Value and the foundation of Economic Dynamics

Alan Freeman

The University of Greenwich

16. March 1994

VALUE AND THE FOUNDATION OF ECONOMIC DYNAMICS

By Alan Freeman. Draft Number 19, 16/3/94

Note: this is an earlier version of the paper entitled 'Reappraising the Classics' submitted to the EEA March 1994 conference. It contains a number of arguments I would not now defend as it was written when the ideas being codified were in the process of development. I have printed it to make a available the mathematical calculations involved in demonstrating a falling dynamic rate of profit. Please do not cite this article without the author's permission.

1. Introduction

Laβ dir von keinem Fachmann imponieren, der dir erzählt: "Lieber Freund, das mache ich schon seit zwanzig Jahren so!" – Man kann eine Sache auch zwanzig Jahre lang falsch machen. – Kurt Tucholsky, ***Schnipsel***

Economists since Ricardo have debated the idea of a universal measure of intrinsic value. This article argues that the enquiry has been crippled by the mathematical formalisation adopted: units of measure established for equilibrium systems, whose parameters do not vary, are then applied to dynamic systems whose parameters all vary. Economics thus has no measure of value free from steady-state assumptions.

This article constructs time-varying labour value measures free of such restrictions and shows that they call for a radical re-evaluation of this century's debate on value. We exhibit a counter-example to the Okishio theorem in which labour-saving innovation leads to a continuously-falling profit rate, and a dynamic approach to the classical problem of transforming values into prices in which the sum of values is equal to the sum of prices while the sum of profits remains equal to the sum of surplus values. Elsewhere we have exhibited a computer simulation model using these values.[1] A dynamic value measure is thus practical, necessary, and the basis for a general reconstruction of economics.

1.1. The critique of 'value-free' economics and its limitations

Critics of neoclassical macroeconomic theory have taken two directions. The Post-Keynesians and others concentrate on the assumption of equilibrium and ask what happens if agents trade at non-equilibrium prices. These critics on the whole ignore the problem of measurement, inviting the objection that their theory has no microfoundation.[2] The Cambridge school have concentrated on measurement, arguing that an objective theory of production requires a measure independent of distribution. Sraffa's search for an invariant standard, however, has led to a construct which, being derived from a system in equilibrium, is open to the same Post-Keynesian objections as neoclassical macroeconomics.[3]

In principle, labour values are invariant with respect to distribution but not time. Yet the most widely accepted derivation of labour values in fact fixes them in time: most Marxists now work from a set of simultaneous equations in which input values are equated to output values. But this rules out variation in time, since outputs emerge after inputs are consumed and no known being eats what it has not yet produced. A time-varying standard can at best be calculated as a sequence of static equilibria.[4] Value theory, determined to equate the inputs of today to the outputs of tomorrow, has been cast into a timeless fantasy world like the legendary Midgard serpent, which spent eternity consuming its tail,

1.2. A restatement of the classical approach: dynamic determination of value

This paper starts from the fact that the inputs of today are the outputs of yesterday. We set out to construct a value measure free of equilibrium assumptions with the question: if it varies from one point in time to the next, what governs the variation? Given a measure of economic size at time t, what will it become at time $t+1$? We define a time-varying measure through *transition rules* which conform to basic principles of measurement in a commodity economy.

2. Capital movement, technical change and measurement

Our approach is axiomatic. We set out the necessary properties of a time-varying measure which is independent of distribution, and ask which measures are compatible with these properties. This rather technical approach is dictated by the current state of the debate. For a long time it has been argued that labour values are logically irrelevant; we are interested in what is logically necessary. If labour values possess the relevant properties, then they are a sufficient basis for economics in its most general form, namely the economics of change; if other measures do not, then labour values are a necessary basis. However, the underlying motive is not a narrow technical one but the need to explain precisely and in a unified framework the relation between technical development and the movement of capital, the motors which all accounts of economic change must sooner or later confront.

Neoclassical growth theory, pioneered by Harrod and Domar[5], places capital accumulation at the centre. It rightly starts from the fact that unconsumed output accumulates. As the Cambridge school point out, its Achilles heel is its theory of technical change, using the neoclassical production function,[6] which makes output a function of a physical quantity – capital – but measures it in price terms. But in alternative accounts that replace the neoclassical production function with a linear production model – such as that given by Okishio (1961) – capital movement is absent. The move from one technical option to another is not conditioned by the rate of investment.

No invention, however compelling, has ever been installed before it was produced. In a real economy, one technology gradually replaces another along a path determined both by technical choice and by the availability and movement of capital. To follow its trajectory one must quantify not only what this capital might produce, but how much of it there is, how it is distributed, how fast it accumulates and where. A measure is required which can quantify both technical advance and the process of its introduction within a single, unified framework. A static measure cannot do so because capital by its very nature accumulates over time, and the unit of its measure changes as it comes into being.

2.1. Stocks and flows; the natural basis of statics and dynamics

At the heart of the problem is a methodological issue illustrated by a well-known lecture where R F Harrod (1963), perhaps the founding father of neoclassical dynamics, says:

> "Now the word Statics being already thoroughly established in economics, we may properly ask in what sense a 'static' economy can be regarded as analogous to a state of rest in the physical world. We do not mean by it one in which no-one does anything at all...but one in which work is steadily going forward day by day and year by year, but without increase or diminution. ' Rest' means that the level of these various quantities remains constant, and that the economy continues to churn over.'

This passage is the key to the equilibrium approach to economic dynamics: statics concerns motion with constant parameters and dynamics concerns motion whose parameters vary. But in the physical sciences, statics is the presence of motion and dynamics is its absence. A genuinely static economy really would do nothing at all. Those states which Harrod defines as static are treated in physics as a restricted dynamic state. Electrostatics, for example, deals with charges and electrodynamics with currents. A state in which 'work is steadily going forward day by day' but 'without increase or diminution' is a direct electrical current, a special case of an alternating current.[7]

This has no impact on what is conventionally termed statics. But in dynamics a genuinely static concept, that of a stock, must be distinguished from its dynamic counterpart, which is a flow. A flow is not a variable stock, but a stock in movement. Capital movement consists of the conversion of flows into stocks and stocks into flows, which dynamics has to quantify and study. A stock's physical size may be fixed, but its composition changes. It is ceaselessly replenished and depleted, so that its variation is the resultant of two flows, production and consumption, each of which must be independently assessed and brought into relation with each other through a single common measure.

This is a practical question: a profit rate is the ratio between a stock and a flow. Of course, if values (or prices) never change from the fall of the auctioneer's hammer till the end of time, such a ratio presents no problems. But if commodities one year ago possessed a different value to their value today, then the physical composition of a stock no longer informs us of its value. The same physical quantity of the same thing does not always stand for the same value; its history becomes relevant. This is not academic: there is a vigorous controversy in accountancy over whether to measure capital in historic-cost or current-price terms. As we shall see, it is the fundamental flaw at the heart of the Okishio theorem. By correcting it, we can explain why profits can fall as physical surplus increases.

The confusion between stocks and flows, rooted in the steady-state foundations of neoclassical theory, has deprived economics of an arsenal of theoretical instruments – above all the powerful devices known to physics as conservation laws. Given a proper distinction between motion and stasis, between mass and momentum, many different kinds of activity can be reduced to a single common substance which is conserved dynamically, exchanges between interacting systems, and aggregates in a linear fashion. The parallel with classical value systems is so striking that it has taken decades of learned work to conceal it.

2.3. Necessary properties of a time-varying measure of value

What are the minimum properties of a measure of value which can quantify both technical change and capital movement? We suggest the following:

1. Technological variation. Inputs to and outputs from production change over time, and the derivation of a unit of measure should neither implicitly nor explicitly require them to be fixed.
2. Backward time-dependence. The measure at any time should depend only on the current and previous state of the system. Simultaneous systems, which necessarily equate the values of inputs at the time of purchase to those of outputs at the time of sale, cannot meet this criterion.
3. Independence from distribution and hence prices. We take this critical axiom, unacceptable to neoclassical theory, as a shared goal of Sraffian and Marxist system. We aim to demonstrate that any other assumption leads to circular reasoning.

2.4. A simple illustration

Consider the following simple economy. Two producers, **P1** and **P2** make two commodities, **C1** and **C2**. Consumption and production is given in the table below, in which **L** represents labour time.

		C1	C2	L		C1	C2
P1	*uses*	10	5	4	*to produce*	18	
P2	*uses*	8	15	2	*to produce*		50

Table 2.4.1 Quantities consumed and produced

The static treatment proceeds as follows: input values must be equal to output values. Use the symbols v_1 and v_2 to represent unit values of **C1** and **C2**. Then the following two equations must be satisfied:

1) $$10v_1+5v_2+4=18v_1$$

2) $$8v_1+15v_2+2=50v_2$$

These equations have only one solution, giving the only unit values compatible with a static system.

