# On the efficiency properties of the Roy's model under uncertainty and market incompleteness

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**Abstract**

We consider Roy's economies with perfectly competitive labor markets and uncertainty. Firms choose their investments in physical capital before observing the characteristics of the workers that they will hire. We provide conditions under which equilibrium allocations are constrained Pareto efficient, i.e., such that it is impossible to improve upon the equilibrium allocation by changing agents' investments in human and physical capital and letting the other endogenous variables adjust to restore market clearing. We also provide a robust example of a class of economies where equilibria are constrained Pareto inefficient due to overinvestments in high skills.

**Keywords:** Roy's model, human capital, constrained Pareto efficiency
**JEL classification:** D60, D82, J24

## 1 Introduction

The Roy's model (1950, 1951) provides a natural setting for the analysis of many labor market phenomena. Its key feature is the emphasis on the role of workers' comparative advantages in different jobs. This allows for a richer set of implications, compared to the ones obtainable in pure efficiency unit models. Additionally, and closer to the issue discussed in this paper, as soon as we move outside the class of perfect market economies, the Roy's model may have welfare properties, and - consequently - policy implications, which are sharply different from the ones obtained in pure efficiency unit economies. From this viewpoint, the key question is how the equilibrium choices at the extensive margin are determined, and how they interact with the ones at the intensive margin in delivering the welfare properties of equilibria.

This issue has been studied in several papers. To relate it to the framework considered here, it is convenient to focus on the simplest example. Let's first look at a pure efficiency unit model. Consider a two-period economy where firms choose their investments in physical capital ex-ante, without knowing exactly the wage rate that will prevail in the labor market that they will face in the second period. This may happen, for instance, because investments in human capital depend upon the realization of some random variable not perfectly observed by the firms when they choose their investments. In the second period, perfectly competitive labor markets open and clear at the equilibrium wages. It is easy to see that, in a pure efficiency unit model, the equilibrium allocation is constrained Pareto optimal (or CPO). By this, we mean that it is impossible to improve upon this allocation by choosing appropriately the individual investments and letting the endogenous variables adjust to restore market clearing. Adopting a different perspective, for this class of economies direct tax/subsidies on investments (or, some nonlinear income taxation) cannot entail a Pareto improvement. Consider now a similar set-up in a Roy's model with two industries. In one, firms only use high skill labor. In the other, they only use low skill workers. As before, investments in physical capital are selected ex-ante and depend upon the distribution of equilibrium wages. Heterogeneous workers choose their skill levels according to their comparative advantage. Also, assume that profits are increasing in the average level of human capital of the two types of workers. In this set up, it is easy to construct examples where equilibria are not constrained Pareto efficient. The argument goes as follows: at the equilibrium, workers are endogenously partitioned into two subsets, defined by the type of human capital they have invested in. Suppose that the agents investing in high skills are the ones with a relatively low cost of their effort in acquiring human capital. Then, a marginal change in the partition, reducing the *size* of the set of agents investing in high skills, simultaneously increases the *average level* of human capital of both low and high skilled workers. This may increase the optimal level of the investments in physical capital in both industries and it may very well be Pareto improving. Since we can increase welfare by shrinking the set of agents with high skills, inefficiency is due to overeducation. Again, this also implies that appropriate systems of tax/subsidies on investments can be Pareto improving.[1]

Results which are, essentially, in the same spirit arise in several, different set-ups. For instance, Charlot and Decreuse (2005) consider a two-sector economy with matching frictions.[2] Firms create vacancies for jobs using either high or low skill labor. Workers optimally choose to enter one of the two labor markets. Under the assumption of complementarity between innate ability and education, high ability workers are the ones investing in high skills at the equilibrium. The authors show that equilibria are characterized by overinvestment in high skills. In this framework, the creation of vacancies in the two sectors plays essentially the same role of the investments in physical capital in the previous example with frictionless labor markets. Again, the key feature of the economy is that the distribution of human capital in the two labor markets matters. If the threshold

[1] An example of economy with these properties is analyzed in detail in Mendolicchio, Paolini, and Pietra (2012b).

[2] See also Mendolicchio, Paolini, and Pietra (2012a).

defining the partition of the workers into the two skills moves up, i.e., if some agents switch from the high to the low skill labor market, the expected human capital of the workers in both markets and, consequently, the vacancy creation also increase. This, in turn, has a positive welfare effect.

The different welfare properties of pure efficiency units and Roy's models can also be verified in the set-up proposed by Acemoglu (1996). This is a full employment economy where wages are determined by bargaining between workers and firms, using as equilibrium concept the Nash bargaining solution with exogenous weights. In his pure efficiency unit model, undereducation always holds at the equilibrium. As in the previous, perfectly competitive, example, Mendolicchio, Paolini, and Pietra (2014) establish that the nature of the inefficiency can be reversed, once workers' choices at the extensive margin are also taken into account.

These results are obtained in different types of economies, but they all share two basic features: First, some variables - investments in physical capital or vacancies - are selected ex-ante by the firms considering the equilibrium distribution of some variables related to the labor supply in the two markets. Second, workers self-select into one of two labor markets by investing in human capital. A change in the equilibrium threshold modifies, at the same time, the distribution of the labor supply in both labor markets. This affects the optimal value of the predetermined variable, e.g., firms' investments, and it may induce a welfare improvement.

In this paper, we extend the analysis of the efficiency properties of the Roy's model of investments in human capital under uncertainty asking the following question: Consider an economy where labor markets are perfectly competitive, but investments in physical capital are selected ex-ante, before the random variable affecting investments in human capital realizes. Let's define an equilibrium allocation to be constrained Pareto efficient if it is impossible to improve welfare by changing the profile of the investments at either margins and adjusting the other equilibrium variables so that market clearing is restored. Under which conditions the equilibrium allocation is constrained Pareto optimal? The bottom line is that constrained optimality is guaranteed if the equilibrium partition of workers is state-contingent. As soon as we depart from this property, we can construct robust examples of economies such that constrained efficiency fails.

Self-selection of workers into different skills is essential because it affects the labour supply in the different labor markets and, therefore, equilibrium wages and the optimal level of the investments in physical capital. However, informational imperfection, or asymmetries, do not play any role. In fact, and to avoid any possible misunderstanding, we can as well assume that, when the labor markets meet, there is full information on the characteristics of each individual worker. Therefore, our results do not depend at all on signalling phenomena. What matters is just the, ex-ante, uncertainty on workers innate abilities which explains why firms choose their investments in physical capital on the basis of the expected distribution of human capital supply for the different types of skills.

We develop our analysis in the framework of Roy's models of the labor markets. However, our results are also related to the more general literature on complementary investments with bargaining and no ex-ante contracting (see Grossman and Hart (1986) and Cole, Mailath and Postlewaite (2001a, 2001b)). It is worthwhile to notice that a classical result of this literature is that failure of efficiency is due to *underinvestment*, caused by the lack of complete appropriability of the surplus generated by the investments. To the contrary, in our model, under appropriate conditions, the lack of constrained efficiency is due to *overinvestments* in human capital. Moreover, this can take place in environments with perfectly competitive markets, so that no hold-up problem may arise.

Finally, bear in mind that, in related literature, the constraints determining the set of attainable welfare levels are the ones which must be satisfied to guarantee an appropriate self-selection of the agents under asymmetric information. Here, workers' characteristics are perfectly observable when labor markets meet. Thus, this type of constraints plays no role whatsoever.

The results proposed in this paper are, we believe, interesting for at least two different reasons. First, they identify the basic features of the economy determining its constrained efficiency properties. This helps to put in a proper perspective the different results previously obtained in the literature. Moreover, they can be immediately applied to many other classes of economies with similar structures. Secondly, they can contribute indirectly to the literature on optimal taxation in Roy's models (see, Saez (2004), and Rothschild and Scheuer (2012, 2014)). Abstracting from the details, assume that a policy vector $\xi$ is selected to maximize some welfare function $E(S(\xi))$ at the associated market equilibrium. For instance, assume that $\xi$ is an optimal linear tax profile. Several contributions in the literature have established that the classical Diamond and Mirlees (1971) results concerning the main, general features of optimal linear taxation break down when labor inputs are not perfectly substitutable and the partition of the agents is exogenously given. Specifically, in the Diamond and Mirlees (1971) framework, production is on the efficient frontier and it is possible - essentially - to ignore the equilibrium price adjustment effects of tax changes, so that optimal tax formulas are the same if prices are treated as fixed, or if they are derived at the equilibrium. When labor inputs are not perfectly substitutable in production and the partition of workers is fixed, both results break down. Stiglitz (1982) shows that with two types of skills, the effects of equilibrium price adjustments cannot be ignored. Naito (1999) shows that efficiency in production also fails. Saez (2004) analyzes the properties of optimal taxation in an economy with several types of labor, imperfectly substitutable in the production function. He establishes that both properties (efficiency in production and irrelevance of the effects of price adjustments) are restored once one considers a long run model, where the partition of workers across skills is selected optimally at the equilibrium, and labor supply is inelastic. In all these papers, different types of labor are the only inputs in production. More recently, Gahvari (2014) considers economies with both labor (with exogenous partition) and capital, and shows that introducing capital as an input may play an important role, reversing some of the results obtained in Stiglitz (1982). One way to interpret our results is as a positive contribution to this literature. Ignoring the technicalities, the key issue for the Saez (2004) result is that the partition of the workers optimally adjust to changes in the policy parameters. This is exactly what will happen in the class of economies analyzed in this paper and having constrained Pareto optimal equilibria, i.e., characterized by a state-contingent partition of the workers across skills. Therefore, we conjecture that results analogous to the ones of Saez (2004) hold for this class of economies. On the other hand, they are bound to fail whenever the workers partition is not state-contingent, as in the class of economies with constrained Pareto inefficient equilibria described below.

The structure of the paper is the following. Next section presents the main, common features of the two classes of economies that we are going to study, and defines our notions of equilibrium and of constrained Pareto optimal allocation. Section 3 introduces conditions such that each equilibrium allocation is constrained optimal (Subsection 3.1) and analyzes a class of economies where equilibria can be constrained inefficient (Subsection 3.2). Some conclusions follow in Section 4.

# 2 The general set-up

We start describing the general structure of the economy.

**Time structure:** Economic activity takes place over three periods. In the first, identical, competitive, firms choose their investments in physical capital maximizing expected profits given their rational expectations on the distribution of future equilibrium wages. In the second period, heterogeneous workers choose their type of skill and their optimal levels of human capital (HC from now on) after observing the investments of the firms and the characteristics of the labor markets. Their expectations on equilibrium wages are rational. In the third, and last, period, before she/he is actually hired, the HC of each worker becomes perfectly observable. Then, labor markets clear and production takes place.

