Asset Pricing in the Resource-Constrained Brain

Siddiqi, Hammad

University of the Sunshine Coast

5 April 2022

Asset Pricing in the Resource-Constrained Brain

Hammad Siddiqi

University of the Sunshine Coast

hsiddiqu@usc.edu.au

This Version: February 2024

First version: April 2022

Abstract

Despite scarcity being central to economics, the scarcity of brain's internal resources has largely been ignored. Neuroscience research increasingly points to the brain evolving as a prediction engine in response to this internal-resource scarcity. The brain meets every situation with subconscious expectations, which are contrasted with information to generate error-signals. Selective processing of such error-signals, in lieu of the entire information-stream, saves brain-resources. I show that applying such predictive-processing to asset pricing gives rise to an alpha, with several empirically observed phenomena (value, momentum, size, high-alpha-of-low-beta, profitability, investment, overnight bias, and time-specific changes in SML slopes) corresponding to either cross-sectional or time-specific variations in this alpha. Additional insights about these phenomena emerge that are consistent with empirical evidence. This indicates that at least a part of the explanation of these puzzles is optimal utilization of the brain's limited internal resources via predictive processing of incoming information.

JEL Classification: G12, G41

Keywords: Predictive Processing, Asset Pricing, CAPM, SML Slope, Betting-Against-Beta, Size Effect, Value Effect, Momentum Effect, Profitability Effect, Investment Effect, Overnight Bias

Asset Pricing in the Resource-Constrained Brain

Even though resource scarcity has long been a defining notion in economics, the fact that the brain resources (neurons and energy) are also finite has largely been ignored.[1] Perhaps, not having a clear framework for analyzing the implications of such internal resource scarcity has played a role in this neglect. However, over the past decade and a half, neuroscience research has been converging to a framework which views the brain as a 'prediction machine' that uses predictions to conserve internal brain resources.[2] In this article, we show that this framework, known as 'predictive processing', provides appropriate conceptual tools for studying the implications of internal-resource scarcity. Specifically, we show that incorporating the predictive-processing framework into asset pricing gives rise to an alpha, with several empirically observed phenomena (value, momentum, size, high-alpha-of-low-beta, profitability, investment, overnight bias, and time-specific changes in SML slopes)[3] corresponding to either cross-sectional or time-specific variations in this alpha.[4]Additional insights about these phenomena emerge that are consistent with empirical evidence. This indicates that at least a part of the explanation of these puzzles is optimal utilization of the brain's limited resources via predictive processing of information.

The predictive processing framework says that the brain uses its prior knowledge of the world to form subconscious expectations in every situation, which are contrasted with available information to generate error signals. Such error signals are then selectively

---

[1] A few exceptions are Alonso et al (2014), Siddiqi and Murphy (2023), and Siddiqi (2023). McKenzie (2018) argues that the neoclassical toolbox extends to behavioral economics if the brain resource scarcity is acknowledged.

[2] There is a large body of literature in the cognitive science/neuroscience that considers the brain to be a prediction machine (Nave et al 2020, Clark 2013, Hohwy 2013, Friston 2010, Bubic et al 2010 among others). A sample based on writings of various cognitive scientists, which is suitable for non-specialist audience, includes Clark (2023), chapter 3 in Hawkins, J. (2021), chapter 4 in Feldman, L. B. (2021a), chapter 4 in Seth, A. (2021), and chapters 4 and 5 in Goldstein (2020). Feldman, L. B. (2021b) also provides a discussion of key ideas.

[3] Fama and French (2016) find deviations from the implications of the CAPM, such as related to beta, size, value, and momentum building on early studies by Black (1972), Stoll and Whaley (1983), Fama and French (1993), and Jegadeesh and Titman (1993) among others. This suggests that there is misspecification in the CAPM, and additional risk factors have been proposed (Fama and French 2016, 2011, 1993).

[4] Specific times when the SML slope is steeper include: Months when inflation is low or negative (Cohen, Polk, and Vuolteenaho 2005), days when news about inflation, unemployment, or Federal Open Markets Committee (FOMC) interest rate decisions are scheduled to be announced (Savor and Wilson 2014), periods of pessimistic investor sentiment (Antoniou et al 2015), and overnight (Hendershott et al 2020).

incorporated into predictions based on the brain's assessment of their relative value. By selectively processing error signals and mostly just leveraging prior knowledge to fill in the gaps, the brain greatly cuts down on the amount of information it processes.[5] All expectations, from the mundane (what you expect to see around the corner) to the sophisticated (risk and reward expectations), are constructed in the brain in this way.[6]

The four components of the predictive-processing framework are: (i) an internal model, which captures typical/average behavior, based on a synthesis of prior experiences in similar situations, (ii) subconscious predictions generated by the internal model (iii) error-signals that result from contrasting such expectations with available information, and (iv) importance weights assigned to error-signals and predictions, leading to adjusted predictions that are consciously experienced.[7]

We keep the same above components in the asset pricing context and specify the following: (i) the relevant internal model (captures average behavior) is based on a synthesis of prior experiences with similar firms[8], (ii) subconscious equity risk and reward expectations are generated by the internal model, (iii) error-signals result from contrasting such expectations with available information, and (iv) relative importance weights on error-signals and initial expectations are such the exploitable arbitrage opportunities against the decision-maker (DM) are prioritized and eliminated, leading to adjusted risk and reward expectations that are consciously experienced.[9]

---

[5] See chapter 4 in Hawkins, J. (2021) (and references therein) for a more detailed discussion on the common observation in neuroscience that much less brain activity happens when (subconscious) expectations match incoming information compared to when they do not (error-signal processing).

[6] Predictive Processing is more appropriately termed Hierarchical Predictive Processing as it views the mind as being organized in layers with a higher-level layer making predictions about the level just below with only the error-signals reaching the higher-level from the level just below. In this way, the lowest level predicts the incoming sensory signals, whereas a higher level makes predictions about the underlying causes such as changing risk and reward. Hence, this framework offers a unified theory of the mind ranging from sensory perception to higher cognition (see Clark (2013)).

[7] For illustrations of how these components work together to create various experiences, see the appendix in Clark (2023).

[8] It is more efficient for the brain to categorize closely related firms together as it reduces information load. Such categorization is a critical part of the way the brain puts the world in order and has a dedicated neuronal mechanism in the brain (Lech et al 2016).

[9] As the marginal benefit of eliminating an exploitable arbitrage opportunity against the DM is very large, it is optimal for the DM's brain to prioritize elimination of such arbitrage opportunities over other error-signals.

In general, the initial subconscious expectations that come from the internal model are adjusted towards rational expectations to the extent that (exploitable) arbitrage opportunities are eliminated without necessarily achieving full convergence. Hence, the consciously experienced final expectations retain some influence of the initial subconscious expectations. It is this influence which gives rise to the alpha term.

The key novel insight is that the relative resource allocation between processing of risk error-signals vs. reward error-signals matters. If the resources are diverted away from the processing of reward error-signals to risk error-signals then the security -market-line (SML) steepens. Hence, the SML is steeper when the brain has strong reasons to consider risk as relatively more important. Empirical evidence supports this novel insight.[10]

With predictive processing of incoming information, the type of investors (retail vs institutional/professional) who trade with the market-makers matters as the internal models of retail investors are likely less sophisticated than the internal models of institutional/professional money managers. Such retail investors tend to concentrate on attention stocks (Barber and Odean 2008, Berkman et al 2012) that are well known brand names frequently mentioned in the media. They typically pick stocks when the market is closed with their orders executed at open. In sharp contrast, institutional investors prefer to trade at close. If the less sophisticated internal models of retail investors generate more favorable predictions about attention stocks, then such stocks should experience a price bump at open, with some of the gains reversing intraday (open-to-close) when institutional investors trade. Consistent with this prediction, Lachance (2023) show that about 1/5th of the US stocks experience such a price bump which tends to reverse during the day. As this 'night effect' emerges from inter-investor variation in internal models, it is predicted to be weaker when reliance on internal models is weaker such as during time periods when events indicate a major break from the past. This prediction is consistent with the empirical findings on the particular weakness of the 'night effect' during the dot.com bubble peak (1999-early 2000), GFC 2008-2009, and during the Covid-19 pandemic (Seibert 2023).

---

[10] Steeper SML is observed when weak inflation/deflation indicates higher macro risks (Cohen et al 2005), in periods of pessimistic investor sentiment (Antoniou et al 2015), and around market open (Hendershott et al 2020) when highly leveraged intraday traders typically enter the market.

If the brain gives more importance to reward error-signals than risk error-signals, then betting-against-beta (BAB) effect arises (Frazzini and Pederson 2014, Black 1972) which gets stronger with the resource tilt favoring reward. It follows that the brain-based model predicts stronger BAB performance during periods of high optimism when the brain likely allocates less resources to risk. This prediction is consistent with empirical evidence (Antoniou et al 2015).

If the resources allocated to the processing of reward error-signal processing are not just larger but sufficiently larger, then the size effect also emerges. Intuitively, subconscious initial expectations (being a cluster average) tend to overestimate both reward and risk for small-size firms. If reward error-signal processing is much stronger in the DM's brain, then the net effect is lower price (and higher alpha), leading to the size effect. One expects to see this for high quality firms (high profits, high growth, and high safety), where available information mostly shows high and growing profitability without any major red flags showing risk related concerns. This prediction matches the empirical findings on the size effect (Asness et al 2018). Another prediction is that the size effect is stronger when ex-ante equity premium, reflecting macroeconomic downturn risk, is high. With monetary policy easing as a proxy for high ex-ante equity premium, the size effect must be stronger in periods of monetary policy easing. Recent empirical findings are consistent with this prediction (Simpson and Grossmann 2024). As, in the brain-based model, the size effect implies BAB (but the reverse is not be true), it is intriguing to note empirical evidence showing that the size effect predicts BAB (Zaremba 2020).

Value effect emerges in the brain-based framework as two firms with identical fundamentals may belong to different clusters; hence, have their initial subconscious expectations generated by different internal models. The impact of such inter-cluster variation is dampened if the brain assigns different importance-weights to error-signals across clusters. However, if such firms are in the same industry (an industry is generally divided into several distinct clusters), then their error-signals are highly correlated, indicating similar importance-weights on their error-signals. This sharpens the inter cluster variation, making value an intra-industry phenomena in the brain-based model, consistent with empirical findings on its intra-industry strength (Campbell, Giglio, and Polk 2023).

As the value effect emerges from inter-cluster variation in internal models, it follows that when reliance on internal models is weaker, the value effect is weaker. So, one expects to see a weaker value effect when events indicate a major break from the past. This prediction is consistent with the empirical findings on the particular weakness of the value effect during the dot.com bubble peak (1999-early 2000), GFC 2008-2009, and during the Covid-19 pandemic (Campbell, Giglio, and Polk 2023).

Overall, value is quite a robust intra-industry phenomenon in the brain-based model, which only disappears completely on occasions, if there is a major break from the past or when the resource allocation decisions in the brain are such that the inter-cluster variation in reward and risk cancel out each other (a knife-edge condition). This appears to be in contradiction with empirical research documenting the poor performance/disappearance of the value effect in the past 20-30 years (see the discussion in Asness et al (2015), Arnott et al (2021), Lev and Srivastava (2022), and Fama and French (2020) among others). However, the value effect has been restored as a robust phenomenon in Wang (2024) who uses a new superior measure, the ratio of cash-based operating profitability to price, suggesting that value's disappearance in earlier research was due to inferior measures of value. The new superior measure restores the robustness of value, in agreement with the prediction here.

