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Abstract

Despite scarcity being central to economics, the scarcity of brain's internal resources has largely been ignored. Neuroscience research increasingly points to the brain evolving as a prediction engine in response to this internal-resource scarcity. The brain meets every situation with subconscious expectations, which are contrasted with information to generate error-signals. Selective processing of such error-signals, in lieu of the entire information-stream, saves brain-resources. We show that applying this predictive-processing framework to asset pricing gives rise to an alpha in CAPM. Several empirically observed phenomena (value, momentum, size, high-alpha-of-low-beta, profitability, investment, and time-specific changes in SML slopes) correspond to either cross-sectional or time-specific variations in this alpha. Additional insights about these phenomena emerge that are consistent with empirical evidence. Hence, potentially, a unified explanation for several asset pricing anomalies emerges as ultimately due to the brain's optimal response to its own internal resource scarcity, suggesting a synthesis of neoclassical and behavioral finance.

JEL Classification: G12, G41

Keywords: Predictive Processing, Asset Pricing, CAPM, SML Slope, Betting-Against-Beta, Size Effect, Value Effect, Momentum Effect

Asset Pricing in the Resource-Constrained Brain

Even though resource scarcity has long been a defining notion in economics, the fact that the brain resources (neurons and energy) are also finite has largely been ignored.[1] Perhaps, not having a clear framework for analyzing the implications of such internal resource scarcity has played a role in this neglect. However, over the past decade and a half, neuroscience research has been converging to a framework which views the brain as a 'prediction machine' that uses predictions to conserve internal brain resources.[2] In this article, we show that this framework, known as 'predictive processing', provides appropriate conceptual tools for studying the implications of internal-resource scarcity. Specifically, we show that incorporating the predictive-processing framework into asset pricing gives rise to an alpha in the CAPM. Several empirically observed phenomena (value, momentum, size, high-alpha-of-low-beta, profitability, investment, and time-specific changes in SML slopes) correspond to either cross-sectional or time-specific variations in this alpha.[3] [4]Additional insights about these phenomena emerge that are consistent with empirical evidence. Hence, potentially, a unified explanation for several asset pricing anomalies emerges as ultimately due to the brain's optimal response to its own internal resource scarcity.

The predictive processing framework says that the brain uses its prior knowledge of the world to form subconscious expectations in every situation, which are contrasted with available information to generate error signals. Such error signals are then selectively

[1] A few exceptions are Alonso et al (2014), Siddiqi and Murphy (2023), and Siddiqi (2023). McKenzie (2018) argues that the neoclassical toolbox extends to behavioral economics if the brain resource scarcity is acknowledged.

[2] There is a large body of literature in the cognitive science/neuroscience that considers the brain to be a prediction machine (Nave et al 2020, Clark 2013, Hohwy 2013, Friston 2010, Bubic et al 2010 among others). A sample based on writings of various cognitive scientists, which is suitable for non-specialist audience, includes Clark (2023), chapter 3 in Hawkins, J. (2021), chapter 4 in Feldman, L. B. (2021a), chapter 4 in Seth, A. (2021), and chapters 4 and 5 in Goldstein (2020). Feldman, L. B. (2021b) also provides a discussion of key ideas.

[3] Fama and French (2016) find deviations from the implications of the model, such as related to beta, size, value, and momentum building on early studies by Black (1972), Stoll and Whaley (1983), Fama and French (1993), and Jegadeesh and Titman (1993) among others. This suggests that there is misspecification in the CAPM, and additional risk factors have been proposed (Fama and French 2016, 2011, 1993).

[4] Specific times when the SML slope is steeper include: Months when inflation is low or negative (Cohen, Polk, and Vuolteenaho 2005), days when news about inflation, unemployment, or Federal Open Markets Committee (FOMC) interest rate decisions are scheduled to be announced (Savor and Wilson 2014), periods of pessimistic investor sentiment (Antoniou et al 2015), and overnight (Hendershott et al 2020).

incorporated into predictions based on the brain's assessment of their relative value. By selectively processing error signals and mostly just leveraging prior knowledge to fill in the gaps, the brain greatly cuts down on the amount of information it processes.[5] All expectations, from the mundane (what you expect to see around the corner) to the sophisticated (risk and reward expectations), are constructed in the brain in this way.[6]

The four components of the predictive-processing framework are: (i) an internal model, which captures typical/average behavior, based on a synthesis of prior experiences in similar situations, (ii) subconscious predictions generated by the internal model (iii) error-signals that result from contrasting such expectations with available information, and (iv) importance weights assigned to error-signals and predictions, leading to adjusted predictions, which are consciously experienced.[7]

We keep the same above components in the asset pricing context and specify the following: (i) the relevant internal model (captures average behavior) is based on a synthesis of prior experiences with similar firms[8], (ii) subconscious equity risk and reward expectations are generated by the internal model, (iii) error-signals result from contrasting such expectations with available information, and (iv) relative importance weights on error-signals and initial expectations are such the exploitable arbitrage opportunities against the decision-maker (DM) are prioritized and eliminated, leading to adjusted risk and reward expectations that are consciously experienced.[9]

[5] See chapter 4 in Hawkins, J. (2021) (and references therein) for a more detailed discussion on the common observation in neuroscience that much less brain activity happens when expectations match incoming information compared to when they do not (error-signal processing).

[6] Predictive Processing is more appropriately termed Hierarchical Predictive Processing as it views the mind as being organized in layers with a higher-level layer making predictions about the level just below with only the error-signals reaching the higher-level from the level just below. In this way, the lowest level predicts the incoming sensory signals, whereas a higher level makes predictions about the underlying causes such as changing risk and reward. Hence, this framework offers a unified theory of the mind ranging from sensory perception to higher cognition (see Clark (2013)).

[7] For illustrations of how these components work together to create various experiences, see the appendix in Clark (2023).

[8] It is more efficient for the brain to categorize closely related firms together as it reduces information load. Such categorization is a critical part of the way the brain puts the world in order and has a dedicated neuronal mechanism in the brain (Lech et al 2016).

[9] As the marginal benefit of eliminating an exploitable arbitrage opportunity against the DM is very large, it is optimal for the DM's brain to prioritize elimination of such arbitrage opportunities over other error-signals.

In general, the initial subconscious expectations that come from the internal model are adjusted towards rational expectations to the extent that (exploitable) arbitrage opportunities are eliminated without necessarily achieving full convergence. Hence, the consciously experienced final expectations retain some influence of the initial internally generated expectations. It is this influence which shows up as the alpha term in the CAPM.

The key novel insight is that the relative resource allocation between processing of risk error-signals vs. reward error-signals matters. We show that if the resources are diverted away from the processing of reward error-signals to risk error-signals then the security -market-line (SML) steepens. The observed times when the SML is steeper are all times when the DM's brain has strong reasons to consider risk as relatively more important, which tilts the internal resource allocation towards risk resulting in a steeper SML. These times include: Months when there is weak inflation/deflation indicating higher macro risks (Cohen et al 2005), periods of pessimistic investor sentiment (Antoniou et al 2015), and around market open (Hendershott et al 2020) when highly leveraged intraday traders enter the market (leverage increases risk by magnifying both gains and losses). Hence, empirical evidence on SML slopes appears to align well with this novel insight.

If the brain gives more importance reward error-signals than risk error-signals, then betting-against-beta (BAB) effect arises (Frazzini and Pederson 2014, Black 1972) which gets stronger with the resource tilt favoring reward. It follows that the brain-based model predicts stronger BAB performance during periods of optimism when the brain likely allocates less resources to risk. This prediction is consistent with empirical evidence (Antoniou et al 2015).