Notice that there is no signalling motivation for the workers' investment in HC, since this is perfectly observable in the last period.[3] Thus, information is symmetric when labor markets meet. The cause of constrained inefficiency is just the interaction between irreversibility of the investments, lack of insurance opportunities and self-selection of the workers into the different skills.

**Uncertainty:** Uncertainty is essentially related to the equilibrium levels of the wages a particular firm will face. To make the story precise, we adopt the usual idea of a collection of labor markets. Each "local" labor market can be thought of as an island populated by a continuum of workers, possibly heterogeneous according to some parameter $\delta$ affecting their optimal investments in HC. We assume that the higher the value of $\delta$, the lower the utility cost of investing

[3] Hence, without any loss of generality, we can consider a two-period model. Otherwise, without full observability of HC, a more extended multiperiod structure could be essential, since firms could learn the actual ability of each worker over time.

in high skills. Also, we assume that there are just two skill levels denoted by $s \in \{ne, e\}$, with $e$ denoting high skills while $e$ denotes high skills.

From the point of view of workers, high and low skill HC differ because of the possible differences in their wages and in the cost of acquiring them. From the point of view of firms, the two skills are different inputs in the production process.[4] The only thing that really matters is that they are not perfectly fungible.

Also, bear in mind that, given the structure of the economy and of the information, workers do not actually face any uncertainty on the wage profile they face when they select their level of HC. Only firms will deal with a substantial uncertainty, because workers in different islands are different.

As already mentioned, we will study two classes of economies. While their time and uncertainty structures are identical, they differ in terms of the structure of the labor markets. This difference will determine their opposite properties in terms of constrained efficiency.[5] To be precise:

- **Economies where all the equilibria are constrained efficient:** After choosing its investment in physical capital, each firm is matched with a single island, characterized by a distribution of the parameter $\delta$ which is identical across island and an island-specific realization of a random variable $\widetilde{T}$, a parameter determining the cost of acquiring high skills. Hence, the supply of high skill (and low skill) labor will vary across island depending upon the realization of $\widetilde{T}$.

- **Economies with constrained inefficient equilibria:** After choosing its level of investment in physical capital, each firm is matched with a pair of islands. Each island is characterized by a specific realization of the parameter $\delta$.[6] The first, where - as we will show - $\delta \in \left[\underline{d}, \delta^t\right]$ is populated by low skilled agents. The second, where $\delta \in \left[\delta^t, \overline{d}\right]$, by high skilled ones. Here, the value of the realization of $\widetilde{T}$ is basically irrelevant and, therefore, fixed and identical in each island. Different values of $\delta$ imply different levels of the labor supply.

Let's now describe the behavior of the two types of agents, workers and firms.

[4] For instance, we will consider the production function $F_j(.) = k_j^{\alpha}\{\phi^{ne}\ell_j^{ne\,\theta} + \phi^e \ell_j^{e\,\theta}\}^{\frac{1-\alpha}{\theta}}$. There, it will be natural to assume that $\phi^e > \phi^{ne}$, so that, at each $\ell_j^e = \ell_j^{ne}$, the marginal product of high skill labor is higher than the one of low skilled labor.

[5] Remember that, informally, an allocation is CPO if it cannot be improved upon by changing the investments in physical and human capital, and adjusting wage rates so to clear the labor markets.

[6] Here, we just report the essential feature driving the model: The measures of the sets of high and low skilled workers are given (the populations of the two islands are identical), while the levels of high and low skills of the actual labor market a firm is facing are random variables. Additional details will be discussed later.

## 2.1 Individual behavior

### 2.1.1 Workers

Each worker is endowed with one unit of time that she/he inelastically supplies. This unit of time is converted into $h^s$ units of HC of skill $s$, where $h^s$ depends upon the worker's effort. Once acquired, HC of type $s$ converts 1-to-1 into efficiency units of labor supply of type $s$. Hence, workers make a choice at both margins, intensive and extensive. To invest in high skills entails, in addition to the disutility cost of effort, a fixed cost $T$, perfectly observed by each worker when choosing her investment. In general, their preferences are described by a utility function $u(c, h; \delta)$ where $c$ is consumption and $h$ is the amount of HC (or, more precisely, the effort applied to acquire HC).[7]

We focus our analysis on the standard case where the optimal investment in HC, given the wage rate, is increasing in the parameter $\delta$, which determines the marginal rate of substitution between consumption and effort. As we will establish in Lemma 1, the labor supply (in terms of efficiency units) is increasing in the wage rate provided that assumption $U$ holds.

*Assumption U:* For each $\delta$, preferences are described by a strictly concave, $C^2$ utility function $u(c, h; \delta) \equiv v(c) - \frac{g(h)}{\delta}$, for some $\delta > 0$. Moreover, $-\frac{\frac{\partial^2 v(.)}{\partial c^2} c}{\frac{\partial v(.)}{\partial c}}$ is "sufficiently" small.

For unskilled labor, the labor supply (in terms of efficiency units) is increasing in the wage rate if the measure of the curvature of $v(.)$, $-\frac{\partial^2 v(.)}{\partial c^2} c / \frac{\partial v(.)}{\partial c}$, is below 1. A slightly stronger condition is necessary to get the same property for skilled labor.[8]

Consider a labor market described by some measurable partition of the set of workers, $\Delta(.)$, and by the level of firms' investments, $k_j$.

Let $w(.) \equiv \{w^{ne}(.), w^e(.)\}$ be any wage map defined on $\Delta(.)$. We can describe workers' behavior as follows. First, given the realization $T$ and the wage map, each worker solves, for each $s$, the optimization problem

$$\max_{h^s} u(c^s, h^s; \delta), \qquad (U^s)$$

with $c^{ne} = w^{ne} h^{ne}$ and $c^e = w^e h^e(.) - T$.

Given $s$, let $\widetilde{h}^s(w^s; \delta, T)$ be the supply of HC of agent $i$ with skill $s$, i.e., the pair of their *notional* supply functions. Also, let $V^s(w^s; \delta, T)$ be the value function of problem $(U^s)$ for agent $\delta$ with skill $s$,. Evidently, a worker may invest in skill $e$ if and only if $V^e(w^e; \delta, T) \geq V^{ne}(w^{ne}; \delta, T)$.

The key properties of workers' behavior are summarized in the following Lemma, whose proof is in Appendix.

[7] The, possible, time-costs of the investment in HC have no relevant implications. Therefore, we ignore them.

[8] The condition is $\frac{\partial^2 v(.)}{\partial c^{s2}} / \frac{\partial v(.)}{\partial c^s} |_{c^e} w^e h^e > -1$, when evaluated at $c^e < w^e h^e(.)$, due to the fixed cost of the investment in HC.

**Lemma 1** *Under assumption (U), for each* $\delta$ *and each* $s$, *(i)* $\frac{\partial \widetilde{h}^s(.)}{\partial w^s} > 0$ *and (ii)* $\frac{\partial \widetilde{h}^e(.)}{\partial T} > 0$. *Moreover, (iii) if* $w^e > w^{ne}$, $\frac{\partial V^e(.)}{\partial \delta} > \frac{\partial V^{ne}(.)}{\partial \delta}$.

As already pointed out, $(i)$ and $(ii)$ are standard results, reported here for completeness. They hold for the *notional* supply functions $\{\widetilde{h}^{ne}(.), \widetilde{h}^e(.)\}$.

Obviously the *actual* demand correspondences $\{h^{ne}(.), h^e(.)\}$ are not continuous functions at the critical set of wage profiles where a worker switches from one skill to the other.

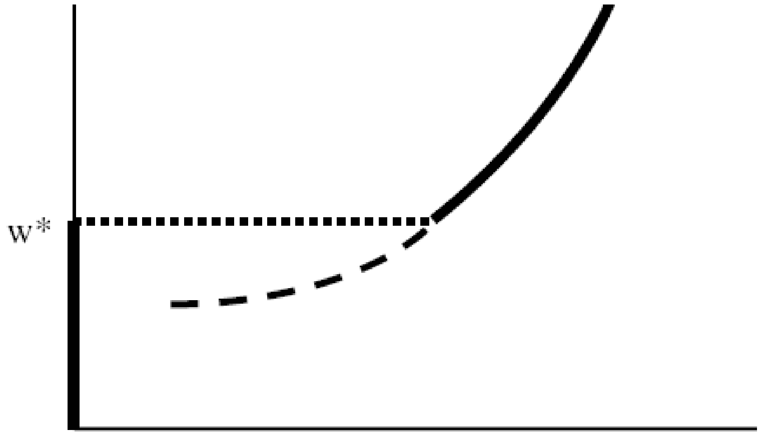


Figure 1: The high skill labor supply

Figure 1 describes the typical individual supply curve for high-skill labor. $w^*$ is the threshold for $w^e$. For $w^e < w^*$, the actual supply of skilled labor is nil. For $w^e > w^*$, it is described by the thick curve. The notional supply curve, $\widetilde{h}^e(.)$, (which coincides with the actual one at $w^e > w^*$) is described by the dashed curve. Evidently, at the threshold, $h^e(.)$ is a (non convex-valued) correspondence. The properties of the low-skill labor supply correspondence are similar. This is always true in Roy's models: the choice at the extensive margin induces a lack of convexity of the individual demand correspondence at the critical wage configurations where a worker switches from one type of skill to the other. To deal with this issue in a straightforward way, we consider a continuum of identical workers for each type $\delta$. Hence, we assume that, when workers are partitioned into the two skill types on each island, there is a continuum of identical individuals, denoted by $i \in [0, 1]$, for each $\delta$. Hence, on each island, the set of agents is described by a square $[0, 1] \times \left[\underline{d}, \overline{d}\right]$, endowed with the Lebesgue measure.[9] Each agent is identified by a pair $(i, \delta)$. Given that all the agents with

[9] As we will see, each equilibrium partition is defined by a threshold $\delta^t \in [\underline{d}, \overline{d}]$. Each worker with ability $\delta$ will invest in low skills if $\delta < \delta^t$, in high skills if $\delta > \delta^t$. Agents with $\delta = \delta^t$ will be indifferent. Since the equilibrium partition is defined up to zero measure subsets, we do not need to be too precise about the actual behavior of agents with $\delta = \delta^t$, a set of measure zero.

the same $\delta$, and all the firms, are identical, we will mostly avoid to make explicit reference to this feature of the model. Its only role is to justify the assumption of perfect competition and, most relevant, to guarantee the convex-valuedness of the labor supply correspondence.