Firms that have shown substantial deviation from the norm, such as recent substantially superior or inferior performance, may see a shift in importance weights towards error-signals and away from internally generated subconscious expectations. This weakening of the importance given to internal models in favor of error-signals generates price momentum (and makes it negatively correlated with value). Hence, the brain-based approach predicts that the momentum effect is ultimately driven by changing fundamentals, which is consistent with empirical findings on the momentum effect (Novy-Marx 2015).

If momentum premium arises due to adjustments in importance-weights on error-signals, it should depend on how much adjustment has already taken place. Hence, after larger (smaller) adjustments in the formation period, smaller (larger) momentum premiums should follow. This prediction matches the empirical findings in Huang (2022). Also, one expects the speed of adjustment to be higher in liquid market states when compared with illiquid market states. So, a novel prediction is that the momentum premiums are higher in liquid market states, which is consistent with the empirical findings in Avramov et al (2016).

The source of profitability effect is underestimation of large expected payoffs and overestimation of small expected payoffs by internal models (with robust profitability indicating large expected payoffs). The brain-based model's prediction that the profitability effect is stronger in market declines and in periods of high volatility has empirical support (Yu, H. et al 2022). The source of investment effect is underestimation of large risks and overestimation of small risks by internal models, with high asset growth (investment) indicating large risks. The brain-based model's prediction that the effect is weaker overnight has empirical support (Chen, J. and Kawaguchi, Y. 2018).

Overall, the ease with which such a diverse range of empirically observed phenomena appear to arise from predictive processing of incoming information, indicates that the optimal utilization of limited brain resources via predictive processing is an important part of the explanations for these phenomena. Of course, other explanations also provide part of the answer, perhaps a bigger part; however, as this article shows, predictive processing provides a thus far neglected dimension to these puzzles.

2. The Brain-Based Capital Asset Pricing Model

Predictive processing is emerging as an integrative and unifying framework in psychology and cognitive science (Friston 2018, Clark 2013, Friston and Kiebel 2009). It has been successfully applied to explain a wide variety of phenomena such as emergence of hallucinations (Griffin and Fletcher 2017), self-recognition (Apps and Tsakiris 2014), placebo-effects (Buchel et al 2014), automobile driving (Engstrom et al 2018), as well as human ability to use and innovate tools (Elk 2021) among others. However, so far, predictive processing has not been applied to issues of direct relevance in economics, despite expectations obviously playing such a hugely important role in human decision making. This paper aims to fill this gap by demonstrating how the framework can be incorporated into asset pricing, and by studying the resulting implications. It uses the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965), and Mossin (1966) to illustrate the power of the predictive processing approach[11]; however, it should be clear from the outset that a similar strategy can be applied to any asset pricing model.

We rely on a standard derivation of CAPM (for example, as in Frazzini and Pedersen (2014)) and consider an overlapping generations (OLG) economy. Each agent lives for two periods. Agents that are born at t aim to maximize their utility of wealth at $t+1$. Their utility functions are identical and exhibit mean-variance preferences. They trade securities $s = 1, \cdots, S$ where security s pays dividends $d_t^s$and has n_S^* shares outstanding and invest the rest of their wealth in a risk-free asset that offers a rate of r_F.

The market is described by a representative agent who is a mean-variance maximizer:

$$\max n'\{E_t(P_{t+1} + d_{t+1}) - (1 + r_F)P_t\} - \frac{\gamma}{2} n'\Omega_t n$$

where P_t is the vector of prices, Ω_t is the variance-covariance matrix of $P_{t+1} + d_{t+1}$, and γ is the risk-aversion parameter.

It follows that the price of a security, s, is given by:

$$P_t^s = \frac{E(X_{t+1}^s) - \gamma Cov(X_{t+1}^s, X_{t+1}^M)}{1 + r_F} \tag{2.1}$$

where security s payoff is $X_{t+1}^s = P_{t+1}^s + d_{t+1}^s$

and the aggregate market payoff is:

$$X_{t+1}^M = n_1^*(P_{t+1}^1 + d_{t+1}^1) + n_2^*(P_{t+1}^2 + d_{t+1}^2) + \cdots\cdots\cdots + n_S^*(P_{t+1}^S + d_{t+1}^S).$$

The brain clusters or categorizes closely related firms together. It is more efficient for the brain to do so as this reduces information load. In fact, such co-categorization is a critical part of the way the brain puts the world in order and has a dedicated neuronal mechanism in the brain (Lech et al 2016). We use q as the cluster identifier and denote the number of firms in cluster q by N_q. In general, the available information about a firm s, I_s, can be split into two subsets. A smaller set Λ_q, which only contains attributes that are

[11] CAPM stands out as the most widely used model of risk-return trade-off in economics (Levi and Welch 2017).

common to all firms in the cluster q, and a larger/richer set Λ_s, which contains firm specific information not already in Λ_q. That is, $I_s = \Lambda_q + \Lambda_s$. Note that compared to Λ_s, Λ_q is relatively stable and only changes slowly overtime.

The brain relies on Λ_q and uses an internal model to generate subconscious reward and risk expectations:[12]

$$E^q = \sum_{i=1}^{N_q} \frac{E[X_{t+1}^i]}{N_q} \tag{2.2}$$

$$Cov^q = \sum_{i=1}^{N_q} \frac{Cov[X_{t+1}^i, X_{t+1}^M]}{N_q} \tag{2.3}$$

The above subconscious expectations are automatically generated (without any conscious control). Prior experiences with similar firms have been synthesized into an internal model that supplies these subconscious expectations.

These subconscious expectations are contrasted with the richer information set, Λ_s, to generate error-signals. Based on the brain's assessment of their relative importance, error-signals are further processed (incorporated into expectations). In particular, error-signals that create exploitable arbitrage opportunities against the DM are prioritized over others. In general, in a resource-constrained brain, the initial subconscious expectations are adjusted in the direction of rational expectations without achieving full convergence. This process, which leads to adjusted expectations that are consciously experienced, is described by introducing a parameter, m_1:

$$E'(X_{t+1}^s) = E^q - m_1 D_1 \tag{2.4}$$

where $D_1 = E^q - E(X_{t+1}^s)$ is the correct adjustment needed, and m_1 is the fraction of correct adjustment reached so $0 \leq m_1 \leq 1$. Rational expectations, $E'(X_{t+1}^s) = E(X_{t+1}^s)$, correspond to processing of all error-signals and achievement of full adjustment: $m_1 = 1$.

[12] As the internal model can be considered a subconscious Bayesian prior on average behavior, predictive processing is sometimes also referred to as the Bayesian brain hypothesis (e.g., Knill and Pouget 2004).

Similarly, the adjusted risk expectation is:

$$Cov'\big((X_{t+1}^{s}, X_{t+1}^{M})\big) = Cov^{q} - m_2 D_2 \tag{2.5}$$

where $D_2 = Cov^{q} - Cov\big((X_{t+1}^{s}, X_{t+1}^{M})\big)$ is the correct adjustment needed, and m_2 is the fraction of correct adjustment, $0 \leq m_2 \leq 1$, achieved. Rational expectations, $Cov'\big((X_{t+1}^{s}, X_{t+1}^{M})\big) = Cov\big((X_{t+1}^{s}, X_{t+1}^{M})\big)$, correspond to elimination of all gaps and achievement of full adjustment: $m_2 = 1$.

If the brain has unlimited resources, then of course, it can process all error-signals and always form rational expectations; however, a resource-constrained brain prioritizes error-signals that create exploitable arbitrage opportunities against the DM over others, which in general adjusts expectations in the direction of rational expectations without necessarily achieving full convergence. A simple re-arrangement of (2.4) and (2.5) leads to:

$$E'(X_{t+1}^{s}) = E(X_{t+1}^{s}) + (1 - m_1)\big(E^{q} - E(X_{t+1}^{s})\big) \tag{2.6}$$

$$Cov'(X_{t+1}^{s}, X_{t+1}^{M}) = Cov(X_{t+1}^{s}, X_{t+1}^{M}) + (1 - m_2)\Big(Cov^{q} - Cov\big((X_{t+1}^{s}, X_{t+1}^{M})\big)\Big) \tag{2.7}$$

The consciously experienced reward and risk expectations, $E'(X_{t+1}^{s})$ and $Cov'(X_{t+1}^{s}, X_{t+1}^{M})$ in (2.6) and (2.7), follow from the predictive processing framework as applied to asset pricing. Rational expectations are a special case in the framework corresponding to $m_1 = 1$ and $m_2 = 1$.

The predictive processing framework gives rise to an alpha term in the CAPM as proposition 1 shows.

Proposition 1 (The Brain-Based CAPM) ***Predictive processing changes the classical CAPM in only one way: an additional term alpha appears whose value depends on the resource allocation decisions in the brain. The brain-based CAPM takes the following form:***

$$\boldsymbol{E[R_{t+1}^{s}] - R_F = \alpha_s + \big(E[R_{t+1}^{M}] - R_F\big) \cdot \beta_s} \tag{2.8}$$

where $E[R^s_{t+1}]$ is the expected (gross) return from stock s, R_F is the (gross) risk-free return, $E[R^M_{t+1}]$ is the expected (gross) return from the aggregate market portfolio, β_s is the beta of the stock s, and α_s takes the form given below:

$$\alpha_s = \left(\frac{\overline{\beta}}{w_s} - \beta_s \frac{(m_1 - m_2)}{(1 - m_2)}\right)\frac{(1 - m_2)\delta_M}{m_1} - \frac{(1 - m_1)}{m_1}\left(\frac{\overline{ER}}{w_s} - R_F\right) \qquad (2.9)$$

where $\overline{\beta} = \sum_{i=1}^{N_q} \frac{\varphi_i w_i \beta_i}{N_q}$ is the average market-value weighted beta in the cluster, $w_i = \frac{n_i^* P_t^i}{P_t^M}$, ($P_t^i$ is the share price of firm i, $n_i^*$is the number of shares of firm i outstanding, and $P_t^M$is the price of the aggregate market portfolio), $\varphi_i = \frac{n_s^*}{n_i^*}$, $\overline{ER} = \sum_{i=1}^{N_q} \frac{\varphi_i w_i E[R^i_{t+1}]}{N_q}$is the average market-value weighted expected return in the cluster, $w_s = \frac{n_s^* P_t^s}{P_t^M}$ is the market-value weight of firm s, and $\delta_M = E[R^M_{t+1}] - R_F$.

Proof:

See Appendix A.

▪

Note, that when the brain has sufficient resources to fully process both the reward error-signals and the risk error-signals, that is, when $m_1 = 1$ and $m_2 = 1$, then $\alpha_s = 0$. So, the classical CAPM is recovered when one restores the common, yet unstated (implicit), assumption in economics of unlimited brain resources. With the realistic allowance made for limited brain resources, an alpha term appears as shown in (2.8) and (2.9). This alpha term is capable of generating a wide range of empirically observed phenomena indicating that at least a part of the explanation of these phenomena is optimal utilization of limited brain resources via predictive processing of incoming information.[13] This suggests a view of the brain as having evolved a mechanism that provides a reasonably good solution to the practical problem of making sense of the constant barrage of incoming information, consistent with the argument made in Page (2022).

