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A Note on the Estimation of Long-Run Relationships in Dependent Cointegrated Panels*

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Abstract. We address the issue of estimation and inference in dependent non-stationary panels of small cross-section dimensions. The main conclusion is that the best results are obtained applying bootstrap inference to single-equation estimators. SUR estimators perform badly, or are even unfeasible, when the time dimension is not very large compared to the cross-section dimension.

Keywords: Panel cointegration, FM-OLS, FM-SUR.

1 Introduction

The estimation of cointegrating relationships in heterogenous, dependent panels can be considered a still largely unsettled problem. Although efficient system methods are available (FIML by Groen and Kleibergen, 2003, DSUR by Mark, Ogaki and Sul, 2005, FM-SUR by Moon, 1999), they are all feasible only when (i) the number of time observations (T) is much larger than that of cross-section observations (N), and, (ii), there is no cointegration across units. Since both conditions are highly unlikely to hold two questions arise. First, is this really a problem from the empirical point of view? The second question stems from considering that efficiency improvements are desirable in order to have more accurate asymptotic interval estimates and tests. Since simulation inference applied to standard single-equation estimators has been shown to deliver good results (see *e.g.*, Psaradakis, 2001, Chang, 2004, Fachin, 2007), do we really need to use a system estimator? The first aim of our paper is thus to compare the estimation performances of single-equation and system estimators in non-stationary panels with short-run dependence across units. Since FM-SUR (contrary to DSUR) is feasible in systems of the dimension typically encountered in practice, we will concentrate on this estimator and

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its single-equation analogue, FM-OLS (Phillips and Hansen, 1990). Second, we will compare simulation and asymptotic inference performance of the FM-OLS estimator with those delivered by asymptotic inference on the FM-SUR estimator. We shall now first discuss the bootstrap inference procedures (section 2), then present the design and results of our Monte Carlo experiment (section 3), while some conclusions are drawn in section 4.

2 Bootstrap procedures

Consider for the sake of simplicity the case of two I(1) variables, Y and X, observed on a panel of N units and T time observations. We assume a linear long-run equilibrium relationship, with possibly heterogenous coefficients, holds in all units. Formally:

$$y_{it} = \theta_i + \beta_i x_{1it} + u_{it}^y \tag{1}$$

where $x_{it} = x_{it-1} + u_{it}^x$. In both cases $i = 1, \ldots, N$, and $t = 1, \ldots, T$. Bootstrap inference involves two key steps: first, constructing the pseudo-datasets; second, defining the test statistics or confidence intervals to be used. Let us examine them in turn.

When constructing pseudo-data sets from non-stationary dependent panels the key point is to reproduce the presence of dependence both in the time series and in the cross-section dimensions. The former aspect has been the subject of the vast debate, whose details are beyond the scope of this paper (for a review, see Politis, 2003). As in Di Iorio and Fachin (2007), we will obtain the bootstrap noises applying the Stationary Bootstrap (SB; Politis and Romano, 1994) to the residuals of the FM regressions. To preserve the cross-unit dependence structure we simply need to resample the entire $T \times N$ matrix of residuals. The systematic part of the bootstrap Data Generating Process (DGP) will depend on the purpose of the exercise: in the case of hypothesis testing it is the result of estimation under the null hypothesis to be tested, while for interval estimation of unconstrained estimation.

Summing up, when the aim is testing the hypothesis $H_{0:}\ \beta_i = \beta_i^{(0)}$ the bootstrap DGP is:

$$y_{it}^* = \widehat{\theta}_i + \beta_i^{(0)} x_{1it} + u_{it}^{*y} \tag{2}$$

while for interval estimation we use

$$y_{it}^* = \widehat{\theta}_i + \widehat{\beta}_i x_{1it} + u_{it}^{*y} \tag{3}$$

where $\widehat{\alpha}_i, \widehat{\beta}_i$ are the unconstrained FM-OLS estimates. In both cases $\widehat{u}_{it}^{*y}$ is obtained applying a Stationary Bootstrap algorithm to the unconstrained residuals $\widehat{u}_{it}^y = y_{it} - \widehat{\theta}_i - \widehat{\beta}_i x_{1it}$. A thorough discussion of the choice of mean block length is included in Paparoditis and Politis (2003). As usual, in the cases of two-tailed tests the bootstrap estimate of the p-value will be

