Optimal Retirement Age: Death Hazard Rate Approach

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Abstract

A model with special attention on the (subjective) survival probability is proposed to understand salient aspects of retirement age decision. Optimal retirement age results are derived with a death hazard rate function having non-negative duration dependence. At the optimum age, the retiree wants to have a compensation in the form of early retirement for his/her evident non-zero death risk. A retiree with large welfare inputs supporting mortality risk decreasing effects delays his/her retirement time. From policy perspective we need to lower the elderly health costs to reduce the death hazard rates leading to higher optimal retirement ages. Some empirical findings with the birth year 1947 cohort in Finland do not conflict the model results. Death hazard rate function estimates show that gender, health, civil status, incomes, and pension affect the death hazard rates. The retirement age has a longevity increasing effect across the different model specification.

JEL classification: C41, J14, I12

Keywords: Optimal retirement age, survival probabilities, death hazard rate function, survival model estimation, frailty.

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1. Introduction

Retirement is a special period in person's life. Typically, it is the last life period before the death. In economics, timing of retirement is usually modelled with the life-cycle approach as a form of labour supply decision. At the time of retirement, leisure and consumption, and disutility of work compensated with incomes are closely interrelated. Higher incomes should induce more leisure and consumption (if leisure is a normal good) implying that higher labour earnings and pension should lead to an earlier retirement. However, if more earnings are gained when retirement is postponed, this can elicit more years of work because of the intertemporal substitution effect. In sum, income effect (early retirement) and substitution effect (post-ponement of retirement) make the retirement date decisions ambiguous (see, e.g. Crawford & Lilien 1981, Fields & Mitchell 1984).

In general, the net total effect on retirement date can be either positive or negative depending on retiree's preferences for work and leisure. However, as a standard result, a larger share in favour of pension income with *a given lifetime* budged lowers the retirement age. Note that this is a sure thing only when work incomes are not directly related to pension income level. When pension level depends on the retiree's work career earnings (i.e. salary and self-employment incomes), the ambiguous net retirement age result can emerge again. In addition, lifetime consumption and wealth, taxes on wealth and incomes, health, and spouse's incomes and pensions also have roles in this context. Different results are derived in the literature on retirement timing in various theoretical and empirical models (see, e.g. Kalemli-Ozcan & Weil 2010, Mao et al. 2014, d'Albis et al. 2012, Kuhn et al. 2015, Bound et al. 2010, Fonseca et al. 2009, Chen et al. 2021, Peijnenburg et al. 2010, Cremer et al. 2004, Dalgaard & Strulik 2014, Fitzpatrick & Moore 2018, French & Jones 2017, Bozio et al. 2021). Any systematic and uniform results from this ongoing research agenda is hard to summarize.

Surprisingly the papers that focus explicitly on how retirement timing depends on death hazard rate or survival probability are almost non-existing. Note that many countries have an eligible age window that permits the retiree delay his/her "optimal" retirement age with rising pension, like between the ages of 63 to 68 years in Finland. This means that (subjective) survival time estimates must have some, but often neglected, role in the formation of retirement decisions. Almost all papers mentioned above base on the result that retiree has a fixed planned death age – typically one very high. Alternatively, he/she takes some estimate of *expected lifetime* as a given parameter in his/her "optimal" retirement date decision. Literature on subjective survival probabilities has shown that persons derive seriously estimates concerning their survival probabilities and let these also affect their retirement decisions (see, e.g. O'Donnell et al. 2008, Palloni & Novak 2016, Wu et al. 2015).

Next, in Section 2, we propose quite elementary model where death hazard rate has a major role in the optimal retirement age decision. Optimal retirement age results are derived with death hazard rate function having a non-negative duration dependence. The model results give us also possibility to propose some policy alternatives concerning elderly health costs and poverty. Section 3 provides some empirical results on the death hazard function rates among the Finnish retirees born in year 1947. Section 4 concludes the paper with a sum-up and discussion on the paper results.

2. Model

2.1. Set-up and the basic model

In the following we assume that person's welfare level (e.g. incomes, pension, consumption) is proportional to his/her retirement age T_R, i.e. vT_R, where $v > 0$. This corresponds to the fact that a person with a long work-career with postponed retirement can earn also higher pension allowing for increases in wealth and health investments. Although the pre-retirement labour supply decisions and full life-cycle optimization approach are not considered here, we pay attention to pre-retirement factors on retirement age, i.e. they are related to the person's value of the parameter *v*.[1)]

Aging means also increasing relative costs to a person as he/she gets older, i.e. $C'(T_R) > 0$ and $C''(T_R) > 0$. Here we mean by costs all the dis-utilities, functional and health problems, which getting older will cause to a person. We stress the importance of probability of death in this context, i.e. retirement age decision most include some estimate of death uncertainty as one of its main forcing variables. We model the probability of survival beyond period T_R as

$$S(t) = Prob(T_R \geq t) = 1 - Prob(T_R < t) = 1 - F(t)$$

where $F(t)$ is the person's estimate for probability of his/her life length less than *t*.

Next, we assume that person faces the following optimization problem with respect to T_R

I) $$\max_{T_R}\{[v \cdot T_R - C(T_R)] \cdot [1 - F(T_R)]\}.$$

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1) We use this simple welfare function instead of e.g. concave function $W(T_R)$ in order to keep the analysis tractable.

The 1st order condition is

$$[v - C'(T_R)][1 - F(T_R)] - [v \cdot T_R - C(T_R)] f(T_R) = 0:$$

$$\text{II)} \qquad [v - C'(T_R)] \;=\; [v \cdot T_R - C(T_R)] \frac{f(T_R)}{[1 - F(T_R)]} \;=\; [v \cdot T_R - C(T_R)] h(T_R).$$

Note when the last term in Eq. II) is positive, $[v \cdot T_R - C(T_R)]h(T_R) > 0$, we have $[v - C'(T_R)] > 0$, meaning that the net marginal welfare at the optimal retirement age is positive. This corresponds to retirement age of T_R^* in the Figure 1 where the slope *v* is *larger* than the slope of cost function. Note that T_R^* is less than T_{R1} where $[v - C'(T_{R1})] = 0$.

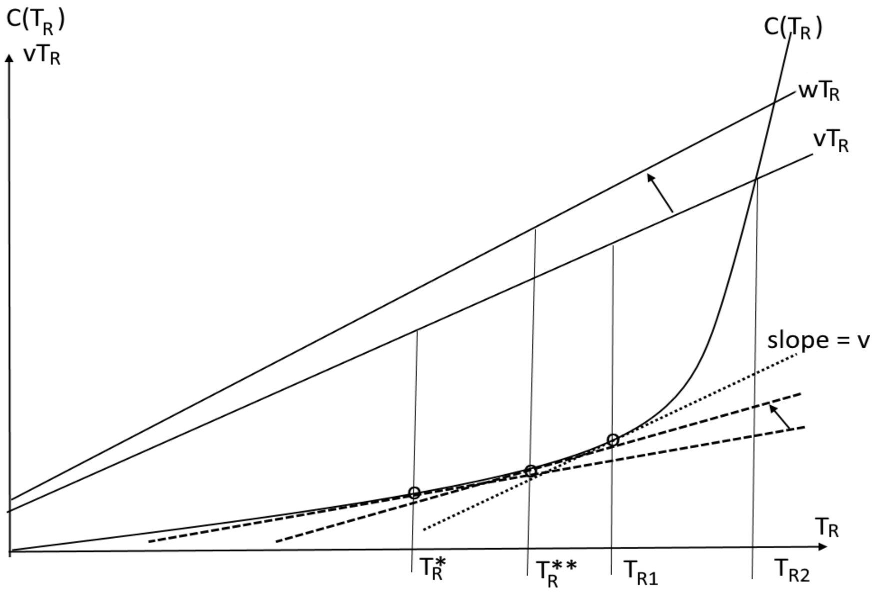

Figure 1. Optimal retirement age as a function of welfare and age costs.

