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**Abstract**

In this paper, we compare the scenarios of exclusive licenses and cross-licenses under the existence of partial vertical integration. To do this, a successive duopoly model is proposed, with two owners and two firms competing in a differentiated product market. Each technology owner has a share in one of the competing firms, so that competition is also extended to the upstream R&D sector. We propose a novel analysis where differences in the size of their innovation process are allowed, extending the results in Sánchez et al. (2021). We find that the cross-licensing scenario is preferred when the size of the innovation is small; this occurs regardless of the participation in the competing companies and how many innovate. If the innovation is very large, the owners may be better off with exclusive licenses.

*Keywords:* Patent Licensing, Exclusive licenses, Market for technology, Asymmetric innovation

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## 1. Introduction

A recent report highlights that the global sales revenue generated by licensed merchandise and services grew to $292.8 billion in 2019, a 4.5 percent increase over the $280.3 billion generated in 2018, a fact that underlines the importance of a growing business.[1] Technology licenses play a crucial role, from an economical and entrepreneurial point of view: i) for licensee firms, they facilitate collective innovation and ii) for licensor firms, they can provide a low-risk way to leverage intellectual property assets, providing them with a framework that complements and enhance their business goals. For example, Apple Inc. complements its technical know-how by acquiring core technology from firms like Qualcomm Inc. and Samsung to create its attractive high-performance devices (**?**).

According to Mendi et al. (2011), there is a coexistence of different patterns in the transference of technology in the market. In fact, most technology transfers occur between firms of the same multinational, parent-firm, and subsidiaries (affiliated firms). Table 1 displays information from the Bureau of Economic Analysis in the US regarding technology transfers, which are classified according to the type of transaction. Specifically, data distinguish transactions between affiliated firms and between non-affiliated firms. The characteristics of technology transfers between affiliated firms, as well as their effects on the level of competition in the industry, are worth exploring due to the significance of such transactions. Table 1 provides evidence of this. It is evident that the majority of revenue and expenditures are derived from affiliated firms, which are vertically linked.

Our aim in this paper is to study the strategic decision of patent holders or technology owners about how many competing firms to license in the context of partial vertical integration. In concrete terms, we consider that technology owners are stakeholders of their clients, so competition is extended to the upstream R&D sector. We propose a successive duopoly model, with two technology owners and two firms competing in a differentiated product market. Furthermore, we assume that innovations are product-specific and independent, and under a cross-licensing scenario, each duopolist becomes a multi-product firm. Our innovation lies in the fact that our analysis encompasses a combination of what is usually studied in the literature, which is mainly concentrated on either fully integrated or completely separate markets. We propose an extension studied previously in Sánchez et al. (2021), but instead we allow for asymmetries in costs through innovations of different sizes.

[1] Retrieved October 22, 2020, from Global Licensing survey, available on https://licensinginternational.org/get-survey/

Table 1: Technology transfers between affiliated and non-affiliated firms in the USA

| | | 1999 | 2009 | 2019 |
|---|---|---|---|---|
| Incomes | Total millions of Dollars | 39,913 | 85,730 | 117,401 |
| | Affiliated % | 71.53 | 66.80 | 67.75 |
| | Unaffliated % | 28.47 | 33.20 | 32.25 |
| Payments | Total millions of Dollars | 12,845 | 29,421 | 42,733 |
| | Affiliated % | 80.21 | 73.48 | 73.00 |
| | Unaffliated % | 19.79 | 26.52 | 27.00 |
| Income-Payment Ratio | Total | 3.11 | 2.92 | 2.75 |
| | Affiliated | 2.77 | 2.65 | 2.55 |
| | Unaffliated | 4.47 | 3.64 | 3.29 |

Source: Bureau of Economic Analysis.

We contribute to two strands of literature: one studies partial vertical integration, and the other focuses on technology licenses. Technology licenses is a topic that has been broadly studied. The literature has focused on two main aspects: the study of the strategic decision of patent holders to whom to assign a license (Badia et al. 2020), and the study of optimal contracts in technology licenses between the licensor and the licensee (Katz and Shapiro 1986). Previous studies about contracts highlight that the optimal mechanism -fixed fees, royalties, or auctions - to the transference of technology may depend whether the owner of technology is an outsider innovator (e.g. Katz and Shapiro 1986; Kamien and Tauman 1986, Kamien 1992; Stamatopoulos and Tauman 2009; Miao 2013) or, on the contrary, the patent holder is a producer in the market (e.g., Wang 1998; Kamien and Tauman 2002; Sen and Tauman 2007). This literature clearly shows that if the owner of the innovation does not compete, the revenue of a fixed fee exceeds that of the royalties. However, incomes from royalties exceed those coming from the fixed quota in cases where the patent holder is a producer in the final market. This is because royalties provide both license revenues and competitive advantages in production. Other interesting studies focus on the role that the expected duration of the relationship between technology owners and firms may play in the election of technology transfer contracts (Mendi 2005; Cebrián 2009). For example, Mendi (2005) finds that a contract where the time horizon is short is more likely to include fixed payments. Under these facts, we consider technology licensing through a fixed payment.

On the other hand, our aim is to study a different approach not considered before in the literature of technology licensing, that is, the incentives of partially vertically integrated firms to license their rivals, given the level of vertical integration. Most theoretical and empirical studies of vertically related markets have focused on two extreme alternatives: full vertical integration and separation. However, in practice, it is quite common to find integrated partial vertical firms, that is, partial ownership agreements in which a firm acquires less than 100% shares in a vertically related firm (Gilo and Spiegel 2011; Hunold and Shekhar 2018). Theoretical studies that focus on partial vertical integration have analyzed different perspectives. For example, Fiocco (2016) investigates the strategic incentives for partial vertical integration with two manufacturer–retailer hierarchies or the case where there is backward ownership, i.e., ownership stakes hold of upstream firms by downstream firms (Greenlee and Raskovich 2006). Other interesting works explore the (anti)competitive effects of partial vertical ownership (Levy et al. 2018; Spiegel et al. 2013, Schmalz 2018). Thus, previous studies usually analyze the incentives to partial vertical integration or the limitation of this phenomenon. However, in our study, the starting point is that the upstream firms (patent holders) are already partially vertically integrated with one competing firm in the downstream market. Then the strategic decision revolves around how many firms license their technology. Therefore, we investigate a new perspective that has not been previously explored in the field of technology licensing, which involves partial vertical integration of patent owners.

To analyze this approach, we propose a model with two technology owners that have to decide to sell one or two licenses, that is, exclusive or non-exclusive licensing. Mendi et al. (2011) evaluate a patent holder on the market and two firms on the downstream market that differ in their level of production costs, where one firm is more efficient than the other. They compare two scenarios to determine whether the affiliate firm is the most efficient firm or not. Moreover, they analyze the implications it has on the market. We extend the preliminary results to Mendi et al. (2011) since we consider two technology owners and each innovator has a share in one of the firms that compete in the market. Furthermore, we assume a differentiated duopoly (see, e.g., Muto 1993; Caballero-Sanz et al. 2002, Mukhopadhyay et al. 1999), where there are cost asymmetries through innovations. Due to the fact that innovators participate in firms' capital shares, cross-licensing generates a trade-off between raising licensing revenues and increasing competition.