$p^* = prop(|t_b^*| > t)$, with $t = s_{\beta_i}^{-1}(\widehat{\beta}_i - \beta_i^{(0)})$, $t_b^* = s_{\beta_i}^{*-1}(\widehat{\beta}_{ib}^* - \widehat{\beta}_i)$, and $\widehat{\beta}$ and $\widehat{\beta}_{ib}^*$ are the FM-OLS estimates of β_i computed respectively on the actual and on the $b-th$ boostrap pseudo-dataset ($b = 1, \ldots, B$). Finally, s_{β_i} and $s_{\beta_i}^*$ are the estimated standard errors of these estimators. One simple way to compute confidence intervals is to take the desired quantiles (Q)of the distribution of the $\widehat{\beta}_{ib}^{*\prime}$s. An α-level confidence interval for β_i is then simply given by $[Q_{\alpha/2}(\widehat{\beta}_i^*), Q_{1-\alpha/2}(\widehat{\beta}_i^*)]$, where $\widehat{\beta}_i^* = \left[\widehat{\beta}_{i1}^* \ldots \widehat{\beta}_{iB}^*\right]$ is the vector of estimates obtained from the B bootstrap datasets. In principle, basing the interval on a pivotal quantity should deliver better results. Psaradakis (2001) suggests the percentile-t interval $[\widehat{\beta}_i - Q_{1-\alpha/2}(\mathbf{t}_b^*)s_{\beta_i}, \widehat{\beta}_i - Q_{\alpha/2}(\mathbf{t}_b^*)s_{\beta_i}]$, where the Gaussian quantiles used in asymptotic inference are replaced by those of the bootstrap distribution (empirical estimate of the unknown small sample distribution of the studentized statistic). The superiority of the second type of interval depends entirely upon the quality of the estimates of the standard errors (see *e.g.*, Kilian, 1999). Hence, in our study we shall compute both type of intervals.

3 Monte Carlo experiment

3.1 Design

Moon and Perron (2004) carried out a simulation study in a traditional seemingly unrelated equations set-up of small system size (at most 4 equations, with up to 300 observations), concluding that system estimators were overall superior to single-equation ones. However, in non-stationary panel analysis the number of equations (units) is typically larger and that of time observations smaller. For instance, an international macroeconomic panel including data at annual frequency starting at the early 1970's for the largest world or european economies will have 10-20 units and 30-40 time observations. Does Moon and Perron's conclusion hold for this sort of systems as well? To shed some light on the issue we will run a simulation experiment based on the DGP by Moon (1999). As we will see, our conclusions will in fact be rather different from Moon and Perron's. The details of the DGP are as follows. In each unit of the panel two right-hand side $I(1)$ variables (X_1, X_2) and a left-hand side variable (Y) are linked by a linear long-run equilibrium relationship:

$$y_{it} = \theta_i + \beta_{1i}x_{1it} + \beta_{2i}x_{2it} + u_{it}^y, i = 1, \ldots, N; \tag{4}$$

$$x_{kit} = x_{kit-1} + u_{kit}^x, k = 1, 2;\ i = 1, \ldots, N. \tag{5}$$

The regression coefficients are generated as Uniform variates, respectively $\theta_i \sim Uniform(2, 4)$ and $\beta_{ki} \sim Uniform(1, 3)$, where $k = 1, 2$. The errors of equations (5) and (4) are drawn from a Multivariate Normal distribution with non-diagonal covariance matrix, so that there is feedback across equations and units. More precisely, letting $\mathbf{u}_t^x = [\mathbf{u}_{1t}^{x\prime}\mathbf{u}_{2t}^{x\prime}\ldots\mathbf{u}_{Nt}^{x\prime}]'$, where $\mathbf{u}_{it}^{x\prime} = [u_{1it}^x u_{2it}^x]'$ and $\mathbf{u}_t^y = [u_{1t}^y u_{2t}^y \ldots u_{Nt}^y]'$, we have

$$\underset{(N+2N)\times 1}{\begin{bmatrix} \mathbf{u}_t^y \\ \mathbf{u}_t^x \end{bmatrix}} = MN\left(\begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \underset{(N+2N)\times(N+2N)}{\begin{bmatrix} R & \Delta \\ \Delta' & \Phi \end{bmatrix}}\right),$$

where R is a full $N \times N$ matrix governing the dependence across units in the $u_{it}^{y\prime}$s, Δ is a $N \times 2N$ matrix governing the dependence between the u^x and u^y noises, and finally Φ is a $2N \times 2N$ matrix governing the dependence in the $u^{x\prime}s$ within and across units. Since Moon and Perron report the performances of both FM-OLS and FM-SUR estimators to be negatively affected by the degree of endogeneity of the X's, we decided to control accurately δ, imposing the homogeneity assumption. In our simulations we considered different values for δ, concluding that the relative performance of system and single-equation estimators does not seem to change with the degree of endogeneity. Here for space constraints we will thus report only results for $\delta = 0.4$. The Δ matrix has a block form ensuring that there is constant correlation between the noise of any X and that of the relevant Y equation, and no correlation across units:

$$\underset{N\times 2N}{\Delta} = \begin{bmatrix} \delta\ \delta\ 0\ 0 \ldots 0\ 0 \\ 0\ 0\ \delta\ \delta \ldots 0\ 0 \\ \vdots\ \vdots\ \vdots \quad \ddots \quad \vdots \\ 0\ 0\ 0\ 0 \ldots \delta\ \delta \end{bmatrix}.$$

