A Value-Added Social Welfare Optimization Problem For Micro-Sustainable Development

Selim, Tarek

8 August 2000

Tenth World Conference on Social Economics
University of Cambridge (UK)

A VALUE-ADDED SOCIAL WELFARE OPTIMIZATION PROBLEM FOR MICRO-SUSTAINABLE DEVELOPMENT

TAREK H SELIM, USAID DOCTORAL FELLOW
DEPARTMENT OF ECONOMICS, GEORGE WASHINGTON UNIVERSITY
(at time of submission)

CURRENT AFFILIATION:
TAREK H. SELIM, PROFESSOR OF BUSINESS ECONOMICS AND STRATEGY
AMERICAN UNIVERSITY IN CAIRO
tarekhassanselim@gmail.com

ABSTRACT

This paper attempts to present a value-added social welfare optimization problem as applied to capital investment projects with environmental amenities. A dynamic model is proposed with the objective of maximizing net social welfare for a desired flow of investment capital, given allowances for uncertainty, risk, sustainability, resource depletion, and social demand constraints. An input-output vector of production is optimized, with the above constraints imposed, and its solution analyzed within the scope of micro-sustainable development. The model's focal outcome has been that long-run sustainability is a function of cumulative expected short-run system failures, implying that environmental sustainability could be viewed as cumulative disturbances of non-optimal deviations from a long-run social extraction path, the latter taken in both stock and flow-rate terms. Contrary to classical theory, the dynamics of social optimality strictly require social re-investments to generate social capital returns exceeding marginal social cost.

Keywords
social welfare, sustainable development, resource depletion, cost-benefit analysis

JEL Classification Codes
D6-H4-Q2

Presented on August 8, 2000

Research paper submitted for the Tenth World Conference on Social Economics, Emmanuel College, University of Cambridge (UK), August 6-9, 2000. My thanks to Arun Malik, John Kwoka, Steven Smith, Harry Watson, Stanley Johnson, Sanjeev Agarwal, and Eva Neterowicz for encouragement, help and valuable suggestions.

A VALUE-ADDED SOCIAL WELFARE OPTIMIZATION PROBLEM FOR MICRO-SUSTAINABLE DEVELOPMENT

INTRODUCTION

A project is "the investment of human, material and capital resources that will provide a specific amount of goods and services to a society, and that will actively contribute to the development process in an economy" (Cusworth and Franks 1993). Project appraisal, on the other hand, can be defined as the feasibility assessment of expected net return from a given supply of social capital, with the objective of maximizing profitability, minimizing risk, and conforming to legal and cultural standards within the project's physical, social and institutional environments (Shaner 1979, see also Selim 1999). Such definitions may narrow the focus of analysis more towards the inner project cycle, rather than considering the total environmental resources required, and ultimately depleted, towards the attainment of given social capital objectives.

Among the interesting dimensions along the lines of this subject, Dasgupta's early social welfare arguments in project appraisal (Dasgupta 1972), calling for a socio-economic assessment of investment capital, also heavily argued by Professor Arrow, has gained much recent attention, albeit relatively late. In addition, the works of Brooke 1990 in uncertainty evaluation, and Zimmerman 1970, Lal 1974 and Olsen 1977 in social impact assessment, and the works of Aulin 1989, 1990, & 1992, DeNeufville 1990, and Beenhakker 1996 in systems analysis and input-output modeling, provide a complementary optimizing technique towards the vision of the early adopters of social welfare evaluation. It must be noted, though, that most of these schools of thought fall under static optimization of maximal social welfare.

It is the objective of this paper to attempt to present a dynamic social welfare optimization problem as applied to capital investment projects with environmental amenities, based on value-added optimization with financial-environmental tradeoffs. The approach is rigorously mathematical, yet economic insights and logical intuition are fairly discussed. The scope of the paper is built on four inter-related modules. The first module generates social capital as a supply function to society, while the second module allows for uncertainty and risk allowances towards the social supply function. The third module, which acts as a vector of dynamic constraints, imposes upper and lower-bound sustainability and dynamic resource depletion constraints on the dynamic flow of investment capital to society. Social re-investments as a shadow price for resource depletion are also considered. The fourth module assumes consumption of social capital via demand. The complete model is then presented, solved, and its implications discussed. A conclusive assessment of the research finishes last.

THE FOUR MODULES

Module #1: The Generation of Social Capital (Supply)

A project's value, using the benefit-cost criterion, and utilizing the weak sustainability constraint for environmental abatement, runs as follows:

(1) $$PV = -I + \sum_{i=1}^{n} \frac{B_t - C_t - E_t}{\prod_{1}^{t}(1+r_t)}$$

(2) $$\sum_{t=0}^{n} \frac{E_t}{\prod_{i=1}^{t}(1+r_t)} \leq \sum_{t=0}^{n} \frac{a_t}{\prod_{i=1}^{t}(1+k_t)} \quad ,$$

whereby private and social benefits are summarized in B_t , social costs in C_t , and environmental abatement efforts in E_t , for a project life of n years, and with an effective social discount rate of r_t per year. Fixed asset allocations are assumed to be internalized in the initial capital investment, I. The magnitude of the shadow project, a_t , is augmented by the social opportunity cost of capital, k_t.

The price level of output, $P_t(Q_t)$, is indexed by the minimum average variable cost in the short-run, whereas in the long-run, by the marginal cost of production (at minimum average total cost), as follows:

(3) $$P_t(Q_t)^{SR} = MC_t = \min AVC_t = \left.\frac{\partial C_{x_t}}{\partial Q_t}\right|_{\min AVC_t} = S^{SR}$$

(4) $$P_t(Q_t)^{LR} = \min AC_t = \min \frac{\sum_{j=1}^{m} C_{x_j}}{Q_t} \quad ,$$

and it follows that the value of investment capital to society can be interpreted as:

(5) $$\int_0^t \left\{ -I + \left[\sum_{i=1}^{n} Q_t P_t(Q_t) - C_t - E_t \right] \left[\frac{1}{\Gamma_{\tau=1}^{t}(1+r_\tau)} \right] \right\} dt$$

and that the value-added supply of social capital can be written as (see Appendix I-3):

(6) $$VA = (\eta)(V) = \left[\frac{\sum_{t=1}^{n} (Q_t P_t(Q_t))}{I + \sum_{t=1}^{n} a_t} \right] \left[-I + \sum_{t=1}^{n} \frac{(Q^*{}_t\ P_t - C_t - E_t)}{\prod_{1}^{\tau} (1+r_\tau)} - A_r\ \frac{\varphi.\sigma_{EPV}.\sigma_Y}{E(Y)} \right]$$

Such a generalization abstains from being comprehensive, since society's total capital resources also include human capital, created capital, and intellectual capital, not mentioning the positive effects of those on social welfare and sustainable development seems rather limiting. However, within the confines of this paper, it will be continually assumed throughout that the supply of social capital constitutes physical and environmental capital resources, and that the supply of human and intellectual capital act as a residual to the physical supply function of output, with the inherent assumption that wages and rents are competitive enough to account for investments in human capital and education.