3) $$v_1 = 5/8 \quad v_2 = 1/5$$

Using these unit values we can rewrite the table so that each entry on the left shows the value this input contributes, and each entry on the right shows the value produced. The new table, 2.4.2, has the obvious property that the quantities on the right are the sum of the quantities on the left. The value contribution of all inputs adds up to the value of the output for each process and for the whole economy.

		C1	C2	L		C1	C2
P1	*uses*	6¼	1	4	*to produce*	11¼	
P2	*uses*	5	3	2	*to produce*		10

Table 2.4.2 Value transferred, added and produced

But now suppose the last cycle of production used different techniques, giving different unit values for the inputs – say v_1=1 and v_2=2. Should these be replaced by the values in equations (3)? If so, they were determined by something which has not yet happened, since outputs have not yet been produced. Moreover v_1 and v_2 would simultaneously be the unique solution to two different sets of equations, which is in general impossible. The static derivation is contradictory: commodities cannot have the same unit value at the beginning and at the end of production unless the economy is in equilibrium.

But we can calculate new unit values in another way. The value contributed by inputs to process **P1**, for example, totals 1× 10+2 × 5+4 = 24. Assume that the value of outputs must equal the value contribution of the inputs that produced them. Then the 18 units of output of **C1** must be valued at 24, and the new unit value of **C1** at the end of the period is $^4/_3$. Similarly the unit value of **C2** must become $^4/_5$.

We can now no longer speak of v_1 and v_2 without saying when they are measured. They are time-varying magnitudes $v_1^{(t)}$ and $v_2^{(t)}$. Moreover we can use exactly the same method to calculate the next pair of unit values, and thus iteratively to calculate unit values at any future time. All we need know are the inputs to and outputs from production over each time period. Moreover

- This calculation requires neither technology nor distribution to remain constant. Inputs, outputs and consumption may vary in any way at all but the calculation remains valid.
- Values at the end of any given period are a function only of values in previous periods and the magnitude of the flows of commodities entering production as inputs.
- Unit values are independent of distribution. Net output – 30 units of **C2** – can be consumed by wage-earners or property-owners in any proportion without affecting the calculation. Neither prices nor profits nor the wage were used: we don't even know what prices are.

Thus even in this simple system, values can be calculated in an obvious manner without equilibrium assumptions. All that is required is an initial condition, the unit values inherited from previous periods of production. This kind of solution, common in physics, is unavoidable in a dynamic theory. We have derived a difference equation that will predict values at any subsequent time T from boundary conditions in the form of unit values at time 0, and technical coefficients for t= 0,...,T.

2.4.1. Fixed-point values and convergence

These values depend on an external unknown, namely the initial conditions. But they bear an important relation to static values which suggests that they are a true generalisation. If there is no technical change, then as table 2.4.3 shows, they converge to stable magnitudes equal to fixed-point values regardless of the initial condition. Whatever values we start with, we end up with the static values provided technical coefficients do not change. This always holds when there is a positive net product, as a consequence of the same theory which guarantees the existence of a static solution. If there is a static solution, the dynamic solution will converge to it if technology remains constant.

t	v1	v2
0	1.0000	2.0000
1	1.3333	0.8000
5	0.8059	0.2776
10	0.6502	0.2108
20	0.6255	0.2002
30	0.6250	0.2000

Table 2.4.3: convergence of dynamic values

This result is usually used to explain how static values can be reduced to dated labour, or as an interesting way of calculating static values[8]. In our view it has a completely different significance because the method remains valid when the static solution does not exist.[9] Dynamic values give the same result under the restrictive conditions needed to calculate static values but remain viable when the restrictions are removed. In short, they are more general, and static values are a special case.

The approach outlined above also seems to us closer to Marx's ideas than that of his 'correctors'. If so, it sheds a different light on his 'failure' to equate input prices to output prices. Perhaps the problem was not his ignorance of mathematics but his knowledge of economics: as anyone not educated by the science of economics knows, prices rise and fall while life proceeds.

2.5. Technical advance with falling profits: depreciation and the Okishio theorem

So far, we have only shown that dynamic values behave like static values in static economies But for changing economies they bring radically different results. We now present an economy in which, despite continuous labour-saving innovation which raises both the transitional and the equilibrium rate of profit, the actual rate of profit falls continuously. This contradicts the Okishio theorem, generally believed to prove that technical innovation necessarily leads to an increasing real rate of profit in the absence of an offsetting rise in real wages.

The counterexample is presented in its simplest possible form: the general result follows from theorems in the second half of this paper. Consider a single sector economy producing a commodity **C** serving as means of production and consumption. In such an economy price and value are the same, so profit rates can be calculated directly in value terms. The economy has capital stock K of which a constant proportion $A=\tau K$ is turned over in each period of production. This produces gross output Y which is in turn a constant proportion of A given by $Y=(1+\mu)A$.

μA is thus the economy's 'physical surplus' out of which will come wages, capitalist consumption and investment. In Marxist terms K represents fixed plus circulating constant capital. Labour inputs L are constant.

The fixed-point unit value of **C** is given by $vK+L=v(K+\mu A)$ or simply $L=v\mu A$, giving $v=L/\mu A$. This says that the unit value of **C** is equal to total labour added in each time period divided by net product. In Sraffian terms this unit value is simply the price of **C** with labour hours as numéraire. In Marxist terms L contributes the sum of V, variable capital and S, surplus value; we do not know the division between V and S because we have not yet fixed the wage. To fix ideas, let τ = 0.2, μ = 0.5, K = 1000 and L = 100. This gives the following system:

	K		**A(= τK)**	**L**		**Y(=A+μA)**
Advanced	1000	*used*	200	100	*produced*	300

Table 2.5.1: production with one sector and fixed capital

This has the fixed-point value 100/100=1, so that physical quantities and values are identical. We cannot calculate the rate of profit without knowing the wage but we can calculate the maximum rate of profit, which would pertain if the wage was zero. According to the Okishio theorem, this must rise with any innovation which increases productivity. It is equal to net new value divided by capital advanced; that is,

4) $r_{EQ}=(Y-A)/K=\mu A/K=\mu\tau$=constant=10% in this case.

Now suppose each period the net physical surplus, after wages and capitalist consumption, is invested in new techniques so that L remains constant, and the proportions μ and τ remain fixed. This is not unnatural since L is limited by the size of the labour force. There is thus gradual labour-saving innovation yielding a higher 'transitional' profit rate: entrepreneurs who introduce the new techniques will use less labourers to produce the same net output, so their costs must be less. If, as the Okishio theorem prescribes, they cost all inputs and all capital at current prices, they must estimate their profits to be higher. To fix ideas further, and to avoid profit-squeeze effects, suppose a constant real wage w of 50 units of the net product. Other wage assumptions do not negate the result.

The calculation is shown in table 2.5.2. In each period, we add up the total value of **C** which has either been preserved as fixed constant capital (K-A), transferred as circulating constant capital (A), or added by labour (L). This, the fifth column of the table, is calculated using the unit value $v^{(t)}$ which held at the beginning of production, so that inputs contribute the value they possessed at time of use.

Now add up the total quantity produced or preserved **C**. This, the sixth column, is equal to $K+\mu A$. Finally calculate the new unit value of **C** by dividing column five, total value of **C**, by column 6, total physical quantity of **C**. This procedure will be fully justified later on: it depreciates fixed capital by averaging all the value which has been preserved or added to **C**, over the total stock of **C**.

Fixed Capital	Turn-over	Net Product	Initial unit value	Total value	Total Quantity	New unit value	V_{eq}	Invest-ment	Value at end	Value at start	Profit	Profit Rate	Okishio Profit Rate
K	$A=\tau K$	μA	*V(t)*	*v(t)K+L*	$K+\mu A$	*V(t+1)*	$L/\mu A$	μA-*W*	$v(t+1)\times(K+\mu A-W)$	$v(t)\times K$	*V(end)-V(beg)*	*Profit/v(t)K*	$(\mu A-W)/K$
1000	200	100	1.0000	1100	1100	1.0000	1.00	50.00	1050.00	1000.00	50.00	5.00%	5.00%
1050	210	105	1.0000	1150	1155	0.9957	0.95	55.00	1100.22	1050.00	50.22	4.80%	5.24%
1105	221	111	0.9957	1200	1216	0.9874	0.90	60.50	1150.85	1100.22	50.63	4.64%	5.48%
1166	233	117	0.9874	1251	1282	0.9757	0.86	66.55	1202.06	1150.85	51.22	4.50%	5.71%

Table 2.5.2: investment, technical change, value and profit rates with real wage 50

2.5.1.Rising static and falling dynamic profit rates

We can now calculate two profit rates. The first, the 'Okishio' or theoretical static rate, is the equilibrium rate which would arise if innovation and investment stopped. This rises continuously because labour productivity is rising continuously and the real wage is constant. The second, the actual dynamic profit rate, is calculated as accountants do by subtracting the worth of the business before production from its worth after production, and dividing by the current worth of advanced capital. This declines continuously as long as technical innovation and investment proceed.