### 2.1.2 Firms

There is a continuum of identical firms, indexed by $j \in [0,1]$. Each firm is endowed with the same concave, $C^2$ production function $F_j(k_j, \ell_j^{ne}, \ell_j^e)$, with $\left\{k_j, \ell_j^{ne}, \ell_j^e\right\} \in \mathbb{R}_+^3$. Returns to scale are constant. Production requires a positive amount of capital and of each type of labor, i.e., $F_j(k_j, \ell_j^{ne}, \ell_j^e) = 0$ for each $\left\{k_j, \ell_j^{ne}, \ell_j^e\right\} \in \partial\mathbb{R}_+^3$. Moreover, $\frac{\partial F_j(.)}{\partial k_j} > 0$ and $\frac{\partial F_j(.)}{\partial \ell_j^s} > 0$, for each $s$, whenever $\left\{k_j, \ell_j^{ne}, \ell_j^e\right\} >> 0$. These properties are either required or, at least, convenient for our efficiency results. More stringent assumptions are required to establish the existence of equilibria. We will come back to the existence problem later on.

Without any essential loss of generality, all commodity prices (for outputs and investments) are equal to 1. As common in the literature, this can be rationalized by making appeal to a "small open economy" assumption.

Firms are expected profit maximizers and face a two-stage decision problem. Let's describe their behavior proceeding backward. In period 3, each firm, given its (sunk) investment in physical capital, $k_j$, and after observing the actual wage profile, chooses its optimal labor demand $\left\{\ell_j^{ne}(w^{ne}, w^e, k_j), \ell_j^e(w^{ne}, w^e, k_j)\right\}$.

In period 1, given a wage profile $\{w^{ne}(.), w^e(.)\}$, each firm chooses its investment $k_j$ solving the optimization problem

$$\max_{\left\{k_j, \ell_j^{ne}(.), \ell_j^e(.)\right\}} E\left(F_j(k_j, \ell_j^{ne}(.), \ell_j^e(.)) - w^{ne}(.)\ell_j^{ne}(.) - w^e(.)\ell_j^e(.)|\Delta(.)\right) - k_j. \quad (\Pi)$$

## 2.2 Equilibrium

The notion of the equilibrium is based on the standard requirements of individual optimization, market clearing and rational expectations.

An allocation is $\chi \equiv \left\{\left\{k_j, \ell_j^s(.)\right\}, \left\{c_i^s(.), h_i^s(.)\right\}, s = ne, e\right\}$.

We will use the expression "associated measurable partition $\overline{\Delta}(.)$" to refer to the measurable partition of the set of islands, or of the single island.[10]

**Definition 2** *An equilibrium is a wage map* $\{\overline{w}^{ne}(.), \overline{w}^e(.)\}$ *with associated measurable partition* $\overline{\Delta}(.)$ *and allocation* $\overline{\chi}$ *such that*[11]
*i.* *for each* $i$, $\delta$ *and* $s$, $\{c^s(.), h^s(.)\}$ *solves* $(U^s)$,
*ii.* $V^e(\overline{w}^e(.); \delta, T) - V^{ne}(\overline{w}^{ne}(.); \delta, T) > 0$ *only if* $\delta \in \overline{\Delta}^e(.)$,
*iii.* $\left\{k_j, \ell_j^{ne}(.), \ell_j^e(.)\right\}$ *solves* $(\Pi)$,
*iv.* *labor markets are in equilibrium in each spot market.*

[10] For economies with CPO equilibria, the partition refers to each single island, for the other set of economies, to the set of islands, as we will clarify later on.

[11] As already mentioned, the individual optimality conditions must hold *a.e.*, however it would just be pedantic to restate this fact over and over again and, therefore, we omit it.

These conditions require that, conditional on the information available, all the agents choose their optimal investments (as stated by $i - iii$), and that labor markets clear at the given wages ($iv$). This, together with the fact that agents make their choices conditional on the equilibrium partition map and on the additional information available to them, if any, implies rational expectations.

The essential difference between the two classes of economies under study is in the definition of the partition $\overline{\Delta}(.)$. For the first type of economies (whose equilibrium allocations are always CPO), all the islands are, ex-ante, identical. A state is defined by a realization of the r.v. $\widetilde{T}$ and $\overline{\Delta}(T)$ is a $T$-contingent partition of the set of agents, $[\underline{d}, \overline{d}] \times [0,1]$. Evidently, the labor market conditions (hence, the wages) a firm is actually facing depend upon the realization $T$ in the island the firm is matched with.

For the other class of economies (where CPO may fail), each island is characterized by a particular value of the parameter $\delta$, and agents within each island are identical. Each firm is matched with a pair of islands, one with skilled workers, the other with unskilled workers. Since the parameter $\delta$ vary across islands, a state is defined by a realization $\{\delta^{ne}, \delta^{e}\}$ and $\overline{\Delta}(.)$ partitions the set of islands, $[\underline{d}, \overline{d}]$.

Consequently, condition ($iv$) takes a different form in the two cases. In the first, an interval $[0,1]$ of identical firms demands labor of the two skills. The market clearing conditions are

*iv.a.* $\int_{\overline{\Delta}^s(T)} \left( \int_0^1 h_i^s(.) di \right) d\delta = \int_0^1 \ell_j^s(.) dj$, for each $s$ and $T$, *a.e.*.

In the second, an interval $[0,1]$ of identical firms demands labor of the two skills, too. However, in each island, agents are identical and labor of skill $s$ is supplied by an interval $[0,1]$ of identical workers with $\delta = \delta^s$. Hence, the market clearing conditions are

*iv.b.* $\int_0^1 h_i^s(.; \delta^s) di = \int_0^1 \ell_j^s(.) dj$, for each $s$ and pair $\{\delta^{ne}, \delta^{e}\}$.

In both set-ups, existence of equilibria is a non trivial issue. As we will see later on, it may require additional restrictions on utility and production functions. We postpone the discussion of this problem, since it is somewhat peripheral to the main concern of this paper.

For future reference, let's also introduce the notion of conditional equilibrium. Here, investments in physical and human capital are exogenously given (for instance, we can think of them as selected by some social planner), while the other equilibrium conditions are the same as before.

**Definition 3** *A* $\left(\overline{K}, \overline{\Delta}\right)$ *−conditional equilibrium is a wage map* $\{\overline{w}^{ne}(.), \overline{w}^{e}(.)\}$ *with associated allocation* $\overline{\chi}$ *such that*

*i.* *for each* $\delta$, $\{c^s(.), h^s(.)\}$ *solves* $(U^s)$, *only if* $\delta \in \overline{\Delta}^s$,

*ii.* $\left\{\ell_j^{ne}(.), \ell_j^{e}(.)\right\}$ *solves* $(\Pi_j)$ *for each* $j$, *given* $k_j = \overline{K}$,

*iii.* *labor markets are in equilibrium in each spot market.*

## 2.3 Constrained Pareto optimality

Given that there are no full insurance opportunities, Pareto efficiency is obviously out of reach. We propose a notion of CPO based on the comparison of the utilities obtained at the equilibrium with the ones that individuals could obtain at a *conditional* equilibrium associated with some alternative pair $\{K, \Delta\}$. Our concept of CPO is related to the one exploited in the GE literature on economies with incomplete markets (see Geanakoplos and Polemarchakis (1986)), since it compares the equilibrium outcome with the allocation obtained changing appropriately the investment portfolios in the first period and adjusting accordingly the equilibrium spot wages in the last period.[12] In our context, this notion presents two key advantages. First, and most relevant from a substantive viewpoint, the policy instruments of the planner are fairly weak: the levels of the investments in physical and human capital. This strengthens the inefficiency result, since a Pareto improvement could, in principle, be easily implemented just by taxing, or subsidizing, appropriately the investments (see Proposition 8 below). The second advantage has a more technical nature: the problem of constrained efficiency can be converted into a planner's optimization problem. This allows us to discuss it in a straightforward way, comparing the first order conditions of the planner's CPO problem with the conditions which must be satisfied at each equilibrium.

Formally, we adopt the following notion of CPO:

**Definition 4** *An equilibrium wage map* $\{\overline{w}^{ne}(.), \overline{w}^{e}(.)\}$ *with associated measurable partition* $\overline{\Delta}(.)$ *and allocation* $\overline{\chi}$ *is constrained Pareto optimal if and only if there is no alternative wage map* $\{\widehat{w}^{ne}(.), \widehat{w}^{e}(.)\}$ *with associated measurable partition* $\widehat{\Delta}(.)$ *and allocation* $\widehat{\chi}$ *such that:*

1. $\{\widehat{k}_j\}$ *is firm invariant, i.e.,* $\widehat{k}_j = \widehat{K}$ *a.e.,*
2. $\{\widehat{w}^{ne}(.), \widehat{w}^{e}(.)\}$ *with associated allocation* $\widehat{\chi}$ *is a* $(K, \Delta)-$*conditional equilibrium for some* $\left(\widehat{K}, \widehat{\Delta}\right)$,
3. $E\left(u(\widehat{c}_i^s(.), \widehat{h}_i^s(.); \delta, T)\right) \geq E\left(u(\overline{c}_i^s(.), \overline{h}_i^s(.); \delta, T)\right)$, *a.e., with* $E\left(u(\widehat{c}_i^s(.), \widehat{h}_i^s(.); \delta, T)\right) > E\left(u(\overline{c}_i^s(.), \overline{h}_i^s(.); \delta, T)\right)$ *for some set of agents of positive Lebesgue measure,*
4. *expected profits are nonnegative:* $E\left[\left(F_j\left(\widehat{k}_j, \widehat{\ell}_j^{ne}, \widehat{\ell}_j^{e}\right) - \sum_s \widehat{w}^s \widehat{\ell}_j^s\right) | \widehat{\Delta}(.)\right] - \widehat{k}_j \geq 0.$

(1) describes the fundamental feature of our economies, (2) restricts the welfare comparison to conditional equilibria, (3) is the usual definition of Pareto dominance. It is expressed in terms of expected utility, so that we are adopting

[12] Evidently, in (1986) investments are portfolios of financial assets. Here, they are portfolios of real assets, the investments in human and physical capital.

the ex-ante perspective. In the first class of economies (with CPO equilibria) expectations are computed with respect to the realization of the r.v. $\widetilde{T}$, which in turn affects the partition of workers and, therefore, equilibrium wages. In the second class, where CPO may fail, $T$ is fixed and expectations are computed with respect to the distribution of wages that, for each type of agents, characterized by their common $\delta$ and their, common, level of skills, also depends upon the value of $\delta$ of the workers with the other skill. For skilled workers expectations are computed with respect to the distribution of these $\delta \in \Delta^{ne}$. For unskilled workers, with respect to the distribution of $\delta \in \Delta^{e}$.

