Sustainability of public debt, investment subsidies, and endogenous growth with heterogeneous firms and financial frictions

MAEBAYASHI, NORITAKA

Nanzan University

May 2024

Sustainability of public debt, investment subsidies, and endogenous growth with heterogeneous firms and financial frictions

Noritaka Maebayashi*

Abstract

This study investigates the effect of public debt on growth, interest rate, and sustaibility of public debt in a very simple endogenous growth model with financial imperfection and the firm heterogeneity. Increases in public debts cause higher real interest rates through financial markets and reduces both the number of firms and private investment, leading to lower long-run growth. It makes public debt less sustainable when public debt is very large. This study also examine the effect of investment subsidy financed by public debt. It hinder economic growth in the long-run although they affect posively on growth in the short run. Therefore, investment subsidy should not be financed by public debt but tax increases.

JEL classification: E62; H20; H60

Keywords: Sustainability of public debt, Finantial frictions, Firm heterogeneity, Investment subsidies

*Faculty of Economics, Nanzan University, 18 Yamasato-cho, Showa-ku, Nagoya 466-8673 JAPAN; e-mail: maebmn@nanzan-u.ac.jp

1 Introduction

Public debt levels have risen significantly in many developed countries, and how to ensure the sustainability of public debt is one of the concerns of policymakers.[1] Economists recognize that higher economic growth and lower interest rates are important factors in stabilizing the debt-to-GDP ratio. However, economic growth, interest rates and public debt accumulation are determined in dynamic processes and are interrelated, the complexity of which makes it somewhat difficult to assess the sustainability of public debt. Indeed, there is no consensus on what a sustainable debt is (e.g., D'Erasmo et al., 2016). On the one hand, Ramsey-type models with representative infinite-lived agents show that public debt cannot be sustainable if the government violates its transversality condition (e.g.,Greiner 2007, 2011, 2012, 2015; Kamiguchi and Tamai 2012). Whether the transversality condition is then tested by many empirical studies (e.g., Hamilton and Flavin 1986; Bohn 1998; Afonso 2005). On the other hand, overlapping generations (OLG) models show that the government loses this constraint and can run a Ponzi game. Acknowledging the possibility of public Ponzi game, many recent studies analyze fiscal sustainability in OLG models and define fiscal sustainability as whether the ratio of public debt to GDP (or capital) converges to a stable level in the long run (Chalk 2000; Bräuninger 2005; Yakita 2008 2014; Arai 2011, Teles and Mussolini 2014, Agénor and Yilmaz 2017, Maebayashi and Konishi 2021).

To my knowledge, few studies have succeeded in studying long-run economic growth, interest rate and sustainability of public debt, simultaneously. Moreover, although the above studies on public debt sustainability use different types of models, they share common features: all the above studies assume that agents are homogeneous and financial markets are perfect.

This study contributes to investigates the effect of public debt on growth, interest rate, and sustainability of public debt in a very simple endogenous growth model with financial imperfection and the firm heterogeneity. We incorporate the Pareto distribution of firm's productivity as in the recent literature on the impact of fiscal policy on growth with firm heterogeneity (e.g., e.g., Mino 2015, 2016; Jaimovich and Rebelo 2017; Arawatari, et al. 2023). Under financial imperfection and firm heterogeneity, high-productivity firms can borrow in the financial market while low-productivity firms cannot because of the credit constraint for the latter firms. Low-productivity agents then become lenders to high-productivity entrepreneurs (borrowers). An increase in government debt increases the issuance of government bonds and reduces the aggregate supply of credit in the financial market, thereby raising the interest rate. This increase in the interest rate reduces the number of firms by increasing the cost of borrowing for entrepreneurs. Public debt would have negative effects on economic growth by crowding out firms and their investments.

[1] One way to address this problem would be fiscal consolidation efforts, especially in the EU.

We analyze these effects of public debt on growth, interest rate, and the sustainability of public debt.

The second contribution of this study is to examine the growth effect of investment subsidies financed by public debt. Governments offer various investment subsidies, including investment cost subsidies, research and development subsidies, investment tax reductions, and direct investment grants as pointed out by Kang (2022). These investment subsidies are recognized as an instrument for boosting economic growth. In 2022, the total government budget allocations for R&D across the EU reached 0.74% of GDP. This was an increase of 5.4% compared to 2021 and an increase of 49.2% compared to 2012. In the United States, according to the World Bank (2023), research and development expenditure increased from 2.67% to 3.46% of GDP between 2012 and 2021. Policy instruments to support young, growing and innovative companies are also expected to boost investment and economic growth (European Commission 2017). The public finances of these developed countries have relied heavily on public debt. However, to my knowledge, the growth-enhancing effect of subsidy policy with heterogeneous firms has been studied only in the case of a balanced government budget. Therefore, it is important to examine the growth effect of investment subsidies financed by public debt. Furthermore, it is worth investigating whether the growth-enhancing effect of investment subsidies can improve the fiscal situation.

The main findings of this study are as follows. First, an increase in public debt has two opposite effects on economic growth. One is the negative growth effect through the crowding-out effect of public debt on investment. The other is the positive effect through an increase in the interest rate. The interest rate rises because an increase in government debt increases the issuance of government bonds and reduces the total supply of credit in the financial market. A rise in the interest rate increases output and wage income because a rise in the interest rate increases the cost of borrowing for entrepreneurs and reduces the number of firms, while increasing capital intensity and the marginal productivity of capital. The positive effect of the former dominates the negative effect of the latter. Therefore, an increase in public debt slows down economic growth.

Second, a rise in the interest rate due to an increase in public debt increases interest payments and the growth of public debt. A crowding-out effect of public debt on private investment reduces the burden of public expenditure on investment subsidies, thereby reducing the growth of public debt. The former effect dominates the latter when the current ratio of public debt to capital is large enough to make public debt unsustainable.

Third, investment subsidies financed by public debt hinder economic growth in the long run although they have a positive impact on growth in the short run. Moreover, investment subsidies make public debt less sustainable or increase the public debt-to-GDP ratio in the long-run steady state. These results are robust unless the credit market is close to perfect. The reasons are as follows. First, investment subsidies lower the barrier to entrepreneurship and increase the number of less productive firms. This lowers aggregate productivity. Second, the increase in these firms increases the aggregate demand for credit in the financial market and leads to upward pressure on the interest rate. This reduces the number of firms because it increases the cost of borrowing for entrepreneurs. Third, the rise in this interest rate increases the cost of servicing public debt and crowds out private investment. These three effects have a significant negative impact on economic growth, even though investment subsidies encourage investment. Public debt increases due to the financing costs of subsidies and due to a rise in the interest rate in the financial market. It worsens the fiscal situation. The policy implication is therefore very clear in the sense that investment subsidy should not be financed by public debt but by tax increases, unless the credit market is close to perfect.

Related Literature

Chalk (2000), de la Croix and Michel (2002), Yakita (2014) examine the sustainability of public debt in OLG models and conclude that a Ponzi game by governments is possible. Fiscal sustainability in OLG models is often defined as the convergence of public debt to a sustainable level in the long run. Chalk (2000) and Maebayashi (2023) examine this issue under some fiscal policy rules in OLG models. The former considers the constant deficit rule, while the latter does so under the fiscal consolidation rule based on the Stability and Growth Pact in the EU. However, these studies are exogenous growth models and therefore ignore the long-run endogenous growth effect of from non-decreasing return to capital (Romer 1986) .

This study is closely related to the studies of Bräuninger (2005), Yakita (2008), Arai and Kunieda (2010), Arai (2011), Teles and Mussolini (2014), Agénor and Yilmaz (2017), Maebayashi and Konishi (2021), and Futagami and Konishi (2023) in the sense that they investigate the sustainability of public debt when the government plays a Ponzi game in OLG models with endogenous growth structure.[2] Bräuninger (2005) Arai (2011), Teles and Mussolini (2014) Maebayashi and Konishi (2021) find a negative effect of public debt on long-run growth (Saint-Paul 1992), but suffer from the assumption of a constant interest rate over time due to the use of the AK model (Romer 1986). Yakita (2008) and Agénor and Yilmaz (2017) include public capital in the final production function (Furagami, et al. 1993) and capture its positive external effects on growth and interest rate. This study differs from them because we consider the positive growth effect of investment subsidy directly to firms and endogenous movements of interest rate. through the

[2]Greiner (2007, 2011, 2012, 2015), Kamiguchi and Tamai (2012), and Miyazawa (2019) investigate the sustainability of public debt in some representative infinitely lived agent models with endogenous growth structure in which a Ponzi game by the government is impossible (by the transversality condition).

financial market structure. Arai and Kunieda (2010) is similar to this study in the sense that they consider the credit market imperfection with heterogeneous agents. However, Arai and Kunieda (2010) assume a uniform distribution of individual productivity, and they all have risk neutrality and ignore the investment subsidy policy. In contrast to Arai and Kunieda (2010), this study incorporates more realistic growth effects under investment subsidy policies through microfoundations with risk-averse utility and Patero distribution of firms' productivity as mentioned in the following literature.

