, the threshold flip of type order happens if and only if

$$\Delta_\theta \leq \frac{\Delta_x + 1}{2}, \tag{6}$$

which is the case when the impact of the buyer's type is relatively small, relative to that of the unknown component. By Theorem 1, when (6) holds, it is optimal to offer a menu of threshold disclosures and posted prices. Otherwise, a posted price, coupled with uniform threshold disclosure, maximizes the seller's revenue.²²

Remark 1. *Theorem 1 and its proof extends readily to the case with a continuum of states. As an example, fix $\Theta = \{l, h\}$ and $X = [0, 10]$, and both θ and x are uniformly distributed. Then, for any state $\hat{x} \in \Omega$, $\mathbb{E}[v(h, x) \mid x < \hat{x}] - \mathbb{E}[v(l, x) \mid x \geq \hat{x}] = \Delta_\theta - 5$. Thus, a menu of prices and information is optimal if $\Delta \geq 5$ and a fixed price coupling with a threshold disclosure (for all types) is optimal if $\Delta < 5$.*

Theorem 1 has two important implications:

Corollary 1. *With $|\Theta| = 2$, privacy of signals does not matter when the threshold flip of type order happens under π_l^* . It matters otherwise.*

Corollary 2. *With $|\Theta| = 2$, the seller does not strictly benefit from using random mechanisms, nor from eliciting signals.*

What leads to the (ir)relevance of signal privacy and the optimality of deterministic mechanisms, signal-independent allocations will be explained when we present the key steps of the proof of Theorem 1, to which we turn next.

5.1 Proof of Theorem 1

To prove Theorem 1, we solve a relaxed problem, denoted by (\mathcal{RP}_b) , ignoring (IC_{lh}) and (IR_h) and provide an implementation. Formally, this relaxed problem is as follows.

$$\begin{aligned}
(\mathcal{RP}_b) \quad & \max_{(\pi, q, U)} \sum_{\theta} f(\theta) \left[\sum_x \sum_s v(l, x) q(\theta, \omega^{\pi_\theta}(\theta, s)) \pi_\theta(s|x) \mu(x) - U(\theta) \right] \\
& \text{s.t.} \quad U(h) - U(l) \geq \sum_x \sum_s \int_{\omega^{\pi_l(l, s)}}^{\omega^{\pi_l(h, s)}} q(l, z) dz \pi_l(s|x) \mu(x) & (IC_{h \rightarrow l}) \\
& \quad U(l) \geq 0 & (IR_l) \\
& \quad q(\theta, \omega) \text{ increases in } \omega. & (MON)
\end{aligned}$$

The characterization of the solution to (\mathcal{RP}_b) is done via the following steps. First, we prove the optimality of deterministic allocation rules. This step, while standard, is helpful in decomposing the buyer's rent into two components: the *ex ante* rent (due to privacy of types) and

²²In particular, when Δ_θ is too high, the seller does not benefit from information disclosure. In this case, the optimal threshold for type l is the highest state ($\hat{x}_l = \bar{x}$), which means no disclosure is provided.

the *posterior* rent (due to privacy of signals). Using this rent decomposition, we establish the optimality of binary-signal experiments and furthermore, of threshold disclosures. Finally, we characterize the optimal allocation and implement it.

First, to obtain the optimality of deterministic allocations, note that $(IC_{h \rightarrow l})$ and (IR_l) must bind in (\mathcal{RP}_b) , i.e.,

$$U(l) = 0, \quad U(h) = \sum_x \sum_s \int_{\omega^{\pi_l(l,s)}}^{\omega^{\pi_l(h,s)}} q(l, z) dz \pi_l(s|x) \mu(x),$$

Thus, transfers have been eliminated, reducing the seller's relaxed problem to

$$\begin{aligned} \max_{q, \pi} \quad & f(h) \sum_x \sum_s v(h, x) q(h, \omega^{\pi_\theta}(\theta, s)) \pi_h(s|x) \mu(x) \\ & + f(l) \sum_x \sum_s \left[v(l, x) q(l, \omega^{\pi_\theta}(\theta, s)) - \int_{\omega^{\pi_l(l,s)}}^{\omega^{\pi_l(h,s)}} q(l, z) dz \right] \pi_l(s|x) \mu(x) \\ \text{s.t.} \quad & q(\theta, \omega) \text{ increases in } \omega. \end{aligned} \tag{MON}$$

Fix π . Given that the objective function is linear and the only constraint is (MON), there exists an optimal allocation rule that is deterministic and exhibits a cut-off structure. Moreover, as $v(h, x)$ is always non-negative, h receives an efficient allocation.

Lemma 5 (Deterministic allocations). *In (\mathcal{RP}_b) , there exists an optimal allocation rule, given by $q(\theta, \omega) = \mathbb{1}_{\omega \geq \hat{\omega}_\theta}$, where $\hat{\omega}_h = v(l, \underline{x})$.*

Second, we derive the sufficiency of binary-signal experiments. As $q(h, s) = 1$ for all s , any π_h is optimal. The relaxed problem reduces to finding the optimal π_l . Let

$$R_l \equiv f(l) \sum_x \sum_s \left[v(l, x) q(l, \omega^{\pi_l(l, s)}) - \int_{\omega^{\pi_l(l, s)}}^{\omega^{\pi_l(h, s)}} q(l, z) dz \right] \pi_l(s|x) \mu(x)$$

denote the term involving π_l in the seller's objective function (revenue) in (\mathcal{RP}_b) . Using $q(l, \omega) = \mathbb{1}_{\omega \geq \hat{\omega}_l}$ by Lemma 5, we obtain

$$\begin{aligned} R_l &= f(l) \sum_x \left[\sum_{\hat{s}_l}^{\bar{s}} v(l, x) \pi_l(s|x) - \sum_{\hat{s}_l}^{\bar{s}} \int_{\omega^{\pi_l(l, s)}}^{\omega^{\pi_l(h, s)}} dz \frac{f(h)}{f(l)} \pi_l(s|x) - \sum_{\underline{s}}^{\hat{s}_l} \int_{\omega^{\pi_l(l, \hat{s}_l)}}^{\omega^{\pi_l(h, s)}} dz \frac{f(h)}{f(l)} \pi_l(s|x) \right] \mu(x) \\ &= f(l) \sum_x \left[\underbrace{\sum_{\hat{s}_l}^{\bar{s}} v(l, x)}_{l \text{'s surplus}} - \underbrace{\sum_{\hat{s}_l}^{\bar{s}} [\omega^{\pi_l(h, s)} - \omega^{\pi_l(l, s)}] \frac{f(h)}{f(l)}}_{h \text{'s ex ante rent}} - \underbrace{\sum_s [\omega^{\pi_l(h, s)} - \omega^{\pi_l(l, \hat{s}_l)] \frac{f(h)}{f(l)}}_{h \text{'s posterior rent}} \right] \pi_l(s|x) \mu(x). \end{aligned}$$

R_l depends on (i) the buyer's expected value (on path for l and off path for h), conditional on whether $s \geq \hat{s}_l$ or $s < \hat{s}_l$, and (ii) the cut-off signal, \hat{s}_l . By (i), there is no revenue loss in replacing all signals $s \geq \hat{s}_l$ with "good news" (s^g) and all $s < \hat{s}_l$ with "bad news" (s^b). At the same time, such

a binary-signal experiment for type l increases the cut-off signal because $\omega^{\pi_l}(l, s^g) = \mathbb{E}[v(l, x) | s \geq \hat{s}_l] \geq \hat{s}_l$. In turn, this improves R_l , which increases in the cut-off signal. We thus obtain the optimality of binary-signal experiments.

Lemma 6 (Binary signals). *In $(\mathcal{R}\mathcal{P}_b)$, there exists an optimal experiment for type l where the signal realization can be either "good news" (s^g) or "bad news" (s^b).*

Third, we prove the optimality of threshold disclosures. By replacing all signals $s \geq \hat{s}_l$ (resp., $s < \hat{s}_l$) with "good news" (resp., "bad news"), R_l becomes

$$\begin{aligned} & f(l) \sum_x \left[\underbrace{v(l, x) \pi_l(s^g | x)}_{l \text{'s surplus}} - \underbrace{[v(h, x) - v(l, x)] \frac{f(h)}{f(l)} \pi_l(s^g | x)}_{h \text{'s ex ante rent}} - \underbrace{[\omega^{\pi_l}(h, s^b) - \omega^{\pi_l}(l, s^g)] \frac{f(h)}{f(l)} \pi_l(s^b | x)}_{h \text{'s posterior rent}} \right] \mu(x) \\ &= \underbrace{f(l) \sum_x \phi(l, x) \pi_l(s^g | x) \mu(x)}_{l \text{'s virtual value}} - \underbrace{f(l) \sum_x \max \left\{ [\omega^{\pi_l}(h, s^b) - \omega^{\pi_l}(l, s^g)] \frac{f(h)}{f(l)}, 0 \right\} \pi_l(s^b | x) \mu(x)}_{h \text{'s posterior rent}}. \end{aligned}$$

Fix $\pi_l(s^b)$. Then, a threshold disclosure minimizes h 's posterior rent by simultaneously maximizing $\omega^{\pi_l}(h, s^b)$ and minimizing $\omega^{\pi_l}(l, s^g)$. Moreover, as $\phi(l, x)$ increases in x , a threshold disclosure maximizes l 's expected virtual value. Therefore:

Lemma 7 (Threshold structure). *In $(\mathcal{R}\mathcal{P}_b)$, a threshold disclosure for l is optimal.*

Last, we characterize the optimal allocation and provide an implementation. Let $\hat{x}_l \in X$ be the cut-off state associated with the optimal threshold disclosure for l and $\lambda \in [0, 1]$ be the probability with which "good news" is sent at \hat{x}_l . Then, by Lemmas 5, 6, and 7, the optimal allocation is given by

$$q(h, x) = 1 \quad \forall x, \quad q(l, x) = \begin{cases} 1 & \text{if } x > \hat{x}_l, \\ 0 & \text{if } x < \hat{x}_l, \\ \lambda & \text{if } x = \hat{x}_l. \end{cases}$$

Solving for the optimal allocation reduces to solving for the optimal (\hat{x}_l, λ) . As will be shown, there are two cases, depending on whether π_l^* triggers the threshold flip of type order. In the first case, when this flip happens, offering π_l^* with $(\hat{x}_l, \lambda) = (x_l, 1)$ is optimal. Not only does it induce zero *posterior* rent for h , given that

$$\omega^{\pi_l^*}(h, s^b) - \omega^{\pi_l^*}(l, s^g) \leq 0$$

when the threshold flip occurs under π_l^* , but it also creates the highest expected virtual value for l 's, given by $f(l) \sum_{x \geq x_l} \phi(l, x) \pi_l(s^g | x) \mu(x)$.

