Numerical Simulation of Economic Depression

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Abstract

In this paper, I numerically simulate the path of economy in an economic depression. It is not easy to perform a numerical simulation of the path to a steady state if households are assumed to behave by generating rational expectations. It is much easier, however, if households are assumed to behave according to a procedure based on the maximum degree of comfortability (MDC), where MDC indicates the state at which a household feels most comfortable with its combination of income and assets. The results of simulations under the supposition of this alternative procedure indicate that, if households do not strategically consider other households' behaviors, consumption jumps upwards immediately after the shock. However, if households strategically select a Pareto inefficient path, large amounts of unutilized economic resources are generated, and the unemployment rate can rise to 30% or higher. These results seem to well match actual historical experiences during severe recessions such as the Great Depression and Great Recession.

JEL Classification: E10, E17, E27, E32, E37

Keywords: Economic depression; Shock; Simulation; Recession; Unemployment; Unutilized economic resources

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1 INTRODUCTION

In this paper, I numerically simulate the path of economy in an economic depression (or severe recession), such as the Great Depression in the 1930s and the Great Recession that occurred around 2008. As a basis for the simulation, I use the completely new method developed in Harashima (2022c, 2023a, 2023b, 2024) to numerically simulate paths to reach steady state. Performing a numerical simulation of the path to a steady state in dynamic economic growth models in which households behave by generating their own rational expectations is a difficult task because there is no closed form solution in these models. However, Harashima (2022c) presented a novel method to perform this simulation by using the concept of maximum degree of comfortability (MDC), where MDC indicates the state at which a household feels most comfortable with its combination of income and assets.

Usually, it is assumed that households behave by generating their own rational expectations to reach a steady state, but Harashima (2018a[1]) showed an alternative procedure for households to reach a steady state. In this procedure, households maintain their capital-wage ratio (CWR) at MDC, and their behavior under the MDC-based procedure is equivalent to that of households who base their behavior on rational expectations. That is, it is equivalent to the behavior under a procedure based on the rate of time preference (RTP) (Harashima 2018a, 2021, 2022a[2]). However, unlike the case of the RTP-based procedure, the path to a steady state can be easily simulated if households behave according to the MDC-based procedure because households are not required to do anything equivalent to computing a complex model.

Indeed, a numerical simulation of the path to a steady state under the MDC-based procedure shows that households can reach a steady state without generating any rational expectations (Harashima 2022c), which conforms with theoretical predictions (Harashima 2010[3], 2012a[4], 2014a), and that a government can achieve a steady state through appropriate intervention, although heterogeneous households cannot necessarily reach their intrinsic CWRs at MDC (a state known as approximate "sustainable heterogeneity" (SH)). Using the same method, Harashima (2023a) simulated the effect of economic rents obtained heterogeneously among households, Harashima (2023b) numerically examined the mechanism underlying why economic inequality can increase in democratic countries, and Harashima (2024) numerically simulated endogenously growing economies and their balanced growth path. In these simulations, a household

[1] Harashima (2018a) is also available in Japanese as Harashima (2019a).
[2] Harashima (2022a) is also available in Japanese as Harashima (2022b).
[3] Harashima (2010) is also available in Japanese as Harashima (2017b).
[4] Harashima (2012a) is also available in Japanese as Harashima (2020b).

was assumed to increase or decrease its consumption according to simple formulae that are assumed to capture and represent a household's behavior under the MDC-based procedure.

However, in these simulations, economic fluctuations were not simulated because it was assumed that there is no shock that generates fluctuations. The purpose of this paper is to numerically simulate and examine what will happen if such a shock does occur, particularly a shock that changes a deep parameter and thereby generates an economic depression (severe recession), by extending the simulation method employed in Harashima (2022c, 2023a, 2023b, 2024).

Although the cause of economic depressions (severe recessions) has long been studied from various points of view (e.g., Temin, 1989; Hall, 2011; Eggertsson and Krugman, 2012; Mian and Sufi, 2012; Christiano et al., 2015; Martin et al., 2015; Guerrieri and Lorenzoni, 2017), no consensus about the cause has yet been reached. However, Harashima (2016a) showed a cause of the Great Recession that was based on the concept of a "Nash equilibrium of a Pareto inefficient path" (NEPIP). This concept is also discussed in other papers by Harashima (2004b, 2009[5], 2012b[6], 2016a, 2016b[7], 2017a, 2018b, 2019b) and can be used to explain a mechanism for why a Pareto inefficient path is rationally chosen by households. Moreover, if such a Pareto inefficient path is rationally chosen, phenomena like the Great Recession and Great Depression can be generated.

In this paper, I simulate the path after a MDC (or equivalently an RTP) shock occurs on the basis of the concept of NEPIP, where the shock occurs and many households suddenly and simultaneously change the CWR at MDC of the entire economy (or the representative household's RTP) that each household subjectively guesses (or expects). Because the MDC- and RTP-based procedures are equivalent, MDC and RTP shocks are also equivalent. In particular, I simulate the impacts of an upward MDC (RTP) shock (i.e., a sudden increase in CWR at MDC (RTP)) because this shock's impacts are expected to well mimic the key phenomena that economic depressions will generate.

The results of the simulations indicate that if households do not select a NEPIP (i.e., they do not strategically consider other households' behaviors after the shock), consumption increases immediately, discontinuously, and largely just after the shock, as predicted theoretically (Harashima, 2004b, 2009, 2012b, 2016a, 2016b, 2017a, 2018b, 2019b). In addition, capital decreases steadily from the level at the prior steady state to a lower level at the posterior steady state after the shock.

On the other hand, if households strategically select a NEPIP, consumption begins to decrease just after the shock and approaches the level at the posterior steady

[5] Harashima (2009) is also available in Japanese as Harashima (2018b).
[6] Harashima (2012b) is also available in Japanese as Harashima (2019c).
[7] Harashima (2016b) is also available in Japanese as Harashima (2020c).

state. Because consumption does not jump upward, a large amount of unutilized economic resources is inevitably generated in each period, also as predicted theoretically by Harashima (2004b, 2009, 2012b, 2016a, 2016b, 2017a, 2018b, 2019b). Because of the unutilized resources, the unemployment rate could rise to roughly 30% or higher. The total accumulated amount of unutilized resources during a depression can almost match the amount of capital at the posterior steady state. These results of the simulation, particularly the one in which a large shock is given, seem to be consistent with the economic conditions actually observed during historical depressions (severe recessions) such as the Great Depression and Great Recession.

2 MDC (RTP) SHOCK

In this section, I explain MDC (RTP) shock on the basis of the concept of NEPIP presented in Harashima (2004b, 2009, 2012b, 2016a, 2016b, 2017a, 2018b, 2019b), the MDC-based procedure developed in Harashima (2018a, 2021, 2022a), and the sustainable heterogeneity (SH) concepts presented in Harashima (2010, 2012a, 2014a). These concepts are explained in more detail in Appendixes 1, 2, and 3 and the references cited above.

2.1 Fundamental shock

An economic depression (severe recession) is highly likely to be caused by a shock that largely changes the steady state because an economy will otherwise quickly come back to the previous steady state and be stable again. This means that such a shock should be a sudden change in a deep parameter that has the potential to change the steady state to a large degree. However, such deep parameters are very limited. Technology is one such parameter, but it will not be the cause of depression because it will not change greatly in a short period, and furthermore, it will never regress greatly and suddenly across an entire economy. An important remaining deep parameter is RTP under the RTP-based procedure (equivalently, CWR at MDC under the MDC-based procedure). If the expected representative household's RTP (equivalently, the guess of the entire economy's CWR at MDC) suddenly shifts upwards, a depression can occur because consumption at the new steady state is lower than that at the posterior steady state. Furthermore, this shock can lead to households' selection of a NEPIP (Harashima, 2004b, 2009, 2012b, 2016a, 2016b, 2017a, 2018b, 2019b).

However, although RTP (or CWR at MDC) is a deep parameter, RTP has not been regarded as a source of shocks for economic fluctuations, possibly because RTP is thought to be constant and not to shift suddenly. There is also a practical reason, however. Models with a permanently constant RTP exhibit excellent tractability (see Samuelson, 1937). However, RTP has been naturally assumed and actually observed to be time-variable. The concept of a time-varying RTP has a long history (e.g., Böhm-Bawerk, 1889; Fisher, 1930). Parkin (1988) showed that RTP is as volatile as technology and leisure preference. Lawrance (1991) and Becker and Mulligan (1997) showed that people do not inherit permanently constant RTPs by nature and that economic and social factors affect the formation of time preference rates.

Harashima (2004a, 2009) presents a model of RTP under the RTP-based procedure, in which RTP can change largely and suddenly if surrounding economic situations change. This model predicts that a large and sudden upwards shift of RTP can be generated, for example, when uncertainty about economic conditions increases greatly. Furthermore, Harashima (2018a, 2021, 2022a) showed that a MDC shock can also occur under the MDC-based procedure, and it is equivalent to an RTP shock under the RTP-based procedure.

2.2 MDC (RTP) shock

Harashima (2018a, 2021, 2022a) showed that, in a heterogeneous population, all households are linked at SH in the sense that a household's behavior must be set so as to be consistent with the behaviors of the other households. Particularly, under the MDC-based procedure, households are linked via "the entire economy's CWR at MDC at approximate SH ($\tilde{S}_{MDC,SH,ap}$)."

On the other hand, Harashima (2018a, 2021, 2022a) showed that $\tilde{S}_{MDC,SH,ap}$ crucially depends on the guessed (estimated) values of a few variables, in particular, the entire economy's CWR at MDC ($\Gamma(\tilde{S}_{MDC,SH})$), the amount of net government transfers (T), and the adjusted CWR (Γ_R). Because these values are generally guessed with incomplete information, $\tilde{S}_{MDC,SH,ap}$ is vulnerable to various shocks and can occasionally fluctuate widely. Vulnerabilities will emerge because of various factors, including households' limited access to information, the difficulty of distinguishing between permanent and temporary incomes, and misconceptions about differences in the natures of capital and wealth.

Because of these vulnerabilities, households will occasionally revise their guessed values of $\Gamma(\tilde{S}_{MDC,SH})$, T, and Γ_R when new pieces of information arrive or some kinds of shocks are recognized. In some cases, the value of $\Gamma(\tilde{S}_{MDC,SH})$ guessed by many households may be simultaneously revised. This revision generates a shock in MDC. Harashima (2015, 2022d[8], 2023d) showed that disinformation disseminated by malicious people through large-scale financial speculations in financial markets can greatly influence people's guessed values of $\Gamma(\tilde{S}_{MDC,SH})$ and subsequently generate a

[8] Harashima (2022d) is also available in Japanese as Harashima (2023c).

large MDC (RTP) shock.

2.3 Nash equilibrium of a Pareto inefficient path (NEPIP)

In this section, I briefly explain the essence of NEPIP following Harashima (2004b, 2009, 2012b, 2016a, 2016b, 2017a, 2018b, 2019b). The concept of NEPIP is discussed in more detail in Appendix 1.

2.3.1 Strategic selection of NEPIP

The presence of a persistently large amount of unutilized resources during an economic depression (severe recession) implies that the economy is not persistently Pareto efficient. Pareto inefficiency usually may not be left as it is for a long period, but a Nash equilibrium can conceptually coexist with Pareto inefficiency. If a Nash equilibrium that consists of strategies generating Pareto inefficient payoffs is selected, unutilized resources as large and persistent as those observed in a depression may exist. Harashima (2004b, 2009, 2012b, 2016a, 2016b, 2017a, 2018b, 2019b) showed that a depression at such a Nash equilibrium—that is, a Nash equilibrium consisting of strategies of choosing a Pareto inefficient transition path of consumption to the steady state (i.e., NEPIP)—is generated even in a frictionless economy if—and probably only if—RTP shifts upwards. In addition, Harashima (2020a[9]) showed that the NEPIP phenomenon after an upward MDC shock under the MDC-based procedure is equivalent to that after an upward RTP shock under the RTP-based procedure.

An essential reason for the generation of NEPIP is that households are intrinsically risk averse and not cooperative. In a strategic environment, this generates the possibility that, if consumption needs to be substantially and discontinuously increased to maintain Pareto efficiency, a non-cooperative household's strategy to deviate from the Pareto efficient path gives a higher expected utility than the strategy of choosing the Pareto efficient path.

Suppose that an upward shift of the guessed entire economy's CWR at MDC (or the expected representative household's RTP) occurs. All households will be knocked off the Pareto efficient path on which they have proceeded until the shift occurred. At that moment, each household must decide in which direction to proceed. Because they are no longer on a Pareto efficient path, each household chooses a path strategically considering the other households' choices. This situation can be described by a non-cooperative mixed-strategy game, and there is a NEPIP in this game.

2.3.2 The shape of NEPIP

[9] Harashima (2020a) is also available in Japanese as Harashima (2023e).

Because NEPIP is not Pareto efficient—that is, because the constraint that Pareto efficiency should be kept does not exist—an infinite number of transition paths can be NEPIPs. However, it is highly likely that a NEPIP will not be a complex winding path but rather a simple monotonously decreasing path from the prior steady state to the posterior one because risk-averse households dislike discontinuous changes in consumption and prefer to smooth it (Harashima, 2019b).