Recent trends in the growth theory literature incorporate heterogeneity of individuals or firms into endogenous growth models (e.g., Mino 2015, 2016; Jaimovich and Rebelo 2017; Arawatari, et al. 2023). Mino (2016), Jaimovich and Rebelo (2017), and Arawatari, et al. (2023) consider the effect of tax and fiscal policies when firms differ in their productivity under Pareto distribution. These studies show that the effect of these public policies on growth is significantly different from the homogeneous individual economy. To my knowledge, studies on investment subsidy policies in this context are somewhat limited. Morimoto (2018) studies R&D subsidy policy in the presence of heterogeneity of individual productivity, and shows that R&D subsidy increases economic growth when the subsidy is not so large. However, Morimoto considers the balanced budget to finance the subsidies. This study contributes to the literature to consider the effect of investment subsidy financed by public debt and shows that the growth effect of subsidy is negative.

2 Model

Consider an economy consisting of two types of households living in two periods. The number of each household is normalized to one. The two types differ in the times when they have access to production. Entrepreneurial households can invest in capital and hire labor in youth and use this capital to produce goods. The production technology follows the form in Mino (2016) and is given by

$$y_{i,t} = A(z_{i,t-1}k_{i,t})^{\alpha}(n_{i,t}K_t)^{1-\alpha}, \quad A > 0 \quad i \in [0,1], \tag{1}$$

where $y_{i,t}$, $k_{i,t}$, $n_{i,t}$, and K_t denote output, capital, labor, and aggregate capital, respectively. Aggregate capital have positive external effect on production (e.g., Romer 1986). We assume that capital $k_{i,t}$ is viewed broadly to include both ICT capital (knowledge capital) related to inovation and development (R&D) and non-ICT capital.[3] Here, $z_{i,t}$ is the production efficiency of the firm owned by the type i entrepreneur (household).

In youth each entrepreneur draws $z_{i,t}$ from a Pareto distribution whose cumulative distribution is given by

$$F(z) = 1 - z^{-\varphi}, \quad \varphi > 1. \tag{2}$$

Here, a lower (higher) value of φ means a higher (lower) degree of heterogeneity in production technology. Following Itskhoki and Moll (2014), Liu and Wang (2014), and Mino (2015), we assume that $z_{i,t}$ is iid (independent and identically-distributed) both over time and across agents.

After realizing z_i, each entrepreneur maximizes lifetime utility:

$$U_t^i = (1-\beta)\ln c_{i,t}^{y,j} + \beta\left[(1-\gamma)\ln c_{i,t+1}^{o,j} + \gamma \ln x_{i,t+1}^{j}\right], \quad j \in \{e, l\} \tag{3}$$

subject to the budget and credit constraints as we will explain the following. Here, $c_{i,t}^{y,e}$, $c_{i,t+1}^{o,e}$, and $x_{i,t+1}^{e}$ are consumption in youth, that in old age, and bequests by entrepreneurs (or borrowers), while $c_{i,t}^{y,l}$, $c_{i,t+1}^{o,l}$, and $x_{i,t+1}^{l}$ are those by non-entrepreneurs (or lenders).

The budget constraint of youth entrepreneurs is given by

$$c_{i,t}^{y,e} = w_t + x_{i,t} + \sigma_k k_{i,t+1} - a_{i,t+1}, \qquad a_{i,t+1} = k_{i,t+1} - d_{i,t} \tag{4}$$

where w_t, $x_{i,t}$, $d_{i,t}$, and $a_{i,t+1}(= k_{i,t+1} - d_{i,t})$ are wages, inheritance from parents, private debt,

[3]ICT capital includes hardware, communication and software and non-ICT capital transport equipment and non residential construction; products of agriculture, metal products and machinery other than hardware and communication equipment; and other products of non-residential gross fixed capital formation (see OECD (2010)).

and net worth of entrepreneurs who produce goods (active entrepreneurs, hereafter), respectively. Furthermore, τ and σ_k are a tax rate on income and a subsidy rate on investment. Note that $x_{i,t}$ depends on the parents' productivity $z_{i,t-1}$ and whether the parents are borrowers or lenders. However, this is not critical to the macroeconomy when we aggregate all agents, as we will see later.[4]

$$c_{i,t+1}^{o,e} = \pi_{i,t+1} - x_{i,t+1}^{e}, \tag{5}$$

$$\pi_{i,t+1} = y_{i,t+1} - w_{t+1}n_{i,t+1} - R_{i,t+1}d_{i,t}. \tag{6}$$

We assume full capital depreciation because we consider a period to be about 30 years.[5] Furthermore, the credit market is assumed to be imperfect in the following sense. Entrepreneurs face a credit constraint such that

$$d_{i,t} \leq \lambda k_{i,t+1}, \tag{7}$$

If $\lambda = 1$, the financial market is perfect, while no borrowing is available if $\lambda = 0$, meaning that λ is the degree of imperfection of the financial market.

From, (1), (3), (4), (5), (6), and (7), the first-order conditions (FOCs) with respect to $n_{i,t+1}$, $d_{i,t}$, and $k_{i,t+1}$ are given by

$$n_{i,t+1}; \qquad w_{t+1} = (1-\alpha)\frac{y_{i,t+1}}{n_{i,t+1}}, \tag{8}$$

$$d_{i,t}; \qquad \frac{1-\beta}{c_{i,t}^{y,e}} = \frac{\beta(1-\gamma)R_{t+1}}{c_{i,t+1}^{o,e}} + \mu_{i,t}, \tag{9}$$

$$k_{i,t+1}; \qquad \frac{(1-\beta)(1-\sigma_k)}{c_{i,t}^{y,e}} = \frac{\beta(1-\gamma)}{c_{i,t+1}^{o,e}}\frac{\partial \pi_{i,t+1}}{\partial k_{i,t+1}} + \lambda\mu_{i,t}, \tag{10}$$

$$\mu_{i,t}(\lambda k_{i,t+1} - d_{i,t}) = 0, \;\; \mu_{i,t} \geq 0, \;\; \lambda k_{i,t+1} - d_{i,t} \geq 0, \tag{11}$$

$$\mu_{i,t} = \frac{\beta(1-\gamma)}{1-\sigma_k-\lambda}\frac{\alpha(y_{i,t+1}/k_{i,t+1}) - (1-\sigma_k)R_{t+1}}{c_{i,t+1}^{o,e}} \tag{12}$$

where $\mu_{i,t}$ is the Lagrangian multiplier associated with the debt constraint and represents the investment wedge between the marginal product of capital and the real interest rate. Note that (12) is derived from (9) and (10) with (1), (6) and (8). If the financial constraint is not binding, $\mu_{i,t} = 0$ holds. We assume that entrepreneurs produce goods as long as their profits are not

[4]Even without the bequest motive $x_{i,t}$, our main results are robust. However, without $x_{i,t}$, the investment levels of all firms become the same, which is somewhat unrealistic. For this reason, and for future reference, we allow for the presence of the bequest motive.

[5]Without full capital depreciation ($\delta \neq 1$), the first term of the RHS of (6) is replaced in $y_{i,t+1} + (1-\delta)k_{i,t+1}$ if we denote $\delta \in [0, 1]$ as capital depreciation.

negative. Thus, the credit constraint (7) bind when

$$\alpha(y_{i,t+1}/k_{i,t+1}) \geq (1-\sigma_k)R_{t+1}. \tag{13}$$

From (1) and (8), we obtain

$$y_{i,t} = Az_{i,t-1}k_{i,t}\left[\frac{(1-\alpha)AK_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}}, \tag{14}$$

which together with (13) yields the cutoff value of z as

$$z_t^* = \frac{(1-\sigma_k)R_{t+1}}{\alpha A}\left[\frac{w_{t+1}}{(1-\alpha)AK_{t+1}}\right]^{\frac{1-\alpha}{\alpha}} \tag{15}$$

This, on the one hand, indicates that the entrepreneurs who draw $z_{i,t} \geq z_t^*$ produce. Because the debt constraints bind the active entrepreneurs, they become borrowers. On the other hand, the financial constraints are ineffective for the entrepreneurs who draw $z_{i,t} < z_t^*$. However, in the competitive final good market, the firms with $z_{i,t} < z_t^*$ cannot compete with the firms with z^*. Thus, the entrepreneurs who own the firms with $z_{i,t} < z_t^*$ give up production and become lenders. We will show later that indifference between becoming borrowers and lenders holds for the agents with $z_{i,t} = z^*$.

Moreover, (15) indicates that investment subsidy reduces the hurdle to become an entrepreneur z^* and increases the number of borrowers for given values of interest rates R_{t+1}. Utility maximization with respect to $x_{i,t+1}^e$ yields

$$x_{i,t+1}^e = \gamma\pi_{i,t+1}. \tag{16}$$

Substituting (8) and $d_{i,t} = \lambda k_{i,t+1}$ into (6), we obtain

$$\pi_{i,t+1} = \alpha y_{i,t+1} - R_{t+1}\lambda k_{i,t+1}. \tag{17}$$

From (9), (12), (16), and (17) with $d_{i,t} = \lambda k_{i,t+1}$, we obtain

$$k_{i,t+1} = \frac{\beta}{1-\sigma_k-\lambda}(w_t + x_{i,t}) \tag{18}$$

(18) indicates that a higher degree of financial market imperfection (a lower value of λ) reduces the investment of each firm. In contrast, a larger investment subsidy to firms increases the investment of each firm.

