Demographic Change, Automation and the Role of Education

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Abstract

This paper analyzes the relationship between demographic change and automation while taking the role of education into account. This is illustrated by incorporating skilled and unskilled labor into a theoretical model. If labor supply by households decreases, for example, due to demographic change, the model states that the optimal level of automation capital increases. However, this relationship depends crucially on the level of education in the workforce. Motivated by this novel prediction derived from the model, a new data set allowing for testing of the prediction is constructed. Patent data are combined with an automation classification to arrive at a novel measure of automation. In a series of analyses, evidence for the theoretical prediction is found. While there is a negative relationship between automation capital and population growth, the results corroborate the theoretical prediction that it is crucial to account for the role of education in that relationship. Doing so yields highly significant results which suggest that population growth is negatively correlated with automation, but that this is only true if the workforce consists of predominantly unskilled workers.

1 Introduction

Demographic change and in particular population aging have put labor markets under pressure in the past decades. Japan is a cautionary example for the rest of the Western world, which is on the same trajectory as Japan was four decades ago.[1] According to data from the World Bank, the share of countries experiencing negative working-age population growth was 7% in 1990 but has increased to 34% in 2015.[2] Balakrishnan *et al.* (2015) calculate for the US economy that aging is responsible for 50% of the decline in the labor force participation rate from 2007-2013, a trend likely to continue. The concern with an aging population is that it reduces the labor force, thus impeding growth (for an overview of the literature on aging and economic growth see Bloom *et al.* (2010)). Kotschy and Sunde (2018) explore how the interplay of population aging and human capital accumulation affect economic growth, concluding that there is potential to offset the negative effects of population aging by increasing education levels. Another potential remedy to counteract the effects of population aging may be close at hand: Japan, the prime example of the adverse effects of a declining population on labor markets, is reported to have successfully invested in automation technology, thereby mitigating the negative impact of the population shrinking on economic growth.[3] This suggests increased automation as a potential solution for problems caused by labor shortages, especially in capital-rich countries facing a decreasing labor force (see also Acemoglu and Restrepo (2017)).

Automation of course is one of the most prevalent topics in the 21st century. However, there are clear limits to how well human labor can be substituted for by machines. It is generally agreed upon in previous reporting and research, that mainly low-skilled jobs are threatened by automation (e.g. Brynjolfsson and McAfee (2011) Frey and Osborne (2017), Nedelkoska and Quintini (2018), De Vries *et al.* (2020), Acemoglu and Loebbing (2022)). This literature emphasizes the role education and skills play in the discussion of automation potential. It also highlights a shortcoming in the literature analyzing the

[1]The Economist (05.12.2019): "Japan's economic troubles offer a glimpse of a sobering future"
[2]For an illustration of the trend using World Bank data, see Figure 2 in the Appendix.
[3]The Economist (27.02.2013): "Doing more with less?"

relationship between demographic change and automation. This shortcoming is a failure to account for the role of education.

This paper extends the existing theory relating population changes and automation by including education in a general equilibrium model. Subsequently, the comparative statics derived from the model are tested empirically. For the empirical part of the paper, a new, freely available measure for automation is constructed. It can facilitate further research contributing to the literature studying the effect of demographics on automation specifically, and automation more generally. There is of course a large literature on structural change and how it depends on human capital and thus education (see for example Teixeira and Queirós (2016), Cruz (2019), and Porzio *et al.* (2022)). However, papers in that literature do not analyze the interplay of demographic change, technology, and human capital. One exception is the working paper by Peralta and Gil (2022), who propose a theoretical model in which individuals choose education and fertility in the presence of automation. The paper analyzes how demographic change affects automation, human capital, and the skill premium, but not how education impacts the effect demographic change has on automation.

In independent papers, Abeliansky and Prettner (2021) and Acemoglu and Restrepo (2022) study the effect of demographic change on automation. This paper differs from those papers in two aspects. First, this paper shows both theoretically and empirically that education has an important impact on the relationship between demographic change and automation. The theoretical prediction regarding the relationship between demographic change and automation changes drastically when extending the model proposed by Abeliansky and Prettner (2021) to include education.[4] While Acemoglu and Restrepo (2022) acknowledge that education may influence the relationship between demographic change and automation, they do not explore the impact theoretically or empirically.

In previous papers, the relationship between automation and population growth has been found to be negative. The intuition behind that finding is that automation capital can

[4]In an extension of their model, Abeliansky and Prettner (2021) differentiate between two skill groups as input factors, but do not analyze the impact changes in education levels have on the relationship between automation and demographic change.

substitute for unskilled labor as a production input. Intuitively, the labor of an unskilled worker at a production line can easily be replaced by an industrial robot. In the case of negative population growth, a shortage in the labor market can be compensated for by increased use of automation capital. In contrast, it is much harder to replace skilled labor with machines. Quite to the contrary, machines such as personal computers are likely to increase the output of skilled labor. In summary, automation capital generally acts as a substitute for unskilled labor and as a complement to skilled labor.

This intuitive understanding of the changing nature of automation capital, depending on the skill level of the labor force, is formalized in an analytically tractable model. The model is used to show the mechanism through which population growth affects automation capital and how that mechanism depends on the education level of the population.

In the case of negative population growth causing a shortage in unskilled labor, automation capital can help to ameliorate the negative effect of unskilled labor shortage on output. If instead negative population growth causes a shortage in skilled labor, the need for automation capital decreases. The mathematical characterization of the mechanism highlights how the relationship between population growth and automation depends on the education level of the population. These results are derived under the assumption of an exogenously given level of education in the population and a fixed stock of capital for a given period, which can be allocated between automation uses and traditional uses within a period.

The second contribution of this paper is the combination of patent data with a classification of patents into automation and non-automation categories, thereby constructing a novel cross-country panel of an automation measure. This data set is subsequently used to test the new theoretical predictions. Acemoglu and Restrepo (2022) also use patent data to measure innovation in automation technology but focus on a very narrow definition of automation patents.[5] Additionally, by relying solely on USPTO data, they only use a subsample of patents filed worldwide, diminishing the patent measure's reliability and

[5] They only use patents classified as 901 under the USPTO as a measure for automation innovation.

representativity. As a more general caveat, their reported evidence comes from estimating long-time differences. As a consequence, they do not use fixed effects and rely on quite small sample sizes of 60 and 31 for their regressions. The first aspect raises questions about omitted variable bias and the second leads to statistically insignificant results.

In comparison, the novel cross-country panel used in this paper has two advantages. One, a much larger database of patents is used. Two, the definition of what constitutes an automation patent is broader and thus well suited to explore the relationship between demographic change and automation in general, instead of being limited to exploring the narrow relationship between demographic change and industrial robot utilization. This aspect seems especially relevant given the rapid advances and utilization of software in production processes. The validity and suitability of appropriately classified patent data as an automation measure are demonstrated by testing similar hypotheses on the relationship between demographic change and automation and arriving at the same results as when using data on industrial robot shipments provided by the International Federation of Robotics.

This paper is related to the growing literature on automation and its economic effects (see e.g. Dechezleprêtre *et al.* (2019), Prettner and Bloom (2020), Krenz *et al.* (2021), and Mann and Püttmann (2023)). While the effect of automation on wages and employment of different skill levels has been studied before (e.g. Acemoglu and Restrepo (2018), Graetz and Michaels (2018)), to the best of my knowledge the effect of education on automation has not.

The existing literature relies heavily on data gathered by the International Federation of Robotics (IFR).[6] It reports the yearly delivery of "multipurpose manipulating industrial robots" for several countries, starting in 1993. This data set has two main drawbacks. First, the data only starts in 1993, and second, it can only be obtained for a high fee, possibly deterring some researchers from engaging with the automation topic. Another feature of the IFR data set is its uniqueness, which guarantees consistency in the automation measure

[6]For an exception see Dechezleprêtre *et al.* (2019) and Mann and Püttmann (2023), who use patent data.

used in the economic literature. On the one hand, this means that different studies and the findings therein can easily be compared, on the other hand, it inhibits testing for out-of-sample consistency of any findings. This paper contributes to the automation literature by making use of a novel and more comprehensive automation measurement.

The paper is structured as follows. In Section 2, the model proposed by Abeliansky and Prettner (2021) is extended to include education. The theoretical analysis suggests that a higher education level reduces the possibility to automate labor as a response to a decreasing labor force. Section 3 derives an estimation equation and details the construction of the new data set. Section 4 consists of three parts. In Section 4.1, results emphasizing the relevance of including education in any reduced form estimation analyzing the relationship between demographic change and automation are presented. The robustness of the results is tested along several lines in Section 4.2. Section 4.3 shows a replication of the analysis done by Abeliansky and Prettner (2021) using patent data to measure automation. It once again emphasizes the importance to account for education, and, by reproducing the original results, shows that patent data provides an apt measure of automation. Section 5 concludes.

2 Theory

This section outlines a neo-classical, general equilibrium model that illustrates a channel through which education affects the relationship between demographic change and automation. To do so, the standard neo-classical model is extended in two ways. One, there are two types of labor used to produce output, skilled labor and unskilled labor.[7] Importantly, the share of skilled labor in the labor force is assumed to be exogenously given and not determined by an endogenous choice of households. And two, there are two types of capital used for production, traditional capital and automation capital. Capital is assumed to be fully mobile between traditional uses and automation uses within a period. The model consists of two parts, an intertemporal utility maximization by a representative household, which pins

[7] This is the crucial aspect in which the model proposed here differs from the one put forward by Abeliansky and Prettner (2021). The model by Abeliansky and Prettner (2021) is a limit case of the model proposed here and discussed in more detail in Section 2.3.

down the capital stock available for production in each period, and an intratemporal output maximization by a representative firm, which takes the available capital stock as given. The intertemporal utility maximization problem is not specific to the novel channel proposed here, such that its discussion is relegated to the Appendix A.1. Its main contribution is to show that the equilibrium capital stock per capita available for production in each period is independent of population growth and, absent technological growth, constant over time. The rest of this section will focus on discussing the intratemporal maximization of output, given demographic change and the possibility to use capital for automation purposes, highlighting the role of education.

2.1 Basic Assumptions

Time is discrete and indexed by $t = 0, 1, 2, ...$ In each period, labor services, capital services, and final output are traded. There is a continuum of infinitely lived households with mass N_t, who are endowed with one unit of labor each. Population grows at rate n_t between time t and time $t + 1$. Households differ in their skill level $S \in \{L, H\}$, where $L_t = (1 - e) \cdot N_t$ and $H_t = e \cdot N_t$, with $e \in [0; 1]$, refer to the unskilled labor force and the skilled labor force, respectively. Importantly, the share of educated population e is modeled to be exogenously given and constant across time, which is why it has no subscript t. Besides being endowed with labor, households also own all capital. Other than in their skill level, households are assumed to be identical. Households maximize their lifetime utility by choosing consumption and investment optimally, taking prices as given. The intertemporal utility maximization results in a constant equilibrium level of capital per capita $\tilde{k}_t$, which is owned by the households and available for production in each period.

2.2 Production

Firms operate under perfect competition, take prices as given and make zero profits in equilibrium. In the following, the actions of one representative firm are considered. Output is produced by combining traditional capital K, automation capital P, and skilled and unskilled labor $H = e \cdot N$ and $L = (1-e) \cdot N$, where e refers to the share of the skilled labor force and is constrained by $e \in [0;1]$. Capital is assumed to be mobile between traditional uses and automation uses. Due to capital mobility, the overall capital stock $\tilde{K}_t$ is divided between automation uses P_t and traditional uses K_t such that output is maximized and $\tilde{K}_t = K_t + P_t$. The production function is assumed to be Cobb-Douglas, ensuring analytical tractability. Specifically, consider a constant returns to scale, nested Cobb-Douglas production function of the form:

$$F(K_t, P_t, N_t) = K_t^{\alpha} \left(\left((1-e)\, N_t + P_t \right)^{\beta} \left(eN_t \right)^{1-\beta} \right)^{1-\alpha}. \tag{1}$$

In the way automation capital P is introduced to the production function, it is a perfect substitute for unskilled labor $(1-e)N$ but acts as a complement to skilled labor eN. This modeling choice is justified for example by the findings presented in Griliches (1969), and, more recently, Krusell *et al.* (2000), Acemoglu and Restrepo (2020), and Prettner and Strulik (2020).

The firm's maximization problem is given by

$$\begin{aligned} \max_{K_t, P_t} \pi_t = \rho_t (Y_t - r_t^{trad} K_t - r_t^{auto} P_t - w_{H,t}(eN_t) - w_{L,t}(1-e)N_t) \\ s.t.\ Y_t = F(K_t, P_t, N_t, e) \\ \tilde{K}_t = K_t + P_t, \end{aligned} \tag{2}$$

where ρ_t refers to the market price of output Y_t, which is normalized to one in the following, such that $\rho \equiv 1$. $w_{H,t}$ refers to the wage rate of skilled labor, and $w_{L,t}$ refers to the wage rate of unskilled labor in period t. The firm takes ρ_t, $w_{H,t}$, $w_{L,t}$, $\tilde{K}_t$ and N_t as given and faces a static optimization problem. Therefore, time subscripts are dropped for the following analysis whenever possible.