If the resources allocated to the processing of reward error-signal processing are not just larger but sufficiently larger, then the size effect also emerges. Intuitively, subconscious initial expectations (being a cluster average) tend to overestimate both reward and risk for small-size firms. If reward error-signal processing is much stronger in the DM's brain, then ultimately, risk overestimation dominates in alpha, leading to the size effect. One expects to see this for high quality firms (high profits, high growth, and high safety), where available information mostly shows high and growing profitability without any major red flags showing risk related concerns. This prediction matches the empirical findings on the size effect (Asness et al 2018).

A novel prediction is that the size effect is stronger when ex-ante equity premium, reflecting macroeconomic downturn risk, is high. Even though ex-ante equity premium is unobservable, central banks (in the US and other advanced economies) generally respond to high downturn risk with monetary policy easing (reduction in discount rate and effective federal funds rate), so monetary policy easing can be considered a proxy for high ex-ante equity premium. Hence, the brain-based model predicts that during monetary policy easing, the size effect must be stronger. Recent empirical findings are consistent with this prediction (Simpson and Grossmann 2024).

Value effect emerges in the brain-based framework as two firms with identical fundamentals may belong to different clusters; hence, have their initial subconscious expectations generated by different internal models. The impact of such inter-cluster variation is dampened if the brain assigns different importance-weights to error-signals across clusters. However, if such firms are in the same industry (an industry is generally divided into several distinct clusters), then their error-signals are highly correlated, indicating similar importance-weights on their error-signals. This sharpens the inter cluster variation, making value an intra-industry phenomena in the brain-based model, consistent with empirical findings on its intra-industry strength (Campbell, Giglio, and Polk 2023).

As the value effect emerges from inter-cluster variation in internal models, it follows that when reliance on internal models is weaker, the value effect is weaker. So, one expects to see a weaker value effect when events indicate a major break from the past. This prediction is consistent with the empirical findings on the particular weakness of the value effect during the dot.com bubble peak (1999-early 2000), GFC 2008-2009, and during the Covid-19 pandemic (Campbell, Giglio, and Polk 2023).

Overall, value is quite a robust intra-industry phenomenon in the brain-based model, which only disappears completely on occasions, if there is a major break from the past or when the resource allocation decisions in the brain are such that the inter-cluster variation in reward and risk cancel out each other (a knife-edge condition). This appears to be in contradiction with empirical research documenting the poor performance/disappearance of the value effect in the past 20-30 years (see the discussion in Asness et al (2015), Arnott et al (2021), Lev and Srivastava (2022), and Fama and French (2020) among others). However, the value effect has been restored as a robust phenomenon in Wang (2024) who uses a new superior measure, the ratio of cash-based operating profitability to price, suggesting that value's disappearance in earlier research was due to inferior measures of value. The new superior measure restores the robustness of value, in agreement with the prediction here.

Firms that have shown substantial deviation from the norm, such as recent substantially superior or inferior performance, may see a shift in importance weights towards error-signals and away from internally generated subconscious expectations. This weakening of the importance given to internal models in favor of error-signals generates price momentum (and makes it negatively correlated with value). Hence, the brain-based approach predicts that the momentum effect is ultimately driven by changing fundamentals, which is consistent with empirical findings on the momentum effect (Novy-Marx 2015).

As, in the brain-based model, the momentum effect arises due to an adjustment in the importance-weights on error-signals, the momentum premium depends on how much adjustment has already taken place. Hence, after larger (smaller) adjustments in the formation period, smaller (larger) momentum premiums should follow. This prediction matches the empirical findings in Huang (2022). Also, one expects the speed of adjustment to be higher in liquid market states when compared with illiquid market states. So, another prediction is that the momentum premiums are higher in liquid market states, which is consistent with the empirical findings in Avramov et al (2016).

In the brain-based model, the profitability effect arises directly from the observation that the internal model tends to underestimate large expected payoffs and overestimate small expected payoffs. The brain-based model's prediction that the profitability effect is stronger in market declines and in periods of high volatility has empirical support (Yu, H. et al 2022). The source of investment effect is underestimation of large risks and overestimation of small risks by the internal model with the prediction that the effect is weaker overnight finding empirical support (Chen, J. and Kawaguchi, Y. 2018).

Intriguingly, a wide range of quite distinct empirically observed phenomena appear consistent with the predictions of the brain-based model.

2. The Brain-Based Capital Asset Pricing Model

We rely on a standard derivation of CAPM (for example, as in Frazzini and Pedersen (2014)) and consider an overlapping generations (OLG) economy. The only innovation is that we use the predictive processing framework to specify expectations, which makes perfectly rational expectations a special case instead of the only case. Each agent lives for two periods. Agents that are born at t aim to maximize their utility of wealth at $t+1$. Their utility functions are identical and exhibit mean-variance preferences. They trade securities $s = 1, \cdots, S$ where security s pays dividends d_t^s and has n_S^* shares outstanding and invest the rest of their wealth in a risk-free asset that offers a rate of r_F.

The market is described by a representative agent who is a mean-variance maximizer:

$$\max n'\{E_t(P_{t+1} + d_{t+1}) - (1 + r_F)P_t\} - \frac{\gamma}{2} n'\Omega_t n$$

where P_t is the vector of prices, Ω_t is the variance-covariance matrix of $P_{t+1} + d_{t+1}$, and γ is the risk-aversion parameter.

It follows that the price of a security, s, is given by:

$$P_t^s = \frac{E(X_{t+1}^s) - \gamma Cov(X_{t+1}^s, X_{t+1}^M)}{1 + r_F} \qquad (2.1)$$

where security s payoff is $X_{t+1}^s = P_{t+1}^s + d_{t+1}^s$

and the aggregate market payoff is:

$$X_{t+1}^M = n_1^*(P_{t+1}^1 + d_{t+1}^1) + n_2^*(P_{t+1}^2 + d_{t+1}^2) + \cdots\cdots\cdots + n_S^*(P_{t+1}^S + d_{t+1}^S).$$

As discussed in the introduction, we apply the predictive processing framework, which says that an internal model (trained on prior experiences with similar firms and capturing average or typical behavior) generates subconscious risk and reward expectations. The DM is not aware of the formation of such subconscious expectations. Nevertheless, they play a critical role in the formation of adjusted expectations that are consciously experienced.

The brain clusters or categorizes closely related firms together. It is more efficient for the brain to do so as this reduces information load. In fact, such co-categorization is a critical part of the way the brain puts the world in order and has a dedicated neuronal mechanism in the brain (Lech et al 2016). We use q as the cluster identifier and denote the number of firms in cluster q by N_q. In general, the available information about a firm s, I_s, can be split into two subsets. A smaller set Λ_q, which only contains attributes that are common to all firms in the cluster q, and a larger/richer set Λ_s, which contains firm specific information not already in Λ_q. That is, $I_s = \Lambda_q + \Lambda_s$. Note that compared to Λ_s, Λ_q is relatively stable and only changes slowly overtime.

The brain relies on Λ_q and uses an internal model to generate subconscious reward and risk expectations:

$$E^q = \sum_{i=1}^{N_q} \frac{E\left[X_{t+1}^i\right]}{N_q} \tag{2.2}$$

$$Cov^q = \sum_{i=1}^{N_q} \frac{Cov\left[X_{t+1}^i, X_{t+1}^M\right]}{N_q} \tag{2.3}$$

The above subconscious expectations are automatically generated (without any conscious control). Prior experiences with similar firms have been synthesized into an internal model that supplies these subconscious expectations.