The last condition, (4), is less obvious. It can be rationalized in two different ways. First, and directly, as a feasibility constraint. Alternatively, we may assume that firms are owned by an additional class of agents, "rentiers", with linear utility functions. At time 0, they have some large initial endowment that can be either consumed or invested as physical capital of the firms they own. With this second interpretation, our economy is embedded into a fully specified general equilibrium model, and our notion of CPO allocation essentially coincides with the one of Pareto optimal allocation constrained by (1) and by the lack of insurance markets. Given that returns to scale are constant, both interpretations could be adopted.

# 3 The constrained Pareto optimality properties of equilibria

As already mentioned, the CPO properties of equilibria crucially depend upon the precise specification of the structure of the labor markets. We consider two cases. In the first, in *each* "island" the distribution of $\delta$, the parameter affecting the utility cost of investments in HC, is non-trivial. These distributions are identical across islands. However, the distributions of workers' individual choices will differ across islands due to the island-specific realization of some random variable, such as the direct costs of education, described by a r.v. $\widetilde{T}$ uniformly distributed on $[\underline{T}, \overline{T}]$. Each firm is matched randomly with a single island, whose population is endogenously partitioned into two measurable subsets of agents, $\{\Delta^{ne}(T), \Delta^{e}(T)\} \equiv \Delta(T)$, dependent upon the realization of $\widetilde{T}$. Islands are different because the realizations of the r.v. $\widetilde{T}$ are so. When this is the case, equilibrium allocations are always CPO. In each subeconomy - defined by an island and a firms -, the equilibrium allocation (defined in terms of labor demand and supply *and* workers' partition) is Pareto efficient, *conditional* on $k_j$, because labor markets are perfectly competitive. Therefore, it can be seen as the optimal solution to a standard problem of surplus maximization, given $k_j$. This is the key property explaining CPO of equilibria, because, by the envelope theorem, in each island, we can ignore the second order effects of changes in investments in physical capital on total surplus. This implies that, the conditions for the individual firms optimal choice of investments entail the first order conditions for the maximization of welfare.

In the second case, fully described in section 3.2, $T$ is always the same[13] and workers are identical within each island (i.e., they have the same $\delta$), but they differ across islands, which are indexed by the associated values $\delta \in [\underline{d}, \overline{d}]$. Firms are matched with a pair of islands, i.e., of labor markets, each one characterized by a value $\delta^s$, $s = ne, e$, invariant across workers of the same island. When choosing $k_j$, each firm $j$ knows the equilibrium partition, but it does not know the actual realization $\{\delta^{ne}, \delta^e\}$ for the pair of labor markets it faces. Consider an equilibrium and the effect of an exogenous increase in the threshold $\delta^t$. Under appropriate assumptions on the technology, this has a positive effect on equilibrium wages - hence on utilities - for all the inframarginal subeconomies (i.e., the ones such that both $\delta^e \neq \delta^t$ and $\delta^{ne} \neq \delta^t$). It also has a positive impact on expected profits.

At the margin, workers are indifferent between investing or not in high skills. Therefore, to switch from one type of skill to the other has no effect on their welfare. On the other hand, an increase in the investments in physical capital improves the welfare of each worker. If the increase is sufficiently small, the sum of the two changes (in $\delta^t$ and $k_j$) on profits is also positive. This shows that we can implement a Pareto improvement using exogenous variations of the threshold and of the investments in physical capital. Hence, equilibria are constrained inefficient.

We now study separately the two cases, showing our main efficiency results.

## 3.1 Economies where equilibrium allocations are constrained Pareto optimal

In this class of economies, each firm deals with a pair of linked labor markets. Linked because workers endogenously and optimally choose their skill levels, so that the partitions of workers are state-contingent. Bear in mind that here a state is defined by the realization of a r.v. $\widetilde{T}$ and that the equilibrium partition is $T-$conditional within each island.

Proposition 6 establishes that equilibria are always CPO. Its proof rests on Lemma 5: equilibrium partitions are always defined by a unique threshold. The proofs of the two results are in Appendix.

**Lemma 5** *Consider any economy such that an equilibrium exists. Then, at each equilibrium,* $\Delta(T) \equiv \{\Delta^{ne}(T), \Delta^e(T)\} = \left\{[\underline{d}, \delta(T)), [\delta(T), \overline{d}]\right\} \times [0,1]$.

**Proposition 6** *Consider any economies with a $T-$conditional partition within each island. Then, each equilibrium allocation is CPO.*

This result is in the spirit of the first fundamental theorem of welfare economics. Hence, the issue of the existence of an equilibrium is beside the point. Indeed, it is quite clear that an equilibrium exists, for some set of appropriate

[13] This may appear as a significant departure from the structure of the previous class of economies. It is not: the same substantive results hold if $\widetilde{T}$ is a random variable.

restrictions on the fundamentals. No matter what these restrictions are, the equilibrium allocation is CPO.

To get the intuition behind the results of the Proposition, let's recast the problem in terms of maximization of a welfare function. We adopt the usual fiction of a benevolent planner choosing the allocation to maximize welfare under the constraints imposed on its choices by the distortions at play in the economy. Given that this is just an heuristic argument, we simply assume that equilibrium variables are differentiable functions of the parameters (this will be established in Appendix). Also, since firms are identical, their optimal investments are always equal. Hence, we will refer to equilibrium allocations conditional on an investment profile $\{k_j\}$ as $K-$conditional equilibrium allocations, for $K = k_j$.

Let's identify the partition of $\Delta$ with an arbitrary - for now - threshold $\delta^t$, $\left\{\Delta^{ne}\left(\delta^t\right), \Delta^{e}\left(\delta^t\right)\right\} = \left\{[\underline{d}, \delta^t], (\delta^t, \overline{d}]\right\}.$

Given $\overline{K}$, and $\overline{k}_j = \overline{K}$, for each $j$, a realization $\overline{T}$, and an arbitrary threshold $\delta^t$, define the map

$$\begin{aligned} S\left(.;\overline{K},\overline{T},\delta^t\right) \quad &\equiv \quad \sum_s \int_{\Delta^s(\delta^t)} \left(\int_0^1 \phi^s\left(\delta\right) u(c_i^s, h_i^s;\delta) di\right) d\delta \\ &+ \int_0^1 \left(F_j\left(\overline{k}_j, \ell_j^{ne}, \ell_j^e\right) dj - \sum_k \overline{k}_j\right) dj \\ &- \sum_s \int_{\Delta^s(\delta^t)} \left(\int_0^1 c_i^s(.) di\right) d\delta - T\int_{\Delta^e(\delta^t)} d\delta \end{aligned}$$

for some map $\phi^s\left(\delta\right)$ such that $\phi^s\left(\delta\right) > 0$ for each $\delta$. Notice that, at an equilibrium, given the individual budget constraints, the sum of the last two terms in $S\left(.;\overline{K},\overline{T},\delta^t\right)$ is the total producers' surplus in state $\overline{T}$. For each $s$, define the constraint

$$\int_{\Delta^s(\delta^t)} \left(\int_0^1 h_i^s di\right) d\delta - \int_0^1 \ell_j^s dj = 0,$$

the material balance conditions for the labor markets. Given $\left(\overline{K},\overline{T},\delta^t\right)$, each sub-economy is a canonical Arrow-Debreu economy. Hence, its equilibrium allocation must be Pareto optimal, and - under standard assumptions - it must maximize $S\left(.;\overline{K},\overline{T},\delta^t\right)$ under the labor market clearing constraints, i.e., it must be an optimal solution to the problem

$$\begin{aligned} &\max_{\{(c^{ne},h^{ne}),(c^e,h^e),(\ell^{ne},\ell^e)\}} S(.;\overline{K},\overline{T},\delta^T) \quad \text{subject to} \\ \int_{\Delta^s(\delta^t)} \left(\int_0^1 h_i^s di\right) d\delta - \int_0^1 \ell_j^s dj \quad &= \quad 0, \text{ for each } s. \end{aligned} \tag{1}$$

Given that spot markets are perfectly competitive, each $K-$contingent equilibrium allocation is PO and it is also an optimal solution to the optimization problem (1), given the functional $\phi^s\left(.\right) = \frac{1}{\frac{\partial v}{\partial c^s}}$.[14]

[14] At the threshold value of $\delta$, typically $\phi^{ne}(\delta) \neq \phi^e(\delta)$. The argument requires us to define

Let $\overline{S}(\delta^t;\overline{K},\overline{T})$ be the value function of problem (1), given $(\overline{K},\overline{T})$. Now, consider

$$\max_{\delta^t} \overline{S}(\delta^T;\overline{K},\overline{T}) \quad \text{subject to}$$

$$\int_{\Delta^s(\delta^t)} \left( \int_0^1 h_i^s di \right) d\delta - \int_0^1 \ell_j^s dj = 0, \text{ for each } s.. \tag{2}$$

By the envelope theorem, the FOC of this problem is

$$\begin{aligned} 0 &= -\phi(\delta)\left[u(c^e,h^e;\delta) - u(c^{ne},h^{ne};\delta)\right] \\ &\quad - \left[ \frac{\partial F_j}{\partial \ell_j^e} \ell_j^e(\delta(.)) - \frac{\partial F_j}{\partial \ell_j^{ne}} \ell_j^{ne}(\delta(.)) + c_h^{ne}(\delta(.)) - c_h^e(\delta(.)) - T \right]. \end{aligned}$$

The first term in square brackets is zero by definition of equilibrium threshold.[15] The second is zero at each equilibrium, because of the budget constraints of the marginal agents (i.e., the ones with $\delta = \delta^t$). Under appropriate assumptions, these FOCs are necessary and sufficient to guarantee that, conditional on $K$, the equilibrium allocation solves (2) at each realization $T$. Finally, let $\overline{S}(K,T)$ be the value function of this problem and define the ex-ante planner's optimization problem:

$$\max_K E(\overline{S}(K,T)|T). \tag{3}$$

By the envelope theorem, its FOCs are $E(\frac{\partial F_j}{\partial k_j}) - 1 = 0$, and, by expected profits maximization, they must be satisfied at each equilibrium. Hence, each equilibrium is CPO.

Notice that, since, at each equilibrium, expected profits are zero, $E(\overline{S}(K,T)|T)$ is a standard welfare function, i.e., the sum of individual expected utilities weighted by some collection of functions $\{\phi^s(\delta)\}$. For quasi-linear utility function, and given $\phi^s(\delta) = 1$, for each $\delta$, $E\left(\overline{S}(K,T)|T\right)$ is the total expected surplus. For general utility functions, we can look at it as the (normalized) Lagrangian of an optimization problem having as objective function the weighted sum of individual utilities, and, as a constraint, the condition that expected profits must be non-negative. Alternatively, we can take $E\left(\overline{S}(K,T)|T\right)$ as a standard social welfare function for the completely specified general equilibrium economy outlined above.