The equilibrium wage rates w_H and w_L are given by the marginal product of the respective labor input, using the definition of $H = e \cdot N$ and $L = (1-e) \cdot N$.

$$
\begin{aligned}
w_H = \frac{\partial Y}{\partial H} &= (1-\alpha)(1-\beta)\frac{Y}{H} \\
w_L = \frac{\partial Y}{\partial L} &= (1-\alpha)\beta\frac{Y}{L+P}
\end{aligned}
$$

$$
\begin{aligned}
\frac{w_H}{w_L} &= \frac{1-\beta}{\beta} \cdot \frac{L+P}{H} \\
\frac{\partial(w_H/w_L)}{\partial P} &= \frac{1-\beta}{\beta} \cdot \frac{L}{H} > 0
\end{aligned} \tag{3}
$$

The ratio of $\frac{w_H}{w_L}$ measures the skill premium paid to skilled labor. If automation capital P increases, the skill premium increases as well. So increased utilization of automation capital affects the skill premium in a similar way as skill-biased technological change does.[8]

To determine how changes in the size of the labor force affect the optimal distribution of $\tilde{K}_t$ between K_t and P_t from the firm's point of view, the assumption of full mobility of capital is used. The return on automation capital P is given by its marginal product:

$$
r^{auto} = \frac{\partial Y}{\partial P} = (1-\alpha)\beta\frac{Y}{(1-e)N+P}. \tag{4}
$$

Likewise, the return on traditional capital K is given by its marginal product:

$$
r^{trad} = \frac{\partial Y}{\partial K} = \alpha\frac{Y}{K}. \tag{5}
$$

As capital is fully mobile between traditional and automation uses, the optimal allocation of $\tilde{K}$, which maximizes output, can be obtained by setting the marginal products of K_t and P_t equal and rearranging. let K^* denote the optimal amount of traditional capital, which can be derived by using the equality of marginal products and plugging in $P = \tilde{K} - K$.

[8]For an overview of the pertinent literature, see Violante (2008).

$$K^* = \left((1-e)N + \tilde{K}\right)\frac{\alpha}{\alpha + (1-\alpha)\beta}$$

Analogously, this can be done for P^*, which denotes the optimal amount of automation capital, in which case $K = \tilde{K} - P$ is plugged into the equalized marginal products.

$$P^* = \tilde{K}\frac{(1-\alpha)\beta}{\alpha + (1-\alpha)\beta} - (1-e)N\frac{\alpha}{\alpha + (1-\alpha)\beta}$$

The maximum output obtainable given $\tilde{K}$ and N can be derived by plugging K^* and P^* into the production function given by (1):

$$Y^* = \left(\frac{\alpha}{\alpha + (1-\alpha)\beta}\right)^{\alpha}\left(\frac{(1-\alpha)\beta}{\alpha + (1-\alpha)\beta}\right)^{\beta(1-\alpha)}\left((1-e)N + \tilde{K}\right)^{\alpha + (1-\alpha)\beta}(eN)^{(1-\alpha)(1-\beta)}.$$

And finally, the equilibrium interest rate r^* can be derived by plugging either K^* or P^* into the respective marginal product and rearranging.

$$r^* = r^{trad} = r^{auto}$$
$$r^* = (\alpha + (1-\alpha)\beta)\frac{Y^*}{(1-e)N + \tilde{K}}$$

To see how an increase in population size N affects the two kinds of capital, consider the respective derivatives with respect to N:

$$\frac{\partial K^*}{\partial N} = (1-e)\frac{\alpha}{\alpha + (1-\alpha)\beta} \geq 0$$

$$\frac{\partial P^*}{\partial N} = -(1-e)\frac{\alpha}{\alpha + (1-\alpha)\beta} \leq 0.$$

The derivative of K^* with respect to N shows, that as the labor force N grows, more of total capital $\tilde{K}$ is used in traditional ways and not for automation purposes. The derivative of P^* with respect to N shows, that as the labor force N grows, less of total capital $\tilde{K}$ is used for automation purposes. This demonstrates clearly that as N increases, more capital is allocated towards traditional uses K and away from automation uses P. Furthermore, for $e = 0$ the size of the effect N has on the capital allocation is at its maximum, with the effect size decreasing as e increases.[9] For $e = 1$, the intra-period allocation of $\tilde{K}$ is independent of the population size N. This can also be demonstrated by looking at the ratio of K^* and P^* and its derivative with respect to N directly. Again, it is obvious that N affects the ratio most if $e = 0$ and the labor force is unskilled, whereas N does not affect the ratio if $e = 1$.

$$\frac{K^*}{P^*} = \frac{\alpha((1-e)N + \tilde{K})}{\tilde{K}(1-\alpha)\beta - \alpha(1-e)N}$$

$$\frac{\partial(K^*/P^*)}{\partial N} = \frac{(1-e)\alpha\tilde{K}(\alpha + (1-\alpha)\beta)}{\left(\tilde{K}(1-\alpha)\beta - \alpha(1-e)N\right)^2} \geq 0$$

A change in the population size N affects the marginal products of the two types of capital, K and P, and the marginal product of the overall capital stock $\tilde{K}$. Specifically, N affects the marginal product of K and P in such a way, that it entails a reallocation of capital from automation uses towards traditional uses, as the positive sign of the derivative above demonstrates. The effect of N on the marginal product of the overall capital stock $\tilde{K}$ is universally positive. The respective derivatives are shown in Appendix A.2.

Note, that the marginal effect of N on $\frac{K^*}{P^*}$ is derived under the implicit assumption of $\frac{\partial \tilde{K}}{\partial N} = 0$. The optimal allocation of $\tilde{K}$ between traditional uses K and automation uses P considered here corresponds to a short-run output maximization problem, as specified in (2).

[9]Mathematically, this can be demonstrated by looking at the limit cases of the partial derivatives of K and P with respect to N.

$$\left.\frac{\partial K}{\partial N}\right|_{e=0} > \left.\frac{\partial K}{\partial N}\right|_{e=1}$$
$$\left.\frac{\partial P}{\partial N}\right|_{e=0} < \left.\frac{\partial P}{\partial N}\right|_{e=1}$$

Output is maximized in each period, taking population size N and the overall capital stock $\tilde{K}$ as given. From the intertemporal utility maximization of households discussed in Appendix A.1, a constant optimal per capita capital stock $\tilde{k}^*$ can be derived, which is independent of population growth. In general, this is not equivalent to a capital stock $\tilde{K}_t$ which is independent of population growth. However, if the variation in n_t (and hence also N_t) is unpredictable, which seems like a reasonable assumption, the capital stock in each period is independent of the unforeseen variation in the contemporaneous population growth and population size. Therefore, it is quite likely that $\tilde{K}_t$ does not react to unpredictable changes in n_t and treating $\tilde{K}_t$ as independent of unsystematic variation in n_t and hence also N_t is, after all, appropriate. In addition, empirical studies have found the capital stock to be quite slow in responding to shocks (see for example Ashraf *et al.* (2008), discussing the effect of demographic changes on capital accumulation). In light of that evidence, disregarding the marginal effect N_t has on $\tilde{K}_t$ is a mild assumption, given that in this model only the short run is considered. In any case, the results derived above carry through when taking into account that $\frac{\partial \tilde{K}_t}{\partial N_t} \neq 0$ under the assumption that $\tilde{K}_t > N_t$. From this, longer-run implications from the model can be derived. For a derivation and discussion of the results, see Appendix A.3.

Education is the new feature of the model and its effect on the return to automation capital is of special interest. The cross derivative of r^{auto} with respect to e and N is universally positive. This implies that education has the potential to mute any negative effects an increase in the population size has on the return on automation capital. The equivalent derivative for the return on traditional capital is always negative. Thus the difference in the effect population size has on the return on traditional- and automation capital diminishes as the share of the educated workforce e increases. The respective derivatives are shown in Appendix A.2. The effect of e on the return on the overall capital stock $\tilde{K}$ is universally positive:[10]

$$\frac{\partial r^*}{\partial e} = (\alpha + (1-\alpha)\beta)\,\frac{1}{\left((1-e)N + \tilde{K}\right)^2}\left(\left((1-e)N+\tilde{K}\right)\cdot\frac{\partial Y^*}{\partial e} + Y^*\cdot N\right) =$$

$$= (\alpha + (1-\alpha)\beta)\,\frac{1}{\left((1-e)N + \tilde{K}\right)^2}\left(\frac{1}{e}Y^*(N+\tilde{K})\right) > 0.$$

2.3 Limit Cases of the Production Function

The elasticity of substitution between labor and automation capital depends on the skilled share of the labor force. This difference in complementarity results in a different sign of $\partial r^{auto}/\partial N$, as discussed in Appendix A.2, depending on whether the population is skilled or unskilled. In this setup, education plays a crucial role in determining the effect of population growth, and hence demographic change, on automation. To better understand how education influences the effect population size has on the incentive to automate, consider the two limit cases of $e = 0$, a fully unskilled labor force, and $e = 1$, a fully skilled labor force.

2.3.1 Fully Unskilled Labor Force

With $e = 0$, the production function reduces to the one proposed by Abeliansky and Prettner (2021).

$$Y = K^{\alpha}\left(N + P\right)^{1-\alpha} \tag{6}$$

The marginal product of automation capital P and the marginal product of traditional capital K are given by

$$r^{auto} = \frac{\partial Y}{\partial P} = (1-\alpha)\frac{Y}{N+P}$$

[10]The derivation makes use of $\frac{\partial Y}{\partial e} = Y \cdot \left(\frac{(1-\alpha)(1-\beta)(N+\tilde{K}) - eN}{e((1-e)N+\tilde{K})}\right)$.

$$r^{trad} = \frac{\partial Y}{\partial K} = \alpha \frac{Y}{K}$$

Analogously to before, we can equate the marginal products due to full capital mobility and set $P = \tilde{K} - K$ and $K = \tilde{K} - P$ to derive the optimal levels of traditional capital K^*, and automation capital P^*. Plugging K^* and P^* into the production function specified in (6) gives the maximum output level obtainable for a given N and $\tilde{K}$.

$$\begin{aligned} K^* &= \alpha(N + \tilde{K}) \\ P^* &= (1-\alpha)\tilde{K} - \alpha N \\ Y^* &= \alpha^\alpha (1-\alpha)^{1-\alpha}(N + \tilde{K}) \end{aligned}$$

Finally, the equilibrium interest rate r^* can be derived by plugging either K^* or P^* into the respective marginal product and rearranging.

$$\begin{aligned} r^* &= r^{trad} = r^{auto} \\ r^* &= \frac{Y^*}{N + \tilde{K}} \end{aligned}$$

To analyze how a change in N affects the optimal allocation of $\tilde{K}$ between K and P, consider the respective derivative of the optimal level with respect to N:

$$\begin{aligned} \frac{\partial K^*}{\partial N} &= \alpha > 0 \\ \frac{\partial P^*}{\partial N} &= -\alpha < 0. \end{aligned}$$

The same can be done for the ratio of optimal K^* to optimal P^*:[11]

$$\begin{aligned}\frac{K^*}{P^*} &= \frac{\alpha(N+\tilde{K})}{(1-\alpha)\tilde{K}-\alpha N} \\ \frac{\partial(K^*/P^*)}{\partial N} &= \frac{\tilde{K}}{\left((1-\alpha)\tilde{K}-\alpha N\right)^2} > 0.\end{aligned}$$

2.3.2 Fully Skilled Labor Force

In the second case of $e = 1$, the whole population is educated and the elasticity of substitution between automation capital and skilled labor is equal to one.

$$Y = K^{\alpha}\left(P^{\beta}N^{1-\beta}\right)^{1-\alpha} \tag{7}$$

The marginal product of automation capital P and the marginal product of traditional capital K are given by

$$r^{auto} = \frac{\partial Y}{\partial P} = (1-\alpha)\beta\frac{Y}{P}$$

$$r^{trad} = \frac{\partial Y}{\partial K} = \alpha\frac{Y}{K}$$

Analogously to before, the optimal levels of K^* and P^* pinning down the optimal allocation of $\tilde{K}$ can be derived by using the full mobility of capital assumption and equating the marginal products. Plugging in $K = \tilde{K} - P$ and $P = \tilde{K} - K$ yields:

$$\begin{aligned}K^* &= \frac{\alpha}{\alpha+(1-\alpha)\beta}\tilde{K} \\ P^* &= \frac{(1-\alpha)\beta}{\alpha+(1-\alpha)\beta}\tilde{K}.\end{aligned}$$

[11] As in Section 2.2, the derivative shown here disregards the effect of N on $\tilde{K}$. The reasoning for the approach is the same as above. Also, the result of $\partial(K^*/P^*)/\partial N > 0$ carries through when taking into account that $\partial\tilde{K}/\partial N \neq 0$ if $\tilde{K} > N$, as demonstrated in Appendix A.3.

The maximum obtainable level of output for given N and $\tilde{K}$ is derived by plugging K^* and P^* into (7)

$$Y^* = \left(\frac{\alpha}{\alpha + (1-\alpha)\beta}\right)^{\alpha} \left(\frac{(1-\alpha)\beta}{\alpha + (1-\alpha)\beta}\right)^{\beta(1-\alpha)} \tilde{K}^{\alpha+(1-\alpha)\beta} N^{(1-\alpha)(1-\beta)}.$$

The equilibrium interest rate r^* is derived by plugging K^* or P^* into the respective marginal product.

$$r^* = r^{trad} = r^{auto}$$
$$r^* = (\alpha + (1-\alpha)\beta)\frac{Y^*}{\tilde{K}}$$

If the population is fully educated, skilled labor is a complementary input to both traditional capital and automation capital. An increase in the labor force due to population growth thus increases the return on automation- and traditional capital equally. The optimal ratio of traditional- and automation capital is in this case determined solely by exogenous parameters and thus independent of population size N.

$$\frac{\partial K^*}{\partial N} = 0$$
$$\frac{\partial P^*}{\partial N} = 0$$
$$\frac{\partial (K^*/P^*)}{\partial N} = 0$$

2.4 Population Growth

So far, it has been shown that the effect of population size on the return on automation capital depends on the education level of the labor force. The effect of population size on traditional capital and its return is always positive. The effect of population size on the return on automation capital however is positive if the labor force is skilled and negative if the labor force is unskilled. This is driven by automation capital acting as a complement to skilled labor and as a substitute for unskilled labor input. In the case of an unskilled labor force, capital is thus shifted from automation to traditional uses if the population level increases. In the case of a skilled labor force, no capital is shifted, since the marginal effects are equal in the optimum.

Next, consider how population growth affects automation capital per capita. Focusing on automation capital per capita has two advantages over considering automation capital levels. One, it is the more natural measure for cross-country analysis. And two, it is more closely linked to population growth, which is the main variable of interest when analyzing demographic change. Turning next to the effect of population growth, it can be shown that it potentially exerts two forces on automation capital per capita.

