Optimal Bidding for a Bundle of Power Transmission Infrastructure Works

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Abstract

We put forward an optimal bidding mechanism for a bundle of power transmission infrastructure works. Specifically, the regulator auctions two works altogether: one is to be developed and operated by the winning bidder, while the other is an owner-operated and financed expansion of an existing work. Participants bid jointly for both contracts, and the package is awarded based on the lowest total bid. The costs are divided into a common developing part for all participants and a private part related to financing. The optimal bidder offers the expected value of the costs, adjusted for the cost advantage over the second lowest bidder. This approach efficiently allocates the works to the firm with the lowest combined costs. However, rents persist due to the informational and cost advantage in financing. When a bidder expects higher costs, it requests a higher payment, which reduces its chances of winning the bid.

Keywords: Auctions, Procurement bidding, Infrastructure works bundling, Electric transmission infrastructure
JEL Classification: D44, H42, H57, L94

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1 Introduction

Electricity is easier to transport and easier to convert than other forms of energy, so it can be used in many different applications. This is why it is used in a wide variety of purposes, such as lighting, heating, computing and refrigeration. These characteristics explain its presence in practically all production processes and the most varied aspects of daily life.

Typically, electricity is not generated at the point where it is used. This is either due to the availability of the primary energy source (wind, sunlight, water flow) or to land use regulation considerations (e.g. thermal power plants). Because of this, in order for electricity to reach consumption centers, it generally has to travel long distances. To transport electrical energy, a process known as transmission, cables are used through which a current of electrons travels.

In this context, the objective of the *electric transmission infrastructure auctions* is to expand the capacity of the system in order to adapt and develop it to maintain the necessary coverage and capacity slack to guarantee the correct operation of the electric system and allow the efficient and safe supply of electricity to its end users. For this, the necessary infrastructure and equipment must be in place to condition the electrical energy at the injection and withdrawal points and conduct it under adequate safety conditions over long distances, reducing the losses associated with the process.

For this purpose, an annual transmission expansion plan must typically be prepared, indicating the expansion works deemed necessary for the development of the electric transmission capacity and its stated objectives. In addition to establishing a list of works and their basic technical characteristics, the regulator must consider in the plan, for each work, estimates of: (1) the necessary Investment Value (I.V.); (2) its annual operation, maintenance and administration costs; (3) the expected useful life of each project; and (4) the budgeted construction period for the referential work of the project.

These power transmission infrastructure works may be *new*, which implies that they involve infrastructure that does not exist in any form at the time the requirement is determined, or *expansion works*, which consist of modifying, expanding or improving the capacity of transmission facilities that are already in operation and therefore have an identifiable owner. In the first case (new works), the bidder to whom the execution is assigned must also operate the works during their useful life; in the second case, the expansion works are developed and constructed by the bidder who is assigned them, but their operation and ownership is typically handed over to the original owner of the work, who must pay the developer the amount stipulated by the latter in its economic proposal, operate it, and maintain it for the foreseeable future.

A good example of these bidding practices is the Chilean case, where the *National Energy Commission (CNE)* prepares an annual expansion plan detailing the works necessary for the development of electricity transmission capacity. The CNE stipulates that the execution (and, in some cases, the operation) of these additional works has to be tendered and start to be executed in a limited period in the near future at the time the plan is made. Specifically, at least since 2019, the Chilean Electricity Coordinator has decided to package new works and expansion works as part of its allocation strategy, so that a package of works can only be tendered as a whole, indissolubly and unconditionally[1].

[1] The rationale for bundling several works together is that expansion works are, in general, less attractive than new ones so they usually didn't receive any bids when auctioned alone, by forcing the bidders to take them in a

It is in this context that this work aims at contributing to the literature by examining the economic desirability of bundling new and expansion infrastructure works as a package that can only be tendered as a whole. While there is an extensive literature on optimal multiple products auctions (see for instance the works of Armstrong (2000), Avery and Hendershott (2000), Li, Sun, Yan, and Yu (2015), Suzuki (2018; 2021) and Huang, Yang and Ho (2022)) to the knowledge of the authors there are is no work squarely dealing with this specific issue. Specifically, following the basic formulation of Goeree and Offerman (2003), we present a model of optimal procurement bidding in which a regulator bids jointly for two works: one of the works must be developed and operated by whoever wins the bid, the other work corresponds to an expansion to be operated by its owner. In bidding for expansion works, we assume that the participants must make an indivisible bid for both works and whoever makes the lowest overall bid is awarded the contract.[2] We also assume that the valuation of the works has a part common to all participants that corresponds to the cost of development, construction and operation of the works. Although all firms face exactly the same costs in this component, they receive a noisy signal from it because the studies and projections they can make contain some degree of unavoidable uncertainty. Finally, we assume that a second component of the development cost is entirely private and corresponds to the financing costs of the works, which depends on the identity of the firm and the conditions under which it accesses the capital market. This component follows a known distribution among firms, but self-realization is observed without noise from the proponent.

Among the main findings of this work, we show that the optimal position of a bidder corresponds to the expected value of the cost of developing and financing the project, conditional on the cost signal it received being the lowest among all bidders. The result of this analysis is that the bidding process thus designed is efficient in that it assigns the work package to the firm that has the lowest combined cost of developing, operating and financing the package. The comparative statics analysis shows that, all else constant, if a bid participant receives a higher cost signal (i.e. expects the development and/or financing of the works to be more expensive) it will react by requesting a higher payment which has the effect of lowering its expected utility and reducing the probability of being awarded the bid[3].

Since the new infrastructure work is associated with its operation and its operation continues after the original bidding period, we extend the model to include the impact of the expected future payment for the operation once it goes through the normal regulatory process. Since the way in which the operation and maintenance payments for these works are regulated is completely independent of the identity of their builder and operator, we model the present value of these payments as constant, certain, and identical for all bidding participants. We find that the only effect that this continuation payment has on bidding participants is to reduce all bids by the same amount, so it does not affect the ordering of bids or who is awarded the bid. Therefore, it does not

package with new works, the authority hopes, they might get more bids. For further details see, for instance: "*Informe Técnico Preliminar Plan De Expansión Anual De Transmisión Año 2022*", March 2023: https://www.cne.cl/wp-content/uploads/2023/03/ITP-Plan-de-Expansion-de-la-Transmision-2022.pdf.

[2] This is in line with the Chilean mechanism thet motivates this research.

[3] We note that the expected utility has two components: the utility obtained by the bidder in case of winning the bid and the probability of winning the bid. Both are negatively affected by increasing the cost signal, the first one directly, since the bidder raises its demand by less than the increase in the expected value of the costs and, on the other hand, a higher demanded payment implies a lower probability of winning the bid. Since the expected utility is the multiplication between both terms, the effect is that the expected utility decreases.

affect the efficiency of the process.

We also examined what would happen if the bidders have some control over part of the project costs. In particular, we assume that construction and operating costs (when applicable) cannot be altered because they are properly optimized from an engineering point of view. On the other hand, we assume that firms can at least partially affect financing costs due to their borrowing capacity and the conditions under which they access the financial market. We also assume that firms observe two interest rates, one for short-term debt for non-collateralizable projects that applies to the expansion work and another long-term interest rate for projects with certain and collateralizable payment flows that is relevant for the financing of the new work. Assuming that each potential bidder observes three elements: a short- term rate, a long-term rate and a maximum amount of financing it can obtain for the new work, then the bidder must choose the combination of charges (between the new work and the expansion work) that allows it to obtain the best composition of debt and rates to maximize its expected return[4].

Finally, we also extend our model to consider the effect of a counterfactual situation in which a minimum investment value to be declared for the expansion works is established. This is precisely the scenario that would arise from the regulator's interpretation of the obligation to internalize costs, which the regulator understands should be applied to the expansion works separately, and not to the package. We found that, in case this restriction is active and affects the bids, this has the direct impact of making the execution of the works more expensive since it prevents companies from fully using the capacity to arbitrage rates when financing the execution of the project, but, in addition, this imposition in the bidding design may break with the efficiency in the allocation of the works so that these are awarded to a company that, in fact, is less efficient and faces a higher execution cost.

