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Behavior-based price discrimination and elastic demand*

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Abstract

Existing studies on Behaviour-based price discrimination (BBPD) typically show that firms offer discounts to encourage consumers located middle of the line segment to switch in a duopoly model. However, in practice, some firms offer both this discount and a discount to encourage consumers with lower preferences for the product itself to buy at the same time. I introduce heterogeneity of consumer willingness to pay and relax the assumption that the market is fully covered. Then, there are three purchase histories: bought from a firm, bought from another firm, and bought nothing. I assume that the two firms offer three different prices according to the purchase histories under BBPD. In the second period, firms offer discounts not only for rival customers but also for customers who bought nothing. On the other hand, firms offer higher prices for consumers who purchase the same goods over two periods in the second period than in the first period. This paper shows that BBPD does not lower all prices in the second period and does not increase consumer surplus.

Keywords: Behavior-based price discrimination, Hotelling model.
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1 Introduction

Behavior-based price discrimination (BBPD) is a price discrimination in which firms set prices according to consumers' purchase histories. Many firms collect consumer data and use them for the price discrimination nowadays. After the seminal works by

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Chen (1997), Villas-Boas (1999), and Fudenberg and Tirole (2000), many studies have discussed the price discrimination. In the standard BBPD models, two firms compete over two periods. In the first period, firms do not engage in price discrimination since there are no purchase histories. After the first period, each firm collects its customers' purchase histories and tracks them. In the second period, firms set two prices based on the collected purchase histories: one for its customers and one for other consumers. In the standard BBPD models, a firm identifies all consumers who did not purchased from the firm as rival customers since the market is fully covered in the first period. Existing studies have shown that BBPD usually intensifies competition and increases consumer surplus.

The standard BBPD models assume that consumers' willingness to pay, which represents the utility of consuming each consumer's ideal brand, is homogeneous, but in several markets, some consumers have a smaller willingness to pay than others. The standard BBPD models assume that consumers vary in their preferences for the two brands. On the other hand, they assume that consumers' willingness to pay is the same for all consumers. However, consumers' willingness to pay differs for different types of consumers in several markets. For example, children's willingness to pay for smartphones is smaller than that of adults. Almost every adult now has a smartphone. It may be difficult for adults to have a social life without smartphones. In contrast, young children usually do not have smartphones since excessive use of smartphones harms their mental and physical health seriously. Many children start using smartphones at school age. This paper aims to investigate the effect of BBPD on the intensification of competition and welfare in such markets.

I extend Fudenberg and Tirole (2000) by introducing heterogeneity of consumers' willingness to pay. In Fudenberg and Tirole (2000), there is one unit of consumers and their willingness to pay is the same. I assume that half of the consumers have a low willingness to pay and the other half have a high willingness to pay. Each type is distributed along the unit interval [0,1]. The high type's willingness to pay is sufficiently large so that the market is fully covered in equilibrium. However, the low type's willingness to pay is not large enough and the market may not be fully covered in equilibrium. Some low-value consumers may choose to buy nothing depending on the offered prices in the first period. If so, after the first period, there are three purchase histories: bought from a firm, bought from another firm, and bought nothing. I assume that each firm offers three different prices for each of the three purchase histories in the second period. Firms can identify their own customers by the collected data. However, they can not distinguish consumers who bought rival goods from consumers who bought nothing only by the collected data. Then, firms distinguish them by checking invoices or screenshots of rival goods submitted by consumers who have bought from their rivals.

I found that the firms always offer discounts not only for consumers who bought from the rival firms but also for consumers who bought nothing in the second period. Moreover, the prices for consumers who bought from rival firms are lower than those for consumers who bought nothing. This relation between the two prices implies that consumers who bought goods in the previous period do not have the incentives to pretend to be consumers who bought nothing. I also found that consumers who buy the same goods over the two periods pay more money in the second period than in the first period, and both these prices are higher than the equilibrium prices under uniform pricing. The reason is that low-value consumers' purchases in the first period reveal their relatively higher preferences for one of the brands, and then firms raise prices in the market segment where they exist. BBPD reduce total demands over two periods since some low-value consumers forgo buying goods in the first period to buy goods cheaper in the second period. Consequently, BBPD worsens the firms' profitability and consumer surplus. Fudenberg and Tirole (2000) show that BBPD intensifies competition by lowering all prices in the second period. However, this paper shows that BBPD lowers two of the three prices but raises the other.