These subconscious expectations are contrasted with the richer information set, Λ_s, to generate error-signals. Based on the brain's assessment of their relative importance, error-signals are further processed (incorporated into expectations). In particular, error-signals that create exploitable arbitrage opportunities against the DM are prioritized over others. In general, in a resource-constrained brain, the initial subconscious expectations are adjusted in the direction of rational expectations without achieving full convergence. This process, which leads to adjusted expectations that are consciously experienced, is described by introducing a parameter, m_1:

$$E'(X_{t+1}^s) = E^q - m_1 D_1 \tag{2.4}$$

where $D_1 = E^q - E(X^s_{t+1})$ is the correct adjustment needed, and m_1 is the fraction of correct adjustment reached so $0 \leq m_1 \leq 1$. Rational expectations, $E'(X^s_{t+1}) = E(X^s_{t+1})$, correspond to processing of all error-signals and achievement of full adjustment: $m_1 = 1$.

Similarly, the adjusted risk expectation is:

$$Cov'\big((X^s_{t+1}, X^M_{t+1})\big) = Cov^q - m_2 D_2 \tag{2.5}$$

where $D_2 = Cov^q - Cov\big((X^s_{t+1}, X^M_{t+1})\big)$ is the correct adjustment needed, and m_2 is the fraction of correct adjustment, $0 \leq m_2 \leq 1$, achieved. Rational expectations, $Cov'\big((X^s_{t+1}, X^M_{t+1})\big) = Cov\big((X^s_{t+1}, X^M_{t+1})\big)$, correspond to elimination of all gaps and achievement of full adjustment: $m_2 = 1$.

If the brain has unlimited resources, then of course, it can process all error-signals and always form rational expectations; however, a resource-constrained brain prioritizes error-signals that create exploitable arbitrage opportunities against the DM over others, which in general adjusts expectations in the direction of rational expectations without necessarily achieving full convergence. A simple re-arrangement of (2.4) and (2.5) leads to:

$$E'(X^s_{t+1}) = E(X^s_{t+1}) + (1 - m_1)\big(E^q - E(X^s_{t+1})\big) \tag{2.6}$$

$$Cov'(X^s_{t+1}, X^M_{t+1}) = Cov(X^s_{t+1}, X^M_{t+1}) + (1 - m_2)\left(Cov^q - Cov\big((X^s_{t+1}, X^M_{t+1})\big)\right) \tag{2.7}$$

The consciously experienced reward and risk expectations, $E'(X^s_{t+1})$ and $Cov'(X^s_{t+1}, X^M_{t+1})$ in (2.6) and (2.7), follow from the predictive processing framework as applied to asset pricing. Rational expectations are a special case in the framework corresponding to $m_1 = 1$ and $m_2 = 1$.

The predictive processing framework gives rise to an alpha term in the CAPM as proposition 1 shows.

Proposition 1 (The Brain-Based CAPM) ***Predictive processing changes the classical CAPM in only one way: an additional term alpha appears whose value depends on the resource allocation decisions in the brain. The brain-based CAPM takes the following form:***

$$\boldsymbol{E[R^s_{t+1}] - R_F = \alpha_s + \left(E\left[R^M_{t+1}\right] - R_F\right) \cdot \beta_s} \qquad (2.8)$$

where $E[R^s_{t+1}]$ is the expected (gross) return from stock s, R_F is the (gross) risk-free return, $E\left[R^M_{t+1}\right]$ is the expected (gross) return from the aggregate market portfolio, β_s is the beta of the stock s, and α_s takes the form given below:

$$\boldsymbol{\alpha_s = \left(\frac{\overline{\beta}}{w_s} - \beta_s \frac{(m_1 - m_2)}{(1 - m_2)}\right)\frac{(1 - m_2)\delta_M}{m_1} - \frac{(1 - m_1)}{m_1}\left(\frac{\overline{ER}}{w_s} - R_F\right)} \qquad (2.9)$$

where $\overline{\beta} = \sum_{i=1}^{N_q} \frac{\varphi_i w_i \beta_i}{N_q}$ is the average market-value weighted beta in the cluster, $w_i = \frac{n_i^* P_t^i}{P_t^M}$, ($P_t^i$ is the share price of firm i, $n_i^*$is the number of shares of firm i outstanding, and $P_t^M$is the price of the aggregate market portfolio), ***$\varphi_i = \frac{n_s^*}{n_i^*}$, $\overline{ER} = \sum_{i=1}^{N_q} \frac{\varphi_i w_i E\left[R_{t+1}^i\right]}{N_q}$is the average market-value weighted expected return in the cluster, $w_s = \frac{n_s^* P_t^s}{P_t^M}$ is the market-value weight of firm s, and $\delta_M = E\left[R_{t+1}^M\right] - R_F$.***

Proof:

See Appendix A.

▪

Note, that when the brain has sufficient resources to fully process both the reward error-signals and the risk error-signals, that is, when $m_1 = 1$ and $m_2 = 1$, then $\alpha_s = 0$.

3. Asset Pricing Anomalies: A Brain-Based Perspective

The enriched CAPM has intriguing implications for the slope of the security-market-line (SML). It also generates betting-against-beta (BAB), size, value, momentum, profitability and investment effects, which generally arise as variations in the alpha term in (2.9) depending on the internal resource allocation decisions. Additional insights emerge, which are empirically supported.

3.1 The Slope of the Security Market Line (SML)

If more (less) resources are allocated to reward error-signal processing or less (more) resources are allocated to risk error-signal processing, that is, when $m_1$rises (falls) or m_2 falls (rises), then the SML rotates in the clockwise (counter clockwise) direction or the SML flattens (steepens). Intuitively, this is due to the changes in the relative underestimation of variation in risk across firms. If the relative underestimation of variation in risk rises, SML flattens, if such underestimation falls, SML steepens. Figure 1 and figure 2 illustrate.

Proposition 2 (SML slope) ***If more (less) resources are allocated to reward error-signal processing or less (more) resources are allocated to risk error-signal processing, that is, when $m_1$rises (falls) or m_2 falls (rises) then the SML rotates in the clockwise (counter clockwise) direction.***

Proof

See appendix B.

▪

The intuition behind proposition 2 is easy to see. If resources are diverted away from risk error-signal processing, then all else constant, the relative underestimation of variation in risk in the cross-section rises. This lowers the observed variation in average returns when plotted against estimated firm betas. This leads to a flatter SML when compared with the SML with rational expectations.

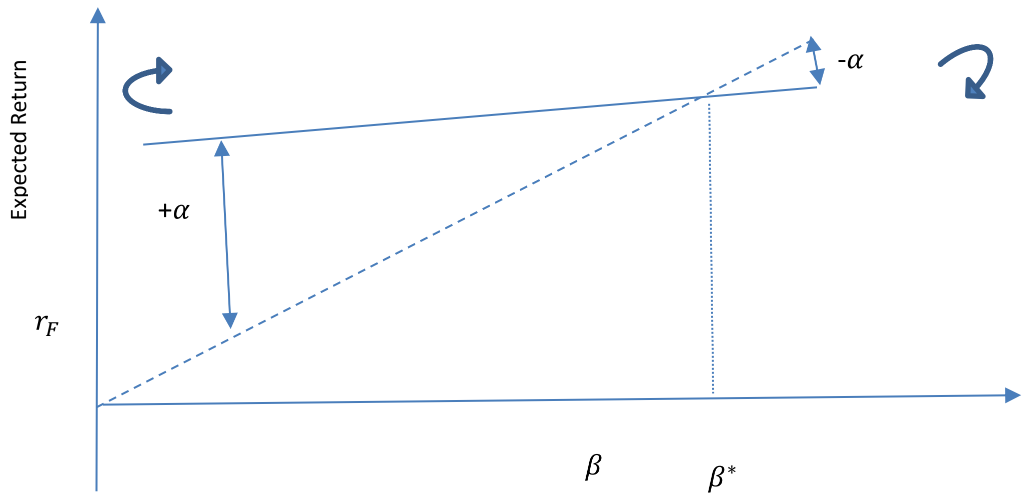

Figure 1 - Slope of the SML when m_1 rises or m_2 falls
When m_1 rises or m_2 falls, SML rotates in the clockwise direction as there is a threshold value, β^*, below which α rises (or becomes less negative) and above which α falls (or becomes more negative). The solid line indicates the brain-based SML whereas the dotted line indicates the classical SML.