With $(\hat{x}_l, \lambda) = (x_l, 1)$, l 's allocation coincides with the benchmark $\mathbb{Q}(l, x) = \mathbb{1}_{x \geq x_l}$. To find out payments, without loss of generality, assume the buyer pays only if "good news" is realized (or trade happens). Thus, $p(h, s^b) = p(l, s^b) = 0$. Then, $p(l, s^g) = \omega^{\pi_l^*}(l, s^g)$ by (IR_l) , and $p(h, s^g)$ is such that $(IC_{h \rightarrow l})$ holds, or $U(h) = U(h, l)$, which implies

$$p(h, s^g) = \mathbb{E}[v(h, x)] - [\omega^{\pi_l^*}(h, s^g) - p(l, s^g)]\pi_l^*(s^g).$$

Now, verify that ignored constraints are satisfied. First, (IR_h) hold because

$$U(h) = [\omega^{\pi_l^*}(h, s^g) - \omega^{\pi_l^*}(l, s^g)]\pi_l^*(s^g) \geq 0$$

Second, $IC_{l \rightarrow h}$ is satisfied given that

$$\begin{aligned} U(l, h) &= \mathbb{E}[v(l, x)] - p(h, s^g) \\ &= \mathbb{E}[v(l, x)] - \mathbb{E}[v(h, x)] + [\omega^{\pi_l^*}(h, s^g) - p(l, s^g)]\pi_l^*(s^g) \\ &= \mathbb{E}[v(l, x)] - \mathbb{E}[v(h, x)] + [\omega^{\pi_l^*}(h, s^g) - \omega^{\pi_l^*}(l, s^g)]\pi_l^*(s^g) \\ &= [\omega^{\pi_l^*}(l, s^b) - \omega^{\pi_l^*}(h, s^b)]\pi_l^*(s^b) < 0 = U(l) \end{aligned}$$

Moreover, under no threshold flip of type order by π^* , $\omega^{\pi_l^*}(h, s^b) \leq \omega^{\pi_l^*}(l, s^g) = p(l, s^g)$. Therefore, if h mimics l , it is optimal for him to report signals truthfully. This deviating behavior is not beneficial for h by the construction of $p(h, s^g)$. We thus obtain Theorem 1(a):

Lemma 8 (With threshold flip by π_l^*). *If π_l^* induces the threshold flip of type order, $q(l, x) = \mathbb{Q}(l, x) = \mathbb{1}_{x \geq \hat{x}_l}$, and $\mathbf{M}^* \equiv \{p^*(\theta), \pi_\theta^*\}_\theta$ is optimal.*

By contrast, when π_θ^* preserves the type order, or $\omega^{\pi_l^*}(h, s^b) > \omega^{\pi_l^*}(l, s^g)$, offering π_l^* to l induces a strictly positive *posterior* rent for h . Consequently, the seller trades off between l 's expected virtual value and h 's *posterior* rent. On the one hand, she wants the threshold to be close to the cut-off x_l , maximizing l 's expected value. On the other hand, she desires to induce a small *posterior* rent for h .

Let π_l^{**} be an optimal experiment for l , associated with $(x^{**}(l), \lambda^{**})$. Suppose, π_l^{**} can flip the type order, i.e., $v^{\pi_l^{**}}(h, s^b) < \omega^{\pi_l^{**}}(l, s^g)$. Then, given that $\omega^{\pi_l^*}(h, s^b) \leq \omega^{\pi_l^*}(l, s^g)$, we can construct $\tilde{\pi}_l$ associated with $(\tilde{\omega}, \tilde{\lambda})$ such that (i) $(x^{**}(l), \lambda^{**})$ is closer to $(x_l^*, 1)$ and (ii) $v^{\tilde{\pi}_l}(h, s^b) \leq v^{\tilde{\pi}_l}(l, s^g)$. By (i), l 's expected virtual value under $\tilde{\pi}_l$ is higher than that under π_l^* , whereas by (ii), h 's *poterior* rent is zero under $\tilde{\pi}_l$. This contradicts with π_l^{**} being optimal. Therefore, π_l^{**} must not affect the type order. Formally:

Claim 1. $\omega^{\pi_l^{**}}(h, s^b) \geq \omega^{\pi_l^{**}}(l, s^g)$.

The detailed proof is in Appendix B.1. By Claim 1, R_l reduces to

$$\begin{aligned} & f(l) \sum_x \phi(l, x) \pi_l^{**}(s^g|x) \mu(x) - f(h) [\omega^{\pi_l^{**}}(h, s^b) - \omega^{\pi_l^{**}}(l, s^g)] \pi_l(s^b) \\ & = \omega^{\pi_l^{**}}(l, s^g) [f(l) \pi_l^{**}(s^g|x) \mu(x) + f(h)] - \mathbb{E}[v(h, x)]. \end{aligned}$$

Therefore,

$$\pi_l^{**} \in \operatorname{argmax}_{\pi_l} \omega^{\pi_l}(l, s^g) [f(l) \pi_l^{**}(s^g, x) + f(h)]. \quad (7)$$

To find optimal transfers, note that by Claim 1, h 's value after "bad news" is higher than l 's after "good news." Hence, if h mimics l , he always reports "good news," and always buys the good. Consequently, l 's allocation is the same as h 's from the latter's perspective. This leads to a bunching solution. As information is of no value for h , the seller can offer π_l^{**} to both types. Moreover, as h always gets the good either on or off-path, by $(IC_{h \rightarrow l})$, both types receive the same posted price.²³ Then, by (IR_l) ,

$$p^{**}(h) = p^{**}(l) = \omega^{\pi_l^{**}}(l, s^g). \quad (8)$$

This bunching mechanism satisfies ignored constraints, and hence, is optimal.

Lemma 9 (No threshold flip by π_l^*). *If π_l^* does not induce the threshold flip of type order, $(\hat{x}_l, \lambda) \neq (x_l, 1)$. A single-option menu, $\{\pi_l^{**}, p^{**}(l)\}$ given by (7) and (8), is optimal.*

6 Optimal mechanism for $|\Theta| \geq 3$

With binary types, there are two scenarios of the optimal mechanism (screening/bunching), depending on whether after information disclosure, the threshold flip of type order occurs or not. With richer type sets, it can be the case that information disclosure flips the order of a group of types but fails to do so for another group. Consequently, the characterization of optimal mechanisms cannot be obtained as a simple extension of that in the binary-type case. Moreover, as we will show, random mechanisms could be used to effectively screen signals and distant types. Despite these complications, we show that the optimality of a rich (respectively, single-option) menu of prices and threshold disclosure extends beyond the binary-type setting to a general model under stronger notions of type order flip (respectively, preservation). This result is presented in Section 6.2, followed by an analysis on the role of random mechanisms in 6.1.

²³With deterministic allocations, it is without loss to offer a menu of posted prices. See Proposition 1.

6.1 Revenue improvement via random mechanisms

Using Example 3 below, we illustrate how random mechanisms outperform their deterministic counterparts in two aspects (i) screening distant types and (ii) screening signals to improve the seller's revenue and efficiency.²⁴

Example 3. $\Theta = \{\theta_1, \theta_2, \theta_3\}$ and $X = \{x_1, x_2\}$. Types and states are equally likely. Valuations are as follows.

$v(\theta, x)$	x_1	x_2
θ_3	6.5	10
θ_2	0	7
θ_1	0	4

Table 1: Example 3(a)

$v(\theta, x)$	x_1	x_2
θ_3	5	5
θ_2	2	5
θ_1	0	4

Table 2: Example 3(b)

Example 3(a) - Screening distant types: In this example, type θ_3 's value is always higher than type θ_1 's. This leaves room for random mechanisms to "separate" these two types. To see this, note the following. If the seller employs deterministic mechanisms, type θ_1 either trades or not at any signal realization. Therefore, if type θ_1 trades (with probability 1) for some signal, it is optimal for θ_3 whose posterior value is always higher than type θ_1 's, having mimicked θ_1 , to (mis)report the realized signal such that he always trades. Then, θ_1 's allocation is the same as θ_3 's from the latter's perspective, leading to bunching these types.²⁵ In turn, this gives too much rent for type θ_3 , making it optimal to *exclude* type θ_1 .

Claim 2. *In Example 3(a), if only deterministic mechanisms are allowed, it is optimal to offer type θ_3 with no disclosure and a posted price $p(\theta_3) = 6.75$, type θ_2 with full disclosure and a posted price $p(\theta_2) = 7$, and to exclude type θ_1 .*

The story, however, is different with random allocations. The key is that if θ_1 trades with a small probability (for any signal), this type's allocation becomes unattractive to θ_3 . To see this, modify the optimal deterministic mechanism by letting θ_1 trade with a probability $\varepsilon \in [0, \frac{3}{4}]$ and

²⁴In the Online Appendix, we fully characterize the optimal random mechanism in several examples.

²⁵In Example 3(a), if the seller employs deterministic mechanisms and serves type θ_1 , a fixed price $p = 4$, associated with full disclosure is optimal.

adjusting transfers such that truth-telling remains satisfied, as follows:

$$q(\theta_3, x) = 1 \forall x, \quad q(\theta_2, x) = \mathbb{1}_{x=b}, \quad q(\theta_1, x) = \begin{cases} \varepsilon & \text{if } x = g, \\ 0 & \text{if } x = b, \end{cases}$$

$$p(\theta_3) = 6.5 - \varepsilon, \quad p(\theta_2) = 7 - 2\varepsilon, \quad p(\theta_1) = 5 \quad \text{paid conditional on trade occurs.}$$

Then, expected payment by θ_3 and θ_2 reduces by ε ; however, that by θ_2 increases by $\frac{5\varepsilon}{2}$. Overall, the seller's revenue increases by $f(\theta_1) \frac{5\varepsilon}{2} - [f(\theta_2) + f(\theta_3)] \varepsilon = \frac{3\varepsilon}{2} > 0$. Therefore, random allocation helps the seller screen effectively distant types (types θ_3 and θ_1), thereby, improving trade surplus extensively as well as the seller's revenue.

Example 3(b) - Screening signals: In this example, type θ_2 's value varies significantly across states. This makes it optimal to exclude type θ_2 at state x_1 , rather than "pooling" the two states under deterministic mechanisms which allow either trade or no trade at any signal realization. Formally, the optimal deterministic mechanism, stated in Claim 3 below, specifies:

$$q(\theta_3, x) = 1 \forall x, \quad q(\theta_2, x) = q(\theta_1, x) = \mathbb{1}_{x=x_2},$$

which are implemented via full disclosure and a fixed price.

Claim 3. *In Example 3(b), if only deterministic mechanisms are allowed, it is optimal to offer full disclosure and a posted price $p = 4$.*

Random mechanisms, on the other hand, arm the seller with the flexibility in designing trade probabilities. This helps her screen realized states by allowing trade to happen at a small probability at low states. To see this, revise the optimal deterministic mechanism by letting θ_2 trade with probability $\delta \leq \frac{1}{3}$, such that now:

$$q(\theta_3, x) = 1 \forall x, \quad q(\theta_1, x) = \mathbb{1}_{x=x_2}, \quad p(\theta_3) = p(\theta_1) = 4,$$

$$(q(\theta_2, x), p(\theta_2, x)) = \begin{cases} (1, 4) & \text{if } x = x_2 \\ (\delta, 2\delta) & \text{if } x = x_1 \end{cases}, \quad \text{with } \delta \leq \frac{1}{3}.$$

This revised mechanism differs from the optimal deterministic mechanism only in the new trade created with type θ_2 at state x_1 . Therefore, as long as this new trade creation preserves incentive compatibility, the seller's revenue increases by $2f(\theta_2)g(x_1)\delta > 0$. We show that this is the case in the Online Appendix.

6.2 Screening vs. Bunching

This section generalizes the finding of optimal mechanisms with binary types (Theorem 1) to a general model with finitely many types.

6.2.1 Optimality of screening

Recall that information disclosure can be used to screen the buyer of binary types when it induces a threshold flip of type order. Similarly, information serves as a screening tool in a richer type space under the following notion of type order flip:

Definition 6 (Partition flip of type order).

The partition flip of type order happens if $\mathbb{E}[v(\theta^+, x) \mid x_{\theta^+} \leq x < x_{\theta}]$ decreases in θ .

Under the partition flip of type order, the expected valuations over relevant partitions of states decrease in types. As the relevant partition for a higher type consists of lower states, such a type order flip requires the new information (about the state) to sufficiently dominate the buyer's initial type in driving valuation fluctuations. Indeed, it coincides with the threshold flip notation when there are only two types. In a richer type set, more than one interior threshold is involved under the menu of threshold disclosure $\{\pi_{\theta}^*\}_{\theta}$, leading to relevant partitions of states.

Theorem 2 below states the optimal mechanism under the partition flip of type order, which features discriminatory information and prices.