Although some firms engage in BBPD by offering a kind of discount, some firms engage in BBPD by offering two kinds of discounts: a discount for rival customers and a discount for other new customers in the real world. These firms identify rival customers by requiring them to submit evidence of their purchase of rival goods and tend to offer larger discounts than other new customers. Japanese cell phone company, Docomo, offers different discounts for customers who switch to it from other cell phone companies and for customers who have a smartphone contract for the first time, respectively. In the United States, T-Mobile offers discounts for new customers, and in addition, it pays the previous carrier's remaining device payment balance for customers who switch from another company if they submit evidence. Shed, which provides event scheduling software, offers switching providers discounts in addition to the free trial for them if they show evidence of contracts with Whova, EventMobi, vFairs, Webex, and others by sending an invoice or a screenshot of these applications. Reclaimai.ai, which provides a calendar application, offers new users a twenty percent discount for six months in addition to a free trial of a paid version if new users show that they switch from Clockwise, Motion, or Calendly. The results derived from this paper are consistent with these facts.

The Japanese government has tightened regulations on discounts for new customers of mobile phone companies in 2023. In Japan, many mobile phone companies habitually offer large discounts to their new customers. The government claimed that the companies compensate for the loss from the discounts by charging high cell phone rates to their old customers. The government has implemented regulations on the discounts several times in an attempt to lower the rates but has yet to be satisfied with the results. This paper shows that the reason for the high rates for old customers is due to heterogeneity of willingness to pay.

This paper is organized as follows. Section 2 describes related literature. In Section 3, I describe the model. In Sections 4 and 5, I provide equilibrium and discuss the effect of BBPD, respectively. Section 6 concludes my paper.

2 Related literature

This paper is related to the strand of the literature on BBPD. There is extensive literature on BBPD (See Fudenberg and Villas-Boas (2006) and Esteves (2009) for review). The standard BBPD models have shown that BBPD intensifies competition and lowers firms' profitability in the duopoly markets. These studies typically discuss discounts that firms offer to encourage consumers located middle of the line segment to switch. On the other hand, firms also offer discounts to encourage consumers with lower preferences for the product itself to buy in practice. Villas-Boas (2004), Acquisti and Varian (2005), and Laussel et al. (2020) discuss this price discrimination in monopoly markets. These studies show that low-value consumers forgo buying goods today not to be recognized as old customers tomorrow. I analyzed the market where the firms offer both kinds of discounts in the duopoly market.

In my model, low-value consumers are loyal to one of the brands and high-value consumers are not. Several studies have discussed BBPD in the market where there are loyal customers. Esteves (2010) assume that all consumers are loyal to either of the two brands and decide which one to purchase depending on prices offered. Their study found that the firms do not share the market in the first period. They have incentives to avoid consumer recognition to relax the second period's competition. Their study indicates that BBPD lowers firms' profitability. On the other hand, Esteves et al. (2022) consider consumer preferences are clustered around the center of the market and show that BBPD raises firms' profitability. Chen and Zhang (2009) introduce loyal consumers who always buy from either of the two brands and switchers who buy from a firm that offers a lower price. Their study shows that firms raises prices to recognize switchers from loyal consumers in the first period and than BBPD raises firms' profitability. My paper shows that BBPD lowers firms' profitability.

Shin and Sudhir (2010) discuss behavior-based pricing where a firm offer more than two prices, similar to my paper. Shin and Sudhir (2010) assume that high-type purchases more than low-type, and firms offer prices for their own low-type customers, their own high-type customers, and their competitor's customers in the second period. Their study shows that price discrimination is beneficial for firms in the duopoly market. Colombo (2018) analyzes a model with high- and low- sensitivity consumers and shows that price discrimination increases the surplus of high-sensitivity consumers with respect to uniform pricing Esteves and Reggiani (2014), and Zhang et al. (2019) discuss elasticities of demand differently from my paper. They assume that the mass of consumers is one, but the amount each consumer buys varies with prices, and indicate that BBPD improves social welfare if demand is sufficiently elastic. My paper shows that some low-value consumers do not purchase goods depending on offered prices and BBPD reduces total demand over two periods.