In addition, on NEPIP, a household does not care about "efficiency" (e.g., it accepts Pareto inefficiency and generation of unutilized resources) because it strategically and intentionally selects a NEPIP even though it knows it is not an efficient path. As a result, a household will single-mindedly behave with the aim of reaching the posterior steady state and move on a very simple and monotonously decreasing path.

3 SIMULATION METHOD

The method of simulations is basically the same as that employed in Harashima (2022c, 2023a, 2023b, 2024), which is explained in Appendix 4, but it is extended to simulate NEPIP.

3.1 Basic simulation assumptions

No technological progress and capital depreciation are assumed, and all values are expressed in real and per capita terms. It is assumed that there are H economies in a country, the number of households in each of economy is identical, and households within each economy are identical. The production function of Economy i ($1 \leq i \leq H$) is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \ , \tag{1}$$

where $y_{i,t}$ and $k_{i,t}$ are the production and capital of a household in Economy i in period t, respectively; ω_i is the productivity of a household in Economy i; A_t is technology in period t; and α ($0 < \alpha < 1$) is a constant and indicates the labor share. All variables are expressed in per capita terms. In simulations, I set $\alpha = 0.65$, $A_t = 1$, and $\omega_i = 1$ for any t and i. The initial capital a household owns is set at 1 for any household.

By equation (1), the production of a household in Economy i in period t ($y_{i,t}$) is calculated, for any i, by

$$y_{i,t} = k_{i,t}^{1-\alpha} \ .$$

The amount of capital used (not owned) by each household (i.e., $k_{i,t}$) is kept identical among households although the amount of capital owned (not used) by each household can be heterogeneous. For any *i*,

$$k_{i,t} = \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H} ,$$

where $\breve{k}_{i,t}$ is the amount of capital a household in Economy *i* owns (not uses).

The capital income of a household in Economy *i* in period *t* ($x_{K,t}$) is calculated by

$$x_{K,i,t} = r_t \breve{k}_{i,t} ,$$

where r_t is the real interest rate in period *t* and

$$r_t = \frac{\partial k_{i,t}}{\partial y_{i,t}} .$$

The labor income of a household in Economy *i* in period *t* ($x_{L,i,t}$) is calculated by extracting its capital income from its production such that

$$x_{L,i,t} = y_{i,t} - r_t k_{i,t} = y_{i,t} - r_t \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H} .$$

Household savings in Economy *i* in period *t* ($s_{i,t}$) are calculated by

$$s_{i,t} = x_{L,i,t} + x_{K,i,t} - c_{i,t} ,$$

where $c_{i,t}$ is the consumption of a household in Economy *i* in period *t*. In period *t* + 1, these savings ($s_{i,t}$) are added to the capital the household owns, and therefore,

$$\breve{k}_{i,t+1} = \breve{k}_{i,t} + s_{i,t} .$$

The following simple consumption formula is used.

Consumption formula 1: The consumption of a household in Economy *i* in period *t* is

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{i,t}}\right)^{\gamma} ,$$

and equivalently

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{\Gamma_{i,t}\dfrac{1-\alpha}{\alpha}}\right)^{\gamma} \quad ,$$

where $\Gamma_{i,t}$ is the capital-wage ratio (CWR) of a household in Economy *i* in period *t*, $\Gamma(\tilde{s}_i)$ is $\Gamma_{i,t}$ of a household in Economy *i* in period *t* when the household is at its MDC, and γ is a parameter. In this paper, I set the value of γ to be 0.5. It is assumed that the intrinsic $\Gamma(\tilde{s}_i)$ (i.e., CWR at MDC) of a household is identical across households and economies, and I set this common $\Gamma(\tilde{s}_i)$ to be $0.04 \times 0.65/(1 - 0.65) = 0.0743$, which corresponds to an RTP of 0.04.

In a heterogeneous population, Consumption formula 1 should be modified to Consumption formula 2. Let $\Gamma_{R,i,t}$ be the adjusted value of $\Gamma_{i,t}$ of a household in Economy *i* in period *t* in a heterogeneous population, and $\Gamma(S_t)$ be the CWR of the country (i.e., the aggregate CWR).

Consumption formula 2: In a heterogeneous population, the consumption of a household in Economy *i* in period *t* is

$$\begin{aligned} c_{i,t} &= \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{R,i,t}}\right)^{\gamma} \\ &= \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{r_t \dfrac{\alpha}{1-\alpha}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)\dfrac{1-\alpha}{\alpha}}{r_t}\right)^{\gamma} \quad , \end{aligned}$$

and equivalently,

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{r_t}\right)^{\gamma} \quad .$$

Let κ_i be the $\check{k}_{i,t}$ that a government aims for in order to induce a household in Economy *i* to own capital at a steady state (i.e., κ_i is the target value set by the government). Under these conditions, the bang-bang (two-step) control rule of government transfers is set as follows.

Transfer rule: The amount of government transfers from a household in Economy *i* to a household in Economy $i + 1$ in period t is T_{low} if $\check{k}_{i,t}$ is lower than κ_i, and T_{high} if $\check{k}_{i,t}$ is higher than κ_i, where T_{low} and T_{high} are constant amounts of capital predetermined by the government, and if $i = H$, $i + 1$ is replaced with 1.

In the simulations, T_{low} is set to be -0.1 and T_{high} to be 0.5. The value of κ_i is varied in each simulation depending on what steady state the government aims to achieve.

3.2 Extension to simulate NEPIP

3.2.1 MDC (RTP) shock

For a household, an upward MDC (RTP) shock means that it suddenly begins to feel that the adjusted CWR that it has guessed is not actually the correct one, and therefore, it has to be corrected immediately by readjusting its value. Accordingly, the estimated adjusted CWR is lowered from the real interest rate to the real interest rate $- \xi_1$ (i.e., from r_t to $r_t - \xi_1$) in Consumption formula 2 in the case of a heterogeneous population, where $\xi_1 > 0$ is a constant that indicates the magnitude of an upward MDC (RTP) shock.

In Consumption formulae 1 and 2, therefore, an upward MDC (RTP) shock with magnitude ξ_1 can be expressed as a sudden change in Consumption formulae 1 and 2 to

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{i,t} - \xi_1}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{\Gamma_{i,t}\dfrac{1-\alpha}{\alpha} - \xi_1}\right)^{\gamma} \qquad (2)$$

and

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)\dfrac{1-\alpha}{\alpha}}{r_t - \xi_1}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{r_t - \xi_1}\right)^{\gamma}, \qquad (3)$$

respectively. Equations (2) and (3) can be transformed to

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i) + \xi_2}{\Gamma_{i,t}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i + \xi_2}{\Gamma_{i,t}\dfrac{1-\alpha}{\alpha}}\right)^{\gamma} \qquad (4)$$

and

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)\dfrac{1-\alpha}{\alpha} + \xi_2}{r_t}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i + \xi_2}{r_t}\right)^{\gamma}, \qquad (5)$$

respectively, where $\xi_2 > 0$ is a constant and another indicator of the magnitude of the upward MDC (RTP) shock, and

$$\xi_2 = \frac{\theta_i}{\frac{r_{t-1}}{\xi_1} - 1}$$

if the shock occurred in period *t*. That is, an upward MDC (RTP) shock can be represented by a shock that increases a household's CRW at MDC (RTP). In simulations, therefore, an upward MDC (RTP) shock is represented by a sudden change in Consumption formula 1 or 2, or to equation (4) or (5), respectively.

3.2.2 NEPIP

As discussed in Section 2.3, a NEPIP is most likely not a complex winding path but rather a simple monotonously decreasing path from the prior steady state to the posterior one. That is, consumption on NEPIP will decrease steadily and continuously from the level at the prior steady state to that at the posterior steady state. However, there can be an infinite number of simply and monotonously decreasing paths.

Nevertheless, considering the importance of "simple" and "monotonous" in the nature of NEPIP, I assume one of the most simple monotonously decreasing paths as the path of consumption on NEPIP in simulations such that

$$c_{i,t} = \left(\overline{c_i} - \underline{c_i}\right)\exp(-\eta t) + \underline{c_i} \ ,$$

where $\overline{c_i}$ is consumption at the prior steady state, $\underline{c_i}$ is that at the posterior steady state, and $\eta(> 0)$ is a parameter.

The speed of decrease in consumption on NEPIP differs depending on the value of parameter η. As η is larger, consumption decreases more rapidly. The value of η will probably differ depending on the preferences of households and may temporally change. Harashima (2019b) showed a mechanism explaining how the shape of the consumption path on NEPIP is determined depending on the preferences of households.

4 RESULTS OF SIMULATIONS

4.1 Setup

For simplicity, a homogeneous population is assumed; thus, all households are identical.

Note, however, that the result is basically the same even if a heterogeneous population is assumed because all households are linked at the approximate SH in a heterogeneous population, as shown in Section 2.2.

To clearly understand and evaluate the results of shock, it would be easiest if the economy were at steady state before a shock occurs. Hence, I first simulate a growing economy by simply applying the simulation method shown in Section 3.1, and then I assume that this simulated economy reaches a steady state before period 300 (see Harashima, 2022c, 2023a, 2023b, 2024). I then reset period 300 to be the new period 0, and the shock is assumed to occur in this new period 0. As discussed in Section 3.2.1, a MDC (RTP) shock is represented by a sudden increase in the guessed CWR of the entire economy at MDC or the expected representative household's RTP (i.e., $\Gamma(\tilde{s}_i)$ or θ_i) in Consumption formulae 1 or 2 in the new period 0.

4.2 Path without strategic consideration

I first simulate the path after an upward MDC (RTP) shock when households do not chose a NEPIP; that is, each of them does not behave strategically considering other households' behaviors (hereafter, I call this the "Jump path").

4.2.1 Base case

A 1 percentage point upward RTP shock (equivalently, a 1.857 percentage point upward MDC shock) is assumed to occur in period 0. The expected representative household's RTP before the shock is set to be 0.035 (equivalently, the guessed entire economy's CWR at MDC is $0.035 \times 0.65/(1 - 0.65) = 0.065$), and in period 0, it is reset to 0.045 (equivalently, the guessed entire economy's CWR at MDC is reset to $0.045 \times 0.65/(1 - 0.65) = 0.0836$). The simulated paths of capital and consumption are shown in Figures 1 and 2.

Figure 1 indicates that capital begins to decrease just after the shock, and then gradually approaches the posterior steady state level. This movement is consistent with the prediction of Ramsey-type growth models. On the contrary, Figure 2 indicates that consumption jumps upwards immediately, discontinuously, and largely just after the shock, but in the next period, it changes direction and begins to decrease gradually to the level at the posterior steady state, which is lower than the level at the prior steady state, similar to capital. This movement of consumption (i.e., a momentary upward jump followed by gradual decreases) is also consistent with the prediction of Ramsey-type growth models.

The reason why households' consumption jumps upwards is that, because $\frac{\theta_i}{\Gamma_{i,t}\frac{1-\alpha}{\alpha}}$ in Consumption formula 1 is suddenly increased to $\frac{\theta_i + \xi_2}{\Gamma_{i,t}\frac{1-\alpha}{\alpha}}$, a large and discontinuous consumption adjustment needs to be abruptly implemented. As shown above, this jump is superficially consistent with the prediction of Ramsey-type growth models, but in Ramsey-type growth models, households' consumption jumps upwards only because they maintain Pareto efficiency. That is, the mechanism that makes households' consumption jump is different in the simulation in this paper and Ramsey-type growth models. It is unknown whether Pareto efficiency is maintained with the jump in the simulation, but the jump can be interpreted to be economically "efficient" in the sense that no unutilized resource is generated. According to the simulation method shown in Section 3, capital decreases in accordance with the increase in consumption on the Jump path; therefore, no unutilized resources are generated.

Note that in the simulations, households are assumed to "consume" existing "capital" to decrease capital on the Jump path, and no capital depreciation is assumed. However, households cannot literally consume capital (e.g., heavy machines in factories or facilities). On the other hand, capital depreciation always occurs naturally, and consequently investments in capital are undertaken even at the steady state to make up for capital depreciation. This means that a consumption jump to decrease capital on the Jump path will in actuality be implemented through increases in production of consumer goods and services and decreases in production for investments in capital to make up for capital depreciation.

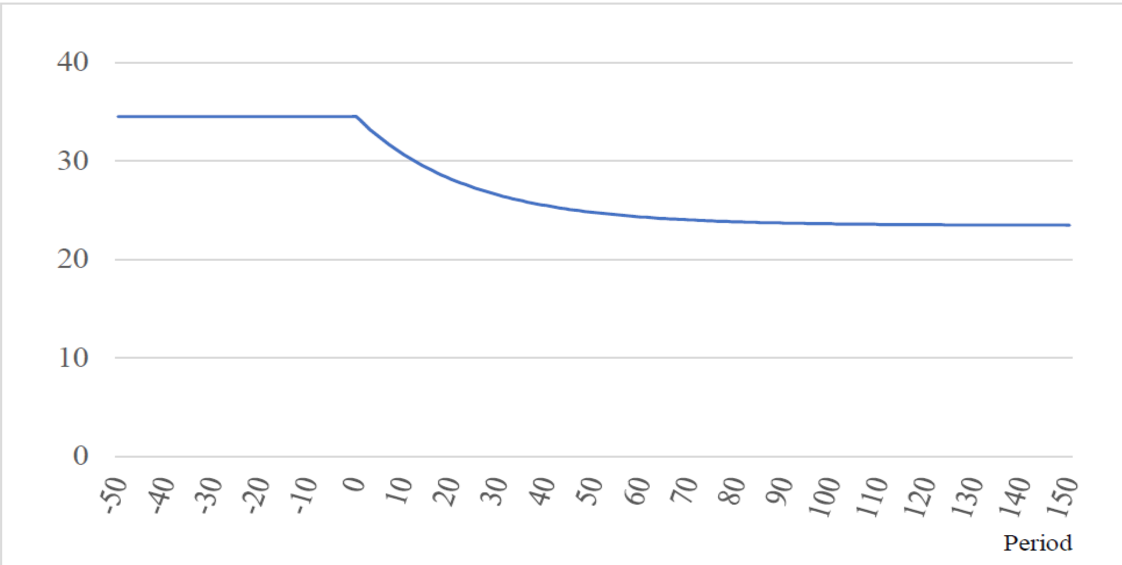

Figure 1: Simulation of capital owned by each household ($\check{k}_{i,t}$) in the base case of the Jump path.