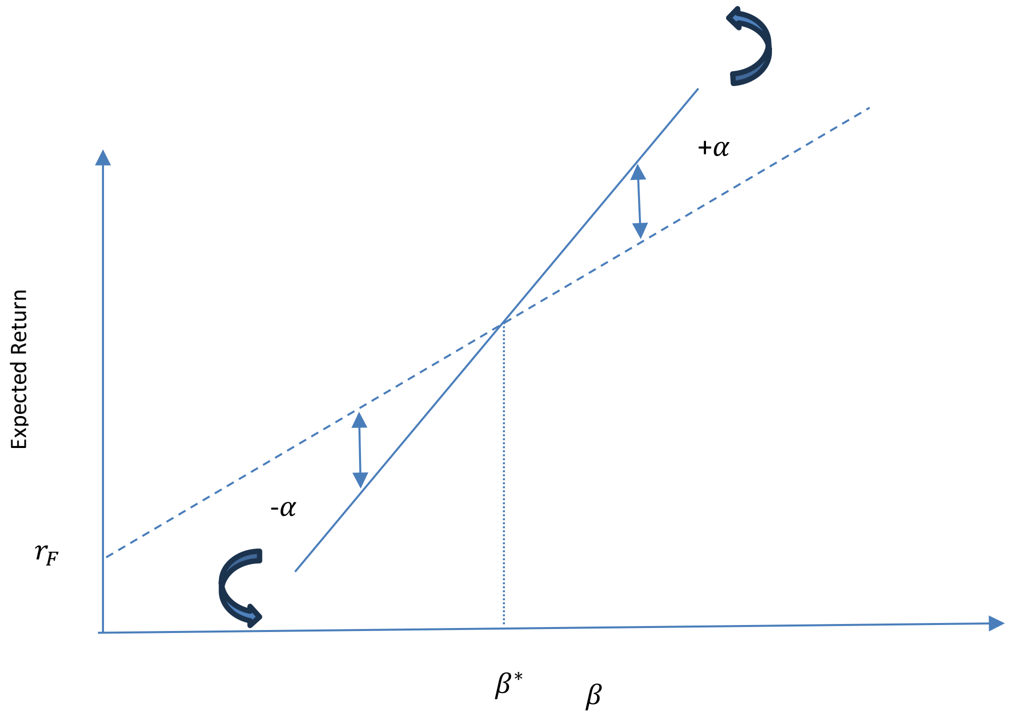

Figure 2 - Slope of the SML when m_1 falls or m_2 rises
When m_1 falls or m_2 rises, SML rotates in the counter clockwise direction as there is a threshold value, β^*, below which α falls (or becomes more negative) and above which α rises (or becomes less negative). The solid line indicates the brain-based SML whereas the dotted line indicates the classical SML.

The empirically observed variation in the SML slope at specific times appears to align well with the brain-based model:

- Around market open, the SML slope typically steepens and then gradually flattens during most of the day (Hendershott et al 2020). Intraday traders who are typically highly leveraged enter around market open and then gradually close out their position during the day (Bogousslavsky 2021). Being highly leveraged, such traders' brains assign higher importance weights to risk error-signals. This increases m_2, which steepens SML as relative underestimation of risk variation across firms falls as a result. SML slope flattens during the day as intraday traders exit the market by closing out their positions for the day, lowering m_2 in the process.
- SML slope is steeper when there is anemic inflation or deflation indicating a weak economy (Cohen et al 2005). It is also steeper in periods of pessimistic investor sentiment (Antoniou et al 2015). It makes sense that the DM's brain gives more importance to risk error-signals during such times. So m_2 rises, which lowers the relative underestimation in risk variation across firms. This steepens the SML slope in the brain-based model.
- SML slope is steeper on macroeconomic announcement days (Savor and Wilson 2014). As most traders have already adjusted their portfolios leading up to the announcement day, trades on the actual announcement day are generally by those whose expectations turned out to be incorrect and, consequently, need to re-adjust their portfolios. The resulting higher importance weights to risk error-signals in the brains of such surprised traders steepens the SML slope (m_2 rises).

High-alpha-of-low-beta effect

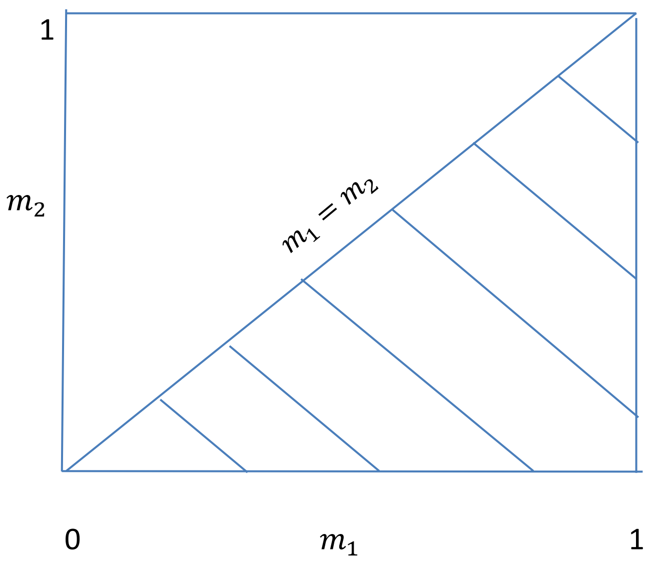

Figure 3 High-alpha-of-low-beta effect is observed in the lined region.

3.2 High-alpha-of-low-beta/ Betting-against-beta

In the brain-based CAPM, high-alpha-of-low-beta or betting-against-beta arises under the following condition (taking the partial derivative of alpha in (2.9) with respect to β_s):

$$\frac{\partial \alpha_s}{\partial \beta_s} = -\frac{\delta_M (m_1 - m_2)}{m_1} < 0 \tag{3.1}$$

Figure 3 shows the region in which high-alpha-of-low-beta or betting-against-beta (BAB) effect is observed in the space of parameters m_1 and m_2. The effect is observed if $m_1 > m_2$.

Proposition 3 (High-alpha-of-low-beta/Betting-against-beta (BAB)) ***High-alpha-of-low-beta effect arises if the importance weights assigned to reward error-signals are higher than the importance weights assigned to risk error-signals such that $\boldsymbol{m_1 > m_2}$.***

The brain-based approach predicts that the high-alpha-of-low-beta effect is not universally observed. The effect is only observed when $m_1 > m_2$, and it gets stronger when $m_1 - m_2$

rises. Intuitively, when underestimation of variation in risk rises relative to underestimation of variation in reward (due to less brain resources going to risk error-signal processing), the SML flattens. One expects to see this doing periods of optimism. It follows that during periods of optimism, BAB premium must be larger, a prediction that matches empirical findings (Antoniou et al 2015). It also follows from (3.1) that BAB premium is predicted to be larger when ex-ante equity premium is high.[10]

3.3 The Size Effect

In the predictive brain, the stock price of firm s is given by:

$$P_t^s = \frac{E(X_{t+1}^s)+(1-m_1)\left(E^q-E(X_{t+1}^s)\right)-\gamma\left[Cov(X_{t+1}^s,X_{t+1}^M)+(1-m_2)\left(Cov^q-Cov\left((X_{t+1}^s,X_{t+1}^M)\right)\right)\right]}{1+r_F} \qquad (3.2)$$

Consider the cross-section of stocks for which $E^q > E(X_{t+1}^s)$ and $Cov^q > Cov\left((X_{t+1}^s, X_{t+1}^M)\right)$. Such stocks are likely to be small-size firms. For such stocks, compared to the rational benchmark, reward is overestimated by $(1-m_1)\left(E^q - E(X_{t+1}^s)\right)$ and risk is overestimated by $(1-m_2)\gamma\left(Cov^q - Cov\left((X_{t+1}^s, X_{t+1}^M)\right)\right)$. If m_1 is sufficiently larger than m_2, then the net effect is lower price (and higher alpha). This is the size effect as it emerges in the brain-based model.