To summarize, the conditions required for the CPO of equilibrium allocations to hold in general are: first, at each $T$, the equilibrium is PO and the partition is optimal, contingent on $K$. This allows us to exploit the envelope theorem and, therefore, to ignore the second order effects of marginal changes in $K$.

$\phi(\delta) = \phi^{ne}(\delta)$ for $\delta \leq \delta(T;K)$, $\phi(\delta) = \phi^e(\delta)$ for $\delta > \delta(T;K)$. Once again, the discontinuity in the value of $\phi(\delta)$ at $\delta = \delta(T;K)$ is irrelevant in our proof, because the set of agents such that $\delta = \delta(T;K)$ has zero measure.

[15] Again, to differentiate between workers with $\delta = \delta^t$ choosing high vs. low skills would not affect the result, since the values of the utility function associated with the two choices are identical.

Second, firms must be maximizing expected profits (i.e., firms' owners must be risk-neutral). In general, state-contingency of the partition is crucial, because it allows us to apply the envelope theorem state-by-state. The logic of the argument breaks down as soon as this theorem does not apply at some stage.[16]

### 3.2 Economies with constrained Pareto inefficient equilibria

We now establish the constrained Pareto inefficiency of the equilibria of a specific class of economies. Constrained inefficiency is due to too large a set of agents investing in high skill, hence to over-education.

We start providing its general description and a parametric example exhibiting the main result. The general proof is in Appendix. The essential feature of these economies is that each firm takes its irreversible investment decision before being matched with two islands, populated - respectively - by high and low skilled workers $\{\delta^{ne}, \delta^{e}\}$. Within each island, workers are identical. Hence, in the first period, each firm chooses under conditions of uncertainty, because it knows the distribution, but not the actual realization of the relevant equilibrium wages. The sets of islands with low and high skills labor are determined endogenously by the optimal equilibrium choices of the workers.

Here, let's focus on a single, competitive firm $j$. When choosing its investments in physical capital, the firm takes into account the equilibrium wage map $\overline{w}(\delta^{ne}, \delta^{e}; K) \equiv \{\overline{w}^{ne}(.), \overline{w}^{e}(.)\}$ and maximizes its expected profits. Expectations are taken with respect to the possible combinations $\{\delta^{ne}, \delta^{e}\} \in \overline{\Delta}^{ne} \times \overline{\Delta}^{e}$, where the partition $\overline{\Delta}$ is determined by the optimal choice of the workers.

To keep the details manageable, and with some abuse of notation, let's assume that effort supply is perfectly inelastic, and that each agent of type $\delta$ supplies $\delta$ units of HC.[17] This entails no substantive loss of generality: different values of the elasticity of the effort supply may change the quantitative results, but they cannot change the qualitative ones we are interested in.

Workers choose their skill level knowing their own innate ability and, hence, the labor supply of all the workers of their own type, and observing the wage rates in the sub-economy they live in (i.e., implicitly, observing the investments in physical capital and the realized pair $\{\delta^{ne}, \delta^{e}\}$). Therefore, their choice is made under conditions of certainty.

As before, when labor markets open, the actual level of human capital of each worker becomes observable to firms too, so that there are no signalling

[16] We have assumed that the relevant r.v. is the cost of education. For this specific structure, it is straightforward to implement the full Pareto optimal allocation. It suffices to allow workers to insure against the variability of $T$, either introducing a $T$-contingent policy of taxes and subsidies or modifying the labor contracts, so that the (risk-neutral) firms are actually bearing the risk. However, one could write down economies which are analytically equivalent, but for which there are no obvious insurance possibilities. For instance, randomness in the marginal utility of consumption may have, analytically, the same effects of randomness of $T$. Evidently, for this kind of uncertainty, implementation of the full Pareto optimum would be problematic, to say the least.

[17] Remember that, up to now, the actual individual labor supply was increasing in $\delta$.

phenomena. We start presenting a parametric example with production function $F_j(.) \equiv k_j^{\alpha}\left[\psi^{ne}\ell_j^{ne\theta}+\psi^{e}\ell_j^{e\theta}\right]^{\frac{1-\alpha}{\theta}}$, $\theta>0$, $\alpha\in(0,1)$. We set $\theta=1-\alpha$. This simplifies a lot the computations, since it gives us the critical property that, given $K$, the wage of each type $\delta^s$ of worker depends, for each $s$, just on the value of $\delta^s$ itself, and it is independent of the level of the labor supply of the other type of workers.

**Example 7** *Let $F_j(.) \equiv k_j^{\alpha}\left[\psi^{ne}\ell_j^{ne\theta}+\psi^{e}\ell_j^{e\theta}\right]^{\frac{1-\alpha}{\theta}}$. Set $(1-\alpha)=\theta=\frac{2}{3}$. The first is the usual estimate for the income share of labor. The value of $\theta$ implies an elasticity of substitution between skilled and unskilled labor equal to 3, which is somewhat large compared to standard estimates. Given that this example has a purely illustrative purpose, the advantage in terms of computational tractability justifies the choice of this value. Since $\frac{1-\alpha}{\theta}=1$, the production function reduces to $F_j(.)=k_j^{\frac{1}{3}}(\psi^{ne}\ell_j^{ne\frac{2}{3}}+\psi^{e}\ell_j^{e\frac{2}{3}})$.*
*By direct computation, the ($K-$conditional) equilibrium wages are $w^s(\delta^s;K)=\frac{2\psi^s}{3}\left(\frac{K}{\delta^s}\right)^{\frac{1}{3}}$ where, as mentioned above, $w^e(\delta^e;K)$ does not depend upon $\delta^{ne}$ (and similarly for $w^e(\delta^e;K)$ and $\delta^{ne}$).*
*Due to constant returns to scale, expected profits at the equilibrium must be zero for each firm. Replace the ($k_j-$conditional) labor demand functions into the condition for expected profit maximization:*

$$\frac{\partial E(\Pi_j(.))}{\partial k_j}=\frac{\int_{\delta^t}^{\overline{d}}\frac{\psi^e}{3}\left(\frac{2}{3}\frac{\psi^e}{w^e(\delta^e;K)}\right)^2 d\delta^e}{\overline{d}-\delta^t}+\frac{\int_{\underline{d}}^{\delta^t}\frac{\psi^{ne}}{3}\left(\frac{2}{3}\frac{\psi^{ne}}{w^{ne}(\delta^{ne};K)}\right)^2 d\delta^{ne}}{\delta^t-\underline{d}}-1=0,$$

*Replacing into these eqs. the $K-$conditional equilibrium wages and solving, we obtain the optimal values of the investments in physical capital as a function of the threshold:*

$$K(\delta^t)=\left(\frac{\psi^e}{5}\frac{\overline{d}^{\frac{5}{3}}-\delta^{t\frac{5}{3}}}{\overline{d}-\delta^t}+\frac{\psi^{ne}}{5}\frac{\delta^{t\frac{5}{3}}-\underline{d}}{\delta^t-\underline{d}}\right)^{\frac{3}{2}}.$$

*Notice that $K(\delta^t)$ is strictly increasing in $\delta^t$ and that the expectations on wages are rational.*
*Assume that preferences in consumption are linear and that the labor supply is perfectly inelastic.[18] Here, let the random variable $\delta$ be the quantity of efficiency units of labor inelastically supplied. Set $\overline{d}=1$, $\underline{d}=0$, $\psi=\{1,1.2\}$ and $T\equiv$*

[18] The linearity assumption obviously simplifies the computations, but plays no substantive role in the argument. Once we obtain an equilibrium, we can perturb preferences, introducing strict concavity in consumption. The new equilibrium has the same efficiency properties of the original one.

0.047006. *The attainable expected utilities of agent* $\delta$ *are*

$$E\left(u\left(c^{e};\delta\right)\right) = \frac{2.4}{3}\left(\frac{1.2}{5}\frac{1-\delta^{t\frac{5}{3}}}{1-\delta^{t}}+\frac{1}{5}\delta^{t\frac{2}{3}}\right)^{\frac{1}{2}}\delta^{\frac{2}{3}}-0.047006,$$

$$E\left(u\left(c^{ne};\delta\right)\right) = \frac{2}{3}\left(\frac{1.2}{5}\frac{1-\delta^{t\frac{5}{3}}}{1-\delta^{t}}+\frac{1}{5}\delta^{t\frac{2}{3}}\right)^{\frac{1}{2}}\delta^{\frac{2}{3}}.$$

*The equilibrium threshold is obtained setting* $u\left(c^{e};\delta^{t}\right)=u\left(c^{ne};\delta^{t}\right)$, *which holds (approximately) at* $\delta^{t}=0.4$.
*To conclude, we now show that an increase of the threshold, together with the optimal adjustment of investments in physical capital, is Pareto improving.*
*For the inframarginal workers, by direct computation, at* $\delta^{t}=4$,

$$\left(\frac{\partial u\left(c^{ne};\delta\right)}{\partial\delta^{t}}|_{\delta^{t}=4},\frac{\partial u\left(c^{e};\delta\right)}{\partial\delta^{t}}|_{\delta^{t}=4}\right)\sim(0.094,0.103)>>0.$$

*Next, consider the workers at the margin. We need to show that the utility of worker* $\delta^{t}$ *switching from e to ne also increases when the threshold is increased. For an agents in the neighborhood of the threshold* $\delta^{t}=0.4$, *the change in the utility due to a switch from e to ne is*

$$g(\delta,\widehat{\delta};\delta^{t} = 0.4)=\frac{2}{3}\left(\frac{1.2}{5}\frac{1-\widehat{\delta}^{\frac{5}{3}}}{1-\widehat{\delta}}+\frac{1}{5}\widehat{\delta}^{\frac{2}{3}}\right)^{\frac{1}{2}}\delta^{\frac{2}{3}}-\frac{2.4}{3}\left(\frac{1.2}{5}\frac{1-0.4^{\frac{5}{3}}}{1-0.4}+\frac{1}{5}0.4^{\frac{2}{3}}\right)^{\frac{1}{2}}\delta^{\frac{2}{3}}$$
$$+0.047006.$$

*By direct computation, its derivative with respect to* $\left[\widehat{\delta},\delta\right]$ *(evaluated in the direction* $[1,1]$*) is strictly positive at* $\delta^{t}=4$. *Hence, the utility of the marginal workers also increases when the threshold increases and they switch from* $s=e$ *to* $s=ne$. *Hence, the stated intervention entails a Pareto improvement.*
*Finally, it is easy to show that the equilibrium value of the threshold,* $\delta^{t}$, *is an increasing function of* $T$, *the direct cost of education. Hence, to obtain a Pareto superior allocation, it suffices to increase, by some small amount, the value of* $T$ *and to allow the endogenous equilibrium variables to adjust to the new value of* $T$.