Theorem 2 (Screening). *Under the partition flip of type order, the optimal allocation is given by $\mathbb{Q}(\theta, x) = \mathbb{1}_{x \geq x_{\theta}}$. A menu of posted prices and threshold disclosures is optimal.*

This result extends Theorem 1 (a) to a model with more than two types, following the same logic: when information disclosure matters sufficiently, it helps screen the buyer. The only difference is that the *partition flip* of type order is required here, taking into account interior types.

The proof proceeds by showing that under the partition flip of type order, \mathbf{M}^* induces truth-telling even if the seller does not observe signals. Therefore, offering \mathbf{M}^* with the buyer privately observing signals is equivalent to offering a menu of posted prices and threshold disclosures $\{p^*(\theta), \pi_{\theta}^*\}_{\theta}$, where the posted price is equal to the payment paid after "good news" in \mathbf{M}^* : $p^*(\theta) = p^*(\theta, s^g)$. This menu helps the seller achieve the upper bound of revenue attained when signals are public signals; hence, it is optimal.

We close this section with an illustrative example.

Example 4. $\Theta = \{h, m, l\}$. X is a finite subset of \mathbb{N} . Types and states are equally likely. Valuations are given by $v(h, x) = x + \Delta_{\theta}$, $v(m, x) = x$, $v(l, x) = x - \Delta_{\theta}$. Accordingly, virtual values are given by $\phi(h, x) = x + \Delta_{\theta}$, $\phi(m, x) = x - \Delta_{\theta}$, $\phi(l, x) = x - 3\Delta_{\theta}$.

In this example, $v(\theta^+, x) - v(\theta, x) = \Delta_{\theta} \forall x$ and $\Delta_x \equiv v(\theta, \bar{x}) - v(\theta, \underline{x}) = \bar{x} - \underline{x} \forall \theta$. In addition,

$x_h^* = \underline{x}$, $x_m^* = \Delta_\theta$, and $x_l^* = 3\Delta_\theta$, which implies

$$\begin{aligned}\mathbb{E}[v(h, x) \mid x_h^* \leq x < x_m^*] &= \frac{3\Delta_\theta - 1 + \underline{x}}{2}, \\ \mathbb{E}[v(m, x) \mid x_m^* \leq x < x_l^*] &= \frac{4\Delta_\theta - 1}{2}, \\ \mathbb{E}[v(l, x) \mid x_l^* \leq x \leq \bar{x}] &= \frac{\Delta_\theta + \bar{x} - 1}{2}\end{aligned}$$

Thus, the partition flip of type order happens if

$$3\Delta_\theta - 1 + \underline{x} \leq 4\Delta_\theta - 1 \leq \Delta_\theta + \bar{x} - 1 \Leftrightarrow \underline{x} \leq \Delta_\theta \leq \Delta_x,$$

which requires the impact of the unknown component to be higher than that of the buyer's type (and is of at least \underline{x}). If this is the case, by Theorem 2, it is optimal to screen the buyer's type using different bundles of posted prices and threshold disclosure.

6.2.2 Optimality of bunching

In the binary-type case, the benefit of screening disappears if the threshold disclosure rule π_l^* fails to flip the ranking of willingness to pay by types. A similar story holds with more than two types under a stronger notion of (no) threshold flip of type order:

Definition 7 (Uniformly no threshold flip of type order). *Under uniformly no threshold flip of type order,*

$$\mathbb{E}[v(\theta^+, x \mid x < \hat{x})] \geq \mathbb{E}[v(\theta, x \mid x \geq \hat{x})] \quad \forall \theta \in \Theta, \forall \hat{x} \in X.$$

In words, this condition satisfies if under *any threshold disclosure* and for *any type* θ : θ^+ 's value after "*bad news*" must be higher than θ 's after "*good news*". This is more likely to hold when valuation heterogeneity is mainly driven by the buyer's type. For instance, when θ^+ 's values are always higher regardless of states, *i.e.*, $v(\theta^+, \underline{x}) \geq v(\theta, \bar{x})$, it is impossible to flip their ranking of valuation after *any rule* of information disclosure, not just the threshold ones.

We are now ready to state the main result of this section.

Theorem 3 (Bunching). *Under uniformly no threshold flip of type order, a posted price, associated with a threshold disclosure, is optimal.*

This result extends Theorem 1(b), carrying the same intuition: when valuation heterogeneity is mainly due to the buyer's types, information about the state becomes inessential for (most types of) the buyer; as a result, its screening function shuts off. The only difference is that no type order flip by *any* threshold disclosure is required here, of which the role is to be explained.

The proof proceeds by solving a relaxed problem considering only deviating behaviors under which all types mimic the lowest type being served. This problem mirrors that for the binary-type case $\Theta = \{h, l\}$, with the lowest type being served representing type l and all the other types echoing type h . The optimality of bunching under *uniformly* no threshold flip of type order follows similar arguments for that in the binary-type setting under no threshold flip by π_l^* . The lowest type being served, and thereby, the optimal posted price and threshold disclosure can be explicitly characterized, leveraging the fact that no threshold flip holds *uniformly* regardless of pairs of types and threshold rule.

To end this section, revisit Example 4 for an illustration. In this example, for any $\hat{x} \in X$,

$$\begin{aligned}\mathbb{E}[v(h, x) \mid x < \hat{x}] - \mathbb{E}[v(m, x) \mid x \geq \hat{x}] &= \left(\Delta_\theta + \frac{\hat{x} - 1 + x}{2}\right) - \left(m + \frac{\bar{x} + \hat{x}}{2}\right) = \Delta_\theta - \frac{\Delta_x - 1}{2}, \\ \mathbb{E}[v(m, x) \mid x < \hat{x}] - \mathbb{E}[v(l, x) \mid x \geq \hat{x}] &= \left(m + \frac{\hat{x} - 1 + x}{2}\right) - \left(l - \Delta_\theta + \frac{\bar{x} + \hat{x}}{2}\right) = \Delta_\theta - \frac{\Delta_x - 1}{2},\end{aligned}$$

where, just to recall, Δ_θ and Δ_x measure the impact of the buyer's private type and the unknown component in valuation variations, respectively. Therefore, uniformly no threshold flip of type order occurs if

$$\Delta_\theta \geq \frac{\Delta_x - 1}{2},$$

which requires the buyer's type to be significantly impactful, relative to the unknown component. If this is the case, by (Theorem 3), information is not leveraged to screen the buyer. A single price-threshold disclosure bundle is optimal.

7 Infinite-type setting

All the proofs of our results extend readily if there is a continuum of states. The extension to the infinite-type case, however, is not trivial. Nevertheless, we find that the previous insights remain valid: Section 7.1 shows that a menu of prices and threshold disclosure is optimal under the partition flip the ranking of willingness to pay across cut-off types; and Section 7.2 shows that a fixed price- threshold disclosure bundle is approximately optimal when the type order is almost preserved.

Throughout this section, consider a continuum of types $\Theta = [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$, endowed with the distribution $F(\theta)$. We assume that $F(\theta)$ is differentiable in θ with density $f(\theta)$, and moreover, $v(\theta, x)$ is differentiable in θ . Then, the virtual value in this environment is given by

$$\phi^c(\theta, x) = v(\theta, x) - v_\theta(\theta, x) \frac{1 - F(\theta)}{f(\theta)}.$$

Similar to the finite-type case, we assume that $\phi^c(\theta, x)$ increases in θ and x .

7.1 Optimality of a screening menu

By the monotonicity of the virtual values, each state x is associated with a cut-off type θ_x above (respectively, below) which the buyer's virtual value is non-negative (respectively, negative). Formally,

$$\theta_x \equiv \inf\{\theta \mid \phi^c(\theta, x) \geq 0\}.$$

Moreover, as $\phi^c(\theta, x)$ increases in x , this cut-off type θ_x decreases in x . We use

$$\Theta_x \equiv \{\theta_x\}_{x \in X}$$

to denote the type space consisting of only cut-off types. Even with a continuum of types, there are *finitely* many cut-off types $\{\theta_x\}_{x \in X}$ due to the finiteness of the state space. Accordingly, \mathbf{M}^* comprises $|\Theta_x|$ options of prices and disclosure rules because each interval of types $[\theta_{x^+}, \theta_x)$ is assigned the same option. Then, the following proposition can be obtained following the proof of Theorem 2 for a type space consisting of only the cut-off types.

Proposition 3. *Fix $\Theta = [\underline{\theta}, \bar{\theta}]$ and $|X| < \infty$. If there is a partition flip of type order within Θ_x , a menu of threshold disclosures and posted prices is optimal.*

This result holds even if there is a continuum of states $X = [\underline{x}, \bar{x}]$ and the valuation function is continuous over states, by approximating an associated finite-state model as the distance between states approaches zero. In this case, the partition flip of type order reduces to the valuation at the cut-off state $v(\theta, x_\theta)$ decreasing in types.²⁶

7.2 (Approximate) optimality of bunching

When valuations shift smoothly across (a continuum of) types, there are always types whose valuations are sufficiently close to others'. This makes it impossible to preserve the ranking of willingness to pay uniformly across the types. Consequently, the optimality of bunching cannot be derived as an extension of Theorem 3 which shows that under the uniformly no threshold flip of valuation ranking across finitely many types, a fixed price-information bundle is optimal. Nevertheless, we establish the *approximate* optimality of bunching under ε -uniformly no threshold flip of type order, formally defined below.

Definition 8 (ε -uniformly no threshold flip of type order). *ε -uniformly no threshold flip of type order occurs if for some $\varepsilon > 0$,*

$$\mathbb{E}[v(\theta + \varepsilon, x) \mid x \leq \hat{x}] \geq \mathbb{E}[v(\theta, x) \mid x \geq \hat{x}] \quad \forall \theta, \hat{x}.$$

²⁶This is the case in, for example, the environments studied in Esó and Szentes (2007) and Wei and Green (2023) under which the valuation function is concave in types and states, and the cross derivative is positive.

The following proposition shows that as ε vanishes, the seller's maximized revenue can be approximated by offering via a fixed price-threshold disclosure bundle. Formally, let R_ε represent the revenue guarantee if the seller offers a single posted price and threshold disclosure rule under the ε -uniformly no threshold flip of type order, we find that:

Proposition 4. $R_\varepsilon \rightarrow V(P)$ as $\varepsilon \rightarrow 0$

Moreover, if there are only two states $\Omega = \{b, g\}$ with $b < g$, we establish the exact optimality of a fixed price and disclosure rule within the class of deterministic mechanisms.

Proposition 5. Fix $\Theta = [\underline{\theta}, \bar{\theta}]$ and $X = \{b, g\}$. If $v(\theta_b, b) > v(\theta_g, g)$ and only deterministic allocations are allowed, a posted price, associated with full disclosure, is optimal.

The idea of the proof is as follows. With binary states $X = \{b, g\}$, there are only two cut-off types θ_g and θ_b . Hence, the partition flip of type order reduces to $v(\theta_b, b) \leq v(\theta_g, g)$. If this is the case, a menu of prices and threshold disclosures is optimal by Proposition 3. If by contrast, $v(\theta_b, b) \leq v(\theta_g, g)$, the seller adjusts the cut-off types to $\tilde{\theta}_b, \tilde{\theta}_g$ just enough to restore the partition flip of type order: $v(\tilde{\theta}_b, b) = v(\tilde{\theta}_g, g)$. In turn, at this boundary of the partition flip, the seller is indifferent between offering a screening menu and a single option of price and information. Put differently, bunching is optimal.

We end this section with a numerical example to illustrate Proposition 5.

Example 5. $v(\theta, x) = 3\theta^2 + 6\theta + x$, $\Theta = [0, 2]$, $X = \{8, 12\}$. Types and states are likely equally.