We move onto the case of lenders. Lenders (the entrepreneurs who draw $z_{i,t} < z_t^*$ and engage no production) maximize their utility (3) subject to

$$c_{i,t}^{y,l} = w_t + x_{i,t} - l_{i,t} - q_t b_{i,t+1}, \tag{19}$$

$$c_{i,t+1}^{o,l} = R_{t+1} l_{i,t} + b_{i,t+1} - x_{i,t+1}^{l}, \tag{20}$$

where $l_{i,t}$ is the loan, while $b_{i,t+1}$ is the quantity of government bonds purchased and q_t is the price of government bonds. The no-arbitrage condition between lending and buying Treasuries equates these rates of return as

$$R_{t+1} = 1/q_t \tag{21}$$

The lenders' FOCs with respect to $l_{i,t} + q_t b_{i,t+1}$ and $x_{i,t+1}^{l}$ result in

$$l_{i,t} + q_t b_{i,t+1} = \beta \left(w_t + x_{i,t} \right), \tag{22}$$

$$x_{i,t+1}^{l} = \gamma (R_{t+1} l_{i,t} + b_{i,t+1}). \tag{23}$$

From the discussion so far, we summerize the following lemma.

Lemma 1. *Indifference between being an entrepreneur (borrower) or a lender holds for households whose productivity is z_t^*. Households with $z_{i,t} \geq z_t^*$ become entrepreneurs (borrowers) while those with $z_{i,t} < z_t^*$ become non-entrepreneurs (lenders) in period t.*

See Appendix A for the proof of Lemma 1.

2.1 Government

The government owes a given amount of debt (denoted by B_t) at the beginning of the period t. The government repays the debt and finances investment by issuing new bonds. This means that the government is playing a "Ponzi game". We assume that government bonds are discount bonds with a maturity of 1 period and a face value of 1. Thus, the government's budget constraint in period t is given by

$$q_t B_{t+1} = B_t + \sigma_k \int_0^1 \int_{z_t \geq z_t^*} k_{i,t+1} dF(z_t) di \tag{24}$$

$$B_{t+1} = \int_0^1 \int_{z_t \leq z_t^*} b_{i,t+1} dF(z_t) di \quad \left(B_t = \int_0^1 \int_{z_{t-1} \leq z_{t-1}^*} b_{i,t} dF(z_{t-1}) di \right) \tag{25}$$

Here, let us define $\tilde{B}_{t+1} \equiv q_t B_{t+1}$ and then we obtain $\tilde{B}_{t+1} = B_{t+1}/R_{t+1}$ by (21). (24) is transformed into

$$\tilde{B}_{t+1} = R_t \tilde{B}_t + \sigma_k \int_0^1 \int_{z_t \geq z_t^*} k_{i,t+1} dF(z_t) di, \tag{26}$$

where we notify that $R_t (= 1 + r_t)$ is the gross interest rate when r_t denotes the interest rate.

2.2 Equilibrium

The aggregate capital stock K_t is held by active entrepreneurs (borrowers) who draw $z_{i,t-1} \geq z_{t-1}^*$ in period $t-1$ and is therefore represented by $\int_0^1 \int_{z_{t-1} \geq z_{t-1}^*} k_{i,t} dF(z_{t-1}) di = K_t$. Using (2), and keeping in mind that z_i is iid, we can rewrite the aggregate capital as

$$K_t = (z_{t-1}^*)^{-\varphi} \int_0^1 k_{i,t} di. \tag{27}$$

Aggregating credit constraint $d_{i,t} = \lambda k_{i,t+1}$, equilibrium condition of the financial market is given by

$$\int_0^1 \int_{z_t \leq z_t^*} l_{i,t} dF(z_t) di = \int_0^1 \int_{z_t \geq z_t^*} d_{i,t} dF(z_t) di = \lambda K_{t+1}. \tag{28}$$

Labor market clears as

$$N_t = \int_0^1 \int_{z_{t-1} \geq z_{t-1}^*} z_{t-1} n_{i,t} dF(z_{t-1}) di = 1, \tag{29}$$

which indicates that total labor demand is equal to labor supply, whose aggregate level is unity.

Using (2) and (27), we can also aggregate the production function (14) as

$$\begin{aligned} Y_t &= \int_0^1 \int_{z_{t-1} \geq z_{t-1}^*} A z_{i,t-1} k_{i,t} \left[\frac{(1-\alpha) A K_t}{w_t} \right]^{\frac{1-\alpha}{\alpha}} dF(z_{t-1}) di \\ &= \frac{A\varphi}{\varphi - 1} z_{t-1}^* \left[\frac{(1-\alpha) A K_t}{w_t} \right]^{\frac{1-\alpha}{\alpha}} K_t. \end{aligned} \tag{30}$$

Substituting (15) into (30), we obtain

$$Y_t = \frac{\varphi(1-\sigma_k)}{\alpha(\varphi - 1)} R_t K_t. \tag{31}$$

From (8), (29), and (31), we obtain $w_t N_t = (1-\alpha) \int_0^1 \int_{z_{t-1} \geq z_{t-1}^*} y_{i,t} dF(z_{t-1}) di = (1-\alpha) Y_t$, leading to

$$w_t = \frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)}(1-\alpha)R_t K_t. \tag{32}$$

Substituting (32) into (15), we obtain

$$z_t^* = \left(\frac{\varphi}{\varphi-1}\right)^{\frac{1-\alpha}{\alpha}} \left(\frac{(1-\sigma_k)R_{t+1}}{\alpha A}\right)^{\frac{1}{\alpha}}. \tag{33}$$

An increase in R_{t+1} reduces the number of firms because it increases the cost of borrowing for entrepreneurs. In addition, investment subsidies σ_k lower the barrier to becoming an entrepreneur z^* and increase the number of less productive firms. This lowers aggregate productivity and reduces both output, (31), and the wage rate, (32).

Let us continue with the aggregation of other elements. Aggregating (18), using (32), and keeping in mind that $x_{i,t}$ is independent of $z_{i,t}$, we obtain

$$K_{t+1} = \frac{\beta}{1-\sigma_k-\lambda}(z_t^*)^{-\varphi}\left[\frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)}(1-\alpha)R_t K_t + \int_0^1 x_{i,t}di\right], \tag{34}$$

where $\int_0^1 x_{i,t}di = \int_0^1 \int_{z_{t-1}\leq z_{t-1}^*} x_{i,t}^l dF(z_{t-1})di + \int_0^1 \int_{z_{t-1}\geq z_{t-1}^*} x_{i,t}^e dF(z_{t-1})di$.

By (16) and (17), we obtain $x_{i,t+1}^e = \alpha y_{i,t+1} - R_{t+1}\lambda k_{i,t+1}$. Aggregating it by using (27) and (31), we obtain

$$\int_0^1 \int_{z_t\geq z_t^*} x_{i,t+1}^e dF(z_t)di = \gamma\left(\frac{\varphi(1-\sigma_k)}{\varphi-1} - \lambda\right) R_{t+1}K_{t+1}. \tag{35}$$

Next, aggregating $x_{i,t+1}^l$ with (2), (25) and (28), we obtain

$$\int_0^1 \int_{z_t\leq z_t^*} x_{i,t+1}^l dF(z_t)di = \gamma(R_{t+1}\lambda K_{t+1} + B_{t+1}). \tag{36}$$

Because of $\int_0^1 x_{i,t+1}di = \int_0^1 \int_{z_t\leq z_t^*} x_{i,t+1}^l dF(z_t)di + \int_0^1 \int_{z_t\geq z_t^*} x_{i,t+1}^e dF(z_t)di$, this associated with (33), (35), and (36) yields

$$\int_0^1 x_{i,t+1}di = \gamma\left[\frac{\varphi(1-\sigma_k)}{\varphi-1}R_{t+1}K_{t+1} + B_{t+1}\right]. \tag{37}$$

From (34) and (37), we obtain

$$K_{t+1} = \frac{\beta}{1-\sigma_k-\lambda}(z_t^*)^{-\varphi}\left[\frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)}(1-\alpha+\alpha\gamma)R_t K_t + \gamma B_t\right]. \tag{38}$$

Aggregating (22) and using (32), we obtain

$$\int_0^1 \int_{z_t \leq z_t^*} l_{i,t} dF(z_t) di = \beta \left[1 - (z_t^*)^{-\varphi}\right] \left[\frac{(1-\alpha)\varphi}{\alpha(\varphi-1)} R_t K_t + \int_0^1 x_{i,t} di\right] - q_t B_{t+1}. \tag{39}$$

Substituting (28) and (37) into (39), we obtain

$$\lambda K_{t+1} = \beta \left[1 - (z_t^*)^{-\varphi}\right] \left[\frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)} (1-\alpha+\alpha\gamma) R_t K_t + \gamma B_t\right] - q_t B_{t+1}. \tag{40}$$

The right-hand side (RHS) of (40) represents the aggregate supply of credit in the financial market whereas the left-hand side (LHS) shows the aggregate demand for it. Increases in public borrowing (public debt issuance), $q_t B_{t+1}$, crowd out the aggregate supply of credit. This leads to an upward pressure on the interest rate R_{t+1} in the financial market as we will see later.

From (38) and (40) associated with (21) and $\tilde{B}_{t+1} \equiv q_t B_{t+1}$, we obtain the following asset market clearing condition:

$$K_{t+1} + \tilde{B}_{t+1} = \beta R_t \left[\frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)} (1-\alpha+\alpha\gamma) K_t + \gamma \tilde{B}_t\right] + \sigma_k K_{t+1} \tag{41}$$

We can confirms that the familiar crowding-out effect of public debt $\tilde{B}_{t+1}$ on private investment K_{t+1} exists.