Module #2: Uncertainty & Risk (Allowances)

Equation (6) above, when aggregated over all society's investments in physical and environmental resources, provide a value-added social supply function to output. The supply of social capital, however, is not *ceteris paribus* in the sense that the change in investment capital from one period to the next can change the level of aggregate social supply with time. It is therefore reasonable to assume a social supply function which incorporates static value-added investments, dynamic flow of capital, and expected likelihood return on social capital, as follows:

(7) $$S = S\left\{ \eta V[\pi(P(Q))], H[\dot{Q}(\dot{x}_c)], \ell[\mu(\overline{Q})] \right\}$$

It is the dynamic maximization of this social supply function, subject to sustainability and resource depletion constraints, that will give rise to an optimal *social extraction path*. The optimal allocation of resources to society obeys a dynamic extraction function, where output and resource utilization are taken in both stock and flow-rate terms, and where society's depletion of its critical resources are traded-off in optimality by value-added capital returns.

Uncertainty analysis of any static function can be written as weighted-average expected values, with a second-order variance correction, depending on the probability distribution of the uncertain random variables in that function. The likelihood function in Equation (7) above is based on such an approximation, yet with a maximum likelihood estimation of the leading variables (physical output Q_t and critical inputs $x_{c,t}$), and with the assumption of non-linear approximations of the form (Lakshminarayan 1996, Freund 1992, Ravindran 1987, and Pouliquen 1970):

$$E\{\pi(\overline{Q})\} = \int_{-\infty}^{\infty} \eta V(Q)\pi[Q,\dot{Q}]dQ \tag{8}$$

$$E\{\pi[Q,\dot{Q}]\} = \ell\left\langle[\pi(Q),\pi(\dot{Q})\right\rangle = \ell\left[a_0 + a_1\pi(\overline{Q}) + a_2(\sigma^2_{\pi(\overline{Q})} + \pi(\overline{Q})^2), \prod_{i=1}^{m} f(x_c,\theta) = \frac{1}{\sigma_\pi\sqrt{2\Pi}}\exp\left\{\frac{-1}{2\sigma^2_\pi}\sum_{t=1}^{m}\pi(\overline{Q}) - \mu(\overline{Q})\right\}\right] \tag{9}$$

$$E(Q) = \overline{Q} = \int_{-\infty}^{\infty} f_1(y)\frac{df_1(y)}{dy} f_1[f_2(Q)]\left|\frac{df_2(Q)}{dy}\right|dy \tag{10}$$

Equations (8)-(10) ensure the validity of statistical invariance, and provide a maximum likelihood estimation for the value-added social supply function in (7), and are also in accordance with the Cramer-Rao lower-bound Inequality theorem, and thus provide a minimum variance unbiased estimate of the proposed function.

Assuming all investors are risk-averse, the static investment supply of capital in (5) can be re-written to be (see Appendix I-3):

$$\pi(\widetilde{Q}) = \pi(\overline{Q}) - \frac{U''[\pi(Q,\dot{Q})]}{U'[\pi(Q,\dot{Q})]}\varphi\sigma_{\pi(Q)}\sigma_y \tag{11}$$

Equation (11) assures that the social supply function in (7) is maximized based on iso-social utility levels, and not solely based on private financial return. The assumptions are that the maximization of the social supply likelihood function is an aggregate of private financial returns augmented to produce social utility levels, which are assumed to consist of cardinal utility maximizations with a uniform composition of relative risk aversions, and that the social welfare function to be maximized has a continuous monotonic form as of (7), and that the static and dynamic flow-rate of investment supply to society are additively separable as in (9).

Module #3: Sustainability & Resource Depletion (Constraints)

Sustainability, as a constraint within the maximization of social welfare, holds in static optimization of a partial equilibrium externality model the result that the shadow price of an externality equals its social marginal damage, and that social marginal abatement costs are equal to the unit price of abatement or mitigation efforts (Mishan 1988, Conrad and Clark 1999, Hanley et.al. 1997, and Pearce and Turner 1990). From a cost-effective standpoint, this also implies that marginal abatement efforts have to be equated across firms, and that the marginal cost of reducing concentration (of an externality) should be equated for each receptor site, for a second-best minimization of a total social cost function.

In a dynamic setting, sustainability can be viewed as having a dual role for social cost minimization. The first role (constraint) is that of system failure. System failure could act as a lower-bound sustainability constraint with the objective of ensuring that the flow-rate of adequate inputs (or socially "critical" inputs) are actually attained over time. A dynamic flow-rate below a critical threshold level of sustainability would then result in lack of an adequate flow of production, and arguably would result in "system failure". Such a generalization would prove helpful only if society's capacity for production can be accurately known, and if the socially critical variable inputs of production are readily identifiable. Such a dynamic sustainability constraint, though critical, can be easily modeled as (see Appendix I):

$$\dot{Q}_L \geq f_Q[\dot{x}_{1c}, \dot{x}_{2c}, \dot{x}_{3c}, ..., \dot{x}_{mc}] \tag{12}$$

The second role (constraint) is that of environmental instability. Abstaining from political and regional risks, social instability from an environmental point of view can simply be treated as a bounded flow function, where an upper sustainability constraint can be imposed on society (via Pigouvian taxes or through a marketable permits scheme), such that the excess flow-rate of concentrated waste is adequately re-cycled at minimum social cost. Such an upper-bound constraint on social capacity can be modeled as (see Appendix I-3):

$$(\dot{Q}_H - \dot{Q}_{rcy}) \leq \dot{Q}_w \tag{13}$$

$$C_g^{ep} = \frac{Q}{\Pi \sigma_y \sigma_Z \bar{u}} \exp\left\langle \frac{-H^2}{2\sigma_Z^2} - \frac{y^2}{2\sigma_y^2} \right\rangle \tag{14}$$

$$C_{ga}^{g} = \frac{Q}{\Pi \bar{u} (\sigma_y^2 + \sigma_{y_0}^2)^{1/2} \sigma_Z} \tag{15}$$

$$D[C_g^{ep}(Q), C_{ga}^{g}(Q), \varphi, U(E(Y))] \equiv Z[Q, \dot{Q}, \varphi] \tag{16}$$

$$r_p^* = t^* = MZ[Q, \dot{Q}, \varphi] \tag{17}$$

Equation (13) follows the second role of environmental sustainability (weakly imposes the instability constraint), whereas Equations (14) and (15) complement the instability constraint by providing for proxy functions of pollution concentration with respect to physical output. Equation (16) assumes an implicit social damage function, of which $U[\cdot]$ denotes the social utility level attained for a value-added aggregate capital supply of *Y*, and is also assumed to be a function of output and its flow-rate to the social supply function in (7) above. The parameter φ acts a correlation coefficient between capital supply and social demand. Equation (17) is an added assumption for the benefit of no market failures in environmental abatement, where Pigouvian taxes and tradable pollution permits are assumed to offset pollution externalities, though not necessarily completely correcting for it, through incentives for mitigation.

Module #4: Social Re-investments & the Consumption of Social Capital (Demand)

Modules 1-thru-3 have not considered the option of social re-investments as a second-best vehicle to combat resource depletion. If the weak sustainability constraints in (2), (12), and (13) all hold true, then it must also be true that there are enough social incentives for mitigation and environmental abatement. Such incentives are assumed to be driven by the social desire to consume. With this "fixation", society is assumed to save enough resources for re-investment of depleted capital resources such that net savings are sufficient to produce a future value-added capital, after *n* years, which can be used as a re-investment mechanism for future resource acquisition. In a sense, society re-invests within itself to re-count depleted capital resources.