The above intuition can be seen more formally by taking the partial derivative of alpha in (2.9) w.r.t the market-cap, w_s:

$$\frac{\partial \alpha_s}{\partial w_s} = -\delta_M \frac{\bar{\beta}}{w_s^2}\frac{(1-m_2)}{m_1} + \frac{(1-m_1)}{m_1}\frac{\overline{ER}}{w_s^2} \qquad (3.2a)$$

$$\Rightarrow \frac{\partial \alpha_s}{\partial w_s} < 0 \text{ if } m_1 > 1 - \frac{\bar{\beta}\delta_M}{\overline{ER}}(1-m_2) \qquad (3.2b)$$

[10] Even though ex-ante equity premium is unobservable, monetary policy easing (lower discount rate and federal funds rate) is likely a good proxy for high ex-ante equity premium as Fed typically engages in such a policy when macro downside risk is high (when ex-ante equity premium is high).

The Size Effect

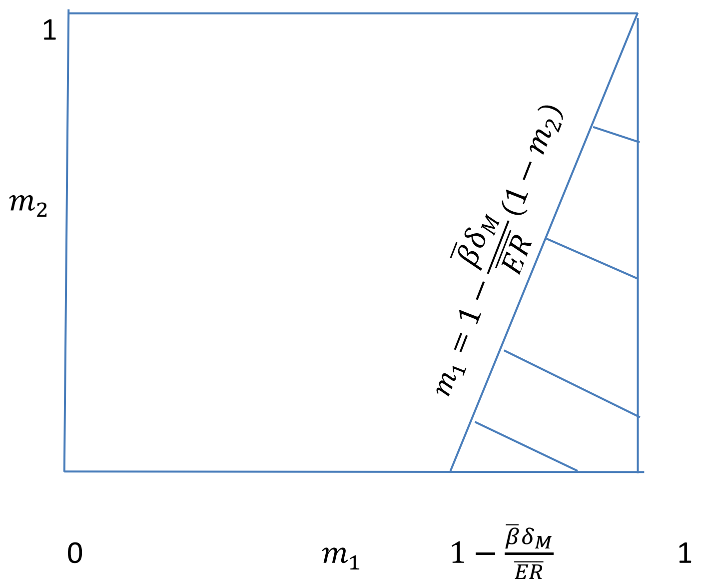

Figure 4 The size effect is observed in the lined region.

So, the size effect arises due to resource allocation decisions in the brain if the importance assigned to reward error-signals is sufficiently larger than the importance assigned to risk error-signals such that m_1 is sufficiently larger than m_2. Figure 4 illustrates.

Proposition 4 (The Size Effect) ***The size effect arises when the importance weights assigned to reward error-signals are sufficiently larger than the importance weights assigned to risk error-signals such that*** $\boldsymbol{m_1 > 1 - \frac{\bar{\beta}\delta_M}{\overline{ER}}(1 - m_2)}$***.***

Corollary 4.1 ***The size effect is stronger when ex-ante equity premium is high.***

A comparison of figure 4 and figure 3 reveals that the size effect is observed in a much smaller region when compared with the BAB effect. The fleeting nature of size effect has been extensively documented in the empirical literature with the effect only observed if certain conditions are met (see Simpson and Grossman (2024), Asness et al (2018) and references therein). It also follows that when the size effect is present, the BAB effect is necessarily present, but the reverse may not be true.

For firms that have high profitability, high growth and high safety, the importance weights assigned by the DM's brain to reward error-signals are likely much larger than the importance weights assigned to risk error-signals. So, the size effect is expected to matter among high quality firms. Empirical evidence shows that this is indeed the case (Asness et al 2018).

It also follows from (3.2b) that the size effect is more likely to be observed, when ex-ante equity premium is higher. Even though ex-ante equity premium is unobservable, monetary easing is a proxy for high ex-ante equity premium as Fed typically lowers the discount rate as well as the effective federal funds rate in response to high macro downturn risk (when ex-ante equity premium is high). So, it is in the monetary policy easing periods when high quality firms are more likely to show the size effect, as even such firms may not show the size effect in periods of monetary policy tightening. Hence, the brain-based model predicts that the size effect is stronger in periods of monetary easing. This prediction is consistent with recent empirical evidence (Simpson and Grossmann 2024).

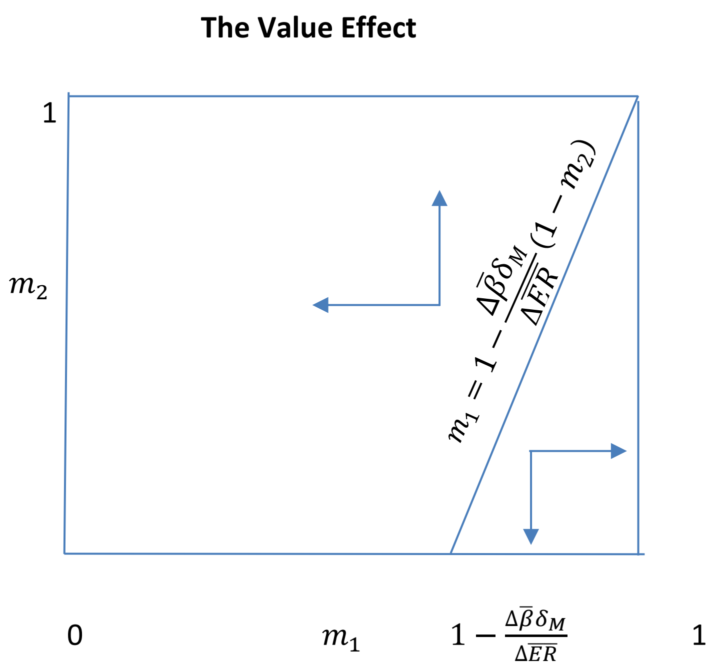

Figure 5 The value effect, which is observed for firms in the same industry, gets stronger in the direction of the arrows in the two regions split by the line $m_1 = 1 - \frac{\Delta\bar{\beta}\delta_M}{\Delta\overline{ER}}(1 - m_2)$.

3.4 The Value Effect

The value effect arises in the brain-based CAPM due to inter-cluster variation in internal models. That is, two firms with identical fundamentals have different prices (and alphas) if they belong to different clusters with each cluster having its own internal model. To fix ideas, consider two firms, a and b, that belong to different clusters. Firm a belongs to cluster q and firm b belongs to cluster l. Their prices are:

$$P_t^a = \frac{E(X_{t+1}^a)+(1-m_{1a})\left(E^q-E(X_{t+1}^a)\right)-\gamma\left[Cov(X_{t+1}^a,X_{t+1}^M)+(1-m_{2a})\left(Cov^q-Cov\left((X_{t+1}^a,X_{t+1}^M)\right)\right)\right]}{1+r_F} \quad (3.3)$$

$$P_t^b = \frac{E(X_{t+1}^b)+(1-m_{1b})\left(E^l-E(X_{t+1}^b)\right)-\gamma\left[Cov(X_{t+1}^b,X_{t+1}^M)+(1-m_{2b})\left(Cov^l-Cov\left((X_{t+1}^b,X_{t+1}^M)\right)\right)\right]}{1+r_F} \quad (3.3a)$$