From (26) and (27), we obtain

$$\tilde{B}_{t+1} = R_t \tilde{B}_t + \sigma_k K_{t+1}. \tag{42}$$

An increase in the current outstanding public debt increases the issuance of public bonds and worsens the fiscal condition. This is also the case for investment subsidies σ_k for a given level of investment K_{t+1}.

Let us define $\theta \equiv \tilde{B}_t / K_t$. (41) with (42) yields

$$\frac{K_{t+1}}{K_t} = R_t \left[\frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)} \beta(1-\alpha+\alpha\gamma) - (1-\beta\gamma)\theta_t\right] \tag{43}$$

$$\frac{\tilde{B}_{t+1}}{\tilde{B}_t} = R_t \left[\frac{\varphi\sigma_k(1-\sigma_k)}{\alpha(\varphi-1)} \beta(1-\alpha+\alpha\gamma)\theta_t^{-1} + [1-\sigma_k(1-\beta\gamma)]\right]. \tag{44}$$

Here, to ensure $K_{t+1}/K_t > 0$, we assume the following condition

$$\theta_t < \bar{\theta} \equiv \frac{\varphi(1-\sigma_k)(1-\alpha+\alpha\gamma)}{\alpha(\varphi-1)(1-\beta\gamma)}. \tag{45}$$

From (43) and (44), we obtain

$$\theta_{t+1} = \frac{\frac{\varphi}{\alpha(\varphi-1)}\beta(1-\alpha+\alpha\gamma)\sigma_k(1-\sigma_k) + [1-\sigma_k(1-\beta\gamma)]\theta_t}{\frac{\varphi}{\alpha(\varphi-1)}\beta(1-\alpha+\alpha\gamma)(1-\sigma_k) - (1-\beta\gamma)\theta_t} \equiv \Lambda(\theta_t;\sigma_k). \tag{46}$$

(46) with (45) characterizes the dynamics of the economy. The LHS of (46) represents the 45 degree line, while the RHS satisfies $\Lambda(0;\sigma_k) = \sigma_k > 0$, $\Lambda'(\theta_t;\sigma_k) > 0$, and $\Lambda''(\theta_t;\sigma_k) > 0$. The RHS of (46) intersects the LHS twice at the steady states at S and U as in Figure 1 if and only if

$$\left[1-\sigma_k(1-\beta\gamma) - \frac{\varphi}{\alpha(\varphi-1)}\beta(1-\alpha+\alpha\gamma)(1-\sigma_k)\right]^2 - 4(1-\beta\gamma)\frac{\varphi}{\alpha(\varphi-1)}\beta(1-\alpha+\alpha\gamma)\sigma_k(1-\sigma_k). \tag{47}$$

and $$\frac{\varphi}{\alpha(\varphi-1)}\beta(1-\alpha+\alpha\gamma)(1-\sigma_k) - [1-\sigma_k(1-\beta\gamma)] > 0. \tag{48}$$

Let us denote the two stationary values of θ_t as θ_S^* at S and θ_U^* at U. From (46), $\theta_S^* < \theta_U^* < \bar{\theta}$ is satisfied if and only if

$$\Lambda(\bar{\theta};\sigma_k) > \bar{\theta}. \tag{49}$$

This gives us the following proposition.

Proposition 1. *Two steady states, represented by S and U in Figure 1, exist under* (47), (48), *and* (49). *The steady state S is stable while U is unstable.*

Proposition 1 indicates that θ_t converges to the stable steady state value θ_S^* as long as the initial value $\theta_0 < \theta_U^*$. Otherwise (the case of $\theta_0 > \theta_U^*$), θ_t continues to grow and eventually violates (45), making fiscal policy with debt financing unsustainable. Therefore, θ_U^* represents the maximum value of the ratio of public debt to capital to ensure the sustainability of public debt.

[Figure 1]

Next, we derive the relationship between the behavior of the entrepreneurs z_t^*, the (gross) interest rate R_{t+1}, and θ_t. Using (38) and (43) with $\tilde{B}_t \equiv B_t/R_t$ yields

$$z_t^* = \left\{\left(\frac{\beta}{1-\sigma_k-\lambda}\right)\frac{\frac{\varphi}{\alpha(\varphi-1)}(1-\sigma_k)(1-\alpha+\alpha\gamma)+\gamma\theta_t}{\frac{\varphi}{\alpha(\varphi-1)}\beta(1-\sigma_k)(1-\alpha+\alpha\gamma)-(1-\beta\gamma)\theta_t}\right\}^{1/\varphi} \equiv z(\theta_t;\sigma_k) \tag{50}$$

Substituting (50) into (33), we obtain

$$R_{t+1} = \frac{\alpha A}{1-\sigma_k}\left(\frac{\varphi-1}{\varphi}\right)^{1-\alpha}\left(\frac{\beta}{1-\sigma_k-\lambda}\right)^{\frac{\alpha}{\varphi}}\left[\frac{\frac{\varphi}{\alpha(\varphi-1)}(1-\sigma_k)(1-\alpha+\alpha\gamma)+\gamma\theta_t}{\frac{\varphi}{\alpha(\varphi-1)}\beta(1-\sigma_k)(1-\alpha+\alpha\gamma)-(1-\beta\gamma)\theta_t}\right]^{\frac{\alpha}{\varphi}}$$
$$\equiv \mathcal{R}(\theta_t;\sigma_k). \tag{51}$$

From (50) and (51), we derive the following lemma:

Lemma 2. $z'(\theta_t;\sigma_k) > 0$ *and* $\mathcal{R}'(\theta_t;\sigma_k) > 0$.

Increases in the public debt to capital ratio θ_t raise the interest rate, $\mathcal{R}'(\theta_t;\sigma_k) > 0$. This is because an increase in government debt increases the issuance of government bonds (see (42)) and decreases the total supply of credit in the financial market (see (40)), raising the interest rate R_{t+1}. Increasing R_{t+1} by increasing θ_t reduces the number of firms z_t^* because it increases the cost of borrowing for entrepreneurs (see (33)). Thus, $z'(\theta_t;\sigma_k) > 0$.

By (46) and (51), we obtain

$$R_t = \Psi(\theta_t;\sigma_k) \;\text{ and }\; \Psi'(\theta_t;\sigma_k) = \frac{\mathcal{R}'(\theta_{t-1};\sigma_k)}{\Lambda'(\theta_{t-1};\sigma_k)} > 0. \tag{52}$$

In contrast to Saint-Paul (1992) and Bräuninger (2005), the interest rate is not constant, but changes over time as θ_t varies.

Applying (52) into (43) and (44) and using $\frac{Y_{t+1}}{Y_t} = \frac{R_{t+1}}{R_t}\frac{K_{t+1}}{K_t}$ (by (31)) and (51), we obtain

$$\frac{K_{t+1}}{K_t} \equiv g_t^K(\theta_t;\sigma_k) = \Psi(\theta_t;\sigma_k)\left[\frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)}\beta(1-\alpha+\alpha\gamma)-(1-\beta\gamma)\theta_t\right], \tag{53}$$
$$\frac{\tilde{B}_{t+1}}{\tilde{B}_t} \equiv g_t^B(\theta_t;\sigma_k) = \Psi(\theta_t;\sigma_k)\left[\frac{\varphi\sigma_k(1-\sigma_k)}{\alpha(\varphi-1)}\beta(1-\alpha+\alpha\gamma)\theta_t^{-1}+1-\sigma_k(1-\beta\gamma)\right], \tag{54}$$
$$\frac{Y_{t+1}}{Y_t} \equiv g_t^Y(\theta_t;\sigma_k) = \mathcal{R}(\theta_t;\sigma_k)\left[\frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)}\beta(1-\alpha+\alpha\gamma)-(1-\beta\gamma)\theta_t\right]. \tag{55}$$

An increase in the public debt to capital ratio θ_t has two opposite effects on $g_t^K(\theta_t;\sigma_k)$ and $g_t^Y(\theta_t;\sigma_k)$. One is the negative growth effect through the crowding out effect of public debt on investment. The other is the positive growth effect from a rise in the interest rate. The interest rate rises because increases in public debt enhance the issuance of public bonds (see (42)) and decrease the aggregate supply of credit in the financial market (see (40)). Output and wage income increase with a rise in the interest rate (see (31) and (32)), because a rise in the interest rate increases the cost of borrowing for entrepreneurs and reduces the number of firms while it increases capital intensity and the marginal productivity of capital. The former positive effect dominates the latter negative one in a numerical example with a reasonable parameter set

$(\alpha, \beta, \gamma, \varphi, A, \lambda, g, \sigma_k,) = (0.4, 0.3, 0.3, 1.5, 17, 0.8, 0.2, 0.01)$ (see Appendix B for the choice of the parameter values) as shown in figure 2.

[Figure 2]

An increase in the public debt to capital ratio θ_t also has two opposite effects on $g_t^B(\theta_t; \sigma_k)$. On the one hand, an increase in the public debt to capital ratio θ_t raises the interest rate by decreasing the aggregate supply of credit in the financial market (see (40)). This increase in the interest rate boosts interest payments and $g_t^B(\theta_t; \sigma_k)$. The positive growth effect of an increase in the interest rate increases private investment and public spending on investment subsidy, leading to increase $g_t^B(\theta_t; \sigma_k)$ as well.