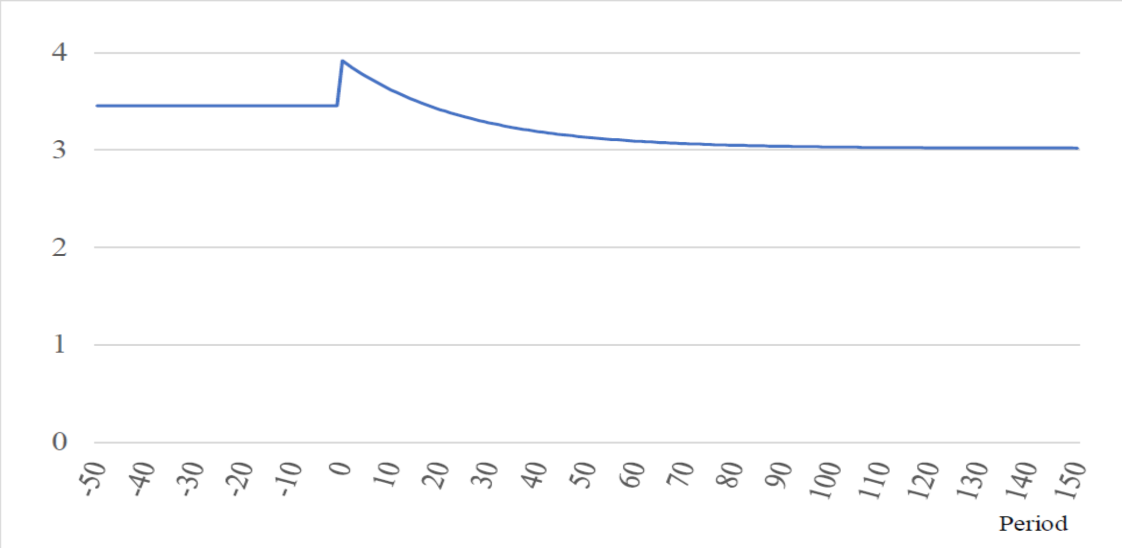

Figure 2: Simulation of consumption ($c_{i,t}$) in the base case of the Jump path.

4.2.2 Effect of magnitude of shock

I further examine whether the consumption jump differs if the magnitude of shock differs. To do so, I examine three different magnitudes of RTP shock: (1) a 0.5 percentage point upwards shift (the expected representative household' s RTP shifts from 0.0375 to 0.0425), (2) a 1 percentage point upwards shift (from 0.035 to 0.045), and (3) a 2 percentage point upwards shift (from 0.03 to 0.05). The other parameter values are the same as those in the base case in Section 4.2.1. The simulated paths of capital and consumption for the three magnitudes are shown in Figures 3 and 4, respectively.

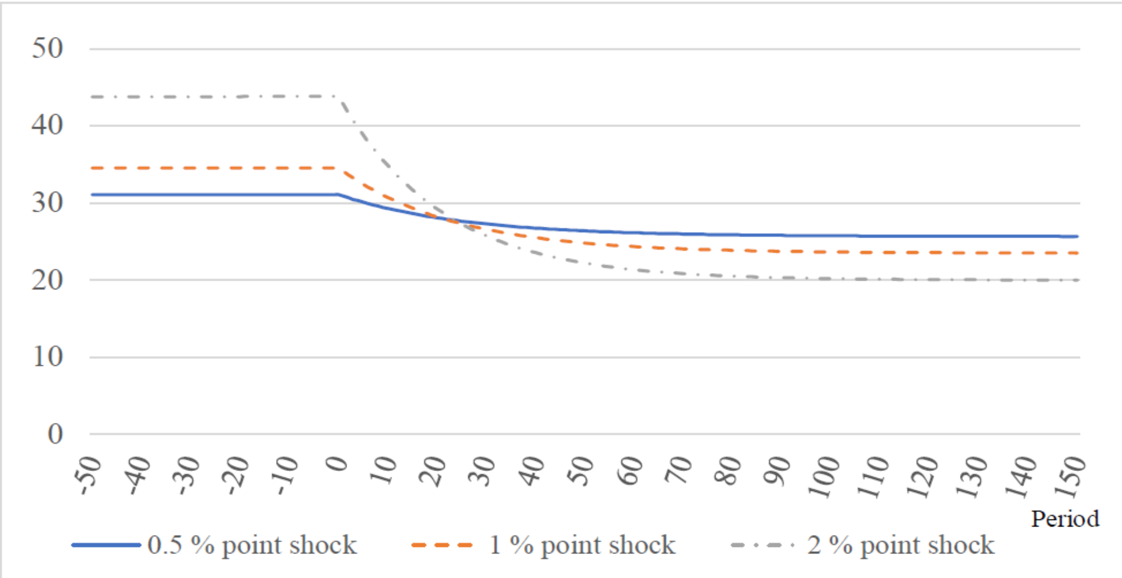

Figure 3: Simulation of the effect of magnitude of shock in the base case of the Jump path for capital owned by each household ($\check{k}_{i,t}$).

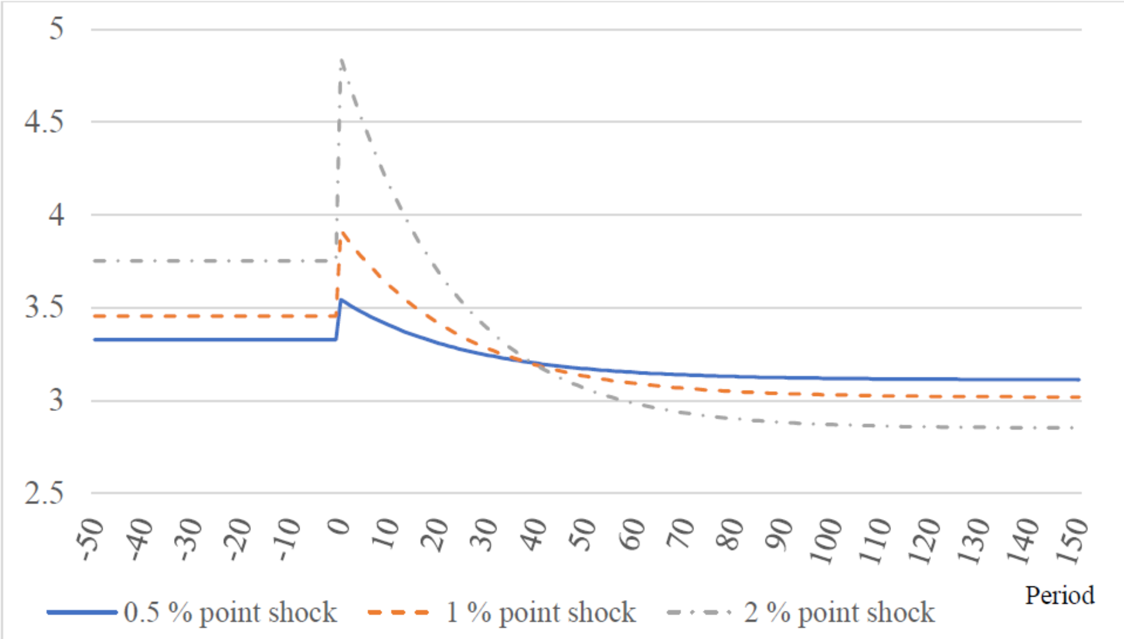

Figure 4: Simulation of the effect of magnitude of shock in the base case of the Jump path for consumption ($c_{i,t}$).

Figures 3 and 4 indicate that, as the magnitude of shock increases from 0.5 to 2 percentage points, the magnitudes of the decreases in capital and consumption jumps are larger. In case (3), consumption increases from 3.75 to 4.85 immediately after the shift. Such an immediate, discontinuous, and large increase in consumption seems to be unimaginable in reality, which strongly suggests that in a strategic situation after the shock, many risk averse and non-cooperative households will not choose the Jump path and will instead choose a NEPIP as Harashima (2004b, 2009, 2012b , 2016a, 2016b, 2017a, 2018b, 2019b) theoretically concluded, particularly if the magnitude of shock is large.

4.3 NEPIP

I next simulate the case that all households choose a NEPIP as a result of strategic considerations after the shock.

4.3.1 Base case

Similar to the base case of the Jump path in Section 4.2.1, I set η = 0.1. In addition, the values of $\overline{c}$ and $\underline{c}$ are set to be "the consumption in period –1" and "that in period 150" on the Jump path, respectively (i.e., $\overline{c}$ =3.45 and $\underline{c}$ = 3.02). The other parameter values are the same as those in the base case of the Jump path in Section 4.2.1; thus, the given shock is a 1 percentage point RTP upwards shift (from 0.035 to 0.045) in period 0. The results of simulations are shown in Figures 5, 6, and 7.

4.3.1.1 Capital

The solid line in Figure 5 is the simulated path of capital. It indicates that capital begins to decrease just after the shock and gradually approaches the level at the posterior steady state. This path of capital decrease is completely the same as that in the base case of the Jump path for a 1 percentage point shock. This outcome was expected because, in simulations, the difference between the Jump path and NEPIP does not lie in decreases in capital but in how these decreases are dealt with (i.e., whether they are consumed or left unutilized).

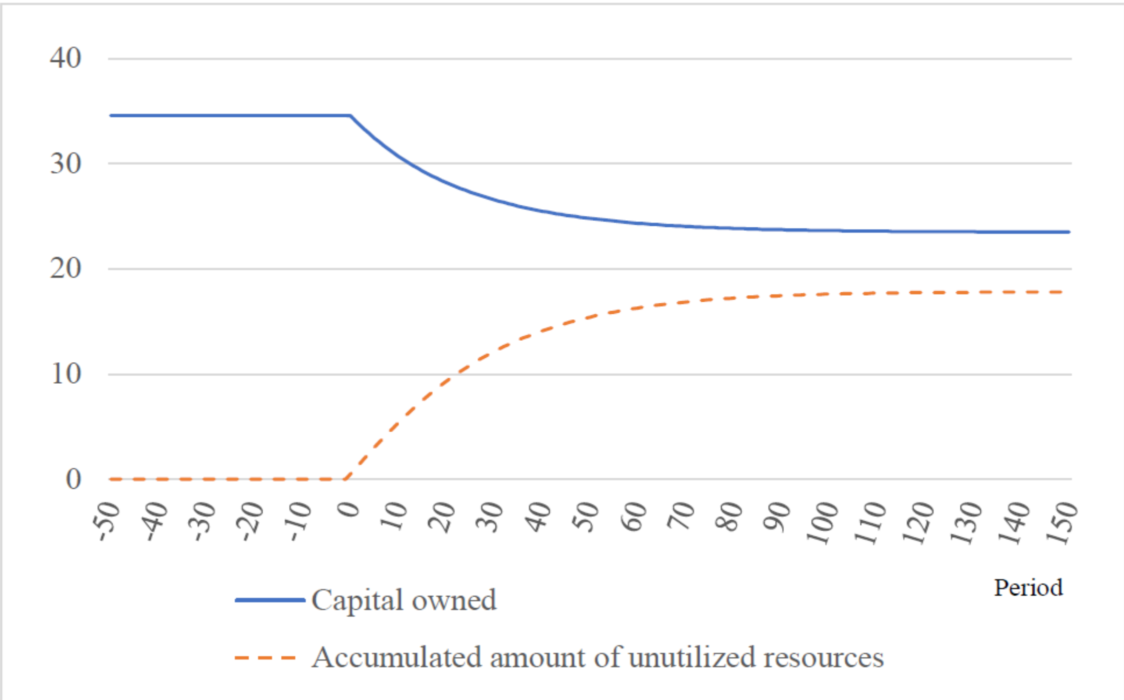

Figure 5: Simulation of capital owned by each household ($\check{k}_{i,t}$) and the accumulated amount of unutilized resources in the base case of NEPIP.

4.3.1.2 Consumption

The simulated path of consumption is shown by the solid line in Figure 6; consumption on the Jump path is also depicted by the dotted line for comparison. Because a NEPIP is strategically chosen, consumption does not jump but immediately begins to decrease just after the shock towards the level at the posterior steady state.