Such a correction, if undertaken, would require an added static constraint, whereby social re-investments are taken to be the shadow price of resource depletion (see Appendix I-3) as follows:

$$\frac{AW(Q)-X(\dot{Q})}{r}\left[(1+r)^{n}-1\right]\geq I_{R}(1+f_{r})^{n} \tag{18}$$

However, the static constraint in (18) needs to be dynamically stable when maximizing for a dynamic social supply function, such as that of (7). For a price-taking firm with a capital supply of Q_t, and a desirable resource extraction flow-rate of $u(t)=-\dot{Q}(t)$, and with an initial capital stock of $x(0)=x_0$, and assuming a market-clearing condition for social demand of $u(t) = SD[P(Q,\dot{Q},t)]$, we can then write the Hamiltonian function for optimal resource depletion as:

$$\underset{u(t),T}{Max}\ H = \int_{0}^{T} e^{-rt}\{P(t)u(t)-C(u(t),x(t))-E(t)SD[P(Q,\dot{Q},t)]\}dt \tag{19}$$

and it follows that the current-value Hamiltonian is of the form:

$$H_c = P(t)SD[\cdot]-C(u(t),x(t))-E(t)SD[\cdot]-m(t)u(t)\,. \tag{20}$$

Using standard dynamic optimization techniques, we solve (19) and (20) for an optimal resource extraction path, governed by the dynamic path of social re-investments as:

$$\dot{m}(t)=rm(t)=\dot{P}(Q,\dot{Q},t)-C_{ux}\dot{x}-C_{uu}SD[P(\cdot)]-\dot{E}(t) \tag{21}$$

$$\dot{u}(t)=\frac{rP(Q,\dot{Q},t)-rC_{u}-E(t)\left[r-\dot{E}(t)\big/E(t)\right]}{1\big/SD'(P(Q))-C_{uu}} \tag{22}$$

Equation (22) specifically dictate the *optimal path of social re-investments* required for the attainment of a dynamic resource depletion constraint, with the additional requirement that the static constraint in (18) has to hold for every period of re-investment.

THE MODEL

The above four modules, after re-formulation, can be integrated together towards a hybrid dynamic optimization model for maximum social surplus, subject to sustainability, resource depletion, and social re-investment constraints, and such that aggregate social supply through the investment of capital resources is always met by aggregate social demand through the consumption of social capital. Excess supply is assumed to be re-invested for future resource acquisition.

The value-added supply of social capital [in (6) above] is augmented by the optimum social extraction path [via the optimal dynamic solution of (19)], and with the imposed sustainability constraints of (2) and (12) through (15), and with the resource depletion constraint of (18), and with uncertainty and risk allowances accounted for by (8), (9), and (11), and with the assumptions of (7), (16), and (17). The social supply function proposed in (7) is maximized over a horizon of time *T*.

The reduced model runs as follows:

(23) $$Max\ S = \int_{t=0}^{T} S\left\{\eta V[\pi(P(Q))], H[\dot{Q}(\dot{x}_c)], \ell[\mu(\overline{Q})]\right\} dt$$

where:

(a) $$E\{\pi(\overline{Q})\} = \int_{-\infty}^{\infty} \eta V(Q)\pi[Q,\dot{Q}]dQ$$

(b) $$V(t) = \int_{0}^{t} \left\{ -I + \left[\sum_{i=1}^{n} Q_t P_t(Q_t) - C_t - E_t \right] \left[\frac{1}{\Gamma_{\tau=1}^{t}(1+r_\tau)} \right] \right\} dt$$

(c) $$\eta(t)V(Q_t) = \int_{0}^{t} \left[\frac{\sum_{t=1}^{n}(Q_t P_t(Q_t))}{I + \sum_{t=1}^{n} a_t} \right] \left[-I + \sum_{t=1}^{n} \frac{(Q*_t\ P_t - C_t - E_t)}{\prod_{1}^{\tau}(1+r_\tau)} - \frac{U''[\pi(Q,\dot{Q})]}{U'[\pi(Q,\dot{Q})]} \varphi \sigma_{\pi(Q)} \sigma_y \right] dt$$

(d) $$\underset{u(t),T}{H}[\dot{Q}(x(t)] = \int_{0}^{T} e^{-rt} \{P(t)u(t) - C(u(t),x(t)) - E(t)SD[P(Q,\dot{Q},t)]\} dt$$

(e) $$\ell\left\langle[\pi(Q),\pi(\dot{Q})\right\rangle = \ell\left[a_0 + a_1\pi(\overline{Q}) + a_2(\sigma^2_{\pi(\overline{Q})} + \pi(\overline{Q})^2), \prod_{i=1}^{m} f(x_c,\theta) = \frac{1}{\sigma_\pi\sqrt{2\Pi}} \exp\left\{ \frac{-1}{2\sigma_\pi^2} \sum_{t=1}^{m} \pi(\overline{Q}) - \mu(\overline{Q}) \right\} \right]$$

subject to:

(f) $$\sum_{t=0}^{n} \frac{E_t}{\prod_{i=1}^{t}(1+r_t)} \le \sum_{t=0}^{n} \frac{a_t}{\prod_{i=1}^{t}(1+k_t)}$$

(g) $$\dot{Q}_L \ge f_Q[\dot{x}_{1c}, \dot{x}_{2c}, \dot{x}_{3c}, ..., \dot{x}_{mc}]$$

(h) $$(\dot{Q}_H - \dot{Q}_{rcy}) \le \dot{Q}_w(C_g^{ep}, C_{ga}^{g})$$

(i) $$\frac{AW(Q) - X(\dot{Q})}{r}\left[(1+r)^n - 1\right] \ge I_R(1+f_r)^n$$

(j) $$\dot{u}(t) = \frac{rP(Q,\dot{Q},t) - rC_u - E(t)\left[r - \dot{E}(t) \big/ E(t) \right]}{1 \big/ SD'(P(Q)) - C_{uu}}$$

(k) $$D[C_g^{ep}(Q), C_{ga}^{g}(Q), \varphi, U(E(Y))] \equiv Z[Q,\dot{Q},\varphi]$$

(l) $$u(t) = -\dot{Q}(t) \quad \forall t \in [0,T)$$

(m) $$u(t) \ge 0 \quad \forall t \in [0,T]$$

(n) $$x(0) = x_0 > 0\,.$$

The inherent objective function to be maximized in (23) is net social surplus over a period of time T. The above dynamic optimization problem, therefore, is not an infinite-horizon problem. Net social surplus, through the supply and sustainable consumption of social capital, represents a finite optimizing solution to social welfare. The dynamic social supply function represented in (23) contains within its boundaries net value-added investment capital, a social extraction path, and a likelihood function for static output.

The model also casts a multitude of static and dynamic constraints, with a hope for a first-best social optimum. The first constraint in (f) is the weak sustainability constraint for environmental abatement. Constraints (g) and (h) represent the lower and upper bound environmental constraints of system failure and environmental instability, respectively. Next, (i) represents a constraint for resource depletion, and (j) is the dynamic constraint for social re-investments. In addition, (k) assumes a social damage function, (l) maintains the presumption of the market-clearing condition for resource scarcity, and (m) and (n) are inequality constraints for the initial stock and flow-rate of resource extraction. Also, for the benefit of a free Hamiltonian-Transversality condition, the terminal time T is assumed to be known.