In details, the condition $[v \cdot T_R - C(T_R)] > 0$ is valid up to time point T_{R2}. However, at the optimum retirement age T_R^* we have $[v - C'(T_R)] - [v \cdot T_R - C(T_R)]h(T_R) = 0$ meaning that $[v - C'(T_R)] > [v \cdot T_R - C(T_R)]h(T_R)$ if $h(T_R) > 0$. The 2nd order conditions for maximum at T_R^* are giving in Appendix 1.

The maximization solution includes also the *death hazard rate function*, *h(T_R)*, that is always *positive*, and specifies the death risk rate estimate for the retirement age before the age of death, given the age T_R.[2)] Note, that hazard rate is a risk measure that is closely related to the probability of dying at age t, $F(t)$, and to the *survivor function* $S(t) = 1 - F(t)$ as

$$h(t) = [dF(t)/dt]/[1-F(t)] = f(t)/[1-F(t)] = -dlnS(t)/dt .$$

These remarks mean that we can give the following interpretation to our optimal *positive net marginal welfare* result $[v - C'(T_R)] > 0$: a person wants to have a compensation for his/her evident non-zero death risk at the retirement age T_R^*. Note that, if the hazard rate function shifts upward (i.e. higher death risk), $[v - C'(T_R)] > 0$ needs to be larger than earlier, i.e. retirement time is less than T_R^*. If the welfare factor v is high, this gives room for higher optimal retirement age as $C'(T_R)$ can also be higher. Alternative, if welfare is very low (i.e. low value of v), or age costs are high at the retirement time, then the sign of $[v - C'(T_R)]$ is negative. Now the optimum retirement time does not exist if $[v \cdot T_R - C(T_R)] > 0$. Person reacts to this by trying to retire as early as possible to keep his/her marginal elderly costs $C'(T_R)$ as low as possible. Clearly this is not socially desirable outcome and policy is needed here to correct negative value of $[v - C'(T_R)]$.[3)]

We argue that death hazard rate function gives us a useful interpretation of retirement age decision problem in a conditional form. Risk to a stay alive until (and after) time T_R is a different thing than the probability to die at time T_R. Most importantly death hazard gives us insight how *aging determines* the *rate* at which retirement age is under the death risk. This means that hazard rate function is a time dependent measure, and we can analyse its *duration dependence,* i.e. how *the risk rate* of life termination *increases*, stays *constant*, or *decreases* as the retirement age T_R increases. Positive duration dependence *PDD* (or increasing hazard) exists at time point t^* if $dh(t)/dt|_{t=t^*} > 0$. Negative duration dependence *NDD* (or decreasing hazard), and constant hazard occur when $dh(t)/dt|_{t=t^*} < 0$ and $dh(t)/dt|_{t=t^*} = 0$.

[2)] A precise definition for hazard function is $\lambda(t) = \lim_{\mu \to 0} Prob(t \le T < t + \mu \mid T \ge t)/\mu$.

[3)] Note that negative value of $[v - C'(T_R)]$ can also mean that $[v \cdot T_R - C(T_R)]$ is negative if we derive the optimum retirement age. The person can survive with condition $[v - C'(T_R)] < 0$ for some time but condition $[v \cdot T_R - C(T_R)] < 0$ refers to cases like the incidence of catastrophic health costs that has some relevance also in Finland (see Tervola et al. 2021).

Comparative statistic results show (see Appendix 2) that $dT_R^* / dv > 0$ happens when aging costs are increasing ($C'' > 0$). This means that if welfare factors *at optimal retirement age* are increased person will delay his/her retirement age because the slope *v* increases to *w* ($vT_R \to wT_R$ in Figure 1), i.e. the marginal gain of postponing *retirement time* increases and the optimal retirement age will rise (T_R** in Figure 1). The opposite result is obtained when the aging cost function $C(T_R)$ switches upward. Note that these results hold only when hazard duration dependence is close to zero, i.e. $h' \simeq 0$. To obtain more general results we have to analyse separately the *NDD* and *PDD* cases where h' is relative large in absolute value supporting either $dT_R^* / dv > 0$ or $dT_R^* / dv < 0$ results.

2.2. Negative duration dependence (*NDD*) and $dT_R^* / dv < 0$

Allowing for increased welfare factors to *decrease* the optimal retirement age ($dT_R^* / dv < 0$) happens when we have large *NDD* effects, i.e. $|h'| > C'' + h[v - C'] / [vT_R^* - C]$ and $h > 1/T_R^*$ (see Appendix 2). However, the typical human death process ("the law of mortality") hardly supports large *NDD*-effects meaning that death occurs (relatively) *less* often when the person gets older. We could proceed here with mild *NDD* effects, e.g. caused by improved medical technology and care, with *h'* close, but below to zero, and $h < 1/T_R^*$, supporting $dT_R^* / dv > 0$ result. Instead, we pursuit in the next sections to model alternatives that concern explicitly inputs or actions that reduce death probability at the individual level under the more natural *PDD* case.

2.3. Positive duration dependence (*PDD*) and $dT_R^* / dv > 0$

Note that when *PDD* or *DD* happens ($h' \geq 0$), we need that $h < 1/T_R^*$ to support result $dT_R^* / dv > 0$ (see Appendix 2). When $h > 1/T_R^*$, we obtain $dT_R^* / dv < 0$. We want to avoid this case as it means that increases in welfare rate lowers the optimal retirement age when the death hazard rate is *PDD*. Although this is a plausible result at the individual level, it is socially unbearable because the target of welfare inputs and medical care (also included in *v*) is to preserve life and reduce death risk which also give the possibility to *rise* the retirement age. [4)]

[4)] The relation between *h* and $1/T_R^*$ has some addition interest here as $1/T_R^*$ gives some estimate for the (subjective) hazard rate in terms of optimal retirement age. As the *objective* (population) hazard rate increases with age (*PDD*) we can refer to results $h < 1/T_R^*$ and $dT_R^* / dv > 0$ without leaning to subjective hazard rates.

2.4. Extended model with *PDD* hazard function

In response to this we introduce a welfare improving function $v(z)$ where input *z*, like higher pension, health promoting life activities, healthy food, and medical care, acts also as quality effect on T_R so that per retirement age the welfare level is higher when *z* increases. These additional inputs are not costless but mostly important they reduce the death risk, i.e. lower the hazard rate. Thus, we assume that for any *z* the function $v(z)$ is concave ($v_z > 0$ and $v_{zz} < 0$), and the cost function is convex in *z*. For hazard function with fixed *z* we keep with *PDD* effects, i.e. $h(T_R, z)$ with $h_{T_R} \geq 0$, but now the risk reducing variable *z* needs special attention. Our extended value function has the following form

$$\text{III)} \qquad V(T_R, z) = \max_{T_R, z}\{[v(z)\cdot T_R - C(T_R, z)]\cdot[1 - F(T_R, z)]\}\,.$$

Optimum conditions are following (Appendix 3 gives the 2nd order conditions)

$$\text{IV-1)} \qquad \frac{\partial V(T_R, z)}{\partial T_R} = [v(z) - C_{T_R}][1 - F(T_R, z)] - [v(z)\cdot T_R - C(T_R, z)]f(T_R, z) = 0$$
$$\Rightarrow \quad [v(z) - C_{T_R}(T_R, z)] = [v(z)\cdot T_R - C(T_R, z)]h(T_R, z).$$

$$\text{IV-2)} \qquad \frac{\partial V(T_R, z)}{\partial z_R} = [v'\cdot T_R - C_z][1 - F(T_R, z)] - [v(z)\cdot T_R - C(T_R, z)]\partial F/\partial z = 0$$
$$\Rightarrow \quad [v'\cdot T_R - C_z] = [v(z)\cdot T_R - C(T_R, z)]g(T_R, z).$$

Last result is non-standard because we allow here for a case where variable *z changes the shape* of distribution function $F(T_R, z)$ by altering its parameters. For example with the Weibull distribution, the survival function is $S(t, z) = 1 - F(t, z) = \exp(-\gamma(z)\cdot t^{\alpha})$ with $\alpha > 0$. Now, the partial derivatives of $-lnS(t, z)$ are $h(t, z) = \gamma(z)\alpha t^{\alpha-1}$ and $g(t, z) = \gamma'(z)\cdot t^{\alpha}$. Note that Weibull hazard function is *PDD* when $\alpha > 1$. Next, we specify that $\gamma'(z) < 0$, i.e. the larger value of welfare input *z* decreases the value of γ and *less* is the hazard rate with the given value of time variable *t*.