Additionally, the technology licensing is based on a fixed-fee mechanism because although this type of contract does not control reaction curves of competing firms, it allows the patent holder to have more room with its decision on the number of licenses granted, that is our main objective in this work. Furthermore, we do not consider any specific duration of the relationship, so we understand that the relationship is one shot, and following the findings of Mendi (2005), fees are more likely to occur and may fit to center our attention in the role of ownership, and therefore, the existence of technology transference with affiliated firms. We extend our analysis to a differentiated duopoly with cost asymmetry through innovations of different sizes. Previous studies highlight situations where technology transfer may not occur at all, depending on the initial cost asymmetry between the firms (see, e.g., Mukhopadhyay et al. 1999; Marjit 1990). Cost asymmetry between competing firms can produce different scenarios in the diffusion of technology, such as transferring the new technology to the efficient firm only, that is, a firm with lower marginal costs (Sinha 2016), transferring the new technology to the inefficient one (see, e.g., in a spatial context, Poddar et al. 2021), or the situation in which cost asymmetry does not matter in the transfer of the new technology (Banerjee and Poddar 2019). These results emphasize that, for the decision-making of a number of licensed firms, the initial marginal costs of production are just as important as the impact of innovation on costs. Therefore, the existence of asymmetric innovations might have further implications. For example, asymmetric innovations may require special attention from policymakers and regulators due to their potential far-reaching effects, such as significant competitive advantage or the shape of an industry's future.[2] We analyze all the possibilities and contribute to the analysis of decision-making where patent holders hold interests in the final market due to partial ownership in the downstream firms.

The results allow us to compare what is the best strategy and the equilibria regarding the number of licenses; exclusive license (one) or cross-licenses (several). We find the main determinants in decision-making that may differ between patent holders depending on the cost of production that firms face in the downstream market. Furthermore, we explore the implications of the asymmetry in the innovation process between patent holders, which could have implications in the diffusion of innovation in the downstream market. As will be seen, the cross-licensing scenario is best when the size of the innovation is small; this occurs, regardless of the participation in the competing firms and how many innovate (Result 4). Technology owners may be better off in a scenario with exclusive licenses. This is so when the size of the innovation is large, both owners have the same innovation, and the initial cost of production is large enough (Result 1). If a patent holder has a share in one of the competing firms and the innovation size between patent holders is the same, she prefers an exclusive license if the cost of production is large enough and, additionally, she holds a minimum share in the firm (Result

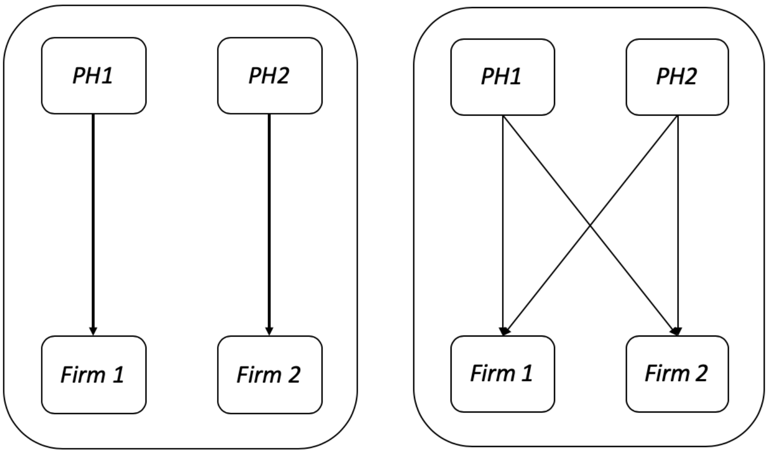
[2]Kaal and Vermeulen (2017) offer a detailed discussion over the effects and implications of disruptive technology from a regulatory point of view.

2). Asymmetry in the innovation process requires further conditions in the differentiation of products (Result 3). The fact that only one of the owners has a stake in a competing firm may lead each innovator to prefer a different scenario (Results 2 and 3).

The rest of the paper is structured as follows. Section 2 introduces the model and examines the equilibrium conditions for both exclusive licenses and cross-licenses. Section 3 compares the two scenarios and describes the main findings. Finally, Section 4 provides the conclusions.

## 2. The model

The modeling adopted is that of a successive duopoly, with two technology owners and two firms competing in a differentiated product market. Each technology owner has a stake in one of the competing firms. As indicated above, our purpose is to compare two scenarios: one in which each owner transfers the technology exclusively to her participating firm, figure 1 (a), and another in which the technologies are transferred to the two competing firms, figure 1 (b). We will refer to the first scenario as that of exclusive licenses and the second as that of cross-licenses.




(a) (b)

Figure 1: Exclusive licensing vs. Cross-licensing

### *2.1. Exclusive licenses*

Consider a duopoly in which each firm initially produces a variety of a differentiated product. The system of inverse demands -which is obtained from the problem of maximizing the utility of a representative consumer subject to the budget constraint- is as follows:

$$p_1 = 1 - q_{11} - dq_{22} \quad (1)$$

$$p_2 = 1 - q_{22} - dq_{11} \quad (2)$$

The first subscript refers to the variety, and the second refers to the firm. The parameter $d$ measures the degree of product differentiation $d \in (0, 1)$, where the varieties are more homogeneous the closer to 1 is $d$. The existing technology allows these varieties to be produced at a constant marginal cost equal to $c$, with $c < 1$. There are also two technology owners, each of whom has a stake, $\alpha_i$, where $\alpha_i \in (0, 1)$, of one of the firms that compete in the market, $i = 1, 2$. The technology owners each have an innovation process. Specifically, owner one, which we denote by $PH_1$, has a process innovation that reduces the marginal cost of production by magnitude $\varepsilon$. This means that if $PH_1$ transfer the innovation to her investee firm, she will be able to produce variety one at a marginal cost $c - \varepsilon$. Similarly, $PH_2$ denotes owner two who has a process innovation of size $\varepsilon - \delta$ thus, the marginal cost of production of variety two, if firm two acquires the innovation, becomes $c - \varepsilon + \delta$. Therefore, competition in the technology market occurs in an asymmetric context, collecting $\delta$ this asymmetry. If $\delta$ is zero, the cost reduction is the same in both cases, since they both have the same innovation in size $\varepsilon$. On the other hand, the closer $\delta$ to $\varepsilon$, the greater the asymmetry between owners; the extreme case of $\delta = \varepsilon$ means that only $PH_1$ has the process innovation; therefore, $\delta \in (0, \varepsilon)$. Table 1 summarizes the sizes of innovation. The transfer of technology is made through a fixed payment $F$.