If they have the same fundamentals, then:

$$E(X_{t+1}^a) = E\left(X_{t+1}^b\right) = E(X_{t+1}) \quad (3.4)$$

$$Cov(X_{t+1}^a, X_{t+1}^M) = Cov\left(X_{t+1}^b, X_{t+1}^M\right) = Cov(X_{t+1}, X_{t+1}^M) \quad (3.4a)$$

In addition, if they also belong to the same industry[11], then their error-signals would be strongly correlated indicating similar importance-weights:

$$m_{1a} = m_{1b} = m_1 \tag{3.4b}$$

$$m_{2a} = m_{2b} = m_2 \tag{3.4c}$$

Substituting from (3.4), (3.4a), (3.4b), and (3.4c) into (3.3) and (3.3a), the difference in the price of the firms is:

$$P_t^a - P_t^b = \Delta P_t = \frac{(1-m_1)(E^q - E^L) - \gamma(1-m_2)(Cov^q - Cov^L)}{1+r_F} \tag{3.5}$$

(3.5) shows that the value effect is an intra-industry phenomenon that arises due to inter-cluster variation in internal models. If the error-signals are uncorrelated (firms belong to different industries which implies $m_{1a} \neq m_{1b}$ and $m_{2a} \neq m_{2b}$), then the impact of such inter-cluster variation is dampened. Within the same industry; however, value is quite robust and disappears only when the inter-cluster variation in reward cancels out the inter-cluster variation in risk, which is the following knife-edge condition:

$$(1-m_1)(E^q - E^L) - \gamma(1-m_2)(Cov^q - Cov^L) = 0$$

$$\Rightarrow m_1 = 1 - \gamma(1-m_2)\frac{Cov^q - Cov^L}{E^q - E^L}$$

$$\Rightarrow m_1 = 1 - \gamma(1-m_2)\frac{\Delta Cov}{\Delta E} \tag{3.6}$$

The above intuition can be seen more formally by using the alpha in (2.9):

$$\alpha_a = \left(\frac{\overline{\beta_q}}{w_s} - \beta_s\frac{(m_1-m_2)}{(1-m_2)}\right)\frac{(1-m_2)\delta_M}{m_1} - \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_q}}{w_s} - R_F\right)$$

$$\alpha_b = \left(\frac{\bar{\beta_l}}{w_s} - \beta_s\frac{(m_1-m_2)}{(1-m_2)}\right)\frac{(1-m_2)\delta_M}{m_1} - \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_l}}{w_s} - R_F\right)$$

[11] An industry typically has several dozen firms so, in general, firms in the same industry are sorted by the brain into a number of distinct clusters/categories, with each cluster having its own internal model.

$$\Rightarrow \Delta\alpha = \frac{\Delta\bar{\beta}}{w_s}\frac{(1-m_2)\delta_M}{m_1} - \frac{(1-m_1)}{m_1}\frac{\Delta\overline{ER}}{w_s} \tag{3.9}$$

As long as $\Delta\alpha$ is different from zero, the value effect is observed with the low price to fundamentals stock outperforming the high price to fundamentals stock. $\Delta\alpha = 0$ if $m_1 = 1 - \frac{\Delta\bar{\beta}\delta_M}{\Delta\overline{ER}}(1-m_2)$(which is a knife-edge condition). Away from this line, the value effect gets stronger. Figure 5 illustrates.

Proposition 5 (The Value Effect) ***If the resource allocation decisions in the brain are such that the inter-cluster variation in risk is not exactly balanced by the inter-cluster variation in reward, then the value effect is observed. The effect is observed as long as*** $\boldsymbol{m_1 \neq 1 - \frac{\Delta\bar{\beta}\delta_M}{\Delta\overline{ER}}(1-m_2)}$.

It directly follows from (3.9) that the value effect is stronger among small-cap stocks. That is, its magnitude rises as w_s falls. This provides a theoretical justification for the small-cap value strategy popular among professional traders. As the value effect in the brain-based CAPM has its roots in inter-cluster variation in internal models, it gets weaker if the brain lowers the importance weights assigned to internally generated predictions coming from the internal models. This is likely if there are major market movements suggesting a substantial break from the norm (making past less of an indicator of the future). This prediction is consistent with the empirical findings on the weakness/disappearance of the value effect in unusual time periods such as during the peak of the dot.com bubble (1999-early 2000), GFC-2008-2009, and the Covid-19 pandemic (Campbell, Giglio, and Polk 2023).

Overall, the value premium emerges as quite a robust intra-industry phenomenon in the brain-based model, only disappearing on occasions when major events compel the brain to weaken its reliance on internal models or when the knife-edge condition that cancels the inter cluster variation is met. As discussed in the introduction, this is apparently in contradiction with empirical research documenting the poor performance/disappearance of the value effect in the past 20-30 years. Recently, Wang (2024) uses a new measure of value, the ratio of cash-based operating profitability to price, to establish the robustness of value, suggesting that value's disappearance in earlier research was due to inferior measures of value. The new superior measure restores the robustness of value, in agreement with the prediction here.

3.5 The Momentum effect

The empirical findings regarding the price momentum show how stocks with superior (inferior) recent performance continue to outperform (underperform) in the short run. In the brain-based framework, the price of a security s from (2.1) is:

$$P_t^s = \frac{E'(X_{t+1}^s) - \gamma Cov'(X_{t+1}^s, X_{t+1}^M)}{1 + r_F} \tag{3.10}$$

Where:

$$E'(X_{t+1}^s) = E(X_{t+1}^s) + (1 - m_1)\big(E^q - E(X_{t+1}^s)\big)$$

$$\Rightarrow E'(X_{t+1}^s) = m_1 E(X_{t+1}^s) + (1 - m_1)E^q \tag{3.11}$$

And,

$$Cov'(X_{t+1}^s, X_{t+1}^M) = \ Cov(X_{t+1}^s, X_{t+1}^M) + (1 - m_2)\left(Cov^q - Cov\big((X_{t+1}^s, X_{t+1}^M)\big)\right) \tag{3.12}$$

$$\Rightarrow Cov'(X_{t+1}^s, X_{t+1}^M) = \ m_2 Cov(X_{t+1}^s, X_{t+1}^M) + (1 - m_2)Cov^q \tag{3.13}$$

In the brain-based CAPM, price momentum arises due to an increase in the importance weights given to error-signals that follow a large change in the fundamentals of momentum winners and losers. To fix ideas, suppose the reward fundamentals of a firm (the momentum winner) improve, so $E(X_{t+1}^s)$ and consequently, $E'(X_{t+1}^s)$ goes up, which increases the stock price immediately. The reward fundamentals of another firm (the momentum loser) deteriorate. So, its price falls. This change in fundamentals, then triggers a change in the importance weights given to reward error-signals. So, m_1 goes up. For momentum winners (drawn from top 10% of firms by recent performance), the internal model typically underestimates reward, $E^q < E(X_{t+1}^s)$, whereas for momentum losers (bottom 10% by recent performance), the internal model typically overestimates reward, $E^q > E(X_{t+1}^s)$. So, this increase in m_1, which follows a large change in fundamentals, increases the price of the momentum winner further and lowers the price of the momentum loser further. Similarly, a large positive (negative) change in fundamentals could be a reduction (an increase) in risk, $Cov(X_{t+1}^{s}, X_{t+1}^{M})$, increasing (decreasing) the price initially, with subsequent increases (decreases) coming from the importance-weight adjustments that increase m_2.

The brain-based model predicts that the price momentum is a robust fundamentals-driven phenomena where an initial large change in fundamentals subsequently triggers an increase in importance weights given to error-signals. This prediction is consistent with the empirical findings on momentum effect being fundamentals driven (Novy-Marx 2015). As the increase in importance weights given to error-signals comes at the expense of the importance weights on initial expectations that come from the internal models, momentum and value (which captures inter-cluster variation in internal models) are negatively correlated.