In the present setting this corresponds to "care adjusted mortality incidence rate" $g(T_R, z) < 0$ with $g_{T_R} \leq 0$ and $g_z \geq 0$. These assumptions mean that for optimal *z** with $g(T_R, z) < 0$ we have

$$\text{IV-2')} \quad \frac{\partial V(T_R, z)}{\partial z_R}|_{z=z^*}: \ [v' \cdot T_R - C_z] = [v(z) \cdot T_R - C(T_R, z)]g(T_R, z) \ \Rightarrow \ [v' \cdot T_R - C_z] < 0.$$

2.5. Interpretation for the optimal T_R and z results

The optimal T_R^* needs that $[v(z) - C_{T_R})] > 0$ as $[v(z) \cdot T_R - C] > 0$ and $h(T_R, z) > 0$ (see result IV-1) above). Thus, at the optimal retirement age T_R^* welfare effects *v(z)* must outweigh the marginal costs of postponing retirement if the death hazard function is *PDD*. This is quite natural outcome in this context, i.e. when the estimate of risk of dying is high (high value of $h(T_R, z)$), the retiree wants to have compensation for it in terms of improved (marginal) net welfare with a lower optimal retirement age. Alternatively, a given high value of z means high value for $v(z)$ and low $h(T_R, z)$, and the optimal T_R^* can increase.

For the optimal welfare input z^* the marginal welfare effect is less than the marginal cost, $[v' \cdot T_R - C_z] < 0$, because large z input means a lower hazard rate as the adjusted mortality incidence rate is negative ($g(T_R, z) < 0$), and $[v(z) \cdot T_R - C(T_R, z)] > 0$ (see results IV-2 and IV-2' above). The result means that a person wants to invest in his/her welfare (e.g. in health and care) with high marginal costs to reduce his/her death hazard rate $h(T_R, z)$. Clearly the result depends on person's income and wealth. This is a sensitive equity question.

From policy perspective above results indicate that policy targeted to increase values of $[v(z) - C_{T_R})]$ or $[v(z) \cdot T_R - C(T_R, z)]h(T_R, z)$ by *lowering* the retirement age T_R is neither desirable nor effective policy as is the *subsidising the cost of welfare* inputs in order to increase their use. Thus, increasing the absolute values of $[v(z) - C_{T_R})]$, $[v' \cdot T_R - C_z]$ and $g(T_R, z)$ with larger z inputs for *all* citizens means also lower individual death hazard rates and higher optimal retirement ages. In general terms this means that *healthy retirement years* are socially more preferred alternative than the longer retirement periods.

The above analysis started with assumption that subjective survival expectations are good approximates to the actual survival values or they average to the "representative" agent's estimate of survival that equals the actual survival probability. The current literature of subjective survival expectations supports this approach. Alternatively, we can use here the “social planner” perspective where the planner takes the life table survival time estimates for representative agents as the starting point and works out the above optimization.

3. Death age hazard rates among the Finnish retirees born in year 1947

3.1. Model implications

Model results above have interesting empirical implications. The optimal retirement age depends on retiree’s net (marginal) welfare level, i.e. on the welfare inputs (e.g. incomes, pensions, and health care), and on the age-related health costs. However, the death hazard rate function has an important role in the model. For the given level of inputs and costs a high hazard rate $h(T_R, z)$ means early retirement age as the net marginal gain of retirement must be large for high $h(T_R, z)$ supporting large $[v(z) - C_{T_R}]$ and low T_R. Similarly, if aging health costs are large, optimal net marginal gain can be sustained only with a lower value of retirement age. However, both these outcomes *can be overruled* if the hazard rate function can be shifted downwards with large welfare inputs. Thus, we have a trade-off situation for the optimal retirement age between welfare inputs and age costs that are mediated through the death hazard function depending on these factors. The shape of hazard rate function is also important here. We sustained our result with a hazard function having a non-negative duration dependence with respect to retirement age, and with hazard function shape sensitivity with respect to welfare inputs.

Although the theory results are quite many-sided and inter-related, the model is based on the following two elementary assumptions: A) death hazard rate is high with health problems, and B) death hazard rate is low with high welfare inputs. These support the following main optimal retirement age results: I) larger welfare inputs mean higher optimal retirement age, and II) large health costs lower the optimal retirement age. Next, we analyse in detail first the assumptions A) and B), and then proceed to empirical death hazard rate modelling with health indicators, income, pensions, and retirement age.

3.2. Data and death hazard rate estimates

We use person level register data on the year 1947 birth cohort in Finland. The sample retirees started their *elderly pension period* as their first and only form of retirement during the follow-up time of 1.1.2007 – 31.12.2019. The sample size is between 35984 and 40662 observations depending on the variables (Appendix 4 gives the related summary statistics). For the year

1947 birth cohort the eligible old-age pension age window was between ages of 63 and 68 years but quite many persons retired before the age of 63 years for different reasons. However, close to 55 percent of sample persons retired between ages of 63 – 64 years, and over 90 percent of persons retired before age of 66 years (see Figure A1 in Appendix 4). Next, we concentrate on the death hazard rate estimates with persons retiring before and after the age of 63 years.

The following Figure 2 (left side) shows the smoothed death hazard rate estimates with 95% CI's for the persons retired before and after age of 63 years. Surprisingly, after 63 age retirees have lower hazard rates compared to the pre-63 age retirees.[5)] Figure 2 (right side) shows that persons that had some health problems before retiring have higher death hazard rates compared to those with-out the health problems. In this context health problems were measured with having zero or non-zero sick on leave (#SOL) days before retirement.

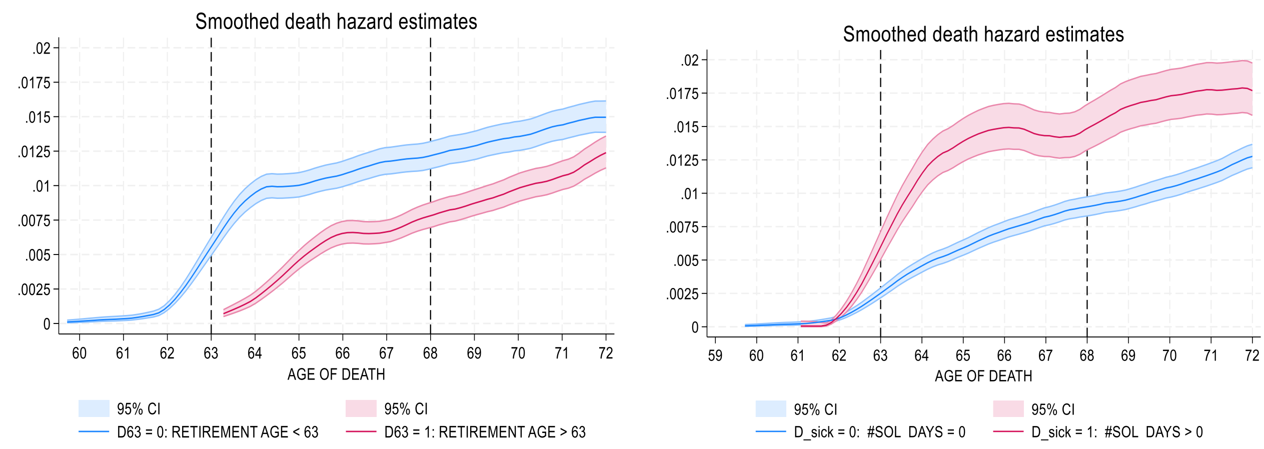

Figure 2. Death hazard rates with retirement age before and after age of 63 years (left), and with zero and non-zero sick on leave days (#SOL) (right).