Table 2: Sizes of innovation of both Patent Holders.

| Innovation sizes | |
|---|---|
| $\delta = 0$ | Same innovation sizes of both PH |
| $\delta \longrightarrow \varepsilon$ | Greater asymmetry between innovations.<br>Only one PH has the technology |
| $\varepsilon \longrightarrow 0$ | Innovation is small |
| $\varepsilon \longrightarrow c$ | Innovation is very large |

Formally, we solve a game in several stages. In the first stage, the $PH1$ and $PH2$ owners simultaneously and non-cooperatively choose the fixed payment for the assignment or license of the innovation. In the second stage, each firm decides whether or not to accept the fixed payment contract offered by its respective patent holder. Finally, given all the above, firms compete in quantities.

Solving backward induction, the profit maximization problem when duopolists have both the respective process innovations are

$$\max_{q_{11}} \pi_1 = (p_1 - c + \varepsilon) q_{11} - F_1 \tag{3}$$

$$\max_{q_{22}} \pi_2 = (p_2 - c + \varepsilon - \delta) q_{22} - F_2 \tag{4}$$

The solution of the system formed by the first order conditions, $\partial\pi_1/\partial q_{11} = 0$ and $\partial\pi_2/\partial q_{22} = 0$, is the following:

$$q_{11}^E = \frac{(2-d)(1-c+\varepsilon)+d\delta}{4-d^2} \tag{5}$$
$$q_{22}^E = \frac{(2-d)(1-c+\varepsilon)-2\delta}{4-d^2} \tag{6}$$

where the super-index $E$ represents the scenario with exclusive licenses. Given the established assumptions, the second order conditions hold.[3] Noting the numerator of $q_{22}^E$, can be deduced that if $\delta$ is large enough, firm two will not produce. Indeed, a lot of asymmetry would be equivalent to the definition of large innovation in the present context. The condition is as follows: $\delta > \frac{(2-d)(1-c+\varepsilon)}{2}$.

Substituting $(5)-(6)$ in the expressions of profits, we get $\pi_1^E = (q_{11}^E)^2$ y $\pi_2^E = (q_{22}^E)^2$. The $PH1$ will design a license contract so that the firm accepts it. To determine the fixed payment, we calculate the opportunity cost of the license, that is, the difference between having it and not having it. The firm is willing to pay an amount $F$ such that $F \leq \pi_1(c-\varepsilon, c-\varepsilon+\delta) - \pi_1(c, c-\varepsilon+\delta) \equiv F_1$. The first term on the right of the inequality refers to the profits $\pi_1^E$ we obtained above. To complete the fee payment, $F_1$, we solve an asymmetric duopoly where the firm 1 produces with the initial marginal cost, $c$, while the rival does it with the corresponding innovation, $c-\varepsilon+\delta$. Solving we get the following profits:

$$\pi_1(c, c-\varepsilon+\delta) = \frac{((2-d)(1-c)-d(\varepsilon-\delta))^2}{(4-d^2)^2} \tag{7}$$

Thus, the fee is

$$F_1^E = \frac{4\varepsilon[(2-d)(1-c)+\varepsilon-d(\varepsilon-\delta)]}{(4-d^2)^2}. \tag{8}$$

Furthermore, we solve an asymmetric duopoly where firm 2 produces with the initial marginal cost, $c$, wile the rival does it with the corresponding innovation $c-\varepsilon$. Thus, we get firstly the following profits:

$$\pi_2(c-\varepsilon, c) = \frac{4(\varepsilon+\delta)\phi - 2(1-c)d(2-d)\varepsilon + (1-c)^2(2-d)^2 + d^2\varepsilon^2}{(4-d^2)^2} \tag{9}$$

where $\phi = (c(2-d)-(1-d)\varepsilon+d+\delta-2)$. Therefore, the fee is:

$$F_2^E = \frac{4(\delta-\varepsilon)\left[c(2-d)-(1-d)\varepsilon+d+\delta-2\right]}{(4-d^2)^2}. \tag{10}$$

[3] $\frac{\partial^2\pi_1^E}{\partial q_{11}} = \frac{\partial^2\pi_2^E}{\partial q_{22}} = -2 < 0$.

**Proposition 1.** *In the scenario with exclusive licenses, the fixed payment is higher for the $PH1$, $F_1^E > F_2^E$.*

**Proof 1.** *See Appendix.*

It is an intuitive result, as the reduction in marginal cost is greater with the technology of $PH1$. In market equilibrium, this allows the firm to obtain a greater market share, so the opportunity cost of not having the innovation is greater. Then $PH1$ is able to charge a greater fix payment to its downstream market.

Thus, due to the fact that $PH1$ has a stake in firm one, her profits are the following:

$$\begin{aligned} \Pi_{PH1}^E &= F_1^E + \alpha_1(\pi_1^E - F_1^E) = \\ &= \frac{4\varepsilon[(2-d)(1-c)+\varepsilon - d(\varepsilon-\delta)] + \alpha_1[(2-d)(1-c) - d(\varepsilon-\delta)]^2}{(4-d^2)^2} \end{aligned} \tag{11}$$

In a similar way, we get the profits for $PH2$:

$$\begin{aligned} \Pi_{PH2}^E &= F_2^E + \alpha_2(\pi_2^E - F_2^E) = \\ &= \frac{4(\varepsilon-\delta)[(2-d)(1-c)+(1-d)\varepsilon - \delta] + \alpha_2[(2-d)(1-c) - d\varepsilon]^2}{(4-d^2)^2} \end{aligned} \tag{12}$$

### *2.2. Cross-licenses*

In this scenario, both patent holders sell their licenses to every firm in the downstream market. This implies that each duopolist becomes a multiproduct firm, that is,they produce variety one at marginal cost $c-\varepsilon$ and variety two at marginal cost $c-\varepsilon+\delta$. Therefore, the reverse demand system is now defined as follows:

$$p_1 = 1-(q_{11}+q_{12}) - d(q_{21}+q_{22}) \tag{13}$$

$$p_2 = 1-(q_{21}+q_{22}) - d(q_{11}+q_{12}) \tag{14}$$

Then, the profit-maximization problem when duopolists have both innovations are given by:

$$\max_{q_{11},q_{21}} \pi_1 = (p_1 - c+\varepsilon)q_{11} + (p_2 - c + \varepsilon - \delta)q_{21} - F_1 - F_2 \tag{15}$$

$$\max_{q_{12},q_{22}} \pi_2 = (p_1 - c+\varepsilon)q_{12} + (p_2 - c + \varepsilon - \delta)q_{22} - F_1 - F_2 \tag{16}$$

The solution of the system formed by the four first-order conditions, where the second-order conditions for maximum are verified[4], yields the following equilibrium quantities for each variety:

$$q_{11}^{NE} = q_{12}^{NE} = \frac{(1-d)(1-c+\varepsilon)+d\delta}{3(1-d^2)} \tag{17}$$

$$q_{21}^{NE} = q_{22}^{NE} = \frac{(1-d)(1-c+\varepsilon)-\delta}{3(1-d^2)} \tag{18}$$

The superscript $NE$ refers to the equilibrium outcomes with cross-licensing or non-exclusive licenses. As in the scenario with exclusive licenses, we write the condition $\delta > (1-d)(1-c+\varepsilon)$ that, if met, would indicate that the process innovation of $PH1$ is large, since the quantity variety two would be negative. The definition of large innovation will be taken into account in the analysis of the cases presented below. The condition of the scenario with exclusive licenses is more demanding, $\delta > \frac{(2-d)(1-c+\varepsilon)}{2} > (1-d)(1-c+\varepsilon)$.