On the other hand, an increase in public debt crowds out private investment. Therefore, the burden of public spending on investment subsidies shrinks as the ratio of public debt to capital θ_t increases, leading to a reduction in $g_t^B(\theta_t; \sigma_k)$. The latter negative (the former positive) effect on $g_t^B(\theta_t; \sigma_k)$ dominates the former (the latter) when θ_t is small (large), as Figure 2 shows.

3 Investment subsidy to firms

In this section, we examine the effects of introducing investment subsidies to firms on growth and fiscal sustainability. In this study, investment subsidies are financed through the issuance of public bonds, rather than through tax finance as in previous studies (e.g., Morimoto 2018).

From (46), we obtain

$$\left.\frac{\partial \Lambda(\theta_t; \sigma_k)}{\partial \sigma_k}\right|_{\sigma_k=0} = 1 + \frac{\beta\varphi(1-\alpha+\alpha\gamma)}{[\beta\varphi(1-\alpha+\alpha\gamma) + \alpha(\varphi-1)(1-\beta\gamma)\theta_t]^2} > 0, \tag{56}$$

from which we arrive at the following proposition:

Proposition 2. *Introducing an investment subsidy for entrepreneurs (a) makes public debt less sustainable and (b) increases the ratio of public debt to capital θ_S^* in the steady state S.*

Investment subsidies to firms, on the one hand, encourage investment of each firm ((18)) and promote economic growth K_{t+1}/K_t. On the other hand, investment subsidies increase the isuance of public bonds and accelerate the accumulation of public debt $\tilde{B}_{t+1}/\tilde{B}_t$. The latter dominates the former and shifts $\theta_{t+1} = \Lambda(\theta_t; \sigma_k)$ upward. θ_S^* increases while θ_U^* decreases. Then, investment subsidies to firms (a) make public debt less sustainable and (b) increase the public debt to capital ratio in the steady state S. Figure 3 illustrates the case of an increase in σ_k from 0.01 to 0.04, justifying that the impact of investment subsidies on economic growth is less than that on fiscal deterioration.

The reasons for the small impact of investment subsidies on economic growth are as follows. First, investment subsidies σ_k lower the barrier to becoming an entrepreneur z^* and increase the number of firms with lower productivity. This lowers aggregate productivity ((33)). Second, the increase in these firms increases the aggregate demand for credit in the financial market ((40)) and leads to an upward pressure on the interest rate R_{t+1}. It reduces the number of firms ((33)) due to an increase in the cost of borrowing for entrepreneurs. Third, an increase in this interest rate increases the cost of repaying public debt and crowds out private investment ((41)). These three have significant negative effects on economic growth.

[Figure 1]

The growth of public debt extends a direct effect through the increase of σ_k and an indirect effect through an increase of the interest rate of the financial market. It amplifies the negative effect on firm entry and the crowding out effect of investment (the second and third effects above) in the long run.

Then, we next investigate the long-run growth effect of investment subsidy. Using (55) and the fact that $\theta_S^* = 0$ for $\sigma_k = 0$, and noting that $\frac{\partial \ln g^Y(\theta_S^*;\sigma_k)}{\partial \sigma_k}|_{\sigma_k=0} = \frac{1}{g^Y(\theta_S^*;\sigma_k)} \frac{\partial g^Y(\theta_S^*;\sigma_k)}{\partial \sigma_k}|_{\sigma_k=0}$, we obtain

$$\left.\frac{\partial \ln g^Y(\theta_S^*;\sigma_k)}{\partial \sigma_k}\right|_{\sigma_k=0} = \left.\frac{\partial \ln R(\theta_S^*;\sigma_k)}{\partial \sigma_k}\right|_{\sigma_k=0} - \frac{\beta\frac{\varphi}{\varphi-1}\left(\frac{1-\alpha}{\alpha}+\gamma\right) + (1-\beta\gamma)\frac{\partial \theta_S^*}{\partial \sigma_k}|_{\sigma_k=0}}{\beta\frac{\varphi}{\varphi-1}\left(\frac{1-\alpha}{\alpha}+\gamma\right)}. \quad (57)$$

From (46) and the fact that $\theta_S^* = 0$ for $\sigma_k = 0$, we obtain

$$\left.\frac{\partial \theta_S^*}{\partial \sigma_k}\right|_{\sigma_k=0} = \frac{\beta\frac{\varphi}{\varphi-1}\left(\frac{1-\alpha}{\alpha}+\gamma\right)}{\beta\frac{\varphi}{\varphi-1}\left(\frac{1-\alpha}{\alpha}+\gamma\right)-1} > 0 \quad (58)$$

Here, note that $\beta\frac{\varphi}{\varphi-1}\left(\frac{1-\alpha}{\alpha}+\gamma\right)-1 > 0$ by the condition (47). Furthermore, from (51) and the fact that $\theta_S^* = 0$ for $\sigma_k = 0$, we obtain

$$\left.\frac{\partial \ln R(\theta_S^*;\sigma_k)}{\partial \sigma_k}\right|_{\sigma_k=0} = 1 + \frac{\alpha}{\varphi}\frac{1}{1-\lambda} + \frac{\alpha}{\varphi}\frac{\frac{\partial \theta_S^*}{\partial \sigma_k}|_{\sigma_k=0}}{\frac{\varphi}{\varphi-1}\left(\frac{1-\alpha}{\alpha}+\gamma\right)} > 0. \quad (59)$$

Substituting (58) and (59) into (57) yields the following proposition:

Proposition 3. *Introducing investment subsidies to firms increases (decreases) long-run growth* $\partial \ln g^Y(\theta_S^*;\sigma_k)/\partial\sigma_k|_{\sigma_k=0} > (<) 0$ *if and only if*

$$\frac{\alpha}{\varphi}\frac{1}{1-\lambda} - \frac{1-\beta\gamma-\frac{\alpha}{\varphi}}{\beta\frac{\varphi}{\varphi-1}\left(\frac{1-\alpha}{\alpha}+\gamma\right)-1} > (<) 0 \quad (60)$$

Investment subsidies encourage investment but reduce the number of firms, which increases capital intensity and the marginal productivity of capital. This works positively for economic growth as shown in the first term of (60). Three negative growth effects mentioned are gathered in the second second term. Reasonable numerical exercises show that the negative growth effects dominate the positive ones as shown in figures 4 and 5, indicating that long-term economic growth decreases in investment subsidy financed by public debt σ_k. We compare this to the case where the investment subsidy is fully financed by income tax revenue and the government maintains a balanced budget. Appendix B shows that in this case, the investment subsidy promotes economic growth for all $\sigma_k(> 0)$. Thus, the policy implication is very clear in the sense that investment subsidy should not be financed by public debt but by tax increases.

Sensitive analyses as to λ (the degree of credit market imperfection) are made because it does not have a widely accepted common value in this literature. Figure 6 combined with (60) shows that the growth effect of σ_k is robust unless λ is very close to 1 (the case of a perfect credit market).

4 Conclusion

This study examines the effect of public debt on growth, interest rates, and the sustainability of public debt in a very simple endogenous growth model with financial imperfection and firm heterogeneity. An increase in public debt leads to higher real interest rates through financial markets and reduces both the number of firms and private investment, leading to lower long-run growth. It makes public debt less sustainable when public debt is very large. This study also examines the effect of investment subsidy financed by public debt. It hampers economic growth in the long run, although it has a positive effect on growth in the short run. Therefore, investment subsidy should not be financed by public debt but by tax increase.

Acknowledgments
I acknowledge the financial support from The Nitto Foundation.

References

[1] Afonso, A, (2005) Fiscal sustainability: The unpleasant European case. ” FinanzArchiv: Public Finance Analysis, 61, 19–44,

[2] Agénor, P., Yilmaz, D. (2017). The simple dynamics of public debt with productive public goods. Macroeconomic Dynamics, 21, 1059–1095.

[3] Arai, R., Kunieda, T. (2010) Sustainable public debt, credit constraints and social welfare. KIER discussion paper series No. 732.

[4] Arai, R. (2011). Productive government expenditure and fiscal sustainability. FinanzArchiv: Public Finance Analysis, 67, 327–351.

[5] Arawatari R., Hori, T., Mino, K. (2023) Government expenditure and economic growth: A heterogeneous-agents approach. Journal of Macroeconomics, 75, 103486.

[6] Chalk, N.A. (2000) The sustainability of bond-financed deficits: An overlapping generations approach. Journal of Monetary Economics, 45, 293–328.

[7] Bohn, H., 1998. The behavior of U.S. public debt and deficits. Quarterly Journal of Economics 113, 949–963.

[8] Bräuninger, M. (2005). The budget deficit, public debt, and endogenous growth. Journal of Public Economic Theory, 7, 827–840.

[9] de la Croix, D., Michel, P. (2002). A Theory of Economic Growth. Cambridge University Press, Cambridge.

[10] Diamond, P., Saez, E. (2011) The case for a progressive tax: from basic research to policy recommendations. Journal of Economic Perspectives, 25, 165–190.

[11] European Commission (2017) Directorate-General for Taxation and Customs Union, Effectiveness of tax incentives for venture capital and business angels to foster the investment of SMEs and start-ups – Final report, Publications Office, 2017, https://data.europa.eu/doi/10.2778/005699.