SOLUTION

The solution to the above model is radically complex. However, after numerous simplifications and assumptions, the reduced form solution follows a *dynamic optimization path*, as follows:

$$\Phi[Q(x_t),\dot{Q}(\dot{x}_{c,t}),t]=\int_0^T e^{-rt}\left\{\int_0^t\int_{-\infty}^{\infty}\left[\frac{\sum_{t=1}^{n}(Q_tP_t(Q_t))}{I+\sum_{t=1}^{n}a_t}\right]\left[-I+\sum_{t=1}^{n}\frac{(Q^*{}_t\,P_t-C_t-E_t)}{\prod_{1}^{\tau}(1+r_\tau)}-\frac{U''[\pi(Q,\dot{Q})]}{U'[\pi(Q,\dot{Q})]}\varphi\sigma_{\pi(Q)}\sigma_y\right]dQdt\right\}dt$$

$$-\int_0^T e^{-rt}E(t)\left\{D[C_g^{ep}(Q),C_{ga}^{g}(Q),\varphi,U(E(Y))]\right\}dt$$

$$-\int_0^T\int_{-\infty}^{\infty}\left|E[\pi(Q),\pi(\dot{Q})]-\ell\left[a_0+a_1\pi(\overline{Q})+a_2(\sigma^2_{\pi(\overline{Q})}+\pi(\overline{Q})^2),\prod_{i=1}^{m}f(x_c,\theta)=\frac{1}{\sigma_\pi\sqrt{2\Pi}}\exp\left\{\frac{-1}{2\sigma_\pi^2}\sum_{t=1}^{m}\pi(\overline{Q})-\mu(\overline{Q})\right\}\right]\right|dQdt$$

Integrating the above simplified dynamic maximization problem with the proposed constraints in (23)(f)-to-(n) above, yields several co-state functions and equations of motion, of which the focal equation of motion amounts to (see Appendix II):

$$\text{(24)}\qquad \dot{\lambda}_1(\lambda_1,Q,r,Z,\dot{u})_{LR-social}=\dot{\lambda}_2[\lambda_2,\eta V\{\pi(Q,\dot{Q})\}]_{SR-\pi(Q^*)}\cdot\left[1-\sum_{m=0}^{T}\frac{\partial\Phi(\dot{Q}_m,\dot{x}_{c,m})}{\partial Q_m}\right]$$

The *focal solution* in (24) basically states that the rate of social re-investments, as a shadow price of long-run resource sustainability, is a cumulative function of short-run system failures arising out of non-optimal production, and that the rate of consumption of social capital (via market-clearing social demand) follows the slack value of the resource depletion constraint, the latter also a function of the dynamics of value-added capital investment returns. In retrospect, optimum social surplus, as a consequence of the maximization of a social supply function, carries with it systematic re-investment alternatives in accordance with the rate of resource extraction, and in addition carries the dynamic burdens of cumulative historical system failures arising out of the depletion of capital resources for the benefit of social consumption.

In addition, as seen in Appendix II, *net returns on social capital* closely follow the following equilibrium conditions, assuming a piecewise continuous interior solution and no bang-bang control for dynamic resource extraction:

$$\text{(25)} \qquad \dot{P}(Q,\dot{Q},t) = rm(t) + C_{ux}\dot{x} + C_{uu}\frac{rP(Q,\dot{Q},t) - rC_u - E(t)\left[r - \dot{E}(t)\big/E(t)\right]}{1\big/SD'(P(Q)) - C_{uu}} + \partial Z[\cdot]\big/\partial Q\,\frac{dQ}{dx_c} + \frac{\partial Z[\cdot]}{\partial u(t)} + \dot{E}(t)$$

$$\text{(26)} \qquad H_c^* = \int \pi(Q,\dot{Q})dQ - m_1(t)\dot{Q}_{\max} + m_2(t)f_Q(\dot{x},u) - Z[\eta V(\cdot),U(Y)]$$

$$\text{(27)} \qquad \left[\partial\pi\big/\partial Q\right]\dot{Q} = \left[\partial Z\big/\partial V(\cdot)\right]$$

Equation (26) shows the optimum Hamiltonian function due to financial-environmental tradeoffs of resource depletion, with systematic short-run and long-run sustainability constraints and social damages internalized.

The implicit solution in Equation (27) has a very important implication: *marginal social benefits due to resource extraction, at equilibrium, have to equal total marginal damages to society divided by the optimum flow rate of social capital resources.*

Therefore, for an increasing flow-rate of value-added investments to society, dynamic optimality requires marginal social benefits to *strictly exceed* marginal social costs due to resource extraction at equilibrium. Such a necessary condition has to take effect in order for social re-investments to sufficiently correct for society's historical non-optimal deviations from a long-run social extraction path.

This implicit solution runs in sharp contrast to the classical theory of environmental economics.

Equation (25) is the pivotal solution for the equilibrium level of social capital returns. The net value-added capital gain from an investment supply of capital [$\dot{P}(Q,\dot{Q},t)$] equals the marginal opportunity cost of keeping social resources non-utilized [$rm(t)$] *plus* the marginal abatement cost due to extraction of critical input resources [$C_{ux}\dot{x}$], *plus* the marginal social cost of resource extraction at the optimum social re-investment rate for long-run sustainability $\left\{C_{uu}\frac{rP(Q,\dot{Q},t) - rC_u - E(t)\left[r - \dot{E}(t)\big/E(t)\right]}{1\big/SD'(P(Q)) - C_{uu}}\right\}$, *plus* marginal social stock damages arising out of the use of input resources for static output production $\left[\partial Z[\cdot]\big/\partial Q\,\frac{dQ}{dx_c}\right]$, *plus* marginal flow damages arising out of the dynamic flow-rate of variable production $\left[\frac{\partial Z[\cdot]}{\partial u(t)}\right]$, *plus* the dynamic flow-rate of social external costs $[\dot{E}(t)]$.

CONCLUSIONS

The core approach of this research has been to maximize the generation of social capital for a given transversal-free social cycle and arrive at an optimal social extraction path with financial-environmental tradeoffs, constrained by the dynamic flow of social re-investments and with implicit formulations for resource depletion and social damages for environmental abatement. The main asymptotic assumption has been that social surplus can be formulated as an aggregate likelihood function for private financial returns augmented to produce additively-separable social utility levels pertaining to the micro-sustainability of physical capital resources.

Based on the value-added social welfare optimization problem proposed, the dynamic conditions for social optimality conclude the following major propositions:

I. Society's production capacity for maximum social welfare can only be attained if both static and dynamic resource depletion arguments are sustainable.

II. Social re-investments, as a shadow price for resource depletion, are dependent on society's current efforts for value-added output in addition to society's historical system failures deviating from an optimal resource depletion path.

III. Systematic expected short-run system failures within a social cycle may reveal actual environmental instability in the long run.

IV. The rate of social re-investments dictate to society an optimal rate of consumption of social capital which basically follows the slack value of the resource depletion constraint.

V. Most importantly, and contrary to the classical theory of environmental economics, dynamic optimality requires social re-investments to generate marginal social benefits, at equilibrium, strictly exceeding marginal damages to society due to resource extraction, in order to arrive at an efficient allocation of resources.