[13] Of course, other explanations provide a part of the answer as well, perhaps even a bigger part; however, this article shows that the scarcity of the brain's internal resources and the brain's evolutionary response to it (predictive processing) also plays a part that should no longer be ignored.

3. Asset Pricing Anomalies: A Brain-Based Perspective

The enriched CAPM has intriguing implications for the slope of the security-market-line (SML). It also generates betting-against-beta (BAB), size, value, momentum, profitability investment, and overnight effects, which generally arise as variations in the alpha term in (2.9) depending on the internal resource allocation decisions. Additional insights emerge, which are empirically supported.

3.1 The Slope of the Security Market Line (SML)

If more (less) resources are allocated to reward error-signal processing or less (more) resources are allocated to risk error-signal processing, that is, when $m_1$rises (falls) or m_2 falls (rises), then the SML rotates in the clockwise (counter clockwise) direction or the SML flattens (steepens). Intuitively, this is due to the changes in the relative underestimation of variation in risk across firms. If the relative underestimation of variation in risk rises, SML flattens, if such underestimation falls, SML steepens. Figure 1 and figure 2 illustrate. The intuition is easy to see: If resources are diverted away from risk error-signal processing, then all else constant, the relative underestimation of variation in risk in the cross-section rises. This lowers the observed variation in average returns when plotted against estimated firm betas. This leads to a flatter SML when compared with the SML with rational expectations.

Proposition 2 (SML slope) ***If more (less) resources are allocated to reward error-signal processing or less (more) resources are allocated to risk error-signal processing, that is, when $m_1$rises (falls) or m_2 falls (rises) then the SML rotates in the clockwise (counter clockwise) direction.***

Proof

See appendix B.

▪

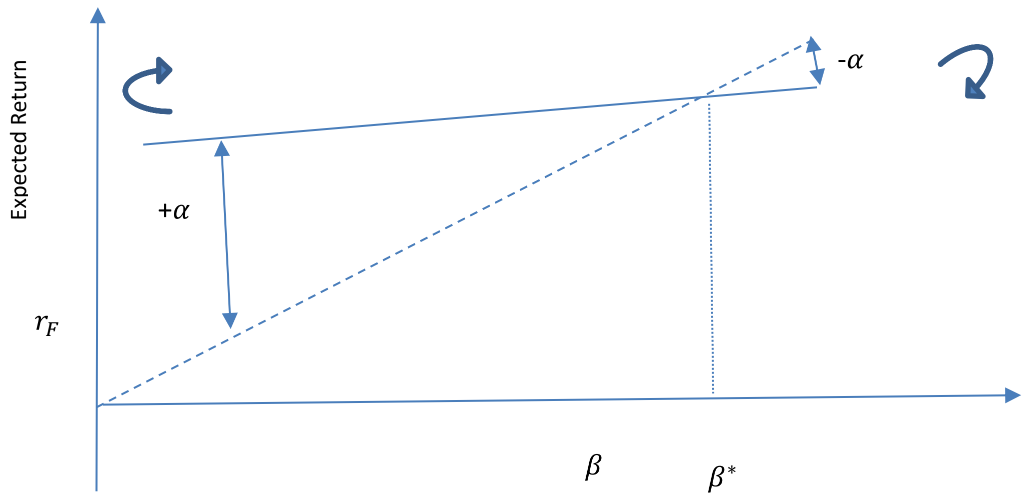

Figure 1 - Slope of the SML when m_1 rises or m_2 falls
When m_1 rises or m_2 falls, SML rotates in the clockwise direction as there is a threshold value, β^*, below which α rises (or becomes less negative) and above which α falls (or becomes more negative). The solid line indicates the brain-based SML whereas the dotted line indicates the classical SML.

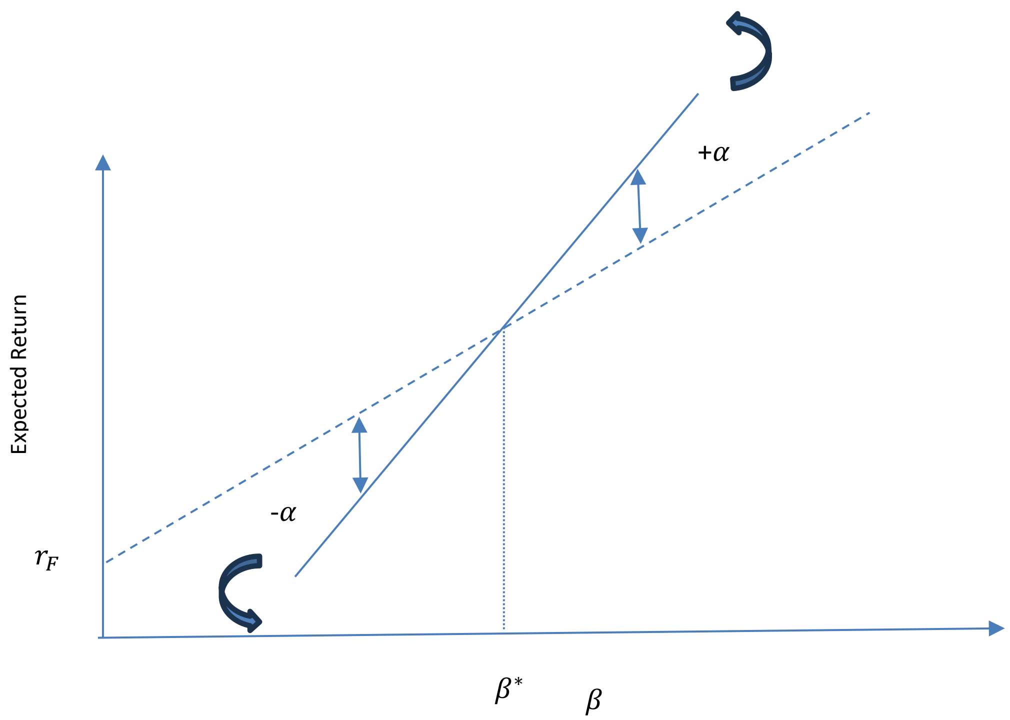

Figure 2 - Slope of the SML when m_1 falls or m_2 rises
When m_1 falls or m_2 rises, SML rotates in the counter clockwise direction as there is a threshold value, β^*, below which α falls (or becomes more negative) and above which α rises (or becomes less negative). The solid line indicates the brain-based SML whereas the dotted line indicates the classical SML.

The empirically observed variation in the SML slope at specific times appears to align well with the brain-based model:

- Around market open, the SML slope typically steepens and then gradually flattens during most of the day (Hendershott et al 2020). Intraday traders who are typically highly leveraged enter around market open and then gradually close out their position during the day (Bogousslavsky 2021). Being highly leveraged, such traders' brains assign higher importance weights to risk error-signals. This increases m_2, which steepens SML as relative underestimation of risk variation across firms falls as a result. SML slope flattens during the day as intraday traders exit the market by closing out their positions for the day, lowering m_2 in the process.
- SML slope is steeper when there is anemic inflation or deflation indicating a weak economy (Cohen et al 2005). It is also steeper in periods of pessimistic investor sentiment (Antoniou et al 2015). It makes sense that the DM's brain gives more importance to risk error-signals during times of high macro risks or when pessimism elevates the importance of such risks. So m_2 rises, which lowers the relative underestimation in risk variation across firms. This steepens the SML slope in the brain-based model.
- SML slope is steeper on macroeconomic announcement days (Savor and Wilson 2014). As most traders have already adjusted their portfolios leading up to the announcement day, trades on the actual announcement day are generally by those whose expectations turned out to be incorrect and, consequently, need to re-adjust their portfolios. The resulting higher importance weights to risk error-signals in the brains of such surprised traders steepens the SML slope (m_2 rises).

3.2 The Overnight Effect

Empirical evidence shows that a significant portion of the stocks show positive returns overnight (close to open) that tend to reverse intraday (open to close) (see Lachance (2023) among others). Such overnight effect readily arises in the brain-based model from different trader types (retail vs professional/institutional) having different internal models. Professional money managers/institutional investors tend to trade at close when the market is most liquid, whereas retail investors pick stocks when the market is closed with their orders executed at open. It is likely that such retail investors, who tend to focus on attention stocks (Barber and Odean 2008, Berkman et al 2012), have less sophisticated internal models that generate more favourable subconscious risk and reward expectation when compared with the professionals. This generates a price bump when the retail investors' orders are executed at open.

To fix ideas, consider an attention stock (selected by the retail traders). The internal model of retails investors generates risk and reward predictions denoted by E^R and Cov^R, whereas the corresponding predictions from professionals/institutional traders are E^P and Cov^P. If retail traders trade with the market-makers at open, then they price the stock as follows:

$$P_t^s = \frac{E(X_{t+1}^s)+(1-m_1)\left(E^R-E(X_{t+1}^s)\right)-\gamma\left[Cov(X_{t+1}^s,X_{t+1}^M)+(1-m_2)\left(Cov^R-Cov\left((X_{t+1}^s,X_{t+1}^M)\right)\right)\right]}{1+r_F} \quad (3.1)$$

The professional money managers/institutional traders, who trade with the market-makers at close, price the stock as:

$$P_t^s = \frac{E(X_{t+1}^s)+(1-m_1)\left(E^P-E(X_{t+1}^s)\right)-\gamma\left[Cov(X_{t+1}^s,X_{t+1}^M)+(1-m_2)\left(Cov^P-Cov\left((X_{t+1}^s,X_{t+1}^M)\right)\right)\right]}{1+r_F} \quad (3.2)$$

So, the price difference between open and the previous close is:

$$\Delta P_t^s = \frac{(1-m_1)(E^R-E^P)-\gamma(1-m_2)(Cov^R-Cov^P)}{1+r_F} \quad (3.3)$$

If the less sophisticated internal models of retail investors generate more favourable predictions about attention stocks, then $E^R > E^P$ and $Cov^R < Cov^P$ leading to $\Delta P_t^s > 0$. This is the overnight effect as it arises in the brain-based model. It also follows from (3.3) that when reliance on internal models is weaker, that is, when m_1 and m_2 rise, the overnight effect is weaker. Consistent with this prediction, empirical evidence shows a particular weaker night effect when events indicate a major break from the past such as during the peak of the dot com bubble (1999 to early 2000), GFC (2008-2009), and during the Covid-19 pandemic (Siebert 2023).