With higher real wages the dynamic rate rises initially because innovation starts more slowly, but always declines eventually for any value of the real wage.[10] Under the alternative hypothesis of a constant labour share in output, the static rate remains constant, but the dynamic rate declines as long as innovation proceeds.

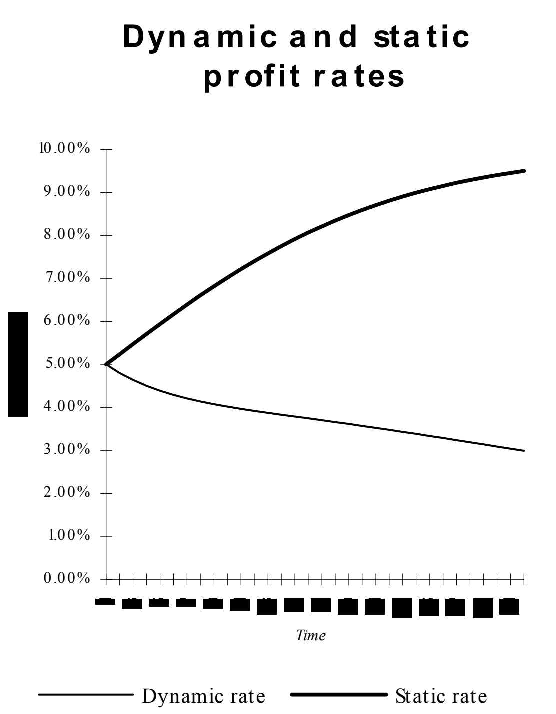

Chart 2.5.1: profit rates

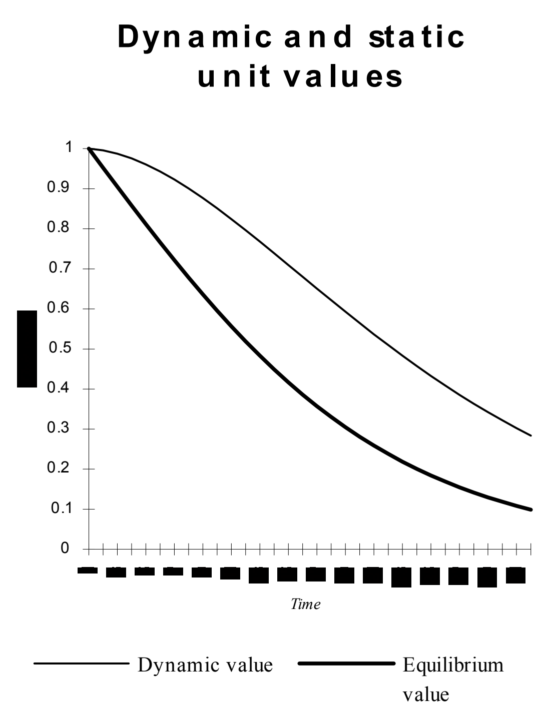

Chart 2.5.2: unit values

2.5.2. Depreciation and profits

Why are the dynamic and static rates in such flat contradiction? Fundamentally, the Okishio theorem neglects the effect of depreciation on *current* profits.[11] Production usually increases physical stocks, but if this stock loses value, the business must absorb the loss. The depreciation of fixed capital involves two elements. The best known is actual wear and tear of the capital, which Marx calls its turnover. But there is a second component, 'moral depreciation' – an allowance against technical obsolescence, that is, stock devaluation caused by technical progress. Rapidly falling computer hardware prices, for example, reflect a very high moral depreciation component. In the extreme case of computer software we have pure moral depreciation, since programmes do not wear out.

A dynamic calculation quantifies both components of depreciation. There appears to be no other sensible way to do it. If, for example, we use the new equilibrium value to calculate the depreciation loss then the rate of profit falls immediately to zero which, though different from the result predicted by static theory, must qualify as a rather catastrophic prognosis.[12]

Using dynamic values, shown in chart 2.5.2, fixed capital gradually absorbs the devaluation effect so that the rate of profit declines continuously until innovation stops (not shown in the charts). If this happens dynamic values converge to a new equilibrium and as fixed capital depreciates profits recover towards a new equilibrium profit rate – as happens in a slump. However, the condition for reaching the equilibrium rate is that all innovative investment stop for a prolonged period – the opposite of the static prediction.[13]

2.5.3. Organic composition, tendency, and value conservation

This suggests a new approach to a heated debate. Marx says that the organic composition of capital tends to rise, producing a tendency for the profit rate to fall which can be offset by countervailing tendencies. Many authors argue it can be fully offset by one particular such tendency, the cheapening of fixed capital.

Value conservation offers an appealing and simple resolution of this debate. Marx argues that there is a limit on labour inputs to production imposed by the size of the labour force and the length of the working day. Unless all net output is consumed, each year some value must be added to advanced capital C+V, which must therefore increase so that the denominator in the rate of profit is growing while the numerator, S, has a maximum limit of S+V. Therefore the rate of profit must decline.

The counter-argument runs that as C devalues this will lower the denominator in the rate of profit. But the discussion has focused on how much, rather than how *fast*, this offsetting will occur. In particular the question never seems to have been asked: when capital is devalued, where does the lost value go? And how fast does it leave the system?

K	*A*=τ*K*	μ*A*	*V(t)*	*v(t)K+L*	*K*+μ*A*	*V(t+1)*	*L*/μ*A*	μ*A-W*	*v(t+1)* × (*K*+μ*A-W*)	*v(t)* ×*K*	*V(end)-V(beg)*	*Profit/ v(t)K*
1000	200	100	1.000	1100	1100	1.0000	1.00	100.00	1100.00	***1000.00***	100.00	10.00%
1100	220	110	1.000	1200	1210	0.9917	0.91	110.00	1200.00	***1100.00***	100.00	9.17%
1210	242	121	0.992	1300	1331	0.9767	0.83	121.00	1300.00	***1200.00***	100.00	8.46%
1331	266	133	0.977	1400	1464	0.9562	0.75	133.10	1400.00	***1300.00***	100.00	7.86%
1464	293	146	0.956	1500	1611	0.9314	0.68	146.41	1500.00	***1400.00***	100.00	7.33%

Table 2.5.3: investment, technical change, value and profit rates with zero real wage

In our model the value of invested goods, and with it the organic composition of capital, rises continuously until investment stops. This process can be seen most clearly from the 'maximum' rate of profit, attained when wages are zero. It is shown in table 2.5.3 where column 11, *v(t)* × *K*, shows the value of invested capital. This now matches Marx's account even though the unit value of capital is falling.[14] The increase in investment exactly compensates for the depreciation and the total value of invested stock rises by exactly 100 in each period, being the value contribution of L, live labour.

In part II we show this is a general result. Dynamic value is a conserved quantity. In this special case where no use values leak from the system because there is no consumption, all new value is simply added to capital stocks. True, some value is lost when there is a wage, or capitalist consumption. But this loss is quantifiable: it is exactly equal to the consumed value. The reader can easily ascertain that the value of the wage, added to the value of invested and circulating capital in table 2.5.2, is equal to the totals of column 11 in table 2.5.3. We can thus trace the value in the system as a substance, like water or energy, which is either preserved as a stock, or destroyed in consumption in a quantifiable manner. We argue that a measure of this type, which we term *value conserving* or simply *conservative*, is the best foundation for economic dynamics.

2.6. Technical change and the formation of abstract value

In the last example we introduced, without justification, a rule for devaluing fixed capital. We shall now explain the argument behind this rule by discussing the way abstract or market values are formed from individual values in a single time period, and then move on to assess the way market values are formed over time. This is also critical to our treatment of the continuous case.

The issue arises the minute we accept – as in a dynamic framework we must – that a commodity can be produced using two different techniques at the same time. Suppose, first, that in a single time period a producer uses 2 tons of corn and 1 hour of labour to make 1 ton of bread. Suppose a second uses 2 tons of corn and 3 hours of labour to make 2 tons of bread. Suppose that $v_{\text{corn}} = 2$ and $v_{\text{labour}}=1$. The value contributions of the first producer's inputs suggest that

5) $$2v_{\text{corn}}+1v_{\text{labour}} = 5 = 1v_{\text{bread}}$$

For the second producer the same calculation yields:

6) $$2v_{\text{corn}}+3v_{\text{labour}} = 7 = 2v_{\text{bread}}$$

So is the unit value of bread 5 or 3½? The value calculation must be modified when outputs from different sources are pooled, either by entering a uniform market, by physical amalgamation, or both. If we cannot distinguish one producer's output from another's, we cannot treat them as separate goods; we must study the total result of the two processes. Together these create 3 tons of bread whose total value is 12; so the new unit value of bread is 4 per ton. Following Marx's usage we call this the *market value* and distinguish it from unit values 5 and 3 which are private to particular producers, and which we term *individual values*. [15].