We instead allow heterogeneity across units in the dependence parameters, generating them as $Uniform(0.3, 0.4)$ random variates; without loss of generality we assume different X's in the same unit to be orthogonal. Letting $\phi_{lk}^{(ij)} = cov(u_{li}^x, u_{kj}^x)$, the covariance between the noise of X_l in the i^{th} unit and X_k in the j^{th} unit, we then have:

$$\underset{2N\times 2N}{\Phi} = \begin{bmatrix} 1 & 0 & \phi_{11}^{(12)} & \phi_{12}^{(12)} & \ldots & \phi_{11}^{(1N)} & \phi_{12}^{(1N)} \\ 0 & 1 & \phi_{21}^{(12)} & \phi_{22}^{(12)} & \ldots & \phi_{21}^{(1N)} & \phi_{22}^{(1N)} \\ \phi_{11}^{(21)} & \phi_{12}^{(21)} & 1 & 0 & \ldots & \phi_{11}^{(2N)} & \phi_{12}^{(2N)} \\ \phi_{21}^{(21)} & \phi_{22}^{(21)} & 0 & 1 & \ldots & \phi_{21}^{(2N)} & \phi_{22}^{(2N)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \phi_{11}^{(N1)} & \phi_{12}^{(N1)} & \phi_{11}^{(N2)} & \phi_{12}^{(N2)} & \ldots & 1 & 0 \\ \phi_{21}^{(N1)} & \phi_{22}^{(N1)} & \phi_{21}^{(N2)} & \phi_{22}^{(N2)} & \ldots & 0 & 1 \end{bmatrix}$$

The choice of the time and cross-section sample sizes have been fixed trying to strike a balance between empirical relevance, which suggests medium $N's$ and small $T's$, and the requirements of the SUR estimator, which is feasible only with a rather large T/N ratio. We thus fixed $N = 5, 10$ and $T = 50, 100$. Finally, we set the number of bootstrap redrawings (B) and Monte Carlo simulations (M) to 1000.

3.2 Results

In Tables 1 and 2 we report summary statistics of the performances of respectively FM-OLS and FM-SUR estimators, further averaging over units and variables the usual Monte Carlo means.
Point estimation performance is evaluated by the average absolute relative bias $100 \times (2N)^{-1} \sum_k^2 \sum_i^N \mid M^{-1} \sum_m^M (\widehat{\beta}_{kim} - \beta_{ki})\beta_{ki}^{-1} \mid$ while the dispersion by the relative Monte Carlo standard error. The first remark in order is that the SUR procedure turned out to be practically unfeasible for $T = 50$ and $N = 10$. The covariance matrix, although not exactly singular, was always so ill-conditioned that the estimators turned out to be highly numerically unstable even using a generalised Moore-Penrose inversion routine. Hence, we do not report them here.

Table 1
FM-OLS: Estimation and Inference Performance

				Coverage			Type I Err	
T	N	$\overline{bias}$	$\overline{s.e.}$	*Asy*	*boot*	*boot-t*	*Asy*	*boot*
50	5	0.64	3.40	84.99	89.71	90.08	15.01	2.78
	10	0.57	4.49	86.00	90.23	90.27	14.00	3.36
100	5	0.25	1.86	90.40	91.99	92.00	9.60	0.78
	10	0.20	1.88	90.55	91.93	93.34	9.45	3.51

$\overline{bias}$: $100 \times (2N)^{-1} \sum_k^2 \sum_i^N \mid M^{-1} \sum_m^M (\widehat{\beta}_{kim} - \beta_{ki})\beta_{ki}^{-1} \mid$

$\overline{s.e.}$: $(2N)^{-1} \sum_i^N \sum_k^2 \left[\left(\sqrt{M^{-1} \sum_m^M (\widehat{\beta}_{kim} - \overline{\widehat{\beta}}_{ki})^2} \right) \beta_{ki}^{-1} \right] \times 100;$