Consequently, the analysis presented shows that it is economically reasonable to group new works with expansion works, since this has the potential to increase competition for all the works. That, therefore, it is reasonable that the allocation be made for the lowest overall bid for the package of works and that this has the virtue of encouraging the firms to seek to minimize the costs of the works as a whole. The theoretical analysis presents a scenario in which it is perfectly rational to offer a corner solution in which the value of the expansion works is taken to the minimum that it is legally possible to offer and that this does not respond to anything other than the incentives that the auction mechanism characteristics create for each bidder.

This paper is structured as follows. In section 2, we review the related literature. In section 3, we introduce the formal model of an optimal bidding for a bundle of power transmission infrastructure works. Here we also present a comparative statics result of what would happen to a participant's optimal proposal and expected profits when the cost signal he receives increases (i.e., when he expects the execution and operation of the works to become more cost-effective). In section 4, we put forward several extensions of the basic model, including: a continuation payment, partial endogeneity in financing costs and a minimum investment value for the expansion project. Section 5 presents a discussion of the main results and policy implications. Finally, in section 6, we end the paper by summing up the main results and conclusions.

[4]Again, this depends on the profit it will make if it is awarded the work and the probability of actually winning the contract.

2 Literature Review

We now present a survey of the literature, focusing on those works on commodity bundling, optimal auction design, and their implications for market efficiency and profitability in various economic contexts, including power transmission infrastructure projects.

We identify in our review several different approaches and focuses. First, works on *commodity bundling and market efficiency* pursued by Adams and Yellen (1976), Palfrey (1983) and McAfee, McMillan and Whinston (1989). Adams and Yellen (1976) introduced the concept of commodity bundling, distinguishing between pure bundling, where goods are sold only as packages, and mixed bundling, where goods are sold individually as well as in packages. They emphasized the profitability of bundling, highlighting its ability to sort customers based on their reservation price characteristics and extract consumer surplus. Palfrey (1983) delved into the impact of bundling decisions on market efficiency, particularly in scenarios with uncertain demand. The study explored how bundling affects buyer welfare and market outcomes, considering factors such as the number of buyers and the nature of competition. McAfee et al. (1989) investigated conditions under which bundling emerges as an optimal strategy, particularly in scenarios with limited information about consumer preferences. The study provides sufficient conditions for bundling to dominate unbundled sales and explored the implications for market outcomes.

Second, we found works on *optimal auction design and price discrimination* with the papers of Bulow and Roberts (1989) and Avery and Hendershott (2000). Bulow and Roberts (1989) drew parallels between optimal auction design and price discrimination strategies. They showed how insights from mechanism design can be applied to standard microeconomic theory, shedding light on the optimal structuring of auctions in various market settings. Avery and Hendershott (2000) extended the relationship between price discrimination and optimal auctions to multiple products, highlighting the unique characteristics of optimal auctions compared to monopoly sales. The study provided insights into how optimal auctions facilitate price discrimination and maximize seller revenue.

Third, some works study the *comparative analysis of auction formats*, such as Armstrong (2000) and Maskin and Riley (2000). Armstrong (2000) analyzed optimal auctions of several objects and considered various auction formats and their implications for market outcomes. The paper explored how auction format selection depends on factors such as bidder asymmetry and market conditions. Maskin and Riley (2000) discussed the revenue-equivalence theorem for auctions and its implications for different auction formats, considering factors like risk neutrality and asymmetry among buyers. They explored how different auction institutions, such as open English auctions and sealed envelope first-price auctions, can be equivalent despite their strategic differences.

Fourth, the *bidding mechanisms in procurement auctions* is studied by Chakraborty (1999), Asker and Cantillon (2010), Li, Sun, Yan, and Yu (2015) and Menicucci, Hurkens and Jeon (2015). Chakraborty (1999) examined the decision-making process of auctioneers regarding bundling different objects before selling them. The paper provides insights into optimal bundling strategies under different bidder scenarios, shedding light on the factors influencing bundling decisions. Asker and Cantillon (2010) characterized the buyer's optimal buying mechanism in procurement auctions with private information about suppliers' costs. They compared it to simpler buying procedures like scoring auctions and bargaining, analyzing how these mechanisms extract buyer surplus and affect market efficiency. Li et al. (2015) examined the principal's bundling decision during a procurement auction for a project consisting of two sequential tasks. They showed how increasing the number of consortiums that can perform both tasks affects the principal's preference for bundling, considering the presence of task externality and sequential information arrival. Menicucci et al. (2015) investigated the optimal sale mechanism for a monopolist selling two objects to a single buyer with privately observed valuations. They proved that under certain conditions, pure bundling is the optimal sale mechanism, considering factors such as the non-negativity of virtual valuations for each object.

Fifth, the *bundling strategies and market outcomes* is investigated by Bakos and Brynjolfsson (1999), Suzuki (2018) and Suzuki (2021). Bakos and Brynjolfsson (1999) focused on optimal bundling strategies for information goods, highlighting the predictive value of bundling in increasing sales and profitability. The study explored the benefits of bundling large numbers of unrelated information goods and provided insights into optimal bundle size and pricing strategies. Suzuki (2018) investigated the effect of bundling contracts on bid price and competition in electricity procurement auctions. The study analyzed bid price regression, participation rates of new entrants, and the bid discounting effect of bundling, providing empirical insights into the impact of bundling behavior on auction outcomes. Suzuki (2021) conducted structural estimations to investigate the effect of bundling contracts on electricity procurement auctions in Tokyo. The study analyzed cost implications, competition dynamics, and auction outcomes, providing insights into the impact of bundling behavior on market efficiency.

Sixth, Huang, Yang and Ho (2022) investigated on *online auction dynamics*. They explored the bidding strategies used in online auctions by sellers and buyers, considering factors such as base price, auction duration, and bidding price. They investigated the effects of these factors on seller profit and buyer utility, aiming to establish a favorable online auction environment for both parties.

Seventh, Zhou (2021) works on *competitive bundling strategies*. Particularly, Zhou (2021) proposed a framework for studying competitive mixed bundling with an arbitrary number of firms. They examined firms' incentive to introduce mixed bundling and equilibrium tariffs when all firms adopt the mixed-bundling strategy, considering factors such as market structure and demand correlation.

Eighth, Chakraborty, Khalil and Lawarree (2021) discussed on *competitive procurement and efficiency*. Specifically Chakraborty et al. (2021) argued for the optimality of limiting competition or using inefficient allocation rules in competitive procurement. They discussed how such mechanisms guard against ex post moral hazard and provide incentives for efficient performance, considering factors like asymmetry between bidders and the presence of ex post moral hazard.

Finally, Lu and Zhao (2023) study *dynamic selling mechanisms*. Particularly, Lu and Zhao (2023) studied intertemporal bundling in optimal dynamic selling mechanisms in a two-period setting. They explored conditions under which intertemporal bundling arises and characterized the optimal bundle design, considering factors such as the correlation between the buyer's values for the two objects.

This paper adds to the literature by showing how the bundling of items of a different nature, cost, development period and payment schedule may affect the bidding strategy and the outcome of the auction, specially when the bidder can affect the overall cost of the combined project by allocating the total payment demanded to the different elements of the auction. We find that, in determined situations, this may in fact, affect the outcome and the efficiency of the auction process.

Figure 1 below provides a structured overview of the literature review, grouping papers based on similar analyses and themes within the context of optimal bidding and commodity bundling for power transmission infrastructure works.

3 The Model

We consider a bidding process in which the regulator assigns the execution of two works O^1 and O^2 in a single and indivisible manner to one bidder.