Define $y_t = \frac{Y_t}{N_t}$, $k_t = \frac{K_t}{N_t}$ and $p_t = \frac{P_t}{N_t}$. If the labor force is partially educated, output per capita is given by

$$y_t = k_t^{\alpha} \left((1 - e + p_t)^{\beta} e^{1-\beta} \right)^{1-\alpha}.$$

Again, the marginal products of the two kinds of capital have to be equal in the optimum. Equalizing the marginal products, plugging in $p = \tilde{k} - k$ or $k = \tilde{k} - p$, and rearranging, the optimal ratio of the two kinds of capital per capita can be derived as

$$\frac{k^*}{p^*} = \frac{\alpha(1 - e + \tilde{k})}{\tilde{k}(1-\alpha)\beta - \alpha(1-e)}, \tag{8}$$

where $\tilde{k} = \frac{\tilde{K}}{N}$ denotes the overall capital stock per capita available for production. In this intensive form formulation of the model, deviations in the population growth rate n from its balanced growth path value result in variation in $\tilde{k}$, which is constant on the balanced growth path. A negative deviation of n from its balanced growth path value leads to a positive deviation of $\tilde{k}$ from its balanced growth path value. This results in a decrease in the optimal ratio of $\frac{k^*}{p^*}$, as the negative sign of the following derivative demonstrates:

$$\frac{\partial(k^*/p^*)}{\partial \tilde{k}} = -\frac{\alpha(1-\alpha)(1-e)}{\left(\tilde{k}(1-\alpha)\beta - \alpha(1-e)\right)^2} \leq 0.$$

A negative deviation of n from its balanced growth path value thus leads to an increased share of overall available per capita capital stock $\tilde{k}$ to be allocated towards automation uses. It is obvious that an increase in the education level e ameliorates this effect. For a fully educated labor force and $e = 1$, a variation in n and $\tilde{k}$ does not affect the optimal ratio of $\frac{k}{p}$.

Intuitively, as population growth decreases, the labor force decreases which, if it is unskilled ($e = 0$), is a perfect substitute for automation capital, thereby increasing the marginal product of automation capital. As capital is allocated endogenously to traditional and automation uses, this results in a capital allocation towards automation uses.[12] If $e \in (0;1)$, population growth not only increases the substitute input for automation capital unskilled labor but also the complement input skilled labor. A higher e can therefore attenuate the positive effect lower population growth has on p. If $e = 1$, the ratio of $\frac{k}{p}$ is unaffected by an increase in n, as both kinds of capital are complements to skilled labor. For a derivation of the results and a separate discussion of the two limit cases of the production function in per capita terms, see Appendix A.4. Neither of the limit cases is relevant anywhere in the world. Therefore, it is important to take education levels into account when analyzing the relationship between population growth and automation capital.

3 Empirical Relevance

The empirical question revolves around the relationship between the incentive to automate and demographic change with a focus on how education influences that relationship. This section first describes the data used for empirical estimation in detail, with an emphasis on the newly created automation measure. In the second step, an estimation equation based on the theoretical analysis in the previous section is derived.

[12]The great advantage automation provides, is that growth can be generated simply by accumulating capital. The size of this additional growth opportunity is determined by how important automation capital is relative to traditional capital. By increasing automation capital, capital per capita can be deepened in a growth-enhancing manner. The size of p effectively measures how much use the economy makes of growth by accumulation. An increase in p thus increases the economic growth potential.

3.1 General Data Description

This section describes the data used to test the theoretical relationship described above empirically. If not indicated otherwise, five-year averages of all data are taken. Doing so reduces noise in the data and partially addresses timeliness concerns regarding patent filings. Given the time span of available data, this results in 9 periods of observation which can be used for estimation.

All data discussed in this paragraph is taken from the World Bank.[13] Information on both total population and population by age group is utilized. The latter is used to construct the working-age population, defined to be aged 20-64. For the regressions, the log of population growth is used as an explanatory variable. One inherent property of growth rates is, that they naturally and frequently take on negative values. To avoid the loss of many observations, the growth rates are transformed linearly by adding the absolute value of the smallest growth rate observed in the data to all observations before taking the log. This is equivalent to a linear rescaling of the variable and does not affect its correlation with any other variable, such that regression results are unaffected by the linear transformation. As a measure of savings, gross fixed capital formation as a share of GDP is used. For robustness checks, some additional variables are considered. GDP per capita is measured in 2015 US Dollars, the openness of the economy is calculated as the external balance on goods and services measured in percent of GDP, and the importance of the service sector is measured as the value added by the service sector in percent of GDP.

Education plays an important role in the theoretical predictions. Specifically, the effect of population growth on automation depends on the share of skilled labor in the economy. The education measure used comes from Barro and Lee.[14] It reports the population share with at least completed secondary education in 5-year intervals.

The core data used is patent data published by the OECD for 59 countries, starting in 1977 and ending in 2020. For a list of all countries with available patent data see Appendix

[13] The data is freely available at `https://databank.worldbank.org/`.

[14] It can be downloaded at `http://barrolee.com/`.

B.4. It is combined with the classification of patent categories into automation and non-automation categories developed by Dechezleprêtre *et al.* (2019) to arrive at a count of automation patents for each country-year observation with available data.

3.2 Patent Data

The vast majority of the literature analyzing the economic effects of automation uses data on industrial robots supplied by the IFR. In this paper, freely available patent data is used to measure automation instead. As this is fairly new, it is discussed in detail in the following.

3.2.1 Classifying Patent Data

Dechezleprêtre *et al.* (2019) use data from patents filed with the EPO to develop a classification of patent categories into automation and non-automation. Two different classifications are proposed. In each patent category, the share of patents described using automation keywords is calculated. The patent categories are then ordered by their share of automation patents. Two cutoff thresholds are considered to classify a patent category as an automation patent category. The stricter one defines all patent categories at or above the 95th percentile of the distribution of the automation patent shares as automation categories. The less strict one defines all patent categories at or above the 90th percentile of the automation patent shares as automation categories. This results in 5% or 10% of all patent categories being defined as automation patent categories. In a final step, all patents belonging to a thus-defined automation patent category are summed at the country-year level to arrive at a raw number of automation patents for each country-year observation. This results in the patent measures $auto95$ and $auto90$.

In addition to those two measures introduced by Dechezleprêtre *et al.* (2019), a third automation measure using patents is proposed here. By counting all patents belonging to a patent category with the highest share of automation patents, considerable noise may be introduced to the automation measures $auto90$ and $auto95$. The newly proposed measure addresses that concern. In the first step, the number of patents within each patent category is multiplied by the share of automation patents in that patent category. In the second step, the resulting number of automation patents belonging to different patent categories is then summed at the country-year level, resulting in one number of automation patents for each country-year observation. This measure is henceforth called $auto1$ and it is the preferred measure for automation patents.

3.2.2 Empirical Considerations

So far, empirical analyses have mainly used data on robots to measure a country's automation level. The IRF data provides information on the yearly installation of multipurpose industrial robots at the country level. Theoretically, using robot data has the advantage of directly measuring how much automation technology is employed. However, it is unclear, how long robots can operate, and at which point in time they are outdated or defunct. Thus, to estimate the stock of robots used in a country, assumptions about the service life of robots have to be made, which is complicated by a likely variation of the service life across time, due to differences in the pace of innovation, and variation in the service life across application areas of the robots. The alternative to using an inevitably noisy estimate of the robot stock is to focus on newly acquired robots. Such a measure will however vary strongly with business cycles, making averaging over several periods necessary and reducing the number of available data points.

This paper proposes an alternative measure of automation, namely automation patents. Conceptually, automation patents measure innovation in the realm of automation and provide an imperfect measure of a country's automation level, just like robots. Berkes *et al.* (2022) evoke the idea of patents as a means "to ensure that investments in new ideas can be recovered with future profits". With that concept in mind, a patent's economic value is equivalent to the present value of the innovation it is protecting. While the market value of patents is in general not known, the number of patents is a helpful, if not perfect, measure of the present value of the ideas protected by them. Automation patents, therefore, provide a measure of investment into research directed toward automation, the level of which is directly linked to the expected present value of such research. One determinant of the present value of automation patents is the demand for automation. In summary, automation patent data provides an alternative and potentially even better measure of the present value of automation in a country than the flow of industrial robots does.

One concern regarding the suitability of patent data as a measure of automation is that they measure ideas, which, contrary to robots, are mobile across countries. In extreme cases, countries may adopt and use automation technology prolifically without registering any automation patents themselves. For such countries, the use of automation capital in the production process is underestimated when relying on automation patents as a measure of automation capital. That, however, is unlikely to occur for two reasons. One, there is a large literature finding that the investment required to adopt foreign technology is similar to the investment required to generate new technology (see, for example, Cohen and Levinthal (1989), Griffith *et al.* (2004) and Aghion *et al.* (2009), p. 151 ff). Therefore, it is unlikely that a country is adopting automation technology without it also generating some automation patents at the same time. This may also be related to a second aspect found in empirical studies, namely that there is a considerable time lag between a technology's invention in one country and its adoption in other countries (see Comin and Hobijn (2010) and Comin and Mestieri (2018), who find a minimum lag of adoption of 5-8 and 7-12 years, respectively). Together, these aspects make it unlikely that the mobility of ideas causes a systematic measurement error or bias in the automation measure constructed using patent data.

Patent data has the great advantage of being widely available, very granular, and detailed. Because so many details are given, most disadvantages that are inherent to patent data can be addressed and possibly dispelled completely.[15] Patent data only provides an indirect measure of automation technology, which certainly is its main disadvantage. Many patents are filed, but only a few are applicable in the industry and thus of real economic value.

[15]The OECD provides an in-depth discussion of the patent data provided by it, see the OECD Patent Statistic Manual (2009) at https://www.oecd-ilibrary.org/science-and-technology/oecd-patent-statistics-manual_9789264056442-en.

This concern can be addressed by using only patents filed under the Patent Co-operation Treaty (PCT), using only patents filed at the EPO, the Japan Patent Office (JPO), and at the USPTO at the same time (referred to as the Triadic family by OECD) or using only patents protected in at least two international patent offices worldwide, one of which within the Five IP offices (IP5), namely the EPO, JPO, USPTO, the Korean Intellectual Property Office (KIPO) and the People's Republic of China National Intellectual Property Administration (CNIPA). Filing a patent is time intensive and costly. Those patents that meet one of the three filing requirements are all but certain to be a subset of the most important and thus economically valuable patents in terms of expected present value. Focusing on this subset of patents also ensures that the patents considered are not affected by different propensities to patent across countries or industries, as only international patents are used in the first place. Another potential drawback of using patents is that changes in patent laws may affect the propensity to patent. As all patents considered here have to be filed under international laws, changes in national patent laws are likely irrelevant. Additionally, as the empirical analysis will be across time, including time-fixed effects will take care of potential problems caused by changes in international patenting laws.

In principle, PCT patents, Triadic family patents, and IP5 patents are equally suited for analysis. However, the count of IP5 patents is suited best for the analysis at hand. While today a patent filed under the PCT is automatically protected in all PCT countries, this is only the case since 2004. Before that, there were fewer member states of the PCT, and the fees for PCT patents increased in the number of countries where the patent was filed. Thus PCT patent data are very well suited for analysis starting in 2004 but less reliable before that and therefore not well suited for the research question at hand. Regarding Triadic family patents, the main drawback is its acknowledged lack of timeliness. Since the goal of this paper is to relate changes in patents and population growth over time, timeliness is relevant. This makes IP5 patents the preferred measure of patenting activity for this paper and the one used if not stated otherwise.

Patents are of course a broad measure of technological progress. Making use of the classification put forward by Dechezleprêtre *et al.* (2019), only those patents that are related to automation technologies are used in the analysis. Thus the lack of specificity can be addressed by using the vast additional data provided with each patent. This makes the invention of automation technology and therefore a competitive edge of economies directly measurable.

And lastly, from a researcher's perspective, it will always be interesting to use different measures for the same underlying object of interest. It not only justifies revisiting old ideas but makes it possible to check their robustness and therefore relevance. In this case, the new data comes with an additional advantage in the time it spans. The patent data is available as far back as 1977 and thus starts much earlier than the robot data provided by the IFR. Using patent data, research over longer time periods is possible. In summary, the classified patent data provide an interesting alternative data set for empirical analysis, which has been underutilized so far.

3.3 Estimation Equation

The model developed in Section 2 demonstrates how the relationship between automation capital density and population growth depends on the level of skills present in the labor force. It illustrates how the interaction between population growth and education affects the allocation of resources toward automation uses. Therefore, education should be included in any specification trying to estimate the relationship between automation and population growth.

The goal is to identify the effect working-age population growth has on automation density, accounting for the effect education has on this relationship. The data set used for estimation has a panel structure, such that it is possible to include country-fixed and time-fixed effects in the regression. Country-fixed effects prevent omitted variables that are constant over time at the country level to bias the coefficients of interest. Additionally, time-fixed effects, which pick up variation over time affecting all countries equally, such as macroeconomic shocks, are included in the regression. This addresses concerns that results are driven by systemic economic shocks. While the inclusion of neither fixed effect guarantees the estimates to be unbiased, it is an important step toward the identification of the true parameter values.

To test the theoretical predictions, a measure for automation in per capita terms is needed as the dependent variable. For that, the variable $auto1$, the construction of which is explained in the previous section, is divided by the working-age population to construct a per capita measure of automation. The main explanatory variables are population growth and education. The economic concern with demographic change is that it affects the size of the working-age population. For that reason, the growth rate of the working-age population is calculated and used in the regressions. The share of the working-age population with at least completed secondary education is used as a measure for education. Reflecting the new theoretical results, it is important to include an interaction term of working-age population growth and education in the regression. As a control variable, gross fixed capital formation measured in percent of GDP is included in the regression to proxy for the savings rate.