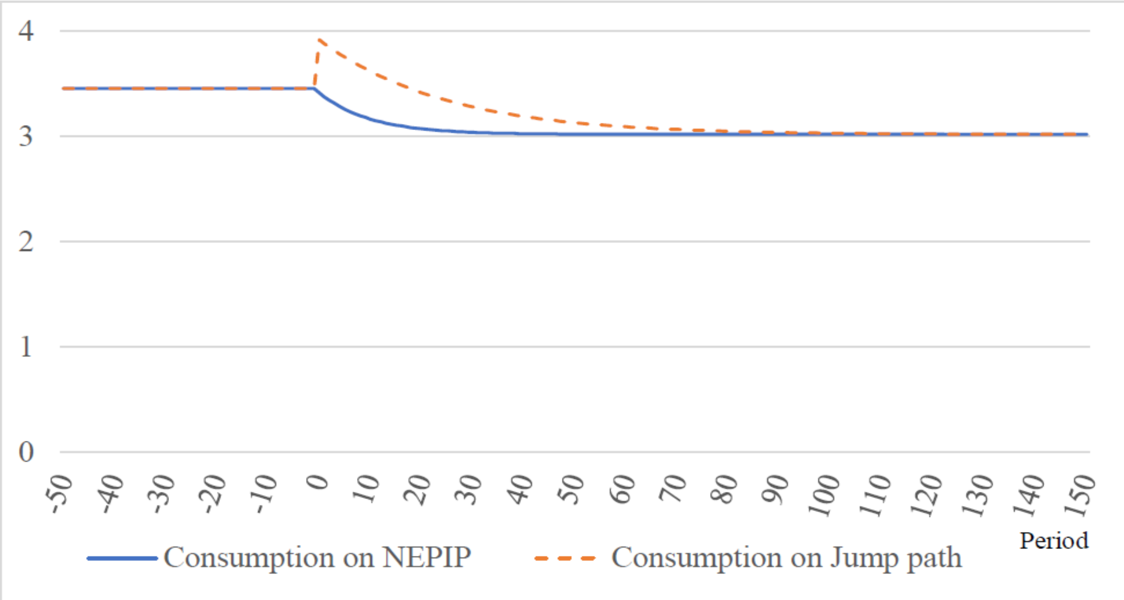

Figure 6: Simulation of consumption ($c_{i,t}$) in the base case of NEPIP and that of the Jump path.

4.3.1.3 Unutilized resources and the unemployment rate

The difference between the dotted line (the Jump path) and the solid line (NEPIP) in Figure 5 indicates the amount of unutilized resources generated in every period. They are unutilized partly because the decreased capital is not consumed on NEPIP, unlike in the case of the Jump path. Figure 7 shows the ratio of "unutilized resources generated on NEPIP" to "consumption on the Jump path" in every period. It indicates that the ratio increases immediately and largely just after the shock, but after several periods, it begins to decrease gradually, eventually approaching zero.

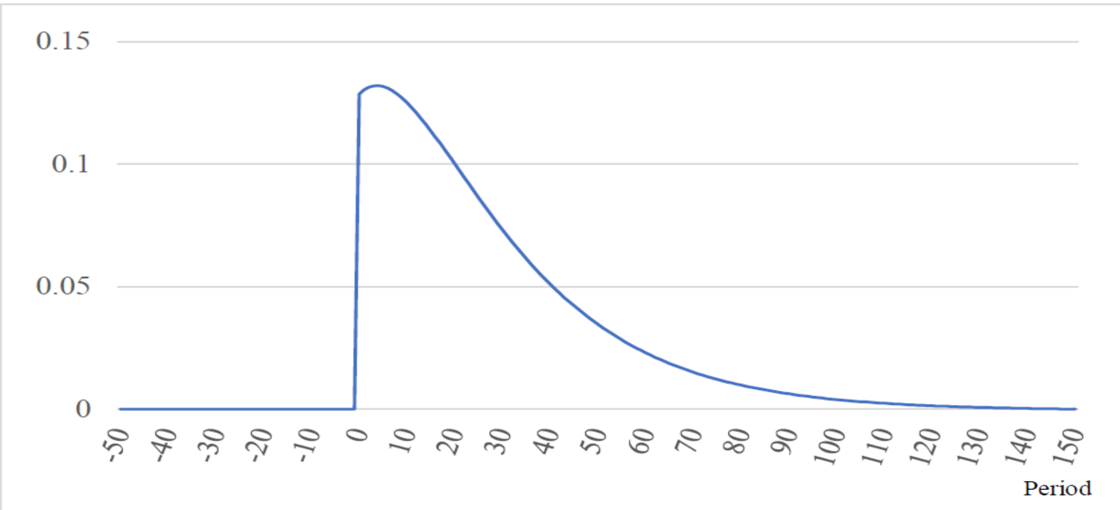

Figure 7: Simulation of unutilized resources (roughly equivalent to increases in the unemployment rate) in the base case of NEPIP.

Generated unutilized resources will be eliminated in every period in various ways, for example, by destroying capital, discarding unsold goods, or preemptively suspending production. As a result, many workers are forced to be idle (unemployed) in every period. Therefore, this ratio may also roughly indicate increases in the unemployment rate due to the generation of unutilized resources. Figure 7 suggests that a 1 percentage point upward jump of RTP roughly raises the unemployment rate 10–15 percentage points immediately after the shock.

What is important is not only the scale of generated unutilized resources but also their persistence and the ensuing high unemployment rate. Economic depressions (severe recessions) like the Great Recession and the Great Depression persist for several years or more (Temin, 1989; Martin et al., 2015; Hall, 2016; Fernald et al., 2017). Figure 7 indicates that long periods are needed until the amount of unutilized resources generated (equivalently, a high unemployment rate) dwindles. This nature of persistence seems to well match the experiences actually observed in historical severe recessions.

The scale and persistence of generation of unutilized resources strongly imply that, if a large upward MDC (RTP) shock occurs, fiscal policies employed by government are extremely important to make these unutilized resources usable. A government should buy and utilize these unutilized resources on a large scale in various ways; as Keynes (1936) famously wrote, "the government should pay people to dig holes in the ground and then fill them up." Harashima (2016b, 2017a) theoretically showed that, if such a shock occurs, audacious and voluminous fiscal policies are indispensable because they are very effective to reduce the amount of unutilized resources and prevent large increases in the unemployment rate.

Note that during the period of the Great Depression, fiscal policies were viewed as a kind of taboo and then employed only on a small scale. On the other hand, in the period of the Great Recession, fiscal policies were no longer taboo and substantial fiscal policies were employed. It seems highly likely that this difference in fiscal policy caused large differences in the eventual economic damages in the two events.

4.3.1.4 Accumulated amount of unutilized resources

The dotted line in Figure 5 is the simulated path of the accumulated amount of unutilized resources. It increases steadily after the shock but is eventually stabilized as the amount of new unutilized resources substantially decreases. Comparing the dotted line (the accumulated amount of unutilized resources) with the solid line (capital) in Figure 5, it is clear that the accumulated amount of unutilized resources eventually almost matches the amount of capital at the posterior steady state.

4.3.2 Effects of the magnitude of shock

I further examine whether the effects of shock differs if the magnitude of the shock differs, by examining the same three magnitudes simulated in Section 4.2.2 for the Jump path. I again set $\eta = 0.1$, and the other parameters including $\overline{c}$ and $\underline{c}$ are also the same as those in the base case of NEPIP in Section 4.3.1 except for the magnitudes of shock.

4.3.2.1 Capital and consumption

Figures 8 and 9 indicate the effects of shocks with different magnitudes on capital and consumption, respectively.

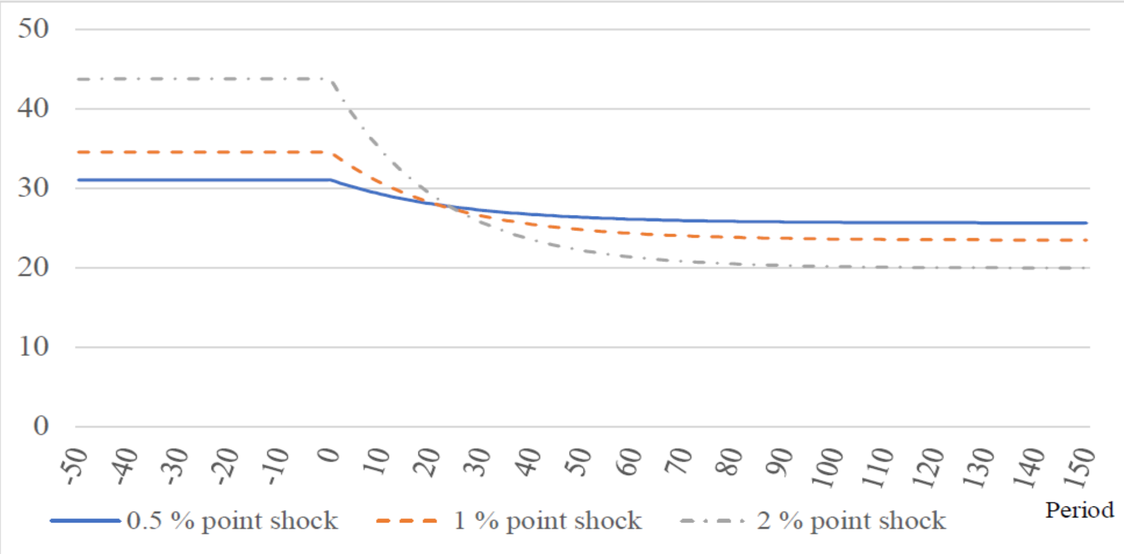

Figure 8: Simulation of the effect of magnitude of shock in the base case of NEPIP for capital owned by each household ($\check{k}_{i,t}$).

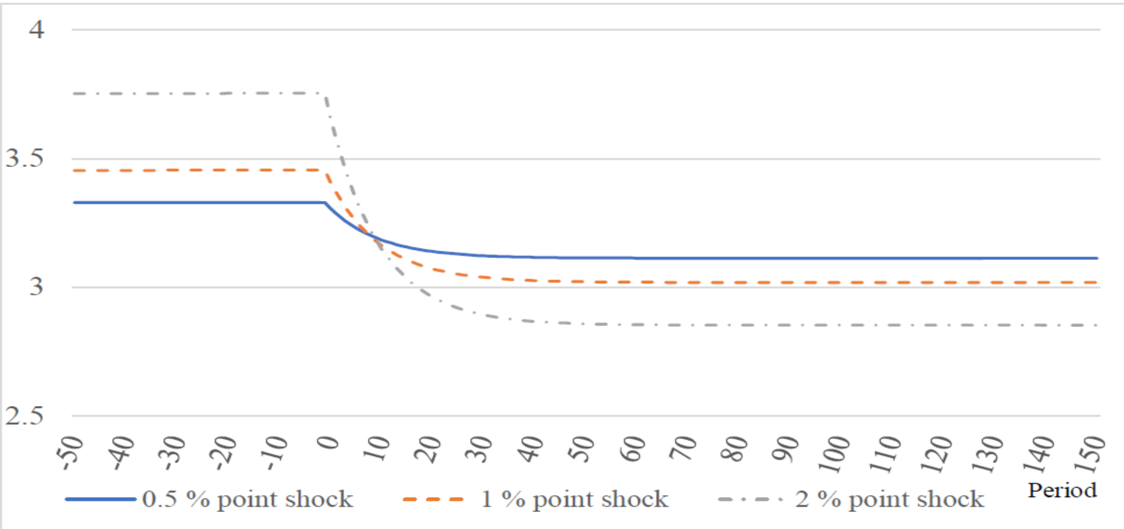

Figure 9: Simulation of the effect of magnitude of shock in the base case of NEPIP for consumption ($c_{i,t}$).

They indicate that, as the magnitude of shock increases, the magnitudes of decrease in capital and consumption increase. Therefore, the magnitude of shock matters. Note that the paths of capital in the three cases are identical to those in the base case of the Jump path shown in Figure 3 for the same reason discussed in Section 4.3.1.1.

4.3.2.2 Unutilized resources and the unemployment rate

Figure 10 indicates the effect of shocks of different magnitudes on the amount of unutilized resources. As discussed in Section 4.3.1.3, this roughly indicates the effect on the unemployment rate. As Figure 10 shows, as the magnitude of the shock increases, the amount of unutilized resources generated in each period increases by an even greater amount, meaning the unemployment rate also increases by a larger amount.

In the case of a 2 percentage point shock, the unemployment rate is simulated to roughly increase by 25 percentage points. As is well known, unemployed workers are generated at all times because of matching friction in labor market. The unemployment rate is therefore always non-zero (e.g., a 5% "natural" unemployment rate). Considering this "natural" unemployment, the total unemployment rate in the case of a 2 percentage point shock may reach 30% or more during the few periods immediately after the shock.