The lack of constrained efficiency of equilibria, induced by overeducation, is a fairly general property of this class of economies. In fact, Proposition 8 shows that it occurs if all inputs are Edgeworth complements (*E-complements* in the sequel), i.e., if $\left(\frac{\partial^2 F_j}{\partial\ell_j^{ne}\partial\ell_j^{e}},\frac{\partial^2 F_j}{\partial\ell_j^{ne}\partial k_j},\frac{\partial^2 F_j}{\partial\ell_j^{e}\partial k_j}\right)>>0$, and if $\frac{\partial^2 F_j}{\partial\ell_j^{ne}\partial\ell_j^{e}}$ is sufficiently small. When this is the case, a decrease in the size of the set of agents acquiring high skills leads to a Pareto improvement. Hence, equilibria are characterized by over-education.

Assume that an equilibrium wage map, $\left\{w^{ne}(\delta^{ne},\delta^{e};\delta^{t}),w^{e}(\delta^{ne},\delta^{e};\delta^{t}))\right\}$, exists.

**Proposition 8** *In addition to the maintained assumptions,*
(*i*) $\left(\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e}, \frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial k_j}, \frac{\partial^2 F_j}{\partial \ell_j^{e} \partial k_j}\right) >> 0$ *and* $\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e}$ *is sufficiently small,*
(*ii*) $\overline{\Delta} = \{\overline{\Delta}^{ne}(T), \overline{\Delta}^{e}(T)\} \equiv \left\{[\underline{d}, \delta^t), \left[\delta^t, \overline{d}\right]\right\}$.
*Then, equilibrium allocations are not CPO and characterized by over-education.*

The proof is in Appendix. Evidently, (*i*) is a restriction on the fundamentals, while (*ii*) is a restriction on the equilibrium properties. Hence, this result is of interest only if there are economies fitting this set-up and such that an equilibrium satisfying (*ii*) exists. We do not provide general conditions for that. Equilibria of this type clearly exist for economies sufficiently close to the one of Example 7. One can show that they also exist for several other open sets of economies. Specifically, in Proposition A3 (in Appendix), we establish their existence for a subset of economies satisfying the assumptions of Proposition 8.

# 4 Conclusions

The paper considers the welfare effects of the interaction between self-selection of workers into different labor markets, segmented by skill levels, and investments in physical capital. The key assumption is that firms, when choosing their investments, are uncertain about the characteristics of the labor markets they will actually face when hiring will take place. The distribution of future equilibrium wages and the partition of workers into the distinct labor markets are endogenously determined at the equilibrium. To evaluate welfare, we consider the effects of changes in the investments in physical capital and in the partition of workers across skills. An allocation is constrained Pareto optimal if it cannot be improved upon by changing these variables and letting the other endogenous variables to adjust to restore the (conditional) equilibrium conditions. Assume that workers investments in human capital depend upon the realization of some random variable such as the monetary cost of their investment, $T$. Then, the equilibrium allocation is constrained Pareto optimal if the equilibrium partition is state-contingent. For each realization of $T$, given the predetermined level of the investments in physical capital, the conditional equilibrium is fully Pareto efficient and it can be expressed as the optimal solution to a standard planner's problem. By, essentially, the envelope theorem, we can ignore the second order effects on welfare of changes in the level of the investments in physical capital. It follows that the first order conditions of the firms' expected profit optimization problems coincide with the FOCs of the (ex-ante) planner optimization problem, so that equilibria, if they exist, are always constrained Pareto optimal. This property is the key for constrained Pareto optimality. It requires that the partition of workers is Pareto efficient (conditional on $K$) in each state of the world.

If this property does not hold, constrained Pareto optimality may fail at the equilibrium. To show that, we consider a class of economies with a somewhat similar time and market structure, but without this property. We show that equilibria may be characterized by overinvestment in high skills, meaning that, by restricting the set of workers acquiring high skills, we can actually implement a Pareto improvement.

The paper presents two contributions: first, it gives precise conditions under which, in perfect labor markets, uncertainty on the actual quality of prospective hirings may, or may not, induce constrained inefficiency, and, specifically, inefficient overeducation. As discussed in the introduction, we also believe that this type of results can be of some interest for the literature on taxation in Roy's economies.

# 5 Appendix

**Proof of Lemma 1.** $(i)$ By the implicit function theorem (*IFT*),

$$\frac{\partial \widetilde{h}^s(.)}{\partial w^s} = -\frac{\frac{\partial v(.)}{\partial c^s} + \frac{\partial^2 v(.)}{\partial c^{s2}} w^s \widetilde{h}^s}{\frac{\partial^2 v(.)}{\partial c^{s2}} w^{s2} - \frac{1}{\delta}\frac{\partial^2 g(.)}{\partial h^{s2}}} = \frac{\partial v(.)}{\partial c^s} \frac{1 + \left(\frac{\partial^2 v(.)}{\partial c^{s2}} / \frac{\partial v(.)}{\partial c^s}\right) w^s \widetilde{h}^s}{\frac{1}{\delta}\frac{\partial^2 g(.)}{\partial h^{s2}} - \frac{\partial^2 v(.)}{\partial c^{s2}} w^{s2}}.$$

By concavity of $v(.)$ and strict convexity of $g(.)$, the denominator is strictly positive. For $s = ne$, if $\left(\frac{\partial^2 v(.)}{\partial c^{ne2}} / \frac{\partial v(.)}{\partial c^{ne}}\right) w^{ne} \widetilde{h}^{ne} > -1$, the numerator is strictly positive. For $s = e$, the same measure must be sufficiently greater than $-1$, so that $\left(\frac{\partial^2 v(.)}{\partial c^{e2}} / \frac{\partial v(.)}{\partial c^e}\right) |_{c^e} w^e \widetilde{h}^e \equiv \left(\frac{\partial^2 v(.)}{\partial c^{e2}} / \frac{\partial v(.)}{\partial c^e}\right) |_{c^e} (c^e + T) > -1$.

$(ii)$ By the *IFT*, $\frac{\partial \widetilde{h}^e(.)}{\partial T} = \frac{-\frac{\partial^2 v(.)}{\partial c^{s2}} w^s}{\frac{1}{\delta}\frac{\partial^2 g(.)}{\partial h^{s2}} - \frac{\partial^2 v(.)}{\partial c^{s2}} w^{s2}} > 0$.

$(iii)$ At $w^e = w^{ne}$ and $T = 0$, $\widetilde{h}^e(w^e, T = 0; \delta) = \widetilde{h}^{ne}(w^{ne}; \delta)$. Since $\frac{\partial \widetilde{h}^e(.)}{\partial T} > 0$, and $\frac{\partial \widetilde{h}^e(.)}{\partial w^e} > 0$, $\widetilde{h}^e(w^e, T; \delta) > \widetilde{h}^{ne}(w^{ne}; \delta)$ at each $(w^e, T) >> (w^{ne}, 0)$. Define $G(.) \equiv u(\widetilde{c}^e(.), \widetilde{h}^e(.); \delta, T) - u(\widetilde{c}^{ne}(.), \widetilde{h}^{ne}(.); \delta)$. By the envelope thm., $\frac{\partial G(.)}{\partial \delta} \equiv \frac{1}{\delta^2}\left[g(\widetilde{h}^e) - g(\widetilde{h}^{ne})\right]$. Since $\widetilde{h}^e(w^e; T, \delta) > \widetilde{h}^{ne}(w^{ne}; \delta)$ (clearly, here we are considering the notional labor supplies), $\frac{\partial G(.)}{\partial \delta} > 0$. ■

**Proof of Lemma 5.** Consider any equilibrium allocation contingent on the profile of investments in physical capital. Fix $k_j = \overline{K} > 0$, for each $j$. Let $\chi(\overline{K}, T)$ denote any $T$-contingent allocation, $\{\Delta(.), (c^s(.), h^s(.))\}$.

First, let's show that $\Delta(\overline{K}, T) \equiv \{[\underline{d}, \delta(\overline{K}, T)), [\delta(\overline{K}, T), \overline{d}]\} \times [0, 1]$. Consider the $\overline{K}-$contingent equilibrium allocation. If $\Delta^e(\overline{K}, T) = \emptyset$, we can set $\delta(\overline{K}, T) = \underline{d}$ and there is nothing to prove. Similarly if $\Delta^{ne}(\overline{K}, T) = \emptyset$. Hence, assume that $\Delta^e(T, \overline{K}) \neq \emptyset$ and $\Delta^{ne}(T, \overline{K}) \neq \emptyset$. By Lemma 1, at each $\delta$,

$$\frac{\partial u(\widetilde{c}^e(.), \widetilde{h}^e(.), \delta; T)}{\partial \delta} - \frac{\partial u(\widetilde{c}^{ne}(.), \widetilde{h}^{ne}(.), \delta; T)}{\partial \delta} = \frac{1}{\delta^2}\left[g(\widetilde{h}^e) - g(\widetilde{h}^{ne})\right] > 0,$$

where "$\widetilde{.}$" denotes notional consumption and labor supply. In particular, this inequality holds at each interior threshold where $u(\widetilde{c}^e(.), \widetilde{h}^e(.), \overline{\delta}; T) = u(\widetilde{c}^{ne}(.), \widetilde{h}^{ne}(.), \overline{\delta})$. Its uniqueness follows immediately. ■

**Proof of Proposition 6.** We proceed by contradiction, exploiting the equivalence between equilibria of the actual economy and equilibria of an economy with an additional class of risk-neutral agents, rentiers, owning the firms. Consider an equilibrium allocation $\overline{\chi}$ with measurable partition $\overline{\Delta}(T)$. Assume that there exists another $\widehat{K}-$conditional equilibrium $\{\widehat{w}^{ne}(.), \widehat{w}^{e}(.)\}$ with allocation $\widehat{\chi}$ and measurable partition $\widehat{\Delta}(T)$ which Pareto dominates $\overline{\chi}$ in the artificial economy with rentiers, i.e., such that $E\left(u(\widehat{c}^s(.), \widehat{h}^s(.); \delta)\right) \geq E\left(u(\overline{c}^s(.), \overline{h}^s(.); \delta)\right)$ *a.e.,* with strict inequality for some positive measure subset of agents and such that a similar property holds for the set of rentiers (which means that expected total profits associated with $\left(\widehat{\chi}, \widehat{\Delta}(T)\right)$ are at least as large as the ones associated with $\left(\overline{\chi}, \overline{\Delta}(T)\right)$. In view of Lemma 4, for each realization $T$, both partitions are defined by a threshold, $\overline{\delta}^t$ and $\widehat{\delta}^t$, respectively.