[12] Eurostat Statistics Explained-Government budget allocations for R&D (GBARD). https://ec.europa.eu/eurostat/statistics-explained/index.php?oldid=553052 (Accessed on March 6, 2024)

[13] Futagami, K., Konishi, K. (2023) Comparative analyses of fiscal sustainability of the budgetary policy rules. Journal of Public Economic Theory, 25, 944–984

[14] Futagami, K., Morita, Y., Shibata, A. (1993). Dynamic analysis of an endogenous growth model with public capital. Scandinavian Journal of Economics, 95, 607—625.

[15] Greiner, A. (2007). An endogenous growth model with public capital and sustainable government debt. Japanese Economic Review, 58, 345–361.

[16] Greiner, A. (2011). Economic growth, public debt, and welfare: Comparing three budgetary rules. German Economic Review, 12, 205–222.

[17] Greiner, A. (2012). Public capital, sustainable debt, and endogenous growth. Research in Economics, 66, 230–238.

[18] Greiner, A. (2015). Public debt, productive public spending and endogenous growth. Japanese Economic Review, 66, 520–535.

[19] Hamilton, J.D., Flavin, M.A., 1986. On the limitations of government borrowing: a framework for empirical testing. American Economic Review 76, 808–816.

[20] Hansen, G.D., İmrohoroğlu, S. (2016). Fiscal reform and government debt in Japan: A neoclassical perspective. Review of Economic Dynamics, 21, 201–224.

[21] Jaimovich N., Rebelo, S (2017). Nonlinear effects of taxation on growth. Journal of Political Economy, 125, 265–291.

[22] Kang, M. (2022) The positive impact of investment subsidies on the economy with present-biased consumers. Quarterly Review of Economics and Finance, 85, 229–235.

[23] Maebayashi, N., Konishi, K. (2021). Sustainability of public debt and inequality in a general equilibrium model. Macroeconomic Dynamics, 25, 874–895.

[24] Maebayashi, N. (2023). The pace of fiscal consolidations, fiscal sustainability, and welfare: An overlapping generations approach. Journal of Macroeconomics, 75, 103485.

[25] Mino, K (2015). A simple model of endogenous growth with financial frictions and firm heterogeneity. Economiics Letters, 127, 20–23.

[26] Mino, K (2016). Fiscal policy in a growing economy with financial frictions and firm heterogeneity. Japanese Economic Review, 67, 3–30.

[27] Miyazawa, K., Ogawa, H., Tamai, T. (2019). Capital market integration and fiscal sustainability. European Economic Review, 120, 103305.

[28] Morimoto, T. (2018) Occupational choice and entrepreneurship: effects of R&D subsidies on economic growth. Journal of Economics, 123, 161–185.

[29] OECD (2010) OECD Factbook 2010: economic, environmental and social statistics. https://doi.org/10.1787/factbook-2010-en.

[30] Romer, P. (1986). Increasing returns and long ‐ run growth. Journal of Political Economy, 94, 1002–1037.

[31] Teles, V., Mussolini, C. (2014). Public debt and the limits of fiscal policy to increase economic growth. European Economic Review, 66, 1—15.

[32] The World Bank (2023). Research and development expenditure (% of GDP). Retrieved from https://data.worldbank.org/indicator/GB.XPD.RSDV.GD.ZS (Accessed on May 08, 2024).

[33] Trabandt, M., Uhlig H. (2011). The Laffer curve revisited, Journal of Monetary Economics, 58, 305–327.

[34] Yakita, A. (2008). Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. Journal of Public Economics, 92, 897–914.

[35] Yakita, A. (2014). Involuntary unemployment and sustainability of bond-financed fiscal deficit. Journal of Macroeconomics, 41, 79–93.

Appendix

A Proof of Lemma 1

Let us denote the items $(c_t^{y,j}, c_{t+1}^{o,j}, x_{t+1}^j, x_t, k_{t+1}, y_{t+1})$ and $j \in \{e, l\}$ of the households whose productivity is z_t^* as $(\bar{c}_t^{y,j}, \bar{c}_{t+1}^{o,j}, \bar{x}_{t+1}^j \bar{x}_t, \bar{k}_{t+1}, \bar{y}_{t+1})$. First, consider the case where the households with $z_{i,t} = z_t^*$ become entrepreneurs.

Substituting (18) into (4) yields

$$\bar{c}_t^{y,e} = (1-\beta)(w_t + \bar{x}_t), \tag{A.1}$$

while inserting (16) and (17) into (5) yields

$$\bar{c}_{t+1}^{o,e} = (1-\gamma)(\alpha \bar{y}_{i,t+1} - \lambda R_{t+1} \bar{k}_{i,t+1}) \tag{A.2}$$

By (14), $\bar{y}_{t+1} = A z_t^* \bar{k}_{t+1} \left[\frac{(1-\alpha)AK_t}{w_t}\right]^{\frac{1-\alpha}{\alpha}}$. Substituting (15) into this, we obtain $\bar{y}_{t+1} = (1 - \sigma_k) R_{t+1} \bar{k}_{t+1}$. This rewrites (A.2) to

$$\begin{aligned}\bar{c}_{t+1}^{o,e} &= (1-\gamma)(1-\sigma_k-\lambda) R_{t+1} \bar{k}_{i,t+1} \\ &= \beta(1-\gamma) R_{t+1}(w_t + \bar{x}_t),\end{aligned} \tag{A.3}$$

where we have used (18). Substituting $\bar{y}_{t+1} = (1 - \sigma_k)R_{t+1}\bar{k}_{t+1}$ and (17) into (16), we obtain

$$\bar{x}^e_{t+1} = \beta\gamma R_{t+1}(w_t + \bar{x}_t) \tag{A.4}$$

Second, consider the case where the households with $z_{i,t} = z^*_t$ become lenders. Substituting (21), (22), and (23) into (19) and (20), we obtain

$$\bar{c}^{y,l}_t = (1 - \beta)(w_t + \bar{x}_t), \tag{A.5}$$

$$\bar{c}^{o,l}_{t+1} = \beta(1 - \gamma)R_{t+1}(w_t + \bar{x}_t), \tag{A.6}$$

From (22) and (23), we obtain

$$\bar{x}^l_{t+1} = \beta\gamma R_{t+1}(w_t + \bar{x}_t) \tag{A.7}$$

Because of $\bar{c}^{y,e}_t = c^{y,l}_{i,t}$, $\bar{c}^{o,e}_{t+1} = c^{o,l}_{t+1}$, and $\bar{x}^e_{t+1} = \bar{x}^l_{t+1}$, Lemma 1 is proved.

B Choice of parameters values for numerical stuides

We set $\beta = 0.3$ because the discount factor should be $\beta/(1-\beta) = 0.97^{30} \approx 0.4$. The parameter α in is set to 0.4 which is near the average values of the US ($\alpha = 0.35$), the EU ($\alpha = 0.38$), and Japan ($\alpha = 0.38$).[6] We select the value of γ to satisfy $\gamma = \beta$ as the benchmark. $g = 0.2$ is follows the same value in Bräuninger (2005). The scale parameter $A = 17$ yields positive plausible values for the long - run growth rates, as we see in Figures 2, 3, and 4. we select $\lambda = 0.8$: the degree of financial imperfection as the benchmark case. Finally, we set $\varphi = 1.5$ as in Diamond and Saez (2011) and Jaimovich and Rebelo (2017). This choice of φ implies that the right tail of the income distribution implied by the model is the same as that estimated by Diamond and Saez (2011) for the US economy.

[6]See Trabandt and Uhlig (2011) for the values of the US and the EU, and Hansen and İmrohoroğlu (2016) for the value of Japan.

C Investment subsidy financed by income tax under a balanced budget regime

Enterpreneurs' budget constraints with constant income tax rate τ in youth and old age are given by

$$c^{y,e}_{i,t} = (1-\tau)w_t + x_{i,t} + \sigma_k k_{i,t+1} - a_{i,t+1}, \qquad a_{i,t+1} = k_{i,t+1} - d_{i,t} \tag{C.1}$$

$$c^{o,e}_{i,t+1} = (1-\tau)\pi_{i,t+1} - x^e_{i,t+1} \qquad \text{with (6)} \tag{C.2}$$

The FOC with respect to $n_{i,t+1}$ is given by (8) and those with respect to $d_{i,t}$ and $k_{i,t+1}$ are replaced by

$$d_{i,t}; \qquad \frac{1-\beta}{c^{y,e}_{i,t}} = \frac{\beta(1-\gamma)(1-\tau)R_{t+1}}{c^{o,e}_{i,t+1}} + \mu_{i,t}, \tag{C.3}$$

$$k_{i,t+1}; \qquad \frac{(1-\beta)(1-\sigma_k)}{c^{y,e}_{i,t}} = \frac{\beta(1-\gamma)(1-\tau)}{c^{o,e}_{i,t+1}}\frac{\partial \pi_{i,t+1}}{\partial k_{i,t+1}} + \lambda\mu_{i,t} \tag{C.4}$$

with (11) and (12).