High-alpha-of-low-beta effect

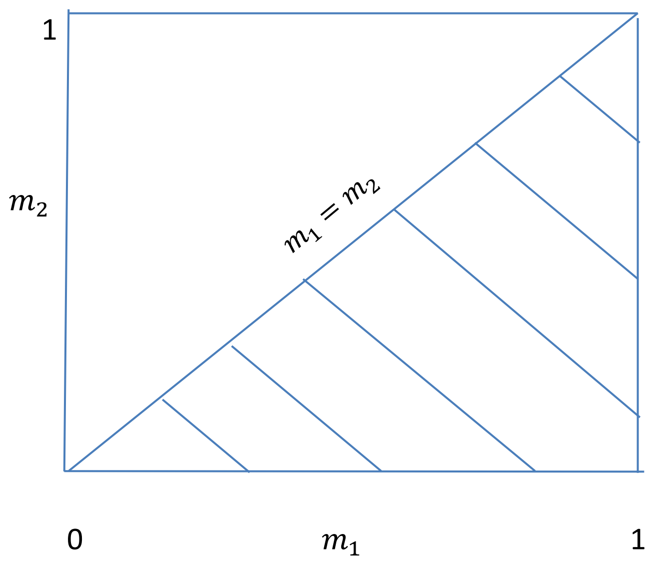

Figure 3 High-alpha-of-low-beta effect is observed in the lined region.

3.3 High-alpha-of-low-beta/ Betting-against-beta

In the brain-based CAPM, high-alpha-of-low-beta or betting-against-beta arises under the following condition (taking the partial derivative of alpha in (2.9) with respect to β_s):

$$\frac{\partial \alpha_s}{\partial \beta_s} = -\frac{\delta_M(m_1 - m_2)}{m_1} < 0 \tag{3.4}$$

Figure 3 shows the region in which high-alpha-of-low-beta or betting-against-beta (BAB) effect is observed in the space of parameters m_1 and m_2. The effect is observed if $m_1 > m_2$.

Proposition 3 (High-alpha-of-low-beta/Betting-against-beta (BAB)) ***High-alpha-of-low-beta effect arises if the importance weights assigned to reward error-signals are higher than the importance weights assigned to risk error-signals such that $\boldsymbol{m_1 > m_2}$.***

The brain-based approach predicts that the high-alpha-of-low-beta effect is not universally observed. The effect is only observed when $m_1 > m_2$, and it gets stronger when $m_1 - m_2$

rises. Intuitively, when underestimation of variation in risk rises relative to underestimation of variation in reward (due to less brain resources going to risk error-signal processing), the SML flattens. One expects to see this doing periods of optimism. It follows that during periods of optimism, BAB premium must be larger, a prediction that matches empirical findings (Antoniou et al 2015). It also follows from (3.4) that BAB premium is predicted to be larger when ex-ante equity premium is high.[14]

3.4 The Size Effect

In the predictive brain, the stock price of firm s is given by:

$$P_t^s = \frac{E(X_{t+1}^s)+(1-m_1)\left(E^q-E(X_{t+1}^s)\right)-\gamma\left[Cov(X_{t+1}^s,X_{t+1}^M)+(1-m_2)\left(Cov^q-Cov\left((X_{t+1}^s,X_{t+1}^M)\right)\right)\right]}{1+r_F} \quad (3.5)$$

Consider the cross-section of stocks for which $E^q > E(X_{t+1}^s)$ and $Cov^q > Cov\left((X_{t+1}^s, X_{t+1}^M)\right)$. Such stocks are likely to be small-size firms. For such stocks, compared to the rational benchmark, reward is overestimated by $(1-m_1)\left(E^q - E(X_{t+1}^s)\right)$ and risk is overestimated by $(1-m_2)\gamma\left(Cov^q - Cov\left((X_{t+1}^s, X_{t+1}^M)\right)\right)$. If m_1 is sufficiently larger than m_2, then the net effect is lower price (and higher alpha). This is the size effect as it emerges in the brain-based model.

The above intuition can be seen more formally by taking the partial derivative of alpha in (2.9) w.r.t the market-cap, w_s:

$$\frac{\partial \alpha_s}{\partial w_s} = -\delta_M \frac{\bar{\beta}}{w_s^2}\frac{(1-m_2)}{m_1} + \frac{(1-m_1)}{m_1}\frac{\overline{ER}}{w_s^2} \quad (3.5a)$$

$$\Rightarrow \frac{\partial \alpha_s}{\partial w_s} < 0 \text{ if } m_1 > 1 - \frac{\bar{\beta}\delta_M}{\overline{ER}}(1-m_2) \quad (3.5b)$$

[14] Even though ex-ante equity premium is unobservable, monetary policy easing (lower discount rate and federal funds rate) is likely a good proxy for high ex-ante equity premium as Fed typically engages in such a policy when macro downside risk is high (which likely correlates with high ex-ante equity premium).

The Size Effect

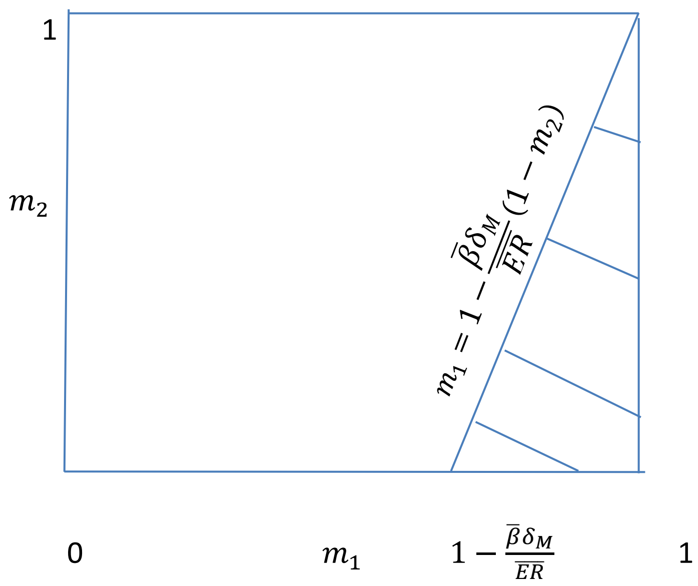

Figure 4 The size effect is observed in the lined region.

So, the size effect arises due to resource allocation decisions in the brain if the importance assigned to reward error-signals is sufficiently larger than the importance assigned to risk error-signals such that m_1 is sufficiently larger than m_2. Figure 4 illustrates.

Proposition 4 (The Size Effect) ***The size effect arises when the importance weights assigned to reward error-signals are sufficiently larger than the importance weights assigned to risk error-signals such that*** $\boldsymbol{m_1 > 1 - \frac{\bar{\beta}\delta_M}{\overline{ER}}(1 - m_2)}$***.***

Corollary 4.1 ***The size effect is stronger when ex-ante equity premium is high.***

A comparison of figure 4 and figure 3 reveals that the size effect is observed in a much smaller region when compared with the BAB effect. The fleeting nature of size effect has been extensively documented in the empirical literature with the effect only observed if certain conditions are met (see Simpson and Grossman (2024), Asness et al (2018) and references therein). It also follows that when the size effect is present, the BAB effect is necessarily present, but the reverse is not be true. It is intriguing to note empirical evidence which shows that the size effect predicts BAB (Zaremba 2020).

For firms that have high profitability, high growth and high safety, the importance weights assigned by the DM's brain to reward error-signals are likely much larger than the importance weights assigned to risk error-signals. So, the size effect is expected to matter among high quality firms. Empirical evidence shows that this is indeed the case (Asness et al 2018).

It also follows from (3.5b) that the size effect is more likely to be observed, when ex-ante equity premium is higher. Even though ex-ante equity premium is unobservable, monetary easing is a proxy for high ex-ante equity premium as Fed typically lowers the discount rate as well as the effective federal funds rate in response to high macro downturn risk (when ex-ante equity premium is likely quite high). So, it is in the monetary policy easing periods when high quality firms are more likely to show the size effect, as even such firms may not show the size effect in periods of monetary policy tightening. Hence, the brain-based model predicts that the size effect is stronger in periods of monetary easing. This prediction is consistent with recent empirical evidence (Simpson and Grossmann 2024).

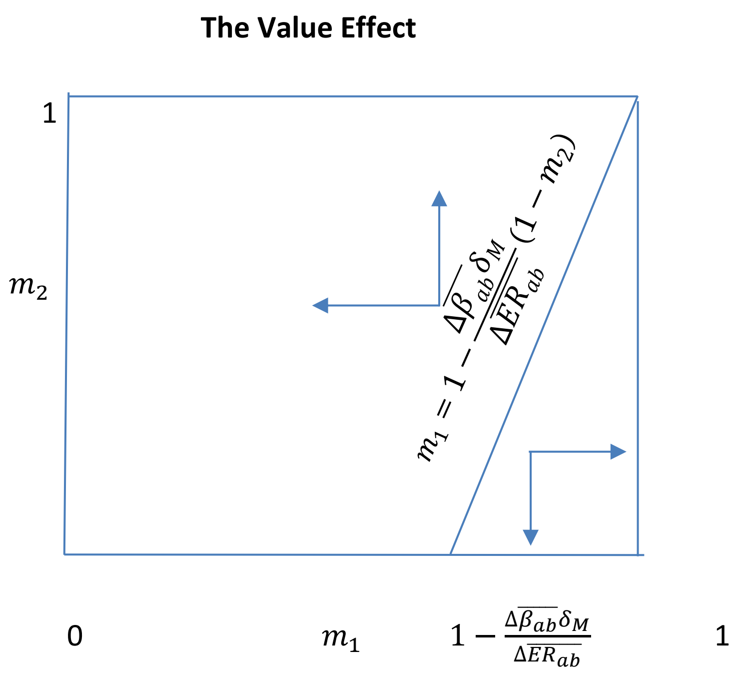

Figure 5 The value effect, which is observed for firms in the same industry, gets stronger in the direction of the arrows in the two regions split by the line $m_1 = 1 - \frac{\Delta\overline{\beta_{ab}}\delta_M}{\Delta\overline{ER}_{ab}}(1 - m_2)$.

3.5 The Value Effect

The value effect arises in the brain-based CAPM due to inter-cluster variation in internal models. That is, two firms with identical fundamentals have different prices (and alphas) if they belong to different clusters with each cluster having its own internal model. To fix ideas, consider two firms, a and b, that belong to different clusters. Firm a belongs to cluster q and firm b belongs to cluster l. Their prices are:

$$P_t^a = \frac{E(X_{t+1}^a) + (1 - m_{1a})\left(E^q - E(X_{t+1}^a)\right) - \gamma\left[Cov(X_{t+1}^a, X_{t+1}^M) + (1 - m_{2a})\left(Cov^q - Cov\left((X_{t+1}^a, X_{t+1}^M)\right)\right)\right]}{1 + r_F} \quad (3.6)$$

$$P_t^b = \frac{E(X_{t+1}^b) + (1 - m_{1b})\left(E^l - E(X_{t+1}^b)\right) - \gamma\left[Cov(X_{t+1}^b, X_{t+1}^M) + (1 - m_{2b})\left(Cov^l - Cov\left((X_{t+1}^b, X_{t+1}^M)\right)\right)\right]}{1 + r_F} \quad (3.6a)$$

If they have the same fundamentals, then:

$$E(X_{t+1}^a) = E\left(X_{t+1}^b\right) = E(X_{t+1}) \quad (3.7)$$

$$Cov(X_{t+1}^a, X_{t+1}^M) = Cov\left(X_{t+1}^b, X_{t+1}^M\right) = Cov(X_{t+1}, X_{t+1}^M) \quad (3.7a)$$