Proposition 6 (The Momentum Effect) ***Firms with recent large positive changes in earning fundamentals show a further increase in their market prices, and firms with recent large negative changes in earning fundamentals show a further decline in their market prices due to an increase in brain resources allocated to their valuations.***

In the brain-based framework, it is the increases in importance-weights on relevant error-signals that generates the price momentum. It immediately follows that the momentum premium should depend on (i) how much room is left to adjust the importance-weights, and (ii) the speed of adjustment.

If most of error-signal adjustment has already taken place in the portfolio formation period, then there is less room to adjust in the evaluation period. However, if little adjustment has taken place in the formation period, then most of the adjustment takes place in the evaluation period. Dividing stock performance in the formation period in percentiles in increasing order of returns, the return difference between the 90th percentile and the 10th percentile (momentum gap) is a measure of adjustment in the formation period, with a small difference indicating that little adjustment has taken place in the formation period. The brain-based model predicts that the subsequent evaluation period returns should be inversely related to the momentum gap in the formation period. That is, a small (large) formation period momentum gap should be followed by a large (small) evaluation period momentum returns. Empirical findings in Huang (2022) are in accord with this prediction.

In liquid market states, the speed of adjustment should be higher. It follows that the brain-based model predicts a higher momentum premium in liquid market states. This prediction is consistent with empirical evidence on the role of market liquidity in momentum premiums (Avramov et al 2016).

3.5.1 The Impact of Financial Constraints

A further novel prediction also follows: Consider a cross-section of firms for which the internal model underestimates both the reward (in (3.11)) as well as the risk (in (3.13)). That is, $E(X_{t+1}^{s}) > E^{q}$ and $Cov(X_{t+1}^{s}, X_{t+1}^{M}) > Cov^{q}$. This cross-section likely consists of large firms. For such firms, if an event triggers a resource re-allocation away from reward error-signal processing to risk error-signal processing, then the reward underestimation rises ($m_1$falls), whereas the risk underestimation falls (m_2 rises). Both of which lead to a reduction in price (in (3.10)). Hence, the brain-based model predicts that, for large firms, such resource re-allocation towards risk error-signal processing lowers price and improves alpha. An event triggering such a re-allocation could be further tightening of financial constraints that a firm face, as this increases the risk of cashflows. It follows that the brain-based model predicts that a portfolio that goes long in large firms that face financial constraints and shorts large firms that are unconstrained should earn excess returns. Consistent with this prediction, by using a novel textual analysis to capture financial constraints, Buehlmaier and Whited (2018) show that financial constraints are indeed priced in this way.

3.6 The Profitability Effect

To fix ideas, consider two firms, R and W, that belong to the same cluster q, and are chosen such that they have similar risk; however, the internal model underestimates the expected reward of R and overestimates the expected reward of W. In other words, the following condition holds: That is, $E(X_{t+1}^{R}) > E^{q} > E(X_{t+1}^{W})$. Compared to the rational benchmark (without brain-resource scarcity), the magnitude of underestimation in the expected reward of R is $(1-m_1)(E(X_{t+1}^{R}) - E^{q})$, whereas the magnitude of overestimation in the expected reward of W is $(1-m_1)\big(E^{q} - E(X_{t+1}^{W})\big)$. So, if one takes a long position in R and a short position in W, then the additional average payoff, $\Delta\pi_{R-W}$, from such a portfolio is:

$$\Delta\pi_{R-W} = (1-m_1)(E(X_{t+1}^{R}) - E^{q}) + (1-m_1)\big(E^{q} - E(X_{t+1}^{W})\big)$$

$$\Rightarrow \Delta\pi_{R-W} = (1-m_1)\Big(E\big(X_{t+1}^{R}\big) - E\big(X_{t+1}^{W}\big)\Big) \qquad (3.14)$$

In a given cross-section of stocks, firms for which the condition $E(X_{t+1}^{s}) > E^{q}$ holds, are likely to be firms with robust profitability, and firm for which the condition $E^{q} > E(X_{t+1}^{s})$ is true, likely have weak profitability. So, if one takes a long position in firms with robust profitability (aggregating across all clusters) and a short position in firms with weak profitability, then such a portfolio earns excess returns. This is the Fama and French (2015) profitability factor as it arises in the brain-based model.

It follows from (3.14) that when resources are diverted away from reward error-signal processing towards risk error-signal processing, that is, when m_1 falls, the profitability premium goes up. Such resource diversion is expected to happen when the market is in a decline and volatility is high indicating rising risk concerns. Hence, the brain-based model predicts stronger profitability premium in periods of market declines and when volatility is high. This prediction has empirical support (Yu, H. et al 2022).

3.7 The Investment Effect

Consider two firms, C and A, that belong to the same cluster q, and are chosen such that they have the same profitability; however, the internal model overestimates the risk of C and underestimates the risk of A. That is, $Cov(X_{t+1}^{C}, X_{t+1}^{M}) < Cov^{q} < Cov(X_{t+1}^{A}, X_{t+1}^{M})$. The magnitude of risk overestimation in firm C is: $\gamma(1-m_2)\left(Cov^{q} - Cov\left((X_{t+1}^{C}, X_{t+1}^{M})\right)\right)$, whereas the magnitude of risk underestimation in firm A (both the overestimation and the underestimation are w.r.t rational benchmark) is: $\gamma(1-m_2)\big(Cov\big((X_{t+1}^{A}, X_{t+1}^{M})\big) - Cov^{q}\big)$. So, if one takes a long position in C and a short position in A, then the additional average payoff, $\Delta\pi_{C-A}$, from such a portfolio is:

$$\Delta\pi_{C-A} = \gamma(1-m_2)\left(Cov^{q} - Cov\left((X_{t+1}^{C}, X_{t+1}^{M})\right)\right) + \gamma(1-m_2)\big(Cov\big((X_{t+1}^{A}, X_{t+1}^{M})\big) - Cov^{q}\big)$$

$$\Rightarrow \Delta\pi_{C-A} = \gamma(1-m_2)\left(Cov\big((X_{t+1}^{A}, X_{t+1}^{M})\big) - Cov\left((X_{t+1}^{C}, X_{t+1}^{M})\right)\right) \quad (3.15)$$

Substituting $\gamma = \frac{(E[R_{t+1}^{M}] - R_F)}{Var(R_{t+1}^{M})P_t^{M}}$ from (A3) in the appendix into (3.15):

$$\Delta\pi_{C-A} = \frac{\delta_M}{Var(R_{t+1}^{M})P_t^{M}}(1-m_2)\left(Cov\big((X_{t+1}^{A}, X_{t+1}^{M})\big) - Cov\left((X_{t+1}^{C}, X_{t+1}^{M})\right)\right) \quad (3.16)$$

In a given cross-section of stocks, firms for which the condition $Cov^{q} < Cov(X_{t+1}^{A}, X_{t+1}^{M})$ holds, are likely to be firms that aggressively invest in their assets (need to aggressively invest in their assets to maintain their level of profitability). Similarly, firms for which the condition $Cov(X_{t+1}^{C}, X_{t+1}^{M}) < Cov^{q}$ holds are likely to be conservative firms that do not need to invest much in their assets to maintain profitability. So, if one takes a long position in conservative firms (aggregating across all clusters) and a short position in firms with high asset growth, then such a portfolio earns excess returns. This is the Fama and French (2015) investment factor as it arises in the brain-based model.