3 Preliminary

I assume that a unit of consumers is uniformly distributed on an interval $[0,1]$. Two firms, Firm A and Firm B, are located at the extremes of the unit interval. That is, Firm A is located at 0 and Firm B is located at 1. The firms produce horizontally differentiated goods with a constant marginal cost. I normalize the marginal cost to zero. I introduce heterogeneity in consumer value. There are two types of consumers. Half of the consumers are a high-value type and the other half is a low-value type. Each type obtains v_H and v_L units of utility for consuming one unit of its ideal good, respectively ($v_H > v_L$). The high-value consumers obtain more utility from each consumer's ideal smartphone or software than low-value consumers. Each type is uniformly distributed along the line segment.

The location of a consumer represents her relative preference for each brand. A consumer located at x incurs a disutility of tx from buying good A and a disutility of $t(1-x)$ from buying good B. The parameter $t > 0$ measures the disutility per unit of distance of purchasing away from the ideal product. Consumer buys at most one unit. The utility for a consumer located at x who purchases from Firm A at price p is given by $u_A(x,p) = v_i - tx - p$ and that of Firm B is given by $u_B(x,p) = v_i - t(1-x) - p$ ($i = H, L$). I assume that v_H is sufficiently large so that all high-value consumers buy products in equilibrium. On the other hand, v_L is not large enough, so not all low-value consumers will buy goods in equilibrium. I assume that $\frac{1}{2}t < v_L < \frac{10929}{11442}t$. When a consumer buys nothing, her utility is defined to be zero.

There are two periods. Firms choose prices simultaneously to maximize their profits, and consumers observe the offered prices and decide to buy goods or nothing in each period. The locations of the consumers and the firms are fixed. In the first period, there are no purchased histories. Firm A offers a_1 and Firm B offers b_1 for all consumers. At the end of the first period, there are three kinds of purchase histories: "bought good A", "bought good B", and "bought nothing". In the second period, firms engage in price discrimination. I assume that Firm A offers a_o for consumers who bought good A, a_n for consumers who bought good B, and α for consumers who bought nothing. Similarly, Firm B offers b_o for consumers who bought good B, b_n for consumers who bought good A, and β for consumers who bought nothing.

Each firm recognizes consumers who bought its own goods by observing collected purchase histories. However, they cannot distinguish consumers who bought its rival goods and who bought nothing based on observation of collected purchase histories only. However, they can recognize them by requiring the consumers who bought rival goods to certifications of previous contracts, for example, receipts or goods themselves, to recognize them. Consumers who bought goods in the first period may pretend to be consumers who bought nothing in the second period. That is, there is a possibility that consumers who bought goods in the first period buy goods at the prices α or β by purposely not submitting the certifications.

I assume that all agents discount their future by the common factor, which is normalized to one. I derive subgame perfect equilibrium by backward induction.

4 Analysis

4.1 Uniform pricing

I consider a case in which the firms do not engage in BBPD as a benchmark case. The two-period model can be reduced to two replications of a static model. I solve the static model.

Since low-type consumers' willingness to pay is sufficiently small, I define three locations of indifferent consumers. The high-type consumer who is indifferent between buying goods A and B is located at x^*. The low-type consumer who is indifferent between buying good A and buying nothing is located at $\underline{y}$, and the low-type consumer who is indifferent between buying good B and buying nothing is located at $\overline{y}$. These indifferent consumers identified by the condition $v_H - tx^* - p_A = v_H - t(1-x^*) - p_B$, $v_L - t\underline{y} - p_A = 0$, $v_L - t(1-\overline{y}) - p_B = 0$. From these equations, we have

$$x^* = \frac{t - p_A + p_B}{2t}, \quad \underline{y} = \frac{v_L - p_A}{t}, \quad \overline{y} = \frac{-v_L + t + p_B}{t}. \tag{1}$$

Since the high-type consumers on $[0, x^*]$ and the low-type consumers on $[0, \underline{y}]$ buy good A, and the high-type consumers on $[x^*, 1]$ and the low-type consumers on $[\overline{y}, 1]$ buy good B, and others buy nothing, the firms' profit are

$$\pi_A^U = \frac{1}{2} p_A (x^* + \underline{y}), \quad \pi_B^U = \frac{1}{2} p_B \{(1 - x^*) + (1 - \overline{y})\}, \tag{2}$$

where the superscript "U" stand for uniform pricing. Firm A chooses p_A to maximize π_A^U and Firm B chooses p_B to maximize π_B^U. Solving these maximization problems, we obtain equilibrium prices as

$$p_A = p_B = \frac{1}{5}(2v_L + t). \tag{3}$$

Introducing the equilibrium prices into (1) yields

$$x^* = \frac{1}{2}, \quad \underline{y} = \frac{3v_L - t}{5t}, \quad \overline{y} = \frac{-3v_L + 6t}{5t}. \tag{4}$$