The problem exists for any value measure including prices. Suppose the same two producers encounter prices given by $p_{\text{corn}} = 2$ and $p_{\text{labour}}=1$, and suppose the rate of profit is 20%, ignoring fixed capital. Output prices for the two producers should be given by

7) $$p_{\text{bread}} = (2p_{\text{corn}}+1p_{\text{labour}})(1+20/100) = 6 \qquad \text{(producer 1)}$$

8) $$p_{\text{bread}} = (2p_{\text{corn}}+3p_{\text{labour}})(1+20/100) = 8.4 \qquad \text{(producer 2)}$$

Either they sell their goods at different prices – which means effectively they are not the same good – or they realise differential profits. This is a critical difference between a static and a dynamic framework. In equilibrium, only an external factor such as monopoly, can sustain profit rate differentials. In a non-equilibrium economy they are always present because technical advance and capital movement take place over a comparable time-scale. Because capital moves in pursuit of surplus profits, abstraction in a multi-technique economy is the key to understanding how technical change takes place.[16]

2.6.2. Abstraction over time and fixed capital

Even with a single technique, new flows are pooled with existing stocks. Society's pool of goods contains more than the result of the last cycle of production. It contains an accumulated hoard of goods produced at various times using various techniques. In an exchange society with a perfect market in all secondary goods, these hoards also contribute to the formation of abstract values. Suppose our breadmakers find that society already has 10 tons valued at 24, left over from yesterday. Assuming society is willing to eat yesterday's bread, there will now be 12 tons of bread with a total value of 36; the new unit value of bread is therefore 3. This suggests treating fixed capital as the conversion of stocks into flows in which value is conserved over the total social stock of each commodity: this was the basis for the treatment of depreciation in our counterexample to the Okishio theorem.

Now, it could be claimed that fixed capital should enter abstraction in a different manner from circulating capital. It could be argued that a machine maintains its value at time of purchase until its is used up:

historic cost valuation. Or it might be considered to acquire the individual value of the most efficiently-produced existing new product: marginal current cost valuation. The difficulty is that both suggestions draw an arbitrary line in time. The leftover bread was produced yesterday. It would be odd to deny that it will meet up with today's bread in determining new values. So what about a machine that was produced yesterday? Or a machine produced before lunch? The distinction between fixed and circulating capital is, in the last analysis, only a question of how soon it is used.[17]

2.7. The transformation of values into prices

Our final example shows that in a dynamic framework, Marx's explanation of the relation between price and value can be extended to the case where inputs are purchased at current prices, while maintaining the two famous 'equalities' for a much wider class of economies than previously believed, and in particular for the case of 'maximum expanded reproduction' when all profits are reinvested. In fact differences between profit and surplus value arise only when there is private capitalist consumption, and it can be shown that the magnitude of these differences is independent of production, being equal to the price-value difference of the consumed (uninvested) surplus.

For this purpose it is necessary to make two corrections to the traditional treatment based on static fixed-point values and prices. Profits must be calculated as above, as the difference between the worth of businesses at the beginning of production, and their worth at the end. Secondly money, as a commodity with value that can serve as a store of value, must be explicitly included in the calculation, and the process of accumulation must make due allowance for money transfers induced by sale and purchases at current prices. Neither of these corrections can be made in a static framework, and we will demonstrate that the economies implied by the static model cannot in fact exist.

Though we happen to believe that commodity money is actually the foundation of credit, paper and electronic money, and though in our example, money is a produced commodity, this assumption is not necessary for the proof.

Suppose there are two circuits of production **M**, money and **G**, goods whose outputs are also called **M** and **G**. Suppose the reproduction **L** of labour power is a circuit of value whose inputs are the consumption of the labourers. We study *proportionate growth*, in which all physical output is re-invested in production and all inputs and outputs expand in a fixed proportion.

The reason for choosing this case is simplicity of exposition. The results, as we shall later prove, are completely general. Moreover according to the standard treatment using fixed-point values, it should be possible for this form of growth to take place on the basis of fixed-point, equal-profit-rate prices. We shall show that this is not so if the requirements of circulation, represented by the movement of money stocks, is taken into account.

Suppose, to fix ideas, that consumption and output is given initially by the following table:

Circuit		**M**	**G**	**L**		**M**	**G**	**L**
M	*Consumes*	0	2	2	*to produce*	2.5	0	0
G	*Consumes*	2	2	4	*to produce*	0	10	0
L	*Consumes*	0	4	0	*to produce*	0	0	7.5

Suppose initially that unit values are given, for example, by $(1\ 1\ {}^{2}/_{5})$ Thus 0.4 is the value of the commodity labour power itself. After one cycle of production, using the method of calculation we have been using, unit values become $(1{}^{3}/_{5}\ {}^{4}/_{5}\ {}^{7}/_{15})$.

2.8. The profit rate in value terms

We can calculate the rate of profit in value terms. This is the rate of profit which would pertain if goods exchanged at values, and it is calculated, as before, as the difference between the total value of all circuits of production before and after production. The following table shows, for each circuit and for the economy as a whole, how this calculation works out:

	M	G	L	Total
Value at start	$2^4/_5$	$5^3/_5$	4	$12^2/_5$
Value at end	4	8	4	16
Profit	$1^1/_5$	$2^2/_5$	0	$3^3/_5$

Note that the 'profit' of the commodity labour power is zero and that total profits 3.6 are equal to the total labour expended, 5, multiplied by 1 minus the value of labour power. Thus this method of calculating the total profit in value terms yields total surplus value. From this table we can also see the total value of commodities in circulation, including money itself. The physical stocks in society after one cycle of production are the outputs of 2.5 units of money, 10 of goods and 7.5 labour hours. Their value is given by the second row of the table and totals 16. Before production there were less commodities in circulation: 2, 8 and 6 respectively. Their total value was $12^2/_5$.

2.9. Price, purchasing power and value

Now assume that initial prices, relative to the money commodity, are given by $P_G=1^1/_5$, $P_L=1$. This choice is arbitrary and the result is the same with any initial prices. We can use this to calculate the initial total price of the stocks in society, consisting of 4 units of money, 8 of goods and 4 of labour-power. The 4 units of money are 'priced' at 1 for the purpose of this calculation. This total price is $17^3/_5$, which is larger than the total value of the same goods. The 'purchasing power' of money is therefore inflated relative to the total value in circulation. This is not just a problem of the unit of measurement: money whose value is $12^2/_5$ exchanges for goods whose price is $17^3/_5$.

Now consider the following problem: suppose we wished to use the money price of a bundle of goods as a measure of the share of social value which it represents. Two distortions would be introduced. The first is because the price of each separate commodity is different from its value. The price of the 8 units of **G** in circulation is $9^3/_5$ and their value is 8. We do not want to lose sight of this difference, because circulation actually does redistribute value and we need a measure which can keep track of this redistribution. But there is a second distortion: the total price of all commodities being greater than their total value, $9^3/_5$ is not an accurate measure of the *share* of total value which this money price represents.

We therefore normalise prices so that the total price of all commodities in circulation, including labour power and money, is equal to their total value. This we take to be the meaning of Marx's famous first equality. [18] In effect, it serves as a formal definition of money of account, a measure of the value which commodity money represents in exchange, adjusted to serve as a measure of the *share* of total social value which commodity money represents in exchange. This involves no requirement that profits must equalise[19], so we shall use the term *generalised price of production* to refer to money prices normalised in this manner. We thus multiply all prices by the quantity

$$\frac{12\,{}^2\!/\!_5}{17\,{}^3\!/\!_5} = \frac{31}{44}$$

giving p_M=0.7, p_G=0.85, p_L=0.7, to two decimal places. The lower case p signifies normalised prices.

We now turn to the effect of reproduction on these generalised prices of production. Since production alters the amount of value in circulation, prices must be renormalised afterwards to give new generalised prices of production at time t+1. This construction is perfectly general and does not demand that profit rates are immediately equalised. It is instructive, however, to choose prices which result in an equal price rate of profit.

To two decimal places the new generalised prices of production are p_M=1.74, p_G=0.83, p_L=0.45. The corresponding money prices are P_G=0.48, P_L=0.26. The profits resulting from these new prices are shown in the table below, using the method of calculation which should by now be familiar.