The works will be assigned to the bidder that demands the lowest payment b for the construction of both works and the operation of work 2 (new work, as opposed to work 1, which is an expansion work whose operation will not be the responsibility of the successful bidder). Although the allocation will be for the lower value of payment b, this is broken down into two parts, a one- time payment against delivery for the work O^1 and an annuity for a total of T periods to cover the construction and operation of the new work O^2, that is:

$$\begin{aligned} b &= b^1 + b^2 \qquad (1) \\ b^1 &= \frac{B^1}{1+r} \\ b^2 &= \sum_{t=1}^{T} \frac{B^2}{(1+r)^t} \end{aligned}$$

obviously, it is irrelevant if the work is assigned to the one who demands a lower payment b or a lower amount $B = B^1 + B^2$ since the relationship between the two is monotonic.

The works are complementary in the sense that it is not possible to operate the work O^2 and, therefore, generate revenue from it, without first completing the work O^1.[5]

We will assume that the cost of construction and operation of the works (as a whole) is a value v common to all bidders but unknown to them, who only observe an unbiased signal v_i.

In addition, each participant $i = 1, \ldots, n$ $(n \geq 2)$ faces a cost c_i of financing the loans, this cost is specific to each participant and depends on its debt rationality and on the bidder's ability to collateralize the flow of payments it expects from the new work.

We will assume that both the signal v_i and the cost c_i come from known distributions with densities $f_v(\cdot)$ and $f_c(\cdot)$ and supports $[v_L, v_H]$ y $[c_L, c_H]$ respectively, with $v_L < v_H$ and $c_L < c_H$. Both distributions are assumed to be log-concave[6] and the signals are assumed to be independent of each other and independent and identically distributed among participants.

Finally, we will assume that the signals on construction and operating costs v_i are averaged to the true value [7](which implies that the quality of the information received by each proposer is the

[5]This eliminates the possibility of bidding for both works with the intention of abandoning O^1 or trying to renegotiate the associated payment later on.

[6]That is, the logarithm of each density is concave, i.e.: $\ln f(\theta x + (1-\theta)y) \geq \theta \ln f(x) + (1-\theta)\ln f(y)$. Most of the usual continuous distributions (e.g. normal, beta, gamma, exponential, uniform) satisfy this condition.

[7]This specification is commonly used in the common value bidding literature such as: Mares and Harstad (2003), De Silva et al. (2008), Goeree and Offerman (2003), or Menezes and Monteiro (2005) another alternative is to use

Group	Paper	Key Contributions
Commodity Bundling and Market Efficiency	Adams and Yellen (1976)	Introduced the concept of commodity bundling, distinguishing between pure and mixed bundling strategies. Explored the profitability of bundling and its ability to extract consumer surplus.
	Palfrey (1983)	Analyzed the impact of bundling decisions on market efficiency under uncertain demand. Investigated how bundling affects buyer welfare and market outcomes, considering factors such as the number of buyers.
	McAfee, McMillan, and Whinston (1989)	Investigated conditions under which bundling emerges as an optimal strategy, particularly in scenarios with limited information about consumer preferences. Explored the profitability of bundling and its implications for market outcomes.
Optimal Auction Design and Price Discrimination	Bulow and Roberts (1989)	Established parallels between optimal auction design and price discrimination strategies. Explored optimal auction structuring in various market settings and the implications for market outcomes.
	Avery and Hendershott (2000)	Extended the relationship between price discrimination and optimal auctions to multiple products. Explored how optimal auctions facilitate price discrimination and maximize seller revenue compared to monopoly sales.
Comparative Analysis of Auction Formats	Armstrong (2000)	Analyzed optimal auctions of several objects, considering various auction formats and their implications for market outcomes. Explored factors such as bidder asymmetry and market conditions in auction format selection.
	Maskin and Riley (2000)	Discussed the revenue-equivalence theorem for auctions and its implications for different auction formats. Explored how different auction institutions can be equivalent despite strategic differences.
Bidding Mechanisms in Procurement Auctions	Chakraborty (1999)	Examined optimal bundling strategies for auctioneers considering different bidder scenarios. Explored factors influencing bundling decisions and the impact on market outcomes.
	Asker and Cantillon (2010)	Characterized the buyer's optimal buying mechanism in procurement auctions with private information about suppliers' costs. Compared to simpler buying procedures like scoring auctions and bargaining, analyzing their implications for market efficiency.
	Menicucci, Hurkens, and Jeon (2015)	Investigated the principal's bundling decision during a procurement auction for a project consisting of two sequential tasks. Explored how increasing the number of consortiums that can perform both tasks affects the principal's preference for bundling, considering the presence of task externality and sequential information arrival.
	Li, Sun, Yan, and Yu (2015)	Examined the principal's bundling decision during a procurement auction for a project consisting of two sequential tasks. Investigated how increasing the number of consortiums that can perform both tasks affects the principal's preference for bundling, considering the presence of task externality and sequential information arrival.
Bundling Strategies and Market Outcomes	Bakos and Brynjolfsson (1999)	Explored optimal bundling strategies for information goods, emphasizing the predictive value of bundling in increasing sales and profitability. Investigated optimal bundle size and pricing strategies.
	Suzuki (2018)	Investigated the effect of bundling contracts on bid price and competition in electricity procurement auctions. Explored empirical insights into the impact of bundling behavior on auction outcomes, including bid price regression and participation rates.
	Suzuki (2021)	Conducted structural estimations to investigate the effect of bundling contracts on electricity procurement auctions. Explored cost implications, competition dynamics, and auction outcomes, providing insights into the impact of bundling behavior on market efficiency.
Online Auction Dynamics	Huang, Yang, and Ho (2022)	Explored bidding strategies used in online auctions by sellers and buyers, analyzing factors such as base price, auction duration, and bidding price. Investigated their effects on seller profit and buyer utility, aiming to establish a favorable online auction environment.
Competitive Bundling Strategies	Zhou (2021)	Proposed a framework for studying competitive mixed bundling with an arbitrary number of firms. Explored firms' incentive to introduce mixed bundling and equilibrium tariffs when all firms adopt the mixed-bundling strategy, considering factors such as market structure and demand correlation.
Competitive Procurement and Efficiency	Chakraborty, Khalil, and Lawarree (2021)	Argued for the optimality of limiting competition or using inefficient allocation rules in competitive procurement. Discussed how such mechanisms guard against ex post moral hazard and provide incentives for efficient performance.

Figure 1: Literature Review on Commodity Bundling and Optimal Auction Design

same), that is:

$$v = \frac{1}{n}\sum_{j=1}^{n} v_j$$

If a bidder i, who receives a cost signal c_i and submits a bid for b_i is assigned the bidding, construction and operation will receive profits for:

$$\pi_i(b_i) = b_i - (v + c_i)$$

since the payoff (and, therefore the strategy) of each proposer depends on two distinct random variables we might face the problem that there is no natural way to order two signals in two distinct dimensions[8], however, in this case, an aggregate statistic is an obvious alternative for such ordering. Indeed, note that:

$$\begin{aligned}\pi_i(b_i) &= b_i - \left(\tfrac{1}{n}\textstyle\sum_{j=1}^{n} v_i + c_i\right)\\ &= b_i - \left(\tfrac{1}{n}v_i + c_i + \tfrac{1}{n}\textstyle\sum_{j\neq i} v_j\right)\\ &= b_i - \left(s_i + \tfrac{1}{n}\textstyle\sum_{j\neq i} v_j\right)\end{aligned}$$

we will refer to the random variable s_i as the "type" of bidder i and note that it has a known distribution that depends on f_c and f_v will be log-concave with support on $[v_L + c_L, v_H + c_H]$.

3.1 Equilibrium of sealed envelope and first price auction.

In the following we characterize a *Bayesian Nash equilibrium* for the game in which the different bidders observe their type $s_{j\neq i}$ and decide, based on this, a bid according to a function $b(\cdot)$. The principal (the regulator) observes all bids and assigns the bid to the bidder who demands the lowest payment $b(\cdot)$.