If firms could distinguish between the high-type consumers and the low-type consumers and offer different prices for each type, they would offer t and $\frac{v_L}{2}$, respectively. We can easily find that $\frac{v_L}{2} < \frac{1}{5}(2v_L + t) < t$. This implies that firms increases their profits if they recognaize consumer types.

4.2 Price discrimination

In this section, I consider a case in which the firms engage in BBPD.

4.2.1 The second period

Suppose that the high-value consumer who is indifferent between buying goods A and B is located at x_1, the low-value consumer who is indifferent between buying good A and buying nothing is located at y_A, and the low-value consumer who is indifferent between buying good B and buying nothing is located at y_B in the first period ($y_A < \frac{1}{2} < y_B$). The high-value consumers on $[0, x_1]$ and the low-value consumers on $[0, y_A]$ bought good A, and the high-value consumers on $[x_1, 1]$ and the low-value consumers on $[y_B, 1]$ bought good B in the first period.

In the second period, high-value consumers on $[0, x_1]$ continue to buy from Firm A again if $v_H - tx - a_o \geq v_H - t(1-x) - b_n$. Otherwise, she or he switches to Firm B. The high-value consumers on $[x_1, 1]$ continue to buy from Firm B again if $v_H - t(1-x) - b_o \geq v_H - tx - a_n$. Otherwise, she or he switches to Firm A. Let x_A be the location of the consumer who is indifferent between buying from Firm A and switching to Firm B, and let x_B be the location of the consumer who is indifferent between buying from Firm B and switching to Firm A. The locations are given by

$$x_A = \frac{t - a_o + b_n}{2t}, \quad x_B = \frac{t - a_n + b_o}{2t}, \tag{5}$$

respectively.

The low-value consumers who bought nothing in the first period buy good A if $v_L - tx - \alpha \geq 0$ or buy good B if $v_L - t(1-x) - \beta \geq 0$. Otherwise, she or he does not buy again. Let z_A be the consumer who is indifferent between buying from Firm A and buying nothing, and z_B be the consumer who is indifferent between buying from Firm B and buying nothing. Those indifferent consumers are located at

$$z_A = \frac{v_L - \alpha}{t}, \quad z_B = \frac{-v_L + t + \beta}{t}. \tag{6}$$

It can be said that there must be the low-value consumers who did not buy goods in the first period buy goods in the second period.[1]

The second-period profits of Firm A and Firm B can be written as

$$\pi_A^{D2} = \frac{1}{2}\{a_o(x_A + y_A) + a_n(x_B - x_1) + \alpha(z_A - y_A)\}, \tag{7}$$

$$\pi_B^{D2} = \frac{1}{2}\{b_o(1 - x_B + 1 - y_B) + b_n(x_1 - x_A) + \beta(y_B - z_B)\}, \tag{8}$$

[1] If $y_A \geq z_A$, the indifferent consumer, y_A satisfies $v_L - ty_A - a_1 + 0 = 0 + 0$. Hence, $v_L - ty_A \geq 0$ unless a_1 is negative. When $v_L - ty_A \geq 0$, Firm A can increase its profit by offering α, no matter how low the price is. Therefore, $y_A \leq z_A$.

where the subscript "D" stands for price discrimination and the subscript "2" stands for period two.

Firm A chooses a_o, a_n, and α to maximize (7), and Firm B chooses b_o, b_n, and β to maximize (8). By solving these maximization problems, we can obtain equilibrium prices as follows:

$$a_o = \frac{t}{3}(1 + 2x_1 + 4y_A), \quad b_o = \frac{t}{3}\{3 - 2x_1 + 4(1 - y_B)\}, \tag{9}$$

$$a_n = \frac{t}{3}\{3 - 4x_1 + 2(1 - y_B)\}, \quad b_n = \frac{t}{3}(-1 + 4x_1 + 2y_A), \tag{10}$$

$$\alpha = \frac{1}{2}(v_L - ty_A), \quad \beta = \frac{1}{2}\{(v_L - t(1 - y_B)\}. \tag{11}$$