First, assume that $\widehat{\delta}^t \geq \overline{\delta}^t$. Consider the measurable set of agents choosing $s = e$ at both allocations. Then, $u(\widehat{c}^e(.), \widehat{h}^e(.); \delta) \geq u(\overline{c}^e(.), \overline{h}^e(.); \delta)$ implies $\widehat{c}^e(.) + T \geq \overline{w}^e \widehat{h}^e(.)$ *a.e.,* with strict inequality for the agents such that the first inequality holds strictly. Similarly, for the agents choosing $s = ne$ at each allocation.

Consider now the set of agents such that $\overline{h}^{ne}(.) = 0$, while $\widehat{h}^e(.) = 0$, i.e., the one switching from $s = e$ to $s = ne$. Still, we must have $\widehat{c}^{ne}(.) \geq \overline{w}^{ne}\widehat{h}^{ne}(.)$, because $u(\widehat{c}^{ne}(.), \widehat{h}^{ne}(.); \delta) \geq u(\overline{c}^e(.), \overline{h}^e(.); \delta) \geq u(\overline{c}^{ne}(.), \overline{h}^{ne}(.); \delta)$.

Finally, for the (risk-neutral) rentiers, denoted $j$, it must be

$$\int_0^1 \widehat{c}_j(.)dj \geq \int_0^1 F_j(\widehat{k}_j, \widehat{\ell}_j^{ne}, \widehat{\ell}_j^e)dj - \int_0^1 \widehat{k}_j dj - \int_0^1 \overline{w}^{ne}\widehat{\ell}_j^{ne} dj - \int_0^1 \overline{w}^e \widehat{\ell}_j^e dj.$$

Integrating over the set of agents, we obtain

$$\int_{\underline{d}}^{\widehat{\delta}^t} \int_0^1 \widehat{c}^{ne}(.) di d\delta^{ne} + \int_{\widehat{\delta}^t}^{\overline{d}} \int_0^1 \widehat{c}^e(.) di d\delta^e + \int_{\widehat{\delta}^t}^{\overline{d}} \int_0^1 T di d\delta^e + \int_0^1 \widehat{c}_j(.)dj$$

$$> \int_0^1 F_j(\widehat{k}_j, \widehat{\ell}_j^{ne}, \widehat{\ell}_j^e)dj - \int_0^1 \widehat{k}_j dj,$$

which implies that the allocation $\widehat{\chi}$ is not feasible.

The argument for $\widehat{\delta}^t < \overline{\delta}^t$ is symmetric.

Hence, the equilibrium allocation is CPO. ■

**Appendix to Subsection 3.2.**

**Proof of Proposition 8.** We will show that we can implement a Pareto improvement by imposing small increases in the values of the equilibrium threshold $\delta^t$ and of the investments in physical capital.

The expected utility of a generic, inframarginal high skilled worker with $\delta = \widehat{\delta}^e > \delta^t$ is

$$E(V^e(w^e(\delta^{ne}, \widehat{\delta}^e; K); \widehat{\delta}^e)) = \frac{\int_{\underline{d}}^{\delta^t} V^e(w^e(\delta^{ne}, \widehat{\delta}^e; K); \widehat{\delta}^e) d\delta^{ne}}{\delta^t - \underline{d}},$$

so that

$$\frac{\partial E\left(V^{e}\left(.\right)\right)}{\partial \delta^{t}}=\frac{V^{e}(w^{e}(\delta^{ne}=\delta^{t},\widehat{\delta}^{e};K);\widehat{\delta}^{e})-E(V^{e}(w^{e}(\delta^{ne},\widehat{\delta}^{e};K);\widehat{\delta}^{e}))}{\delta^{t}-\underline{d}}$$

and

$$\frac{\partial E\left(V^{e}\left(.\right)\right)}{\partial K}=\frac{\int_{\underline{d}}^{\delta^{t}}\frac{\partial V^{e}(.)}{\partial w^{e}}\frac{\partial w^{e}}{\partial K}d\delta^{ne}}{\delta^{t}-\underline{d}}.$$

Similarly, the expected utility of a generic, inframarginal low skilled worker with $\delta=\widehat{\delta}^{ne}<\delta^{t}$ is

$$E(V^{ne}(w^{ne}(\widehat{\delta}^{ne},\overline{\delta}^{e};K);\widehat{\delta}^{ne}))=\frac{\int_{\delta^{t}}^{\overline{d}}V^{ne}(w^{ne}(\widehat{\delta}^{ne},\delta^{e};K);\widehat{\delta}^{ne})d\delta^{e}}{\overline{d}-\delta^{t}},$$

and thus

$$\frac{\partial E(V^{ne}(.))}{\partial \delta^{t}}=\frac{E(V^{ne}(w^{ne}(.);\widehat{\delta}^{ne}))-V^{ne}(w^{ne}(\widehat{\delta}^{ne},\delta^{e};K);\widehat{\delta}^{ne})}{\overline{d}-\delta^{t}}$$

and

$$\frac{\partial E(V^{ne}(.))}{\partial K}=\frac{\int_{\delta^{t}}^{\overline{d}}\frac{\partial V^{ne}(.)}{\partial w^{ne}}\frac{\partial w^{ne}(.)}{\partial K}d\delta^{e}}{\overline{d}-\delta^{t}}$$

Lemma A2 below establishes that, under the maintained assumptions,

$$\left(\frac{\partial E(V^{e}(.))}{\partial \delta^{t}},\frac{\partial E(V^{ne}(.))}{\partial \delta^{t}}\right)>>0,\ \text{for each } s.$$

Therefore, for all the inframarginal workers, an increase in the threshold increases expected utility.

Let's now consider the marginal worker. Since we are just considering increases of the threshold, this is the one switching from $s=e$ to $s=ne$. We must show that their expected utility increases, too. By definition of threshold, for the agent with $\delta=\delta^{t}$, $E(V^{ne}(w^{ne}(.);\delta)=E(V^{e}(w^{e}(.);\delta)$. Therefore, the direct effect of the switch is zero. On the other hand, as explained in Lemma A1, we can pick a $dK>0$ so that $\frac{\int_{\delta^{t}}^{\overline{d}}\frac{\partial V^{ne}(.)}{\partial w^{ne}}\frac{\partial w^{ne}(.)}{\partial K}d\delta^{e}}{\overline{d}-\delta^{t}}dK>0$. Hence, the total impact of the increase in the value of $\delta^{t}$ is positive for the agent with $\delta=\delta^{t}$. By continuity, there is some open neighborhood of $\delta^{t}$ such that, for each agent in this nbd, the expected utility also increases when $\delta^{t}$ increases.

Finally, in Lemma A2 we also show that we can choose $dK>0$ such that, at the new $\left(K',\delta^{t\prime}\right)$ −conditional equilibrium, expected profits are non-negative.

Hence, an increase in the pair $\left(K,\delta^{t}\right)$ is feasible and entails a Pareto improvement. ■

The proof is based on two steps. We first show that $\left(K,\delta^{t}\right)$ −conditional equilibrium wages are continuously differentiable functions of $\left(K,\delta^{t}\right)$ and establish some of their properties properties. Next, we show that, given an equilibrium, we can select an appropriate change in the pair $\left(K,\delta^{t}\right)$ such that the new $\left(K,\delta^{t}\right)$ −conditional equilibrium entails a Pareto improvement.

**Lemma A1** Under the maintained assumptions, and if $\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e}$ is sufficiently small, at each $(K, \delta^t)$ −conditional equilibrium, $\left(\frac{\partial w^{ne}}{\partial \delta^e}, \frac{\partial w^e}{\partial \delta^{ne}}, \frac{\partial w^{ne}}{\partial K}, \frac{\partial w^e}{\partial K}\right) >> 0$.

**Proof of Lemma A1.** First, consider the matrix $D_{(w,k_j)}\ell_j$. By the FOCs of firm $j$'s (ex-post) optimization problem and the *IFT*,

$$D_{(w^{ne},w^e,k_j)}\ell_j = \begin{bmatrix} \frac{\partial \ell_j^{ne}}{\partial w^{ne}} & \frac{\partial \ell_j^{ne}}{\partial w^e} & \frac{\partial \ell_j^{ne}}{\partial k_j} \\ \frac{\partial \ell_j^e}{\partial w^{ne}} & \frac{\partial \ell_j^e}{\partial w^e} & \frac{\partial \ell_j^e}{\partial k_j} \end{bmatrix} = \frac{1}{\frac{\partial^2 F_j}{\partial \ell_j^{ne2}} \frac{\partial^2 F_j}{\partial \ell_j^{e2}} - \left(\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e}\right)^2}$$
$$\times \begin{bmatrix} \frac{\partial^2 F_j}{\partial \ell_j^{e2}} & -\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e} & -\frac{\partial^2 F_j}{\partial \ell_j^{e2}} \frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial k_j} + \frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e} \frac{\partial^2 F_j}{\partial \ell_j^e \partial k_j} \\ -\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e} & \frac{\partial^2 F_j}{\partial \ell_j^{ne2}} & -\frac{\partial^2 F_j}{\partial \ell_j^{ne2}} \frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial k_j} + \frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e} \frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial k_j} \end{bmatrix}.$$

Concavity of $F_j(.)$ implies that $\left[\frac{\partial^2 F_j}{\partial \ell_j^{ne2}} \frac{\partial^2 F_j}{\partial \ell_j^{e2}} - \left(\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e}\right)^2\right] > 0$. By assumption, $\left(\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e}, \frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial k_j}, \frac{\partial^2 F_j}{\partial \ell_j^e \partial k_j}\right) >> 0$. Therefore, the coefficients of the matrix $D_{(w^{ne},w^e,k_j)}\ell$ have the pattern of signs $\begin{bmatrix} - & - & + \\ - & - & + \end{bmatrix}$.