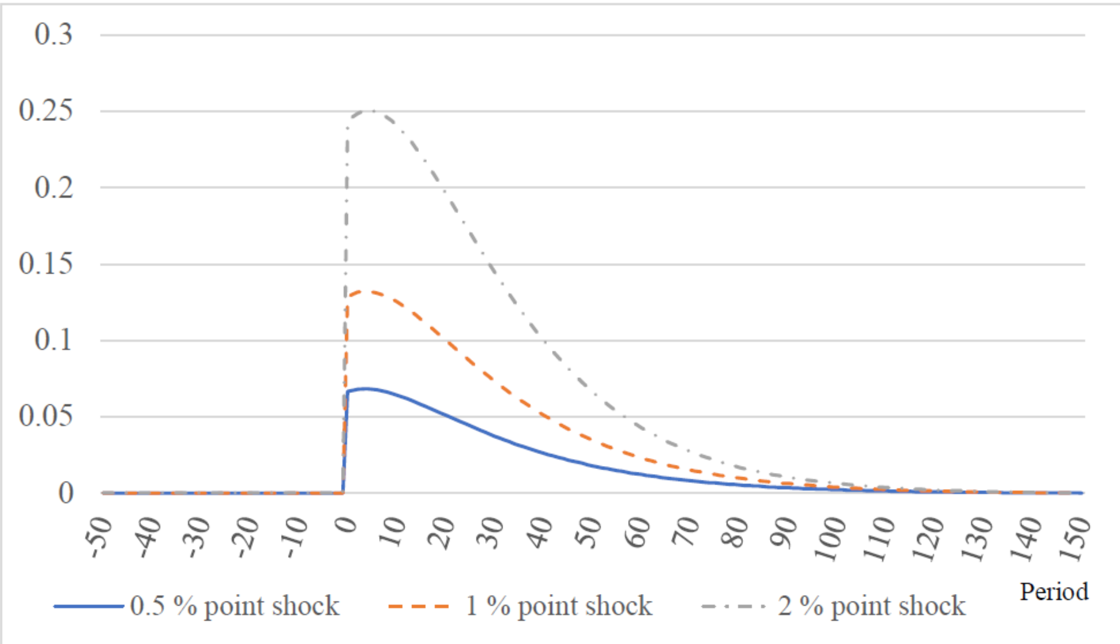

Figure 10: Simulation of the effect of magnitude of shock in the base case of NEPIP for the ratio of unutilized resources on NEPIP to consumption on the Jump path (roughly equivalent to increases in the unemployment rate).

4.3.2.3 Accumulated amount of unutilized resources

Figure 11 shows the accumulated amount of unutilized resources for three shocks, and as the magnitude of the shock increases, the accumulated amount of unutilized resources also increases. In the case of a 2 percentage point shock, the eventually accumulated amount (37.1) is larger than the amount of capital at the posterior steady state (19.9), while the amount of capital at the prior steady state is 43.9.

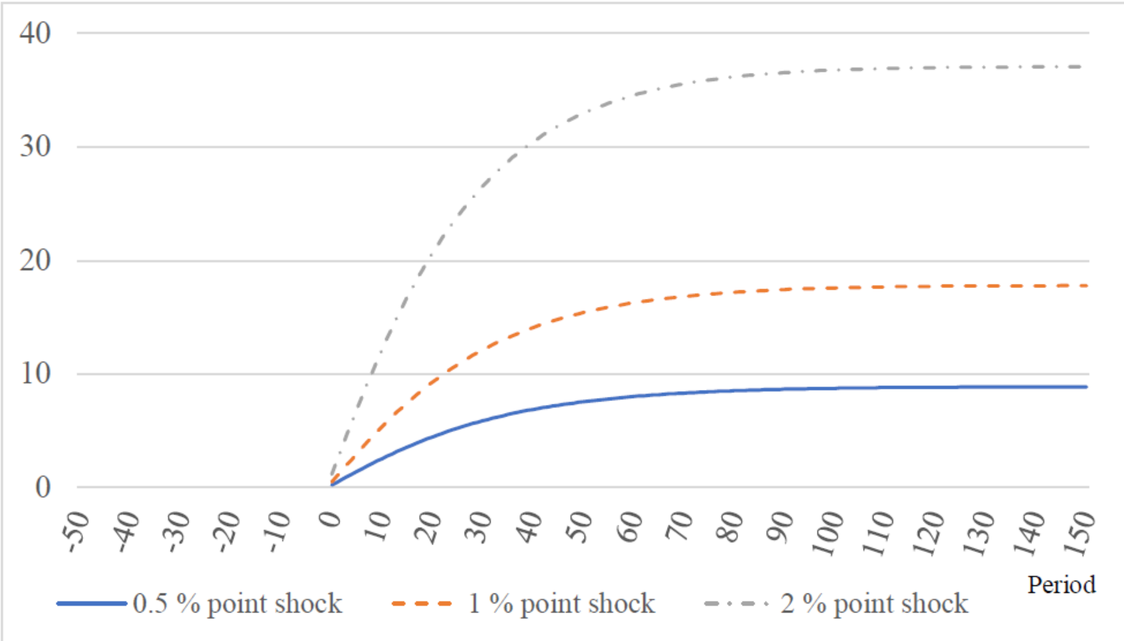

Figure 11: Simulation of the effect of magnitude of shock in the base case of NEPIP for the accumulated amount of unutilized resources.

4.3.3 Effect of adjustment speed of consumption

As shown in Section 3.2.2, the speed of decrease in consumption on NEPIP differs depending on the value of parameter η. Here, I examine the following three cases: η = 0.05, 0.1, and 0.5.

4.3.3.1 Capital

Figure 12 shows the paths of capital in these three cases. By the same reason shown in Section 4.3.1.1, the path of capital does not differ whether households behave strategically or not, and therefore the path of capital is not affected by η.

4.3.3.2 Consumption

Figure 13 shows the paths of consumption and indicates that as the value of η is larger, consumption decreases more rapidly. In the case of $\eta = 0.5$, consumption decreases almost to the level of the posterior steady state within a few periods after the shock.

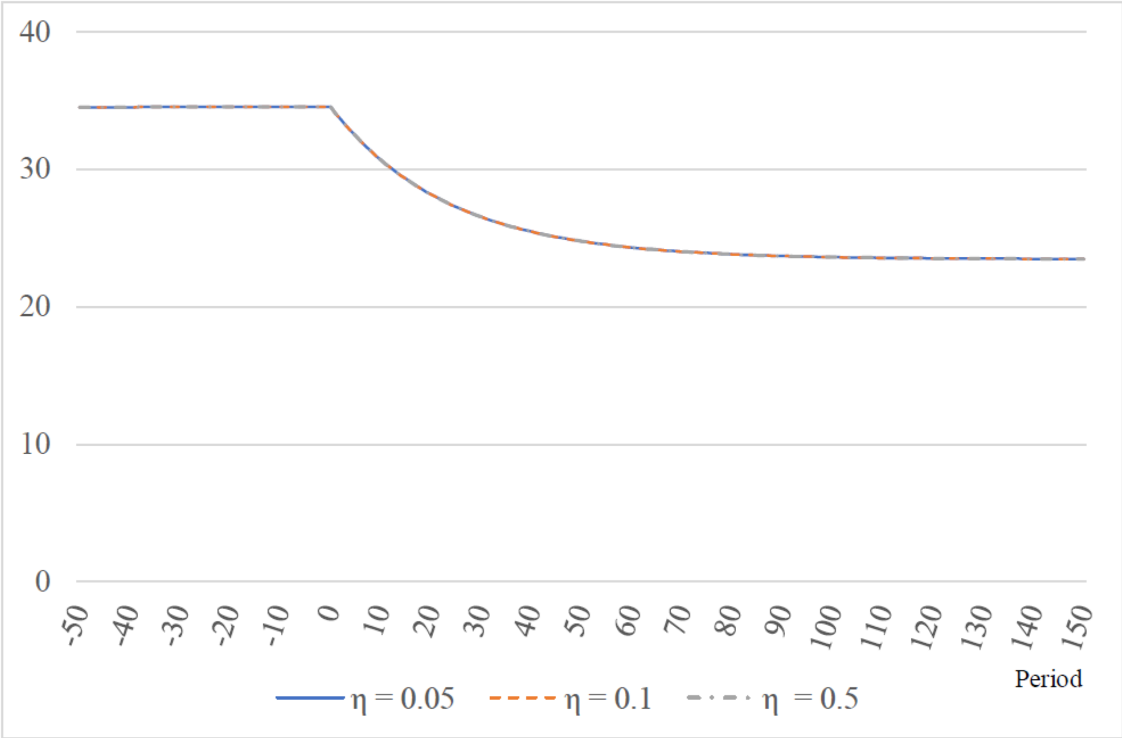

Figure 12: Simulation of the effect of adjustment speed in the base case of NEPIP for capital owned by each household ($\check{k}_{i,t}$).

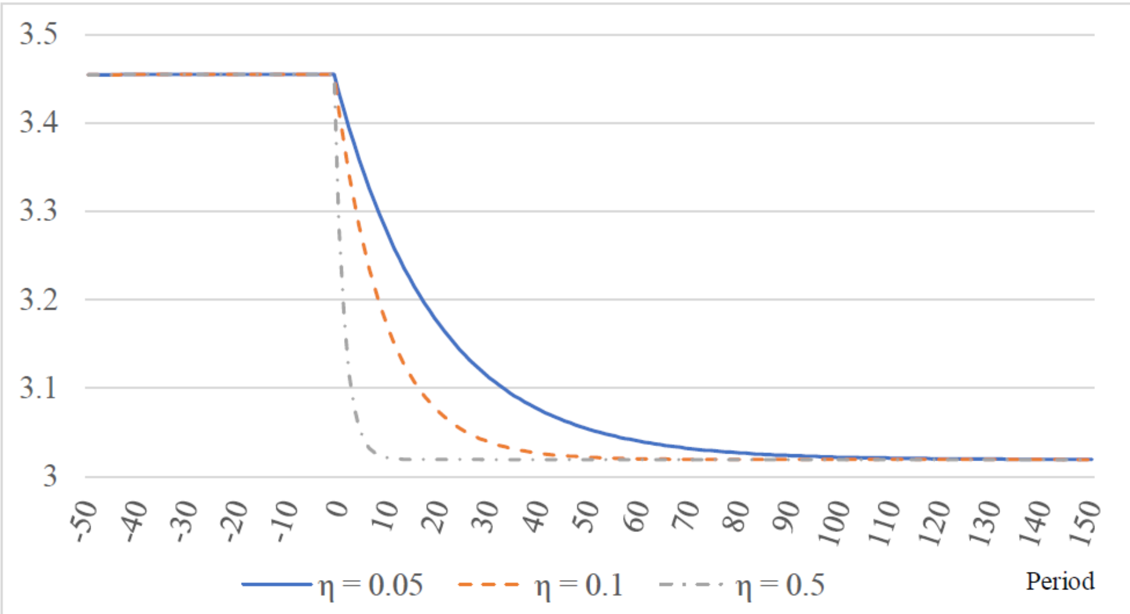

Figure 13: Simulation of the effect of adjustment speed in the base case of NEPIP for consumption ($c_{i,t}$).

4.3.3.3 Unutilized resources and unemployment rate

Figure 14 shows the ratio of the unutilized resources on NEPIP to consumption on the

Jump path; it indicates that, as the value of η increases, unutilized resources are generated at an even greater rate in each period, which implies that the unemployment rate also increases at a greater rate. Figure 14 implies that, in the case of η = 0.5, the unemployment rate will rise roughly 20 percentage points immediately after the shock.

Figure 13 indicates that consumption more quickly approaches the level at the posterior steady state as the value of η increases. This may mean that the economy is more quickly stabilized again, in the sense that consumption reaches and stays at the steady state level. However, Figure 14 indicates that this is not true because periods with a high unemployment rate persist longer at higher values of η.

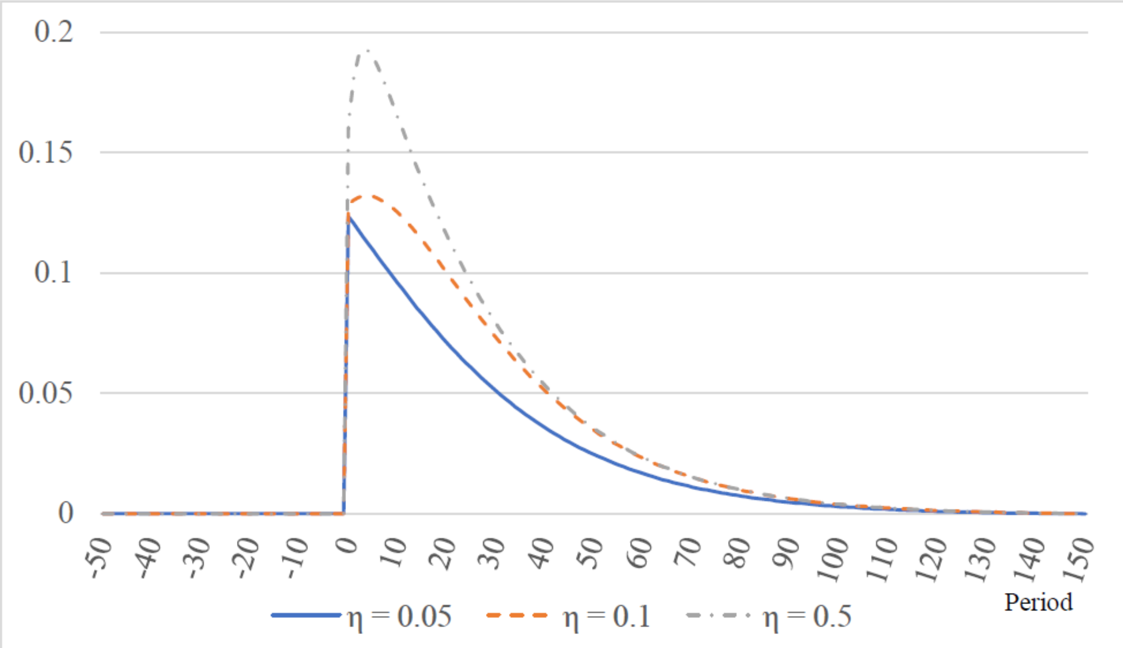

Figure 14: Simulation of the effect of adjustment speed in the base case of NEPIP for the ratio of the unutilized resources on NEPIP to the consumption on the Jump path (roughly equivalent to increases in the unemployment rate).