	M	G	L	Total
Price at start	3.10	5.92	3.38	12.4
Price at end	4.34	8.28	3.38	16
Profit in price terms	1.24	2.36	0	3.6

Now, the profit in price terms is the same as the profit in value terms, as indeed it must be since the prices in each period are normalised so that the sum of prices is equal to the sum of values. But, as we showed earlier, the sum of profits in value terms is equal to the sum of surplus values over the same period. Thus the second of Marx's two equalities, the equality of profits in price terms and the sum of surplus values, also holds.

This result is so simple that it is almost trivial. The only non-trivial step is proving that the sum of profits in value terms is equal to the sum of surplus values, a result we shall give later on. The result is moreover perfectly general and applies to *any* set of initial and final prices whether or not they equalise profits[20]. Yet it is free from all the weaknesses normally ascribed to Marx's procedure for transforming values into prices. This appears to be in flat contradiction with static solutions such as that of von Bortkiewicz. How can this be? The answer is that, whereas static values can be generalised to dynamic values, the static price calculation is simply wrong. Fixed-point values can easily be found: they are respectively 1.50, 0.87 and 0.47 to two decimal places, and they give total surplus value equal to 3.2. However static prices are not a special case of dynamic prices; static prices correspond to an economy which cannot exist, because it cannot accumulate.

To explain this consider first what happens if businesses trade at the same initial prices as before, that is 1.2 and 1 in money terms. What are the results of circulation? This appears at first sight to depend on two issues, namely what inputs producers require in the next period and what the capitalists consume. In static formulations these are the only two determinants of prices. But once a distinction between stocks and flows is introduced one must enquire not only what flows of commodities will be necessary in the next period, but also what distribution of stocks.

Assume the capitalists consume nothing and all outputs are re-invested. This still does not tell us what inputs are required, since we do not know in which circuit of production they are invested. Let us treat the simplest possible case, which is to expand all production proportionately by 25%, which exactly uses up the entire physical surplus. According to fixed-point theory expansion along this path, a so-called von Neumann ray, can continue indefinitely on the basis of a fixed equilibrium price. Suppose, therefore, that producers trade at the equal profit rate prices so as to give them the inputs required to start production on a 25% higher scale. This requires physical stocks distributed as follows:

Circuit		M	G	L		M	G	L
M	*Consumes*	0	2.5	2.5	*to produce*	3.125	0	0
G	*Consumes*	2.5	2.5	5	*to produce*	0	12.5	0
L	*Consumes*	0	5	0	*to produce*	0	0	9.375

Can they acquire these stocks at the equal profit rate prices? The money price of their sales and purchases will be as follows:

	M	G	L	Total
Sales	2.50	4.77	1.95	9.22
Purchases	1.84	4.99	2.39	9.22
Net transfers	-0.66	0.22	0.44	0

But if the new stock levels are the result of circulation at the given prices, the net transfer in price terms should be zero. This only happens if money itself is left out of the account. Once we introduce money, not just as money of account but as means of circulation and payment, we find that an exchange economy in which all stocks expand in proportion *cannot exist*, even if production expands in proportion. The requirements of circulation contradict the requirements of circulation.

A von Neumann ray is not a simplification but a fiction, indeed a fantasy. In a real exchange economy the price imbalances above would have to be counterbalanced by monetary transfers. The producers **M** would suffer a decline of 0.66 in the money balances in their possession. The producers **G** would enjoy a gain of 0.22 and the labourers would also gain 0.44. The total money in society would not change, but its distribution would. The next period of production would open with stocks distributed differently, and in different proportions, to the last [GIVE THIS]

Previous treatments of the transformation problem on the whole ignore the triple role of money which serves as a measure but is also used to circulate other values and itself acts as a store of value In short it not only measure value but possesses it. It may be argued that this assumption demands a commodity money theory. But the result is hard to avoid even without this assumption. We could have constructed the circuits **M** and **G**, for example, so that money was not produced or consumed but simply stayed in existence with a fixed value. But it must still serve as a store of value whether its form is gold, paper or megabytes. Why should people maintain bank accounts if they have no value? Money, as generalised purchasing power, must have a value – whether directly or by proxy – precisely because it exchanges for value.

As production expands along a von Neumann ray at a constant and equal rate of profit, its actual course is as follows: the requirements of production expand the stocks in circulation by a fixed proportion equal to the rate of profit; while money stocks adjust continually according to the requirements of circulation. If, on the other hand, both stocks and flows expanded in strict proportionality as the von Bortkiewicz solution requires, then circulation could not take place on the basis of equal profit rate prices. We have moved into a terrain where the static analysis produces no meaningful results at all, whereas a dynamic analysis produces the exact result proposed by Marx.

3. Value measures and their properties

This section tries to establish, more or less axiomatically, what is meant by a time-varying measure independent of distribution. Consider the following thought-experiment. Suppose we know from observation the physical quantities of every commodity, including labour power, which have been consumed and produced between in each time interval over an entire period. Suppose we also know the initial physical stocks of these same commodities and their distribution. We may also assume known, though our theory does not require it, the prices at which they were sold and any other time-dependent data which an arbitrary theory might require. For now let us take this data as given.

There are many ways to construct a time-varying measure for this economy. We could measure everything at current prices. These change from day-to-day, but nevertheless measure any collection of objects at any given time. We can use this measure to answer questions such as 'how much capital was invested in car production in 1990 or 'how much did labourers consume in June 1992'.

But current prices are not the only such measure. As soon as we ask questions such as 'were labourers better off in 1990 than in 1985' or 'was the nation's stock of capital greater or smaller last year or this year', all economists agree that current prices are inadequate, because they vary, and because baskets of consumption and investment also vary, over time. They construct price indices and other instruments of revelation or deception, depending on one's point of view, to answer questions which current prices cannot. These are nothing more than alternative measures of economic size, or value.

All such indices suffer two drawbacks. First, they do not compare like with like. If labourers do not consume the same bundle of goods in 1990 as in 1985, there *is* no single price measure that can compare the two. In fact the Paasche and Laspeyres price indices are just as much a '93% measure of value' as poor Ricardo's labour values were a '93% theory of price'. Secondly, the instrument of measure itself – price – depends on who consumes the product and on their income. In short, it depends on distribution.

Is there a class of measures which do *not* depend on distribution, so that, no matter what the relative consumption and wealth of the population, and no matter what prices are, the measure of the same bundle of goods will be the same, provided only that it is measured at the same time? In short, can we establish time-varying measures which are functionally dependent only on the inputs flows to, and outputs flows from, production up to the point of measurement?

This means something quite precise: if the economy followed an alternative trajectory in which the inputs and outputs to production were exactly the same, but the private consumption of the population was distributed differently, then relative prices could not be the same. Using price as a measure of size, the objects in the economy would seem to have undergone some kind of relativistic distortion.

The Cambridge school's objection to the neoclassical production function can be expressed as follows: even though the economy has consumed the same inputs and produced the same outputs, changes in private consumption make it seem as if there has been a change in the technical relations of production. We seek a measure of value for which this is not so.

3.1. Definition of a measure

The first step is to define what a measure actually consists of. Intuitively it is a mapping from a bundle of commodities onto a single real number. In fact this is not quite enough. A measure has a unit of attached to it: seconds, pounds, feet, amps or whatever, that relates it to some standard of comparison. Moreover an economic measure has an intuitively obvious property: it adds up. When two objects are combined, their collective size is the sum of their individual sizes. Mathematically, this can be expressed thus: value is a linear mapping. Can we justify this intuitively reasonable idea?

Not all measures are linear. There are many non-linear physical measures – for example, vector distances. The distance from A to C is not always equal to the distance AB plus the distance BC. Is there any special reason why a measure of economic size should aggregate linearly? In fact to be independent of distribution a measure must at least be linear at each point in time, or the value of a collection of goods could be changed by transferring commodities from one owner to another. A non-linear measure, incidentally, would be subjective, since the value of a commodity would depend on who owned it.

This is only relevant, however, to a measure specified for a single point in time, which we will term an *instantaneous* measure. In a dynamic economy a measure is parameterised by time. It is an instantaneous measure specified at each point in time, calculated from the physical quantities produced and consumed up to that time, and from any other parameters which the theory concerned considers relevant. When we want to distinguish such a generalised measure we will refer to it as a *dynamic* measure. An instantaneous measure is thus a dynamic measure at a particular point in time, and a dynamic measure is a set of instantaneous measures specified at all intermediate points over a period of time.[21]

Note, in parenthesis, that we do not require a measure to be fully specified for all commodities or even bundles of commodities. If a measure tells us the size of only one commodity, it is still a measure.