Note that (13), (14), and (15) remain unchanged. Maximizing utility with respect to $x^e_{i,t+1}$ yields

$$x^e_{i,t+1} = \gamma(1-\tau)\pi_{i,t+1}. \tag{C.5}$$

From (C.3), (12), (C.5), and (17) with $d_{i,t} = \lambda k_{i,t+1}$, we obtain

$$k_{i,t+1} = \frac{\beta}{1-\sigma_k-\lambda}[(1-\tau)w_t + x_{i,t}] \tag{C.6}$$

Because of the balanced budget without public debts, the lenders' budget constraints are

$$c^{y,l}_{i,t} = (1-\tau)w_t + x_{i,t} - l_{i,t}, \tag{C.7}$$

$$c^{o,l}_{i,t+1} = (1-\tau)R_{t+1}l_{i,t} - x^l_{i,t+1}, \tag{C.8}$$

The FOCs of the lenders with respect to $l_{i,t}$ and $x^l_{i,t+1}$ result in

$$l_{i,t} = \beta\left[(1-\tau)w_t + x_{i,t}\right], \tag{C.9}$$

$$x^l_{i,t+1} = \gamma(1-\tau)R_{t+1}l_{i,t}. \tag{C.10}$$

The government collects income tax revenue and maintains a balanced budget to finance investment subsidies:

$$\tau\left[w_tN_t+\int_0^1\int_{z_{t-1}\geq z_{t-1}^*}\pi_{i,t}dF(z_t)di+R_t\int_0^1\int_{z_{t-1}\leq z_{t-1}^*}l_{i,t}dF(z_t)di\right]$$
$$=\sigma_k\int_0^1\int_{z_t\geq z_t^*}k_{i,t+1}dF(z_t)di \tag{C.11}$$

The equations from (27) to (33) remain unchanged. Aggregating (C.6) and using (32), we obtain

$$K_{t+1}=\frac{\beta}{1-\sigma_k-\lambda}(z_t^*)^{-\varphi}\left[\frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)}(1-\alpha)(1-\tau)R_tK_t+\int_0^1 x_{i,t}di\right], \tag{C.12}$$

By (C.5) and (17), we obtain $x_{i,t+1}^e=(1-\tau)[\alpha y_{i,t+1}-R_{t+1}\lambda k_{i,t+1}]$. Aggregating it by using (27) and (31), we obtain

$$\int_0^1\int_{z_t\geq z_t^*}x_{i,t+1}^e dF(z_t)di=\gamma\left(\frac{\varphi(1-\sigma_k)}{\varphi-1}-\lambda\right)(1-\tau)R_{t+1}K_{t+1}. \tag{C.13}$$

Next, aggregating $x_{i,t+1}^l$ with (2), and (28), we obtain

$$\int_0^1\int_{z_t\leq z_t^*}x_{i,t+1}^l dF(z_t)di=\gamma(1-\tau)R_{t+1}\lambda K_{t+1}. \tag{C.14}$$

Because of $\int_0^1 x_{i,t+1}di = \int_0^1\int_{z_t\leq z_t^*}x_{i,t+1}^l dF(z_t)di+\int_0^1\int_{z_t\geq z_t^*}x_{i,t+1}^e dF(z_t)di$, this associated with (33), (C.13), and (C.14) yields

$$\int_0^1 x_{i,t+1}di=\gamma(1-\tau)\frac{\varphi(1-\sigma_k)}{\varphi-1}R_{t+1}K_{t+1}. \tag{C.15}$$

From (C.12) and (C.15), we obtain

$$K_{t+1}=\frac{\beta}{1-\sigma_k-\lambda}(z_t^*)^{-\varphi}\frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)}(1-\alpha+\alpha\gamma)(1-\tau)R_tK_t. \tag{C.16}$$

Aggregating (C.9) and using (32), we obtain

$$\int_0^1\int_{z_t\leq z_t^*}l_{i,t}dF(z_t)di=\beta\left[1-(z_t^*)^{-\varphi}\right]\left[\frac{(1-\alpha)\varphi}{\alpha(\varphi-1)}(1-\tau)R_tK_t+\int_0^1 x_{i,t}di\right]. \tag{C.17}$$

Substituting (28) and (C.15) into (C.17), we obtain

$$\lambda K_{t+1} = \beta \left[1 - (z_t^*)^{-\varphi}\right] \frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)}(1-\alpha+\alpha\gamma)(1-\tau)R_t K_t. \tag{C.18}$$

From (C.16) and (C.18), we obtain

$$\frac{K_{t+1}}{K_t} = \beta \frac{\varphi}{\alpha(\varphi-1)}(1-\alpha+\alpha\gamma)(1-\tau)R_t. \tag{C.19}$$

From (C.16) and (C.19) we obtain

$$z_t^* = z^* = \left(\frac{\beta(1-\sigma_k)}{1-\sigma_k-\lambda}\right)^{\frac{1}{\varphi}}. \tag{C.20}$$

Substituting (C.20) into (33) yields

$$R_t = R = \frac{\alpha A}{1-\sigma_k}\left(\frac{\varphi-1}{\varphi}\right)^{1-\alpha}\left(\frac{\beta(1-\sigma_k)}{1-\sigma_k-\lambda}\right)^{\frac{\alpha}{\varphi}}. \tag{C.21}$$

Substituting (17), $\int_0^1 \int_{z_{t-1} \geq z_{t-1}^*} k_{i,t} dF(z_{t-1}) di = K_t$, (28) (29) (31), and (32) into (C.11), we obtain

$$\tau \frac{\varphi(1-\sigma_k)}{\alpha(\varphi-1)} R_t K_t = \sigma_k K_{t+1} \tag{C.22}$$

From (C.19) and (C.22), we have

$$\tau = \frac{\beta\sigma_k(1-\alpha+\alpha\gamma)}{1-\sigma_k+\beta\sigma_k(1-\alpha+\alpha\gamma)} \tag{C.23}$$

Substituting (C.21) and (C.23) into (C.19), we obtain

$$\frac{K_{t+1}}{K_t} = \frac{\beta A}{1-\sigma_k}\left(\frac{\varphi}{\varphi-1}\right)^{\alpha} \frac{1-\alpha+\alpha\gamma}{1-\sigma_k[1-\beta(1-\alpha+\alpha\gamma)]}\left(\frac{\beta(1-\sigma_k)}{1-\sigma_k-\lambda}\right)^{\frac{\alpha}{\varphi}} \tag{C.24}$$

Because of $\varphi > 1$, $\partial(K_{t+1}/K_t)/\partial\sigma_k > 0$ for all $\sigma_k > 0$.

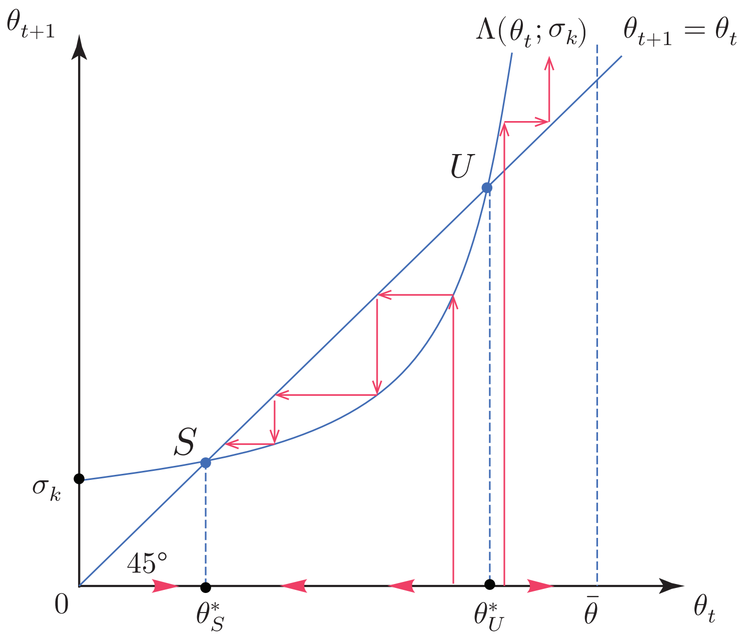

Figure 1: The dynamics of θ_t

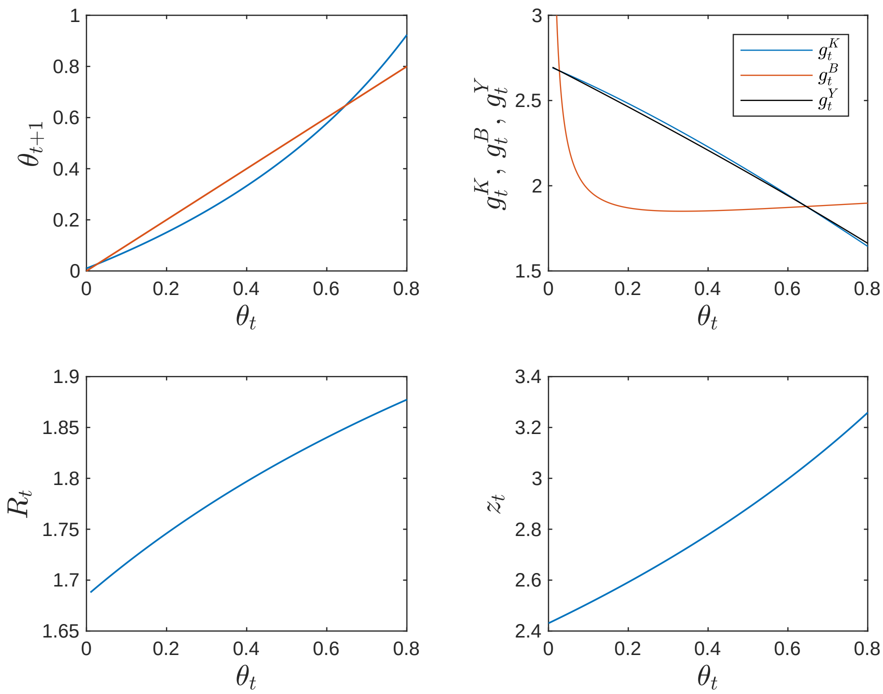

Figure 2: $\theta_{t+1} = \Lambda(\theta_t, \sigma_k)$, $R_t = \Psi(\theta_t, \sigma_k)$, $z(\theta_t, \sigma_k)$, $g^K(\theta_t, \sigma_k)$, $g^B(\theta_t, \sigma_k)$, and $g^Y(\theta_t, \sigma_k)$ under the numerical example