In this example, $\phi(\theta, x) = 3\theta^2 + 6\theta + x - (6\theta + 6)(2 - \theta) = 9\theta^2 + x - 12$. Thus, $\theta_{12} = 0$ and $\theta_8 = \frac{2}{3}$. Hence, $v(\theta_8, 8) = \frac{43}{3}$ and $v(\theta_{12}, 12) = 12$. As $v(\theta_8, 8) > v(\theta_{12}, 12)$, no flip of type order occurs. By Proposition 5, within the class of deterministic mechanism, offering a fixed bundle of price and threshold disclosure to all types is optimal.

8 Discussion

8.1 Posterior rent and privacy of signals

As explained in the binary-type model, not observing signals generally hurts the seller due to the presence of the buyer's *posterior* rent. Specifically, implementing the benchmark allocation requires the seller to pay the buyer's *posterior* rent (apart from his *ex ante* rent), making $V(P) < V(\bar{P})$. When valuation shifts smoothly across (infinite) types, the relevance of signal privacy comes from a different reason. Indeed, any allocations *implementable with private signals* can

be implemented without generating *posterior* rent to the buyer.²⁷ Therefore, if the seller fails to achieve the upper bound of revenue $V(\bar{P})$, it is due to an implementability issue. In such a scenario, information design can expand the set of implementable allocations. To illustrate, consider the following example where the benchmark allocation is implementable with private signals only if uninformative experiments are possible.

Example 6. $v(\theta, x) = \theta^2 + \theta + x - 2$. Types and states are uniformly distributed over $\Theta = [0, 1]$ and $\Omega = [0, 3]$.

In this example, $p^*(\theta, s^g) = -\theta^2 + \frac{2}{3}\theta + 1$. Moreover, $p^*(\theta, s^g)$ is a concave function in $[0, 1]$ with $p(0, s^g) = 1, p(1, s^g) = \frac{2}{3}$. Thus, $p^*(\bar{\theta}, s^g) = \min_{\theta} p^*(\theta, s^g)$. Then by Proposition 2, the seller implements the benchmark allocation via \mathbf{M}^* . Suppose the seller provides full disclosure to all types. To implement the benchmark allocation, it must be that for any θ and x , $q(\theta, x) = \mathbb{1}_{x \geq x_{\theta}}$. For the buyer to report truthfully their states, it is necessary that

$$p(\theta, x) = \begin{cases} \bar{p}(\theta) & \text{if } x \geq x_{\theta}, \\ \underline{p}(\theta) & \text{otherwise.} \end{cases}$$

To prevent the lowest type $\underline{\theta}$ from mimicking some type θ and always report $x < x_{\theta}$, it must be that $\underline{p}(\theta) \geq 0$. Therefore,

$$\int_{x \geq x_{\theta}} \mu(x) dx p^*(\theta, s^g) = \int_{x \geq x_{\theta}} \mu(x) dx \bar{p}(\theta) + \underline{p}(\theta) \int_{x \leq x_{\theta}} \mu(x) dx \geq \int_{x \geq x_{\theta}} \mu(x) dx \bar{p}(\theta)$$

where the equality uses the fact that all mechanisms implementing the benchmark allocation share the same expected payment. Thus, $p^*(\theta) \geq \bar{p}(\theta)$ for all type θ .

Consider $\theta = \frac{1}{3}$, we have $\bar{p}^*(\frac{1}{3}, s^g) = \frac{10}{9}$, and $v(\frac{1}{3}, x_{\frac{1}{3}}) = \frac{13}{9}$. Thus, $v(\frac{1}{3}, x(\frac{1}{3})) > p^*(\frac{1}{3}, s^g) \geq \bar{p}(\frac{1}{3})$. Then, if the buyer observes any state $x \in (\bar{p}(\frac{1}{3}), v(\frac{1}{3}, x(\frac{1}{3})))$, it is optimal for him to misreport state \bar{x} , receiving the good at a price lower than his valuation. Thus, the benchmark allocation is not implementable under full disclosure.

8.2 Alternative proof for Wei and Green (2023)

Wei and Green (2023) revisit Esó and Szentes (2007)'s "continuous" model, adding a twist that the buyer can walk away after information disclosure. In this section, we solve the former's problem by directly modifying the latter's optimal mechanism.²⁸

²⁷We omit the formal proof, which extends the arguments in Krämer and Strausz (2015a) to a setting with information design and possibly finitely many states.

²⁸Indeed, this modified mechanism coincides with Wei and Green (2023)'s solution.

Under Esó and Szentes (2007)'s optimal mechanism, the seller offers full disclosure and a menu of "information fees" $\hat{c}(\cdot)$ and "strike prices" $\hat{p}(\cdot)$ for the good to implement the benchmark optimal allocation. Thus, $(q(\theta), p(\theta)) \in \{(0, \hat{c}(\theta)), (1, \hat{c}(\theta) + \hat{p}(\theta))\}$. This menu is a deterministic mechanism. Therefore, following the arguments in the proof of Proposition 1, it is revenue-equivalent to a persuasive-posted price mechanism which offers type θ (i) a binary-signal experiment which sends "good news" if $x \geq x_\theta$ and "bad news" otherwise, and (ii) a posted price.

$$\tilde{p}(\theta) = \hat{c}(\theta) + \hat{p}(\theta) + \frac{\hat{c}(\theta) [1 - \mathcal{Q}(\theta)]}{\mathcal{Q}(\theta)} = \hat{p}(\theta) + \frac{\hat{c}(\theta)}{\mathcal{Q}(\theta)}.$$

In addition, Wei and Green (2023) show that information design leads to reverse price discrimination in the continuous model. This feature can also be obtained by leveraging the properties of Esó and Szentes (2007)'s optimal mechanism. Let $\mathbf{X}(\theta) \equiv \frac{1}{\mathcal{Q}(\theta)}$ represent the inverted trade probability for θ . Then, $\tilde{p}(\theta) = \hat{p}(\theta) + \hat{c}(\theta)\mathbf{X}(\theta)$, and

$$\tilde{p}'(\theta) = \hat{p}'(\theta) + \hat{c}'(\theta)\mathbf{X}(\theta) + \hat{c}(\theta)\mathbf{X}'(\theta) = \hat{c}(\theta)\mathbf{X}'(\theta) < 0,$$

where the second equality uses the fact that under Esó and Szentes (2007)'s optimal mechanism, $\hat{c}(\theta)$ and $\hat{p}(\theta)$ solves $\hat{c}'(\theta) = \hat{p}'(\theta)\mathcal{Q}(\theta) = \hat{p}'(\theta)\frac{1}{\mathbf{X}(\theta)}$, and the last uses $\mathbf{X}'(\theta) < 0$. Thus, $\tilde{p}(\cdot)$ is a decreasing function.

8.3 On the number of signals

As we have seen, it is without loss of generality to offer binary-signal experiments with deterministic allocation. This is no longer true when random mechanisms are necessary. When the valuations vary significantly across states, a rich menu is needed to screen the states effectively. As a result, binary-signal experiments are not sufficient. In this section, we illustrate this with a simple example where an optimal experiment sends at least three signals to some type.

Example 7. $\Theta = \{\theta_3, \theta_2, \theta_1\}$, $X = \{x_1, x_2, x_3, x_4\}$. *Types and states are equally likely.*

$v(\theta, x)$	x_1	x_2	x_3	x_4
θ_3	7	7	7	7
θ_2	0	3	7	7
θ_1	0	0	0	6

In this example, θ_2 's valuation varies significantly across states with that at state x_1 being sufficiently low. If restricted to binary-signal experiments, the seller can only separate the state space for type θ_2 into two partitions which, under the optimal mechanism, include $\{x_1, x_2\}$ and $\{x_3, x_4\}$. Armed with three signals, the seller can distinguish a very unfavorable state x_1 from a better one x , fine-tuning the design of allocations. The formal proof is in the Online Appendix.

A Preliminary results: omitted proofs

A.1 Proof of Lemma 1

Let $\mathbf{M} \equiv \{\pi_\theta, q(\theta, s), p(\theta, s)\}_{\theta, s}$ be an optimal mechanism. Toward a contradiction, assume that there exists x such that $q(\bar{\theta}, x) < 1$. Then, the seller can improve her revenue by revising type $\bar{\theta}$ contract to $\tilde{\mathbf{C}} \equiv \{\tilde{\pi}_{\bar{\theta}}, \tilde{p}(\bar{\theta})\}$ in which $\tilde{\pi}_{\bar{\theta}}$ provides no information and $\tilde{p}(\bar{\theta})$ is a posted price for the good, given by:

$$\tilde{p}(\bar{\theta}) = \sum_s p(\bar{\theta}, s) \pi_{\bar{\theta}}(s) + \mathbb{E}[v(\theta, x)] - \sum_x v(\bar{\theta}, x) \sum_s q(\bar{\theta}, s) \pi_{\bar{\theta}}(s|x) g(x)$$

To see this, note the following. If type $\bar{\theta}$ buys the good at the price $\tilde{p}(\bar{\theta})$ and no disclosure, he obtains:

$$\mathbb{E}[v(\bar{\theta}, x)] - \tilde{p}(\bar{\theta}) = \sum_x v(\bar{\theta}, x) \sum_s [q(\bar{\theta}, s) - p(\bar{\theta}, s)] \pi_{\bar{\theta}}(s|x) g(x),$$

which is equal to that under the original mechanism \mathbf{M} . Moreover, if type θ mimics $\bar{\theta}$, he either does not buy the good to get a zero payoff or buys the good, to obtain

$$\begin{aligned} & \mathbb{E}[v(\theta, x)] - \tilde{p}(\bar{\theta}) \\ &= \mathbb{E}[v(\theta, x)] - \mathbb{E}[v(\bar{\theta}, x)] + \sum_x \sum_s [v(\bar{\theta}, x) q(\bar{\theta}, s) \pi_{\bar{\theta}}(s|x) g(x) - \sum_x \sum_s p(\bar{\theta}, s) \pi_{\bar{\theta}}(s|x) g(x)] \\ &= \mathbb{E}[v(\theta, x)] - \sum_x \sum_s v(\bar{\theta}, x) [1 - q(\bar{\theta}, s)] \pi_{\bar{\theta}}(s|x) g(x) - \sum_x \sum_s p(\bar{\theta}, s) \pi_{\bar{\theta}}(s|x) g(x) \\ &\leq \mathbb{E}[v(\theta, x)] - \sum_x \sum_s v(\theta, x) [1 - q(\bar{\theta}, s)] \pi_{\bar{\theta}}(s|x) g(x) - \sum_x \sum_s p(\bar{\theta}, s) \pi_{\bar{\theta}}(s|x) g(x) \\ &= \sum_x \sum_s [v(\theta, x) q(\bar{\theta}, s) - p(\bar{\theta}, s)] \pi_{\bar{\theta}}(s|x) g(x), \end{aligned}$$

which is exactly type θ 's from mimicking $\bar{\theta}$ and report signals truthfully under the \mathbf{M} . Therefore, $\tilde{\mathbf{C}}$ weakly increases type $\bar{\theta}$'s on-path payoff and weakly reduces the other types' off-path payoff. Consequently, the buyer reveals his true type. While payments by the other types remain unchanged, type $\bar{\theta}$ now pays

$$\begin{aligned} \tilde{p}(\bar{\theta}) &= \sum_s p(\bar{\theta}, s) \pi_{\bar{\theta}}(s) + \mathbb{E}[v(\theta, x)] - \sum_x v(\bar{\theta}, x) \sum_s q(\bar{\theta}, s) \pi_{\bar{\theta}}(s|x) g(x) \\ &> \sum_s p(\bar{\theta}, s) \pi_{\bar{\theta}}(s), \end{aligned}$$

which his expected payment under \mathbf{M} . This contradicts with \mathbf{M} being optimal.