Coverage: proportion of 5% confidence intervals including the true value of the coefficient of interest;

Since this (T, N) combination can be considered rather representative of the sample sizes used in applied work on non-stationary panels (with indeed the time sample often actually smaller than this one) this is an important finding. In the other cases both estimators are essentially unbiased even with the smaller time sample, although the SUR estimator is always somehow more biased than the OLS one. For instance, for $T = 50$, $N = 5$ and $\delta = 0.2$ the average relative bias is 0.40% for the former and 0.34% for the latter. The Monte Carlo standard errors are very similar, with again the single-equation estimator always slightly superior to the SUR one (for the same case, respectively 3.98% and 3.41%). Coverage and Type I errors of Gaussian inference on FM-OLS are both disappointing, with severe overrejection and undercoverage. For the same parameters combination quoted above the Type I error of a 5% test is 14.63% and the coverage, as a consequence, 85.37%. Both problems are partially solved using the bootstrap, although coverage is inferior to nominal for both the basic and the studentized intervals (respectively, 89.38% and 90.11%) and the test underrejects (Type I error 2.36%). On the other hand, the performance of asymptotic inference on the SUR estimator is simply disastrous, with Type I errors close to 50% when T is not large with respect to N (that is, always for $T = 50$ and when $N = 10$ for $T = 100$) and around 20% even in the more favorable case of $T = 100$, $N = 5$. The reason for this extremely poor performance, not obvious from the bias and Monte Carlo variability statistics, is found in Table 3: the detailed results for the case $T = 50$, $N = 5$, $\delta = 0.4$[1] show that the standard formulas for the variance of the SUR estimator grossly underestimate its actual variance.

Table 2
FM-SUR: Estimation and Inference Performance

				Coverage	Type I Err
T	N	$\overline{bias}$	$\overline{s.e.}$	Asy	Asy
50	5	0.79	4.04	52.56	47.44
	10	-	-	-	-
100	5	0.35	1.94	79.72	20.28
	10	0.51	2.22	55.48	45.52

-: not available (numerical overflow);
all symbols and abbreviations: see Table 1

Table 3
Bias and Variability of FM-OLS and FM-SUR estimators
$T = 50, N = 5, \delta = 0.4$

		$bias$		$MC\ s.e.$		$\overline{\widehat{\sigma}}$		$\overline{\widehat{\sigma}} - MC\ s.e$	
Unit		OLS	SUR	OLS	SUR	OLS	SUR	OLS	SUR
1	β_1	-0.13	0.44	5.1	5.3	3.8	2.2	1.2	3.1
	β_2	0.25	0.57	6.4	7.9	5.2	3.0	1.2	4.9
2	β_1	-0.33	0.21	3.0	3.2	2.3	1.4	0.7	1.9
	β_2	1.09	1.27	6.0	6.6	4.7	2.7	1.4	3.9
3	β_1	0.65	0.60	10.7	13.6	8.1	4.3	2.6	9.3
	β_2	-0.38	0.35	5.7	6.8	4.3	2.3	1.4	4.5
4	β_1	0.52	2.34	9.8	12.7	8.2	4.5	1.6	8.2
	β_2	-0.38	0.01	3.8	4.4	3.1	1.8	0.8	2.7
5	β_1	2.35	0.96	11.1	13.5	8.5	4.6	2.6	9.0
	β_2	0.23	1.10	4.7	5.3	3.6	2.1	1.1	3.2

MC $s.e.$: Monte Carlo s.e.; $\overline{\widehat{\sigma}}$: average estimated standard error $\times 100$;
other symbols and details: see Table 1.

4 Conclusions

Our main conclusion is very simple: on the basis of our simulation exercise the best option in non-stationary panel analysis seems to be given by single-

[1] Detailed results for the other cases do not provide any additional insights and thus are not reported here, but, as customary, available on request.

equation estimators with bootstrap inference. The potential efficiency gains of SUR-type estimators remain such even in the restrictive case of no long-run relationships across units. In fact, when the time dimension is not very large relatively to the cross-section dimension the covariance matrix is likely to be so ill-conditioned to make the resulting estimates essentially meaningless. Further, even when some meaningful point estimates can be obtained, their variance is likely to be grossly underestimated by standard formulas, with disastrous effects on inference. These conclusions are in stark contrast to Moon and Perron's (2004). However, this should not come as a surprise. The properties of SUR estimators depend critically upon the quality of the estimate of the covariance matrix. Obtaining good estimates may be an easy task in small systems, such as those examined by Moon and Perron, but may became very difficult in even slightly larger systems, such those considered in our study.

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