To address reverse causality concerns, all regressions use a lag of one period (which is equivalent to five years) for all explanatory variables. For interpretation purposes, the log of the dependent variable and the log of the working-age population growth rate is used in the regression. Based on these considerations, the following baseline estimation equation is derived

$$\begin{aligned}\log(p_{c,t}) = \eta_0 + \eta_1 \cdot \log(n_{c,t-1}) + \eta_2 \cdot e_{c,t-1} + \eta_3 \cdot (\log(n_{c,t-1}) \times e_{c,t-1}) + \eta_4 \cdot s_{c,t-1} \\ + FE_c + FE_t + \varepsilon_{c,t}, \qquad (9)\end{aligned}$$

where $p_{c,t}$ measures automation patents per capita, $n_{c,t-1}$ refers to working-age population growth, $e_{c,t-1}$ is the share of working-age population with at least completed secondary education, and $s_{c,t-1}$ is the savings rate. The subscript c indicates that variables are measured at the country level and subscript t refers to the period of observation. $\varepsilon_{c,t}$ is the error term.

4 Empirical Results

This section analyzes the empirical relevance of the theoretical results derived in Section 2. First, results from estimating the baseline regression are reported and discussed in Section 4.1. Subsequently, these results are shown to be robust to using different measures for the outcome and explanatory variables and including additional control variables in Section 4.2. And lastly, Section 4.3 demonstrates the adequacy of the new automation measure proposed by replicating previous findings with this new data.

4.1 Main Results

The theory presented in Section 2 makes clear predictions about the relationship between population growth and automation capital per capita, and the crucial way in which this relationship is influenced by the overall education level. Using Equation (9) as an estimation equation, a fixed effects regression is run to test the model predictions. The theory makes clear predictions about the signs of the estimated coefficients. If the assumed production function is a good representation of the real world, $\hat{\eta}_1 < 0$ and $\hat{\eta}_2 > 0$ are expected. The coefficient of the interaction term is predicted to be positive $\hat{\eta}_3 > 0$ if $e \in (0; 1)$. Finally, $\hat{\eta}_4 > 0$ is predicted.

Results are reported in Table 1. Throughout, the investment variable is included as a control variable. Its coefficient is consistently estimated to be positive and it is highly significant, as expected. To emphasize the contribution made by including an interaction term between education and population growth, the explanatory variables of interest are added one by one. In column (1) the only explanatory variable is working-age population growth. Its coefficient has the expected negative sign but remains statistically insignificant. The specification corresponds to the limit case of the production function if the whole workforce is uneducated. The coefficient of working-age population growth stays statistically insignificant when education is included as an explanatory variable in column (2), the coefficient of which is also insignificant. This specification has no clear correspondence to the theoretical hypotheses. However, a cautious interpretation of the insignificant coefficient

Table 1: *Working-age Population Growth and Automation Density*

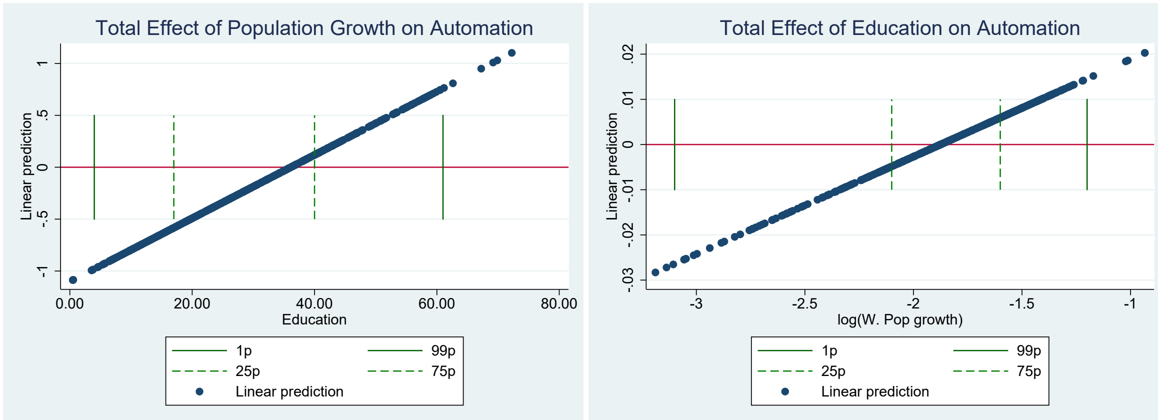
	(1)	(2)	(3)
log(W. Pop growth)	-0.1646	-0.1636	-1.0837***
	(-1.14)	(-1.13)	(-3.97)
Education		0.0004	0.0588***
		(0.07)	(3.65)
log(W. Pop growth) × Education			0.0322***
			(3.94)
Investment Share	0.0362***	0.0361***	0.0345***
	(3.47)	(3.43)	(3.36)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
R^2	0.60	0.60	0.62
Observations	328	325	325

Note: Dependent variable is the log of the automation measure *auto*1, constructed from patent data reported by the OECD and divided by working-age population to arrive at a per-capita measure. All explanatory variables are lagged by one period. log(W. Pop growth) is the log of working-age population growth. Education measures the share of the working-age population with at least completed secondary education as reported by Barro and Lee. Investment Share refers to gross fixed capital formation as a share of GDP. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

of education is that the density of automation patents does not vary with education when analyzed in isolation from population growth. In column (3), an additional interaction term between working-age population growth and education is included. This is the specification that is closest to the one derived from the theory. The theoretical hypothesis states that an increase in working-age population growth has a negative effect on automation capital per capita but that the effect is smaller the higher the education level of the workforce. This last aspect is picked up by the interaction term, which the significance of the coefficient indicates to be highly relevant.

Focusing on the estimation results reported in column (3), the results in Table 1 indicate that including the interaction effect is crucial. Given that the whole population is unskilled (implying $e = 0$ in the model), the coefficient of the interaction term can be neglected and working-age population growth is estimated to have a negative effect on the incentive to automate. The effect is quantitatively substantial. A 1% decrease in working-age population growth is associated with an increase of 1.1% in the automation measure. If in turn working-age population growth is zero, the effect of a 1% increase in the skilled population share is associated with an increase of the automation measure by 0.6%. The coefficient of the interaction term is positive as expected and highly significant. It attenuates the negative effect of working-age population growth on the automation measure, such that its effect is smaller in countries with a higher educated population share.

To interpret the effect of working-age population growth on automation, it may be helpful to look at how the effect changes depending on the level of education. Over the whole period 1977-2019, the education share takes on the values 16.7%, 28.2%, and 39.3% at the 25th, 50th, and 75th percentile of the distribution across countries. Over time, the average value of the education measure increased from 18% in 1977-1979 to 35% in 2016-2019. This is visualized in Figure 4a in the Appendix. The total effect of working-age population growth at different levels of education can be calculated as $\frac{\partial automate}{\partial popgrowth} = \eta_1 + \eta_3 \cdot educ$. The result is plotted in Figure 1a (for the sake of clarity predicted values are plotted). The effect of working-age population growth on automation density is estimated to be negative if the



(a) *Total Effect of Population Growth* **(b)** *Total Effect of Education*

Figure 1: *Visualization of Regression Results in Table 1*

Note: The Figure illustrates the regression results reported in Table 1. Panel (a) shows the predicted total effect log population growth has on automation density for different values of the education variable. Panel (b) shows the predicted total effect an increase in education has on automation density for different values of working-age population growth. In both cases, solid lines mark the 1st and 99th percentile of the distribution of the variable plotted at the x-axis and dashed lines mark the 25th and 75th percentile of the distribution.

share of the educated population is low. As the share of the educated population increases, which is equivalent to a movement along the x-axis of Figure 1a, the size of the negative effect decreases. If the share of the educated population is at 35%, the effect of working-age population growth on automation density flips sign and becomes positive. The ambiguity of the effect working-age population growth has on automation density is relevant in the real world, as the sign of the estimated coefficient flips between the 25th and 75th percentile of the distribution of the education variable. This means that there are many countries where the effect is negative, but also many where the effect is positive. It also helps to explain why the estimated coefficient of working-age population growth in column (1) of Table 1 is insignificant. If education and especially the interaction between education and working-age population growth are omitted from the regression, the resulting coefficient estimates the average effect working-age population growth has on automation density, which across countries is not statistically different from zero.

Figure 1b visualizes the total effect of education on automation density for different values of working-age population growth. Again, the sign of the relationship changes from negative to positive between the 25th and 75th percentile of the working-age population growth distribution. The effect of education on automation density thus depends on the level of working-age population growth. Averaging it across countries experiencing different levels of working-age population growth results in an estimated average effect that is insignificant, as shown in column (2) of Table 1. In addition to looking at the total effect, marginal effects of population growth at certain levels of education, and vice versa, can be plotted as well. The resulting figures are relegated to the Appendix, see Figure 3.

In summary, the empirical findings are in line with the theoretical hypotheses derived from the model. The predicted negative effect of working-age population growth on automation can be demonstrated in the data. This is especially true when the role of education is taken into account. Specifically, the model predicts a negative relationship between automation and population growth, which is mitigated by education. The empirical analysis suggests that this effect is even reversed to the positive if the education level of the workforce is sufficiently high.

4.2 Robustness

In this section, the robustness of the results presented above is tested. They are robust to using different measures of the education variable. Specifically, the share of the population with some tertiary education and the share of the population with completed tertiary education is considered instead of the share of the population with completed secondary education. Both are strict subsets of the originally used education measure. The results are reported in Table 8 and Table 9 in the Appendix. The magnitude of the coefficients of interest changes slightly but the pattern and statistical significance stay the same. The results are also robust to using total population growth instead of working-age population growth (reported in Table 10). The coefficients are all significant at the 1% level and even higher in magnitude than in the original specification.

4.2.1 Alternative Patent Measures

For reasons laid out in Section 3.2.2, of the three available measures for patent data, those patents reported under the IP5 were used for analysis so far. A good robustness check is thus to analyze the theoretical relationship using those patents counted towards the Triadic Family and the PCT as well, to see if similar results can be obtained.

Reassuringly, the analysis results in very similar estimates using both alternative patent measures, which are reported in Table 11 in the Appendix. The coefficients are estimated at the same significance level and even slightly higher in magnitude for both Triadic patent data and PCT patent data. In both cases, the coefficient of the investment share is insignificant but the point estimate remains positive.

As explained in Section 3.2.2, three different measures for automation arise from the Dechezleprêtre *et al.* (2019) classification, *auto*1, *auto*90, and *auto*95. So far, *auto*1 has been used to derive a density measure of automation. As a robustness check, the estimations reported in Section 4.1 are repeated using automation density measures constructed from *auto*90 and *auto*95 as the dependent variable. The results are reported in Table 12. The main results can be replicated with the significance and magnitude of the coefficients very similar to those in the baseline regression.

4.2.2 Additional Control Variables

The estimations discussed so far have only included variables suggested by the theory to be of importance. Despite using time- and country-fixed effects in all specifications, there might be concerns regarding omitted variable bias. This section reports results from including several additional control variables in the regression to test the robustness of the results discussed so far.

Three additional control variables are included. One is the service share of the economy. It measures changes in the focus on manufacturing or services of individual economies which are not picked up by time-fixed effects. Two, the log of GDP is included to control for booms or recessions in individual economies not picked up by time-fixed effects. And three, the external balance as a measure of openness is included. Openness likely affects the pressure to keep up with technological advances and to stay competitive in general. Trade liberalization was ongoing in the period considered, which potentially makes openness a confounding factor if it is not included in the regression.

Results are reported in Table 2. The coefficients of all control variables are positive and highly significant when incorporated into the regressions. The coefficients of the main explanatory variables appear robust to the inclusion of further control variables, as displayed in column (3). The significance of the coefficients remains at the 1% level and the magnitude of the coefficients of working-age population growth, education, and the interaction effect even increase slightly.

4.2.3 The Relationship of Education and Population Growth

In the Unified Growth Theory, the interplay of population growth and education plays an important role. According to the literature, sustained economic growth was only made possible once fertility rates declined. With the onset of industrialization, human capital became more important and valuable. In a Unified Growth framework, the fertility choice is often described to feature a quantity-quality trade-off, referring to the number and education of children. A key assumption and often confirmed finding in this literature is a negative correlation between education levels and population growth rates.[16]

A strong negative correlation between education and population growth, as proposed in the Unified Growth Theory, potentially leads to imprecise estimates of the coefficients reported in Table 1. Indeed, the correlation between education and working-age population growth is quite strong at $\rho = -0.59$ across the whole sample. If some of the countries used for the estimation of the main specification are still undergoing the demographic transition, this can confound the estimates.

The data set used for estimation comprises information on 59 countries, including all

[16]The Unified Growth Theory was founded by Oded Galor. It has produced a large body of literature and is an actively researched topic in economics. For a comprehensive treatment of the theory and empirical findings see Galor (2011).

Table 2: *Adding Control Variables*

	(1)	(2)	(3)
log(W. Pop growth)	-0.2135	-0.2285	-1.9831***
	(-1.10)	(-1.18)	(-5.23)
Education		0.0068	0.0891***
		(1.14)	(5.39)
log(W. Pop growth)× Education			0.0451***
			(5.29)
Investment Share	0.0470***	0.0466***	0.0466***
	(3.85)	(3.82)	(4.05)
log(GDP p.c.)	1.4353***	1.4580***	1.4733***
	(5.70)	(5.77)	(6.19)
Openness	0.0514***	0.0519***	0.0466***
	(4.53)	(4.58)	(4.34)
Service Share	0.0116	0.0127	0.0194**
	(1.23)	(1.34)	(2.15)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
R^2	0.74	0.74	0.77
Observations	282	282	282

Note: Dependent variable is the log of the automation measure $auto1$, constructed from patent data reported by the OECD and divided by working-age population to arrive at a per-capita measure. All explanatory variables are lagged by one period. log(W. Pop growth) is the log of working-age population growth. Education measures the share of the working-age population with at least completed secondary education as reported by Barro and Lee. Investment share refers to gross fixed capital formation as a share of GDP. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

member countries of the G20. While some of the developing countries present in the data may still be undergoing the demographic transition at the start of the observation period in 1977, this is unlikely to be the case for the G20 member countries. Overall, the correlation between education and working-age population growth is lower in G20 member countries than in the rest of the sample countries, with respective values of $\rho = -0.47$ and $\rho = -0.62$.[17] When repeating the regressions reported in Table 1 in the sub-sample of G20

[17] There is a large variation in the correlation between education and population growth within the G20 member countries. For example, in Argentina, Australia, Russia, the UK, and the US, the two variables are positively correlated, and in all other countries negatively correlated. There is however no discernible or concerning pattern in the variation of the correlation.

member countries, the results (reported in Table 13 in the Appendix) are very similar to the ones obtained when using the whole sample for estimation. Based on these results, it is unlikely that an ongoing demographic transition drives the overall results.