Without loss of generality, we focus on the strategy of bidder 1. We will assume that the bidder observes its type s_1 and submits a bid b_1. Conditional on his type and on his bid being the lowest of all (otherwise he does not win the bid) his expected utility is:

$$\begin{aligned}\pi_1^e(b_1) &= \mathrm{E}\left[b_1 - s_1 - \tfrac{1}{n}\textstyle\sum_{j=2}^{n} v_j \middle| s_1, b_1 \leq b_j \forall j \neq 1\right] \Pr\left[b_1 \leq b_j \forall j \neq 1\right]\\ \pi_1^e(b_1) &= \left(b_1 - s_1 - \tfrac{1}{n}\textstyle\sum_{j=2}^{n} \mathrm{E}[v_j | s_1, b_1 \leq b_j \forall j \neq 1]\right) \Pr\left[b_1 \leq b_j \forall j \neq 1\right] \qquad (2)\end{aligned}$$

Now, assuming that each of the other $n-1$ participants decides its bid according to a function $b(s_j)$ we note that this function must be strictly increasing[9] and, given that the support of s_j is bounded, then so must be the path of $b(\cdot)$, that is, $b \in [b_L, b_H]$.

the more classical Wilson (1968) specification in which v distributes according to some known distribution F_v and participants receive an unbiased signal z from a distribution $F_{Z|V=v}$. Qualitatively, both specifications generate the same results, but the one used here simplifies the calculations considerably.

[8] See, for example, the discussion in Goeree and Offerman (2003).

[9] In this regard, it is sufficient to note that the signals of the different participants are affiliated, so that observing a higher value of v_1 makes it more likely that v will be higher for all participants, making it reasonable to request a higher payment when observing a higher rate s_1.

Given this, it is natural to assume that b_1 must also belong to this interval since if $b_1 > b_H$ participant 1 will never win the bidding and, if $b_1 < b_L$ participant 1 will always win the bidding. Notice that the participant 1 would also win if it offers $b_1 = b_L$ but, in this case, will have higher profits.

Then, we can safely assume that there exists some type ω for which $b(\omega) = b_1$. With this we transform the game in which proposer 1 chooses an offer b_1 to one in which player 1 chooses which type to declare ω and the system automatically chooses what his bid will be based on the function $b(\cdot)$. The idea is, then, to implement bidding as a direct mechanism[10].

Accordingly, then:

$$\begin{aligned}
\pi_1^e(\omega) &= \left(b(\omega) - s_1 - \frac{1}{n}\sum_{j=2}^{n} \mathrm{E}[v_j|s_1, b(\omega) \leq b(s_j)\forall j \neq 1]\right) \Pr\left[b(\omega) \leq b(s_j)\forall j \neq 1\right] \\
\pi_1^e(\omega) &= \left(b(\omega) - s_1 - \frac{n-1}{n}\mathrm{E}[v|s_1, \omega \leq s_j\forall j \neq 1]\right) \Pr\left[\omega \leq s_j\forall j \neq 1\right] \\
\pi_1^e(\omega) &= \left(b(\omega) - s_1 - \frac{n-1}{n}\mathrm{E}[v|\omega \leq s]\right) \Pr\left[\omega \leq s\right] \\
\pi_1^e(\omega) &= \left(b(\omega) - s_1 - \frac{n-1}{n}\mathrm{E}[v|\omega \leq s]\right) \left[1 - F_s(\omega)\right]^{n-1}
\end{aligned} \tag{3}$$

In the second step we again used the condition that $b(\cdot)$ is strictly increasing. In the third step we defined $s = \min_j s_j$ and in the last step we used the fact that the $n-1$ types of the other participants are i.i.d to each other $F_s(\cdot)$ is the cumulative probability function of each type s_j.

Now, we are able to differentiate this expression and impose the first order condition to obtain the optimal strategy of bidder 1 given that the other bidders are generating bids according to the function $b(\cdot)$. In this case, we are interested in finding the functional form of $b(\cdot)$ that bidder 1 should use.

$$\begin{aligned}
\pi_1^e(\omega)' &= \left(b'(\omega) - \frac{n-1}{n}\frac{d}{d\omega}\mathrm{E}[v|s \geq \omega]\right) \left[1 - F_s(\omega)\right]^{n-1} \\
&\quad -(n-1)\left(b(\omega) - s_1 - \frac{n-1}{n}\mathrm{E}[v|\omega \leq s]\right) \left[1 - F_s(\omega)\right]^{n-2} f_s(\omega) = 0 \\
&= \left(b'(\omega) - \frac{n-1}{n}\frac{d}{d\omega}\mathrm{E}[v|s \geq \omega]\right) \left[1 - F_s(\omega)\right] \\
&\quad -(n-1)\left[b(\omega) - s_1 - \frac{n-1}{n}\mathrm{E}[v|\omega \leq s]\right] f_s(\omega) = 0
\end{aligned}$$

After some rearrangements we can obtain an expression that defines a differential equation to determine the optimal function to be used by the bidder to determine its bid.

Proposition 1 *The optimal bidding strategy is given by the following equation:*

$$\pi_1^e(\omega)' = \left\{\frac{d}{d\omega}\left(b(\omega) - \frac{n-1}{n}E[v|s \geq \omega]\right)\right\}\frac{[1 - F_s(\omega)]}{(n-1)f_s(\omega)} - \left(b(\omega) - \frac{n-1}{n}E[v|s \geq \omega] - s_1\right) = 0 \tag{4}$$

[10] That is, it "asks" each proposer directly what its type is. Note, however, that nothing prevents the proposer from declaring a type other than its own.

Proof of Proposition 1 *Follows from the analysis above.*

From **Proposition 1** and following the *Revelation Principle* (see, for example, Fudenberg and Tirole, 1995) it is possible to reach this *Perfect Bayesian Nash Equilibrium* with a mechanism that is "direct" (each proposer simply reveals a type) and incentive compatible (the revealed type is the one that each proposer actually has). This means that we can concentrate exclusively on the equilibrium in which $\omega = s_1$ implies:

$$\pi^e(s_1)' = \left\{\frac{d}{ds_1}\left(b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \geq s_1]\right)\right\}\frac{[1-F_s(s_1)]}{(n-1)f_s(s_1)} - \left(b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \geq s_1] - s_1\right) = 0 \tag{5}$$

We define $y_1 = \min_{j \neq 1} s_j$ and notice that $\frac{f_s(s_1)(n-1)}{1-F_s(s_1)} = \frac{f_{y_1}(s_1)}{1-F_{y_1}(s_1)}$. Using the following result from applying the Fundamental Theorem of Calculus to a conditional expectation on a truncated distribution:

$$\begin{aligned}
\mathrm{E}[Y|X \geq x] &= \int_x^\infty \mathrm{E}[Y|X=t]\frac{f_X(t)}{1-F_X(x)}dt \\
\frac{d}{dx}\mathrm{E}[Y|X \geq x] &= -\mathrm{E}[Y|X=x]\frac{f_X(x)}{1-F_X(x)} + \int_x^\infty \mathrm{E}[Y|X=t]\frac{f_X(t)}{(1-F_X(x))^2}f_X(x)dt \\
&= \left(-\mathrm{E}[Y|X=x] + \mathrm{E}[Y|X \geq x]\right)\frac{f_X(x)}{1-F_X(x)}
\end{aligned} \tag{6}$$

we get:

$$s_1 = \mathrm{E}[y_1|y_1 = s_1]$$

Given that:

$$\begin{aligned}
\frac{d}{ds_1}E[y_1|y_1 \geq s_1] &= \left[-\mathrm{E}[y_1|y_1 = s1] + \mathrm{E}[y_1|y_1 \geq s_1]\right]\frac{f_{y_1}(s_1)}{1-F_{y_1}(s_1)} \\
\Rightarrow s_1 = \mathrm{E}[y_1|y_1 = s_1] &= -\frac{d}{ds_1}E[y_1|y_1 \geq s_1]\frac{1-F_s(s_1)}{f_s(s_1)(n-1)} + \mathrm{E}[y_1|y_1 \geq s_1]
\end{aligned}$$