BBPD divides the second period's market into two market segments according to their purchase histories: bought goods in the first period and bought nothing in the first period. Firms set prices for the former segment as duopolists and set prices in the latter segment as monopolists. A decrease in y_A and an increase in y_B means a shrinkage of the segment of consumers who bought goods and an expansion of the segment of consumers who did not buy goods. Equations (9), (10), and (11) show that firms set higher prices as the segment size is larger in each segment. Introducing the equilibrium prices into (7) and (8), we have

$$\pi_A^{D2} = \frac{t}{36}(1 + 2x_1 + 4y_A)^2 + \frac{t}{36}(5 - 4x_1 - 2y_B)^2 + \frac{1}{8t}(v_L - ty_A)^2, \tag{12}$$

$$\pi_B^{D2} = \frac{t}{36}(7 - 2x_1 - 4y_B)^2 + \frac{t}{36}(-1 + 4x_1 + 2y_A)^2 + \frac{1}{8t}(v_L - t + ty_B)^2. \tag{13}$$

The first terms are profits from old customers, the second terms are profits from rival customers, and the third terms are profits from consumers who bought nothing in equations (12) and (13). The profits from the market segment of consumers who bought goods decreases and the profits from the market segment of consumers who bought nothing increases as y_A decreases or y_B increases.

4.3 The first period

In the first period, consumers make decisions to purchase goods anticipating the second period's prices. The indifferent high-value consumer located at x_1 is indifferent between buying from Firm A in the first period at a price a_1 and then buying from Firm B in the second period at a price b_n, or buying from Firm B in the first period at a price b_1 and then buying from firm A in the second period at a price a_n. Thus, x_1 satisfies $v_H - tx_1 - a_1 + v_H - t(1 - x_1) - b_n = v_H - t(1 - x_1) - b_1 + v_H - tx_1 - a_n$. Introducing prices in (10) into this equation, we get

$$x_1 = \frac{6t - 2t(y_A + y_B) - 3a_1 + 3b_1}{8t}. \tag{14}$$

The indifferent low-value consumer who is located at y_A is indifferent between buying from Firm A in the first period at a price a_1 and buying from Firm A again in the second period at a price a_o, or buying nothing in the first period and then buying from Firm A in the second period at a price α. Thus, y_A satisfies $v_L - ty_A - a_1 + v_L - ty_A - a_o = v_L - ty_A - \alpha$. In the same way, y_B satisfies $v_L - t(1-y_B) - b_1 + v_L - t(1-y_B) - b_o = v_L - t(1-y_B) - \beta$. Introducing prices in (9) and (11) into the equations, we have

$$y_A = \frac{9v_L - 2t - 4tx_1 - 6a_1}{17t}, \quad y_B = \frac{-9v_L + 23t - 4tx_1 + 6b_1}{17t}. \tag{15}$$

Introducing equations in (15) into (14), we can obtain x_1 as a function of prices, a_1 and b_1, as

$$x_1 = \frac{20t - 13a_1 + 13b_1}{40t}. \tag{16}$$

Introducing (16) into (15) yields

$$y_A = \frac{90v_L - 40t - 47a_1 - 13b_1}{170t}, \quad y_B = \frac{-90v_L + 210t + 13a_1 + 47b_1}{170t}. \tag{17}$$

The first period's profits of Firm A and Firm B are

$$\pi_A^{D1} = \frac{1}{2}a_1(x_1 + y_A), \quad \pi_B^{D1} = \frac{1}{2}b_1(1 - x_1 + 1 - y_B). \tag{18}$$

where the subscript " 1 " stands for period one. Firm A chooses a_1 to maximize $\Pi_A^D = \pi_A^{D1} + \pi_A^{D2}$, and Firm B chooses b_1 to maximize $\Pi_B^D = \pi_B^{D1} + \pi_B^{D2}$ in the first period. Solving these maximization problems, we have

$$a_1 = b_1 = \frac{1828}{8325}(2v_L + t). \tag{19}$$

Furthermore, introducing the equilibrium prices into (16) and (17), we have

$$x_1 = \frac{1}{2}, \quad y_A = \frac{3117v_L - 2604t}{8325t}, \quad y_B = 1 - y_A. \tag{20}$$