The next step is to calculate the fixed payment that firms must pay each patent holder. Let us see how we obtain the fixed payment that the firm will pay to $PH1$ for the cession of the process innovation. As we have pointed out above, $PH1$ designs the contract for the firm to accept it, that is, the fixed payment cannot exceed the opportunity cost of acquiring the technology. Thus, firm one is willing to pay an amount $F$ such that $F \leq \pi_1(c-\varepsilon, c-\varepsilon+\delta; c-\varepsilon, c-\varepsilon+\delta) - \pi_1(c, c-\varepsilon+\delta; c-\varepsilon, c-\varepsilon+\delta) \equiv F_1^{NE}$. The profits of the first term on the right of the inequality correspond to the profits $\pi_1^{NE}$. We need to solve an asymmetric duopoly with a single-product firms (firm one) and another that is a multi-product firm (firm two), that is,

$$\max_{q_{21}} \pi_1 = (p_2 - c + \varepsilon - \delta) q_{21} \tag{19}$$

$$\max_{q_{12}, q_{22}} \pi_2 = (p_1 - c + \varepsilon) q_{12} + (p_2 - c + \varepsilon - \delta) q_{22} \tag{20}$$

taking the inverse demands in (13)-(14) where $q_{11} = 0$. Once the equilibrium quantities has been calculated, it is replaced in profits. Making the difference between profits with and without innovation of size $\varepsilon$ we get the fixed fee:

$$F_1^{NE} = \frac{[(1-d)(1-c+\varepsilon)+d\delta]^2}{9(1-d^2)} \tag{21}$$

This same fixed fee is the one that the owner $PH1$ will charge to firm 2. We derive the fixed fee for $PH2$ in the same way, obtaining:

$$F_2^{NE} = \frac{[(1-d)(1-c+\varepsilon)-\delta]^2}{9(1-d^2)} \tag{22}$$

[4] $\frac{\partial^2 \pi_1^{NE}}{\partial q_{11}} = \frac{\partial^2 \pi_2^{NE}}{\partial q_{12}} = \frac{\partial^2 \pi_1^{NE}}{\partial q_{21}} = \frac{\partial^2 \pi_2^{NE}}{\partial q_{22}} = -2 < 0$

As in Proposition 1, $F_1^{NE} > F_2^{NE}$. This is because the impact of innovation when using $PH1$ technology is greater, then the reduction of costs or the overall downstream profits, and that is why $PH1$ can extract a higher fee.

Finally, profits of the technology owner one, $PH1$, are given by

$$\begin{aligned}\Pi_{PH1}^{NE} &= 2F_1^{NE} + \alpha_1(\pi_1^{NE} - F_1^{NE}) = \\ &= 2\frac{[(1-d)(1-c+\varepsilon)+d\delta]^2}{9(1-d^2)} + \alpha_1\frac{(1-c+\epsilon-\delta)^2}{9}.\end{aligned} \tag{23}$$

Proceeding in the same way, we derive the technology owner two's profits:

$$\begin{aligned}\Pi_{PH2}^{NE} &= 2F_2^{NE} + \alpha_2(\pi_2^{E} - F_2^{NE}) = \\ &= 2\frac{[(1-d)(1-c+\varepsilon)-\delta]^2}{9(1-d^2)} + \alpha_2\frac{(1-c+\epsilon)^2}{9}.\end{aligned} \tag{24}$$

## 3. Results

Once we have solved the two scenarios, we analyze the decision of the patent holders on whether to sell an exclusive license or two licenses. To answer this main question, we compare patent holders' profits depending on the degree of the innovation process, that is, if innovation is large or small. In concrete terms, we consider two scenarios related to process innovation: (i) when the innovation does not present a cost reduction, that is, $\varepsilon = 0$, and (ii) when there is a large innovation that makes production costs zero, and therefore $\epsilon = c$. In the next section, we present the main results and determinants of the strategic decision of the licensing programs for both patent holders.

### *3.1. Large innovation*

Large innovation is considered when the effect on reducing costs in downstream market firms is very strong. In this case, we consider the extreme case where the cost is reduced to the maximum, that is, $\varepsilon = c$, therefore, $c = 0$.

Furthermore, to present optimal conclusions, we distinguish three scenarios that show different levels of asymmetry in the game between the patent holders:

- *Case I: Symmetry.* In this case, the patent holders do not have a share in the downstream firms, that is, $\alpha_i = 0$. Thus, firms in the technology market are not present in the product market. On the other hand, the size of innovation of both patent holders is equal; thus, $\delta = 0$.

- *Case II: Stake asymmetry* We assume that $PH_1$ has a share in one of the competing firms in the market, $\alpha_1 \epsilon (0, 1)$, but $PH_2$ has not, $\alpha_2 = 0$. Therefore, $PH_1$ is present on the product market. We hold the symmetry in the size of innovation for both patent holders.

- *Case III: Full asymmetry* We add asymmetry in the size of innovation to the scenario in Case II. Thus, innovation $\delta = \varepsilon$ and $PH2$ does not reduce costs.

*3.1.1. Case I*

This case is the simplest and shows a symmetric situation, where both patent holders do not have a stake in the competing firms, and the impact of their technology is the same, i.e., the innovation size of both patent holders is equal ($\delta \longrightarrow 0$).