Next question concerns how the welfare inputs – here income and pension – affect the death hazard rates. Note that the policy of *accelerated accrual rate* (AAR) to rise the retirement age was valid in Finland during our sample follow-up. AAR schema provided a 4.5% increase on old-age pension based on the full work time incomes per year between the ages of 63 to 68 years. Thus, the income and pension variables here include an additional AAR effect to delay retirement age, but our focus is on their effects on the death hazard rate. Figure 3 (left side) shows that persons having incomes *above* 100.000 euros per year have the lowest death hazard

5) Note that you cannot retire after you have died, i.e. the condition $AGE - T_R \geq 0$ is always valid in our sample. However, this "older you retire, older you die" -condition does not mean that death hazard rates are always less with higher retirement ages. Appendix 5 shows that when retirement happen at the age of 66 years or later the death hazard rates are higher than for pre-66 age retirees.

rates at the given age of death compared to rates with less income takers. Figure 3 (right side) replicates these results in the metric of yearly average old-age pensions. Note that between death ages of 66 and 68 years the death hazard rates do not increase among persons having incomes above 100.000 euros. In addition, the hazard rate functions start only after the age of death of 63 years because the sample contains only a few observations with the age of death less than 63 years in the highest income and pension categories.

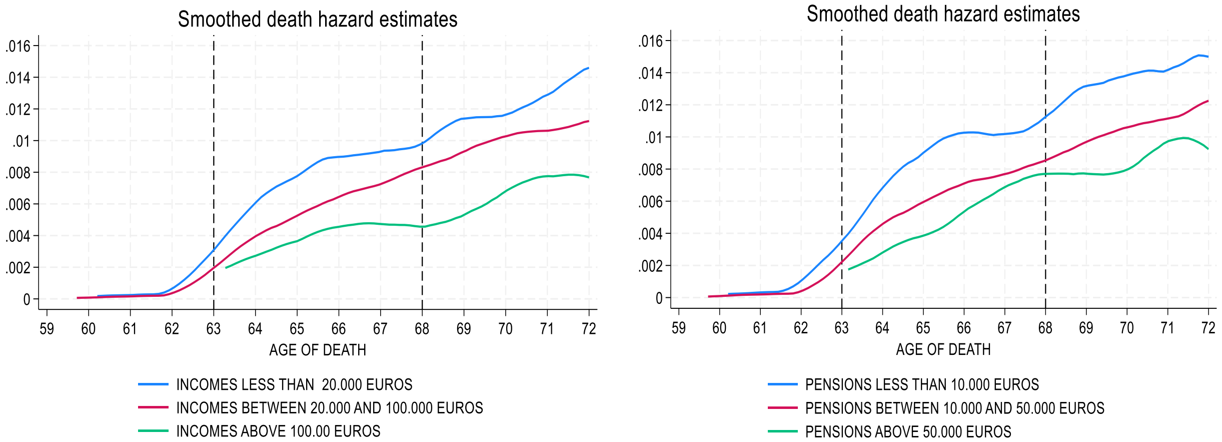

Figure 3. Death hazard rates with incomes (left), and with pension (right)

To sum-up, above graphical analysis shows that persons retiring after age of 63 years have a *lower* death hazard rates than pre-63 age retirees have showing that death hazard is related to the retirement age. Secondly, death hazard rates are sensitive to welfare inputs and health problems in the expected way: Higher incomes (and pension) sustain lower hazard rates, but health problems mean increased death risk.

3.3. Modelling death hazard rate

Next, we propose some survival time models to test our theory model implications. We model death hazard rate function depending on retirement age, health risks, incomes, pensions, and on some additional time independent covariates. Our focus here is not to model how retirement age is related to incomes and pensions but on how retirement age conditions death hazard rate function.

Models presented below are so-called “slope change” models where persons retiring before the age of 63 years are expected to have different coefficient estimates on income and pension variables than retirement taking place after the age of 63 years. As the target of AAR schema was to postpone retirement age with full-time work after age of 63 years, we can argue that incomes before retirement, and especially AAR pensions when retiring, have different impacts on death hazard rates before and after of retirement age of 63 years (see above, Figures 2 and 3). The framework allows us to test for possible pre-retirement income and post-retirement pension effects on the death hazard rates. To identify the health aspects of our theory model we include into empirical models a category variable (*D_sick*) that measures the number of sick-on leave days before retirement. Next, we estimate parametric accelerated failure time (*AFT*) models with *ML*-method for the death age variable AGE .

The covariate parts of *AFT* models are [6)]

Model A1: $d_0 + d_1 T_{R,i} + \boldsymbol{d}_2^{'} D_{SICK,i} + \boldsymbol{d}_3^{'} \boldsymbol{X}_i$

Model A2: $\alpha_0 + \alpha_1 T_{R,i} + \boldsymbol{\alpha}_2^{'} D_{SICK,i} + \boldsymbol{\alpha}_3^{'} D_{63} \times INCOME_i + \boldsymbol{\alpha}_4^{'} \boldsymbol{X}_i$

Model A3: $\beta_0 + \beta_1 T_{R,i} + \boldsymbol{\beta}_2^{'} D_{SICK,i} + \boldsymbol{\beta}_3^{'} D_{63} \times PENSION_i + \boldsymbol{\beta}_4^{'} \boldsymbol{X}_i$

Model A4: $c_0 + c_1 T_{R,i} + \boldsymbol{c}_2^{'} D_{SICK,i} + \boldsymbol{c}_3^{'} D_{63} \times INCOME_i + \boldsymbol{c}_4^{'} D_{63} \times PENSION_i + \boldsymbol{c}_5^{'} \boldsymbol{X}_i$

where

AGE_i : age of death during the sample follow-up or age at the end of sample follow-up (i.e. the censored observations)

$T_{R,i}$: retirement age after age of 59 years

$$D_{SICK} = \begin{cases} 1, & \text{if number of sick-on leave days is } 0 - 60 \\ 2, & \text{if number of sick-on leave days is } 60 - 240 \\ 3, & \text{if number of sick-on leave days is } > 240 \end{cases}$$

$$D_{63} = \begin{cases} 1, & \text{if retirement age is } > 63 \text{ (AAR policy is valid)} \\ 0, & \text{if retirement age is } \leq 63 \text{ (AAR policy is not valid)} \end{cases}$$

$INCOME_i$ = mean of yearly salary incomes before retirement (1000 euros)

$PENSION_i$ = mean of yearly pension after retirement (1000 euros)

$\boldsymbol{X_i}$ = some additional covariates (e.g. gender and civil status)

Note that in our theory setting retirement age was an *endogenous* variable derived as an optimum solution to the model. In this sense our empirical models can be interpretated as "structural". However, our interest here is on the empirical effect of (optimum) retirement age on the death hazard rates specified with the death as the event variable and the age of death is the survival time variable. In our theory model death hazard function was specified on the

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6) *AFT* models have form of $lnAGE = \mu + \boldsymbol{\alpha}' \boldsymbol{X} + \sigma\varepsilon$ where μ and σ are distribution location and scale parameters. This leads to the following survival function $S(t \mid X) = exp(\sigma\varepsilon) \cdot (AGE \cdot exp(-\mu - \boldsymbol{\alpha}' \boldsymbol{X})) = S_0(AGE \cdot exp(-\mu - \boldsymbol{\alpha}' \boldsymbol{X}))$ (e.g. see Legrand 2021, Chapter 2).

retirement age. We expect that death hazard rates with the age of death are somewhat larger than with the retirement age because the age of death cannot be less than the retirement age is ($AGE^{\dagger} - T_R \geq 0$). Note that the retirement age as first order Taylor approximation of the death hazard function is positive.[7)] However, this reservation does not solve our potential endogeneity problem in the empirical models.