Firm A supplies good A for all low-value consumers who bought from Firm A in equilibrium (see Appendix A). The same is true for Firm B. From (5), (6), (9), (10), and (11), we can derive the second period's equilibrium prices as

$$a_n = b_n = \frac{1039}{8325}(2v_L + t), \; a_o = 2a_n, \; b_o = 2b_n, \; \alpha = \beta = \frac{1302}{8325}(2v_L + t), \tag{21}$$

and the locations of the second period's indifferent consumers as

$$x_A = \frac{-1039v_L + 3643t}{8325t}, \quad x_B = 1 - x_A, \quad z_A = \frac{5721v_L - 1302t}{8325t}, \quad z_B = 1 - z_A. \tag{22}$$

From (21), we can find that $a_o > \alpha > a_n$ and $b_o > \beta > b_n$. Therefore,

Proposition 1 *The firms offer discounts for consumers who bought from rival firms and bought nothing in the second period under BBPD. The firms offer lower prices for consumers who bought from rival firms than those who have bought nothing.*

Fudenberg and Tirole (2000) and other standard BBPD models show that firms offer discounts for consumers who bought from rival firms. I indicate that firms offer discounts not only for consumers who bought from rival firms but also for consumers who bought nothing. The difference between a_n and α or b_n and b_β is not so large. However, this proposition indicates that consumers who bought goods in the first period do not have incentives to buy goods at α or β by pretending not to buy anything in the first period. This is because they can buy goods at lower prices a_n or b_n by revealing their purchase histories correctly.

By comparing (3), (19), and (21), we can find that $a_o > a_1 > p_A$ and $b_o > b_1 > p_B$, which can be summarized the following proposition.

Proposition 2 *The first period's prices and the second period's prices for consumers who buy the same goods over two periods are higher than the equilibrium prices under uniform pricing.*

If Firm A raises the first period price, a_1, its second period profit from the market segment of consumers who bought goods decreases and its profit from the market segment of consumers who bought nothing increases in the second period, as shown in (12). The former effect is smaller than the latter effect. Then, Firm A sets a_1 higher than the price under uniform pricing. In the former segment, there are the high-value consumers and the low-value consumers who have relatively high preferences for good A and Firm A competes with Firm B. On the other hand, in the latter segment, all of the consumers are the low type and Firm A supplies good A as a monopolist. Firm A raises a_1 to expand its monopoly market. This leads to an increase in α and decreases in a_n and a_o. However, the price for its old consumers, a_o, is higher than p_A. This is due to the low-value consumers on $[0, y_A]$. Since these consumers strongly prefer good A, all of them repurchase good A even if Firm A offers a high price when y_A is sufficiently small. The converse of Proposition 2, that is $a_o < p_A$ and $b_o < p_B$, would be established if all of the consumers were the high type in this model.[2]

5 Welfare

In this section, I compare the amount of demand under uniform pricing and that under BBPD. The per-period total demand under uniform pricing is $1 + 2\underline{y}$ since the firms are symmetric. Similarly, the first period's total demand and the second period's total

[2]The firms set $p_A = p_B = t$ under uniform pricing, and $a_1 = b_1 = \frac{4}{3}t$, $a_o = b_o = \frac{2}{3}t$, $a_n = b_n = \frac{1}{3}t$ if all of the consumers are the high type.

demand are $1 + 2y_A$ and $1 + 2z_A$, respectively. From (4), (20), and (22), we can find that $y_A < \underline{y} < z_A$ and $y_A + z_A < 2\underline{y}$. Then, I can state the following proposition.

Proposition 3 *The first period's total demand under BBPD is smaller than the per-period total demand under uniform pricing, and the second period's total demand is larger than that. The total demand over two periods under BBPD is smaller than that under uniform pricing.*

As shown in Acquisti and Varian (2005) and Chen and Zhang (2009), low-value consumers may have incentives to forgo buying goods to avoid establishing purchase histories of buying goods in the first period. The low-value consumers anticipate that they face high prices, a_o and b_o, in the second period if they buy goods in the first period. They also anticipate that they will be able to buy goods at low prices, α or β in the second period if they do not buy goods in the first period. Since some low-value consumers who have relatively weak preferences for good A do not buy goods if a_1 is high, Firm A can identify them by raising a_1. Equation (15) shows that the more firms raise the prices, the less low type buy goods in the first period. Firm A engages in price discrimination by using the collected data and increase the demand from the market segment of consumers who did not purchase good A in the second period.