A.2 Proof of Proposition 1

We complete the arguments in the main text by showing that under Case 2: $\underline{p}(\theta) > 0$, the seller's revenue remains weakly higher under the revised mechanism. Fix type θ . If θ mimics type θ' , he receives weakly less information and pays weakly higher for each action (buy or do not buy the good). Thus, his off-path payoff is weakly lower under the revised mechanism. On the other hand, by revealing his type and buying the good if and only if s^g is realized, he obtains:

$$\tilde{U}(\theta) \equiv \sum_x [v(\theta, x) - \tilde{p}(\theta)] \mathcal{Q}(\theta, x) g(x) = \sum_x [v(\theta, x) - \bar{p}(\theta)] \mathcal{Q}(\theta, x) g(x) - \underline{p}(\theta) [1 - \mathcal{Q}(\theta)], \quad (9)$$

which is his payoff under the original mechanism \mathbf{M}^d . As \mathbf{M}^d induces truth-telling, it follows that θ finds it optimal to reveal his type under the revised mechanism. Moreover, by (9) and the fact that the buyer's payoff is non-negative under \mathbf{M}^d , $\tilde{U}(\theta) \geq 0$. As a result, the buyer's payoff from buying the good upon observing s^g , given by $\frac{\tilde{U}(\theta)}{\mathcal{Q}(\theta)}$, is non-negative. Hence, the buyer buys the good after s^g , and accordingly, pays the seller

$$\tilde{p}(\theta) \mathcal{Q}(\theta) = \bar{p}(\theta) \mathcal{Q}(\theta) + \underline{p}(\theta) [1 - \mathcal{Q}(\theta)], \quad (10)$$

which is exactly type θ 's expected payment under \mathbf{M}^d . Thus, the seller's revenue cannot decrease under the revised mechanism.

A.3 Proof of Proposition 2

Part (a): Given that $\phi(\theta, x_\theta) > 0 \forall \theta$, the optimal allocation in $(\bar{\mathcal{P}})$ is uniquely given by $Q(\theta, x) = \mathbb{1}_{x \geq x_\theta}$. Therefore, if there exists \mathbf{M} such that $R_{\mathbf{M}} = V(\bar{\mathcal{P}})$, (i) \mathbf{M} must be deterministic and (ii), upon observing any signal, type θ knows whether $x \geq x_\theta$ or not. Because of (i) and the fact that the buyer pays the same expected payment under \mathbf{M} and \mathbf{M}^* , the buyer pays more to get the good under \mathbf{M}^* . Because of (ii), the buyer receives weakly less information under \mathbf{M}^* . To sum up, the buyer pays more to get the good and gets weakly less information under \mathbf{M}^* . Therefore, if \mathbf{M} is incentive compatible with private signals, so is \mathbf{M}^* . Thus, $R_{\mathbf{M}^*} = V(\bar{\mathcal{P}})$.

Part (b): "If": Suppose $p^*(\bar{\theta}, s^g) = \min_{\theta} \{p^*(\theta, s^g)\}$, we now show that \mathbf{M}^* induces truth-telling even if the seller does not observe signals. Note that \mathbf{M}^* , as a solution to $(\bar{\mathcal{P}})$, induces truth-telling with public signals. Therefore, it suffices to show that under \mathbf{M}^* , for any type θ and θ' , it is not beneficial for θ to report θ' and then either (i) always report s^b , (iii) always report s^g or (ii) always misreporting signals. By always reporting s^b off-path, θ obtains a zero payoff; hence, (i) is not beneficial. If (ii) is beneficial, then θ also benefits from mimicking $\bar{\theta}$ and reporting signals truthfully (type $\bar{\theta}$ always observes s^g and pays the least after s^g), which contradicts \mathbf{M}^* being incentive compatible with public signals. Now, consider the last deviating behavior. Note that

if type θ , who reports θ' , prefers to misreport s^g (buys the good) rather than truthfully s^b (and gets a zero payoff), it must be optimal for him to buy the good (or report s^g upon observing this signal). Hence, if (ii) is beneficial, so is (iii), a contradiction,

Part (b): “*Only If*”: Suppose $\exists \theta$ such that $p(\bar{\theta}, s^g) > p(\theta, s^g)$. By mimicking θ and always reporting s^g , type $\bar{\theta}$ always gets the good at a lower price $p(\theta, s^g)$. Therefore, θ prefers to misreport θ than truth-telling. Consequently, if the seller offers \mathbf{M}^* with private signals, she obtains $R_{\mathbf{M}^*} < V(\bar{\mathcal{P}})$.

B Screening vs. Bunching: omitted proofs

B.1 Proof of Claim 1

Let

$$\alpha_l \equiv \max \{x' \mid x \leq \hat{x}_l^* : \mathbb{E}[v(h, x) \mid x > x'] < \mathbb{E}[v(l, x) \mid x > x']\},$$

$$\beta_l \equiv \min \{x' \mid x \geq \hat{x}_l^* : \mathbb{E}[v(h, x) \mid x < x'] < \mathbb{E}[v(l, x) \mid x > x']\}.$$

If $x^{**}(l) \in [\underline{x}, \alpha_l]$, the seller can do strictly better by offering a threshold disclosure $\bar{\pi}(l)$ under which (i) the threshold is α_l^+ and (ii) with probability λ , “good news” is sent at α_l^+ , such that such that $\omega^{\bar{\pi}}(h, s^g) = \omega^{\bar{\pi}}(h, s^b)$. Note that λ exists because by definition of α_l ,

$$\mathbb{E}[v(h, x) \mid x > \alpha_l^+] > \mathbb{E}[v(l, x) \mid x > \alpha_l^+],$$

$$\mathbb{E}[v(h, x) \mid x > \alpha_l] < \mathbb{E}[v(l, x) \mid x > \alpha_l].$$

Thus, it must be that $x^{**}(l) > \alpha_l$. By similar arguments, we also have $x^{**}(l) < \beta_l$. Thus, $x^{**}(l) \in (\alpha_l, \beta_l)$. Hence,

$$\mathbb{E}[v(h, x) \mid x > x^{**}(l)] > \mathbb{E}[v(l, x) \mid x > x^{**}(l)],$$

which implies $\omega^{\pi^{**}(l)}(h, s^b) \geq \omega^{\pi^{**}(l)}(l, s^g)$.

B.2 Proof of Theorem 2

The proof leverages Lemma B.1 below, which provides two expressions of the price gap between two adjacent types under \mathbf{M}^* .

Lemma B.1. *There exist positive functions $\lambda(\theta)$ and $\beta(\theta)$ such that:*

$$(a) \quad p^*(\theta^+, s^g) - p^*(\theta, s^g) = \left[\mathbb{E}[v(\theta^+, x) \mid x_{\theta^+} \leq x <] - p^*(\theta, s^g) \right] \lambda(\theta), \quad \forall \theta \geq \underline{\theta}.$$

$$(b) \quad p^*(\theta, s^g) - p^*(\theta^-, s^g) = \left[\mathbb{E}[v(\theta, \omega) \mid x_{\theta} \leq x < x_{\theta^-}] - p^*(\theta, s^g) \right] \beta(\theta), \quad \forall \theta \geq \underline{\theta}^+.$$

Proof of Lemma B.1. To examine the ranking of $p^*(\cdot)$, we start with the expected payment $\mathbb{P}(\theta) = p^*(\theta) \sum_{x \geq x_\theta} \mu(x)$. By its definition (see equations (5)),

$$\begin{aligned}\mathbb{P}(\theta^+) - \mathbb{P}(\theta) &= \sum_{x_{\theta^+} \leq x < x_\theta} v(\theta^+, x) \mu(x) \quad \forall \theta \geq \underline{\theta}, \\ \mathbb{P}(\underline{\theta}) &= \sum_{x_{\underline{\theta}} \leq x < x_{\underline{\theta}^-}} v(\underline{\theta}, x) \mu(x).\end{aligned}$$

Using

$$\sum_{x_{\theta^+} \leq x < x_\theta} v(\theta^+, x) \mu(x) = \mathbb{P}^*(\theta^+) - \mathbb{P}^*(\theta) = p^*(\theta^+, s^g) \sum_{x \geq x_{\theta^+}} \mu(x) - p^*(\theta, s^g) \sum_{x \geq x_\theta} \mu(x) \quad (11)$$

Part (a). Write the RHS of (11) as $\sum_{x \geq x_{\theta^+}} \mu(x) [p^*(\theta^+, s^g) - p^*(\theta, s^g)] + p^*(\theta, s^g) \sum_{x_{\theta^+} \leq x < x_\theta} \mu(x)$. Then, we obtain

$$\begin{aligned}p^*(\theta^+, s^g) - p^*(\theta, s^g) &= \frac{\sum_{x_{\theta^+} \leq x < x_\theta} v(\theta^+, x) \mu(x) - p^*(\theta, s^g) \sum_{x_{\theta^+} \leq x < x_\theta} \mu(x)}{\sum_{x \geq x_{\theta^+}} \mu(x)} \\ &= \left[\mathbb{E}[v(\theta^+, x) \mid x_{\theta^+} \leq x < x_\theta] - p^*(\theta, s^g) \right] \frac{\sum_{x_{\theta^+} \leq x < x_\theta} \mu(x)}{\sum_{x \geq x_{\theta^+}} \mu(x)} \\ &\propto \mathbb{E}[v(\theta^+, x) \mid x_{\theta^+} \leq x < x_\theta] - p^*(\theta, s^g).\end{aligned}$$

Part (b). Write the RHS of (11) as $\sum_{x \geq x_\theta} \mu(x) [p^*(\theta^+, s^g) - p^*(\theta, s^g)] + p^*(\theta^+, s^g) \sum_{x_{\theta^+} \leq x < x_\theta} \mu(x)$, and the rest followed by similar arguments. \square

Armed with Lemma B.1, we now show that the highest type pays the lowest price under \mathbf{M}^* . It follows from Lemma B.1 that if for all $\theta \geq \underline{\theta}$,

$$\mathbb{E}[v(\theta^+, x) \mid x(\theta^+) \leq x < x_\theta] \leq \mathbb{E}[v(\theta, x) \mid x_\theta \leq x < x_{\theta^-}], \quad (12)$$

then the sign of $[p^*(\theta^+, s^g) - p^*(\theta, s^g)]$ is decreasing in θ . Moreover, this sign is non-positive because by (6.2.1) for $\underline{\theta}$,

$$\mathbb{E}[v(\underline{\theta}^+, x) \mid x(\theta^+) \leq x < x(\underline{\theta})] \leq \mathbb{E}[v(\underline{\theta}, x) \mid x(\underline{\theta}) \leq x < x(\underline{\theta}^-)] = p^*(\underline{\theta}, s^g),$$

implying $p^*(\underline{\theta}^+, s^g) - p^*(\underline{\theta}, s^g) \leq 0$, by part (1) of Lemma B.1. Therefore,

$$p^*(\theta^+, s^g) - p^*(\theta, s^g) \leq 0 \quad \forall \theta \geq \underline{\theta}. \quad (13)$$

This implies that $p^*(\theta, s^g)$ is the lowest price. Then by Proposition 2, $R_{\mathbf{M}^*} = V(\bar{P})$. Moreover, as \mathbf{M}^* induces truth-telling with private signals, the seller can simply offer a menu of posted prices and threshold disclosure $\{\pi_\theta^*, p^*(\theta)\}_\theta$, where $p^*(\theta) = p^*(\theta, s^g)$.