4.3.3.4 Accumulated amount of unutilized resources

Figure 15 shows the paths of the accumulated amount of unutilized resources; it indicates that larger amounts of unutilized resources are accumulated at higher values of η. The accumulated amount of unutilized resources in the case of η = 0.5 eventually almost matches the amount of capital at the posterior steady state (i.e., the former is 21.3 and the latter is 23.5), while the capital at the prior steady state is 34.6. This indicates the tremendous negative impact an upward MDC (RTP) shock can have.

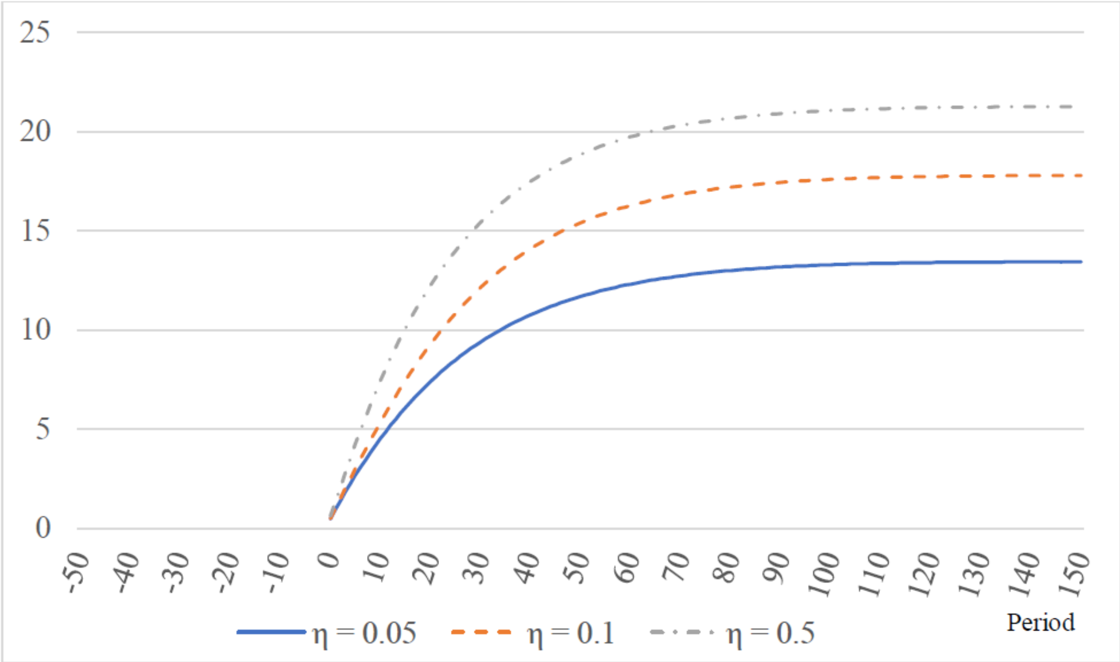

Figure 15: Simulation of the effect of adjustment speed in the base case of NEPIP for the accumulated amount of unutilized resources.

5 CONCLUDING REMARKS

In this paper, I numerically simulate the path of an economy in an economic depression on the basis of the simulation method employed in Harashima (2022c, 2023a, 2023b, 2024), in which a completely new method to numerically simulate paths to reach steady state was developed. Using this simulation method, Harashima (2022c) simulated the path to a steady state and showed that households can reach a steady state without generating any rational expectations. Furthermore, Harashima (2023a) simulated the effect of economic rents obtained heterogeneously among households, Harashima (2023b) numerically examined the mechanism why economic inequality can increase in democratic countries, and Harashima (2024) numerically simulated endogenously growing economies and their balanced growth path.

However, in these studies, economic fluctuations were not simulated because it was assumed that there is no shock that generates fluctuations. In this paper, I examine what would happen if a shock does occur, particularly, a shock that generates a severe recession on the basis of the concept of NEPIP by extending the simulation method used in the previous studies. In particular, I simulate the path after an upward MDC (equivalently an RTP) shock occurs.

The results of simulation indicate that if households do not select a NEPIP, consumption jumps upwards immediately, discontinuously, and largely immediately after the shock, as predicted theoretically, and capital decreases steadily after the shock. On the other hand, if households strategically select a NEPIP, consumption immediately begins to decrease just after the shock and approaches the level consistent with the posterior steady state. Because consumption does not jump upward, a large amount of unutilized resources is generated in each period. Because of these unutilized resources, the unemployment rate could reach 30% or higher. The total accumulated amount of unutilized resources during a depression can almost match the amount of capital at the posterior steady state. These simulation results seem to well match or be consistent with economic conditions actually observed during economic depressions (severe recessions) such as the Great Depression and the Great Recession.

APPENDIX 1:
Nash equilibrium of a Pareto inefficient path (NEPIP)

A1.1 The model

Households are assumed to be non-cooperative, risk averse, and infinitely living. They are also assumed to be identical in the sense that their preferences, labor incomes, and initial financial assets are identical. In addition, there is assumed to be a sufficiently large number of them. Each household maximizes its expected utility

$$E\int_0^\infty u(c_t)\exp(-\theta t)dt$$

subject to

$$\frac{dk_t}{dt} = f'(A, k_t) - c_t$$

where c_t, k_t, and y_t are consumption, capital, and production per capita in period t, respectively; A is technology; θ (> 0) is the rate of time preference (RTP); u is the utility function; $y_t = f(A, k_t)$ is the production function; and E is the expectation operator.

Suppose that there is a shock that makes the RTP of a household shift upward (i.e., increase) in period $t = 0$. After the shock, the steady state is changed from the prior (original) one to the posterior one. There are two options for each household with regard to consumption just after the shock. The first is a jump option **J**, in which a household's consumption jumps upwards and then proceeds on the posterior Pareto efficient saddle path to the posterior steady state. The second is a non-jump option **NJ**, in which a household's consumption does not jump but instead gradually decreases from the prior steady state to the posterior steady state. This transition path is not Pareto efficient. The household that chose the **NJ** option reaches the posterior steady state in period s (≥0). The difference in consumption between the two options in period t is b_t (≥ 0). The existence of b_t indicates that unutilized resources and excess capital exist, and they have to be somehow eliminated.

The probability that households choose option **NJ** will not necessarily be low because option **J** requires a discontinuous large and sudden increase in consumption, but risk-averse households intrinsically dislike this type of discontinuous change in consumption and want to smooth the stream of consumption. The expected utility of a household after the shock depends on whether the household chooses option **J** or **NJ**. Let

Jalone indicate that a household chooses the **J** option but other households choose the **NJ** option, **NJalone** indicate that the household chooses the **NJ** option but other households choose the **J** option, **Jtogether** indicate that all households choose the **J** option, and **NJtogether** indicate that all households choose the **NJ** option. Let p ($0 \leq p \leq 1$) be the subjective probability of a household that the other households choose the **J** option. With p, the expected utility of the household when it chooses option **J** is

$$E(J) = pE(Jtogether) + (1 - p)E(Jalone) ,$$

and when it chooses option NJ is

$$E(NJ) = pE(NJalone) + (1 - p)E(NJtogether) ,$$

where $E(Jalone)$, $E(NJalone)$, $E(Jtogether)$, and $E(NJtogether)$ are the expected utilities of the household when choosing **Jalone**, **NJalone**, **Jtogether**, and **NJtogether**, respectively. A household determines whether to choose option **J** or **NJ** by strategically considering other households' choices.

A1.2 The existence of NEPIP

Harashima (2009) proved that, under reasonable conditions, there is a p^* ($0 \leq p^* \leq 1$) such that if $p = p^*$, $E(J) - E(NJ) = 0$, and if $p < p^*$, $E(J) - E(NJ) < 0$. That is, it is possible that a Pareto inefficient path (i.e., a NEPIP) can be rationally chosen by households.

Suppose that there are $H(\in N)$ identical households in the economy and H is sufficiently large. Households' strategic choices between options **J** and **NJ** are well described by a H-dimensional symmetric mixed strategy game. Let q_η ($0 \leq q_\eta \leq 1$) be the probability that a household $\eta(\in N)$ chooses option **J**. Harashima (2009) showed that strategy profiles

$$(q_1,q_2,\ldots,q_H) = \{(1,1,\ldots,1), (p^*, p^*, \ldots, p^*), (0,0,\ldots,0)\}$$

are Nash equilibria of this game.

A1.3 The preference of worst-case aversion

As shown by Harashima (2009), refinements of the Nash equilibrium are required to determine which Nash equilibrium, **NJtogether** (0,0,…,0) or **Jtogether** (1,1,…,1), is dominant, and these refinements necessitate additional criteria. If households are worst-case averse in the sense that they prefer to avoid options that include the worst-case scenario when its probability is not known, they suppose a very low p and select the

NJtogether (0,0,…,0) equilibrium (i.e., a NEPIP), because **Jtogether** is the best choice in the sense of the amount of payoff, followed by **NJalone** and **NJtogether**, whereas **Jalone** is the worst. The outcomes of choosing option **J** are more dispersed than those of choosing option **NJ**. If households are worst-case averse in the above-mentioned sense, a household will prefer option **NJ** that does not include the worst-case scenario **Jalone**, because it fears the worst-case scenario that, after the shock, it alone will substantially increase consumption while the other households will substantially decrease consumption. This behavior is rational because it is consistent with the household's preference.

A1.4 NEPIP and severe recessions

Because NEPIP is Pareto inefficient and excess capital and b_t exist, unutilized resources are successively generated and eliminated—that is, a recession is generated. In this situation, as Harashima (2012b) showed, the unemployment rate rises by frictions in the job search and matching process. Note that Harashima (2014b) also showed the generation mechanism of the shock on RTP. The main underlying factor that generates this shock is that households need to generate an expected RTP under sustainable heterogeneity, as shown by Harashima (2014b, 2014c).

APPENDIX 2: The MDC-based procedure

A2.1 "Comfortability" of CWR

Let k_t and w_t be per capita capital and wage (labor income), respectively, in period t. Under the MDC-based procedure, a household should first subjectively evaluate the value of $\frac{\tilde{w}_t}{\tilde{k}_t}$ where $\tilde{k}_t$ and $\tilde{w}_t$ are household k_t and w_t, respectively. Let Γ be the subjective valuation of $\frac{\tilde{w}_t}{\tilde{k}_t}$ by a household and Γ_i be the value of $\frac{\tilde{w}_t}{\tilde{k}_t}$ of household i ($i = 1, 2, 3, \dots,$ M). Each household assesses whether it feels comfortable with its current Γ (i.e., its combination of income and capital expressed by CWR). "Comfortable" in this context means "at ease," "not anxious," and other similar feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. The higher the value of DOC, the more a household feels comfortable with its Γ. For each household, there will be a most comfortable CWR value because the household will feel less comfortable if CWR is either too high or too low. That is, for each household, a maximum DOC exists. Let $\tilde{s}$ be a household's state at which its DOC is the maximum (MDC). MDC therefore indicates the state at which the combination of revenues and assets is felt most comfortable. Let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$ is household i's Γ_i when it is at $\tilde{s}_i$.

A2.2 Homogeneous population

I first examine the behavior of households in a homogeneous population (i.e., all households are assumed to be identical).

A2.2.1 Rules

Household i should act according to the following rules:

Rule 1-1: If household i feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any i.
Rule 1-2: If household i feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any i.

A2.2.2 Steady state

Households can reach a steady state even if they behave only according to Rules 1-1 and 1-2. Let S_t be the state of the entire economy in period t and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ of the entire economy at S_t (i.e., the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. Let also $\tilde{S}_{RTP}$ be the steady state under the RTP-based procedure; that is, it is the steady state in a Ramsey-type growth model in which households behave based on rational expectations generated by discounting utilities by θ, where θ (> 0) is the RTP of a household. In addition, let $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$.

Proposition 1: If households behave according to Rules 1-1 and 1-2, and if the value of θ that is calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ under the RTP-based procedure in an economy where θ is identical for all households, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.
Proof: See Harashima (2018a).

Proposition 1 indicates that we can interpret $\tilde{S}_{MDC}$ to be equivalent to $\tilde{S}_{RTP}$. This means that both the MDC-based and RTP-based procedures can function equivalently and that CWR at MDC can be substituted for RTP as a guide for household behavior.

A2.3 Heterogeneous population

In actuality, however, households are not identical—they are heterogeneous—and if heterogeneous households behave unilaterally, there is no guarantee that a steady state other than corner solutions exists (Becker 1980; Harashima 2010, 2012a). However, Harashima (2010, 2012a) has shown that SH exists under the RTP-based procedure. In addition, Harashima (2018a) has shown that SH also exists under the MDC-based procedure, although Rules 1-1 and 1-2 have to be revised, and a rule for the government should be added in a heterogeneous population.