4.2.4 Time Series Analysis

In the cross-country analysis described and reported in Section 4.1, country-fixed effects were included in all regressions to control for unobserved heterogeneity across countries. These fixed effects take care of time-constant heterogeneity, such as cultural values or institutions, which might be correlated with the dependent and explanatory variables. Spanning 43 years, the time period used for estimation is quite long. Over such a long time span, even country characteristics considered quite stable across time, such as the education system, may change, potentially weakening the effectiveness of fixed effects in controlling for cross-country heterogeneity.

To address such concerns, this section reports results from an empirical analysis focusing on the US. The data on population size by age group and the education variable taken from Barro and Lee are only available at 5-year intervals. However, when using only 8 periods for estimation it is unlikely that reasonably reliable estimates can be obtained. Therefore, the data used so far is augmented by data on education taken from the PSID. The PSID gathers information of 5,000 representative households in the US, among other things on the highest level of education attained. Starting in 1997, the education variable is only surveyed biyearly. From the individual-level data, education measures corresponding to those provided by Barro and Lee are constructed. Reassuringly, the respective correlations are quite high at 0.76, 0.92, and 0.91 for completed secondary education, some tertiary education, and completed tertiary education, respectively. Since the correlation of the two measures of some tertiary education is the highest, this is the education variable used for analysis in the following. Data on population size by age group, from which a measure of working-age population is constructed, is only available at the 5-year interval. To avoid losing many time periods for estimation due to that data limitation, the total population size and its growth

Table 3: *Time Series Analysis using US Data*

	Automation Density	
	(1)	(2)
log(Pop growth)	-8.445	-3.909
	(-1.55)	(-0.59)
Education	0.994**	0.209
	(2.46)	(0.28)
log(Pop growth) × Education	0.315**	0.069
	(2.36)	(0.28)
Investment Share	-0.008	-0.012
	(-0.23)	(-0.25)
R^2	0.80	
Observations	24	13

Note: Dependent variable is the log of automation patents per capita. All explanatory variables are lagged by one period. log(Pop growth) is the log of population growth. Education measures the share of the population with some tertiary education. Investment share refers to gross fixed capital formation in % of GDP. Column (1) reports results from a regression in levels. Column (2) reports results from a regression in first differences. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

rate is used for the following analysis instead.

The estimation equation is the same as the one used for the panel data. Results from running an OLS regression using the US data are reported in column (1) of Table 3. When restricting the sample to the US data, a similar pattern of relationships is found. The coefficient of population growth is negative, though only significant at the 14% level. The coefficients of the education variable and the interaction term have the same sign as before but have a lower significance level as well. Given the small sample size, low significance levels of coefficients are not surprising.

One concern in time series analysis is a potential serial correlation of the error term. A graphical analysis of the residuals however finds no significant autocorrelation or partial autocorrelation of the error terms (see Figure 5 in the Appendix). Several tests are available to assess if residuals from linear regression are serially correlated. Two of the most common test are Engle's Lagrange multiplier test and the Durbin-Watson test. In both cases, the null hypothesis is that there is no serial correlation in the errors. A rejection of the null hypothesis, therefore, indicates that the error terms are indeed serially correlated. When applying Engle's Lagrange multiplier test and a Durbin-Watson test to the error terms of the regression, the respective test statistics are given by $\chi^2 = 0.37$, which corresponds to a $p - value = 0.54$, and $d = 0.19$. In both cases, the null hypothesis is not rejected, signifying that there is no statistically significant evidence for serial correlation of the error terms. So neither the graphical analysis nor the analytic tests of the error terms indicate that they are serially correlated.

Another concern in time series analysis is that variables may be non-stationary, in which case a regression can lead to spurious results. The standard test for non-stationarity is the Augmented Dickey-Fuller test. Applying it to the dependent variable and all explanatory variables, the null hypothesis of non-stationarity cannot be rejected. Likewise, the Engle-Granger test, designed to test for cointegration of variables, indicates that the dependent variable and the explanatory variables are indeed cointegrated. For the first differences of all variables, the Augmented Dickey-Fuller test rejects the null hypothesis of non-stationarity. This suggests that the variables are non-stationary in levels but stationary in first difference form. Due to the only bi-yearly availability of the education measure starting in 1997, the sample available for estimation if first differences of the data are used is much reduced to only 13 observations. Results from such an estimation are reported in column (2) of Table 3. The signs of the estimated coefficients stay the same, the magnitude however is much reduced and none of the coefficients is significantly different from zero. Given the small sample size, this is not surprising.

The time series analysis of US data finds a similar pattern as the panel analysis in Section 4.1. The advantage of using only US data is that the results cannot be biased by time-varying unobserved heterogeneity across countries. The disadvantage is that the sample size is much smaller, reducing statistical power, and that time series analysis is accompanied by its own confounding factors, such as serial correlation, non-stationarity, and cointegration.

Given the fact that the analysis in this section is so different from the baseline regression, it is striking that the results from it are similar to the previously reported results. In view of the differences in the empirical approach, the tentative results reported in this section reinforce the confidence that the analysis indeed reveals a systematic relationship between demographic change, automation, and education.

4.3 Replication

One contribution of this paper is to show that patent data combined with a classification system of patent categories into automation and non-automation classes constitute a new, appropriate, and high-quality resource to measure automation. To assess and test the usefulness of patent data, a replication study of Abeliansky and Prettner (2021) is done. This study is replicated because it addresses a similar empirical question. By trying to replicate the study with different data, both the patent data and the empirical results derived from it are tested.

The estimation equation used in Abeliansky and Prettner (2021) is:

$$\log(p_{i,t}) = a + \beta_1 \log(n_{i,t-1}) + \beta_2 \cdot \log(s_{i,t-1}) + \beta_3 \cdot \log(p_{i,t-1}) + \varepsilon_{i,t}$$

where p_t and p_{t-1} refer to an automation measure in t and $t-1$, the latter of which is only included in some specifications. n refers to population growth, and s refers to investment. Based on their theory (which omits education), the following signs are expected for the coefficients: $\hat{\beta_1} < 0$, $\hat{\beta_2} > 0$, $\hat{\beta_3} > 0$.

There are a few issues with this approach. First, their model predicts a negative effect of population growth on automation capital density. Irrespective of that, they use the growth rate of robot density (their automation measure) as a dependent variable, rather than the level of robot density. The estimation specification is thus not suited to test the theoretical hypothesis derived from their model (which corresponds to the limit case of $e = 0$ also discussed in Section 2.3). That they use the growth rate of robot density as a dependent variable causes a second issue, namely that the dependent variable, as well as the main explanatory variable, naturally and frequently takes on negative values. According to Abeliansky and Prettner (2021), they apply a box-cox transformation to the outcome variable and the main explanatory variable to deal with zero and negative values. However, a box-cox transformation does not alleviate the problem of zero and negative values but only ensures a zero-skew distribution of a variable, once negative and zero values are dropped.[18] Therefore, by applying a box-cox transformation, many observations will be dropped. Despite these issues, the patent data is transformed in the same way, first calculating growth rates and then applying a box-cox transformation to it, to replicate their analysis as closely as possible. Additionally and in line with the approach taken by Abeliansky and Prettner (2021), three-year rather than five-year averages of all data are taken for the replication exercise, to ensure maximal comparability of the findings.

When using patent data covering the same period as in the original study, the regression results, in particular the significance of the estimated coefficients, cannot be replicated (see Table 4 in the Appendix). However, when running the same regressions using the whole period the patent data is available, the finding of a significantly negative relationship between population growth and automation growth can be replicated (see Table 5 in the Appendix). This is robust to using different measures of automation patents.

Next, the baseline regression considered here is extended by successively including education and an interaction term between education and population growth, as done in Section 4.1. The results reported in Table 7 in the Appendix show that including education, and especially the interaction term with population growth, is important. The coefficients of both education and the interaction term are statistically significant and positive. Additionally, the point estimate for the population growth variable has a higher significance and a higher magnitude if education and an interaction term are included. This indicates that any specification omitting education and the interaction term most likely fails to estimate the true relationship between population growth and automation.

[18] A box-cox transformation creates a new variable z in the following manner: $z = (x^{\lambda} - 1)/\lambda$. λ is chosen such that the skewness of z is zero. However, for the transformation to work, x has to be strictly positive (see Stata Manual `https://www.stata.com/manuals13/rlnskew0.pdf`).

According to the model put forward by Abeliansky and Prettner (2021), automation today is a function of automation in the last period. Therefore, the fixed effects regressions may be misspecified. To account for that, Abeliansky and Prettner (2021) test a dynamic specification as well, using corrected fixed effects.[19] The same is done using the patent data, results of which are reported in Table 6 in the Appendix. As in the original paper, the magnitude of the autocorrelation coefficient is small. In the replication it is also statistically insignificant, indicating that neglecting to account for it is unproblematic, a conclusion also drawn by Abeliansky and Prettner (2021). However, in the replicated dynamic specification the estimated coefficient of population growth is smaller and no longer statistically significant, contrary to the original paper in which the coefficient of population growth stays significantly negative when including an autocorrelation term.

It should be noted here, that the model proposed by Abeliansky and Prettner (2021) can be simplified considerably when assuming a steady state. If in steady state, their derived expression for p_{t+1} can be simplified by setting $p_{t+1} = p_t = p^*$:

$$p_{t+1} = s(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\frac{1+p_t}{1+n}$$
$$p^* = s(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\frac{1+p^*}{1+n}.$$

Subsequently, the steady state condition can be solved for p^*:

$$p^* = \frac{s(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}}{(1+n) - s(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}}.$$

The relationship between population growth and automation density can now be derived by taking the derivative of p^* with respect to $(1+n)$:

$$\frac{\partial p^*}{\partial(1+n)} = -\frac{s(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}}{\left((1+n) - s(1-\alpha)\left(\frac{\alpha}{1-\alpha}\right)^{\alpha}\right)^2} < 0.$$

By assuming that the model economy is in a steady state or on a Balanced Growth Path,

[19]For the estimation, the stata command *xtbcfe* developed by De Vos *et al.* (2015) is used.

an expression for p^* can thus be derived. With this, it can unambiguously be shown that the effect of population growth on automation density is always negative, making dynamic specifications obsolete.

In summary, the patent data can replicate the findings of Abeliansky and Prettner (2021) well. Since the empirical model is misspecified, the results in and of themselves should not be considered reliable. Beyond and more important than replicating empirical results, this section has shown that freely available patent data from the OECD are a well-suited measure of automation across countries and time.

5 Conclusion

Demographic change, especially the shrinking of the working-age population, poses a threat to economic growth in many developed economies. With the retirement of the baby boomer generation imminent, politicians struggle to counteract the drainage of the labor market by means of immigration, improving family and work compatibility, or encouraging the elderly to stay in the labor market longer. Another possibility to fill the void baby boomers are leaving in the labor market is by automating labor.

This paper studies the link between population growth, especially working-age population growth, and automation. Importantly, the proposed model distinguishes between skilled and unskilled labor. Assuming that automation capital is a closer substitute to unskilled labor than skilled labor, an economy's automation potential is predicted to depend on the population's education level.

The theoretical prediction is then tested empirically and verified using patent data from the OECD. A decrease in working-age population growth by 1% is associated with a 1.1% increase in automation, given the population is unskilled. The effect of population growth however depends crucially on the education level, such that the relationship between population growth and automation is even positive if a large enough share of the population is skilled.

The empirical results are derived using a new measure of automation based on patent data. While data on automation patents measures something slightly different than robot data, it still contains valuable information about automation at the country year level, which can be used for empirical analysis. In the replication exercise, I show that findings using robot data can be replicated with patent data. This automation measure based on patent data has several advantages over robot data. It covers a longer time horizon, as far back as 1977 instead of 1993. It is publicly available, meaning free of charge, contrary to the quite costly robot data. And lastly, it constitutes a broader measure of automation. Compared to robot data, patent data is much more likely to capture technological advances based on AI. In the face of rapid developments in automation in general and AI in particular, having a measure other than industrial robots for automation seems more important than ever.

The theoretical results were derived under two assumptions. One, that the education level is exogenously given and two, that capital is fully mobile between automation and traditional uses in the intratemporal maximization. In the next step, it will be interesting to explore how relaxing those assumptions affects the theoretical results. Intuitively, imposing friction in the mobility of capital between uses does not alter the results, as long as mobility of capital between uses is in general possible. The friction will lead to a sluggish response to changes in exogenous model parameters, which affects the transition between equilibria but not the direction or size of the effect population growth has on capital allocation. Endogenizing the education level of the labor force in contrast is likely to affect the model results in more complex ways. As derived in Section 2.2, the skill premium paid to skilled labor increases as more capital is used for automation. Therefore, an increase in automation capital, caused by changes in population growth, also affects a household's incentive to seek higher education. The interplay of demographic change, automation, and education will thus be more nuanced, once the share of educated labor in the workforce is the result of an endogenous choice by households. Extending the model by relaxing the two assumptions as discussed here is an interesting avenue for future work.

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A Theory Appendix

A.1 Intertemporal Maximization

Consider an infinite horizon economy in discrete time indexed by $t = 0, 1, 2, ...$ with competitively producing firms and one representative household. Population grows at rate n_t, such that $N_{t+1} = N_t(1 + n_t)$. Both the household and the firms have perfect foresight and there is no risk in the model.