In retrospect, environmental sustainability could be viewed as cumulative disturbances of non-optimal deviations from a long-run social extraction path corrected by social re-investments that carry social capital returns exceeding marginal social cost.

APPENDIX I

1. Private Financial Payoffs from a Given Supply of Capital Investment

MONEY WORTH INDICATORS

Present Value

$$PV = -I + \sum_{t=1}^{n} \frac{B_t - C_t}{\prod_{1}^{t}(1 + r_t)}$$

Annual Worth

$$AW = (PV)(C_r)$$

$$C_r = \frac{r(1+r)^n}{(1+r)^n - 1}$$

Future Worth

$$FW = (PV)(1+r)^n$$

Capitalized Worth

$$CW = (PV)(r)$$

RATE OF RETURN INDICATORS

Internal Rate of Return

$$-I + \sum_{t=1}^{n} \frac{B_t - C_t}{(1+\rho)^t} = 0$$

External Rate of Return

$$(-I)(1+r_e)^n = \left(\sum_{t=1}^{n} \frac{B_t - C_t}{(1+r)^t} \right)(1 + r^n)$$

Growth Rate of Return

$$\left\{ I + \left(\sum_{t=1}^{tt} \frac{C_t - B_t}{(1+r)^t} \right) \right\} \left(1 + r_g\right)^{tt} = \sum_{t=1}^{n} (B_t - C_t)(1+r)^{tt-t}$$

PAYBACK INDICATORS

Payback Period

$$I = \sum_{t=1}^{p} (B_t - C_t)$$

Discounted Payback Period

$$I + \frac{RI_t}{\prod_{1}^{t}(1+r_t)} = \sum_{t=1}^{p_d} \frac{B_t - C_t}{\prod_{1}^{t}(1+r_t)}$$

Project Balance

$$CPB_{t_b} = -I(1+r)^{t_b} + \left(\sum_{t=1}^{t_b} \frac{B_t - C_t}{(1+r)^t} \right)(1 + r^{t_b})$$

ACCOUNTING INDICATORS

Premium

$$Pr = \frac{\left(-I + \sum_{t=1}^{n} \frac{B_t - C_t}{(1+r)^t} \right)}{I}$$

Profit Margin

$$P_m = \frac{\sum_{t=1}^{n} (B_t - C_t)}{I}$$

Return on Investment $$R_I = \frac{\frac{n+1}{n} \sum_{t=1}^{n} (B_t - C_t)}{\sum_{t=0}^{n} \left[I - \frac{t}{n}(I - S_v) \right]}$$

2. Summary and Comparison of Financial Project Evaluation Methods

Evaluation Method	Identical to Present Value?	Time Value of Money?	Ease of Computation	Advantages	Disadvantages	Indicator	Criteria for Feasibility
Money Worth							
Present Value	Yes	Yes	Easy to compute	Discounts all cash flows to present date	Does not include repeated projects calculations	Feasibility	Positive
Annual Worth	Yes	Yes	Easy to compute	Shows net discounted annual earnings	Same + does not represent ‘net’ gain at a single point in time	Feasibility	Positive
Future Worth	Yes	Yes	Easy to compute	Shows net discounted future gain	Does not include repeated projects calculations	Feasibility	Positive
Capitalized Worth	Yes	Yes	Easy to compute	Shows yearly net gain for an infinite time horizon	Same as above + ignores project time span	Feasibility	Positive
Rate of Return							
Internal Rate	Yes	Yes	Slightly difficult to compute	Shows project yield without comparison with ‘external’ factors	Multiple solutions if net cash flow crosses zero more than once	Feasibility	$> i^*$
External Rate	No	Yes	Slightly difficult to compute	Measures minimum guaranteed return	Very conservative	Feasibility	$> i$
Growth Rate	No	Yes	Difficult to compute	Measures the project yield at any specified future date	Solely depends on specified year of analysis	Feasibility	$> i$

Evaluation Method	Identical to Present Value?	Time Value of Money?	Ease of Computation	Advantages	Disadvantages	Indicator	Criteria for Feasibility
Payback							
Payback Period	No	No	Easy to compute	Offers a quick risk determination and can be used as an effective comparison index	No set criteria, assumes re-investments as costs, and ignores cash profiles beyond the payback period	Payback	no set criteria, but usually compared with a standard industry ‘norm’
Discounted Payback	No	Yes	Slightly difficult to compute	Same as above	No set criteria	Payback	Same as above
Project Balance	No	Yes	Slightly difficult to compute	Gives year by year analysis of net project gain	Only set criteria at end year of the project	Feasibility	Positive at end year
Accounting							
Premium	No	Yes	Easy to compute	Determines financial profit return	Does not show net money gain	Profitability	$> i$
Profit Margin	No	No	Easy to compute	Same as above	Same as above	Profitability	$> i$
Return on Investment	No	No	Slightly difficult to calculate	Same as above + incorporates salvage value	Same as above	Profitability	No set criteria
Book Accounting	No	No	Easy to compute	Determines year by year returns	Same as above	Profitability	No set criteria

* market interest rate

3. Value and Net Value-Added

Demand: $D_t = f_1(Y_t, POP_t, P_t, P_t^c)$ and $Q_t = f_3(Y_t, POP_t, P_t, P_t^c, X_{1t}, X_{2t}, \ldots, X_{mt})$

Supply: $P_t = MC_t = min\ AVC_t = \left.\frac{\partial C_{x_1}}{\partial Q_t}\right|_{min\ AVC_t} = S^{SR}$, and $P_t = MC_t = min\ AC_t = min \frac{\sum_{j=1}^{m} C_{x_j}}{Q_t}$

Max Profit (Net Present Value): $PV = -I + \sum_{t=1}^{n} \frac{(Q_t P_t - C_t)}{\prod_{1}^{\tau}(1 + r_t)}$

MAXIMIZE $PV = -I + \sum_{t=1}^{n} \frac{(Q_t P_t - C_t)}{\prod_{1}^{\tau}(1 + r_t)}$

where $Q_t = f_{Q_t}(Y_t, POP_t, P_t, P_t^c, X_{1t}, X_{2t}, \ldots, X_{mt}, \ldots)$,

subject to:

(sustainability) $Q_t \geq f_{Q_t}(\dot{x}_{1c}, \dot{x}_{2c}, \ldots, \dot{x}_{mc})\Delta t$ ***for t=1,2,...,n***

$Q_t \leq (\dot{Q}_{rcy} + \dot{Q}_w)\Delta t$ ***for t=1,2,...,n***

(resource depletion)

$$\frac{(AW - X)}{r}\left[(1 + r)^n - 1\right] \geq I_{NR}(1 + f_{NR})^n$$

(air environment) $f_{A1}(Q_t) \leq APS_1$; ... ; $f_{Am}(Q_t) \leq APS_m$

(water environment) $f_{W1}(Q_t) \leq WQS_1$; ... ; $f_{Wm}(Q_t) \leq WQS_m$

Model Re-Formulation:

Maximize **g(x)**
Subject to: $h_j(x) \leq b_j$; $X \geq 0$

Maximize **g(x)**
Subject to: $h_j(x) + S_j^{\,2} = b_j$; $X \geq 0$

MAXIMIZE $PV = -I + \sum_{t=1}^{n} \frac{(Q_t P_t - C_t)}{\prod_{1}^{\tau}(1 + r_t)}$

subject to:

$-Q_t + S^2{}_1 = -f_{Q_t}(\dot{x}_{1c}, \dot{x}_{2c}, \ldots, \dot{x}_{mc})\Delta t$, *for t=1,2,...,n*

$Q_t + S^2{}_2 = (\dot{Q}_{rcy} + \dot{Q}_w)\Delta t$, *for t=1,2,...,n*

$\frac{(X - AW)}{r}\left[(1 + r)^n - 1\right] + S^2{}_3 = -I_{NR}(1 + f_{NR})^n$

$f_{A1}(Q_t) + S_4^{\,2} = APS_1$; ... ; $f_{Am}(Q_t) + S^2{}_{(4+m)} = APS_m$

$f_{W1}(Q_t) + S^2{}_{(5+m)} = WQS_1$; ... ; $f_{Wm}(Q_t) + S^2{}_{(5+2m)} = WQS_m$

$Q_t \geq 0$, *for t=1,2,...,n*

Generalized Lagrangian:

$$L = \sum_{t=1}^{n} \frac{(Q_t P^o{}_t - C^o{}_t)}{\prod_{1}^{\tau}(1+r^o{}_t)} - I^o$$

$$- \lambda_1 \{-Q_t + S^2{}_1 + f_{Q_t}(\dot{x}^o{}_{1c}, \dot{x}^o{}_{2c}, \ldots, \dot{x}^o{}_{mc})\Delta t\}$$

$$- \lambda_2 \{Q_t + S^2{}_2 - (\dot{Q}_{rcy} + \dot{Q}^o{}_w)\Delta t\}$$

$$- \lambda_3 \{\frac{(X^o - AW)}{r}\left[(1+r)^{n^o} - 1\right] + S^2{}_3 + I^o{}_{NR}(1+f^o{}_{NR})^{n^o}\}$$

$$- \lambda_4 \{f_{A1}(Q_t) + S_4{}^2 - APS^o{}_1\} - \ldots - \lambda_{(4+m)} \{f_{Am}(Q_t) + S^2{}_{(4+m)} - APS^o{}_m\}$$

$$- \lambda_{(5+m)}\{ f_{W1}(Q_t) + S^2{}_{(5+m)} - WQS^o{}_1\} - \ldots$$

$$- \lambda_{(5+2m)}\{ f_{Wm}(Q_t) + S^2{}_{(5+2m)} - WQS^o{}_m\}$$

Kuhn-Tucker Conditions:

$\frac{\partial L}{\partial \lambda_j} = 0$, *for j=1 to 5+2m , yielding (5+2m) derivatives*

$\frac{\partial L}{\partial S_j} = 0$, *for $nS_1+nS_2+S_3+nmS_4+nmS_5$, yielding (2n+2nm+1) derivatives*

$\frac{\partial L}{\partial Q_t} = 0$, *for t=1 to n, yielding n derivatives*

$\frac{\partial L}{\partial r} = 0$, *yielding one derivative*

$\frac{\partial L}{\partial AW} = 0$, *yielding one derivative*

$\frac{\partial L}{\partial \dot{Q}_{rcy}} = 0$, *for t=1 to n, yielding n derivatives*

for a total of *(5+2m+2n+2nm+1+n+1+1+n) =(4n+2nm+2m+8)* conditions.

$$PV^* = -I^o + \sum_{t=1}^{n} \frac{(Q^*{}_t\, P^o{}_t - C^o{}_t)}{\prod_{1}^{\tau}(1+r^o{}_t)}$$

Optimum Lagrangian multipliers = Shadow Prices of the Constraints; & are assumed to be the implicit price to be paid, in terms of changes in net financial payoff, per unit change of the environmental constraints.

Exogenous variables:

I^o , $I_{NR}{}^o$, $P_t{}^o$, $C_t{}^o$, r_t^o , n^o, $(\dot{x}^o{}_{1c}, \dot{x}^o{}_{2c}, \dot{x}^o{}_{3c}, \ldots, \dot{x}^o{}_{mc})$, $\dot{Q}^o{}_w$, X^o , $f_{NR}{}^o$,
$(APS^o{}_1, APS^o{}_2, APS^o{}_3, \ldots, APS^o{}_m)$, and $(WQS^o{}_1, WQS^o{}_2, WQS^o{}_3, \ldots, WQS^o{}_m)$.

PROJECT VALUE

Expected Present Value: $$EPV = -I + \sum_{t=1}^{n} \frac{\sum_i \int_{-\infty}^{\infty} B_{it} f(B_{it}) dB_{it} - \sum_i \int_{-\infty}^{\infty} C_{it} f(C_{it}) dC_{it}}{(1+r)^t} - A_r \frac{Cov(EPV, Y)}{E(Y)}$$

Coefficient of Relative Risk Aversion: $$A_r = \frac{-Yu''(Y)}{u'(Y)}$$

Covariance Function: $$Cov(EPV, Y) = \varphi \sigma_{EPV} \sigma_Y$$

Net Value: $$EPV = -I + \sum_{t=1}^{n} \frac{\sum_i \int_{-\infty}^{\infty} B_{it} f(B_{it}) dB_{it} - \sum_i \int_{-\infty}^{\infty} C_{it} f(C_{it}) dC_{it}}{(1+r)^t} - A_r \frac{\varphi . \sigma_{EPV} . \sigma_Y}{E(Y)}$$

Project Value: $$V = -I + \sum_{t=1}^{n} \frac{(Q^*{}_t \, P_t(Q_t) - C_t - E_t)}{\prod_1^{\tau} (1 + r_t)} - A_r \frac{\varphi . \sigma_{EPV} . \sigma_Y}{E(Y)}$$

NET VALUE-ADDED

Net Value-added: $$VA = (\eta)(V) = \left[\frac{\sum_{t=1}^{n} (Q_t P_t(Q_t))}{I + \sum_{t=1}^{n} a_t} \right] \left[-I + \sum_{t=1}^{n} \frac{(Q^*{}_t \, P_t - C_t - E_t)}{\prod_1^{\tau} (1 + r_\tau)} - A_r \frac{\varphi . \sigma_{EPV} . \sigma_Y}{E(Y)} \right]$$

Project Value is assumed to yield an estimate of social capital surplus from a given investment project, assuming no social damages (at this point of analysis), and the efficiency criterion (heavily based on White 1982 and DeNeufville 1990) imply that the effectiveness of this surplus has to be augmented by the shadow price of environmental abatement and by the initial sunk cost of investment. Imposing the efficiency criterion on project value yields net social value-added to society (see also Aulin 1992 and Aulin-Ahmavaraa 1990).

Social surplus here only includes environmental factors related to resource depletion and environmental sustainability (based on upper-bound and lower-bound system failure sustainability constraints), in addition to private financial payoffs from a given supply of capital. It is also assumed that all investment capital is in the hands of risk-averse entrepreneurs and that the labor market is competitive in wages paid. The degree of relative risk aversion is assumed constant for all investors, and the private benefits associated with investment capital are assumed to be normally distributed (with known expected value).

It should be noted here again that the above static optimization problem does not consider a capital flow function and assumes no social damages besides utilizing the resource depletion and weak sustainability arguments. The optimum Lagrangian multipliers are therefore themselves the shadow prices of the environmental constraints.