Suppose that households are identical except for their MDCs (i.e., their values of $\Gamma(\tilde{s})$). Let $\tilde{S}_{MDC,SH}$ be the steady state at which MDC is achieved and kept constant by any household (i.e., SH in a heterogeneous population under the MDC-based procedure), and let $\Gamma(\tilde{S}_{MDC,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC,SH}$. In addition, let Γ_R be a household's numerically adjusted value of Γ for SH based on its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ and several other related values. Specifically, let $\Gamma_{R,i}$ be Γ_R of household i, T be the net transfer that a household receives from the government with regard to SH, and T_i be the net transfer that household i receives (i = 1,2,3, … , M).

A2.3.1 Revised and additional rules

Household i should act according to the following rules in a heterogeneous population:

Rule 2-1: If household i feels that the current $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption as before for any i.
Rule 2-2: If household i feels that the current $\Gamma_{R,i}$ is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption or revises its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ so that it perceives that $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$ for any i.

At the same time, the government should act according to the following rule:

Rule 3: The government adjusts T_i for some i if necessary so as to make the number of votes cast in elections in response to increases in the level of economic inequality equivalent to the number cast in response to decreases.

A2.3.2 Steady state

Even if households and the government behave according to Rules 2-1, 2-2, and 3, there is no guarantee that the economy can reach $\tilde{S}_{MDC,SH}$. However, thanks to the government's intervention, SH can be approximately achieved. Let $\tilde{S}_{MDC,SH,ap}$ be the state at which $\tilde{S}_{MDC,SH}$ is approximately achieved (an approximate SH), and $\Gamma(\tilde{S}_{MDC,SH,ap})$ be $\Gamma(S_t)$ at $\tilde{S}_{MDC,SH,ap}$ on average. Here, let $\tilde{S}_{RTP,SH}$ be the steady state that satisfies SH under the RTP-based procedure, that is, in a Ramsey-type growth model in which households that are identical except for their θs behave generating rational expectations by discounting utilities by their θs. Furthermore, let $\Gamma(\tilde{S}_{RTP,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP,SH}$.

Proposition 2: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 2-1 and 2-2, if the government behaves according to Rule 3, and if the value of θ_i that is calculated back from the values of variables at $\tilde{S}_{MDC,SH,ap}$ is used as the value of θ_i for any i under the RTP-based procedure in an economy where households are identical except for their θs, then $\Gamma(\tilde{S}_{MDC,SH,ap}) = \Gamma(\tilde{S}_{RTP,SH})$.
Proof: See Harashima (2018a).

Proposition 2 indicates that we can interpret $\tilde{S}_{MDC,SH,ap}$ as being equivalent to $\tilde{S}_{RTP,SH}$. No matter what values of T, Γ_R, and $\Gamma(\tilde{S}_{MDC,SH})$ are estimated by households, any $\tilde{S}_{MDC,SH,ap}$ can be interpreted as the objectively correct and true steady state. In addition, a government need not necessarily provide the objectively correct T_i for $\tilde{S}_{MDC,SH,ap}$ even though the $\tilde{S}_{MDC,SH,ap}$ is interpreted as objectively correct and true.

APPENDIX 3: Sustainable heterogeneity

A3.1 SH

Here, three heterogeneities—RTP, degree of risk aversion (DRA), and productivity—are considered. Suppose that there are two economies (Economy 1 and Economy 2) that are identical except for RTP, DRA, and productivity. Each economy is interpreted as representing a group of identical households, and the population in each economy is constant and sufficiently large. The economies are fully open to each other, and goods, services, and capital are freely transacted between them, but labor is immobilized in each economy. Households also provide laborers whose abilities are one of the factors that determine the productivity of each economy. Each economy can be interpreted as representing either a country or a group of identical households in a country. Usually, the concept of the balance of payments is used only for international transactions, but in this paper, this concept and the associated terminology are used even if each economy represents a group of identical households in a country.

The production function of Economy i (= 1, 2) is

$$y_{i,t} = A_t^{\alpha} k_{i,t}^{1-\alpha} \ ,$$

where $y_{i,t}$ and $k_{i,t}$ are the production and capital of Economy i in period t, respectively; A_t is technology in period t; and α ($0 < \alpha < 1$) is a constant and indicates the labor share. All variables are expressed in per capita terms. The current account balance in Economy 1 is τ_t and that in Economy 2 is $-\tau_t$. The accumulated current account balance

$$\int_0^t \tau_s ds$$

mirrors capital flows between the two economies. The economy with current account surpluses invests them in the other economy. Since $\frac{\partial y_{1,t}}{\partial k_{1,t}} \left(= \frac{\partial y_{2,t}}{\partial k_{2,t}} \right)$ is returns on investments,

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds \ \text{ and } \ \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

represent income receipts or payments on the assets that an economy owns in the other economy. Hence,

$$\tau_t - \frac{\partial y_{2,t}}{\partial k_{2,t}} \int_0^t \tau_s ds$$

is the balance on goods and services of Economy 1, and

$$\frac{\partial y_{1,t}}{\partial k_{1,t}} \int_0^t \tau_s ds - \tau_t$$

is that of Economy 2. Because the current account balance mirrors capital flows between the economies, the balance is a function of capital in both economies such that

$$\tau_t = \kappa(k_{1,t}, k_{2,t}) \ .$$

This two-economy model can be easily extended to a multi-economy model. Suppose that a country consists of H economies that are identical except for RTP, DRA, and productivity (Economy 1, Economy 2, … , Economy H). Households within each economy are identical. $c_{i,t}$, $k_{i,t}$, and $y_{i,t}$ are the per capita consumption, capital, and output of Economy i in period t, respectively; and θ_i, $\varepsilon_q = -\frac{c_{1,t}u_i''}{u_i'}$, ω_i, and u_i are the RTP, DRA, productivity, and utility function of a household in Economy i, respectively (i = 1, 2, …, H). The production function of Economy i is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \ .$$

In addition, $\tau_{i,j,t}$ is the current account balance of Economy i with Economy j, where i, j = 1, 2, … , H and $i \neq j$.

Harashima (2010) showed that if, and only if,

$$\lim_{t\to\infty} \frac{\dot{c}_{i,t}}{c_{i,t}} = \left(\frac{\sum_{q=1}^{H} \varepsilon_q \omega_q}{\sum_{q=1}^{H} \omega_q} \right)^{-1} \left\{ \left[\frac{\varpi \alpha \sum_{q=1}^{H} \omega_q}{Hmv(1-\alpha)} \right]^{\alpha} - \frac{\sum_{q=1}^{H} \theta_q \omega_q}{\sum_{q=1}^{H} \omega_q} \right\} \qquad \text{(A3.1)}$$

for any i (= 1, 2, … , H), all the optimality conditions of all heterogeneous economies are satisfied, where m, v, and ϖ are positive constants. Furthermore, if, and only if, equation (A3.1) holds,

$$\lim_{t\to\infty}\frac{\dot{c}_{i,t}}{c_{i,t}}=\lim_{t\to\infty}\frac{\dot{k}_{i,t}}{k_{i,t}}=\lim_{t\to\infty}\frac{\dot{y}_{i,t}}{y_{i,t}}=\lim_{t\to\infty}\frac{\dot{A}_{t}}{A_{t}}=\lim_{t\to\infty}\frac{\dot{\tau}_{i,j,t}}{\tau_{i,j,t}}=\lim_{t\to\infty}\frac{\frac{d\int_0^t \tau_{i,j,s}ds}{dt}}{\int_0^t \tau_{i,j,s}ds}$$

is satisfied for any i and j ($i \neq j$). Because all the optimality conditions of all heterogeneous economies are satisfied, the state at which equation (A3.1) holds is SH by definition.

A3.2 SH with government intervention

As shown above, SH is not necessarily naturally achieved, but if the government properly transfers money or other types of economic resources from some economies to other economies, SH is achieved.

Let Economy 1+2+…+ (H – 1) be the combined economy consisting of Economies 1, 2, …, and (H – 1). The population of Economy 1+2+… + (H – 1) is therefore (H – 1) times that of Economy i (= 1, 2, 3, …, H). $k_{1+2+\cdots+(H-1),t}$ indicates the capital of a household in Economy 1+2+…+ (H – 1) in period t. Let g_t be the amount of government transfers from a household in Economy 1+2+…+ (H – 1) to households in Economy H, and $\overline{g}_t$ be the ratio of g_t to $k_{1+2+\cdots+(H-1),t}$ in period t to achieve SH. That is,

$$g_t=\overline{g}_t k_{1+2+\cdots,+(H-1),t} \ .$$

$\overline{g}_t$ is solely determined by the government and therefore is an exogenous variable for households.

Harashima (2010) showed that if

$$\lim_{t\to\infty}\overline{g}_t=\left(\frac{\sum_{q=1}^{H}\varepsilon_q\omega_q}{\omega_H}\right)^{-1}\left\{\frac{\varepsilon_H\sum_{q=1}^{H}\omega_q-\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H-1}\omega_q}\left[\frac{\varpi\alpha\sum_{q=1}^{H}\omega_q}{Hmv(1-\alpha)}\right]^{\alpha}-\frac{\varepsilon_H\sum_{q=1}^{H}\theta_q\omega_q-\theta_H\sum_{q=1}^{H}\varepsilon_q\omega_q}{\sum_{q=1}^{H-1}\omega_q}\right\}$$

is satisfied for any i (= 1, 2, …, H) in the case that Economy H is replaced with Economy i, then equation (A3.1) is satisfied (i.e., SH is achieved by government interventions even if households behave unilaterally). Because SH indicates a steady state, $\lim_{t\to\infty}\overline{g}_t$ = constant.

Note that the amount of government transfers from households in Economy 1+2+ … + (H – 1) to a household in Economy H at SH is

$$(H-1)g_t=(H-1)\ k_{1+2+\cdots+(H-1),t}\lim_{t\to\infty}\overline{g}_t \ .$$

Note also that a negative value of g_t indicates that a positive amount of money or other type of economic resource is transferred from Economy H to Economy $1+2+\cdots+(H-1)$ and vice versa.

APPENDIX 4: Simulation method

A4.1 Simulation assumptions

A4.1.1 Environment

No technological progress and capital depreciation are assumed, and all values are expressed in real and per capita terms. It is assumed that there are H economies in a country, the number of households in each of economy is identical, and households within each economy are identical.

A4.1.2 Production

The production function of Economy i ($1 \leq i \leq H$) is

$$y_{i,t} = \omega_i A_t^{\alpha} k_{i,t}^{1-\alpha} \quad , \tag{A4.1}$$

where ω_i is the productivity of a household in Economy i. Because α indicates the labor share, I set α = 0.65. In addition, I set $A_t = 1$ and $\omega_i = 1$ for any t and i. The initial capital a household owns is set at 1 for any household.

With $A_t = 1$ and $\omega_i = 1$, by equation (A4.1), the production of a household in Economy i in period t ($y_{i,t}$) is calculated, for any i, by

$$y_{i,t} = k_{i,t}^{1-\alpha} \ . \tag{A4.2}$$

A4.1.3 Capital

Because the marginal productivity is kept equal across economies within the country through arbitrage in markets, the amount of capital used (not owned) by each household (i.e., $k_{i,t}$) is kept identical among households in all economies in any period; that is, $k_{i,t}$ is identical for any i although the amount of capital each household owns (not uses) can be heterogeneous. Hence, by equation (A4.2), the amount of production ($y_{i,t}$) is always identical across households and economies regardless of how much capital a household in Economy i owns, when $\omega_i = 1$. In addition, for any i,

$$k_{i,t} = \frac{\sum_{i=1}^{H} \breve{k}_{i,t}}{H} \ ,$$

where $\breve{k}_{i,t}$ is the amount of capital a household in Economy i owns (not uses). As shown above, I set the initial capital of a household owns to be 1 (i.e., $\breve{k}_{i,0} = 1$ for any i)

throughout simulations in this paper.

A4.1.4 Incomes

The capital income of a household in Economy *i* in period *t* ($x_{K,t}$) is calculated by

$$x_{K,i,t} = r_t \check{k}_{i,t} \ ,$$

where r_t is the real interest rate in period *t* and

$$r_t = \frac{\partial k_{i,t}}{\partial y_{i,t}} \ . \tag{A4.3}$$

Hence, by equations (A4.1) and (A4.3), the real interest rate r_t is calculated by

$$r_t = (1-\alpha) k_{i,t}^{-\alpha} = (1-\alpha)\left(\frac{\sum_{i=1}^{H} \check{k}_{i,t}}{H}\right)^{-\alpha} \ .$$

The labor income of a household in Economy *i* in period *t* ($x_{L,i,t}$) is calculated by extracting its capital income from its production such that

$$x_{L,i,t} = y_{i,t} - r_t k_{i,t} = y_{i,t} - r_t \frac{\sum_{i=1}^{H} \check{k}_{i,t}}{H} \ .$$

Because the amount of capital used and the amount of labor inputted by a household is identical for any household in any economy when $\omega_i = 1$, household labor income is identical across economies. Note that if productivity ($\omega_{i.t}$) is heterogeneous among economies, production and labor income differ in proportion to their productivities. Note also that in a homogeneous population, the labor income becomes equal to $\alpha y_{i,t}$ for any household.