B.3 Proof of Theorem 3

Let L be the lowest type being served under an optimal mechanism. Consider the following relaxed problem (\mathcal{RP}_L) , under which all types mimics L off-path:

$$\begin{aligned}
(\mathcal{RP}_L) \quad & \max_{(\pi, q, U)} \sum_{\theta \geq L} \left[\sum_x \sum_s v(\theta, x) q(\theta, \omega^{\pi_\theta}(\theta, s)) \pi_\theta(s|x) \mu(x) - U(\theta) \right] f(\theta) \\
& s.t. \quad U(\theta) - U(L) \geq \sum_s \int_{\omega^{\pi_L(L, s)}}^{\omega^{\pi_L(\theta, s)}} q(L, z) dz \pi_L(s) \quad \forall \theta > L \quad (IC) \\
& \quad U(L) \geq 0 \quad (IR_L) \\
& \quad q(\theta, \omega) \text{ increases in } \omega. \quad (\text{MON})
\end{aligned}$$

We will show that the solution to this relaxed problem, which features a posted price and a threshold disclosure, solves the original problem. Obviously, (IR_L) and $(IC_{\theta \rightarrow L})$ bind for all $\theta > L$ under (\mathcal{RP}_L) , reducing the seller's relaxed problem to

$$\begin{aligned}
& \max_{q, \pi} \sum_\theta \sum_x v(\theta, x) q(\theta, \omega^{\pi_\theta}(\theta, s)) \pi_\theta(s|x) \mu(x) f(\theta) - \sum_\theta \sum_x \sum_s \int_{\omega^{\pi_L(L, s)}}^{\omega^{\pi_L(\theta, s)}} q(L, z) dz \pi_L(s|x) \mu(x) f(\theta) \\
& s.t. \quad q(\theta, \omega) \text{ increases in } \omega.
\end{aligned}$$

Fix π , it is a linear problem in q with (MON) being the only constraint. Thus, the optimal allocation is generally unique, given by

$$q(L, \omega) = \mathbb{1}_{s \geq \hat{s}_L}, \quad q(\theta, x) = 1 \forall x \forall \theta > L.$$

Fix $q(L, s) = \mathbb{1}_{s \geq \hat{s}_L}$. The term involving π_L in the seller's objective (revenue) is given by

$$\begin{aligned}
\mathbf{R}(\pi_L) & \equiv \sum_x \sum_{\hat{s}_L}^{\bar{s}} v(L, x) \pi_L(s|x) f(L) g(x) \\
& - \sum_{\theta \geq L^+} \left[\sum_{\hat{s}_L}^{\bar{s}} [\omega^{\pi_L}(\theta, s) - \omega^{\pi_L}(L, \hat{s}_L)] - \sum_{\theta \geq L^+} \sum_{\underline{s}}^{\hat{s}_L} \max\{\omega^{\pi_L}(\theta, s) - \omega^{\pi_L}(L, \hat{s}_L), 0\} \right] \pi_L(s|x) f(\theta) \\
& = \sum_x \sum_{\hat{s}_L}^{\bar{s}} v(L, x) \pi_L(s|x) f(L) g(x) \\
& - \sum_{\theta \geq L^+} \left[\sum_{\hat{s}_L}^{\bar{s}} [\omega^{\pi_L}(\theta, s) - \omega^{\pi_L}(L, \hat{s}_L)] - \sum_{\theta \geq L^+} \sum_{\underline{s}}^{\hat{s}_L} [\omega^{\pi_L}(\theta, s) - \omega^{\pi_L}(L, \hat{s}_L), 0] \right] \pi_L(s|x) f(\theta) \\
& = \sum_x \sum_{\hat{s}_L}^{\bar{s}} v(L, x) \pi_L(s|x) f(L) g(x) - \sum_{\theta \geq L^+} \sum_{\underline{s}}^{\bar{s}} [\omega^{\pi_L}(\theta, s) - \omega^{\pi_L}(L, \hat{s}_L)] \pi_L(s|x) f(\theta) \\
& \equiv \bar{\mathbf{R}}(\pi_L).
\end{aligned}$$

$\bar{\mathbf{R}}(\pi_L)$ is an upper bound of $\bar{\mathbf{R}}(\pi_L)$. We now show that this bound is tight. By replacing all signals $s \geq \hat{s}_L$ with "good news" and all signals $s < \hat{s}_L$ with "bad news," $\bar{\mathbf{R}}(\pi_L)$ weakly increases, and $\bar{\mathbf{R}}(\pi_L)$ reduces to

$$\bar{\mathbf{R}}(\pi_L) = \omega^{\pi_L}(L, s^g) \left[\sum_{\theta \geq L^+} f(\theta) + f(L)\pi_L(s^g) \right] - \sum_{\theta \geq L^+} \mathbb{E}[\nu(\theta, x)]f(\theta),$$

Let

$$\pi_L^{**} \equiv \operatorname{argmax}_{\pi_L} \omega^{\pi_L}(L, s^g) \left[\sum_{\theta \geq L^+} f(\theta) + f(L)\pi_L(s^g) \right] - \sum_{\theta \geq L^+} \mathbb{E}[\nu(\theta, x)]f(\theta)$$

By the same arguments used for the binary-type case, π_L^{**} features a disclosure rule. Next, we find an optimal payment schedule. As optimal allocation is deterministic, without loss of generality to focus on posted-price mechanisms. By (IR_L) , $p^{**}(L) = \omega^{\pi_L^{**}}(L, s^g)$. Consider type θ who mimics L . Under no uniformly no threshold flip of type order, $\omega^{\pi_L^{**}}(\theta, s^g) \geq \omega^{\pi_L^{**}}(\theta, s^b) \geq \omega^{\pi_L^{**}}(L, s^g)$. Hence, it is optimal for θ to always buy the good after mimicking L . Hence, by (IR) $p^{**}(\theta) = p^{**}(L)$ for all $\theta > L$. Obviously, this single option of price and information $\{\pi_L^{**}, p^{**}(L)\}$ satisfies ignored constraints and hence, solves the original problem.

Remark 2. Let $V(\mathcal{RP}_\theta)$ denote the value of program \mathcal{RP}_θ in which θ is the lowest type being served. Under no threshold flip of type order, it is optimal to serve only types above (including) L , where L solves $L \in \operatorname{argmax}_\theta V(\mathcal{RP}_2(\theta))$.

C Random mechanisms: omitted proofs

This section provides proofs of Claims 2 and 3, which rely on the following lemma.

Lemma C.1. *If only deterministic mechanisms are allowed, it is optimal to offer a menu $\{\alpha(\theta, x), p(\theta)\}_{\theta, x}$ under which each type θ trades with probability $\alpha(\theta, x)$ at state x and pays $p(\theta)$ for the good.*

The proof for Lemma C.1 is as follows. If only deterministic mechanisms are allowed, by Proposition 1, it is optimal to offer each type θ receives a posted price $p(\theta)$ for the good and a binary-signal experiment with $S = \{s^g, s^b\}$. Hence, an experiment can be represented by the probability that signal s^g is realized at state x for type θ , $\alpha(\theta, x)$.

C.1 Proof of Claim 2

To characterize the optimal deterministic mechanism, or the optima menu $\{\alpha(\theta, x), p(\theta)\}_{\theta, x}$, consider the following relaxed problem in which (i) only IR condition for θ_1 is kept, and (ii) off

the equilibrium path, θ_2 mimics θ_1 and buys the good only after s^g whereas θ_3 mimics θ_1 and always buys the good.

$$\begin{aligned} \max_{\{p, \alpha\}} \quad & \sum_{\theta} \sum_x p(\theta) \alpha(\theta, x) \mu(x) f(\theta) \\ \text{s.t.} \quad & \sum_x [v(\theta_2, x) - p(\theta_2)] \alpha(\theta_2, x) \mu(x) \geq \sum_x [v(\theta_2, x) - p(\theta_1)] \alpha(\theta_1, x) \mu(x) & (IC_{21}) \\ & \sum_x [v(\theta_3, x) - p(\theta_3)] \geq \mathbb{E}[v(\theta_3, x)] - p(\theta_1) & (IC_{21}) \\ & \sum_x [v(\theta_1, x) - p(\theta_1)] \alpha(\theta_1, x) \mu(x) \geq 0 & (IR_1) \end{aligned}$$

As trading with θ_3 and θ_2 generates no rent for others, these types receive efficient allocations: $\alpha(\theta_3, x_2) = \alpha(\theta_3, x_1) = 1$, $\alpha(\theta_2, x) = \mathbb{1}_{x=g}$.

If $\alpha(\theta_1, x_1) > 0$, then reduce $\alpha(\theta_1, x_1)$ and increase $p(\theta_1)$ such that $\sum_x p(\theta_1) \alpha(\theta_1, x) \mu(x)$ remains unchanged. By doing so, the seller's revenue increases. Moreover, no constraints are violated because (i) the right-hand side of (IC_{21}) decreases (as $v(\theta_3, x_1) > 0$) and (ii) the right-hand side of (IC_{21}) and left-hand side of (IR_1) remains unchanged (as $v(\theta_2, x_1) = v(\theta_1, x_1) = 0$). Thus, $\alpha(\theta_1, x_1) = 0$.

If $\alpha(\theta_1, x_2) < 1$, then reduces $\alpha(\theta_1, x_2)$ by ε and increases $p(\theta_3)$ by $[v(\theta_3, x_2) - p(\theta_2)]\varepsilon$ and $p(\theta_2)$ by $\frac{[v(\theta_3, x_2) - p(\theta_2)]\varepsilon}{\alpha(\theta_1, x_2)\mu(x_2)}$. By doing so, no constraint is affected while the seller's revenue increases by $[f(\theta_3) + f(\theta_2)][v(\theta_3, x_2) - p(\theta_2)]\mu(x_2)\varepsilon - f(\theta_1)v(\theta_1, x_2)\mu(x_2)\varepsilon = \phi(\theta_1, x_2)f(\theta_1)\mu(x_2) > 0$. Thus, $\alpha(\theta_1, x_2) = 1$.

If (IR_1) does not bind, increase $p(\theta_1)$ until it binds. This increases the seller's revenue while not violating any constraints. Thus, (IR_1) binds and hence, $p(\theta_1) = v(\theta_1, x_2) = 4$.

If (IC_{31}) does not bind, increase $p(\theta_3)$ until it binds. This increases the seller's revenue while not violating any constraints. Thus, (IR_3) binds and hence, $p(\theta_3) = p(\theta_1) = 4$.

If (IC_{21}) does not bind, increase $p(\theta_2)$ until it binds. This increases the seller's revenue while not violating any constraints. Thus, (IC_{21}) binds, or $p(\theta_2) = p(\theta_1) = 4$.

To sum up, we obtain $\alpha(\theta_3, x_2) = \alpha(\theta_3, x_1) = 1$, $\alpha(\theta_2, x) = \alpha(\theta_1, x) = \mathbb{1}_{x=x_2}$, and $p(\theta_3) = p(\theta_2) = p(\theta_1) = 4$. The seller's revenue is $4 \cdot [f(\theta_3) + [f(\theta_2) + f(\theta_1)]\mu(x_2)] = \frac{10}{3}$. As $p(\theta_3) = 4$, type θ_3 buys the good at any state. Thus, the maximized revenue can be obtained via a posted price of 4 and full disclosure to all types.