3.2. Measurement and production

When production or consumption takes place, the physical quantity and ownership of goods in the economy changes. But this requires the passage of time, and one instantaneous measure is replaced by another. The issue now becomes; what governs the transition from one measure to the next? If we can establish a transition rule which is itself independent of distribution, then distribution cannot affect the value which the new measure will assign to any bundle of goods. We therefore have a time-varying, linear measure of value which is independent of distribution.

3.3. The transition rules of labour value systems

Classical (Marxist) labour values are a function of the flows of value and quantity entering and leaving production. In fact, as is well-known, all commodities other than labour contribute value equal to their value, and labour contributes value proportional to labour time. This means that a labour value measure is a linear function of the quantities, but not the values, entering production. Consider, for example, a business which consumes commodities **C1, C2** and labour power, and produces **C3**. Suppose the unit value of **C1** and **C2** are 3 and 2 respectively and that the value of labour power (the value consumed in producing labour power) is ½. The quantities and values of the flows entering production are given by

	C1	C2	Labour		C3
Quantity	4	9	6	*produces*	27
Value consumed	12	18	3	*produces*	36

The value of the 27 units of **C3** is not given by the sum of values consumed. It is the sum of

C1	C2	Labour Power
12	18	6

This is a linear function of the vector of quantity inputs (4 9 6), multiplying it by the vector (3 2 1). But this is not a linear function of the vector of value inputs (12 18 3). The 'work' is done by a change in the instantaneous value measure applied to the inputs entering production. Inputs themselves are valued at (3 2 ½), but their contribution to value are given by a new value measure (3 2 1) in which the value of labour power is replaced by the quantity of labour power. These contributions to value are conserved from then on. A new measure of value is derived for the commodity **C3** on the basis that the total value of the produced **C3** must be equal to the value contributions of its inputs. The value of 27 units of **C3** being 36, the new measure is based on a unit value of ¾.

Let us call the initial value measure $V^{(t)}$, the measure of value contributions $V^{(t)}_{IN}$, and the measure of value outputs $V^{(t+1)}_{OUT}$.[22] Finally abstraction creates a new stock value measure $V^{(t+1)}$. There are three transitions to consider: $V^{(t)}\rightarrow V^{(t)}_{IN}$; $V^{(t)}_{IN}\rightarrow V^{(t+1)}_{OUT}$; and finally $V^{(t)}_{OUT}\rightarrow V^{(t+1)}$. These correspond to the phases of circulation which Marx designates as C—P, P...P', and P'—C', thus:

C—P	P...P'	P'—C'
$V^{(t)}\rightarrow V^{(t)}_{IN}$	$V^{(t)}_{IN}\rightarrow V^{(t+1)}_{OUT}$	$V^{(t)}_{OUT}\rightarrow V^{(t+1)}$

The second two transitions do not destroy value by their very definition. $V^{(t)}_{IN}\rightarrow V^{(t+1)}_{OUT}$ is defined so that the value of outputs is equal to the value of inputs. $V^{(t)}_{OUT}\rightarrow V^{(t+1)}$ is defined so that new value is distributed as over all produced and consumed value, regardless of who owns it.

Now consider how distribution affects these value transitions. Prices do not enter the calculation and a functional dependence on them can only occur via their effect on consumption. Changes in consumption affect the data in only one place, which is the consumption of the labourers themselves, producing the commodity labour power. Other final demand figures only through its relation to labourers' consumption. All components of final demand affecting value are determined by the value of labour power.

True, labour power is an element of $V^{(t)}$. But in the transition $V^{(t)}\rightarrow V^{(t)}_{IN}$ it was replaced by a magnitude independent of private consumption. In the transition $V^{(t)}_{IN}\rightarrow V^{(t+1)}_{OUT}$, it figures only in determining the value of labour power itself. And the only feature of distribution which might affect the transition $V^{(t)}_{OUT}\rightarrow V^{(t+1)}$ is the effect of consumer durables on abstraction. But abstraction operates over the whole stock of goods in society, and is therefore independent of who owns them.

Thus if $V^{(t)}$ is used to measure a bundle of commodities, the resulting magnitude cannot be affected by distribution in any previous time period unless this bundle includes a stock of the commodity labour power. But by its very definition, this commodity is sold by one class and purchased by another, and cannot be redistributed between them. We conclude that the value measure is entirely independent of distribution between the classes and that as far as distribution within the capitalist class is concerned, the only element affected by distribution is the value of labour power itself.

3.4. Price as a measure of value

Now consider some alternative measures of value. What about price? It is linear: the price of two independent collections of goods is equal to the sum of their separate prices. Indeed current prices must be proportional to current values (though the factor of proportionality changes). So why not use prices as a value measure? The principal reason, which we have already discussed, is that the variation of prices over time depends on distribution. Sraffa had to eliminate time from his system because its invariance properties disappear in a dynamic framework. However this applies not just to neoricardian prices but to any price measure. The size of an arbitrary bundle of commodities, measured in price terms, must in general vary when prices vary.[23]

We can approach this a different and instructive way from the standpoint of value conservation. It is a reasonable question to ask of any value measure what happens to the value of various aggregates as time progresses. Now, we can always divide up the total stocks of society K, in a given period, into those which serve in the next period as inputs to production, those which remain unconsumed, and those which enter final consumer demand. Call these A, F and D respectively. Thus

$$K=F+A+D$$

which we term the fundamental stock accounting identity, defined rigorously in part II. Changes in distribution may affect who owns the elements of K, but not the total stock of each good it contains.

Our dynamic value measure can only be altered by changes in commodity flows. It is functionally dependent on A alone. It follows that the total value of everything *not* consumed in production – final consumer demand plus unconsumed stocks, or $F+D$ – must remain constant. In short, value is conserved in distribution. If, on the other hand, a measure does *not* conserve value in distribution, it cannot possibly be independent of it.

Are price measures conserved in distribution? At first sight they are, since no physical commodity is destroyed or created by simply exchanging goods. But in order to explain what happens in production, neoclassical price theory is obliged to put a price, not just on the consumption of a commodity but on its ownership. Such a measure cannot be conserved in distribution, since individuals can acquire assets just by owning other assets. The mere existence of a warehouse serves to create extra value.

We believe that this is a feature not just of neoclassical prices but of any attempt to use price as a measure of value. Consider a producer using iron, carbon and labourers to make steel. Let the quantities consumed be I,C, and L respectively and the output S. Suppose prices and the wage to be p_I, p_C, p_L and p_S respectively. The price is not in general given by

9) $$Sp_S = Ip_I + Cp_C + Lp_L$$

because input and output prices are determined independently in the market, so a profit or loss intervenes. Now we can either argue that profit arises outside of production, or we can modify the price equation with some source of value – the factor of capital, or the factor of land – which is priced as a means of production. The output price can be explained via the rent of capital K, priced at p_K to give

10) $$Sp_S = Ip_I + Cp_C + Lp_L + Kp_K$$

But this extra source of value, unlike all others, is not consumed in production. p_K does not depend on what is used up in production. It is a rent, a charge levied on exclusive possession or access, not on consumption.[24] It is therefore functionally dependent on the distribution of a crucial element of K which in general has little to do with production, namely money. p_K is the price of a price – the money value of K – and not its size.

But if a capital sum of £10,000 can contribute £1,000 per year to the product, it can contribute £1,000 a year to me, whether or not I have anything to do with production. Moreover it must make this contribution every time a flow comes to rest, whether in a factory or a warehouse. This, it seems to us, will happen for any measure of economic size based on the price of inputs to production, all attempts to evade it amounting to mathematical or verbal subterfuge. Thus suppose for example, in deference to the Cambridge critique of heterogeneous capital, we resort to the price equation

11) $$Sp_S = Ip_I + Cp_C + Lp_L + r\mathbf{p}.\mathbf{K}$$

where **K** is the vector of invested capital, **p** the vector of prices and r the rate of profit or the 'cost of capital'. Is not **K** a 'technological factor of production? Is $r\mathbf{p}$ not just a rather complicated price expression? Perhaps – but **p.K** is not equal to the capital advanced, which is the historic, and not the current expenditure which the company has undertaken. Their bankers are not influenced by the fact that the £10,000 borrowed in 1985 bought a machine which now costs £2,000. Interest is charged on the money, not the machine.

3.5. Why labour? The possibility of alternative conservative systems

We believe we have proved that for a measure of value to be independent of distribution, the transition rules which evaluate contributions to value should be a function of input and output commodity flows alone. There could be measures of this type other than labour values. The physiocratic system, for example, treats corn as a factor of production which adds more than its own value,[25]and it is sometimes argued that any input to production can in principle be treated in this way as a source of value.