Finally, I discuss the effect of BBPD on welfare. Appendix B shows that the second period profit under BBPD is larger than the per-period profit under uniform pricing, but the first period profit is smaller than that. The firms recognize consumer types by raising the first period's price in the first period and try to increase the profits by price discrimination in the second period, but rival firms poach their customers. It also shows that the profits over two periods under BBPD are smaller than those under uniform pricing. Appendix B shows that BBPD reduces consumer surplus. This is due to price increases, reduced demand, and switching. Therefore,

Proposition 4 *BBPD decreases firms' profitability and consumer surplus.*

Since social welfare is the sum of consumer surplus and firms' profits, we can state that BBPD reduces social welfare. The standard BBPD models show that BBPD reduces profits but increases consumer surplus. This paper shows that BBPD decreases consumer surplus when there are low-value consumers.

6 Conclusion

This paper analyzes BBPD in a market where there are half of the consumers are low-value consumers and the others are high-value consumers. This paper assumes that the willingness to pay of low-value consumers is sufficiently low and the market is not fully covered depending on prices offered by firms in equilibrium. Thus, there are three kinds of purchase histories: "bought good A", "bought good B", and "bought nothing" at the end of the first period under both uniform pricing and BBPD. This paper assumes that under BBPD, firms set different prices according to the consumers' purchase histories in the second period.

I show that firms offer discounts for consumers who bought their rival goods and consumers who bought nothing under BBPD. On the other hand, consumers who buy the same goods over two periods pay more money in the second period than in the first period. BBPD reduces total demand, firms' profitability, and consumer surplus.

Throughout the analysis, I have assumed that there are two types of consumers and have fixed the proposition of high type and low type in this paper. It will be interesting for future research to analyze the situation where there are more types and how the effect of changes in the proportion on the results.

A The proof that Firm A supplies good A for all low-value consumers in $[0, y_A]$ in the second period

The low-value consumers on $[0, y_A]$ buy good A in the first period and Firm A offers a_o for those consumers in the second period. Firm A needs to set a_o lower to supply good A for all consumers in $[0, y_A]$ as y_A is larger. Then, there is a possibility that Firm A earns a larger profit by supplying its good only for $[0, y]$ than by supplying for $[0, y_A]$ if y_A is sufficiently large ($0 < y < y_A$). I prove that Firm A supplies good A for all consumers on $[0, y_A]$ in equilibrium by contradiction.

Suppose Firm A offers p and supplies good A only to low-value consumers on $[0, y]$. Since the consumer located on y is indifferent between buying good A and buying nothing, y satisfies $v_L - ty - p = 0$, that is $y = \frac{v_L - p}{t}$. Firm A's profit from the low-value consumers on $[0, y]$ is $\frac{1}{2}py$. It is maximized at $p = \frac{v_L}{2}$ and $y = \frac{v_L}{2t}$. Firm A maximizes its profit from low-value consumers when it supplies to consumers on $[0, \frac{v_L}{2t}]$. Hence, Firm A does not supply for all consumers on $[0, y_A]$ if $y_A > \frac{v_L}{2t}$. From (20), we can check that $y_A < \frac{v_L}{2t}$. Therefore, Firm A supplies good A for all low-value consumers in $[0, y_A]$.

B Profits and Consumer surpluses

Introducing (3) and (4) into (2), we can obtain per-period equilibrium profits under uniform pricing as

$$\pi_A^U = \pi_B^U = \frac{3}{100}\frac{(2v_L + t)^2}{t}. \tag{A4}$$

Then, each firm's profit over two periods is $0.06(2v_L + t)^2/t$. Per-period equilibrium consumer surplus under uniform pricing can be defined as

$$cs^U = \frac{1}{2}\int_0^{x^*} (v_H - tx - p_A)dx + \frac{1}{2}\int_{x^*}^1 (v_H - t(1-x) - p_B)dx + \frac{1}{2}\int_0^{\underline{y}} (v_L - tx - p_A)dx + \frac{1}{2}\int_{\overline{y}}^1 (v_L - t(1-x) - p_B)dx. \quad \text{(A5)}$$

Introducing (3) and (4) into (A5), we have

$$cs^U = \frac{v_H}{2} - \frac{t}{8} + \frac{9v_L^2 - 16v_L t - 4t^2}{50t}. \quad \text{(A6)}$$