C.2 Proof of Claim 3

To characterize the optimal deterministic mechanism, or the optima menu $\{\alpha(\theta, x), p(\theta)\}_{\theta, x}$, consider the following relaxed problem in which (i) only IR condition for θ_1 is kept, and (ii) off the equilibrium path, m mimics θ_1 and buys the good only after s^g whereas θ_3 mimics θ_1 and always buys the good.

$$\begin{aligned}
& \max_{\{p, \alpha\}} \sum_{\theta} \sum_x p(\theta) \alpha(\theta, x) \mu(x) f(\theta) \\
& \text{s.t.} \quad \sum_x [v(\theta_2, x) - p(\theta_3)] \alpha(\theta_2, x) \mu(x) \geq \mathbb{E}[v(\theta_3, x)] - p(\theta_2) \quad (IC_{32}) \\
& \quad \sum_x [v(\theta_2, x) - p(\theta_2)] \alpha(\theta_2, x) \mu(x) \geq \sum_x [v(\theta_2, x) - p(\theta_1)] \alpha(\theta_1, x) \mu(x) \quad (IC_{21}) \\
& \quad \sum_x [v(\theta_1, x) - p(\theta_1)] \alpha(\theta_1, x) \mu(x) \geq 0 \quad (IR_1)
\end{aligned}$$

If $\alpha(\theta_1, x_1) > 0$, reduce $\alpha(\theta_2, b)$ and increase $p(\theta_1)$ such that $p(\theta_1) \sum_x \alpha(\theta_1, x) \mu(x)$ remains unchanged, and increase $p(\theta_2)$ such that (IC_{21}) remains satisfied. By doing so, no constraints are affected, whereas the seller's revenue strictly increases. Thus, $\alpha(\theta_1, x_1) = 0$.

Note that to ensure that type θ_2 's on-path payoff is non-negative, it is necessary that $v(\theta_2, x_2) \geq p(\theta_2)$. Hence, if $\alpha(\theta_2, x_2) < 1$, by increasing $\alpha(\theta_2, x_2)$, we strictly improve the seller's revenue while not violating any constraints. Thus $\alpha(\theta_2, x_2) = 1$.

If $\alpha(\theta_1, x_2) < 1$. Then, set $p(\theta_1) = v(\theta_1, x_2) = 4$, increase $\alpha(\theta_1, x_2)$ by ε and reduce $p(m)$ and $p(h)$ by $\frac{\varepsilon}{\sum_x \alpha(\theta_2, x) \mu(x)}$. Under this change, no constraints are violated. Moreover, the seller's revenue increases by $f(\theta_1)4\varepsilon - f(\theta_3) \frac{\varepsilon}{\sum_x \alpha(\theta_2, x) \mu(x)} - f(\theta_2)\varepsilon > 0$. Thus $\alpha(\theta_1, x_2) = 1$.

If (IR_1) does not bind, we can increase $p(\theta_1)$ up to it becoming binding, thereby increasing the seller's revenue without violating any constraints. Thus, (IR_1) binds. Given that $\alpha(\theta_1, x_1) = 0$, we thus have $p(\theta_1) = v(\theta_1, x_2)$.

If (IC_{32}) does not bind, increase $p(\theta_3)$ until it binds. This increases the seller's revenue while not violating any constraints. Thus, (IC_{21}) binds, and hence, $p(\theta_3) = p(\theta_2)$.

If (IC_{21}) does not bind, increase $p(\theta_2)$ until it binds. This increases the seller's revenue while not violating any constraints. Thus, (IC_{21}) binds. Given that $\alpha(\theta_2, x_2) = \alpha(\theta_1, x_2) = 1$, $\alpha(\theta_1, x_1) = 0$ and $p(\theta_1) = v(\theta_1, x_2)$, this implies

$$\begin{aligned}
& [v(\theta_2, x_1) - p(\theta_2)] \alpha(\theta_2, x_1) \mu(x_1) + [v(\theta_2, x_2) - p(\theta_2)] \mu(x_2) = [v(\theta_2, x_2) - v(\theta_1, x_2)] \mu(x_2) \\
& \Leftrightarrow p(\theta_2) = \frac{v(\theta_2, x_1) \alpha(\theta_2, x_1) \mu(b) + v(\theta_2, x_2) \mu(x_2) - [v(\theta_2, x_2) - v(\theta_1, x_2)] \mu(x_2)}{\alpha(\theta_2, x_1) \mu(x_1) + \mu(x_2)}. \quad (14)
\end{aligned}$$

Then, the objective problem of the relaxed problem becomes

$$\begin{aligned} & \left[f(\theta_3) + f(\theta_2)[\alpha(\theta_2, x_1)\mu(x_1) + \mu(x_2)] \right] \frac{v(\theta_2, x_1)\alpha(\theta_2, x_1)\mu(x_1) + v(\theta_2, x_2)\mu(x_2) - [v(\theta_2, x_2) - v(\theta_1, x_2)]\mu(x_2)}{\alpha(\theta_2, x_1)\mu(x_1) + \mu(x_2)} \\ & \equiv H(\alpha(\theta_2, x_1)) \end{aligned}$$

Under the specification in Example 3(b), the relaxed problem becomes

$$\max_{\alpha(\theta_2, x_1)} H(\alpha(\theta_2, x_1)) \equiv (\alpha(\theta_2, x_1) + 3) \frac{(\alpha(\theta_2, x_1) + 2)}{\alpha(\theta_2, x_1) + 1} = \alpha(\theta_2, x_1) + 2 + \frac{2(\alpha(\theta_2, x_1) + 2)}{\alpha(\theta_2, x_1) + 1}$$

Thus, $H'(\alpha(\theta_2, x_1)) = 1 - \frac{1}{(\alpha(\theta_2, x_1) + 1)^2}$ and $H''(\alpha(\theta_2, x_1)) = \frac{2}{(\alpha(\theta_2, x_1) + 1)^3} > 0$. Therefore, $H(\alpha(\theta_2, x_1))$ is a convex function. Moreover, $R(0) = R(1) = 6$. Thus, $\alpha(\theta_2, x_1) = 0$ is optimal. This implies that $p(\theta_3) = p(\theta_2) = p(\theta_1) = 4$. Hence, a posted price $p = 4$ and full disclosure is an optimal deterministic mechanism.

D Infinite types: omitted proofs

D.1 Proof of for Proposition 3

We first solve the seller's benchmark problem with public signals when $\Theta = [\underline{\theta}, \bar{\theta}]$. With $\mathbb{P}(\theta)$ and $\mathbb{Q}(\theta, x)$ representing the expected payment and allocation over signals, this problem writes:

$$\begin{aligned} & (\bar{P}^c) \quad \sup_{\mathbb{P}, \mathbb{Q}} \int_{\theta} \mathbb{P}(\theta) dF(\theta) \\ & s.t. \quad \forall \theta, \theta': \quad \sum_x v(\theta, x) \mathbb{Q}(\theta, x) \mu(x) - \mathbb{P}(\theta) \geq \sum_x v(\theta, x) \mathbb{Q}(\theta', x) \mu(x) - \mathbb{P}(\theta') \quad (15) \\ & \quad \quad \quad \sum_x v(\theta, x) \mathbb{Q}(\theta, x) \mu(x) - \mathbb{P}(\theta) \geq 0. \quad (16) \end{aligned}$$

By the Envelope condition, (15) implies $U'(\theta) = \sum_x v_{\theta}(\theta, x) \mathbb{Q}(\theta, x) \mu(x) \forall \theta \geq \tilde{\theta}_g$. By integration by parts,

$$U(\theta) = U(\tilde{\theta}_g) + \int_{\tilde{\theta}_g}^{\theta} \sum_x v_{\theta}(\theta', x) \mathbb{Q}(\theta', x) \mu(x) d\theta'. \quad (17)$$

Consider a relaxed problem which keeps (17) and the partition constraint for the lowest type $\underline{\theta}$. Using $U(\underline{\theta}) = 0$ at optimum, this relaxed problem becomes:

$$\sup_{\pi, q} \int_{\theta} \phi^c(\theta, x) \mathbb{Q}(\theta, x) \mu(x) dF(\theta),$$

where $\phi^c(\theta, x) \equiv v(\theta, x) - v_\theta(\theta, x) \frac{1-F(\theta)}{f(\theta)}$. As $\phi^c(\theta, x)$ increases in θ and x , it is optimal to set $\mathbb{Q}(\theta, x) = \mathbb{1}_{\theta \geq \theta_x}$ or equivalently, $\mathbb{Q}(\theta, x) = \mathbb{1}_{x \geq x_\theta}$. Fix an arbitrary $x \in X$. For any $\theta \in [\theta_x, \theta_{x^-}]$, $\mathbb{Q}(\theta) = \mathbb{Q}(\theta_x)$ and $\mathbb{P}(\theta) = \mathbb{P}(\theta_x)$. Payments are backed out using $U(\tilde{\theta}) = 0$ and (17), given by:

$$\mathbb{P}(\theta) = \sum_{x \geq x_\theta} v(\theta, x) \mu(x) - \int_{\theta} \sum_{x \geq x_{\theta'}} v_\theta(\theta', x) \mu(x) d\theta'$$

Moreover,

$$\int_{\theta_x}^{\theta_x^-} \sum_{x \geq x_{\theta'}} v_\theta(\theta', x) \mu(x) d\theta' = \int_{\theta_x}^{\theta_x^-} \sum_{x \geq x_{\theta_x}} v_\theta(\theta', x) \mu(x) d\theta' = \sum_{x \geq x_{\theta_x}} \int_{\theta_x}^{\theta_x^-} v_\theta(\theta', x) \mu(x) = \sum_{x \geq x_{\theta_x}} [v(\theta_x^-, x) - v(\theta_x, x)] \mu(x),$$

which implies

$$\mathbb{P}(\theta) = \sum_{x \geq x_\theta} v(\theta, x) \mu(x) - \sum_{\theta_x \leq \theta} \sum_{x \geq x_{\theta_x}} [v(\theta_x^-, x) - v(\theta_x, x)] \mu(x).$$

Therefore, \mathbf{M}^* for this problem consists of Θ_x options $\{\pi_\theta^*, p^*(\theta, s^g), p^*(\theta, s^b)\}_{\theta \in \Theta_x}$. Then, following similar arguments in the proof of Theorem 2 for the type space Θ_x , we get

$$p^*(\theta_x, s^g) = \min_{\theta \in \Theta} p^*(\theta, s^g)$$

under the partition flip of type order within Θ_x . As a result, \mathbf{M}^* induces truth-telling even if the seller does not observe signals. Then, offering \mathbf{M}^* is equivalent to offering a menu of threshold disclosures and posted prices $\{\pi_\theta^*, p^*(\theta)\}_{\theta \in \Theta_x}$, where $p^*(\theta) = p^*(\theta, s^g)$. Hence, this menu helps the seller achieve the upper bound of revenue $V(\bar{P}^c)$; hence, it is optimal.