then, replacing in (5):

$$\begin{aligned}
\left\{\frac{d}{ds_1}\left(b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \geq s_1]\right)\right\}\frac{[1-F_s(s_1)]}{(n-1)f_s(s_1)} & \\
-\left(b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \geq s_1] + \frac{d}{ds_1}E[y_1|y_1 \geq s_1]\frac{1-F_s(s_1)}{f_s(s_1)(n-1)} - \mathrm{E}[y_1|y_1 \geq s_1]\right) &= 0 \\
\left\{\frac{d}{ds_1}\left(b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \geq s_1] - E[y_1|y_1 \geq s_1]\right)\right\}\frac{[1-F_s(s_1)]}{(n-1)f_s(s_1)} & \\
-\left(b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \geq s_1] - \mathrm{E}[y_1|y_1 \geq s_1]\right) &= 0
\end{aligned} \tag{7}$$

defining:

$$\phi(s_1) = b(s_1) - \frac{n-1}{n}\mathrm{E}[v|s \geq s_1] - E[y_1|y_1 \geq s_1] \tag{8}$$

then (7) reduces to:

$$\begin{aligned} \phi'(s_1)\frac{f_{y_1}(s_1)}{1-F_{y_1}(s_1)} - \phi(s_1) &= 0 \\ \phi'(s_1) - \phi(s_1)\frac{f_{y_1}(s_1)}{1-F_{y_1}(s_1)} &= 0 \end{aligned}$$

Defining:

$$P(s_1) = \exp\left(-\int_{\frac{1}{n}v_L+c_L}^{s_1} \frac{f_{y_1}(u)}{1-F_{y_1}(u)}du\right)$$

Since, we know that:

$$P'(s_1) = -\frac{f_{y_1}(s_1)}{1-F_{y_1}(s_1)}\exp\left(-\int_{\frac{1}{n}v_L+c_L}^{s_1} \frac{f_{y_1}(u)}{1-F_{y_1}(u)}du\right) = -\frac{f_{y_1}(u)}{1-F_{y_1}(u)}P(s_1)$$

We can obtain:

$$\begin{aligned} \phi'(s_1)P(s_1) - \phi(s_1)\frac{f_{y_1}(s_1)}{1-F_{y_1}(s_1)}P(s_1) &= 0 \\ [\phi(s_1)P(s_1)]' &= 0 \\ \phi(s_1)P(s_1) &= K \end{aligned} \tag{9}$$

Where K is some constant.

Given that:

$$P\left(s_L = \frac{1}{n}v_L + c_L\right) = 1$$

replacing in (9) we have:

$$\phi'(s_L) - \phi(s_L)f_{y_1}(s_L) = 0$$

Noting that this equality can only hold if every term in the sum is null (and that $f_{y_1}(s_L) \neq 0$ by definition), we conclude that $K = 0$ and we have, then, that the optimal strategy of the proponent is given by:

$$b(s_1) = \frac{n-1}{n}\mathrm{E}[v|s \geq s_1] + \mathrm{E}[y_1|y_1 \geq s_1]$$

Hence, we can get the following result:

Proposition 2 *The Perfect Bayesian Nash Equilibrium of this game is that all bidders use the strategy given by the following equation:*

$$b_i = b(s_i) = \frac{n-1}{n}E[v|s \geq s_i] + E[y_1|y_1 \geq s_i] \tag{10}$$

Proof of Proposition 2 *Follows from the analysis above.*

Given the result put forward in **Proposition 2** we can now obtain the expected profit of all participants.

Corollary 1 *A firm's profit expectation is given by the following equation:*

$$\begin{aligned}
\pi_i^e(s_i) &= \left(b(s_i) - s_i - \frac{n-1}{n}E[v|s \geq s_i]\right)[1 - F_s(s_i)]^{n-1} \\
&= \left(\frac{n-1}{n}E[v|s \geq s_i] + E[y_1|y_1 \geq s_i] - s_i - \frac{n-1}{n}E[v|s \geq s_1]\right)[1 - F_s(s_i)]^{n-1} \\
&= (E[y_1|y_1 \geq s_i] - s_i)[1 - F_s(s_i)]^{n-1} \qquad (11)
\end{aligned}$$

Corollary 1 implies that (reasonably) firm i's profit expectation is given by its expected cost advantage (with respect to the second competitor with the lowest expected cost advantage) times the probability that its cost signal is the lowest of all participants (further noting that this indicates that the bidding mechanism is such that the work is allocated efficiently, i.e., it will be developed by the bidder that expects to be able to do it at the lowest cost).

It is also interesting to note that equation (10) can be rewritten as:

$$\begin{aligned}
b_i = b(s_i) &= \frac{n-1}{n}\mathrm{E}[v|s \geq s_i] + \mathrm{E}[y_1|y_1 \geq s_i] \\
&= \mathrm{E}\left[\frac{n-1}{n}v \,\middle|\, s \geq s_i\right] + \mathrm{E}[y_1|y_1 \geq s_i] \\
&= \mathrm{E}\left[\frac{1}{n}\sum_{j=1}^{n} v_j - \frac{1}{n}v_i \,\middle|\, s \geq s_i\right] + \mathrm{E}[y_1|y_1 \geq s_i] \\
&= \mathrm{E}\left[V - s_i + c_i \middle| \, s \geq s_i\right] + \mathrm{E}[y_1|y_1 \geq s_i] \\
&= \mathrm{E}\left[V + c \middle| \, s \geq s_i\right] - s_i + \mathrm{E}[y_1|y_1 \geq s_i] \\
&= \mathrm{E}\left[V + c \middle| \, s \geq s_i\right] + \mathrm{E}[y_1 - s_i|y_1 \geq s_i]
\end{aligned}$$

the first term of this expression corresponds to the expected value of the development, operation and financing costs, given the signals received by the participant, and the second term corresponds to the expected value of its informational rent and lower private costs (financing). This facilitates the economic interpretation of the supply strategy.

3.2 Comparative statics

Based on the result presented in **Proposition 2**, we analyze what happens to a participant's optimal proposal and expected profits when the cost signal he receives increases (i.e., when he expects the execution and operation of the works to become more costly).

First, we know that:

$$\begin{aligned}
b_i(s_i) &= \frac{n-1}{n}\mathrm{E}[v|s \geq s_i] + \mathrm{E}[y_1|y_1 \geq s_i] \\
\Rightarrow \frac{\partial b_i(s_i)}{\partial s_i} &= \frac{n-1}{n}\left(-\mathrm{E}[v|s = s_i] + \mathrm{E}[v|s \geq s_i]\right)\frac{f_s(s_i)}{1-F_s(s_i)} \\
&\quad + \left(-\mathrm{E}[y_1|y_1 = s_i] + \mathrm{E}[y_1|y_1 \geq s_i]\right)\frac{f_y(y_1)}{1-F_y(y_1)} \\
&= \frac{n-1}{n}\left(-\mathrm{E}[v|s = s_i] + \mathrm{E}[v|s \geq s_i]\right)\frac{f_s(s_i)}{1-F_s(s_i)} + \left(-s_i + \mathrm{E}[y_1|y_1 \geq s_i]\right)\frac{f_y(y_1)}{1-F_y(y_1)}
\end{aligned}$$

We know that random variables $(X_1, X_2, \ldots, X_n)$ with support in a set D are said to be affiliated if the joint density $f : D \to \Re$ exhibits the monotonic multivariate likelihood ratio property.

That is, given two vectors x and y and defining the supremum and infimum by coordinates as follows:

$$\begin{aligned}
x \vee y &= (\max\{x_1, y_1\}, \max\{x_2, y_2\}, \ldots, \max\{x_n, y_n\}) \\
x \wedge y &= (\min\{x_1, y_1\}, \min\{x_2, y_2\}, \ldots, \min\{x_n, y_n\})
\end{aligned}$$

then:

$$f(x \vee y)f(x \wedge y) \geq f(x)f(y)$$

Intuitively, this means that, as the value of X_i increases, it is more likely to observe higher realizations of the other X_j, $j \neq i$.