In addition, if they also belong to the same industry[15], then their error-signals would be strongly correlated indicating similar importance-weights:

$$m_{1a} = m_{1b} = m_1 \tag{3.7b}$$

$$m_{2a} = m_{2b} = m_2 \tag{3.7c}$$

Substituting from (3.7), (3.7a), (3.7b), and (3.7c) into (3.6) and (3.6a), the difference in the price of the firms is:

$$P_t^a - P_t^b = \Delta P_t = \frac{(1-m_1)(E^q - E^L) - \gamma(1-m_2)(Cov^q - Cov^L)}{1+r_F} \tag{3.8}$$

(3.8) shows that the value effect is an intra-industry phenomenon that arises due to inter-cluster variation in internal models. If the error-signals are uncorrelated (firms belong to different industries which implies $m_{1a} \neq m_{1b}$ and $m_{2a} \neq m_{2b}$), then the impact of such inter-cluster variation is dampened. Within the same industry; however, value is quite robust and disappears only when the inter-cluster variation in reward cancels out the inter-cluster variation in risk, which is the following knife-edge condition:

$$(1-m_1)(E^q - E^L) - \gamma(1-m_2)(Cov^q - Cov^L) = 0$$

$$\Rightarrow m_1 = 1 - \gamma(1-m_2)\frac{Cov^q - Cov^L}{E^q - E^L}$$

$$\Rightarrow m_1 = 1 - \gamma(1-m_2)\frac{\Delta Cov}{\Delta E} \tag{3.9}$$

The above intuition can also be seen by using the alpha in (2.9) ($with\ \beta_a = \beta_b = \beta_s$):

$$\alpha_a = \left(\frac{\overline{\beta_q}}{w_a} - \beta_s\frac{(m_1-m_2)}{(1-m_2)}\right)\frac{(1-m_2)\delta_M}{m_1} - \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_q}}{w_a} - R_F\right)$$

$$\alpha_b = \left(\frac{\bar{\beta_l}}{w_b} - \beta_s\frac{(m_1-m_2)}{(1-m_2)}\right)\frac{(1-m_2)\delta_M}{m_1} - \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_l}}{w_b} - R_F\right)$$

[15] An industry typically has several dozen firms so, in general, firms in the same industry are sorted by the brain into a number of distinct clusters/categories, with each cluster having its own internal model.

$$\Rightarrow \Delta\alpha = \left(\frac{\overline{\beta_q}}{w_a} - \frac{\bar{\beta}_l}{w_b}\right)\frac{(1-m_2)\delta_M}{m_1} - \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_q}}{w_a} - \frac{\overline{ER}_l}{w_b}\right) \quad (3.9a)$$

$$\Rightarrow \Delta\alpha = \Delta\overline{\beta_{ab}}\frac{(1-m_2)\delta_M}{m_1} - \frac{(1-m_1)}{m_1}\Delta\overline{ER_{ab}} \quad (3.9b)$$

where $\Delta\overline{\beta_{ab}} = \left(\frac{\overline{\beta_q}}{w_a} - \frac{\bar{\beta}_l}{w_b}\right)$ and $\Delta\overline{ER_{ab}} = \left(\frac{\overline{ER_q}}{w_a} - \frac{\overline{ER}_l}{w_b}\right)$.

As long as $\Delta\alpha$ is different from zero, the value effect is observed with the low price to fundamentals stock outperforming the high price to fundamentals stock. $\Delta\alpha = 0$ if $m_1 = 1 - \frac{\Delta\overline{\beta_{ab}}\delta_M}{\Delta\overline{ER_{ab}}}(1-m_2)$(which is a knife-edge condition). Away from this line, the value effect gets stronger. Figure 5 illustrates.

Proposition 5 (The Value Effect) ***If the resource allocation decisions in the brain are such that the inter-cluster variation in risk is not exactly balanced by the inter-cluster variation in reward, then the value effect is observed. The effect is observed as long as*** $\boldsymbol{m_1 \neq 1 - \frac{\Delta\overline{\beta_{ab}}\delta_M}{\Delta\overline{ER_{ab}}}(1-m_2)}$.

As the value effect in the brain-based CAPM has its roots in inter-cluster variation in internal models, it gets weaker if the brain lowers the importance weights assigned to internally generated predictions coming from the internal models. That is, the value effect generally gets weaker if both m_1 and m_2 rise in (3.8) or (3.9b). This is likely if there are major market movements suggesting a substantial break from the norm (making past less of an indicator of the future). This prediction is consistent with the empirical findings on the weakness/disappearance of the value effect in unusual time periods such as during the peak of the dot.com bubble (1999-early 2000), GFC-2008-2009, and the Covid-19 pandemic (Campbell, Giglio, and Polk 2023).

Overall, the value premium emerges as quite a robust intra-industry phenomenon in the brain-based model, only disappearing on occasions when major events compel the brain to weaken its reliance on internal models or when the knife-edge condition that cancels the inter cluster variation is met. As discussed in the introduction, this is apparently in contradiction with empirical research documenting the poor performance/disappearance of the value effect in the past 20-30 years. Recently, Wang (2024) uses a new measure of value, the ratio of cash-based operating profitability to price, to establish the robustness of value, suggesting that value's disappearance in earlier research was due to inferior measures of value. The new superior measure restores the robustness of value, in agreement with the prediction here.

3.6 The Momentum effect

The empirical findings regarding the price momentum show how stocks with superior (inferior) recent performance continue to outperform (underperform) in the short run. In the brain-based framework, the price of a security s from (2.1) is:

$$P_t^s = \frac{E'(X_{t+1}^s) - \gamma Cov'(X_{t+1}^s, X_{t+1}^M)}{1 + r_F} \tag{3.10}$$

Where:

$$E'(X_{t+1}^s) = E(X_{t+1}^s) + (1 - m_1)\big(E^q - E(X_{t+1}^s)\big)$$

$$\Rightarrow E'(X_{t+1}^s) = m_1 E(X_{t+1}^s) + (1 - m_1)E^q \tag{3.11}$$

And,

$$Cov'(X_{t+1}^s, X_{t+1}^M) = Cov(X_{t+1}^s, X_{t+1}^M) + (1 - m_2)\left(Cov^q - Cov\big((X_{t+1}^s, X_{t+1}^M)\big)\right) \tag{3.12}$$

$$\Rightarrow Cov'(X_{t+1}^s, X_{t+1}^M) = m_2 Cov(X_{t+1}^s, X_{t+1}^M) + (1 - m_2)Cov^q \tag{3.13}$$

In the brain-based CAPM, price momentum arises due to an increase in the importance weights given to error-signals that follow a significant change in the fundamentals of momentum winners and losers. To fix ideas, suppose the reward fundamentals of a firm (the momentum winner) improve, so $E(X_{t+1}^s)$ and consequently, $E'(X_{t+1}^s)$ goes up, which increases the stock price immediately. The reward fundamentals of another firm (the momentum loser) deteriorate. So, its price falls as a result. This change in fundamentals, then triggers a change in the importance weights given to reward error-signals. So, m_1 goes up. For momentum winners (drawn from top 10% of firms by recent performance), the internal model typically underestimates reward, $E^q < E(X_{t+1}^s)$, whereas for momentum losers (bottom 10% by recent performance), the internal model typically overestimates reward, $E^q > E(X_{t+1}^s)$. So, this increase in m_1, which follows a significant change in fundamentals, increases the price of the momentum winner further and lowers the price of the momentum loser further. Similarly, a large positive (negative) change in fundamentals could be a reduction (an increase) in risk, $Cov(X_{t+1}^s, X_{t+1}^M)$, increasing (decreasing) the price initially, with subsequent increases (decreases) coming from the importance-weight adjustments that increase m_2.

The brain-based model predicts that the price momentum is a robust fundamentals-driven phenomena where an initial large change in fundamentals subsequently triggers an increase in importance weights given to error-signals. This prediction is consistent with the empirical findings on momentum effect being fundamentals driven (Novy-Marx 2015). As the increase in importance weights given to error-signals comes at the expense of the importance weights on initial expectations that come from the internal models, momentum and value (which captures inter-cluster variation in internal models) are negatively correlated.

Proposition 6 (The Momentum Effect) ***Firms with recent large positive changes in earning fundamentals show a further increase in their market prices, and firms with recent large negative changes in earning fundamentals show a further decline in their market prices due to an increase in brain resources allocated to their valuations.***

In the brain-based framework, it is the increases in importance-weights on relevant error-signals that generates the price momentum. It immediately follows that the momentum premium should depend on (i) how much room is left to adjust the importance-weights, and (ii) the speed of adjustment.

If most of error-signal adjustment has already taken place in the portfolio formation period, then there is less room to adjust in the evaluation period. However, if little adjustment has taken place in the formation period, then most of the adjustment takes place in the evaluation period. Dividing stock performance in the formation period in percentiles in increasing order of returns, the return difference between the 90th percentile and the 10th percentile (momentum gap) is a measure of adjustment in the formation period, with a small difference indicating that little adjustment has taken place in the formation period. The brain-based model predicts that the subsequent evaluation period returns should be inversely related to the momentum gap in the formation period. That is, a small (large) formation period momentum gap should be followed by a large (small) evaluation period momentum returns. Empirical findings in Huang (2022) are in accord with this prediction.

In liquid market states, the speed of adjustment should be higher. It follows that the brain-based model predicts a higher momentum premium in liquid market states, which is consistent with empirical findings (Avramov et al 2016).

3.6.1 The Impact of Financial Constraints

A further novel prediction also follows: Consider a cross-section of firms for which the internal model underestimates both the reward (in (3.11)) as well as the risk (in (3.13)). That is, $E(X_{t+1}^{s}) > E^{q}$ and $Cov(X_{t+1}^{s}, X_{t+1}^{M}) > Cov^{q}$. This cross-section likely consists of large firms. For such firms, if an event triggers a resource re-allocation away from reward error-signal processing to risk error-signal processing, then the reward underestimation rises ($m_1$falls), whereas the risk underestimation falls (m_2 rises). Both of which lead to a reduction in price (in (3.10)). Hence, the brain-based model predicts that, for large firms, such resource re-allocation towards risk error-signal processing lowers price and improves alpha. An event triggering such a re-allocation could be further tightening of financial constraints that a firm face, as this increases the risk of cashflows. It follows that the brain-based model predicts that a portfolio that goes long in large firms that face financial constraints and shorts large firms that are unconstrained should earn excess returns. Consistent with this prediction, by using a novel textual analysis to capture financial constraints, Buehlmaier and Whited (2018) show that financial constraints are indeed priced in this way.

3.7 The Profitability Effect

To fix ideas, consider two firms, R and W, that cannot be distinguished on the basis of risk; however, one firm's expected payoff is larger, say $E(X_{t+1}^R) > E(X_{t+1}^W)$. If their prices are also similar, that is, $P_t^R \approx P_t^W$, then it follows that the expected payoff of R is underestimated and the expected payoff of W is overestimated by the relevant internal models such that:

$$E(X_{t+1}^R) + (1 - m_1)\big(E^q - E(X_{t+1}^R)\big) \approx E(X_{t+1}^W) + (1 - m_1)\Big(E^l - E(X_{t+1}^W)\Big) \qquad (3.14)$$

where the internal model in cluster q underestimates the expected reward of R and the internal model in cluster l overestimates the expected reward of W. That is, $E^q < E(X_{t+1}^R)$ and $E^l > E(X_{t+1}^W)$.