In this subcase, $PH_1$ will prefer a scenario with cross-licensing to one of exclusive licensing if $2F_1^{NE} > F_1^E$. And since both patent holders have the same size of process innovation, $2F_1^{NE} - F_1^E = 2F_2^{NE} - F_2^E$. In this extreme case where the size of the innovation is very large such that the marginal cost is zero, ($\varepsilon \longrightarrow c$), we investigate the sign of the difference:

$$2F_1^{NE} - F_1^E\big|_{\varepsilon \longrightarrow c} = \frac{2\,(1-d)}{9\,(1+d)} - \frac{4c(2-c-d)}{(4-d^2)^2} \tag{25}$$

Let $c_I$ represents the threshold that makes $2F_1^{NE} - F_1^E = 0$ given by

$$c_I = \frac{2-d}{2} - \frac{1}{6}\sqrt{\frac{4+32d-11d^2-7d^3-2d^4+2d^5}{1+d}}. \tag{26}$$

We find the following result:

**Result 1.** *When patent holders i) do not have a share in the firms that compete in the market, $\alpha_i = 0$, ii) have the same process innovation, $\delta = 0$, and iii) the innovation is very large, $\varepsilon = c$, they prefer a cross-licensing scenario when $c < c_I$, while they prefer a scenario with exclusive licenses when $c_I < c < 1$.*

**Proof 2.** *See the Appendix.*

When the costs of the firms are high and the impact of innovation is greater, patent holders would prefer to sell only one license. On the other hand, in the case that the costs are low and the impact of the innovation in firms is smaller, the fees charged are much smaller and the benefit of cross-licensing greater. Thus, it suggests that in a symmetrical situation, the greater the impact of innovation, due to the cost structure of the companies in the downstream market, the more likely that the patent holders will sell exclusive licenses.

As can be seen in the value of $c_I$ above, this cost depends on the relationship between the products of the competing firms in the downstream market, i.e., the level of substitutability of the product that sets the level of competition in the market $d$.

**Proposition 2.** *In Case I: Symmetry, the value of $c_I$ decreases as the products are more substitutes ($d \longrightarrow 1$). Therefore, a higher competitive level favors patent holders' preference to sell exclusive licenses.*

**Proof 3.** *See the Appendix.*

Figure 2: Value of $c_I$ under changes in $d$

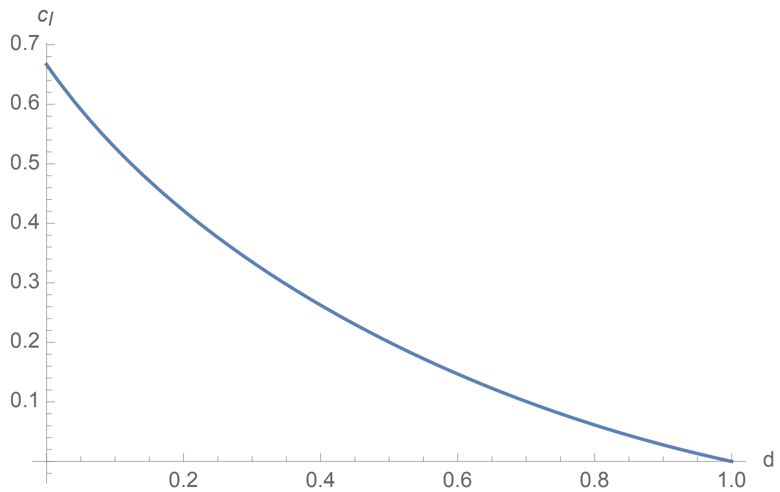



Figure 2 reflects the finding in Proposition 2, and it can be easily seen that as long as the level of competitiveness increases in the downstream market ($d \longrightarrow 1$), the required value of marginal costs $c_I$ under which patent holders decide to offer exclusive licenses decreases. The following table reflects this result numerically:

| | ***d=0*** | ***d=0.5*** | ***d=0.8*** | ***d=1*** |
|---|---|---|---|---|
| $c_I$ | 0.67 | 0.20 | 0.06 | 0 |

When products are independents, that is, $d = 0$, patent holders decide exclusive licenses as long as $0.67 < c < 1$, and cross-licenses otherwise. However, if $d = 0.8$, patent holders choose exclusive licenses if $0.06 < c < c_I$. Then, it can be seen that as long as the products become more homogeneous, it is easier to find a situation with exclusive licenses.

**Proposition 3.** *In Case I: Symmetry, when products are perfect substitutes, that is, $d = 1$, patent holders prefer to sell just one license, exclusive licenses.*

**Proof 4.** *See the Appendix.*

Proposition 3 shows an extreme case where products are perfect substitutes. In this case, marginal costs do not affect the decision-making of patent holders. Furthermore, as a result of the high level of competition in the product market, firms are willing to pay higher fees for greater innovations with the objective of not losing market share against its competitors. The optimal decision for patent holders is to offer exclusive licenses.

### *3.1.2. Case II*

This subcase analyzes an asymmetric situation where only one patent holder has a stake in one of the competing firms in the downstream market. Suppose that only $PH_1$ has a share in firm 1, that is, $\alpha_1 > 0$ and $\alpha_2 = 0$. Our aim is to study how this specific asymmetry affects the decision-making and licensing strategy of both $PH_1$ and $PH_2$. This is a mixed case with respect to what is normally analyzed in the literature, who has studied, on the one hand, the case of an innovator outside the industry and, on the other, the case of an innovative competitor in the industry.[5]

As before, for $PH_1$ the sign of (25) minus (11) is not defined. Following the procedure we have just followed when competing firms are not investees (Case I), the difference in profits for $PH1$, when patent holders have the same process innovation ($\delta \longrightarrow 0$) and in a case of large innovation ($\varepsilon \longrightarrow c$), is reduced to:

$$\begin{aligned}\Pi^{NE}_{PH1} - \Pi^{E}_{PH1} = & \frac{(2-d)^2(1-d)\left(\alpha_1(1+d)(5+d) - 2(2+d)^2\right)}{9(1+d)\left(4-d^2\right)^2} \\ & + \frac{36(1-\alpha_1)c^2(1+d) - 36(1-\alpha_1)c(2-d)(1+d)}{9(1+d)\left(4-d^2\right)^2}.\end{aligned}$$

Similarly to Case I, we find two thresholds or conditions that mark the decision of $PH_1$ in the marginal cost (the same threshold $c_I$ of Case I in equation 28) and in the share that $PH_1$ has in firm 1, $\alpha_1$. Let $\alpha_{1(II)}$ be

$$\alpha_{1(II)} = \frac{36c(1+d)(2-c-d) - 2(1-d)\left(4-d^2\right)^2}{(1+d)\left(2-6c-3d+d^2\right)\left(6c-(2-d)\left(5+d\right)\right)}.$$

[5] See, for example, Sandonís and Faulí-Oller (2006) or Sandonís and Faulí-Oller (2008) where they study the incentives of an external innovator to merge with an insider firm using other mechanisms for licensing: two-part tariff and auctions.

On the other hand, the difference between profits of $PH2$ when $(\delta \longrightarrow 0)$ and $(\varepsilon \longrightarrow c)$ reduces to:

$$\Pi_{PH2}^{NE} - \Pi_{PH2}^{E} = \frac{2\,(1-d)}{9\,(1+d)} - \frac{4c(2-c-d)}{(4-d^2)^2} \equiv 2F_1^{NE} - F_1^E|_{\varepsilon \longrightarrow c}.$$

For $PH_2$, given that there is no interdependence between $\alpha_1$ and $\alpha_2$, her decision is the same as that analyzed in Case I with no share in competing firms.