In survival analysis some interest has focused on the problem of endogenous covariates. The basic result is that knowing the value of the time-dependent covariate up to time *s* (e.g. retirement age, incomes, or pensions), the future values of covariate up to time $t > s$ are *not impacted by the occurrence of event* (e.g. death) *at time s* (Legrand 2021 p. 274, Lancaster 1990, p. 28). In other words, the time-dependent variables for which the evolution over time is *known or fixed in advance*, independently of possible values of event (death/alive), are exogenous. Here the death interrupts retirement spell, income, and pension plans. Thus, these variables are not necessarily exogenous in our survival models although the variables have known values before and after the random death. We can say that the death has an impact on these variables but in a fixed knowable way.

Another potential problem in estimating hazard rate function is related to frailty or unobserved heterogeneity. Frailty is present when some persons will experience the event earlier than others because of some unobserved factors, and this heterogeneity will bias downwards the model parameter estimates. This problem can be handled by introducing a non-negative random variable (i.e. frailty distribution) for everyone in sample that multiplies the hazard function. Next, we estimate the AFT models with Gamma frailty distribution and test if the frailty is present in the models.

3.4. Model estimation results

The BIC/AICC specification values for model A4 with additional covariates showed that *Generalized Gamma* (GG) error distribution is the most preferred alternative (see Table 1).[8)] The results were similar with models A1-A3. However, when we correct estimates for the

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[7)] By taking the 1st order Taylor -approximation for the death hazard function around the optimal retirement age with $\Delta T \geq 0$ we get $h(AGE^{\dagger}) = h(T_R^* + \Delta T) \approx h(T_R^*) + h'(T_R^*)(\Delta T) = h_0 + h_1 \Delta T$ where $h'(T_R^*) = h_1 \geq 0,$ if *h* is non-*NDD.* Note also that the pairwise correlation between death and retirement ages is 0.222 in the sample.

[8)] We added to models following variables: *GENDER* = 1 (male), 2 (female), and *CIVIL STATUS* = 1 (unmarried), 2 (married etc.), 3 (divorced), 4 (widow). The former interacts with the D_{SICK} variable.

possible frailty, we use *LogNormal* (LN) approach because GG estimations under different frailty distribution were not converging. Finally, the interpretation of coefficient estimates follows the *AFT* -model structure, i.e. when $\alpha > 0$ ($\alpha < 0$) log time to event increases (decreases) (see footnote 6).

Table 1. Model information criterions for alternative distributions of age of death (Model A4)

Model	N	LL(null)	LL(model)	df	AIC	BIC
exponential	35,886	-10756.850	-10360.661	14	20749.331	20868.162
weibull	35,886	-4941.865	-4492.011	15	9014.021	9141.342
gompertz	35,886	-4995.093	-4544.908	15	9119.817	9247.139
lognormal	**35,886**	**-4814.241**	**-4353.512**	**15**	**8737.024**	**8864.346**
llogistic	35,886	-4927.556	-4469.389	15	8968.779	9096.101
ggamma	**35,886**	**-4583.041**	**-4247.279**	**16**	**8526.559**	**8662.368**

In the estimation result Table 2, the first model (A1) uses T_R as the only continuous predictor. The sign of its coefficient estimate is positive with value of 0.0079 (p-value < 0.001). This means that postponing retirement time has an increasing impact on the log time to death, i.e. death hazard function is lower for persons delaying their retirement. Having sick-on leave days over two months (60 days) increases the death hazard risk for both sexes but interestingly having short or zero period of sick-on leave days lowers the death risk. Being married or widow lowers the risk compared to unmarried.

When incomes and pension are added into the model (models A2 – A4), the effect of retirement age on the death hazard rates is smaller and less precise than earlier. We observe that it is the level of pension that predicts the death risk, less the incomes. The AAR pension schema (*D63* = 1) lowers the death hazard rates with larger pension coefficient estimate compared to non-AAR period (*D63 = 0*), i.e. retiring before age of 63 years. In the model specification A3 the t-test for equality of *D63* pension coefficient estimates (0.0011 and 0.0016) is – 3.22 (p-value = 0.001), and in the model A4 the equality is also rejected (p – value = 0.005). Note that *D63* income coefficient estimates change their signs between models A2 and A4. In the final model alternative A4 income coefficient estimates are negative both in AAR and non-AAR periods. This means that incomes before retirement increase the death risk contrasting the pension effects. The LR test ($\bar{\chi}^2(1)$ test) for non-frailty is rejected in all model alternatives. Appendix 6 gives goodness-of-fit plot between the Cox-Snell (CS) residuals and cumulative hazard.

Model fit is good up-till the large values of CS residuals corresponding to large values of *AGE* variable. Table 3 reports the model information criterions for models A1-A4.

Table 2. AFT model estimation results with LogNormal distribution (*LN*) and Gamma (*G*) frailty distribution (dependent variable: *lnAGE*)

Variable	MODEL A1	MODEL A2	MODEL A3	MODEL A4
GENDER and NUMBER OF SICK-ON-LEAVE DAYS				
MALE & DAYS 60-240	-0.0328***	-0.0408***	-0.0299***	-0.0351***
MALE & DAYS > 240	-0.0368***	-0.0476***	-0.0340***	-0.0404***
FEMALE & DAYS 0-60	0.0437***	0.0394***	0.0486***	0.0406***
FEMALE & DAYS 60-240	-0.0062	-0.0196***	0.0009	-0.0148**
FEMALE & DAYS > 240	-0.0253***	-0.0316***	-0.0120	-0.0193*
SOCIAL STATUS				
MARRIED	0.0392***	0.0281***	0.0325***	0.0232***
DIVORCED	0.0101**	0.0081	0.0071*	0.0047
WIDOW	0.0327***	0.0292***	0.0242***	0.0223***
RETIREMENT AGE (T_R)	**0.0079*** **	**0.0026***	**0.0050*** **	**0.0027***
INCOME				
D63=0: $T_R \leq 63$		**0.0000**		**-0.0002***
D63=1: $T_R > 63$ (AAR)		**0.0004*** **		**-0.0004***
PENSION				
D63=0: $T_R \leq 63$			**0.0011*** **	**0.0010*** **
D63=1: $T_R > 63$ (AAR)			**0.0016*** **	**0.0019*** **
CONSTANT	3.8304***	4.1840***	4.0001***	4.1585***
LN: lnsigma	-2.4517***	-2.4202***	-2.4339***	-2.4847***
G: lntheta	2.2063***	2.2914***	2.0167***	2.4117***
LR test of frailty: theta = 0, $\bar{\chi}^2(1)$	181.67***	120.96***	149.87***	137.40***

Legend: * p<0.05; ** p<0.01; *** p<0.001

Table 3. Model information criterions for alternative LN -distribution models

Model	N	LL(null)	LL(model)	df	AIC	BIC
A1	40,653	-5691.533	-5242.794	12	10509.590	10612.941
A2	35,976	-4766.497	-4464.512	14	8957.024	9075.892
A3	40,542	-5528.782	-4985.086	14	9998.173	10118.710
A4	**35,886**	**-4617.424**	**-4284.811**	**16**	**8601.623**	**8737.432**

Table 4 gives the average marginal effects (AME's) of variable coefficient estimates on the median age of death, i.e. $dAGE_{MED} /dx$ –effects. Thus, AME's give the direct effect of one unit increase in the value of predictors on the median age of death. For example, being a female person increases median death age with 3.77 years. Having large number of sick-on leave days before retirement decreases 4.5 – 5.5 years the median death age. 10.000€ increase in yearly income and retiring after age of 63 years means –0.3 decrease in the median death age. However, 10.000€ increase in pension means 1.4 increase in the median death age. Finally, a one-year increase in the retirement age supports close to a half year increase in the median death age. When this happens after the age of 63 year this gives additional 1.35 years to the median age of death. Note that marginal effects on the *mean* death ages are close to the median effects (see Appendix 7).