From (3.14):

$$E(X_{t+1}^R) - E(X_{t+1}^W) = (1 - m_1)\Big(E^l - E(X_{t+1}^W) - E^q + E(X_{t+1}^R)\Big)$$

$$\Rightarrow E(X_{t+1}^R) - E(X_{t+1}^W) = \frac{(1 - m_1)}{m_1}(E^l - E^q) \qquad (3.15)$$

Note that E^l must be larger than E^q, that is, $E^l > E^q$.

Dividing both sides by P_t^R which is the same as P_t^W and substituting for E^l and E^q from (2.2):

$$E(R_{t+1}^R) - E(R_{t+1}^W) = \frac{(1 - m_1)}{m_1}\left(\sum_{i=1}^{N_l} \frac{E\left[X_{t+1}^i\right]}{P_t^W N_l} - \sum_{j=1}^{N_q} \frac{E\left[X_{t+1}^j\right]}{P_t^R N_q}\right)$$

$$\Rightarrow E(R_{t+1}^R) - E(R_{t+1}^W) = \frac{(1 - m_1)}{m_1}\left(\sum_{i=1}^{N_l} \frac{P_t^i E\left[R_{t+1}^i\right]}{P_t^W N_l} - \sum_{j=1}^{N_q} \frac{P_t^j E\left[R_{t+1}^j\right]}{P_t^R N_q}\right)$$

$$\Rightarrow E(R_{t+1}^R) - E(R_{t+1}^W) = \frac{(1 - m_1)}{m_1}\left(\sum_{i=1}^{N_l} \frac{n_i P_t^i E\left[R_{t+1}^i\right]\left(\frac{n_W}{n_i}\right)}{n_W P_t^W N_l} - \sum_{j=1}^{N_q} \frac{n_j P_t^j E\left[R_{t+1}^j\right]\left(\frac{n_R}{n_j}\right)}{n_R P_t^R N_q}\right)$$

Inserting $\varphi_i = \frac{n_W}{n_i}$, $\varphi_j = \frac{n_R}{n_i}$, $w_i = \frac{n_i P_t^i}{P_t^M}$, $w_j = \frac{n_j P_t^j}{P_t^M}$, $w_W = \frac{n_W P_t^W}{P_t^M}$, and $w_R = \frac{n_R P_t^R}{P_t^M}$:

$$E(R_{t+1}^R) - E(R_{t+1}^W) = \frac{(1-m_1)}{m_1}\left(\sum_{i=1}^{N_l}\frac{w_i E\left[R_{t+1}^i\right]\varphi_i}{w_W N_l} - \sum_{j=1}^{N_q}\frac{w_j E\left[R_{t+1}^j\right]\varphi_j}{w_R N_q}\right) \tag{3.16}$$

Using $\overline{ER_l} = \sum_{i=1}^{N_l}\frac{\varphi_i w_i E\left[R_{t+1}^i\right]}{N_l}$ and $\overline{ER_q} = \sum_{j=1}^{N_q}\frac{\varphi_j w_j E\left[R_{t+1}^j\right]}{N_q}$:

$$E(R_{t+1}^R) - E(R_{t+1}^W) = \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_l}}{w_W} - \frac{\overline{ER_q}}{w_R}\right) \tag{3.17}$$

Fama and French (2015) profitability factor is a mechanism for distinguishing between firms whose expected payoffs are underestimated and firms whose expected payoffs are overestimated. This is done by sorting stocks in size (market-cap) buckets and then attempting to separate high expected payoff stocks and low expected payoff stocks in each bucket. The typical indicator used for this purpose is operating profitability, with high operating profitability pointing to high expected payoff firms. A long position is then taken in a diversified portfolio of robust operating profitability stocks, and a short position is taken in a diversified portfolio of weak operating profitability stocks. If diversification keeps risks comparable between the long leg and the short leg, the excess return from such a portfolio is (from 2.9):

$$\Delta\alpha_{R-W} = \alpha_R - \alpha_W \approx \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_l}'}{w_S} - \frac{\overline{ER_q}'}{w_S}\right) \tag{3.18}$$

where $\overline{ER_l}'$ is the portfolio long-leg average of $\overline{ER_l}$, and $\overline{ER_q}'$ is the portfolio short-leg average of $\overline{ER_q}$.

Proposition 7 (The Profitability Premium): ***Among comparable size (market-cap) firms, a portfolio that takes a long position in robust profitability firms and a short position in weak profitability firms generates an excess alpha given by:***

$$\boldsymbol{\alpha_R - \alpha_W \approx \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_l}'}{w_s} - \frac{\overline{ER_q}'}{w_s}\right)}$$

It follows that when resources are diverted away from reward error-signal processing, that is, when m_1 falls, the profitability premium goes up. Such resource diversion is expected to happen when the macro conditions indicate rising risk concerns, for example, in periods of large market declines/high volatility. Hence, the brain-based model predicts stronger profitability premium in periods of market declines and when volatility is high. This prediction is in line with empirical evidence (Yu, H. et al 2022).

It also directly follows from proposition 7 that both macro risks and expectation errors contribute to the profitability premium. Rising macro risks/worsening macro conditions may compel the brain to allocate more resources to risk error-signal processing. Such resource re-allocation lowers m_1, which increases the profitability premium. At the same time, it is the expectation errors embedded in the subconscious expectations (generated by an internal model) that give rise to the profitability premium in the first place. Hence, from the perspective of the brain-based approach, it is futile to distinguish between macro risk-based vs behavioral (expectation errors) explanations of the profitability premium, as it is the interplay between the two that generates the profitability premium.

3.8 The Investment Effect

Consider two firms, C and A, that have similar profitability; however, the payoff covariance of A is larger than the payoff covariance of C. That is, $Cov(X_{t+1}^{C}, X_{t+1}^{M}) < Cov(X_{t+1}^{A}, X_{t+1}^{M})$. If their prices are also similar, that is, $P_t^A \approx P_t^C$, then it follows that the payoff covariance of A is underestimated and the payoff covariance of C is overestimated by the relevant internal models such that:

$$\begin{aligned} Cov(X_{t+1}^{A}, X_{t+1}^{M}) + (1-m_2)\Big(Cov^{l} - Cov\big((X_{t+1}^{A}, X_{t+1}^{M})\big)\Big) \\ = Cov(X_{t+1}^{C}, X_{t+1}^{M}) \\ + (1-m_2)\Big(Cov^{q} - Cov\big((X_{t+1}^{C}, X_{t+1}^{M})\big)\Big) \end{aligned} \quad (3.19)$$

$$\Rightarrow Cov(X_{t+1}^{A}, X_{t+1}^{M}) - Cov(X_{t+1}^{C}, X_{t+1}^{M}) = \frac{(1-m_2)}{m_2}(Cov^{q} - Cov^{l})$$

$$\Rightarrow \beta_A - \beta_C = \frac{(1-m_2)}{m_2}\left(\frac{\overline{\beta_q}}{w_C} - \frac{\overline{\beta_l}}{w_A}\right) \tag{3.20}$$

Using (2.9) and substituting from (3.20), it follows that a portfolio which is long C and short A has an excess alpha:

$$\alpha_C - \alpha_A = \frac{(1-m_2)\delta_M}{m_2}\left(\frac{\overline{\beta_q}}{w_C} - \frac{\overline{\beta_l}}{w_A}\right) - \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_q}}{w_C} - \frac{\overline{ER_l}}{w_A}\right) \tag{3.21}$$

Fama and French (2015) investment factor attempts to create an exposure to excess returns similar to the excess returns in (3.21). It does so by sorting stocks in size (market-cap) buckets, which suggests that $w_C \approx w_A \approx w_s$, and then using investment (change in total assets) as an indicator of payoff covariance, with high investment indicating high payoff covariance. In each size bucket, a long position is taken in low investment firms and high investment firms are shorted. If both the long leg as well as the short leg are sufficiently diversified, then the portfolio long-leg average of $\overline{ER_q}$ is likely similar to the portfolio short-leg average of $\overline{ER_l}$. It follows that, within a size-bucket, a long position in low investment firms (C) and a short position in high investment firms (A), if investment is indeed a credible indicator of payoff covariance, leads to excess returns:

$$\alpha_C - \alpha_A = \frac{(1-m_2)\delta_M}{m_2}\left(\frac{\overline{\beta_q}'}{w_s} - \frac{\overline{\beta_l}'}{w_s}\right) \tag{3.22}$$

where $\overline{\beta_q}'$ is the portfolio long-leg average of $\overline{\beta_q}$, and $\overline{\beta_l}'$ is the portfolio short-leg average of $\overline{\beta_l}$.

Proposition 8 (The Investment Effect): ***Within a size bucket (market-cap), if high investment (large increase in total assets) credibly indicates high payoff covariance firms, then taking a long position in firms that invest conservatively (C) combined with a short position in firms that invest aggressively (A) leads to excess returns given by:***

$$\boldsymbol{\alpha_C - \alpha_A = \frac{(1-m_2)\delta_M}{m_2}\left(\frac{\overline{\beta_q}'}{w_s} - \frac{\overline{\beta_l}'}{w_s}\right)}$$

If the brain resources are diverted towards risk error-signal processing, which increases m_2, and, at the same time, macro conditions deteriorate, which increases δ_M then the net impact on investment premium is ambiguous. That is, it is unclear what happens to the investment premium if resources are diverted towards risk error-signal processing due to macro conditions worsening. However, if resources are diverted towards risk error-signal processing without macro conditions worsening, then the investment premium weakens. This happens, for example, around market open, when highly leveraged intra-day traders enter the market, whose brains are compelled to allocate more resources to risk error-signal processing due to the embedded leverage in such traders' portfolios (irrespective of the macro conditions). So, the brain-based model predicts that the investment premium is weaker overnight (close to open). This prediction has empirical support (Chen, J. and Kawaguchi, Y. 2018).

3.9 The Relationship between Value, Profitability, and Investment

Value, profitability, and investment effects are closely related in the brain-based approach. To fix ideas, consider two stocks, a and b, that have the same fundamentals (and betas) but are in different clusters. Using (2.9), the difference between their alphas is:

$$\Delta\alpha = \left(\frac{\overline{\beta_a}}{w_a} - \frac{\overline{\beta_b}}{w_b}\right)\frac{(1-m_2)\delta_M}{m_1} - \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_a}}{w_a} - \frac{\overline{ER_b}}{w_b}\right) \tag{3.23}$$

The various ways in which stock $'a'$ can have a higher alpha include: $\frac{\overline{\beta_a}}{w_a} > \frac{\overline{\beta_b}}{w_b}$ and $\frac{\overline{ER_a}}{w_a} \approx \frac{\overline{ER_b}}{w_b}$, $\frac{\overline{\beta_a}}{w_a} \approx \frac{\overline{\beta_b}}{w_b}$ and $\frac{\overline{ER_a}}{w_a} < \frac{\overline{ER_b}}{w_b}$, $\frac{\overline{\beta_a}}{w_a} \gg \frac{\overline{\beta_b}}{w_b}$ and $\frac{\overline{ER_a}}{w_a} > \frac{\overline{ER_b}}{w_b}$, $\frac{\overline{\beta_a}}{w_a} < \frac{\overline{\beta_b}}{w_b}$ and $\frac{\overline{ER_a}}{w_a} \ll \frac{\overline{ER_b}}{w_b}$ among others. That is, there are several ways in which the value effect can arise, some of which may correspond with the profitability and the investment effects.