APPENDIX II

1. Summary of the Reduced Form Solution

$$Max\ S = \int_{t=0}^{T} S\left\{\eta V[\pi(P(Q))], H[\dot{Q}(\dot{x}_c)], \ell[\mu(\overline{Q})]\right\} dt$$

$$\underset{u(t),T}{H[\dot{Q}(x(t)]} = \int_{0}^{T} e^{-rt}\{P(t)u(t) - C(u(t), x(t)) - E(t)SD[P(Q,\dot{Q},t)]\}dt$$

$$\eta(t)V(Q_t,\dot{Q}) = \int_{0}^{t} \left[\frac{\sum_{t=1}^{n}(Q_t P_t(Q_t,\dot{Q},t))}{I + \sum_{t=1}^{n} a_t}\right]\left[-I + \sum_{t=1}^{n}\frac{(Q_t P_t - C_t(u_t) - E_t)}{\prod_{1}^{\tau}(1+r_\tau)} - \frac{U''[\pi(Q,\dot{Q})]}{U'[\pi(Q,\dot{Q})]}\varphi\sigma_{\pi(Q)}\sigma_y\right]dt$$

$$H_c(\cdot) = V(\cdot) + m(t)f(\cdot) - Z(Q,\varphi)$$

$$H_c^* = \int \pi(Q,\dot{Q})dQ - m_1(t)\dot{Q}_{\max} + m_2(t)f_Q(\dot{x},u) - Z[\eta V(\cdot), U(Y)]$$

$$MAC(Q^*) = MZ(\dot{Q})$$

$$\left[{\partial\pi}/{\partial Q}\right]\dot{Q} = \left[{\partial Z}/{\partial V(\cdot)}\right]$$

$$\frac{\dot{m}_2}{m_2} = r - {m_1}/{m_2}$$

$$\dot{m}(t) = rm(t) = \dot{P}(Q,\dot{Q},t) - C_{ux}\dot{x} - C_{uu}SD[P(\cdot)] - \dot{E}(t)$$

$$-rm(t) - \dot{E}(t) + \dot{P}(Q,\dot{Q},t) = C_{ux}\dot{x} + C_{uu}\frac{rP(Q,\dot{Q},t) - rC_u - E(t)\left[r - {\dot{E}(t)}/{E(t)}\right]}{{1}/{SD'(P(Q))} - C_{uu}} + {\partial Z[\cdot]}/{\partial Q}\frac{dQ}{dx_c} + \frac{\partial Z[\cdot]}{\partial u(t)}$$

$$\dot{u}(t) = \frac{rP(Q,\dot{Q},t) - rC_u - E(t)\left[r - {\dot{E}(t)}/{E(t)}\right]}{{1}/{SD'(P(Q))} - C_{uu}}$$

$$\frac{\partial H(\cdot)}{\partial m_1(t)} = -u(t) - \dot{Q}_{\max}$$

$$\frac{\partial H(\cdot)}{\partial m_2(t)} = f_Q(\dot{Q}, \dot{x}_{c,t})$$

$$m_2(t) = m(0)e^{rt}$$

$$\frac{\dot{m}_1}{m_1} = 1 - r$$

2. Sub-Solution to the Value-Added Social Welfare Optimization Problem

EPV/IRR approach when maximizing net profit $\Pi(\boldsymbol{Q}_t, \dot{\boldsymbol{Q}}_t)$

$$\Pi = \boldsymbol{EPV} = -\boldsymbol{I}_{\boldsymbol{o}} + \Sigma_{\boldsymbol{t}=1}^{\boldsymbol{N}} \left[\frac{\Sigma_{\boldsymbol{i}} \overline{\boldsymbol{B}}_{\boldsymbol{it}} - \Sigma_{\boldsymbol{i}} \overline{\boldsymbol{C}}_{\boldsymbol{it}}}{(1+\boldsymbol{r})^{\boldsymbol{t}}} \right] - \left[\frac{\boldsymbol{Y}\boldsymbol{U}''(\boldsymbol{Y})}{\boldsymbol{U}'(\boldsymbol{Y})} \right] \left[\frac{\boldsymbol{Cov}(\Pi, \boldsymbol{Y})}{\boldsymbol{E}(\boldsymbol{Y})} \right]$$

$$= -\boldsymbol{I}_{\boldsymbol{o}} + \left[\sum_{\boldsymbol{i}=1}^{N} \boldsymbol{Q}_t \boldsymbol{P}_t - \boldsymbol{C}_t \right] \left[\frac{1}{\Gamma_{\tau=1}^{\boldsymbol{t}} (1+\boldsymbol{r}_\tau)} \right] - \left[\frac{\boldsymbol{U}''(\boldsymbol{Y})}{\boldsymbol{U}'(\boldsymbol{Y})} \right] [\varphi \sigma_\Pi \sigma_{\boldsymbol{Y}}]$$

Max $\Pi(\boldsymbol{Q}_t, \dot{\boldsymbol{Q}}_t)$ subject to:

Uncertainty $$E(x) = \int_{-\infty}^{\infty} x f(x) dx = a_0 + a_1 \bar{x} + a_2 \{\sigma^2 + (\bar{x})^2\}$$

Inverse Transformation (1) $$x^{-1}(y) = \frac{dy}{dx}$$

Inverse Transformation (2) $$E(x) = \int_{-\infty}^{\infty} x f(x) dx = \int_{-\infty}^{\infty} f_1(y) \cdot \frac{df_1(y)}{dy} \cdot f_1[f_2(x)] \left| \frac{df_2(x)}{dy} \right| dy$$

Maximum Likelihood $$L(x) = L(f(x);\theta) = \prod_{i=1}^{m} f(x;\theta) = \left(\frac{1}{\sigma\sqrt{2\pi}} \right)^m e^{-\frac{1}{2\sigma^2} \sum_{i=1}^{m} (x_i - \mu)^2}$$

System Failure $$\dot{Q}_L \geq f_Q(\dot{x}_{1c}, \dot{x}_{2c}, \dot{x}_{3c}, ..., \dot{x}_{mc})$$

Environmental Instability $$(\dot{Q}_H - \dot{Q}_{rcy}) \leq \dot{Q}_w$$

Social Re-investments $$\frac{(AW - X)}{r} \left[(1+r)^n - 1 \right] \geq I_{NR} (1 + f_{NR})^n$$

Pollution Concentration (1) $$C_g^{ep} = \frac{Q}{\pi \sigma_y \sigma_z \bar{u}} exp\left(\frac{-H^2}{2\sigma_z^2} - \frac{y^2}{2\sigma_y^2} \right)$$

Pollution Concentration (2) $$C_{ga}^{g} = \frac{Q}{\pi \bar{u} \left(\sigma_y^2 + \sigma_{y_o}^2 \right)^{0.5} \sigma_z},$$

where $U(\bullet)$ is the value-added measure of market demand (or industry supply), $D[C_g^{ep}(Q), C_{ga}^{g}(Q), \varphi, U(Y)] = Z[Q, \varphi]$ *is the implicit social damage function,*

λ_1 *is the shadow price of long-run sustainability,* λ_2 *is the shadow price of short-run system failure,* φ *is the correlation coefficient between capital supply and social demand, and* $r_p^* = t^* = MZ(Q^*, \varphi)$ *(assuming no market failures in environmental abatement).*