It is worth recalling the precise property of labour power which ensures the independence of labour values from distribution: the labourers' consumption is a component of final demand. Being neither slave nor machine, the labourer enters the market with an income and with purchasing power. Labour power is the only commodity which walks around purchasing other commodities. If robots had income they would create value. Now, precisely because the inputs to the production of labour power are a component of final demand, if labour power made a contribution to value which in any way depended on them, then we would not have a measure independent of distribution. From this it follows that, although commodities other than labour might conceivable make a contribution to value, labour power itself *must* make a contribution independent of the consumption of the labourers to satisfy our axioms.

This does not rule out the possibility that some additional input might also make a contribution to value independent of its value. This is why we have been careful to refer to the value measures which satisfy our axioms as a 'class' of value measures rather than a single such measure. But if a commodity contributes an amount of value different from its own, some justification is required.

A 'corn measure of value' might make sense where corn production does not involve waged labour, since the corn producer does indeed then appear directly as the producer of a net surplus, albeit produced by means of non-waged labour. Labour, however, is the only commodity which purchases in addition to being purchased. Nevertheless, the possibility perhaps remains open of creating dynamic value systems in which value is absorbed or created at the interface with non-capitalist sectors.[26]

Finally we should note that the condition for a conservative labour measure of value is that the contribution to value made by labour power be independent of distribution. It is conceivable, first, that different concrete labour powers make a different contribution to value, and second that the contribution of labour power to value be some other function of labour time than its simple magnitude. We proceed using Occam's Razor.

(the second part of this paper, which presents some of the above results more formally, has not been printed here.)

References

Alberro, J and Persky, J, 1979. "The Simple Analytics of Falling Profit Rates, Okishio's Theorem and Fixed Capital", *Review of Radical Political Economics,*vol7, Spring.

Andrews, M, 1984. "On the Importance of the Equilibrium Assumption to the Okishio Theorem Result". Working Paper, Rutgers University.

Andrews, M, 1987. "Technical Change in Postwar U.S. Agriculture and the Falling Rate of Profit." Working Paper, Rutgers University, Cook College, New Brunswick, New Jersey.

Baumol, B. *Economic Dynamics* (?date/pubs?)

Birkhoff, G and MacLane, S, 1963. *A Survey of Modern Algebra*. New York:Macmillan.

von Bortkiewicz, L, 1951. "Value and Price in the Marxian System", *International Economic Papers* 2.

Carchedi, G, 1992. *Frontiers of Political Economy*. London:Verso

Carchedi, G, 1984 'The Logic of Prices as Values', *Economy and Society* Vol 13 No 4, pp431-455

Carchedi, G, 'Marxian Price Theory and Modern Capitalism', *International Journal of Political Economy*, 1988, Fall, pp6-109

Carnap, R, 1958. *Introduction to Symbolic Logic and its Applications*. New York: Dover

Fine, B and Harris, L, 1976 "Controversial Issues in Marxist Economic Theory", in Milliband and Saville, eds, *Socialist Register*.

Fine, B and Harris, L, 1979. *Rereading Capital.* London:McMillan; New York:Comlumbia University Press.

Freeman, A, 1990. *A Dynamic Simulation Model of a Capitalist Economy*. CSE conference 1991

Gandolfo. *Economic Dynamics* (?date?pubs?)

Gantmacher, F. R., *The Theory of Matrices*, Volume II. New York:Chelsea.

Gershgorin, S, 1931. *Über die Abrenzung der Eigenwerte einer Matrix*. Izv Akad Nauk SSSR Ser Mat. **7**, 749-754; 16,

Giussani, P 1992 "The Determination of Prices of Production" in *International Journal of Political Economy*, Vol 21, No 4, Winter 1991-2

Goodwin, R and Punzo, L 1987. *The Dynamics of a Capitalist Economy*. Boulder, Colorado: Westview and Cambridge:Polity

Hahn, F, 1984. *Keynesian Economics and General Equilibrium Theory: Reflections on Some Current Debates* in Hahn, F *Equilibrium and Macroeconomics*. Oxford: Blackwell
Harcourt, G C, 1972. *Some Cambridge Controversies in the Theory of Capital*. Cambridge: Cambridge University Press.
Harrod, R F, 1937. *An Essay on Dynamics*.
Hunt, E and Schwartz, J. *The Subtle Anatomy of Capitalism*
Laibman, D, 1987 "Growth, Technical Change and Cycles: Simulation Models in Marxist Economic Theory", *Science and Society*, Vol 51, Winter 1987-1988, pp 414-438
Mandel, 1974 *Late Capitalism*. London, Verso
Mandel 1984. "Gold, Money and the Transformation Problem" in Mandel and Freeman (1984)
Mandel & Freeman, (eds) 1984 *Marx, Ricardo and Sraffa* London:Verso
Marx, 1968a. *Theories of Surplus Value*, Part I. London:Lawrence and Wishart
Marx, 1968b. *Theories of Surplus Value*, Part II. London:Lawrence and Wishart
Marx, 1968c. *Theories of Surplus Value*, Part III. London:Lawrence and Wishart
Marx, 1977a. *Capital*, Volume I. London:Penguin
Marx , 1972. *Capital*, Volume III. London:Penguin
Marx, 1976. *Value Studies by Marx*. London:New Park
Marx 1977b *Results of the Immediate Production Process*, printed in Marx 1977a
May, K, 1951. "Value and Price of Production: a Note on Winternitz's Solution." *Economic Journal,* December.
Mendelson, E, 1964. *Introduction to Mathematical Logic*. New Jersey: Van Nostrand.
Miconi, B, "On Harrod's Model and Instability", (?date & source?)
Morishima and Catephores, 1979. 'The Transformation Problem: a Markov Process', *Value, Exploitation and Growth*, McGraw-Hill.
Morishima, M, 1973. *Marx's Economics*, Cambridge
Nakicenovic, N, 1988. *Dynamics of Change and Long Waves*. Working Paper,Laxenburg (Austria): International Institute for Applied Systems Analysis(IIASSA)
Naples, M, 1985. "Dynamic Adjustment and Long-run Inflation in a Marxian Model", *Journal of Post-Keynesian Economics* Fall 1985, Vol 8 No 1
Naples, M, 1988. "Is a Uniform Profit Rate Possible? A Logical-Historical Analysis", *Science and Society* Vol 52 No. 1, Spring 1988, pp83-93.
Naples, M, "Time, Money, Equilibrium and the Labour Theory of Value." Working Paper, Rutgers University, New Brunswick, New Jersey.
Naples, M "Methodology and the Transformation Problem", Working Paper, Rutgers University
Okishio, N, 1961. "Technical Change and the Rate of Profit", *Kobe University Economic Review*, No. 7, 86-99
Okishio, N, 1972. 'On Marx's Production Prices', *Keizaigaku Kenkyu*, 19.
Pasinetti, (?). *Lectures in the Theory of Production*. Cambridge:(?)
Salama, 1979. *Sur La Valeur,* Paris
Savran, S, 1978. "Surplus, Wages and Reproduction: Contradictions of Sraffian Economics". Working Paper, Centro Studi di Economia Politica, 20124 Milano via Tadino 33.
Savran, S, 1979. "On the Theoretical Consistency of Sraffa's Economics." *Capital and Class*, 12.
Savran, S, 1980. "Confusions Concerning Sraffa (and Marx): Reply to Critics", *Capital and Class*, 7.
Savran, S, 1984 in E Mandel and A Freeman (eds), op cit.
Semmler, 1984. *Competition, Monopoly, and Differential Profit Rates*. New York: Columbia University Press
Seneta, E, 1973 *Non-negative Matrices: an Introduction to Theory and Applications*, London:George Allen and Unwin
Seton, F, 1957. "The Transformation Problem", *Review of Economic Studies*, Vol. 24 pp149-160.
Shaikh, A, 1973. 'The So-called Transformation Problem: Marx vindicated', mimeograph, New School for Social Research, New York
Shaikh, A, 1977, 'Marx's Theory of Value and the "Transformation Problem"' in J Schwartz (ed) *The Subtle Anatomy of Capitalism*. Goodyear.
Shaikh, A, 1981 "The Poverty of Algebra" in I. Steedman(ed) *The Value Controversy*. London: Verso
Shaikh, A, 1984. "The Transformation from Marx to Sraffa" in Mandel and Freeman (eds) op cit
Sraffa, P, 1960. *The Production of Commodities by means of Commodities*. London: Cambridge University Press.
Steedman, I 1977. *Marx After Sraffa*. London: Verso
Varga, R *Matrix Iterative Analysis*, 1963. New Jersey:Prentice-Hall.
Winternitz, J, 1948. "Values and Prices: a Solution of the So-called Transformation Problem." *The Economic Journal*, June
Wolff, R, Callari, A and Roberts, B, 1984 "A Marxian Alternative to the Traditional 'Transformation Problem'" in *Review of Radical Political Economics* Vol 16(2/3) 115-135 (1984)

---

[1]See Freeman (1990). In this model, values are determined dynamically by production, and prices are determined dynamically by values and by distribution, through the movement of capital between branches of the economy and between individual capitals within each branch. The model exhibits regular business cycles whose frequency is related to the turnover time of fixed capital and which can be sustained indefinitely by processes of technical change.