Table 4. Average marginal effects on the median age of death

	dy/dx	Delta-method std. err.	z	Prob>|z|	[95% conf.	interval]
FEMALE	3.7709	0.269	14.00	0.000	3.243	4.299
#SOL DAYS 60-240	-4.5324	0.376	-12.05	0.000	-5.270	-3.794
#SOL DAYS > 240	-5.4795	0.553	-9.82	0.000	-6.573	-4.386
MARRIED	2.5175	0.413	6.09	0.000	1.707	3.328
DIVORCED	0.6050	0.456	1.33	0.185	-0.289	1.499
WIDOW	2.5465	0.526	4.84	0.000	1.514	3.578
D63=1: T_R > 63	1.3512	0.313	4.32	0.000	0.738	1.964
INCOMES	-0.0312	0.009	-3.25	0.001	-0.050	-0.012
PENSIONS	0.1441	0.019	7.47	0.000	0.106	0.182
RETIREMENT AGE (T_R)	0.4786	0.150	3.19	0.001	0.185	0.773

Note: dy/dx for factor levels is the discrete change from the base level.

3.5. Prediction with model results

The following (counter factual or ceteris paribus) figures give summaries of our main results. Figure 4 shows the shapes of death hazard rate function at retirement ages of 61, 63, 65 and 68, years. The function estimates are based on the *LN*-model estimation with Gamma frailty distribution (Model A4) for death ages between ages of 59 – 73.7 years including the censored observations (i.e. the alive persons at the end of sample follow-up). For a given age of death (say 67 years) the higher retirement age gives a lower dead hazard rate function, and for the given retirement age the hazard rate function is increasing in the age of death showing the positive duration dependence (*PDD*). Note that our theory argument was that the retirement age optimization happens with a given *PDD* hazard function, not that the retirement age shifts the function. The slowdown in the increase rate of hazard functions at the high values of death age corresponds to frailty correction, high number of censored observations (still alive persons at the end of the sample follow-up), and on poor model fit at the high death ages (see Appendix 6).

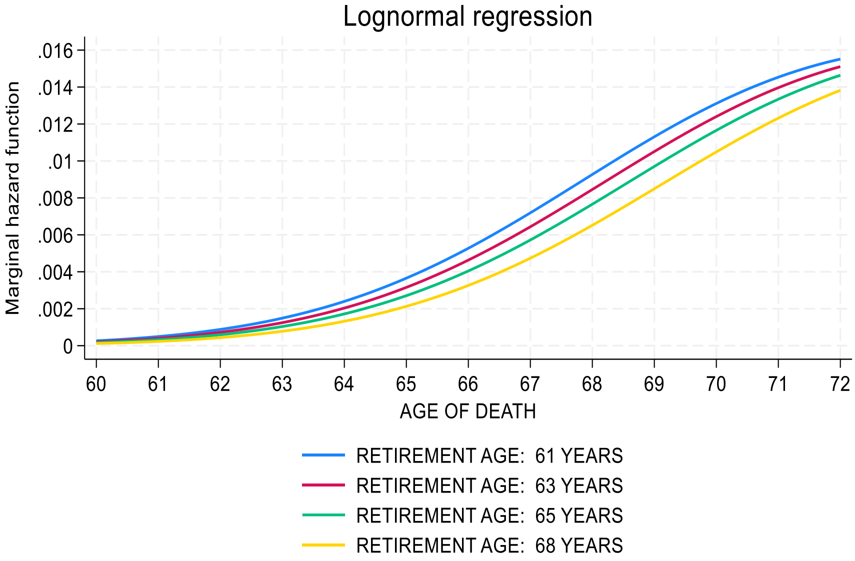


Figure 4. Death hazard functions with retirement age at 61, 63, 65, and 68 years

Figure 5. gives the hazard rate functions at the different level of pensions before and after retirement age of 63 years. The pension level effects on the shape and the level of death hazard rate function are noticeable especially during the AAR schema period (i.e. post-63 retirement age). Large AAR pension protects (the yellow line in the right figure) from high death hazard rates compared to low pension without AAR (the blue line in the left figure).

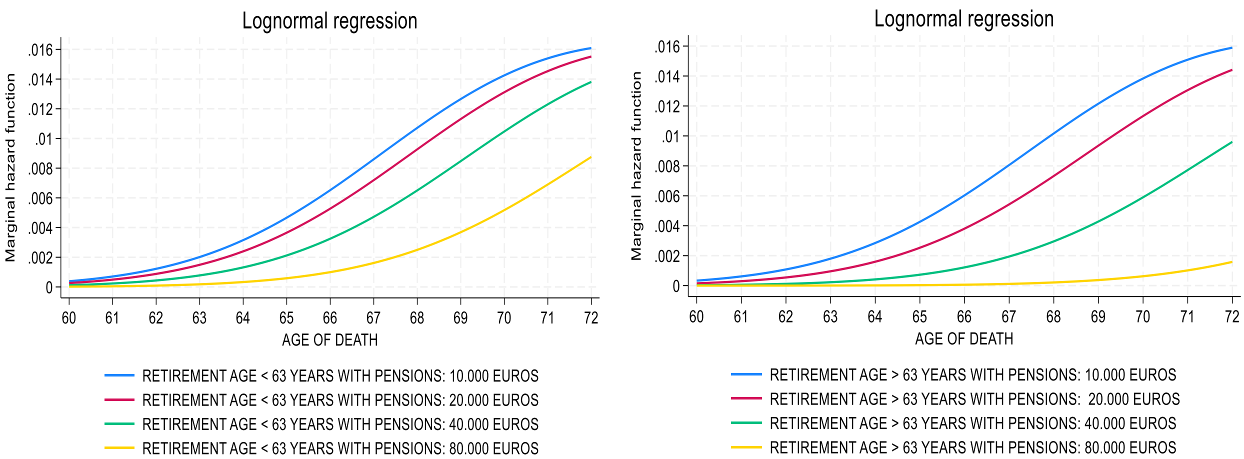


Figure 5. Death hazard functions with pensions before (left) and after (right) the retirement age of 63 years.

Finally, we produce a graph that sum-ups our main results in a very contrasting way. We build on the estimation results four (potential) different hazard rate functions that correspond to the largest and smallest death risk at the retirement age of 63 years (see Figure 6). We notice that if the retiree had severe health problems before the retirement (#SOL above 240 days), his/her yearly incomes and pension were low (20.000 and 10.000 euros), the hazard rate function is with all death ages at much higher level compared with a *female* with non-health problems (#SOL = 0) with high incomes and pensions (120.000 and 60.000 euros). A male with equal characteristics has the same level of death risk when the death happens at early age but as the age of death increases the hazard rates rise to the level that is obtained by the less fortune retirees with health problems. The figure shows that it is the poor health that shapes and determines the level of death hazard rate function, but the welfare inputs (incomes and pension) have also a major role in this context.