From (3.16), one can see that if the resources are diverted towards risk error-signal processing, which increases m_2, and, at the same time, macro conditions deteriorate, which increases δ_M and/or decreases P_t^{M}, then the net impact on investment premium is ambiguous. That is, it is unclear what happens to the investment premium if resources are diverted towards risk error-signal processing due to macro conditions worsening. However, if resources are diverted towards risk error-signal processing without macro conditions worsening, then the investment premium weakens. This happens, for example, around market open, when highly leveraged intra-day traders enter the market, whose brains are compelled to allocate more resources to risk error-signal processing due to the embedded leverage in such traders' portfolios irrespective of the macro conditions. So, the brain-based model predicts that the investment premium is weaker overnight (close to open). This prediction has empirical support (Chen, J. and Kawaguchi, Y. 2018).

4. Conclusions

Camerer, Lowenstein, and Prelec (2005) emphasize that neuroscience research suggests that the human behavior requires fluid dynamics between 'automatic' and 'consciously controlled' processes. Consistent with this early realization, in the past decade and a half, predictive processing has emerged as a dominant paradigm in neuroscience for thinking about the brain. This paradigm has 'subconscious expectations' that are automatically generated by an internal model. Error-signals are selectively incorporated into such initial expectations in a more 'consciously controlled' way to arrive at final adjusted expectations. In this way, by leveraging the fluid dynamics between 'automatic' and 'consciously controlled' processes, predictive processing offers a window into how internal resource allocation decisions about what type of 'error-signals' to prioritize matter.

We show that, in asset pricing context, applying the predictive processing framework gives rise to an alpha term in the CAPM, which reflects the internal tension between competing demands on limited brain resources. We show that a wide range of quite distinct empirical phenomena can be seen as arising out of shifting priorities regarding the types of 'error-signals' to process.

Overall, this article shows that predictive processing potentially offers a synthesis of behavioral and neoclassical finance as, in this framework, behavioral biases ultimately can be thought of as emerging from the brain's optimal response to its own internal resource scarcity.[12]Perhaps, the most intriguing aspect is that applying the predictive processing framework to asset pricing provides a parsimonious way of making sense of quite a wide range of distinct anomalies within a unified framework, consistent with the vision set out in Camerer et al (2005) of neuroscience providing a radical contribution to the science of economics.

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[12] In particular, the anchoring bias in Siddiqi (2019) and Siddiqi (2018), small-risk neglect in Siddiqi and Quiggin (2019), and zero-risk bias in Siddiqi (2017) can all be readily modelled as directly arising from predictive processing.

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Appendix A

Substituting from (2.6) and (2.7) into (2.1) and solving for expected return of s, $E[R_{t+1}^{s}]$:

$$E[R_{t+1}^{s}] = R_F + \frac{\gamma}{P_t^s}\left[Cov(X_{t+1}^{s}, X_{t+1}^{M}) + (1-m_2)\left(Cov^q - Cov\big((X_{t+1}^{s}, X_{t+1}^{M})\big)\right)\right] - \frac{(1-m_1)}{P_t^s}[E^q - E(X_{t+1}^{s})] \qquad (A1)$$

Multiplying (A1) by the market-value weight, $w_s = \frac{n_s^* P_t^s}{P_t^M}$, and aggregating across all firms in the market:

$$E[R_{t+1}^{M}] = R_F + \frac{\gamma}{P_t^M}\, Var(X_{t+1}^{M}) \qquad (A2)$$

(A2) follows as $Cov^q - Cov\big((X_{t+1}^{s}, X_{t+1}^{M})\big)$, which is the difference between cluster average covariance and firm s covariance, aggregates to zero. Similarly, $E^q - E(X_{t+1}^{s})$ aggregates to zero. One can solve for γ as follows:

$$\gamma = \frac{(E[R_{t+1}^{M}] - R_F)}{Var(R_{t+1}^{M})P_t^M} \qquad (A3)$$

Substituting (A3) into (A1):

$$E[R_{t+1}^{s}] = R_F + \frac{(E[R_{t+1}^{M}] - R_F)}{Var(R_{t+1}^{M})P_t^M P_t^s}\left[Cov(X_{t+1}^{s}, X_{t+1}^{M}) + (1-m_2)\left(Cov^q - Cov\big((X_{t+1}^{s}, X_{t+1}^{M})\big)\right)\right] - \frac{(1-m_1)}{P_t^s}[E^q - E(X_{t+1}^{s})] \quad (A4)$$

Substituting $\delta_M = E[R_{t+1}^{M}] - R_F$, recognizing that $\beta_s = \frac{Cov(R_{t+1}^{s}, R_{t+1}^{M})}{Var(R_{t+1}^{M})}$, $\beta_i = \frac{Cov(R_{t+1}^{i}, R_{t+1}^{M})}{Var(R_{t+1}^{M})}$, substituting from (2.2) for E^q, and from (2.3) for Cov^q leads to:

$$E[R_{t+1}^{s}] = R_F + \delta_M\,\beta_s + (1-m_2)\delta_M\left[\frac{\sum_i^{N_q} P_t^i \beta_i}{P_t^s N_q} - \beta_s\right] - (1-m_1)\left[\frac{\sum_i^{N_q} P_t^i E\big[R_{t+1}^{i}\big]}{P_t^s N_q} - E[R_{t+1}^{s}]\right] \qquad (A5)$$

$$\Rightarrow E[R^s_{t+1}] = R_F + \delta_M\,\beta_s + (1-m_2)\delta_M\left[\frac{\sum_i^{N_q} w_i\beta_i\left(\frac{n_s^*}{n_i^*}\right)}{w_sN_q} - \beta_s\right]$$
$$- (1-m_1)\left[\frac{\sum_i^{N_q} w_iE\left[R^i_{t+1}\right]\left(\frac{n_s^*}{n_i^*}\right)}{w_sN_q} - E[R^s_{t+1}]\right] \qquad (A6)$$

Inserting $\varphi_i = \frac{n_s^*}{n_i^*}$, defining $\bar{\beta} = \sum_{i=1}^{N_q}\frac{\varphi_i w_i\beta_i}{N_q}$, and $\overline{ER} = \sum_{i=1}^{N_q}\frac{\varphi_i w_i E\left[R^i_{t+1}\right]}{N_q}$ leads to:

$$E[R^s_{t+1}] = R_F + \delta_M\,\beta_s + (1-m_2)\delta_M\left[\frac{\bar{\beta}}{w_s} - \beta_s\right] - (1-m_1)\left[\frac{\overline{ER}}{w_s} - E[R^s_{t+1}]\right] \qquad (A6)$$

$$\Rightarrow m_1E[R^s_{t+1}] = R_F + \delta_M\,\beta_s + (1-m_2)\delta_M\left[\frac{\bar{\beta}}{w_s} - \beta_s\right] - (1-m_1)\frac{\overline{ER}}{w_s} \qquad (A7)$$

Dividing both sides by m_1 and re-arranging leads to (2.8).

Appendix B

$$\frac{\partial\alpha_s}{\partial m_1} = \frac{1}{m_1^2}\left[\frac{\overline{ER} - \bar{\beta}(1-m_2)\delta_M}{w_s} - R_F - \beta_s\delta_M m_2\right] \qquad (B1)$$

For low values of β_s, $\frac{\partial\alpha_s}{\partial m_1} > 0$ and for high values of β_s, $\frac{\partial\alpha_s}{\partial m_1} < 0$. It follows that when m_1 rises, SML rotates in the clockwise direction. That is, SML flattens.

$$\frac{\partial\alpha_s}{\partial m_2} = \frac{-\bar{\beta}\delta_M}{m_1w_s} + \frac{\beta_s\delta_M}{m_1} \qquad (B2)$$

For low values of β_s, $\frac{\partial\alpha_s}{\partial m_2} < 0$ and for high values, $\frac{\partial\alpha_s}{\partial m_2} > 0$. It follows that when m_2 rises, SML rotates in the counter-clockwise direction. That is, SML steepens.