Introducing (19) and (20) into (12), (13), and (18), we can obtain the profits under BBPD as

$$\pi_A^{D1} = \pi_B^{D1} = \frac{1424469}{(8325)^2}\frac{(2v_L + t)^2}{t}, \quad \text{(A7)}$$

$$\pi_A^{D2} = \pi_B^{D2} = \frac{8788013}{4(8325)^2}\frac{(2v_L + t)^2}{t}, \quad \text{(A8)}$$

$$\Pi_A^{D} = \Pi_B^{D} = \frac{14485889}{4(8325)^2}\frac{(2v_L + t)^2}{t}. \quad \text{(A9)}$$

We can see that $\frac{14485889}{4(8325)^2} \simeq 0.052254$. Then, BBPD reduces the firms' profitability. As the same matter, consumer surplus under BBPD can be calculated as

$$cs^{D1} = \frac{v_H}{2} - \frac{t}{8} + \frac{1}{2(8325)^2 t}\left[19390857v_L^2 - 49914768v_L t - 12478692t^2\right], \quad \text{(A10)}$$

$$cs^{D2} = \frac{v_H}{2} - \frac{t}{8} + \frac{1}{4(8325)^2 t}\left[50427430v_L^2 - 88183820v_L t - 22045955t^2\right]. \quad \text{(A11)}$$

Consumer surplus over two periods is the sum of consumer surplus in the first period and that in the second period. From (A10) and (A11), consumer surplus over two periods under BBPD is

$$CS^D = v_H - \frac{t}{4} + \frac{1}{4(8325)^2 t}\left[89209144v_L^2 - 188013356v_L t - 47003339^2\right]. \quad \text{(A12)}$$

References

Acquisti, A. and H. Varian (2005) "Conditioning prices on purchase history," *Marketing Science*, 24 (3), 367–381.

Chen, Y. (1997) "Paying customers to switch," *Journal of Economics and Management Strategy*, 6 (4), 877–897.

Chen, Y. and Z.J. Zhang (2009) "Dynamic targeted pricing with strategic consumers," *International Journal of Industrial Organization*, 27 (1), 43–50, 10.1016/j.ijindorg.2008.03.002.

Colombo, S. (2018) "Behavior- and characteristic-based price discrimination," *Journal of Economics and Management Strategy*, 27 (2), 237–250, 10.1111/jems.12244.

Esteves, R.-B. (2009) "A survey on the economics of behavior-based price discrimination," *NIPE Working Papers*, 5, NIPE, Universidade do Minho.

——— (2010) "Pricing with customer recognition," *International Journal of Industrial Organization*, 28 (6), 669–681.

Esteves, R.-B., Q. Liu, and J. Shuai (2022) "Behavior-based price discrimination with nonuniform distribution of consumer preferences," *Journal of Economics and Management Strategy*, 31 (2), 324–355, 10.1111/jems.12466.

Esteves, R.-B. and C. Reggiani (2014) "Elasticity of demand and behaviour-based price discrimination," *International Journal of Industrial Organization*, 32 (1), 46–56.

Fudenberg, D. and J. Tirole (2000) "Customer poaching and brand switching," *RAND Journal of Economics*, 31 (4), 634–657.

Fudenberg, D. and J.M. Villas-Boas (2006) "Chapter 7: Behavior‐based price discrimination and customer recognition," *Handbook of Economics and Information Systems*, 1, 377–436.

Laussel, D., N.V. Long, and J. Resende (2020) "Quality and price personalization under customer recognition: A dynamic monopoly model with contrasting equilibria," *Journal of Economic Dynamics and Control*, 114, 10.1016/j.jedc.2020.103869.

Shin, J. and K. Sudhir (2010) "A customer management dilemma: When is it profitable to reward one's own customers?" *Marketing Science*, 29 (4), 671–689, 10.1287/mksc.1090.0547.

Villas-Boas, J.M. (1999) "Dynamic competition with customer recognition," *RAND Journal of Economics*, 30 (4), 604–631.

——— (2004) "Price cycles in markets with customer recognition," *RAND Journal of Economics*, 35 (3), 486–501, 10.2307/1593704.

Zhang, T., Y. Huo, X. Zhang, and J. Shuai (2019) "Endogenous third-degree price discrimination in Hotelling model with elastic demand," *Journal of Economics/ Zeitschrift fur Nationalokonomie*, 127 (2), 125–145.