A.1.1 Firms

Firms produce output using labor L_t and capital $\tilde{K}_t$ as inputs, with the production function given by[20]

$$Y_t = F(L_t, \tilde{K}_t),$$

which in intensive form with $y_t = \frac{Y_t}{L_t}$ and $\tilde{k}_t = \frac{\tilde{K}_t}{L_t}$ can be written as

$$y_t = f(\tilde{k}_t).$$

Firms operate under perfect competition and make zero profits in equilibrium. Therefore, production factors are paid their marginal products, such that $R_t = f'(\tilde{k}_t)$ and $w_t = f(\tilde{k}_t) - f'(\tilde{k}_t)\tilde{k}_t$. Firms maximize their profit, taking prices $\{p_t, w_t, r_t\}_{t=0}^{\infty}$ as given. The firms' maximization problem is given by

$$\max_{y_t, \tilde{k}_t, n_t} \pi_t = \sum_{t=0}^{\infty} p_t(y_t - r_t\tilde{k}_t - w_t)$$
$$s.t.\ y_t = f(\tilde{k}_t)$$

[20] In Section 2, it is important to distinguish between traditional capital K and automation capital P, which together make up the total capital stock $\tilde{K}$. It is this stock of total capital $\tilde{K}$ which is determined in the intertemporal maximization discussed here.

where $r_t = R_t - \delta$ is the interest rate net of depreciation and $\delta \in (0, 1)$. Since the input factors labor and capital are owned by the households, firms take them as given such that the maximization is static. How the firms solve this infinite number of static maximization problems and the implications this has for the demand of sub-classes of capital is the focus of Section 2.

A.1.2 Households

The representative household owns all production factors and rents them to the firms, receiving marginal products as remuneration in return, which constitutes its income. Output can be consumed or invested. Taking into account a constant rate of population growth $n = const$, the representative household maximizes the lifetime utility of the entire dynasty by choosing consumption and savings optimally. The lifetime utility of the dynasty is given by

$$\sum_{t=0}^{\infty}(1+n)^t N_0 \left(\frac{1}{1+\rho}\right)^t u(c_t),$$

where $u(c_t)$ denotes the instantaneous per capita utility and ρ is the rate of time preference. For simplicity, assume $u(c_t) = log(c_t)$. In that case, the intertemporal elasticity of substitution is equal to one. The budget constraint faced by the household is given by the law of motion of capital

$$\tilde{K}_{t+1} = \tilde{K}_t(1-\delta) + F(\tilde{K}_t, L_t) - c_t L_t.$$

Dividing both sides by $L_{t+1} = L_t(1+n)$ gives the law of motion of the per capita capital stock

$$(1+n)\tilde{k}_{t+1} = \tilde{k}_t(1-\delta) + f(\tilde{k}_t) - c_t.$$

By choosing c_t, the household also determines $\tilde{k}_{t+1}$ through the law of motion. The intertemporal maximization problem of the household can be solved using a Lagrangian, which is set up and solved in the next section.

A.1.3 Solving the Household Problem

The intertemporal maximization problem of the household can be solved using a Lagrangian, which is set up as follows:

$$\max_{c_t, c_{t+1}, k_{t+1}} \mathcal{L} = \sum_{t=0}^{\infty} (1+n)^t N_0 \left(\frac{1}{1+\rho}\right)^t u(c_t) + \lambda_t (\tilde{k}_t (1-\delta) + f(\tilde{k}_t) - c_t - \tilde{k}_{t+1}(1+n))$$

The first-order conditions are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial c_t} &= (1+n)^t N_0 \left(\frac{1}{1+\rho}\right)^t u'(c_t) - \lambda_t \stackrel{!}{=} 0, \\ \frac{\partial \mathcal{L}}{\partial c_{t+1}} &= (1+n)^{t+1} N_0 \left(\frac{1}{1+\rho}\right)^{t+1} u'(c_{t+1}) - \lambda_{t+1} \stackrel{!}{=} 0, \\ \frac{\partial \mathcal{L}}{\partial \tilde{k}_{t+1}} &= -\lambda_t (1+n) + \lambda_{t+1}((1-\delta) + f'(\tilde{k}_{t+1})) \stackrel{!}{=} 0. \end{aligned}$$

In addition to the first order conditions, a terminal value condition is necessary, with can simply be stated as $\tilde{k}_\infty \geq 0$. It ensures that the representative household does not accumulate negative wealth and it is also referred to as a "No-Ponzi Game" condition.

Combining the first-order conditions and rearranging, the Euler Equation can be derived:

$$\begin{aligned} \lambda_t &= \lambda_{t+1} \frac{(1-\delta) + f'(\tilde{k}_{t+1})}{1+n} \\ \left(\frac{1}{1+\rho}\right)^t u'(c_t) = \lambda_t &= \lambda_{t+1} \frac{(1-\delta) + f'(\tilde{k}_{t+1})}{1+n} \\ \frac{u'(c_t)}{u'(c_{t+1})} &= \left(\frac{1}{1+\rho}\right) ((1-\delta) + f'(\tilde{k}_{t+1})) \end{aligned}$$

Intuitively, if the time discount rate ρ increases, the marginal utility of consumption in period $t+1$ decreases relative to the marginal utility of consumption in period t, which, under the assumption of decreasing marginal utility, is equivalent to an increase in per capita consumption c_t relative to per capita consumption c_{t+1}. If however additional production possibility with one more unit of capital in $t+1$, given by $f'(\tilde{k}_{t+1})$, increases, per capita consumption in period t decreases relative to per capita consumption in period $t+1$.

A.1.4 Equilibrium and Steady State

Let p_t denote the price of the final output and consumption good, w_t the wage rate paid for labor services and r_t the rental rate for capital paid in period t. Taking the prices $\{p_t, w_t, r_t\}_{t=0}^{\infty}$ as given, firms maximize their profits and the household maximizes its intertemporal utility. In equilibrium, the markets for input factors capital and labor, and the final consumption good clear.

In steady state, the capital-labor ratio $\tilde{k}^*$ is constant, as well as consumption c^* and output per capita $f(\tilde{k}^*)$. The Euler Equation together with the law of motion of capital fully describes the dynamics of the neoclassical growth model.[21]

$$\frac{u'(c^*)}{u'(c^*)} = \left(\frac{1}{1+\rho}\right)((1-\delta) + f'(\tilde{k}^*)) = 1$$
$$(1+n)\tilde{k}^* = \tilde{k}^*(1-\delta) + f(\tilde{k}^*) - c^*$$

From the law of motion of capital, the steady state level of consumption can be derived as

$$c^* = f(\tilde{k}^*) - (\delta + n)\tilde{k}^*.$$

The marginal product of the steady state per capita capital stock $\tilde{k}^*$ can be derived as

$$f'(\tilde{k}^*) = \delta + \rho$$

Note, that if $f(\tilde{k}_t)$ is concave, its inverse exists, which is denoted by $f^{-1}(\tilde{k}_t)$. Making use of

[21] Note, that due to $u(c_t) = \log(c_t)$, the intertemporal elasticity of substitution is equal to one and does not scale the Euler Equation.

the invertibility of $f'(\tilde{k}_t)$ and rearranging yields the equilibrium capital stock per capita

$$\tilde{k}^* = f^{'-1}(\delta + \rho).$$

A change in the population growth rate n reduces the equilibrium consumption level c^*. The representative household reacts to a change in n since it maximizes the lifetime utility of the entire dynasty. A change in n however leaves the equilibrium per capita capital level $\tilde{k}^*$ unchanged, as n does not factor into its equilibrium level. To see why, note that the Euler Equation consists of exogenous and invariant parameters δ and ρ as well as the marginal product of the equilibrium capital stock $f(\tilde{k}^*)$. For consumption to be constant, the marginal product of capital has to be equal to some constant value given by $(\delta + \rho)$. Therefore, only one value of $\tilde{k}$ is consistent with constant consumption levels, which is given by $\tilde{k}^*$.

A.2 The effect of N and e on K, P, and $\tilde{K}$

To distinguish the effect N has on the marginal product of the overall capital stock $\tilde{K}$ which is optimally allocated between K and P, such that the marginal products of K and P are equalized, and the effect N has on the two types of capital K and P, respectively, K and P have to be analyzed in isolation. This allows to derive the effect N has on K and P if the allocation of $\tilde{K}$ between capital uses does not change.

To derive the effect N has on the marginal product of traditional capital K, the value of K is held constant. This isolates the effect N has on the marginal product of K, by shutting down the effect N has on the allocation of $\tilde{K}$ between K and P. The same holds for the derivation of the effect N has on automation capital P. The respective derivatives are given by

$$\frac{\partial r^{trad}}{\partial N} = \alpha(1-\alpha)\frac{Y}{K \cdot N}\left((1-e-e\beta)N + (1-\beta)P\right) > 0$$

$$\frac{\partial r^{auto}}{\partial N} = (1-\alpha)\beta\frac{Y}{((1-e)N+P)^2 N}\left((1-\alpha)(1-\beta)P - \alpha(1-e)N\right).$$

The sign of $\partial r^{trad}/\partial N$ is universally positive. The sign of $\partial r^{auto}/\partial N$ is indeterminate. On the one hand, an increase in N increases the skilled labor force, a complement to automation capital, and with it the return on automation capital. On the other hand, it increases unskilled labor, a substitute for automation capital, and with it decreases the return on automation capital. The derivative can be decomposed into the effect population size has on output and unskilled labor supply:

$$\frac{\partial r^{auto}}{\partial N} = (1-\alpha)\beta\frac{1}{((1-e)N+P)^2}\left(((1-e)N+P)\frac{\partial Y}{\partial N} - Y\frac{\partial((1-e)N+P)}{\partial N}\right).$$

Formally, the positive effect of population size on r^{auto} is captured by the first term in the large brackets, and the negative effect of population size on r^{auto} by the second term in the large brackets. Which effect dominates depends on the size of the skilled labor force.

Full mobility of capital entails, that the marginal product of both kinds of capital is equal at all times. Therefore, an increase in N is accompanied by a reallocation of $\tilde{K}$ between traditional uses and automation uses. If the ratio of K/P adjusts to the increase in N, the effect of an increase in N on the equilibrium interest rate r^* is strictly positive, reflecting the increase in the input factor labor which is available for production:

$$\frac{\partial r^*}{\partial N} = ((1-\alpha)\beta + \alpha) \left(((1-e)N + \tilde{K}) \cdot \partial Y / \partial N - Y \cdot (1-e)\right) > 0$$

$$\frac{\partial Y}{\partial N} = (1-\alpha)Y \left(\frac{\beta(1-\alpha)(1-e)\beta}{(1-\alpha)\beta + \alpha} + \frac{1-\beta}{N} \right) + \alpha Y \frac{(1-e)}{((1-\alpha)\beta + \alpha)((1-e)N + \tilde{K})} > 0$$

Holding constant the allocation of $\tilde{K}$ between traditional uses and automation uses, the cross derivatives of r^{auto} and r^{trad} with respect to N and e are given by

$$\begin{aligned}\frac{\partial^2 r^{auto}}{\partial N \partial e} = r^{auto} \cdot ((1-\alpha)(1-\beta)eN + (1-(1-\alpha)\beta)\,((1-e)N + P)) \cdot \\ \left(\frac{(1-e)N(1-\alpha)(1+(1-\alpha)(1-\beta))P}{((1-e)N + P)} \right) + \\ + r^{auto} \cdot N\,((1-\alpha)(1-\beta)eN + (1-(1-\alpha)\beta)(1-e)) > 0\end{aligned}$$

$$\frac{\partial^2 r^{trad}}{\partial N \partial e} = \alpha(1-\alpha)\frac{Y}{K \cdot N}(\beta - 1)N < 0.$$

Having shown that education influences the effect population size has on the return on automation capital and hence the incentive to automate, the direct effect of education on automation incentives is also of interest. The effect can be derived by taking the derivative of r^{auto} with respect to education, which is always positive.

$$\frac{\partial r^{auto}}{\partial e} = r^{auto} \cdot N \cdot ((1-\alpha)(1-\beta)eN + (1-(1-\alpha)\beta)((1-e)N + P)) > 0$$

Education increases r^{auto} for two reasons: First, it increases the supply of skilled labor, which is a complement to automation capital. Second, it decreases unskilled labor, which is a substitute for automation capital.

A.3 Taking into Account the effect of N on $\tilde{K}$

In Section 2.2 it has been derived that $\frac{\partial K/P}{\partial N} > 0$ under the implicit assumption of $\frac{\partial \tilde{K}}{\partial N} = 0$. Here it is shown, that the results carry through when taking into account the second-order effect of $\frac{\partial \tilde{K}}{\partial N}$.

$$\frac{K}{P} = \frac{\alpha(1-e)N + \tilde{K}}{\tilde{K}(1-\alpha)\beta - \alpha(1-e)N}$$

$$\begin{aligned}\frac{\partial K/P}{\partial N} = &\frac{(\tilde{K}(1-\alpha)\beta - \alpha(1-e)N)\left(\alpha(1-e) + \frac{\partial \tilde{K}}{\partial N}\right)}{\left(\tilde{K}(1-\alpha)\beta - \alpha(1-e)N\right)^2} - \\ &- \frac{(\alpha(1-e)N + \tilde{K})\left(\frac{\partial \tilde{K}}{\partial N}(1-\alpha)\beta - \alpha(1-e)\right)}{\left(\tilde{K}(1-\alpha)\beta - \alpha(1-e)N\right)^2} \\ &= \frac{\alpha(1-e)(1+(1-\alpha)\beta)\left(\tilde{K} - \frac{\partial \tilde{K}}{\partial N}\cdot N\right)}{\left(\tilde{K}(1-\alpha)\beta - \alpha(1-e)N\right)^2}\end{aligned}$$

Note, that $\tilde{k} = const$ on the balanced growth path implies that $\tilde{K}$ and N grow at the same rate, which is given by n. Therefore, $\frac{\partial \tilde{K}}{\partial N} = 1$. This results in $\frac{\partial K/P}{\partial N} > 0 \iff \tilde{K} > N$. There is no reason why the opposite should be true, such that it is an innocuous assumption for $\tilde{K} > N$ to be true. In that case, $\frac{\partial K/P}{\partial N} > 0$ holds even when allowing for $\frac{\partial \tilde{K}}{\partial N} \neq 0$.