The combination of the above conditions leads to the following result.

**Result 2.** *When i) only the owner $PH1$ has a share in the firm that competes in the market, that is, $\alpha_1 > 0$ and $\alpha_2 = 0$, ii) both owners have the same process innovation, $\delta = 0$, and iii) the innovation is very large, $\varepsilon = c$, then if the cost c is high enough, $c > c_I$ and $\alpha_1 < \alpha_{1(II)}$, the $PH_1$ prefers a scenario with exclusive licenses. Otherwise, $PH_1$ will prefer cross-licensing.*

**Proof 5.** *See the Appendix.*

With stake asymmetry, the optimal licensing programs of both patent holders are not lined up and differ. Decision-making for $PH_2$ does not change with respect to the Case I analyzed above. Thus, the optimal strategy for patent holder two remains as in Case I. However, the decision $PH_1's$ has changed because under Case II it has a stake in a downstream firm. The entry of $PH_1$ as owner of firm 1 reveals an additional requirement in order to sell exclusive licenses to his participating firm, which involves specific values in the participation in the firm.

Generally, the cost structure of businesses is of great significance. For example, if the industry has a low-cost structure or is less intensive, such as the technology or IT sector, then a cross-licensing strategy is the preferred option. On the other hand, if the cost structure is large or higher, technology owners have incentives to sell exclusive licenses, since the impact that innovation can have on it is greater, resulting in higher fixed payments for patent holders. By introducing asymmetry in Case II, the possibility of participation in the product market by innovative firms hardens the requirement to carry out an exclusive license. This is so because patent holders are now able to derive profits through ownership in the firm in an additional way, and accepting to give the technology exclusively requires a minimum profit coming through product market share. The intuition for this is that the more advantage and greater competitiveness the investee firm has (a large innovation that is exclusively licensing to one firm), the greater the income, not only through fees but also through the share of profits.

*3.1.3. Case III*

This case introduces an additional asymmetry in the innovation process to the one studied in Case II. Suppose that $\delta = \varepsilon$. This assumption means that only $PH_1$ has innovation and therefore there is asymmetry in the innovation process between both patent holders. The fixed payment would be equal to the profits of having the innovation (monopoly with zero cost) less the profits of not having it (differentiated duopoly with cost $c$).

If $\delta = \varepsilon$, then for the technology owner one, $PH1$, we have that

$$\Pi_{PH1}^{E} - \Pi_{PH1}^{NE}|_{\delta \longrightarrow \varepsilon} = \frac{4c\,(2 - c\,(1-d) - d)}{(4-d^2)^2} - \frac{2(1-(1-c)d)^2}{9(1-d^2)} + \frac{(1-c)^2\,(1-d)\,(5+d)\,\alpha_1}{9(2+d)^2}.$$

Following the same procedure as before, the quadratic inequality must be solved to obtain under what conditions $PH1$ decides to sell an exclusive license. Let us call the roots of the polynomial $c_{III}^{-}$ and $c_{III}^{+}$ the positive root (see the Appendix for the expressions in (A.11) and (A.12) and proofs). Both are positive, that is, $c_{III}^{-}, c_{III}^{+} > 0$. However, there are conditions under which $c_{III}^{+}$ is fewer than 1, and it is when $0.442891 < d < 1$.

If $\delta = \varepsilon$, then for the $PH2$, we have that

$$\Pi_{PH2}^{E} - \Pi_{PH2}^{NE}|_{\delta \longrightarrow \varepsilon} = \frac{2(c+d-1)^2}{9\,(d^2-1)} < 0.$$

The combination of the above conditions leads to the following result.

**Result 3.** *When i) only $PH1$ has innovation, and ii) $PH1$ has a stake in one of the downstream firms, $PH1$ sells an exclusive license as long as $c_{III}^{-} < c < c_{III}^{+}$ and $0.442891 < d < 1$, or $c_{III}^{-} < c < 1$ if $0 < d < 0.442891$. On the other hand, $PH_2$ prefers to offer cross-licenses.*

**Proof 6.** *See the Appendix.*

This result shows the possibility of vertical integration between the technology provider $PH_1$ and firm 1, which operates on the product market. By entering $PH_1$ as the owner of one of the participating firms in the product market, it can open up the possibility of a change in the market structure towards a vertically integrated monopoly. Clearly, it follows from the conditions of Result 3 that this will be the case as long as there is a product differentiation that allows it. Thus, if there are high levels of $d$, (which implies greater competitiveness and therefore product substitutability), the requirement in cost levels is lower than if there is low competitive intensity (there is only one condition). However, for high levels of competitive intensity, the cost-level requirements are more demanding (two conditions). Once again, the intensity or level of marginal costs of the firms play an important role in decision making. In this section, the possibility of evolving into a monopoly entails gaining market share and, therefore, an increase in the possible income of $PH_1$, since under monopoly the income increases drastically.

### *3.2. Small innovation*

Small innovation is considered when the impact of the innovation on cost reduction is small. In this case, to carry out the analysis, we consider the extreme case where the cost reduction is zero; therefore, $\varepsilon = 0$. Thus, the effect that produces this innovation is null or negligible. In fact, the distinction of cases is not necessary because the decision is the same for all of them.

If the innovation is very small, ($\varepsilon \longrightarrow 0$), then the difference in profits for $PH1$ remains as:

$$\Pi_{PH1}^{NE} - \Pi_{PH1}^{E}|_{\varepsilon \longrightarrow 0} = \frac{(1-c)^2(1-d)[2(2+d)^2 - (1+d)(5+d)\alpha_1]}{9(1+d)(2+d)^2} > 0 \tag{27}$$

On the other hand, the difference between the profits of $PH2$ is the expression in equation (30) in the limiting case of small innovation. Therefore,

$$\Pi_{PH2}^{NE} - \Pi_{PH2}^{E}|_{\varepsilon \longrightarrow 0} = \frac{(1-c)^2(1-d)[2(2+d)^2 - (1+d)(5+d)\alpha_2]}{9(1+d)(2+d)^2} > 0. \tag{28}$$

**Result 4.** *With small innovation, both patent holders prefer to sell two licenses, that is, to cross-license both firms in the downstream market.*

In the limiting case of ($\varepsilon \longrightarrow 0$), if patent holders decide to offer exclusive licenses, no firm would have incentives to acquire the innovation, as the effect is null. Therefore, there will be no market for technology. On the other hand, given the existence of cross-licensing, despite the fact that there is no positive effect from innovation, there is a competitive advantage when producing with two different technologies, that is, firms are now able to produce both varieties of products in the downstream market and get profits from them. That is why firms do have incentives to acquire licenses, since if they did not do so, their competitive situation would worsen. This effect has not much to do with innovation itself, but with multi-production.