D.2 Proof of for Proposition 4

Suppose it is optimal to exclude all types below L , or $q(\theta, x) = 1$ for all x and $\theta < L$. Then, the seller's revenue must be weakly lower than that obtained from selling to the buyer whose types is distributed by \hat{f} over Θ , where $\hat{f}(\theta) = f(\theta) \forall \theta \notin [L, L + \varepsilon]$, $\hat{f}(\theta) = 0 \forall \theta \in [L, L + \varepsilon]$, and $\hat{f}(L + \varepsilon) = \int_{\theta=L}^{\theta=L+\varepsilon} f(\theta) d\theta$. Let (\hat{P}) represent the seller's problem when $\theta \sim \hat{f}$ and $V(\hat{P})$ the corresponding value. Consider the following relaxed problem of (\hat{P}) where all types mimic $L + \varepsilon$ off the equilibrium path:

$$\begin{aligned} (\mathcal{RP}_{L+\varepsilon}) \quad & \max_{(\pi, q, U)} \sum_{\theta \geq L+\varepsilon} \sum_x \sum_s p(\theta, \omega^{\pi_\theta}(\theta, s)) \pi_\theta(s|x) \mu(x) \hat{f}(\theta) \\ \text{s.t.} \quad & U(\theta) - U(L + \varepsilon) \geq \sum_s \int_{\omega^{\pi_L(L+\varepsilon, s)}}^{\omega^{\pi_L(\theta, s)}} q(L + \varepsilon, z) dz \pi_{L+\varepsilon}(s) \quad \forall \theta > L + \varepsilon \quad (IC_{\theta \rightarrow L+\varepsilon}) \\ & U(L + \varepsilon) \geq 0 \quad (IR_{L+\varepsilon}) \\ & q(\theta, \omega) \text{ increases in } \omega. \quad (\text{MON}) \end{aligned}$$

By the same arguments as the proof of Theorem 3, a posted price $\hat{p}_{L+\varepsilon}$, associated with a threshold disclosure $\hat{\pi}_{L+\varepsilon}$, solves this relaxed problem. Note that $(\hat{\pi}_{L+\varepsilon}, \hat{p}_{L+\varepsilon})$ does not necessarily solve the original problem. In case it does, the seller's revenue is the value of problem $(\mathcal{RP}_{L+\varepsilon})$, denoted by $V((\mathcal{RP}_{L+\varepsilon}))$. Let R_ε represent the seller's revenue if she offers $(\hat{\pi}_{L+\varepsilon}, \hat{p}_{L+\varepsilon})$ (regardless of whether it solves the original problem or not). Then,

$$\begin{aligned} R_\varepsilon &\geq V(\mathcal{RP}_{L+\varepsilon}) - \mathbb{E}[v(L+2\varepsilon, x)] \int_{L+\varepsilon}^{L+2\varepsilon} \hat{f}(\theta) d\theta \\ &\geq V(\hat{P}) - \mathbb{E}[v(L+2\varepsilon, x)] \int_{L+\varepsilon}^{L+2\varepsilon} \hat{f}(\theta) d\theta \end{aligned}$$

Therefore,

$$\lim_{\varepsilon \rightarrow 0} R_\varepsilon \geq V(\hat{P}) - \lim_{\varepsilon \rightarrow 0} \mathbb{E}[v(L+2\varepsilon, x)] \int_{L+\varepsilon}^{L+2\varepsilon} \hat{f}(\theta) d\theta = V(\hat{P})$$

On the other hand, $\lim_{\varepsilon \rightarrow 0} R_\varepsilon \leq V(\hat{P})$. Therefore, $\lim_{\varepsilon \rightarrow 0} R_\varepsilon = V(\hat{P})$.

D.3 Proof of Proposition 5

Let \mathbf{M} be an arbitrary optimal (deterministic) mechanism, which is, without loss of generality, a menu of trade probabilities and posted prices $\mathbf{M} = \{p(\theta), \alpha(x, \theta)\}_{x, \theta}$. Let $\tilde{\theta}_b \equiv \inf\{\theta \mid \alpha(\theta, x) = 1 \forall x\}$ represents the lowest type who receives an efficient allocation under \mathbf{M} , and $\tilde{\theta}_g \equiv \inf\{\theta \mid \alpha(\theta, x) > 0 \text{ for some } x\}$ be the lowest type being served. With $\tilde{\Theta} \equiv \{\theta \mid \theta \geq \tilde{\theta}_b\}$, \mathbf{M} must solve the following problem:

$$\begin{aligned} (\mathcal{P}) \quad &\sup_{p, \alpha} \int_{\tilde{\Theta}} p(\theta) dF(\theta) \\ &s.t. \quad \alpha(\theta, x) = 1 \quad \forall x, \theta \geq \tilde{\theta}_b \\ &\quad \sum_x [v(\theta, x) - p(\theta)] \alpha(\theta, x) \mu(x) \geq \sum_x [v(\theta, x) - p(\theta')] \alpha(\theta', x) \mu(x) \quad \forall \theta, \theta' \in \tilde{\Theta} \\ &\quad \sum_x [v(\theta, x) - p(\theta)] \alpha(\theta, x) \mu(x) \geq 0 \quad \forall \theta \in \tilde{\Theta}. \end{aligned}$$

By IR condition for $\tilde{\theta}_g$, $p(\tilde{\theta}_g) \leq v(\tilde{\theta}_g, g)$. Consider $\theta \in [\tilde{\theta}_b, \bar{\theta}]$. If $p(\theta) > v(\tilde{\theta}_g, g)$, then θ prefers to mimic $\tilde{\theta}_g$ and always buy the good at a lower price. Thus, to incentivize θ to reveal his type, it must be that

$$p(\theta) \leq v(\tilde{\theta}_g, g) \quad \forall \theta \in [\tilde{\theta}_b, \bar{\theta}].$$

Suppose $v(\tilde{\theta}_b, b) > v(\tilde{\theta}_g, g)$. Then, $\exists \hat{\theta}$ such that for any $\theta' \in [\hat{\theta}, \tilde{\theta}_b]$,

$$v(\theta', b) \geq v(\tilde{\theta}_g, g) \geq p(\tilde{\theta}_g).$$

It then induces θ' to mimic $\bar{\theta}$ and always buy the good. By doing so, he gets the good a higher expected surplus at a lower price. Therefore, it must be that

$$v(\tilde{\theta}_b, b) \leq v(\tilde{\theta}_g, g).$$

By Envelope condition, for the buyer to report truthfully his type, it is necessary that $U'(\theta) = \sum_x v_\theta(\theta, x)\alpha(\theta, x)\mu(x)$. Then, by integration by parts,

$$U(\theta) = U(\tilde{\theta}_g) + \int_{\tilde{\theta}_g}^{\theta} \sum_x v_\theta(\theta, x)\alpha(\theta, x)\mu(x)d\theta. \quad (18)$$

Consider a relaxed problem that keeps only the IR condition for $\tilde{\theta}_g$ and the necessary envelope condition for truth-telling. Using (18) and the fact that $U(\tilde{\theta}_g) = 0$ at optimum, this relaxed problem reduces to

$$\sup_q \int_{\theta} \phi^c(\theta, x)q(\theta, x)\mu(x)f(\theta) \quad s.t. \quad q(\theta, x) = 1 \quad \forall x, \forall \theta \geq \tilde{\theta}_b,$$

where $\phi^c(\theta, x) \equiv v(\theta, x) - v_\theta(\theta, x)\frac{1-F(\theta)}{f(\theta)}$. Solving this point-wise maximization problem yields

$$q(\theta, x) = \begin{cases} 1 & \text{if } \theta \geq \min\{\theta_b, \tilde{\theta}_b\} \\ \mathbb{1}_{x=g} & \text{if } \max\{\theta_g, \tilde{\theta}_g\} \leq \theta \leq \min\{\theta_b, \tilde{\theta}_b\}. \end{cases}$$

Prices are pinned down using binding constraints, given by

$$p(\theta) = \begin{cases} v(\min\{\theta_b, \tilde{\theta}_b\}, b)\mu(b) + v(\max\{\theta_g, \tilde{\theta}_g\}, g)\mu(g) & \text{if } \theta \geq \min\{\theta_b, \tilde{\theta}_b\} \\ v(\max\{\theta_g, \tilde{\theta}_g\}, g) & \text{if } \max\{\theta_g, \tilde{\theta}_g\} \leq \theta \leq \min\{\theta_b, \tilde{\theta}_b\} \end{cases}$$

Consider $\theta \geq \min\{\theta_b, \tilde{\theta}_b\}$ and $\theta' \in [\max\{\theta_g, \tilde{\theta}_g\}, \min\{\theta_b, \tilde{\theta}_b\}]$. As $v(\tilde{\theta}_b, b) \leq v(\tilde{\theta}_g, g)$, we have

$$v(\min\{\theta_b, \tilde{\theta}_b\}, b) \leq v(\max\{\theta_g, \tilde{\theta}_g\}, g),$$

which implies $p(\theta) \leq p(\theta')$. Thus, this two-option menu of prices and threshold disclosure induces participation and truth-telling. As \mathbf{M} solves the original problem, by definition of $\tilde{\theta}_b$ and $\tilde{\theta}_g$, it must be that

$$\min\{\theta_b, \tilde{\theta}_b\} = \tilde{\theta}_b, \quad \max\{\theta_g, \tilde{\theta}_g\} = \tilde{\theta}_g.$$

Suppose $v(\min\{\theta_b, \tilde{\theta}_b\}, b) < v(\max\{\theta_g, \tilde{\theta}_g\}, g)$, then $p(\theta) < p(\theta')$. Then, it is optimal to set $\hat{\theta} \equiv \inf\{\theta \geq \theta_g \mid v(\min\{\theta_b, \tilde{\theta}_b\}, b) \leq v(\max\{\hat{\theta}, \tilde{\theta}_g\}, g)\}$ as the lowest type being served, a contradiction. Therefore,

$$v(\min\{\theta_b, \tilde{\theta}_b\}, b) = v(\max\{\theta_g, \tilde{\theta}_g\}, g).$$

This implies that all types receive the same price. Moreover, all types $\theta \geq \min\{\theta_b, \tilde{\theta}_b\}$ always buy the good regardless of signal realization. Thus, it is optimal to offer full disclosure for all types.

References

- Bang, S. H. and Kim, J. (2013), 'Price discrimination via information provision', *Information Economics and Policy* **25**(4), 215–224.
- Battaglini, M. (2005), 'Long-term contracting with markovian consumers', *American Economic Review* **95**(3), 637–658.
- Battaglini, M. and Lamba, R. (2019), 'Optimal dynamic contracting: The first-order approach and beyond', *Theoretical Economics* **14**(4), 1435–1482.
- Bergemann, D., Heumann, T. and Morris, S. (2022), 'Screening with persuasion', *arXiv preprint arXiv:2212.03360*.
- Bergemann, D. and Pesendorfer, M. (2007), 'Information structures in optimal auctions', *Journal of economic theory* **137**(1), 580–609.
- Bergemann, D. and Wambach, A. (2015), 'Sequential information disclosure in auctions', *Journal of Economic Theory* **159**, 1074–1095.
- Courty, P. and Li, H. (2000), 'Sequential screening', *The Review of Economic Studies* **67**(4), 697–717.
- Eső, P. and Szentes, B. (2007), 'Optimal information disclosure in auctions and the handicap auction', *The Review of Economic Studies* **74**(3), 705–731.
- Guo, Y., Li, H. and Shi, X. (2022), Optimal discriminatory disclosure, Technical report, Working paper.
- Kamenica, E. and Gentzkow, M. (2011), 'Bayesian persuasion', *American Economic Review* **101**(6), 2590–2615.
- Kolotilin, A., Mylovanov, T., Zapechelnyuk, A. and Li, M. (2017), 'Persuasion of a privately informed receiver', *Econometrica* **85**(6), 1949–1964.
- Krähmer, D. (2020), 'Information disclosure and full surplus extraction in mechanism design', *Journal of Economic Theory* **187**, 105020.
- Krähmer, D. and Strausz, R. (2015a), 'Ex post information rents in sequential screening', *Games and Economic Behavior* **90**, 257–273.

- Krähmer, D. and Strausz, R. (2015*b*), 'Optimal sales contracts with withdrawal rights', *The Review of Economic Studies* **82**(2), 762–790.
- Lewis, T. R. and Sappington, D. E. (1994), 'Supplying information to facilitate price discrimination', *International Economic Review* pp. 309–327.
- Li, H. and Shi, X. (2017), 'Discriminatory information disclosure', *American Economic Review* **107**(11), 3363–3385.
- Myerson, R. B. (1981), 'Optimal auction design', *Mathematics of operations research* **6**(1), 58–73.
- Myerson, R. B. (1986), 'Multistage games with communication', *Econometrica: Journal of the Econometric Society* pp. 323–358.
- Pavan, A., Segal, I. and Toikka, J. (2014), 'Dynamic mechanism design: A myersonian approach', *Econometrica* **82**(2), 601–653.
- Skreta, V. (2006), 'Mechanism design for arbitrary type spaces', *Economics Letters* **91**(2), 293–299.
- Smolin, A. (2023), 'Disclosure and pricing of attributes', *The Rand Journal of Economics* .
- Wei, D. and Green, B. (2023), '(reverse) price discrimination with information design', *American Economic Journal: Microeconomics* .
- Zhu, S. (2023), 'Private disclosure with multiple agents', *Journal of Economic Theory* **212**, 105705.