Focal Solution: $$\dot{\lambda}_1(\lambda_1, X, I_{nr}, f_{nr})_{LR-social} = \dot{\lambda}_2(\lambda_2, \Pi(Q_t))_{SR-\Pi_{\max}} \cdot \left[1 - \sum_{m=1}^{t} \frac{\partial \Phi(\dot{Q}_m, \dot{x}_{c,m})}{\partial Q_m} \right]$$

REFERENCES

Aulin, A., **Foundations of Economic Development**, Springer-Verlag, Berlin, 1992
Aulin, A., **Foundations of Mathematical System Dynamics**, Pergamon Press, Oxford, 1989
Aulin-Ahmavaraa, P., **Dynamic Input-Output and Time**, Economic Systems Research, vol 2, 1990
Aulin-Ahmavaraa, P., **A Complete Dynamic Input-Output Model including the Production of Human Capital and Labor**, Economic Systems Research, vol 1, 1989
Austin, J., **Agro-Industrial Project Analysis: Critical Design Factors**, EDI Series in Economic Development, Johns Hopkins, Baltimore, 1992
Barrett, L., et.al., **Cost of Air Pollution Damage: A Status Report**, Environmental Protection Agency, Research Park, North Carolina, 1973
Beenhakker, H., **Investment Decision Making in the Private and Public Sectors**, Quorun, Connecticut, 1996
Bell, C. et.al., **Project Evaluation in Regional Perspective**, World Bank Research Publications, Johns Hopkins Publishers, Baltimore, 1982
Brokensha, D., and Hodge, P., **Community Development: an Interpretation**, Chandler, San Francisco, 1969
Brooke, M., **Handbook of International Financial Management**, Macmillan, Hants (England), 1990
Canter, L., **Environmental Impact Assessment**, McGraw Hill, New York, 1977
Chanlett, E., **Environmental Protection**, McGraw Hill, New York, 1973
Conrad, J.M. and Clark, C.W., **Natural Resource Economics**, Cambridge University Press, Cambridge (UK), 1999.
Burkey, S., **People First: A Guide to Self-Reliant Participatory Rural Development**, Zed Books, London, 1993
Cusworth, J.W. and Franks, T.R., **Managing Projects in Developing Countries**, Longman UK, Essex, 1993
Dasgupta, P. et.al., **Guidelines for Project Evaluation**, UNIDO, Vienna, 1972
De Neufville, R., **Applied Systems Analysis: Engineering Planning and Technology Management**, McGraw-Hill, New York, 1990
Fabrycky,W. et.al., **Applied Operations Research and Management Science**, Prentice-Hall, New Jersey, 1984
Freund, J., **Mathematical Statistics**, Prentice-Hall, New Jersey, 1992
Gilpin, A. **Environmental Impact Assessment (EIA)**, Cambridge University Press, Hong Kong, 1995
Grilli, E. and Salvatore, D., **Economic Development**, Greenwood Press, Connecticut, 1994
Hanley, D., Shogren, J.F., and White, B., **Environmental Economics in Theory and Practice**, Oxford University Press, 1997.
Harberger, A., **Project Evaluation Collected Papers**, Macmillan, London, 1972
Hayden, F., **Project Evaluation in a Futures Real Time System**, Journal of Economic Issues, vol 16, no.2, 1982
Huylenbroeck, G. and Martens, L., **The Average Value Ranking Multi-Criteria Method for Project Evaluation in Regional Planning**, European Journal of Agricultural Economics, vol 19, 1992
Jacoby, H. and Laughton, D., **Project Evaluation: A Practical Asset Pricing Method**, Energy Journal, vol 13, no.2, 1992
Kreith, F., **Lack of Impact**, Environment, vol 15, no.1, 1975
Lakshminarayan, P., **Tradeoffs in Balancing Multiple Objectives of an Integrated Agricultural Economic and Environmental System**, Center for Agricultural and Rural Development (CARD), Monograph # 96-M8, Iowa State University, Ames, 1996
Lal, D., **Methods of Project Analysis: A Review**, IBRD, World Bank, Staff Occasional Papers #16, Johns Hopkins, Baltimore, 1974
Leistritz, F., and Ekstrom, B., **Social Impact Assessment: an Annotated Bibliography**, Garland Publishing, New York, 1986
Mikesell, R., **Project Evaluation and Resource Sustainability**, Contemporary Policy Issues, vol 10, 1992
Mikkelsen, B., **Methods for Development Work and Research: A Guide for Practitioners**, Sage Publications, New Delhi, 1995
Mishan, E.J., **Cost-Benefit Analysis**, Routledge & Times, London, 1988.
Munn, R., **Environmental Impact Assessment**, John Wiley, Surrey, 1979
Odum, H., **Environment, Power and Society**, John Wiley, New York, 1971
Olsen, M. and Melvin, D., **Towards a Methodology for Conducting Social Impact Assessments Using Quality of Social Life Indicators**, Methodology of Social Impact, Stroudburg, 1977

Overseas Development Administration, **A Guide to Social Analysis for Projects in Developing Countries**, ODA Publication, London, 1995
Pearce, D., and Turner, R.K., **Economics of Natural Resources and the Environment**, Johns Hopkins University Press, 1990.
Perkins, H., **Air Pollution**, McGraw Hill, New York, 1974
Pouliquen, L., **Risk Analysis in Project Appraisal**, World Bank Occasional Papers, #11 (1970), Johns Hopkins, Baltimore, 1970
Ravindran, A., and Phillips, D., **Operations Research: Principles and Practice**, John Wiley, New York, 1987
Remer, D., and Nieto, A., **A Compendium and Comparison of 25 Project Evaluation Techniques, Parts 1 and 2**, International Journal of Production Economics, vol 42, 1995
Reutlinger, S., **Techniques for Project Appraisal under Uncertainty**, World Bank Occasional Papers, #10 (1970), Johns Hopkins, Baltimore, 1984
Schwartz, H. and Berney, R., **Social and Economic Dimensions of Project Evaluation**, Inter-American Development Bank, Proceedings of the Use of Socio-Economic Criteria in Project Evaluation (March 1973), Washington D.C., 1977
Selim, H., **Development Assistance Policies and the Performance of Aid Agencies**, Macmillan, London, 1983
Selim, T.H., **Cost-Benefit Evaluation for Capital Investment Projects with Pecuniary Externalities and Sustainability Constraints**, Proceedings of the AEAS 25th Silver Anniversary Conference on Knowledge & Sustainable Development, Washington DC, October, 1999.
Shaner, W., **Project Planning for Developing Countries**, Praeger Publishers, New York, 1979
Smith, C., **Economic Development, Growth and Welfare**, Macmillan, Hong Kong, 1994
Wathern, P., **Environmental Impact Assessment: Theory and Practice**, Hyman, London, 1988
White, D., **Optimality and Efficiency**, John Wiley, Chichester, 1982
Whitehouse, G. and Wechsler,B., **Applied Operations Research: A Survey**, John Wiley, New York, 1976
Wilson, E., **Sociology: Rules, Roles and Relationships**, Dorsey Press, Illinois, 1966
Zimmerman, C. and Duwors, R., **Sociology of Underdevelopment**, Copp Clark, Vancouver, 1970