## 4. Conclusions

The objective of this work has been to introduce us in the context of competition in the technology market, with the existence of an asymmetry between the innovation in the process collected with the parameter $\delta$. The fact that technology owners have a stake in one of the companies that operates in the market of differentiated products (measured by $d$) has been studied. As indicated at the beginning, the work is a contribution to the literature of technology licenses that combines a series of elements that, as far as we know, have not been studied before. In addition, the treatment of special cases allows recovering scenarios previously discussed in the literature, which are part of a more comprehensive model here. For example, when one of the $\alpha_i$ is zero and $\delta = \varepsilon$ we have a structure in which a single innovator (who may or may not be present in the market through participation) has to decide whether to sell one or two licenses through a fixed payment.

Our analysis has consisted of comparing two possible scenarios to highlight this possible dilemma that innovative companies face when deciding between exclusive licenses or not. The results reveal that, in the context of exclusive licenses, the fixed payment will be higher for the larger technology, since it supposes less lower production costs for companies; the demand it will be higher, so the opportunity cost of not having it is greater than in the case of a smaller technology.

However, when we introduce cross-licenses, and in this case, the downstream firms become multiproduct, the results show us various options. As the model has a high number of parameters, we have proceeded to study particular cases. At first, we have analyzed what happens when there is no participation of the patent holders as owners, $\alpha_1 = \alpha_2 = 0$, we assume that both have the same process innovation and we leave the costs and degree of substitution as free parameters between varieties. The cross-licensing scenario has been shown to be preferred when the size of the innovation is small. If the innovation is very large, the owners may be better off with exclusive licenses: this occurs when the initial cost of production is large enough - the requirement is less, the more differentiated the varieties are. In this way, the firm that competes in duopoly can achieve a greater market share, and therefore the owner can achieve a higher income. If this is not the case, then you prefer to sell to both companies.

The results change when the possibilities of stake by patent holders in downstream firms are introduced. Now, if the participating firm is the only one that owns the innovation, it will only sell an exclusive license if the innovation is large and for certain participation shares. This does not happen when innovators are out of competition in the market. Finally, if both owners have the same innovation, the cross-licensing scenario is preferred when the innovation is small; the degree of substitution between varieties does not play a role in this decision.

Therefore, our work suggests a series of determinants to explain the observation of scenarios with exclusive licenses and cross licenses. Among these, the ownership positions of innovators in competing companies and the size of process innovations are particularly relevant. From an applied point of view, it will be interesting to identify particular cases of licensing policies that coincide with the predictions of our theoretical model. From a formal point of view, the analysis can be extended to other types of contracts, as well as other areas of competition, such as price and levels of investment in R&D.

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## Appendix

### Proofs

In order to proof the former results, comparison between patent holders profits in both scenarios, exclusive licenses and cross-licenses, must be made. However, those profits are adjusted to each case made and evaluated in those different situations. All proofs are computed with Wolfram Mathematica software.

*Proof of Result 1*

For this result, there is a large innovation, $\varepsilon = c$, and symmetry among both patent holders, such as neither patent holder has a stake in the downstream market, $\alpha_i = 0$, and the effect of innovation is the same, $\delta = 0$. Then, Equations (11), (12), (25) and (26) evaluated in those terms are:

$$\Pi^E_{PH1} = \Pi^E_{PH2} = \frac{4c(2-c-d)}{(4-d^2)^2} \tag{A.1}$$

$$\Pi^{NE}_{PH1} = \Pi^{NE}_{PH2} = \frac{2(1-d)}{9(1+d)} \tag{A.2}$$

Then, any patent holder would prefer to sell just one license as long as $(A.1) > (A.2)$. Once the inequality is reordered, we have the next quadratic equation: $c^2 - c(2-d) + \frac{(1-d)(4-d^2)^2}{18(1+d)} < 0$. As expected, once the inequality is solved there are two values of $c$, however, one of them is above the maximum value of $c$, because $0 < c < 1$.

Thus, patent holders sell a private license, that is, $A.1 > A.2$, as long as $c > c_I = \frac{1}{6}(3(2-d) - \sqrt{\frac{(2-d)^2(1+d(9+2d(3+d)))}{1+d}})$, and a cross-license otherwise.

*Proof of Result 2*

For this result, there is a large innovation, $\varepsilon = c$, but an asymmetry is introduced between both patent holders in the sense that the patent holder 1 has a stake in a firm in the downstream market, $\alpha_1 > 0$ and $\alpha_2 = 0$. However, the effect of innovation is the same $\delta = 0$. Then, Equations (11), (12), (25) and (26) evaluated in those terms are:

$$\Pi^{E}_{PH1} = \frac{4c(2-c-d) + \alpha_1 (2(1-c)-d)^2}{(4-d^2)^2} \tag{A.3}$$

$$\Pi^{E}_{PH2} = \frac{4c(2-c-d)}{(4-d^2)^2} \tag{A.4}$$

$$\Pi^{NE}_{PH1} = \frac{1}{9}\left(\frac{2(1-d)}{(1+d)} + \alpha_1\right) \tag{A.5}$$

$$\Pi^{NE}_{PH2} = \frac{2(1-d)}{9(1+d)} \tag{A.6}$$

In this case, because of the asymmetry, the decisions of both patent holders are different. The patent holder 2 has no stake in a downstream firm, then its result remains as in Case I, also contrasted in Proof of Result 1, where it can be seen that $(A.1) = (A.4)$ and $(A.2) = (A.6)$. Then, it has to be proved when the patent holder 1 prefers exclusive licenses versus cross-licenses, that is, when $(A.3) > (A.5)$.

$$(A.3) - (A.5) = \frac{36c(1+d)(1-\alpha_1)(2-c-d) - (2-d)^2(1-d)\left(2(2+d)^2 - (1+d)(5+d)\alpha_1\right)}{9\,(1+d)\,(4-d^2)^2} \tag{A.7}$$

Following the same procedure as before, the following quadratic inequality must be solved in order to obtain under what conditions $PH1$ decides to sell and exclusive license, then once is reordered:

$$\begin{aligned} 36c^2(1+d)(1-a_1) \\ -36c(2-d)(1+d)(1-a_1) \\ +(2-d)^2(1-d)\left(2(2+d)^2 - (1+d)(5+d)\alpha_1\right) < 0 \end{aligned}$$

Once the inequality is solved, two conditions must be fulfilled in order for patent holder 1 to decide over an exclusive license.

1. $c > c_I$
2. $\alpha_1 < \alpha_{1(II)} = \frac{36c(1+d)(2-c-d)-2(1-d)\left(4-d^2\right)^2}{(1+d)(2-6c-3d+d^2)(6c-(2-d)(5+d))}$

Then $PH1$ sells an exclusive license as long as $c > c_I$, its stake in the downstream market is low enough, $\alpha_1 < \alpha_{1(II)}$. In any other case, $PH1$ would decide to sell cross-licenses.