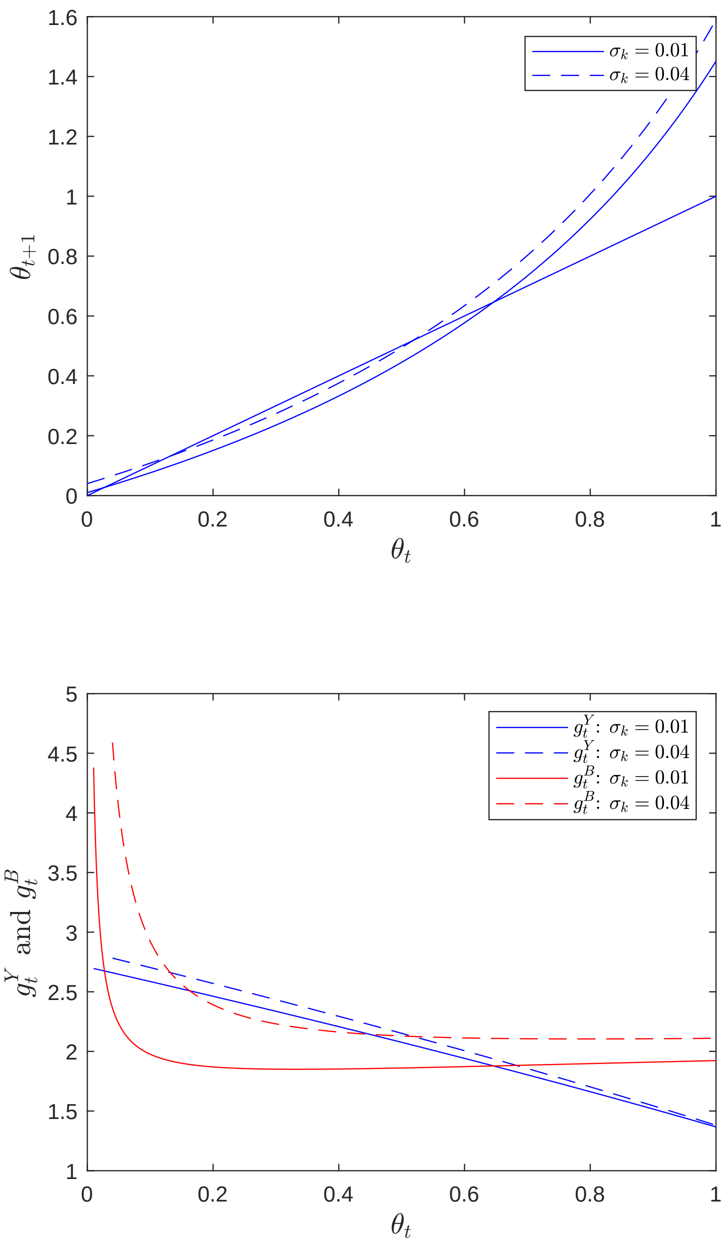

Figure 3: The dynamics of θ_t

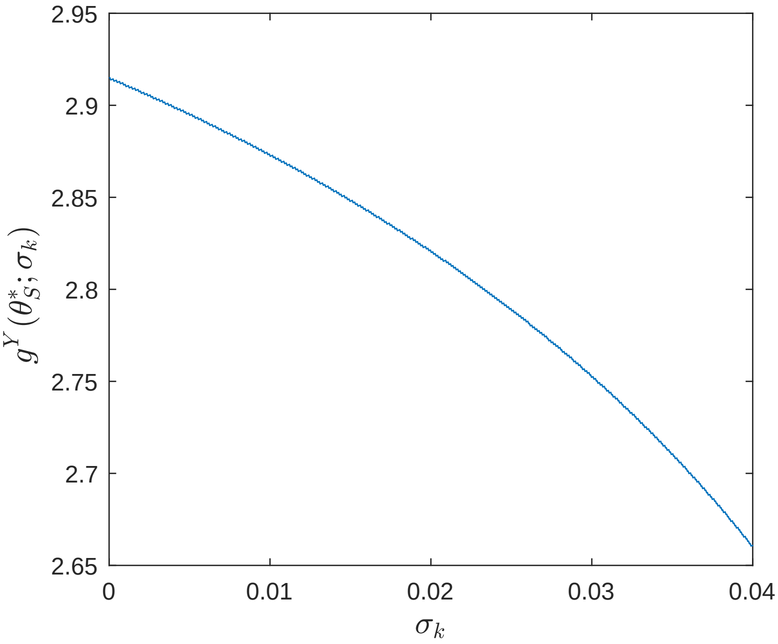

Figure 4: The relationship between σ_k and growth

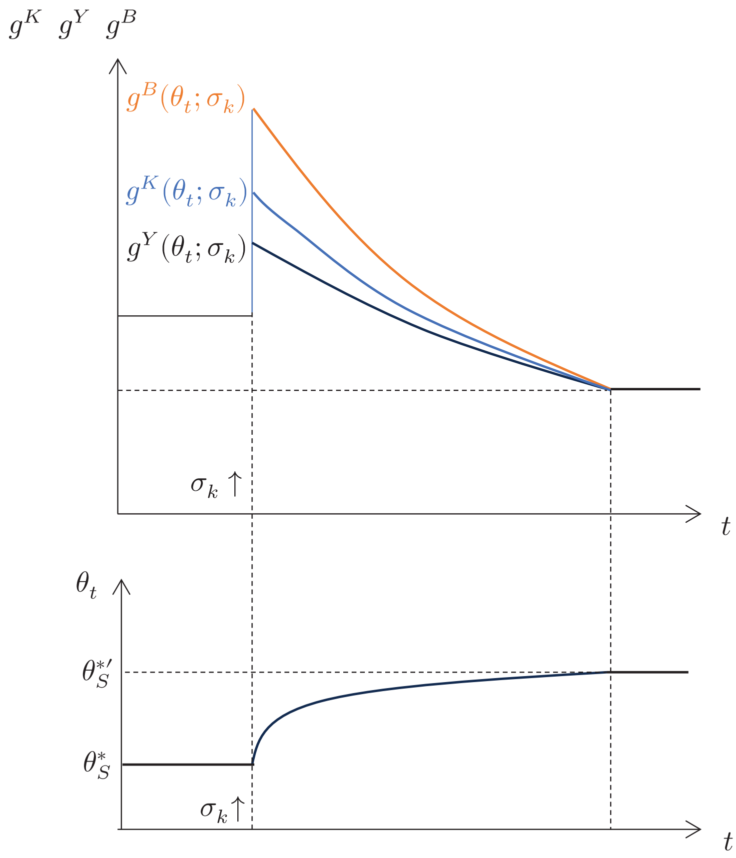

Figure 5: Time path of θ_t and growth when σ_k increases

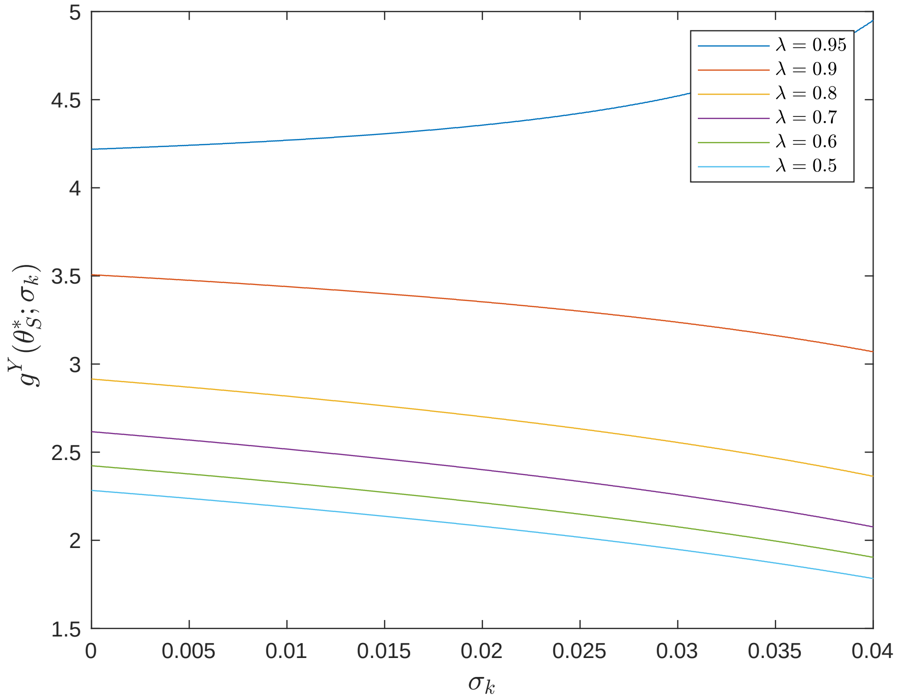

Figure 6: The relationship between λ and the growth effect of σ_k