By taking the derivative of $\frac{\partial (K/P)}{\partial N}$ with respect to e, the role the share of skilled labor in the labor force plays can be determined.

$$\begin{aligned}\frac{\partial^2 K/P}{\partial N \partial e} = &\frac{\left(\tilde{K}(1-\alpha)\beta - \alpha(1-e)N\right)^2 \alpha(1+(1-\alpha)\beta(-1)}{\left(\tilde{K}(1-\alpha)\beta - \alpha(1-e)N\right)^4} - \\ &- \frac{\alpha(1-e)(1+(1-\alpha)\beta)\cdot 2\left(\tilde{K}(1-\alpha)\beta - \alpha(1-e)N\right)(\alpha N)}{\left(\tilde{K}(1-\alpha)\beta - \alpha(1-e)N\right)^4} \\ &= \frac{\alpha(1+(1-\alpha)\beta)(-1)\left(\tilde{K}(1-\alpha)\beta + (1-e)\alpha N\right)}{\left(\tilde{K}(1-\alpha)\beta - \alpha(1-e)N\right)^3} < 0\end{aligned}$$

The denominator of the cross derivative is positive since it is the denominator of $\frac{K}{P}$, which is positive, to the power of three. The numerator is negative, since $\alpha(1 + (1 - \alpha)\beta)(-1) < 0$. Thus the cross derivative is negative.

Since $\frac{\partial(K/P)}{\partial N} > 0$, the effect of the population size N on the ratio $\frac{K}{P}$ is positive. For the limit case of $e = 1$, the ratio of $\frac{K}{P}$ is fully determined by the parameters α and β and thus independent of N. Therefore, in the limit case of $e = 1$, $\frac{\partial(K/P)}{\partial N} = 0$. The negative sign of the cross derivative demonstrates that as e increases, the effect of N on $\frac{K}{P}$ decreases. This is equivalent to the results derived in the main part of the paper, which neglects the effect of N on $\tilde{K}$.

A.4 Limit Cases of Per Capita Production Functions

To better understand the effect of population growth on per capita output and per capita capital, which is made up of traditional and automation capital, consider the limit cases of the per-capita production function discussed in Section 2.4.

If the labor force is unskilled ($e = 0$), per capita output is given by

$$y = k^{\alpha}(1+p)^{1-\alpha}.$$

Deriving the marginal product of traditional and automation capital per capita:

$$r^{trad} = \frac{\partial y}{\partial k} = \alpha \frac{y}{k},$$
$$r^{auto} = \frac{\partial y}{\partial p} = (1-\alpha)\frac{y}{(1+p)}.$$

Setting the two marginal products equal due to capital mobility and plugging in $k = \tilde{k} - p$ and $p = \tilde{k} - k$, the optimal values of traditional capital per capita k^* and automation capital per capita p^* can be derived.

$$r^{trad} = r^{auto}$$
$$k^* = \alpha\tilde{k} + \alpha$$
$$p^* = (1-\alpha)\tilde{k} - \alpha$$

Note, that $\tilde{k} = \frac{\tilde{K}}{N}$. If the growth rate of N, denoted by n, deviates from its balanced growth path value, this affects $\tilde{k}$. Specifically, if n decreases, $\tilde{k}$ increases. This also affects the optimal ratio of k and p, which decreases. Thus, if the population decreases, the optimal ratio of

k^*/p^* decreases.

$$\frac{k^*}{p^*} = \frac{\alpha\tilde{k} + \alpha}{(1-\alpha)\tilde{k} - \alpha},$$
$$\frac{\partial(k^*/p^*)}{\partial\tilde{k}} = \frac{-\alpha}{\left((1-\alpha)\tilde{k} - \alpha\right)^2} < 0.$$

If the labor force is completely skilled $(e = 1)$, output per capita is given by

$$y = k^\alpha p^{\beta(1-\alpha)}.$$

Deriving the marginal effect of traditional and automation capital per capita:

$$r^{trad} = \frac{\partial y}{\partial k} = \alpha\frac{y}{k},$$
$$r^{auto} = \frac{\partial y}{\partial p} = \beta(1-\alpha)\frac{y}{p}.$$

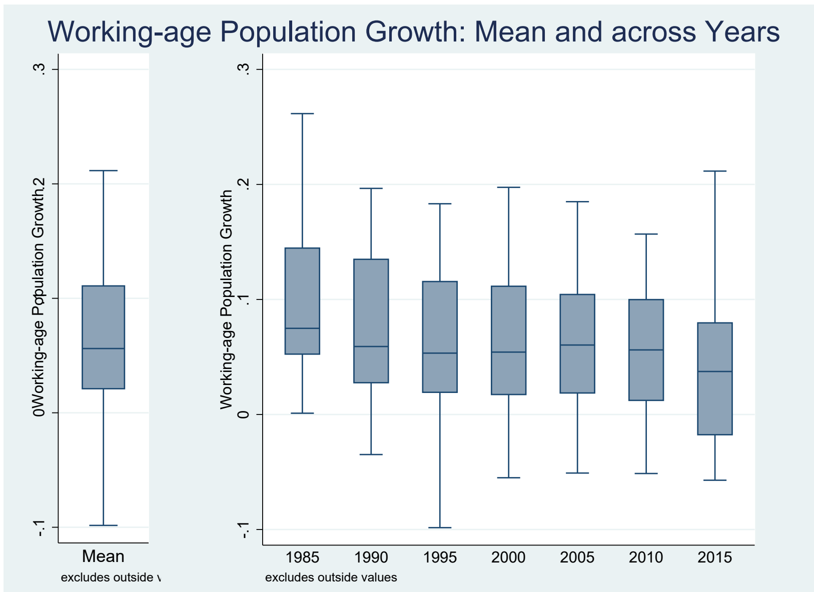
Setting equal due to capital mobility and reformulating yields

$$r^{trad} = r^{auto},$$
$$k^* = \frac{\alpha}{\alpha + (1-\alpha)\beta} \cdot \tilde{k},$$
$$p^* = \frac{(1-\alpha)\beta}{\alpha + (1-\alpha)\beta} \cdot \tilde{k},$$
$$\frac{k^*}{p^*} = \frac{\alpha}{\beta(1-\alpha)}.$$

If the workforce is fully educated, the two types of capital k and p are both allocated a constant share of total capital per capita $\tilde{k}$. Thus, the ratio $\frac{k}{p}$ is unaffected by population growth.

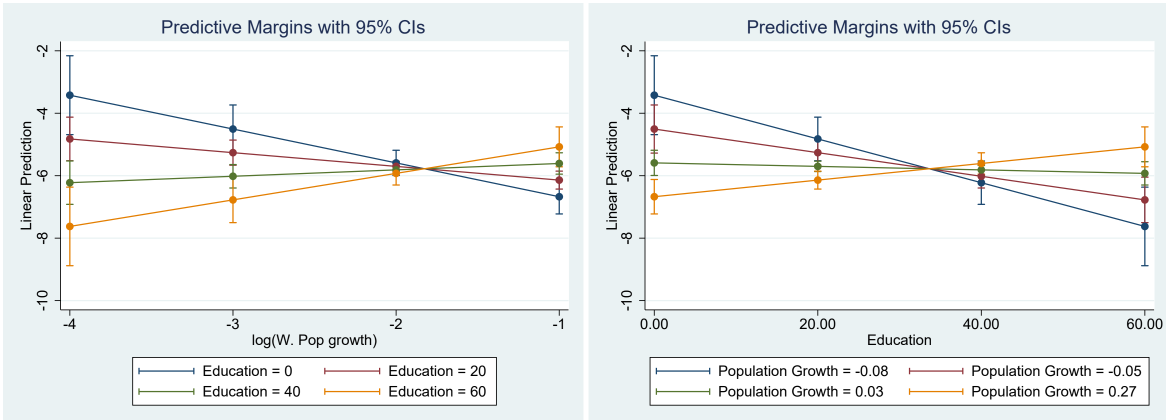
B General Appendix

B.1 Additional Figures



Note: The figure illustrates the development of the working-age population growth rate over the years using a boxplot. In the left panel, the 25th, 50th, and 75th percentile of the distribution across years and countries is shown. The right panel shows the respective statistics across countries in 5-year intervals.

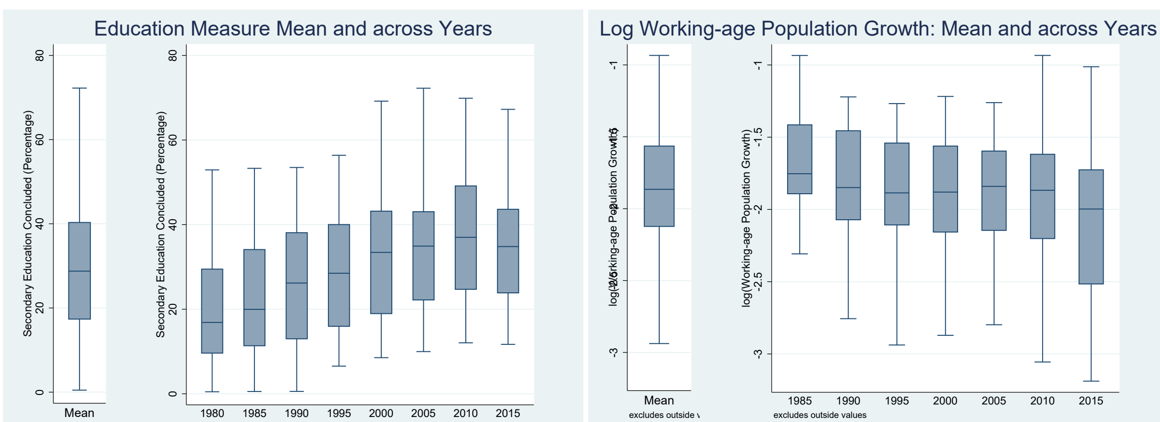
Figure 2: *Visualization of Working-age Population Growth over Time*



(a) *Marginal Effect of Population Growth* **(b)** *Marginal Effect of Education*

Figure 3: *Visualization of Marginal Effects from Table 1*

Note: The Figure illustrates the regression results reported in Table 1. Panel (a) shows the predicted marginal effect an increase in log working-age population growth has on automation density for different values of the education variable. Panel (b) shows the predicted marginal effect an increase in education has on automation density for different values of working-age population growth. In both cases, 95% confidence intervals of the predicted effect are reported.



(a) *Education* **(b)** *Log Working-age Population Growth*

Figure 4: *Visualization of Variable Distribution Across Time*

Note: The figure illustrates the development of the education measure and working-age population growth over the years using a boxplot. In the respective left panel, the 25th, 50th, and 75th percentile of the distribution across years and countries is shown. The right panel shows the respective statistics across countries in 5-year intervals

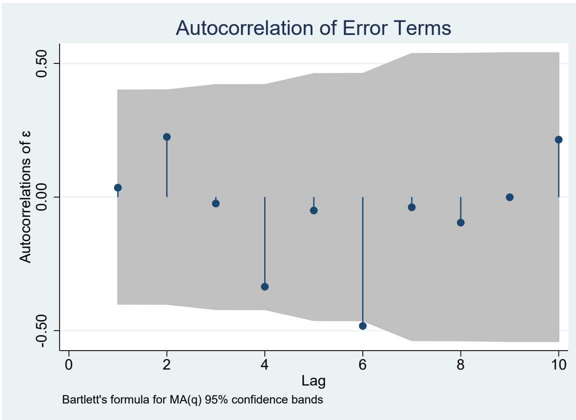
To interpret the effect of working-age population growth on automation, it may be helpful to look at the marginal effect at certain levels of education. The distribution of education across years is visualized in Panel (a) of Figure 4. Fixing the share of the educated population at

0%, 20%, 40%, and 60%, the respective marginal effects of working-age population growth are -1.1, -0.4, 0.2, and 0.8. These numbers are calculated as $\frac{\partial automate}{\partial popgrowth} = \eta_1 + \eta_3 \cdot educ$, inserting the values 0, 20, 40, and 60 for education. The analysis can also be visualized, which is done in Figure 3. It shows the estimated marginal effect of working-age population growth on automation for the same fixed shares of the educated population. For low levels of education, represented by Education = 0 and Education = 20, the marginal effect of working-age population growth is negative and for high levels of education (Education = 40 and Education = 60) it is positive.

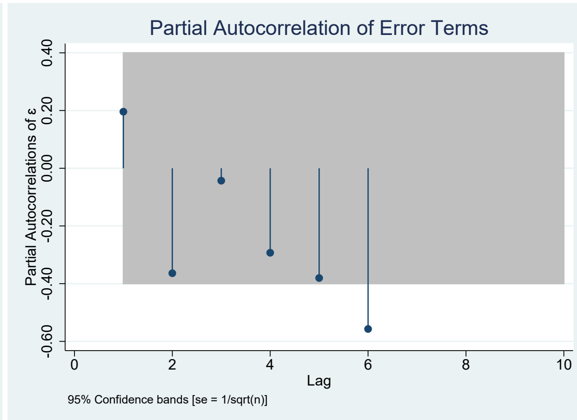
The same analysis of marginal effects is done for the education variable at different levels of working-age population growth. In that case, the marginal effect is calculated as $\frac{\partial automate}{\partial educ} = \eta_2 + \eta_3 \cdot popgrowth$. Again, this is calculated for some meaningful values of working-age population growth. In the data, the working-age population growth rate ranges from -0.10 to 0.30. The distribution of the growth rate across years and by years is visualized in Panel (b) of Figure 4. For the visualization in Figure 3b, the values -0.08, -0.05, 0.03, 0.27 are used.[22] At these values, the marginal effect of an increase in education on the density of automation patents is given by -0.07, -0.04, -0.01, 0.03, which corresponds to the respective slope of the lines. The negative sign for low levels of working-age population growth is driven by the effect working-age population growth has on automation density, whose negative effect outweighs the positive effect education has on automation density.

[22]Note, that these are the values before the rescaling of the population growth variable. $\log(-4)$ in the figure corresponds to a rescaled working-age population growth rate of 0.18, which, taking into account the rescaling, is equivalent to a growth rate of -0.08.