*Proof of Result 3*

For this result, another asymmetry is introduced where the patent holder 2 does not make any innovation, such as $\delta = \varepsilon$. Then, equations (11) and (25) must be reevaluated, such as:

$$\Pi_{PH1}^{E} = \frac{4c\left(2 - c\left(1 - d\right) - d\right) + (1 - c)^2(2 - d)^2\alpha_1}{\left(4 - d^2\right)^2} \tag{A.8}$$

$$\Pi_{PH1}^{NE} = \frac{1}{9}\left(\frac{2(1 - (1 - c)d)^2}{1 - d^2} + (1 - c)^2\alpha_1\right) \tag{A.9}$$

We are going to proved when the patent holder 1 prefers exclusive licenses versus cross-licenses, that is, when $(A.8) > (A.9)$

$$(A.8) - (A.9) = \frac{4c\left(2 - c\left(1 - d\right) - d\right)}{\left(4 - d^2\right)^2} - \frac{2(1 - (1 - c)d)^2}{9(1 - d^2)} + \frac{(1 - c)^2\left(1 - d\right)\left(5 + d\right)\alpha_1}{9(2 + d)^2}.$$

Following the same procedure as before, the following quadratic inequality must be solved in order to obtain under what conditions $PH1$ decides to sell and exclusive license, then once is reordered:

$$\begin{aligned} 2c^2\left(18 - d\left(18 + d\left(2 - d\left(18 - 8d + d^3\right)\right)\right)\right) \\ -4c\left(2 - d\right)\left(1 - d\right)\left(9 + d\left(1 - d\right)\left(1 - d\right)\left(3 + d\right)\right) \\ (2 - d)^2(1 - d)^2\left(2(2 + d)^2 - (1 - c)^2\left(1 + d\right)\left(5 + d\right)\alpha_1\right) < 0 \end{aligned}$$

The previous quadratic inequality throws the following results, where $PH1$ sells an exclusive license as long as $c_{III}^{-} < c < c_{III}^{+}$ and $0.442891 < d < 1$, or $c_{III}^{-} < c < 1$ if $0 < d < 0.442891$.

Then, these are the values of $c$ required:

$$c_{III}^{-} = 1 - \frac{2d\left(7 + d^2 + d^4\right) - \sqrt{2(2 - d)^2(1 - d)^2\left(1 + d\right)\left(18\left(1 + d - 2d^2\right) - \left(5 + d\right)\left(2 - 10d^2 - d^4\right)\alpha_1\right)}}{36 - 2d\left(18 + d\left(2 - d\left(18 - 8d + d^3\right)\right)\right) - (2 - d)^2(1 - d)^2\left(1 + d\right)\left(5 + d\right)\alpha_1} \tag{A.10}$$

$$c_{III}^{+} = 1 + \frac{2d\left(7 + d^2 + d^4\right) - \sqrt{2(2 - d)^2(1 - d)^2\left(1 + d\right)\left(18\left(1 + d - 2d^2\right) - \left(5 + d\right)\left(2 - 10d^2 - d^4\right)\alpha_1\right)}}{36 - 2d\left(18 + d\left(2 - d\left(18 - 8d + d^3\right)\right)\right) - (2 - d)^2(1 - d)^2\left(1 + d\right)\left(5 + d\right)\alpha_1} \tag{A.11}$$

Both are positive, that is, $c_{III}^{-}, c_{III}^{+} > 0$. However, there are conditions under which $c_{III}^{+}$ is fewer than 1, and is when $0.442891 < d < 1$.

*Proof of Result 4*

In this case, there is a small innovation and we analyze the extreme scenario for $\varepsilon = 0$. Then, cross-license is the chosen by both patent holders. To prove this, Equations (29) and (30) must be positive. Note that both equations only differ in $\alpha_1$ and $\alpha_2$. Generally, we need to check that

$$\frac{(1-c)^2(1-d)[2(2+d)^2-(1+d)(5+d)\alpha_i]}{9(1+d)(2+d)^2} > 0.$$

Every term is positive by inspection except for the last term in the numerator. Taking into account that $\alpha_i = 1$, we have, in order to be positive, $2(2+d)^2 - (1+d)(5+d) > 0$. Once it is computed and reordered, we have $d^2 - 2d + 3 > 0$. If $d$ is evaluated to its maximum, $d = 1$, it can be easily seen that the inequality is fulfilled because $1 - 2 + 3 = 2 > 0$.

*Proof of Proposition 1*

For this proposition, we want to prove which patent holder's fee is higher in the case of exclusive licenses, that is, if $F_1^E > F_2^E$, or otherwise. Then

$$F_1^E - F_2^E = \frac{4\delta\left((2-d)(1-c) + 2\varepsilon - \delta\right)}{\left(4-d^2\right)^2} > 0$$

Every term between brackets is positive by definition. The only term that may be negative is $2\varepsilon - \delta$; however, by definition $\varepsilon \geq \delta$. Thus, we prove that $F_1^E > F_2^E$.

Similarly, in the cross-licensing scenario, we can derive if $F_1^{NE} > F_2^{NE}$, or otherwise. Then,

$$F_1^{NE} - F_2^{NE} = \frac{\delta\left(2(1-c) + (2\varepsilon - \delta)\right)}{9} > 0$$

Every term between brackets is positive by definition. The only term that can be negative is $(2\varepsilon - \delta)$; however, by definition $\varepsilon \geq \delta$. Thus, we prove that $F_1^{NE} > F_2^{NE}$.

This proposition shows that the payment (fee) for the best innovation is always higher.

*Proof of Proposition 2*

To prove Proposition 2, $\frac{\partial c_I}{\partial d} < 0$. Figure 2 shows this relationship as well as the table does numerically.

$$\frac{\partial c_I}{\partial d} = \frac{-(4d^5+2d^4-11d^3-16d^2-11d+14)}{6(1+d)\sqrt{(2-d)^2(1+d)(1+d(9+2d(3+d)))}} - \frac{1}{2} < 0$$

To prove that the former inequality holds, can be made by pointing out different values of product differentiation, $d$. The first term is always negative as long as $d < 0,78$. For higher values, the first term is positive, but never greater than $\frac{1}{2}$. For example, when evaluated at a maximum of $d = 1$, the value of the first term is $\frac{1}{9} < \frac{1}{2}$.

*Proof of Proposition 3*

When products are perfect substitute, equation (A.1) and (A.2) are as follows:

$$\Pi^{E}_{PH1} = \Pi^{E}_{PH2} = \frac{4c(1-c)}{9}$$

$$\Pi^{NE}_{PH1} = \Pi^{NE}_{PH2} = 0$$

Then, it is easily seen that the choice of any patent holder under this situation is always to sell just one license.