[2]See, for example, Hahn (1984). The underlying problem is that neoclassical equilibrium prices actually furnish a theory of value. Notional equilibrium prices are different from observed market prices. There is thus a price-value distinction and a transformation problem, since real prices are determined by equilibrium prices but not equal to them.

[3]'The investigation is concerned exclusively with such properties of an economic system as do not depend on changes in the scale of production or in the properties of "factors" ... In a system in which, day after day, production continued unchanged in these respects, the marginal product of a factor (or alternatively the marginal cost of a product) would not merely be hard to find — it just would not

be there to be found'[3]. (Sraffa 1960, preface) This approach, designed to purge economics of marginalist obstructions, also purges it of change. The result is an equilibrium price system. Technology is given and fixed externally; from it a time-invariant measure of value, the standard commodity, is derived.

[4]Here we refer to the mainstream treatment of value theory and not to the small number of Marxist theorists attempting to break from the straitjacket of static values and prices. First mention must be made of Robert Langston, whose work inspired the collection of works in Mandel & Freeman (1984), all of which in one way or another explore aspects of a non-equilibrium approach to value theory. Other thoughtful contributions of which we are aware (we are sure they are not the only ones) have also been made by Andrews, Laibman, and Naples; Shaikh has frequently emphasised the time-dependent nature of value calculation. In (1973) he proposed the now well-known reduction to dated labour to which our concept of value convergence owes a substantial debt, and in (1984) proposed the notion of a circulation model of reproduction. Michio Morishima (1979) provides an interative interpretation of transformation and explicitly rejects the von Bortkiewicz assumptions, deriving a solution in which both Marx's equalities are dynamically satisfied under certain assumptions concerning the scale of outputs. Morishima attributes an early version of the iterative approach to Okishio, 1972. Michel de Vroey (1981) developed an extended critique of mainstream value theory centering on the issues of time and money. Fine and Harris (1976,1979) have also attempted to frame the concept of organic composition and its relation to technical composition in terms of relations at different points in time. No bibliography on value dynamics can fail to note the pioneering work of Goodwin (1987, and many others); however, this work is virtually all in a comparative static framework.

[5]See Baumol, Gandolfo, or Miconi

[6]See G C Harcourt() and also Hunt and Schwartz() for summaries of the discussion on this point, as well as Robinson (1962)

[7]This is also central to the concept of *cause* in the physical sciences. Magnetic fields are produced by electrical currents – charges in motion. When these currents vary a reciprocal relation emerges, and magnetic fields in turn produce electrical effects. If the relation between price and value is dynamically determined, all arguments fall which are directed at value theory but derived from the steady state. Had Mr Faraday been irresponsible enough to sire a political revolution instead of an industrial one, we might well be ploughing through learned tracts on the logical impossibility of the dynamo.

[8]Thus, for example, Steedman (1977) p73: "the backward resolution of means of production into dated labour quantities is a purely conceptual resolution and *not* a 'historical story', for it is seen at a glance that the a and A [technical coefficients - AF] quantities at every step are those of the current period."

[9]For example, if technology changes. It may be argued that in such cases comparative statics still produces usable results. But dynamic values are also defined where even comparative static values simply do not exist. Giussani (1992), points out that no meaningful static solution exists when the Hawkins-Simons conditions are violated – for example if there is negative net production in any product in a particular year. But this can easily happen in a real economy, for example following a poor harvest

[10]This model is not kind to profit-squeeze theories. However, it should of course be recognised that any instantaneous *increase* in the wage rate will, of course, result in an instantaneous fall in profits. If a rise in the real wage takes place over a period, then the actual impact on profits depends on the relative speed of the various effects at work.

[11]This point was made by Alberro and Persky (1979)

[12]It is, however possible, to depreciate capital using dynamic values calculated directly from turnover, that is, without assuming abstraction of value over fixed capital. In this case profits fall faster than with abstraction over stocks. However, conservation of value no longer holds unless the devaluation of fixed capital is estimated as a loss of value akin to consumption, rather than a value transfer to newly-produced but technically-superior goods as it is in the system we propose.

[13]This may shed some light on the role of unproductive consumption in rebuilding profits after a slump. It suggests that, for example, arms spending or public works could play a limited counter-cyclical role by siphoning off accumulated value, permitting invested capital to depreciate without a collapse in demand and without a rise in wages, not as a general or permanent feature of capitalism but as a specific and limited measure which alternates with high investment.

[14]'By relative cheapening of machinery, I mean that the absolute value of the amount of machinery employed increases, but that it does not increase in the same proportion as the mass and efficiency of the machinery' Marx (1968c) p221, footnote.

[15]Marx (1968b; 1977b)[get page references and perhaps quotation]

[16]The most developed discussion on this is Mandel (1974)

[17]If large items of fixed capital are embedded in production with no secondary market this could modify the analysis. The old machine is the a different commodity from the new. But in this case we have to be consistent. The equivalent replacements have to be regarded as a new stock of machines with their own, independent valuation.

[18]There are some parallels with Sraffa's price measure based on his standard commodity, an ideal commodity bundle which the economy could produce if net output were proportional to net input. But our standard is not a theoretical ideal commodity but the actual commodity bundle $\mathbf{K}'$, consisting of the entire existing stock of value in society.

[19] We regard the common assumption that profits are immediately equal as an unacceptable simplification which has been adhered to only because a static solution cannot be derived without it. First, any *general* formulation of transformation must deal, not just with equal profit rate prices but with aribtrary prices. Second, the process of equalisation and its counter-tendencies are part of the mechanism of economic growth, and they cannot be wished out of existence before the enquiry begins. Finally, in real life profit rates

are not equal. cf Marx (1968c, p462): 'the average profit in each sphere becomes evident only in the average profit rates obtained, for example, over a cycle of seven years.'

[20]Marx (1968c, p167) suggests a calculation of this type in a passage which seems to have escaped attention in the literature: 'the cost price of constant capital – or of the commodities which enter into the value of the newly produced commodity as raw materials, auxiliary materials and machinery [or] labour conditions – may likewise be either above or below its value. The commodity comprises a portion of the price which differs from value, and this portion is independent of the quantity of labour newly added...Variable capital, whatever difference between value and cost-price it may contain, is replaced by a certain quantity of labour which forms a constituent part of the value of the new commodity, irrespective of whether its price expresses its value correctly or stands above or below the value.' The cost-price (generalised price of production) of the output is thus equal to the cost-price of constant capital inputs plus a share of new surplus value. See Giussani (1992) and Roberts, Wolff & Callari.

[21]There is a possible analogy with the formalism of state space theory. An economy can be treated as a dynamic system if we regard the physical quantities consumed and produced as 'inputs' (not in the economic sense but in the terminology of state space theory), and the dynamic measure represents the 'state' of the system.

[22]Throughout this article, the term 'technical coefficient' refers strictly to the proportion of a given input which is *used up during the course of production*. This is not exactly the same as the 'technical possibility coefficients' which cannot change while production is going on. Our definition is perfectly general and remains valid if technical changes taking place during production; simply add up what has been consumed.

[23]It is a substantive claim of Sraffa's to have discovered a unit of measure – the standard commodity – in terms of which distribution between labour and capital can be studied independent of price variation. Even so, as we move along the wage-profit frontier the total price of the net product – measured in the standard commodity – varies, as does the total price of inputs to production, and the total price of outputs. The measure of *size* that results, even from the static system, is not independent of distribution. [CHECK THIS VERY CAREFULLY]

[24]The difficulty is highlighted by the fact that the magnitude of p_K (or r$\boldsymbol{p}$) depends on the time-span of production. The *dimensions* of p_K are different from the dimensions of the other prices. The terms representing flows have a time dimension (dollars per hour, day, or per week) because the flows themselves have a time dimension (tons per hour, day or week). The prices themselves have no time dimension, being pure ratios. But what of capital? This is not a flow but a stock. It has no time dimension. The term Kp_K can have a time dimension only if it has a totally distinct kind of price, a price with a time dimension built in..

[25]'The Physiocrats transferred the inquiry into the origin of surplus value from the sphere of circulation into the sphere of direct production, and thereby laid the foundation for the analysis of capitalist production.' Marx (1968a), p45

[26]One interesting possibility, beyond the scope of this article, is to define the value of labour power as equal to the value of the total private consumption of society. In this case the value of labour power is no longer constant, but all values become independent of distribution. Such a measure has the further property that for an economy whose technical coefficients are fixed, the value of capital remains constant while values converge on the fixed point.