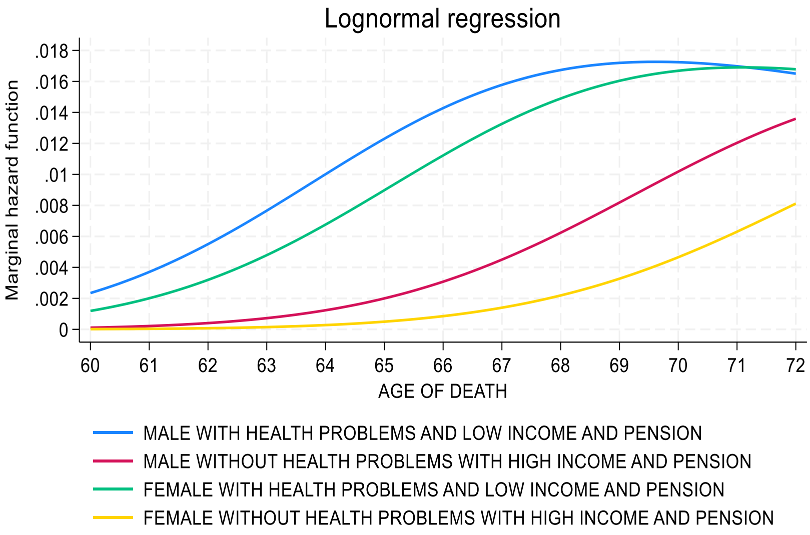


Figure 6. Death hazard rates with different combinations of covariate values at the retirement age of 63 years

In summary our empirical findings do not conflict the theory model implications. Note, that our main target in theory modelling was to show that the (optimal) retirement age was sensitive to welfare inputs and health costs that also adjusted the death hazard rate function. We found above that the higher retirement age means higher median age of death, and the lower dead hazard rate function was identified with the higher retirement ages. Thus, a lower death risk gives room for the postponed retirement.

4. Conclusions

The problem of person level “optimal” retirement age or time was analysed with models where the special attention was on the (subjective) survival probability that the retiree can’t ignore when deciding to retire. Sensible optimization results were derived supporting death hazard rate function with the non-negative duration dependence. The main theory model result was that the person wants to have a compensation for his/her evident non-zero death in form of early retirement. This can be avoided if retiree’s welfare inputs are large, and the death hazard rate is low at the retirement age. In the augmented model the retiree can use welfare inputs also to reduce his/her mortality risk leading to a later retirement age. If access to inputs is limited, the state should subside the input costs or provide them to delay retirement age with reduced death hazard rate. From policy perspective this means that public health care and pension policy reducing elderly poverty lower the death hazard rate. This allows for the higher optimal retirement ages with healthy retirement years.

Some empirical findings with the birth year 1947 cohort in Finland did not conflict the model results. Our survival model estimations showed that gender, health, civil status, incomes, and pension affected death risk in different ways but still the retirement age had the longevity increasing effect. In this context especially the AAR pension schema starting at the retirement age of 63 years had the death risk decreasing effect. This can be – once again – evidence of the fact that in the average wealthier and healthier people live longer. However, we showed also that these people postpone their retirement in response to their high income and health levels. Our empirical strategy was to estimate the death hazard rate function and its dependence on welfare inputs, health costs, and retirement age, not on the dependency of (optimal) retirement age on e.g. income, pensions, and health. This will be studied in the future research in details, especially putting focus on the AAR pension schema.

Appendix 1. 2nd order maximum conditions for model $\max_{T_R}\{[v\cdot T_R - C(T_R)]\cdot[1-F(T_R)]\}$

Derivate of the 1st order optimal result

$$\frac{dV}{dT_R}|_{T_R=T_R^*} = (v - C'(T_R)) - [vT_R - C(T_R)]h(T_R) = 0$$

is

$$\frac{d^2V}{dT_R{}^2} = -C''(T_R) - (v - C'(T_R))h(T_R) - (vT_R - C(T_R))h'(T_R).$$

This is negative at $T_R = T_R^*$ because we have $h' > 0$ (*PDD*), $C'' > 0$, $(v - C') > 0$, and $(vT_R - C) > 0$ supporting the maximum result.

Appendix 2. Derivation of dT_R^* / dv **effect.**

Total differentiation of the 1st order maximum condition $v - C' - vT_R^* h + Ch = 0$ gives

$$dv - C''dT_R^* - T_R^* h dv - vhdT_R^* - vT_R^* h' dT_R^* + C' hdT_R^* + Ch' dT_R^* = 0 \quad \Leftrightarrow$$

$$[1 - T_R^* h]dv = [C'' + vh + vT_R^* h' - C'h - Ch']dT_R^*$$

$$\Rightarrow \quad \frac{dT_R^*}{dv} = \frac{1 - T_R^* h}{C'' + h[v - C'] + h'[vT_R^* - C]}.$$

At T_R^*, when *NDD* ($h' < 0$) and, if $h < 1/T_R^*$ and $C'' + h[v - C'] > |h'|[vT_R^* - C] > 0$, we have $\frac{dT_R^*}{dv} > 0.$

At T_R^*, when *non-DD* ($h' = 0$) and, if $h < 1/T_R^*$, we have $\frac{dT_R^*}{dv} = \frac{1 - T_R^* h}{C'' + h[v - C']} > 0.$

At T_R^*, when *PDD* ($h' > 0$) and, if $h < 1/T_R^*$, we have $\frac{dT_R^*}{dv} = \frac{1 - T_R^* h}{C'' + h[v - C'] + h'[vT_R^* - C]} > 0.$

Appendix 3. 2nd order maximum conditions for the extended model

$$\frac{\partial V(T_R, z)}{\partial T_R}|_{T_R=T_R^*} = [v(z) - C_{T_R}(T_R, z)][1 - F(T_R, z)] - [v(z)\cdot T_R - C(T_R, z)]\partial F(T_R, z)/\partial T_R = 0.$$

A) $$\frac{\partial^2 V(T_R, z)}{\partial T_R^2}|_{T_R=T_R^*} = -\underset{(+)}{C_{T_R T_R}}[1 - F] - 2[v \underset{(+)}{-} C_{T_R}]\partial F/\partial T_R - [vT_R \underset{(+)}{-} C]\partial^2 F/\partial T_R^2 < 0.$$

B) $$\frac{\partial^2 V(T_R, z)}{\partial T \partial z}|_{T_R=T_R^*, z=z^*} = [v' - C_{T_R z}][1 - F] - [v \underset{(+)}{-} C_{T_R}]\partial F/\partial z - [v'T_R - C_z]\partial F/\partial T_R - [vT_R \underset{(+)}{-} C]\partial^2 F/\partial T_R \partial z.$$

$$\frac{\partial V(T_R,z)}{\partial z_R}\Big|_{z=z^*} \;=\; [v'(z)T_R - C_z(T_R,z)][1-F(T_R,z)] - [v(z)\cdot T_R - C(T_R,z)]\partial F(T_R,z)/\partial z = 0\,.$$

$$C)\quad \frac{\partial^2 V(T_R,z)}{\partial z^2}\Big|_{z=z^*} \;=\; [\underset{(-)}{v''T_R} - \underset{(+)}{C_{zz}}][1-F] - 2[\underset{(-)}{v'T_R - C_z}]\partial F/\partial z - [\underset{(+)}{vT_R - C}]\partial^2 F/\partial z^2 < 0.$$

$$D)\quad \frac{\partial^2 V(T_R,z)}{\partial z\partial T_R}\Big|_{T_R=T_R^*,z=z^*} = [v' - C_{zT_R}][1-F] - [v'T_R - C_z]\partial F/\partial T_R - [\underset{(+)}{v - C_{T_R}}]\partial F/\partial z - [\underset{(+)}{vT_R - C}]\partial^2 F/\partial z\partial T_R.$$

A) and *C*) are negative when $\partial F/\partial T_R > 0$, $\partial F/\partial z < 0$, and second order terms are close to zero. This implies that the Hessian is negative semidefinite supporting the local maximum at (T_R^*, z^*).