A4.1.5 Savings

Household savings in Economy *i* in period *t* ($s_{i,t}$) are calculated by

$$s_{i,t} = x_{L,i,t} + x_{K,i,t} - c_{i,t} \ .$$

In period *t* + 1, these savings ($s_{i,t}$) are added to the capital the household owns, and therefore,

$$\breve{k}_{i,t+1} = \breve{k}_{i,t} + s_{i,t} \ .$$

A4.2 Consumption formula

A4.2.1 Consumption formula in a homogeneous population

For a simulation to be implemented, the consumption formula that describes how a household adjusts its consumptions needs to be set beforehand. However, under the MDC-based procedure, there is no strict consumption formula for households. A household just has to behave roughly feeling and guessing (i.e., not exactly calculating) its CWR and CWR at MDC in each period. It increases its consumption somewhat if it feels that $\Gamma(\tilde{s}_i)$ is larger than $\Gamma_{i,t}$ and decreases its consumption somewhat if it feels the opposite way. The amount of the increase/decrease will differ by period. In this sense, the actual formula of consumption under the MDC-based procedure is lax and vague; therefore, it is difficult to set a strict consumption formula with a mathematical functional form.

Nevertheless, if we consider the average consumption over some periods (i.e., moving averages), it will be possible to describe a mathematical form of the consumption formula because households will behave in a similar manner on average. Considering this nature, I introduce the following simple consumption formula because it seems to simply but correctly capture the behavior of households under the MDC-based procedure on average. Please note that that this consumption formula is not the only possible choice. Other, possibly more complex and subtle, functional forms could be chosen.

Consumption formula 1: The consumption of a household in Economy *i* in period *t* is

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{i,t}}\right)^{\gamma} \ , \qquad \text{(A4.4)}$$

where $\Gamma_{i,t}$ is the CWR of household in Economy *i* in period *t* and γ is a parameter.

Because

$$\theta_i = \left(\frac{1-\alpha}{\alpha}\right)\Gamma(\tilde{s}_i) \ , \qquad \text{(A4.5)}$$

as shown in Harashima (2018a, 2021, 2022a), by equation (A4.5), equation (A4.4) is equal to

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{\Gamma_{i,t}\frac{1-\alpha}{\alpha}}\right)^{\gamma} .$$

Athough a household is set to precisely follow equation (A4.4) in the simulations, in reality, they do not behave by calculating equation (A4.4). Furthermore, they are not even aware of Consumption formula 1 itself and cannot know the exact numerical value of each $\Gamma(\tilde{s}_i) = \theta_i \alpha/(1-\alpha)$. Instead, households feel and guess whether they should increase or decrease consumption considering their income and wealth.

That is, Consumption formula 1 is set only for the convenience of calculation in the simulation. It seems to well capture the essence of household behavior in that it increases or decreases consumption depending on a household's feelings with regard to $\Gamma_{i,t}$ and $\Gamma(\tilde{s}_i)$. In this context, the value of parameter γ represents the average adjustment velocity of increase or decrease in consumption.

Consumption formula 1 means that a household's consumption is roughly equal to the sum of its incomes $(x_{L,i,t} + x_{K,i,t})$. The reason for this equality is that there is no technological progress and capital depreciation, so savings stay around zero at the stabilized (steady) state. As mentioned above, the adjustment velocity of consumption in each period is determined by the value of γ in equation (A4.4). As the value of γ is larger, a stabilized (steady) state can be achieved more quickly (if it can be achieved). In this paper, I set the value of γ to be 0.5.

A4.2.2 Consumption formula in a heterogeneous population

As shown in Harashima (2018a, 2021, 2022a), in a heterogeneous population, a household behaving under the MDC-based procedure does not use its CWR ($\Gamma_{i,t}$) to make decisions about its consumption. Instead, it uses an adjusted value of CWR considering the behaviors of other heterogeneous households and the government because the entire economic state of the country depends on these heterogeneous behaviors in a heterogeneous population. Accordingly, in a heterogeneous population, Consumption formula 1 has to be modified to accommodate the adjusted CWR. Let $\Gamma_{R,i,t}$ be the adjusted value of $\Gamma_{i,t}$ of a household in Economy i in period t and $\Gamma(S_t)$ be the CWR of the country (i.e., the aggregate capital-wage ratio).

A4.2.2.1 Consumption formula 2

Unilateral behavior implies that a household behaves supposing that other households must behave in the same manner as it does. In other words, it assumes that other households' preferences are almost identical to its preferences, or at least, its preferences are not exceptional but roughly the same as the preferences of the average household

(Harashima, 2018a). If all households behaved in the same manner as a household in Economy *i* did, the real interest rate (r_t) would be equal to the household's $\Gamma_{R,i,t}(1-\alpha)/\alpha$ and eventually converge at its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. Hence, if a household in Economy *i* behaves unilaterally in a heterogeneous population, it feels and guesses that its $\Gamma_{R,i,t}$ $(1-\alpha)/\alpha$ is roughly identical to the real interest rate (r_t). That is, the real interest rate will be used as $\Gamma_{R,i,t}(1-\alpha)/\alpha$, and $r_t\alpha/(1-\alpha)$ will be used as its adjusted CWR ($\Gamma_{R,i,t}$).

Therefore, even if a unilaterally behaving household's raw (unadjusted) CWR is accidentally equal to its CWR at MDC, the household does not feel that it is at its MDC unless at the same time r_t is accidentally equal to its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. The household will instead feel that the value of r_t will soon change, and accordingly, its raw (unadjusted) CWR will also change soon. That is, it feels and guesses that the entire economic state of the country is not yet stabilized because r_t is not equal to its $\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. As a result, the household will still continue to change its consumption to accumulate or diminish capital (see Lemma 2 in Harashima, 2018a).

Considering the above-shown nature of the adjusted CWR, Consumption formula 1 can be modified to Consumption formula 2 to use in simulations with a heterogeneous population.

Consumption formula 2: In a heterogeneous population, the consumption of a household in Economy *i* in period *t* is

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{\Gamma_{R,i,t}}\right)^{\gamma}$$

$$= \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)}{r_t\frac{\alpha}{1-\alpha}}\right)^{\gamma} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\Gamma(\tilde{s}_i)\frac{1-\alpha}{\alpha}}{r_t}\right)^{\gamma} \qquad \text{(A4.6)}$$

and equivalently, by equations (A4.5) and (A4.6),

$$c_{i,t} = \left(x_{L,i,t} + x_{K,i,t}\right)\left(\frac{\theta_i}{r_t}\right)^{\gamma} \ .$$

As with $\Gamma_{i,t}$ in Consumption formula 1, the use of r_t in equation (A4.6) does not mean that households always actually behave by paying attention to r_t. What Consumption formula 2 means is that, on average, unilaterally behaving households will feel and guess that r_t represents their adjusted CWRs.

Under the RTP-based procedure, a household changes its consumption according to

$$\frac{\dot{c}_{i,t}}{c_{i,t}}=\varepsilon^{-1}(r_t-\theta_i)\ ,$$

where ε is the degree of relative risk aversion. That is, a household changes its consumption by comparing r_t and its $\theta_i=\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. The household changes consumption as r_t increasingly differs from $\theta_i=\Gamma(\tilde{s}_i)(1-\alpha)/\alpha$. This household's behavior under the RTP-based procedure is very similar to that according to Consumption formula 2, which means that the formula is basically consistent with a household's behavior under the RTP-based procedure.

In addition, in a homogeneous population, r_t is always equal to a homogenous household's $\Gamma_{i,t}(1-\alpha)/\alpha$ because all households behave in the same manner. Hence, equation (A4.4) is practically identical to equation (A4.6) (i.e., Consumption formula 1 is practically identical to Consumption formula 2) because $\Gamma_{i,t}$ in equation (A4.4) can be replaced with $r_t\frac{\alpha}{1-\alpha}$.

A4.2.2.2 Consumption formula 2-a

In Consumption formula 2, a household is supposed to feel that its preferences are not exceptional and almost the same as the preferences of the average household, but it may not actually feel that way. It may instead feel that its preferences are different from those of the average household. In this case, the household will not only feel its preferences are different, but it will also have to guess how far its preferences are from the average (i.e., by how much its adjusted CWR is different from the real interest rate).

For example, a household in Economy *i* may feel and guess that its adjusted CWR is

$$\Gamma_{R,i,t}=\frac{\alpha}{1-\alpha}\ (r_t+\chi_i) \qquad \text{(A4.7)}$$

instead of $\Gamma_{R,i,t}=r_t\frac{\alpha}{1-\alpha}$ in Consumption formula 2, where χ_i is a constant and $\chi_i\neq\chi_j$ for any *i* and *j*. χ_i represents the magnitude of how much a household in Economy *i* feels it is different from the average household. I refer to a modified version of Consumption formula 2 in which $r_t\frac{\alpha}{1-\alpha}$ is replaced with $\frac{\alpha}{1-\alpha}\ (r_t+\chi_i)$ shown in equation (A4.7) as Consumption formula 2-a. In this case, a household in Economy *i* behaves feeling that

$$\Gamma_{R,i,t}=\frac{\alpha}{1-\alpha}\ (r_t+\chi_i)=\Gamma_{i,t} \qquad \text{(A4.8)}$$

holds at a stabilized (steady) state that will be realized at some point in the future.

A4.2.2.3 Consumption formula 2-b

In both Consumption formulae 2 and 2-a, the raw (unadjusted) CWR is not included and therefore plays no role. Nevertheless, a household may utilize a piece of information derived from its raw (unadjusted) CWR because past behaviors may contain some useful information for guiding future behavior. As indicated in Section A4.2.2.2, χ_i is a parameter that indicates how far a household is from the average household. In general, the value of the parameter should be adjusted if households obtain any new and additional pieces of information. This implies that a piece of information derived from the raw (unadjusted) CWR may be used to adjust the value of parameter χ_i.

For example, a household in Economy *i* may use its raw (unadjusted) CWR ($\Gamma_{i,t}$) to adjust the value of χ_i such that

$$\chi_{i,t} = \chi_{i,t-1} + \zeta_i \left(\Gamma_{i,t} \frac{1-\alpha}{\alpha} - r_{t-1} - \chi_{i,t-1} \right) , \qquad \text{(A4.9)}$$

where $\chi_{i,t}$ is χ_i in period *t*, and ζ_i is a positive constant and its value is close to zero. Equation (A4.9) means that a household in Economy *i* increases the value of $\chi_{i,t}$ a little if its raw (unadjusted) CWR is higher than its adjusted CWR ($r_{t-1} + \chi_{i,t-1}$) in the previous period and vice versa. It fine-tunes $\chi_{i,t}$ in this manner because it feels that equation (A4.8) will eventually hold at some point in the future, as shown in Section A4.2.2.2. The value of ζ_i is close to zero because $\Gamma_{i,t}$ is highly likely to be almost equal to $\Gamma_{i,t-1}$, and therefore, the guess of $\chi_{i,t}$ in period *t* will not change largely from that of $\chi_{i,t-1}$ in period $t-1$. I refer to the modified version of Consumption formula 2-a in which χ_i is replaced with $\chi_{i,t}$ shown in equation (A4.9) as Consumption formula 2-b.

A4.3 Rule of government transfer

Although governments implement transfers among households in complex and subtle manners, a simple bang-bang (two-step) control is adopted in simulations in this paper as the rule of government transfer for simplicity. In addition, government transfers in each period are assumed to be added to or extracted from the capital of each relevant household in the next period.

Let κ_i be the $\check{k}_{i,t}$ that a government aims for in order to induce a household in Economy *i* to own capital at a steady state (i.e., κ_i is the target value set by the government). Under these conditions, the bang-bang (two-step) control rule of government transfers is set as follows.

Transfer rule: The amount of government transfers from a household in Economy i to a household in Economy i + 1 in period t is T_{low} if $\check{k}_{i,t}$ is lower than κ_i, and T_{high} if $\check{k}_{i,t}$ is higher than κ_i, where T_{low} and T_{high} are constant amounts of capital predetermined by the government, and if $i = H$, i + 1 is replaced with 1.

In the simulations, T_{low} is set to be -0.1 and T_{high} to be 0.5. The value of κ_i is varied in each simulation depending on what steady state the government aims to achieve. Note that because of the discontinuous control signal in bang-bang (two-step) control, flow variables may show discontinuous zigzag paths but stock variables can move relatively smoothly. These zigzag paths may look unnatural, but they are generated only because of the bang-bang (two-step) control method that is adopted for simplicity.

Even if a household knows about the existence of government transfers, it still behaves based on Consumption formula 2 (or 2-a and 2-b) with no government transfer. That is, a household uses $x_{L,i,t} + x_{K,i,t}$, not $x_{L,i,t} + x_{K,i,t}$ + government transfers (T_{low} or T_{high}), as the "base" consumption in determining whether it should increase or decrease its consumption. This behavior superficially may mean that a household does not consider government transfers in the process of adjusting its CWR. However, it is implicitly assumed that a household knows that government transfers exist and that they are an exogenous factor. Therefore, the household feels that the transfers should be removed from the elements that it can change or control freely. Furthermore, it is implicitly assumed that a household correctly knows the exact amount of government transfers.

However, these assumptions may be oversimplifications, and they can be relaxed to allow for incorrect guesses on the amount of government transfers. This relaxation enables a household to use $x_{L,i,t} + x_{K,i,t}$ + government transfers (T_{low} or T_{high}) instead of $x_{L,i,t} + x_{K,i,t}$ in determining its consumption.

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