There are two important results that we will use here, the first is that if the variables $(X_1, X_2, \ldots, X_n)$ are independent of each other, then they are affiliated. The second is that if $u(x)$ is an increasing function of its arguments and the random variables X are affiliated then:$\mathrm{E}[u(X)|x_k = a]$ is increasing in a. See, for example, Menezes and Monteiro (2005), for the respective proofs.

We can also show that $E[u(x)|X \geq z]$ is non-decreasing in z. Note that:

$$\begin{aligned}
\mathrm{E}[u(x)|X \geq z_1] &= \int_{z_1}^{\infty} \mathrm{E}[u(x)|x = t]\frac{f_X(t)}{1-F_X(z_1)}dt \\
\mathrm{E}[u(x)|X \geq z_2] &= \int_{z_2}^{\infty} \mathrm{E}[u(x)|x = t]\frac{f_X(t)}{1-F_X(z_2)}dt \\
\Rightarrow E[u(x)|X \geq z_2] - E[u(x)|X \geq z_1] &= \int_{z_1}^{z_2} \mathrm{E}[u(x)|X = t]f_X(t)\left[\frac{1}{1-F_X(z_2)} - \frac{1}{1-F_X(z_1)}\right]dt \\
&= \frac{F_X(z_2) - F_X(z_1)}{(1-F_X(z_2))(1-F_X(z_1))}\int_{z_1}^{z_2} \mathrm{E}[u(x)|X = t]f_X(t)dt
\end{aligned}$$

It is straightforward to see that, as long as $u(x)$ is non-decreasing and $z_2 \geq z_1$ then the latter expression will be non-negative from which it follows that $E[u(x)|Z \geq z]$ is non-decreasing in z.

Now, because of the affiliation between the random variables s_j $j = 1, 2, \ldots, n$ and given that $v = n(s - c)$ and $y_1 = min_{j \neq i} s_j$ are (at least) non-decreasing functions of s_i and that, moreover,

$E[u(x)|X \geq z]$ is non-decreasing in z then we have that $b_i(s_i)$ is non-decreasing in s_i. That is, all else kept constant, if a firm receives a larger s_i signal (indicating that it expects the project to be more costly) then its optimal response is to demand a larger payment for its completion. In other words, the optimal posture of any bidding participant is non-decreasing in the cost signal it receives.

Now, we obtain the expected profit equation, based on (11):

$$\pi^e(s_i) = (\mathrm{E}[y_1|y_1 \geq s_i] - s_i)\,[1 - F_s(s_i)]^{n-1} \tag{12}$$

Equation (12) allows us to differentiate the expected profit with respect to the cost signal, given by s_i, which produces:

$$\begin{aligned}
\pi^e(s_i) &= (\mathrm{E}[y_1|y_1 \geq s_i] - s_i)\,[1 - F_s(s_i)]^{n-1} \\
\Rightarrow \frac{\partial \pi^e(s_i)}{\partial s_i} &= \left[(-\mathrm{E}[y_1|y_1 = s_i] + \mathrm{E}[y_1|y_1 \geq s_i])\,\frac{f_s(s_i)}{1 - F_s(s_i)} - 1\right][1 - F_s(s_i)]^{n-1} \\
&\quad - (n-1) f_s(s_i)\,(\mathrm{E}[y_1|y_1 \geq s_i] - s_i)\,[1 - F_s(s_i)]^{n-2} \\
&= [1 - F_S(s_i)]^{(n-2)} \left\{ \left[(-s_i + E[y_1|y_1 \geq s_i])\,\frac{f_s(s_i)}{1 - F_s(s_i)} - 1\right][1 - F_s(s_i)] \right\} \\
&\quad - [1 - F_S(s_i)]^{(n-2)} \left\{(n-1) f_s(s_i)(\mathrm{E}[y_1|y_1 \geq s_i] - s_i)\right\}
\end{aligned}$$

Hence, we get the following result:

Proposition 3 *The effect of a change in the cost signal received by a firm on the equilibrium expected profit from winning the project is negative or equal to zero, that is:*

$$\frac{\partial \pi^e(s_i)}{\partial s_i} \leq 0$$

Proof of Proposition 3 *Follows from the analysis above.*

Proposition 3 shows that, *ceteris paribus*, when a firm receives a higher cost signal, that is when it expects the cost of developing the project to be higher, its expected profit from winning the project decreases. The rationale behind this result comes from the double effect: there is a lower margin but, in addition to the higher cost of developing the project, there is a lower profit margin. Consequently, the probability of winning the bid decreases, as it is likely that another participant will have a lower cost signal, or will be able to finance the project at a lower cost.

4 Extensions of the Model

4.1 Extension 1: Continuation Payment

Suppose that the contract stipulates that the firm developing the new work should demand a payment for the construction, operation, and maintenance of the infrastructure for the first T periods and that, after that, provided that the useful life of the new work is greater than T, there will be a fixed continuation payment with a present value of e to be paid once the initial contract has elapsed.

We will assume that this payment e will be determined in a regulatory process that is independent of the identity of the developer and the actual construction costs of the project, since the payment will be determined according to an efficient company tariff process that will consider the cost of development for a fictitious company that develops the project in the most economical way possible at the time of pricing.[11]

This implies that the expected value of the amount e is constant and the same for all firms, independent of the cost signals received.

In this case the assignee firm's profit would be:

$$\pi(b) = b - V - c_i + e$$

which implies that the expected payment, when bidding according to a type ω is given by (compare with equation 3):

$$\pi_1^e(\omega) = \left(b(\omega) - \frac{n-1}{n} \mathrm{E}[v|\omega \leq s] - s_1 + e \right) [1 - F_s(\omega)]^{n-1}$$

Thus, the first-order condition for determining the optimal proposal is, in this case:

$$\left\{ \frac{d}{d\omega} \left(b(\omega) - \frac{n-1}{n} \mathrm{E}[v|s \geq \omega] \right) \right\} \frac{[1 - F_s(\omega)]}{(n-1) f_s(\omega)} - \left(b(\omega) - \frac{n-1}{n} \mathrm{E}[v|s \geq \omega] - s_1 + e \right) = 0$$

which leads to the differential equation:

$$\phi'(s_1) - [\phi(s_1) + e] \frac{f_{y_1}(s_1)}{1 - F_{y_1}(s_1)} = 0$$

where $\phi(s_1)$ is defined in (8).

The solution of this differential equation produces the following result:

Proposition 4 *The optimal bidding of participants, under a continuation payment, is given by:*

$$b(s_i) = \frac{n-1}{n} E[v|s \geq s_i] + E[y_1|y_1 \geq s_i] - e$$

Proof of Proposition 4 *Follows from the analysis above.*

Proposition 4 shows that, the only effect of the continuation payment is to lower the bids of all participants by exactly the continuation payment. In other words, this makes all bids lower by the same amount and does not alter the composition or order of the bids and preserves the efficient allocation of the work in the bidding process.

4.2 Extension 2: Partial Endogeneity in Financing Costs

We minimally extend the model to assume that firms have some control over c_i the private cost of financing the project as follows: instead of receiving a c_i signal of the total cost of financing the project, we assume that firms each receive interest rate signals r^1 and r^2 associated with the part of

[11] This is the way natural monopolies are usually regulated in Chile.

the cost that they can finance with a collateralizable future income stream (associated with the new work, r^2) and another short-term rate (r^1) associated with the part of the cost of the expansion work that must be financed for a period and that will not generate a collateralizable payment stream. We naturally assume that $r^1 > r^2$.

In addition, each firm receives a signal of the limit $0 < \lambda \leq 1$ for the total cost it can finance against the flow it can offer as collateral. These signals depend, among other things, on the specific characteristics of the firm, like its access to financial markets and its reputation.