The profitability effect arises from taking a long position in robust profitability stocks and a short position in weak profitability stocks provided that such profitability variation credibly indicates expected payoff variation. The investment effect arises from taking a long position in firms with conservative investment in their assets and shorting firms that invest aggressively provided that asset growth is a reliable indicator of payoff covariance.

Combining the two factors together:

$$\Delta\alpha = \frac{(1-m_1)}{m_1}\left(\frac{\overline{ER_l}'}{w_s} - \frac{\overline{ER_q}'}{w_s}\right) + \frac{(1-m_2)\delta_M}{m_2}\left(\frac{\overline{\beta_q}'}{w_s} - \frac{\overline{\beta_l}'}{w_s}\right) \tag{3.24}$$

Even though (3.24) and (3.23) have similarities, they are only identical when the following conditions hold:

$$\frac{1}{m_2}\left(\frac{\overline{\beta_q}'}{w_s} - \frac{\overline{\beta_l}'}{w_s}\right) \approx \left(\frac{\overline{\beta_a}}{w_a} - \frac{\overline{\beta_b}}{w_b}\right)\frac{1}{m_1} \tag{3.25}$$

$$\left(\frac{\overline{ER_l}'}{w_s} - \frac{\overline{ER_q}'}{w_s}\right) \approx \left(\frac{\overline{ER_b}}{w_b} - \frac{\overline{ER_a}}{w_a}\right) \tag{3.26}$$

Intuitively, profitability factor aims to selects stocks that have high expected payoffs (such expected payoffs are underestimated by the internal model) with average prices. Investment factor aims to select stocks that have low payoff covariances (such payoff covariances are overestimated by the internal model) with average prices. Value factor picks stocks that have similar (average) fundamentals with low prices. Such low prices can be due to differences in expected payoff underestimation/overestimation, differences in payoff covariance overestimation/underestimation, or some combination of both. It follows that, depending on the data set and the empirical methodology, it is possible that the conditions (3.25) and (3.26) may hold; however, there is no guarantee that they hold in general.

Empirical evidence is consistent with the above as the early findings of Fama and French (2016) suggest that the profitability and investment factors make value redundant with later findings (different data sets) restoring value even in the presence of profitability and investment factors (Foye, J. 2018, Fama and French 2017).

4. Conclusions

Camerer, Lowenstein, and Prelec (2005) emphasize that neuroscience research suggests that the human behavior requires fluid dynamics between 'automatic' and 'consciously controlled' processes. Consistent with this early realization, in the past decade and a half, predictive processing has emerged as a dominant paradigm in neuroscience for thinking about the brain. This paradigm has 'subconscious expectations' that are automatically generated by an internal model. Error-signals are selectively incorporated into such initial expectations in a more 'consciously controlled' way to arrive at final adjusted expectations. In this way, by leveraging the fluid dynamics between 'automatic' and 'consciously controlled' processes, predictive processing offers a window into how internal resource allocation decisions about what type of 'error-signals' to prioritize matter.

This article shows that, in asset pricing context, applying the predictive processing framework gives rise to an alpha, which reflects the internal tension between competing demands on limited brain resources. A wide range of quite distinct empirical phenomena can be seen as arising out of shifting priorities regarding the types of 'error-signals' to process.

Overall, this article shows that predictive processing potentially offers a synthesis of behavioral and neoclassical economics as, in this framework, behavioral biases ultimately can be thought of as emerging from the brain's optimal response to its own internal resource scarcity.[16]In particular, both macro-risks and expectation errors play a part in generating various anomalies. Macro-risks matter by shifting the resource allocation inside the brain between risk error-signal processing and reward error-signal processing, whereas expectation errors directly arise from reliance on subconscious expectations that come from an internal model. So, both macro-risk based and behavioral aspects (expectation errors) matter and are components of a unified picture that emerges from pushing the notion of scarcity inside the brain.

Perhaps, the most intriguing aspect is that applying the predictive processing framework to asset pricing provides a parsimonious way of making sense of quite a wide range of distinct anomalies within a unified framework, consistent with the vision set out in Camerer et al (2005) that neuroscience could potentially provide a radical contribution to economic science.

---

[16] In particular, the anchoring bias in Siddiqi (2019) and Siddiqi (2018), small-risk neglect in Siddiqi and Quiggin (2019), and zero-risk bias in Siddiqi (2017) can all be readily modelled as directly arising from predictive processing.

References

Alonso, Brocas, and Carrillo (2014), "Resource allocation in the brain", *Review of Economic Studies*, Vol. 81, pp. 501-534.

Antoniou, C., Doukas, J., and Subrahmanyam, A. (2015), "Investor Sentiment, Beta, and the Cost of Equity Capital", *Management Science,* Vol. 62, No. 2.

Apps, M. A. J, and Tsakiris, M. (2014), "The free-energy self: A predictive coding account of self-recognition", *Neuroscience and Biobehavioral Review,* Vol. 41, pp. 85-97.

Arnott, Robert, Campbell Harvey, Vitali Kalesnik, and Juhani Linnainmaa (2019), "Reports of value's death may be greatly exaggerated", *Financial Analysts' Journal,* Vol. 77, Issue 1, pp. 44-67.

Asness, Clifford, Andrea Frazzini, Ronen Israel, and Tobias Moskowitz (2015), "Fact, fiction, and value investing", *Journal of Portfolio Management* 42 (1): 34-52.

Asness, Frazzini, Israel, Moskowitz, and Pedersen (2018), "Size matters, if you control your junk", *Journal of Financial Economics,* Vol. 129, Issue 3, pp. 479-509.

Avramov, D., Cheng, S., and Hameed, A. (2016), "Time varying liquidity and momentum profits", *Journal of Financial and Quantitative Analysis,* Vol. 51, No. 6, pp. 1897-1923.

Barber, B. and Odean, T. (2008). "All That Glitters: The Effect of Attention and News on the Buying Behavior of Individual and Institutional Investors." *The Review of Financial Studies*, Vol. 21, Issue 2, pp. 785-818.

Berkman, H., PD Koch, L. Tuttle, and YJ Zhang. (2012). "Paying Attention: Overnight Returns and the Hidden Cost of Buying at the Open." *Journal of Financial and Quantitative Analysis*, Vol. 47, No. 4, pp. 715-741.

Black, F. (1972), "Capital market equilibrium with restricted borrowing", *Journal of Business,* Vol. 45, Issue 3, pp.444-455.

Buchel, C., Geuter, S., Sprenger, C., and Eippert, F. (2014), "Placebo analgesia: A predictive coding perspective", *Neuron,* Vol. 81, Issue 6, pp. 1223-1239.

Bogousslavsky, V. (2021), "The cross-section of intraday and overnight returns", *Journal of Financial Economics,* Vol. 141, Issue 1, pp. 172-194.

Bubic, A., Von Cramon, D. Y., & Schubotz, R. I. (2010), "Prediction, cognition and the brain", *Frontiers in Human Neuroscience*, 4, 25.

Buehlmaier, M., and Whited, T. (2018), "Are financial constraints priced? Evidence from textual analysis", *The Review of Financial Studies,* Vol. 31, Issue 7, pp. 2693-2728.

Camerer, Colin, George Loewenstein, and Drazen Prelec. 2005. "Neuroeconomics: How Neuroscience Can Inform Economics." *Journal of Economic Literature*, 43 (1): 9-64**.**

Campbell, J., Giglio, S., and Polk, C. (2023), “What drives booms and busts in value?”, *NBER Working Paper No. 31859,* DOI: 10.3386/w31859

Chen. J., and Kawaguchi, Y. (2018), “A revisit of the cross-section of overnight and intraday abnormal returns: Evidence from the Japanese REIT market”, *International Journal of Economics and Finance,* Vol. 10, No. 1, pp.

Clark, A. (2023), “The Experience Machine: How Our Minds Predict and Shape Reality”, Allen Lane (1st Edition).

Clark, A. (2015), “Surfing Uncertainty: Prediction, Action, and the Embodied Mind”, Oxford University Press.

Clark, A. (2013), “Whatever next? Predictive brains, situated agents, and the future of cognitive science”, *Behav. Brain Sci.* 36, 181-204, doi: 10.1017/S0140525X12000477

Cohen, R. B., C. Polk, and T. Vuolteenaho (2005), “Money illusion in the stock market: The Modigliani Cohn hypothesis”, *Quarterly Journal of Economics*, Vol. 120, Issue 2, pp. 639 – 668.

Elk, M. (2021), “A predictive processing framework of tool use”, *Cortex,* Vol. 139, pp. 211-221.

Engstrom, J., Bargman, J., Nilsson, D., Seppelt, B., Markkula, G., Piccinini, G. B., and Viktor, T. (2018), “Great expectations: A predictive coding account of automobile driving”, *Theoretical Issues in Ergonomics Science,* Vol. 19, Issue 2, pp. 156-194.

Fama, E. and French, K. (2020), “The Value Premium”, *Fama-Miller Working Paper No. 20-01*, Available at SSRN: https://ssrn.com/abstract=3525096 or http://dx.doi.org/10.2139/ssrn.3525096

Fama, E. and French, K. (2017), “International tests of a five-factor asset pricing model”, *Journal of Financial Economics,* Vol. 123, Issue 3, pp. 441-463.

Fama, E., and French, K. (2016), “Dissecting anomalies with a five-factor model”, *The Review of Financial Studies,* Vol. 29, Issue 1, pp. 69-103.

Fama, E., and French, K. (2015), “A five factor asset pricing model”, *Journal of Financial Economics,* Vol. 116, Issue 1, pp. 1-22.

Fama, E., and French, K. (2011), “Size, Value, and Momentum in International Stock Returns.” *Journal of Financial Economics,* 105, 457-472

Fama, E. F. and K. R. French (2004), “The capital asset pricing model: Theory and evidence”, *The Journal of Economic Perspectives*, Vol. 18, pp. 25–46.

Fama, E., and French. K. (1993), “Common risk factors in the returns on stocks and bonds”, *Journal of Financial Economics,* Vol. 33, Issue 1, pp. 3-56.

Feldman, L. B. (2021a), “Seven and a Half Lessons about the Brain”, Mariner Books (26 October, 2021)

Feldman, L. B. (2021b), “Your brain predicts (almost) everything you do”, available at: https://www.mindful.org/your-brain-predicts-almost-everything-you-do/

Foye, J. (2018), “A comprehensive test of the Fama-French five-factor model in emerging markets”, *Emerging Markets Review,* Vol. 37, pp. 199-222.

Frazzini and Pedersen (2014), “Betting against beta”, *Journal of Financial Economics,* Vol. 114, pp. 1-25.

Friston, K. (2018), “Does predictive coding have a future?”, *Nature Neuroscience,* Vol. 21, Issue 8, pp. 1019-1021.

Friston, K. (2010), “The free-energy principle: A unified brain theory?”, *Nature Reviews Neuroscience*, Volume 11, Issue 2, pp. 127–138.