Appendix 4. Data sources and variable summary statistics

Data sources

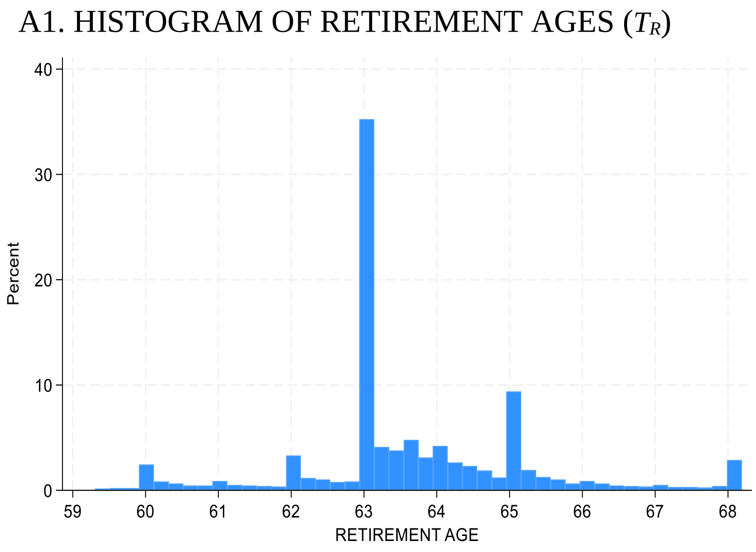
Person level register follow-up data. Starting 1.1.2007 and ending 31.12.2019.
Statistics of Finland: birthday in year 1947, gender, civil status, date of death.
ETK (Finnish Centre for Pensions): date of retirement, pensions, earnings.
KELA (The Social Insurance Institution of Finland): number of days of sick on-leave (#SOL).

Summary statistics of data variables

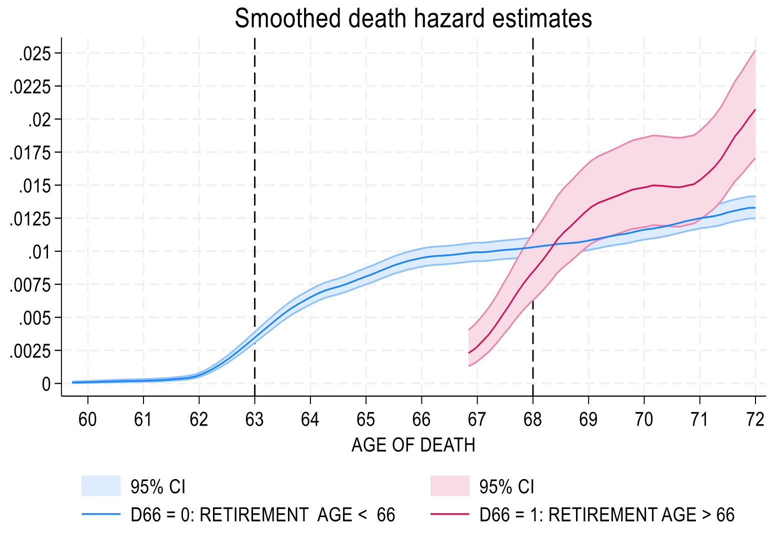
(*AGE* and T_R in years, *INCOME* and *PENSION* in 1000€, number of sick on-leave days (*#SOL*) in 100days)

DEAD	Stats	AGE	T_R	INCOME	PENSION	#SOL
NO	Mean	73.28	63.36	29.64	20.24	0.21
	Median	73.28	63.08	26.15	17.75	0.00
	CV	0.00	0.03	0.76	0.59	2.95
	Min	72.77	59.00	0.00	0.18	0.00
	Max	73.77	72.38	568.22	310.66	9.52
	N	36586	36586	32838	36583	36586
YES	Mean	68.71	62.98	26.63	17.38	0.46
	Median	69.00	63.00	23.99	15.23	0.00
	CV	0.04	0.03	0.91	0.57	1.98
	Min	59.72	59.00	0.01	1.13	0.00
	Max	73.62	71.61	830.75	180.13	8.18
	N	4076	4076	3146	3968	4076

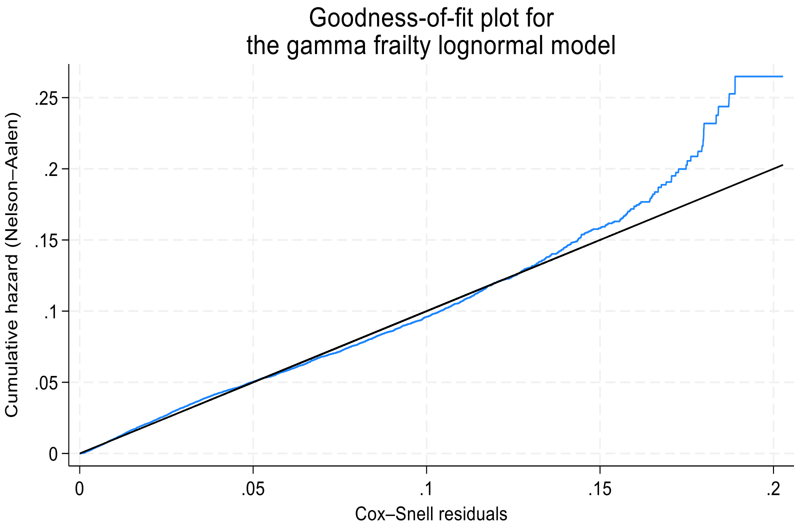
DEAD	Stats	D63	GENDER	CIVIL STATUS	D_{SICK}
NO	Mean	0.46	1.53	2.34	1.14
	Min	0.00	1.00	1.00	1.00
	Max	1.00	2.00	4.00	3.00
	N	36586	36586	36578	36586
YES	Mean	0.34	1.35	2.30	1.30
	Min	0.00	1.00	1.00	1.00
	Max	1.00	2.00	4.00	3.00
	N	4076	4076	4075	4076



Appendix 5. Death hazard rates with retirement ages below and above of age of 66 years



Appendix 6. Cox-Snell residuals with cumulative hazard



Appendix 7. Average marginal effects on the mean age of death

	Delta-method dy/dx	std. err.	z	Prob>|z|	[95% conf.	interval]
FEMALE	3.6574	0.218	16.71	0.000	3.2286	4.086
#SOL DAYS 60-240	-3.9942	0.310	-12.86	0.000	-4.602	-3.385
#SOL DAYS > 240	-5.3403	0.423	-12.62	0.000	-6.169	-4.511
MARRIED	2.4471	0.321	7.62	0.000	1.818	3.076
DIVORCED	0.5679	0.360	1.58	0.115	-0.138	1.274
WIDOW	1.8647	0.419	4.44	0.000	1.041	2.687
D63=1: T_R > 63	1.3860	0.267	5.18	0.000	0.861	1.910
INCOMES	-0.0201	0.009	-2.16	0.031	-0.038	-0.001
PENSION	0.1213	0.018	6.63	0.000	0.085	0.156
RETIREMENT AGE (T_R)	0.2622	0.081	3.32	0.001	-0.134	0.186

Note: dy/dx for factor levels is the discrete change from the base level.

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