Given the above, the cost c_i becomes:

$$c_i = \Big[(1 + r^2)\delta + (1 + r^1)(1 - \delta)\Big]\, V$$

where δ is the fraction of the infrastructure works that the firm decides to finance by offering as collateral the expected flow of payments. Obviously $\delta \leq \lambda$.

Now, given that:

$$s_i = \frac{1}{n} v_i + c_i$$

it follows that if s_i is strictly increasing in c_i and, given that $\pi^e(s_i)$ is decreasing in s_i, it is straightforward to note that the firm will want to minimize c_i as much as possible so the solution to its choice problem of δ will be, simply, to set $\delta = \lambda$.

Therefore, a company that plans to finance its investment primarily using future cash flows will choose to do so.

However, equation (1) shows that this is not exogenous, the firm can affect the value of future collateralizable flows if it allocates a larger portion of its bid collection to new construction (i.e., it decreases b^1 and feeds b^2) and, rationally, it should choose to do so since nothing in the bidding process structure prevents it from doing so.

Consequently, we can put forward the following result:

Proposition 5 *A firm that observes a relatively high value of the signal on the limit for the total cost it can finance against the flow it can offer as collateral, will direct a larger portion of the payment to the future flows associated with the new work.*

Proof of Proposition 5 *Follows from the analysis above.*

The rationale behind the result presented in **Proposition 5** is that whenever a firm receives a high value of (λ), will devote a larger portion of the payment to the future flows associated with the new work, because this action does increase its chances of winning the bid and, at the same time, increases its expected benefits from doing so.

Incidentally, the presence of a continuation payment will have the effect of making it easier *for all the firms* the financing of both works' with a higher charge to the new work.

4.3 Extension 3: Minimum Investment Value for the Expansion Project

Finally, we explore the counterfactual effect that the auction rules could have if the bidding conditions considered a minimum investment value to be declared for the expansion work. This is precisely the scenario that would arise from the regulator's interpretation of the obligation to internalize costs, which it understands must be applied to the expansion works separately, and not as a package.[12]

Recall that our assumption, according to the design of the auction under study, is that each bidder demands full payment for the work package according to (1) which is not subject to any minimum value restrictions.

By imposing a minimum investment value for the amplification work, say $\bar{b_1}$, so that $b_1 \geq \bar{b_1}$ we have to consider several impact possibilities depending on what are the optimal values b_1^* and b_2^* that the company would declare in the absence of this constraint.

Indeed, it is easy to see that if $b_1^* \geq \bar{b_1}$ then the constraint has no effect on the bidder's optimal strategy and the bidder will demand the same payoff, with the same structure as if no minimum declared investment value is imposed.

The situation changes when $b_1^* < \bar{b_1}$ as this case forces to reduce the financing of the work package that is directed towards the new work which reduces the company's ability to arbitrage rates between the two and, ultimately, increases the costs of execution.

Consequently, the firm will choose $b_1^{**} = \bar{b_1}$ and $b_2^{**} < b_2^*$ as a position to build the works package but, as decreasing b_2 has an impact on the financing cost c it is expected that $b_1^{**} + b_2^{**} > b_1^* + b_2^*$ since the constraint forces it to choose a combination of financing sources that is different from the one it would choose if it did not face the constraint of declaring a value of investment for the expansion work which is, at least, $\bar{b_1}$.

An interesting result is that the inclusion of a minimum investment requirement may break the efficiency of the bidding process in the sense that it may result in assigning the execution of the works to a bidder who, in the absence of the requirement, would have faced a higher cost of financing the works. This would imply that the mandate to execute the works at the lowest possible cost would not be fulfilled.

Indeed, let us consider the following situation. In the absence of restrictions on the value b_1 the bidder i would have submitted a proposal for the amount:

$$b_i^* = b_{1,i}^* + b_{2,i}*$$

while applicant j would have submitted a proposal for:

$$b_j^* = b_{1,j}^* + b_{2,j}*$$

we assume that $\bar{b_1} > b_{1,i}^*$ and $\bar{b_1} > b_{1,j}^*$, , and assume, without loss of generality, that $b_i^* > b_j^*$ so that the works would have been efficiently assigned to applicant j before applicant i.

[12]The reason to consider this normative change emerges from the previous result. Since bidding firms may have an incentive to demand a low payment for the expansion work this may affect the owner of the existing infrastructure who must pay, operate, and maintain the addition and is assigned a payment from the system that is proportional to the payment it made. This implies that the owner of the existing work may have to pay an unrealistically low price for the expansion at the cost of receiving a low payment for its operation and mainenance for some time (until the next regulatory process). In this way, the incentives of the bidder and those of the owner of the infrastructure to be expanded may not be aligned.

Suppose, consistent with this result, that the financing costs are such that:

$$\begin{aligned} c_i(b^*_{1,i}, b^*_{2,i}) &> c_j(b^*_{1,j}, b^*_{2,j}) \\ r_{1,i}b^*_{1,i} + r_{2,i}b^*_{2,i} &> r_{1,j}b^*_{1,j} + r_{2,j}b^*_{2,j} \end{aligned}$$

and suppose that firms' j advantage comes mainly from having a lower cost of equity capital, so that $r_{2,j} < r_{2,i}$ but $r_{1,i} < r_{1,j}$[13] then, by imposing the constraint $b_1 \geq \bar{b_1}$ and assuming that it is active for both firms[14], then it suffices that:

$$\frac{r_{2,i}b^{**}_{2,i} - r_{2,j}b^{**}_{2,j}}{r_{1,j} - r_{1,i}} \leq \bar{b_1}$$

to verify that:

$$r_{1,i}b^{**}_{1,i} + r_{2,i}b^{**}_{2,i} < r_{1,j}b^{**}_{1,j} + r_{2,j}b^{**}_{2,j}$$

This would allow firm i to make a cheaper proposal than firm j which would lead to the bidding result being reordered and now the work package is more likely to be assigned to bidder i rather than bidder j which would introduce inefficiency and force a higher cost for the work than would have been the case without the restriction.

Hence, we can summarize this discussion in the following result:

Proposition 6 *Requiring as part of the bidding process a minimum value of the investment for one of the works may have the effect of, first, making the execution of the package of works more expensive (if it forces bidders to change the way they will finance them with respect to what would be optimal without the requirement) and, second, it may even lead to the works being allocated inefficiently.*

Proof of Proposition 6 *Follows from the analysis above.*

It should be noted that this is not to say that a stated minimum investment value could not be required for the expansion work, but the regulator should be reasonably sure that the social benefit that this restriction placed on the efficient use of financing alternatives will bring in is at least large enough to offset the cost imposed on society by the possibly inefficient allocation of expansion works.

5 Conclusion and Policy Implications

In this work we present a procurement bidding model in which a regulator auctions two complementary works together. One of the works is to be developed and operated by whoever is awarded the bid, the other work corresponds to an expansion of an existing piece of infrastructure to be operated by its current owner. In expansion works tenders, we assume that participants must make an indivisible bid for both works and the lowest overall bid is awarded.

We assume that the valuation of the works has a part common to all participants that corresponds to the cost of development, construction and operation of the works. Although all firms face exactly

[13]This does not necessarily imply that $b^*_{2,j} \geq b^*_{2,i}$ and $b^*_{1,j} < b^*_{1,i}$, although this is to be expected if companies can choose how to allocate financing from one work to the other.

[14]This should be the case since we assumed that both firms chose optimal values of b_1 below the minimum in the unconstrained equilibrium.

the same costs in this component, they receive a noisy signal from it because the studies and projections they can make contain some degree of unavoidable uncertainty.

We assume that a second component of the development cost is entirely private and corresponds to the financing costs of the works, which depends on the identity of the firm and the conditions under which it accesses the capital market. This component follows a known distribution among firms, but self-realization is observed without noise from the proponent.