Friston, K., and Kiebel, S. (2009), “Predictive coding under the free energy principle”, *Philosophical Transactions of the Royal Society B*, Vol. 364, Issue 1521, **https://doi.org/10.1098/rstb.2008.0300**

Griffin, J. D., and Fletcher, P. C. (2017), “Predictive processing, source monitoring, and psychosis”, *Annual Review of Clinical Psychology,* Vol. 13, Issue 13, pp. 265-289.

Goldstein, B. (2020), “The Mind: Consciousness, Prediction, and the Brain”, Publisher: The MIT Press, ISBN-10: 0262044064

Hawkins, J. (2021), “A Thousand Brains: A New Theory of Intelligence”, Publisher: Basic Books, ISBN-10: 1541675819

Hohwy, J. (2013), “The Predictive Mind”, Oxford University Press: ISBN: 9780199682737

Hendershott, T., Livdan, D., and Rosch, D. (2020), “Asset pricing: A tale of night and day”, *Journal of Financial Economics*, Vol. 138, Issue 3, pp. 635-662.

Huang, S. (2022), “The momentum gap and return predictability”, *The Review of Financial Studies,* Vol. 35, Issue 7, pp. 3303-3336.

Jegadeesh, N. and Titman S. (1993), “Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency.” *Journal of Finance*, Vol. 48, pp. 65-91.

Knill, D. and A. Pouget. (2004), “The Bayesian Brain: The Role of Uncertainty in Neural Coding and Computation”, *Trends in Neuroscience,* Vol. 27, Issue 12, pp. 712–719.

Lachance. M. (2023), “Night trading: Lower risk but higher return?”, *Review of Financial Economics,* Vol. 41, Issue 4, 347-363.

Lech, R., Güntürkün, O., and Suchan, B. (2016), “An interplay of fusiform gyrus and hippocampus enables prototype- and exemplar-based category learning”, *Behavioural Brain Research,* Vol. 311, pp. 239-246.

Lev, B., and Srivastava A. (2022), "Explaining the Recent Failure of Value Investing", *Critical Finance Review*, Vol. 11: No. 2, pp 333-360.

Levi, Y. & Welch, I. (2017), “Best Practice for Cost-of-Capital Estimate”, *Journal of Financial and Quantitative Analysis*, Vol. 52, Issue 2, pp. 427-463.

Lintner, J. (1965), “The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets”, *Review of Economics and Statistics*, Vol.47, pp. 13-37.

Lou, D., Polk, C., and Skouras, S. (2019), “A tug of war: Overnight versus intraday expected returns”, *Journal of Financial Economics,* Vol. 134, Issue 4, pp. 192-213.

McKenzie, R. (2018), “A Brain-Focused Foundation for Economic Science”, Palgrave Macmillan, Cham.

Mossin, J. (1966), “Equilibrium in a capital asset market”, *Econometrica,* Vol. 34, pp. 768-783

Nave, K., Deane, G., Miller, M., and Clark, A. (2020), “Wilding the Predictive Brain”, *Wiley Interdisciplinary Review of Cognitive Science,* Volume 11, Issue 6: e1542. *DOI:10.1002/wcs.1542*

Novy-Marx, R., and Velikov, M. (2022), “Betting Against Betting Against Beta”, *Journal of Financial Economics,* Vol. 143, Issue 1, pp. 80-106.

Novy-Marx, R. (2015), “Fundamentally, momentum is fundamental momentum”, *NBER Working Paper Series,* Working Paper No: 20984.

Page, L. (2022), “Optimally irrational: The good reasons we behave the way we do”, Cambridge University Press, 1st Edition.

Savor, P., and Wilson, M. (2014), “Asset pricing: A tale of two days”, *Journal of Financial Economics,* Vol. 113, Issue 2, pp. 171-201.

Seibert, M. (2023), “Trading the night effect”, (July 7) Available at https://www.linkedin.com/pulse/trading-night-effect-moritz-seibert/

Seth, A. (2021), “Being You: A New Science of Consciousness”, Dutton Books (19 October 2021)

Sharpe, W. (1964), “Capital asset prices: A theory of market equilibrium under conditions of risk”, *The Journal of Finance,* Vol. 19, No. 3, pp. 425-442.

Siddiqi, H. and Murphy, A. (2023), “The Resource-Constrained Brain: A New Perspective on the Equity Premium Puzzle”, *Journal of Behavioral Finance*, Vol. 24, Issue 3, DOI:10.1080/15427560.2021.1975716

Siddiqi, H. (2023), “Information Processing in the Brain and Financial Innovations”, *Journal of Behavioral Finance,* doi.org/10.1080/15427560.2023.2198718

Siddiqi, H., and Quiggin, J. (2019), “The pricing kernel puzzle: A behavioral explanation”, *Cogent Economics and Finance,* Vol. 7, Issue 1.

Siddiqi, H. (2019), “Anchoring-Adjusted Option Pricing Models”, *Journal of Behavioral Finance,* Vol. 20, Issue 2, pp. 139-153.

Siddiqi, H. (2018), “Anchoring-Adjusted Capital Asset Pricing Model”, *Journal of Behavioral Finance,* Vol. 19, Issue 3, pp. 249-270.

Siddiqi, H. (2017), “Certain and Uncertain Utility: A New Perspective on Financial Innovation”, *Economics Letters,* Vol. 158, pp. 7-9.

Simpson, M., and Grossmann, A. (2024), “The resurrected size effect still sleeps in the (monetary) winter”, *International Review of Financial Analysis,* March, 103081.

Stoll, H. and Whaley (1983), “Transaction Costs and the Small Firm Effect”, *Journal of Financial Economics* 12 (1983), 57-79.

Yu, H., Chen, L., and Chen, C. (2022), “The profitability effect: An evaluation of alternative explanations”, *Pacific-Basin Finance Journal,* Vol. 72, April.

Wang, B. (2024), “A new value strategy”, *The Review of Asset Pricing Studies*, Volume 14, Issue 1, pp. 40–83**.**

Zaremba, A. (2020), “Small-minus-big predicts betting-against-beta: Implications for international equity allocation and market timing”, *Investment Analysts Journal,* Vol. 49, Issue 4, pp. 322-341.

Appendix A

Substituting from (2.6) and (2.7) into (2.1) and solving for expected return of s, $E[R^s_{t+1}]$:

$$E[R^s_{t+1}] = R_F + \frac{\gamma}{P^s_t}\left[Cov(X^s_{t+1}, X^M_{t+1}) + (1-m_2)\left(Cov^q - Cov\big((X^s_{t+1}, X^M_{t+1})\big)\right)\right] - \frac{(1-m_1)}{P^s_t}[E^q - E(X^s_{t+1})] \qquad (A1)$$

Multiplying (A1) by the market-value weight, $w_s = \frac{n_s^* P_t^s}{P_t^M}$, and aggregating across all firms in the market:

$$E[R_{t+1}^M] = R_F + \frac{\gamma}{P_t^M} Var(X_{t+1}^M) \qquad (A2)$$

(A2) follows as $Cov^q - Cov\big((X_{t+1}^s, X_{t+1}^M)\big)$, which is the difference between cluster average covariance and firm s covariance, aggregates to zero. Similarly, $E^q - E(X_{t+1}^s)$ aggregates to zero.

One can solve for γ as follows:

$$\gamma = \frac{(E[R_{t+1}^M] - R_F)}{Var(R_{t+1}^M) P_t^M} \qquad (A3)$$

Substituting (A3) into (A1):

$$E[R_{t+1}^s] = R_F + \frac{(E[R_{t+1}^M] - R_F)}{Var(R_{t+1}^M) P_t^M P_t^s} \Big[Cov(X_{t+1}^s, X_{t+1}^M) + (1 - m_2)\Big(Cov^q - Cov\big((X_{t+1}^s, X_{t+1}^M)\big)\Big)\Big] - \frac{(1 - m_1)}{P_t^s} [E^q - E(X_{t+1}^s)] \quad (A4)$$

Substituting $\delta_M = E[R_{t+1}^M] - R_F$, recognizing that $\beta_s = \frac{Cov(R_{t+1}^s, R_{t+1}^M)}{Var(R_{t+1}^M)}$, $\beta_i = \frac{Cov(R_{t+1}^i, R_{t+1}^M)}{Var(R_{t+1}^M)}$, substituting from (2.2) for E^q, and from (2.3) for Cov^q leads to:

$$E[R_{t+1}^s] = R_F + \delta_M\, \beta_s + (1 - m_2)\delta_M \left[\frac{\sum_i^{N_q} P_t^i \beta_i}{P_t^s N_q} - \beta_s\right] - (1 - m_1)\left[\frac{\sum_i^{N_q} P_t^i E\big[R_{t+1}^i\big]}{P_t^s N_q} - E[R_{t+1}^s]\right] \qquad (A5)$$

$$\Rightarrow E[R_{t+1}^s] = R_F + \delta_M\, \beta_s + (1 - m_2)\delta_M \left[\frac{\sum_i^{N_q} w_i \beta_i \left(\frac{n_s^*}{n_i^*}\right)}{w_s N_q} - \beta_s\right] - (1 - m_1)\left[\frac{\sum_i^{N_q} w_i E\big[R_{t+1}^i\big] \left(\frac{n_s^*}{n_i^*}\right)}{w_s N_q} - E[R_{t+1}^s]\right] \qquad (A6)$$

Inserting $\varphi_i = \frac{n_s^*}{n_i^*}$, defining $\bar{\beta} = \sum_{i=1}^{N_q} \frac{\varphi_i w_i \beta_i}{N_q}$, and $\overline{ER} = \sum_{i=1}^{N_q} \frac{\varphi_i w_i E[R_{t+1}^i]}{N_q}$ leads to:

$$E[R_{t+1}^s] = R_F + \delta_M \beta_s + (1 - m_2)\delta_M \left[\frac{\bar{\beta}}{w_s} - \beta_s\right] - (1 - m_1)\left[\frac{\overline{ER}}{w_s} - E[R_{t+1}^s]\right] \qquad (A6)$$

$$\Rightarrow m_1 E[R_{t+1}^s] = R_F + \delta_M \beta_s + (1 - m_2)\delta_M \left[\frac{\bar{\beta}}{w_s} - \beta_s\right] - (1 - m_1)\frac{\overline{ER}}{w_s} \qquad (A7)$$

Dividing both sides by m_1 and re-arranging leads to (2.8).

Appendix B

$$\frac{\partial \alpha_s}{\partial m_1} = \frac{1}{m_1^2}\left[\frac{\overline{ER} - \bar{\beta}(1 - m_2)\delta_M}{w_s} - R_F - \beta_s \delta_M m_2\right] \qquad (B1)$$

For low values of β_s, $\frac{\partial \alpha_s}{\partial m_1} > 0$ and for high values of β_s, $\frac{\partial \alpha_s}{\partial m_1} < 0$. It follows that when m_1 rises, SML rotates in the clockwise direction. That is, SML flattens.

$$\frac{\partial \alpha_s}{\partial m_2} = \frac{-\bar{\beta}\delta_M}{m_1 w_s} + \frac{\beta_s \delta_M}{m_1} \qquad (B2)$$

For low values of β_s, $\frac{\partial \alpha_s}{\partial m_2} < 0$ and for high values, $\frac{\partial \alpha_s}{\partial m_2} > 0$. It follows that when m_2 rises, SML rotates in the counter-clockwise direction. That is, SML steepens.