B.1.1 Graphical Analysis of Error Terms



(a) *Autocorrelation*



(b) *Partial Autocorrelation*

Figure 5: *Graphical Analysis of Time Series Estimation Error Terms*

Note: The figure shows the autocorrelation and partial autocorrelation of error terms from the time series analysis of US data, results of which are reported in Table 3.

Figure 5 shows the autocorrelation and the partial autocorrelation of the error terms resulting from time series analysis, the results of which are reported in Table 3. There is no distinctive pattern in the error terms for different lag times. Furthermore, none of the correlations across lags are statistically significant, indicating that there is no serial correlation of the error terms.

B.2 Replication Regression Results

Table 4: *Replication of Abeliansky and Prettner (2021) using Patent data 1993-2020*

	(1)	(2)	(3)
log(Pop Growth)	0.0112	-0.1481	-0.2754
	(0.06)	(-0.68)	(-1.35)
Investment Share	-0.0014	0.0011	0.0076
	(-0.20)	(0.14)	(1.03)
Constant	-1.7019***	-1.6279***	0.0000
	(-3.31)	(-2.82)	(.)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
R^2	0.25	0.10	0.08
Observations	311	247	250

Note: Dependent variable in column (1), column (2) and column (3) is the box-cox transformed growth rate of automation patents per capita as defined by the $auto1$, $auto90$ and $auto95$ measure, respectively. All explanatory variables are lagged by one period. log(Pop growth) is the box-cox transformed population growth rate. Investment share refers to gross fixed capital formation in % of GDP. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

Tables 4 and 5 show regression results from replicating Abeliansky and Prettner (2021) using patent data. The outcome of similar regression results using patent data covering the same time period cannot be obtained (Table 4). However, one of the advantages of using patent data is that it covers a much longer time period than the robot data. While reliable data on robots starts in 1993, the OECD patent data goes back to 1977. When running the same regressions using the whole time period the patent data is available, the finding of a significantly negative relationship between population growth and automation growth can be replicated (Table 5). This is robust to using different classifications and thus measures of automation patents.[23]

Table 6 reports results from running a dynamic corrected fixed effects regression using log growth rates of automation measures as a dependent variable as in Abeliansky and

[23] Probably due to reasons detailed in Section 3.2.2, this finding is limited to an analysis using IP5 patents, while analyses using Triadic family patents or PCT patents do not result in statistically significant coefficients.

Table 5: *Replication of Abeliansky and Prettner (2021) using Patent data 1977-2020*

	(1)	(2)	(3)
log(Pop Growth)	-0.2419*	-0.2987**	-0.3484***
	(-1.94)	(-2.20)	(-2.62)
Investment Share	0.0057	0.0038	0.0097
	(1.07)	(0.63)	(1.64)
Constant	-0.5952*	-0.4360	-0.5800*
	(-1.86)	(-1.26)	(-1.70)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
R^2	0.59	0.56	0.59
Observations	429	336	336

Note: Dependent variable in column (1), column (2) and column (3) is the box-cox transformed growth rate of automation patents per capita as defined by the *auto*1, *auto*90 and *auto*95 measure, respectively. All explanatory variables are lagged by one period. log(Pop growth) is the box-cox transformed population growth rate. Investment share refers to gross fixed capital formation in % of GDP. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

Prettner (2021). The autocorrelation is insignificant. The estimated coefficient of log population growth though is smaller and no longer significant. However, that may be due to the much-reduced sample size.

Tabel 7 uses the same dependent variable as column (1) in Tables 4 and 5. In columns (2) and (3) education and an interaction term of education with the population growth variable are added. The results show clearly that including education is important, especially the interaction term. The coefficient of both education and the interaction term is statistically significant and positive. Additionally, the point estimate for the population growth variable has a higher significance and a higher magnitude.

Table 6: *Corrected FE Estimates 1977-2020*

	(1)	(2)	(3)
$\log(\Delta auto1)_{t-1}$	0.114		
	(1.37)		
$\log(\Delta auto90)_{t-1}$		0.002	
		(0.02)	
$\log(\Delta auto95)_{t-1}$			0.059
			(0.36)
log(Pop Growth)	-0.124	0.099	-0.091
	(-0.63)	(0.27)	(-0.32)
Investment Share	0.003	0.002	0.014
	(0.39)	(0.20)	(1.36)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
N	289	141	144

Note: Dependent variable in column (1), column (2), and column (3) is the log growth rate of automation patents per capita as defined by the *auto*1, *auto*90, and *auto*95 measure, respectively. A three-year lag of the dependent variable is included as an explanatory variable in each regression. All other explanatory variables are lagged by one period. log(Pop growth) is the log of population growth. Investment share refers to gross fixed capital formation in % of GDP. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

Table 7: *Replication of Abeliansky and Prettner (2021) adding Education data 1977-2020*

	(1)	(2)	(3)
log(Pop Growth)	-0.2419*	-0.2381*	-0.3663***
	(-1.94)	(-1.92)	(-2.61)
Education		0.0029	0.0151**
		(0.71)	(1.99)
log(Pop growth) $\times$ Education			0.0062*
			(1.91)
Investment Share	0.0057	0.0082	0.0083
	(1.07)	(1.50)	(1.52)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
R^2	0.59	0.59	0.60
Observations	429	426	426

Note: Dependent variable in all columns is the box-cox transformed growth rate of automation patents per capita as defined by the *auto*1. All explanatory variables are lagged by one period. log(Pop growth) is the box-cox transformed population growth rate. Education measures the share of the population with some tertiary education. Investment share refers to gross fixed capital formation in % of GDP. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

B.3 Additional Regression Tables

B.3.1 Using Different Education Measures

Table 8: *Using Some Tertiary Education as Education Measure*

	(1)	(2)	(3)
log(W. Pop growth)	-0.1646	-0.1551	-0.5227**
	(-1.14)	(-1.07)	(-2.20)
Some Tertiary Education		-0.0053	0.0433
		(-0.56)	(1.63)
log(W. Pop growth) × Education			0.0259*
			(1.95)
Investment Share	0.0362***	0.0351***	0.0352***
	(3.47)	(3.29)	(3.32)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
R^2	0.60	0.60	0.61
Observations	328	325	325

Note: Dependent variable is the log of the automation measure *auto*1, constructed from patent data reported by the OECD and divided by working-age population to arrive at a per-capita measure. All explanatory variables are lagged by one period. log(W. Pop growth) is the log of working-age population growth. Education measures the share of the working-age population with at least some tertiary education as reported by Barro and Lee. Investment share refers to gross fixed capital formation as a share of GDP. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

Tables 8 and 9 report results from estimating the main regressions but using some tertiary education or completed tertiary education as an education measure. The coefficient on working-age population growth decreases both in magnitude and significance. The coefficient of the interaction term also varies in significance and magnitude. The overall findings however can be replicated quite well.

Table 9: *Using Completed Tertiary Education as Education Measure*

	(1)	(2)	(3)
log(W. Pop growth)	-0.1646	-0.1660	-0.5611**
	(-1.14)	(-1.15)	(-2.41)
Completed Tertiary Education		0.0054	0.0881**
		(0.41)	(2.18)
log(W. Pop growth) × Education			0.0441**
			(2.16)
Investment Share	0.0362***	0.0368***	0.0371***
	(3.47)	(3.47)	(3.52)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
R^2	0.60	0.60	0.61
Observations	328	325	325

Note: Dependent variable is the log of the automation measure *auto*1, constructed from patent data reported by the OECD and divided by working-age population to arrive at a per-capita measure. All explanatory variables are lagged by one period. log(W. Pop growth) is the log of working-age population growth. Education measures the share of the working-age population with at least completed tertiary education as reported by Barro and Lee. Investment share refers to gross fixed capital formation as a share of GDP. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

B.3.2 Using Total Population Growth

Table 10: *Total Population Growth and Automation Density*

	(1)	(2)	(3)
log(Pop growth)	-0.7332***	-0.7563***	-2.0402***
	(-3.23)	(-3.40)	(-5.11)
Education		-0.0008	0.0661***
		(-0.13)	(3.58)
log(Pop growth) $\times$ Education			0.0344***
			(3.83)
Investment Share	0.0390***	0.0381***	0.0467***
	(3.72)	(3.69)	(4.53)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
R^2	0.61	0.63	0.65
Observations	341	334	334

Note: Dependent variable is the log of the automation measure *auto*1, constructed from patent data reported by the OECD and divided by population to arrive at a per-capita measure. All explanatory variables are lagged by one period. log(Pop growth) is the log of total population growth. Education measures the share of the total population with at least completed secondary education as reported by Barro and Lee. Investment share refers to gross fixed capital formation as a share of GDP. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

Table 10 reports results from estimating the main regressions but using total population growth instead of working-age population growth. All coefficients remain significant and even slightly increase in magnitude.

B.3.3 Using PTC and Triadic Patent Data

Table 11: *Main Regression using Triadic and PCT Patents*

	Triadic			PCT		
	(1)	(2)	(3)	(4)	(5)	(6)
log(W. Pop growth)	-0.2155	-0.2149	-1.5596***	-0.4690**	-0.4747**	-2.3359***
	(-1.10)	(-1.09)	(-4.21)	(-1.97)	(-1.99)	(-5.31)
Education		0.0007	0.0859***		0.0125	0.1241***
		(0.08)	(3.93)		(1.28)	(5.08)
Interaction			0.0471***			0.0617***
			(4.23)			(4.94)
Investment Share	0.0217	0.0217	0.0189	0.0193	0.0183	0.0164
	(1.52)	(1.50)	(1.35)	(1.21)	(1.15)	(1.07)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.43	0.43	0.47	0.74	0.74	0.77
Observations	324	321	321	323	320	320

Note: Dependent variable is the log of automation patents per capita as reported by the OECD. In columns (1), (2), and (3), only automation patents registered with the EPO, the JPO, and the USPTO are used for the analysis. In columns (4), (5), and (6), only automation patents filed with the PCT are used for analysis. All explanatory variables are lagged by one period. log(W. Pop growth) is the log of working-age population growth. Education measures the share of the working-age population with at least completed secondary education as reported by Barro and Lee. Investment share refers to gross fixed capital formation as a share of GDP. The variable Interaction is defined as follows: Interaction = log(W. Pop growth) × Education. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

Table 11 reports regression results from running the baseline specification but using Triadic patent data and PCT patent data instead. The estimated coefficients are very similar in magnitude and significance.

B.3.4 Using auto90 and auto95 Data

Table 12: *Main Regression using auto90 and auto95*

	auto90			auto95		
	(1)	(2)	(3)	(4)	(5)	(6)
log(W. Pop growth)	-0.2070	-0.2097	-1.0128***	-0.5406*	-0.5432*	-2.0826***
	(-1.16)	(-1.17)	(-2.94)	(-1.83)	(-1.83)	(-3.88)
Education		0.0045	0.0594***		0.0067	0.0855***
		(0.58)	(2.76)		(0.86)	(3.52)
Interaction			0.0306***			0.0439***
			(2.72)			(3.42)
Investment Share	0.0383***	0.0378***	0.0380***	0.0359***	0.0352**	0.0407***
	(2.94)	(2.88)	(2.93)	(2.58)	(2.52)	(2.96)
Time FE	Yes	Yes	Yes	Yes	Yes	Yes
Country FE	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.32	0.32	0.35	0.31	0.31	0.34
Observations	304	301	301	300	297	297

Note: Dependent variable is the log of automation patents per capita as reported by the OECD. In columns (1), (2), and (3), the dependent variable is calculated using automation patents as measured by the variable *auto*90. In columns (4), (5), and (6), the dependent variable is calculated using automation patents as measured by the variable *auto*95. All explanatory variables are lagged by one period. log(W. Pop growth) is the log of working-age population growth. Education measures the share of the working-age population with at least completed secondary education as reported by Barro and Lee. Investment share refers to gross fixed capital formation as a share of GDP. The variable Interaction is defined as follows: Interaction = log(W. Pop growth) × Education. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

Table 12 reports regression results from running the baseline specification but using *auto*90 and *auto*95 patent data as dependent variables instead. The pattern of the different estimation specifications is similar to the baseline case. The estimated coefficients however are smaller and have a lower significance level. This is likely due to higher noise in the automation measures *auto*90 and *auto*95, compared to the preferred measure of *auto*1.

B.3.5 Subsample of G20 Member Countries

Table 13: *Main Regression using only G20 Countries*

	(1)	(2)	(3)
log(W. Pop growth)	-0.6435	-0.6764	-2.3878***
	(-1.47)	(-1.56)	(-2.96)
Education		-0.0140	0.0615*
		(-1.57)	(1.95)
log(W. Pop growth) $\times$ Education			0.0408**
			(2.49)
Investment Share	0.1467***	0.1464***	0.1343***
	(7.29)	(7.34)	(6.72)
Time FE	Yes	Yes	Yes
Country FE	Yes	Yes	Yes
R^2	0.71	0.72	0.74
Observations	113	113	113

Note: Dependent variable is the log of automation patents per capita as reported by the OECD. Only the subsample of G20 member states is used for regression. All explanatory variables are lagged by one period. log(W. Pop growth) is the log of working age population growth. Education measures the share of the working-age population with at least completed secondary education as reported by Barro and Lee. Investment share refers to gross fixed capital formation as a share of GDP. Significance stars are defined as follows: * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. t-statistics are reported in parentheses.

Table 13 reports regression results from repeating the regressions from Table 1 in the sub-sample of G20 member states. The pattern of results is the same as in the full sample.

B.4 List of Countries

There are 59 countries for which all variables necessary for estimating the main specification are available. They are:

Argentina, Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Chile, China, Colombia, Croatia, Czech Republic, Denmark, Egypt, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Malaysia, Mexico, Morocco, Netherlands, New Zealand, Norway, Pakistan, Peru, Philippines, Poland, Portugal, Romania, Russia, Saudi Arabia, Singapore, Slovenia, South Africa, Spain, Sweden, Switzerland, Thailand, Tunisia, Turkiye, Ukraine, United Kingdom, United States.