The first thing we show is that the optimal position of a bidder corresponds to the expected value of the cost of developing and financing the work, conditional on the cost signal it received being the lowest among all bidders. This offer, which is the most aggressive it could make, is then obscured in a term that depends on the expected cost advantage over the bidder with the second lowest signal (conditional, again, on having received the lowest). The fundamental tension faced by a bidder in a procurement bid is that increasing the payment it demands increases the economic benefit of being awarded the work but, at the same time, decreases the probability of being awarded the work. By maximizing his expected profit (which is, in practice, the interaction between the two terms), he is forced to compensate the two effects by considering the positive and negative impacts of increasing the payment requested.

The result of this analysis is that the bidding process thus designed is efficient in that it assigns the package of works to the firm that has the lowest combined cost of developing, operating and financing the package.

The bidding process does not fully dissipate the rents for the successful bidder as there remains an informational and cost advantage for the firm that can finance the works at a lower cost. This is due, in part, to the obscuring of bids.

The comparative statics analysis shows that, all else constant, if a bid participant receives a higher cost signal (i.e. expects the development and/or financing of the works to be more expensive) it will react by requesting a higher payment which has the effect of lowering its expected utility and reducing the probability of being awarded the bid. We notice that the expected utility has two components: the utility obtained by the bidder in case of winning the bid and the probability of winning the bid. Both are negatively affected by increasing the cost signal, the first one directly, since the bidder raises its demand by less than the increase in the expected value of the costs and, on the other hand, a higher demanded payment implies a lower probability of winning the bid. Since the expected utility is the multiplication between both terms, the effect is that the expected utility decreases.

With regard to the obscuring of bids, let us remember that in tenders that have a common value component there is the risk of the winner's curse. That is, being awarded the bid may be bad news for the bidder because this could imply that he was overoptimistic when valuing the object being tendered (or estimating the cost of providing it, in the case of the type of bids we are studying) so he runs the risk of suffering losses (overpaying for the object or requesting a payment lower than the cost of providing it, in the case of procurement bids). This effect does not occur in private value bidding because the bidder can avoid it by simply not bidding above its valuation.

One way to minimize the risk of falling victim to the winner's curse is to reduce (obscure) your position by bidding less aggressively so that, if you win, you reduce your potential loss. This is particularly clear in the case of transmission infrastructure bids, as it is explicitly stated in the bidding conditions that any error in the estimation of development, construction or operation costs will be the sole responsibility of the winning bidder.

On the other hand, a bidder who expects to make a profit from bid participation has no incentive to bid too aggressively as this reduces his chances of making a profit, so in a sealed-bid, first-price bid, he must partially reduce (obscure) his proposal below his valuation. At the same time, this shading cannot be too large without significantly reducing the bidder's chances of winning the bid and, in fact, the optimal shading decreases with the number of bidders[15].

From the above, it follows that the bidder should be interested in reducing the obscuring of bids and that, under certain conditions, the best way to achieve this is to attract more participants to the bidding process (Goeree and Offerman, 2003).

Since the new work is associated with its operation and its operation continues after the original bidding period, we extend the model to include the impact of the expected future payment for the operation once it goes through the normal regulatory process. Since the way in which the operation and maintenance payments for these works are regulated is completely independent of the identity of their builder and operator, we model the present value of these payments as constant, certain and identical for all bidding participants. Here we assume, as in the Chilean electricity transmission sector, that this payment will be determined in a regulatory process that is independent of the identity of the developer and the actual construction costs of the project, since the payment will be determined according to an efficient company tariff process that will consider the cost of development for a fictitious company that develops the project in the most economical way possible at the time of pricing.

We found that the only effect that this continuation payment has on the bidding participants is to reduce all bids by the same amount, so it does not affect the order of the bids or who is awarded the bid. Therefore, it does not affect the efficiency of the process.

The next question we ask is what happens if the bidders have some control over part of the project costs. In particular, we assume that construction and operating costs (where applicable) cannot be altered because they are properly optimized from an engineering point of view.

On the other hand, we assume that companies can at least partially affect financing costs based on their borrowing capacity and the conditions under which they access the financial market.

In particular, we assume that firms observe two interest rates, one for short-term debt for uncollateralizable projects that applies to expansion work and another long-term interest rate for projects with certain and collateralizable payment streams that is relevant for financing new work.

To the extent that this rate, which corresponds to the company's WACC[16], is lower than the short-term rate (a condition to be expected both because of the difference in terms and because of the existence of collateral that can be used to guarantee payments), the opportunity for arbitrage arises by directing the cost of financing as much as possible towards the new construction, which can be achieved by decomposing the total payment demanded (on which the award of the bid depends)

[15] See, for instance, Menezes and Monteiro (2008).

[16] Weighted Average Capital Cost is the interest rate at which the company can raise resources, either in the form of debt or equity, weighted by the specific type of capital, considering the different tax treatment that both forms of financing may receive.

so as to redirect as much of it as possible towards new construction.

Assuming that each potential bidder observes three elements: a short-term rate, a long-term rate and a maximum amount of financing it can obtain for the new work, then the bidder must choose the combination of charges (between the new work and the expansion work) that allows it to obtain the best composition of debt and rates to maximize its expected return. Again, this depends on the profit it will make if it is awarded the work and the probability of actually winning the contract.

This implies that the solution to this financing problem should, to the extent that the opportunity to arbitrage rates between works is effective, have a corner solution: the firm should transfer as much of the financing from the expansion work to the new work as possible. This leads to one of two possible outcomes: If the maximum amount that the bidder can charge to the new work is less than the total cost of the work, then the declared investment value for the expansion work should be reduced to the limit given by the borrowing capacity against the revenue stream of the new work (this would generate an I.V. of the expansion work below the benchmark I.V. but certainly above the lowest possible book value). If, on the other hand, it is possible to finance the entire package of works by borrowing against the payment stream associated with the new work, then the PV of the expansion work should be brought to the lowest admissible book value, that is, the equivalent of one monetary unit.

It should be noted that the existence of a continuation payment decreases the project financing requirements included in the economic proposal. While these flows do not affect the financing requirement for the construction of the works, it does have the impact of reducing the amount of debt that can be incurred against the new work making it less likely that the maximum debt constraint will be active and, therefore, making it easier to minimize the investment value of the expansion work.

As a last model extension exercise, we consider the effect of a counterfactual situation in which a minimum investment value to be declared for the expansion works is established. This is precisely the scenario that would arise from the regulator's interpretation of the obligation to internalize costs, which it understands should be applied to the expansion works separately, and not to the package. We found that, in case this restriction is active and affects the bids, this has the direct impact of making the execution of the works more expensive since it prevents the companies from fully using the capacity to arbitrage rates when financing the execution of the project, but, in addition, this imposition in the bidding design can break with the efficiency in the allocation of the works so that these are awarded to a company that, in fact, is less efficient and faces a higher execution cost.

To sum up, the theoretical model put forward in this work suggests that the type of public bidding with multiple bundled works is an optimal mechanism design, which maximizes social welfare and "forces" companies to submit realistic bids that are aligned with their possibilities of compliance. It also follows from the model that it is rational for companies to demand a higher payment for new works, which are more attractive, and a lower one for extension works, less attractive options and, at the margin, under certain conditions, they could reach an investment value close to zero for extension works.

The bundling of works in indivisible bids is desirable because it increases participation and competition, since the expansion works alone can not be very attractive for new potential participants in the electricity transmission market to be interested in submitting proposals; by grouping them with new works, this effect is mitigated, since the package of works becomes more profitable. However, for the allocation of the package of works to be efficient, it is necessary that it is awarded to the bidder that submits the lowest overall bid for the set of works, this encourages the most efficient financing alternatives and combinations to be sought and the most aggressive bids to be submitted.

This result should be taken into consideration by regulators who may be tempted by, for instance political reasons, to disqualify very low investment values for the expansion works, which makes no sense in that it distances the regulator from expanding the system efficiently, guaranteeing the operation of the system at the lowest possible cost.

6 References

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