Consider now the conditional equilibria. For a given pair $\{\delta^{ne}, \delta^e\} \in \{\Delta^{ne}, \Delta^e\}$, the $(K, \delta^t)$ −conditional equilibrium is described by the equations:

$$\Psi(w^{ne}, w^e, \delta^{ne}, \delta^e; K) \equiv \begin{bmatrix} L^{ne}(w^{ne}, w^e) - H^{ne}(w^{ne}; \delta^{ne}) \\ L^e(w^{ne}, w^e; K) - H^e(w^e; \delta^e) \end{bmatrix} = 0,$$

where, for economy of notation, we use capital letters to denote the integral of the labor demand ($L^s(.)$) and supply ($H^s(.)$) over the intervals $[0,1]$ of identical agents. Also, bear in mind that in computing derivatives with respect to $K$, we are implicitly taking the derivative with respect to each $k_j$, with $k_j = K$ for each $j$. By the *IFT*,

$$[D_{(\delta,K)}w] \equiv \begin{bmatrix} D_{(\delta^{ne},\delta^e,K)}w^{ne} \\ D_{(\delta^{ne},\delta^e,K)}w^e \end{bmatrix}$$
$$= -\begin{bmatrix} \frac{-\frac{\partial L^e}{\partial w^e} - \frac{\partial H^e}{\partial w^e}}{\det D_{(w^{ne},w^e)}\Psi(.)} & \frac{-\frac{\partial L^{ne}}{\partial w^e}}{\det D_{(w^{ne},w^e)}\Psi(.)} \\ \frac{-\frac{\partial L^e}{\partial w^{ne}}}{\det D_{(w^{ne},w^e)}\Psi(.)} & \frac{-\frac{\partial L^{ne}}{\partial w^{ne}} - \frac{\partial H^{ne}}{\partial w^{ne}}}{\det D_{(w^{ne},w^e)}\Psi(.)} \end{bmatrix} \begin{bmatrix} -\frac{\partial H^{ne}}{\partial \delta^{ne}} & 0 & \frac{\partial L^{ne}}{\partial k_j} \\ 0 & -\frac{\partial H^e}{\partial \delta^e} & \frac{\partial L^e}{\partial k_j} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\left(\frac{\partial L^e}{\partial w^e} - \frac{\partial H^e}{\partial w^e}\right)\frac{\partial H^{ne}}{\partial \delta^{ne}}}{\det D_{(.)}\Psi(.)} & \frac{-\frac{\partial L^{ne}}{\partial w^e}\frac{\partial H^e}{\partial \delta^e}}{\det D_{(.)}\Psi(.)} & \frac{\left(-\frac{\partial L^e}{\partial w^e} + \frac{\partial H^e}{\partial w^e}\right)\frac{\partial L^{ne}}{\partial k_j} + \frac{\partial L^{ne}}{\partial w^e}\frac{\partial L^e}{\partial k_j}}{\det D_{(w^{ne},w^e)}\Psi(.)} \\ \frac{-\frac{\partial L^e}{\partial w^{ne}}\frac{\partial H^{ne}}{\partial \delta^{ne}}}{\det D_{(.)}\Psi(.)} & \frac{\left(\frac{\partial L^{ne}}{\partial w^{ne}} - \frac{\partial H^{ne}}{\partial w^{ne}}\right)\frac{\partial H^e}{\partial \delta^e}}{\det D_{(.)}\Psi(.)} & \frac{\left(-\frac{\partial L^{ne}}{\partial w^{ne}} + \frac{\partial H^{ne}}{\partial w^{ne}}\right)\frac{\partial L^e}{\partial k_j} + \frac{\partial L^e}{\partial w^{ne}}\frac{\partial L^{ne}}{\partial k_j}}{\det D_{(w^{ne},w^e)}\Psi(.)} \end{bmatrix}.$$

Under the maintained assumptions, for for $|\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e}|$ (hence, $\frac{\partial L^s}{\partial w^{s'}}$) sufficiently small, the pattern of signs of this matrix is

$$\begin{bmatrix} - & + & + \\ + & - & + \end{bmatrix}.$$

Its determinant is

$$\left[\frac{\partial L^{ne}}{\partial w^{ne}}\frac{\partial L^e}{\partial w^e} - \frac{\partial L^{ne}}{\partial w^e}\frac{\partial L^e}{\partial w^{ne}}\right] - \left[\frac{\partial L^{ne}}{\partial w^{ne}}\frac{\partial H^e}{\partial w^e} + \frac{\partial L^e}{\partial w^e}\frac{\partial H^{ne}}{\partial w^{ne}} - \frac{\partial H^{ne}}{\partial w^{ne}}\frac{\partial H^e}{\partial w^e}\right],$$

which is positive, because the first term in square brackets is positive (it is the determinant of the negative definite square matrix $D_w L(.)$), while the second term is negative.

The signs of the coefficients then follows by $\left(\frac{\partial h_j^{ne}}{\partial w^{ne}}, \frac{\partial h_j^e}{\partial w^e}\right) >> (0,0)$, that we have shown in Lemma 1, by the signs of $D_{(w^{ne},w^e,k_j)}\ell_j$, established above, and by the assumption that $|\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e}|$ (hence, $\frac{\partial L^{ne}}{\partial w^e}$) is sufficiently small. ■

**Lemma A2** Under the assumption of Proposition 8, at each equilibrium, $\left(\frac{\partial E(V^e)}{\partial \delta^t}, \frac{\partial E(V^{ne})}{\partial \delta^t}, \frac{\partial E(\Pi_j)}{\partial \delta^t}\right) >> 0$.

**Proof of Lemma A2.** In this proof we will exploit the properties of $(K, \delta^t)$ −conditional equilibria and use the pair $(K, \delta^t)$ as policy instruments to attain a Pareto improvement.

First, by Lemma A1, $w^s(\delta^s, \delta^{s'}, K)$ is strictly increasing in $\delta^{s'}$, for each $K$. This immediately implies

$$\begin{bmatrix} \frac{V^e(w^e(\delta^t,\widehat{\delta}^e;K)) - E(V^e(w^e(\delta^{ne},\widehat{\delta}^e;K)))}{\overline{d}-\delta^t} \\ \\ \frac{V^{ne}(w^{ne}(\widehat{\delta}^{ne},\delta^t;K)) - E(V^{ne}(w^{ne}(\widehat{\delta}^{ne},\delta^e;K))}{\delta^t - \underline{d}} \end{bmatrix} >> 0.$$

Next, we claim that, at each conditional equilibrium, the indirect effect of an increase of $\delta^t$ on the expected utilities of the inframarginal agents is also positive, i.e., that

$$\left[\frac{\int_{\underline{d}}^{\delta^t} \frac{\partial V^e(.)}{\partial w^e}\frac{\partial w^e}{\partial K} d\delta^{ne}}{\delta^t - \underline{d}}, \frac{\int_{\delta^t}^{\overline{d}} \frac{\partial V^{ne}(.)}{\partial w^{ne}}\frac{\partial w^{ne}(.)}{\partial K} d\delta^e}{\overline{d} - \delta^t}\right] >> 0.$$

Obviously, $\frac{\partial V^s(.)}{\partial w^s} > 0$ for each $s$. Under the maintained assumptions, by Lemma A1, $\frac{\partial w^s}{\partial K} > 0$, for each $s$. Hence, to establish our claim, it suffices to show that, given the increase in $\delta^t$, we can also increase the value of $K$ so that expected profit, computed at the new conditional equilibrium, are nonnegative.

Let $\Pi(\delta^{ne}, \delta^e; K)$ be the producers' surplus in state $(\delta^{ne}, \delta^e)$. Then,

$$E(\Pi(\delta^{ne}, \delta^e; K), \delta^t) \equiv \frac{1}{\delta^t - \underline{d}} \int_{\underline{d}}^{\delta^t} \left(\frac{\int_{\delta^t}^{\overline{d}} \Pi(\delta^{ne}, \delta^e; K) d\delta^e}{\overline{d} - \delta^t}\right) d\delta^{ne}.$$

Because of constant return to scale, given $\delta^t$, at each equilibrium, $E\left(\Pi\left(\delta^{ne}, \delta^e; K\right), \delta^t\right) = 0$. Consider its derivative with respect to the threshold $\delta^t$ :

$$\frac{\partial E(\Pi(\delta^{ne}, \delta^e; K), \delta^t))}{\partial \delta^t} = \frac{\frac{\int_{\delta^t}^{\overline{d}} \Pi(\delta^{ne}=\delta^t, \delta^e; K) d\delta^e}{\overline{d}-\delta^t} - E(\Pi(\delta^{ne}, \delta^e; K), \delta^t)}{\delta^t - \underline{d}}$$
$$-\frac{1}{\delta^t - \underline{d}} \int_{\underline{d}}^{\delta^t} \frac{\Pi(\delta^{ne}, \delta^e = \delta^t; K) - \frac{\int_{\delta^t}^{\overline{d}} \Pi(\delta^{ne}, \delta^e; K) d\delta^e}{\overline{d}-\delta^t}}{\overline{d} - \delta^t} d\delta^{ne}.$$

Given $\left(K, \delta^t\right)$, using the results of Lemma A1 and the assumption that $|\frac{\partial^2 F_j}{\partial \ell_j^{ne} \partial \ell_j^e}|$ is sufficiently small,

$$\begin{aligned} \frac{\partial \Pi(\delta^{ne}, \delta^e; K)}{\partial \delta^{ne}} &= -\frac{\partial w^{ne}}{\partial \delta^{ne}} L^{ne}(.) + \frac{\partial w^e}{\partial \delta^{ne}} L^e(.) \\ &= \frac{\left[\frac{\partial H^e}{\partial w^e} L^{ne}(.) + \frac{\partial L^e}{\partial w^{ne}} L^e(.) - \frac{\partial L^e}{\partial w^e} L^{ne}(.)\right]}{\det D_{(w^{ne}, w^e)} \Psi(.)} \frac{\partial H^{ne}}{\partial \delta^{ne}} > 0. \end{aligned}$$

Since $E(\Pi(\delta^{ne}, \delta^e; K), \delta^t) = 0$, this implies $\frac{\int_{\delta^t}^{\overline{d}} \Pi(\delta^{ne}=\delta^t, \delta^e; K) d\delta^e}{\overline{d}-\delta^t} > 0$. Essentially the same argument implies that $\int_{\underline{d}}^{\delta^t} \frac{\Pi(\delta^{ne}, \delta^e=\delta^t; K) - \frac{\int_{\delta^t}^{\overline{d}} \Pi(\delta^{ne}, \delta^e; K) d\delta^e}{\overline{d}-\delta^t}}{\overline{d}-\delta^t} d\delta^{ne} < 0$.

Hence, $\frac{\partial E(\Pi(\delta^{ne}, \delta^e; K), \delta^t)}{\partial \delta^t} > 0$ and, for each $dK$ sufficiently small,

$$\frac{\partial E(\Pi(\delta^{ne}, \delta^e; K), \delta^t)}{\partial \delta^t} + \frac{\partial E(\Pi(\delta^{ne}, \delta^e; K), \delta^t)}{\partial K} > 0. \blacksquare$$

An heuristic existence argument is summarized in the following two figures. Figure A2 shows the expected marginal product of capital, $E(MP_{k_j}(.))$, for a typical firm, as a function of *aggregate* investments, given several arbitrary values of the threshold, $\{\delta^1, \delta^2, \delta^3\}$. These curves are obtained exploiting the labor market clearing conditions in the different islands. Since they are decreasing, the individual firm's profit maximizing condition identifies the unique levels of the equilibrium aggregate investments associated with the different thresholds, so that $K(\delta^t)$ is a well-defined, increasing, function.

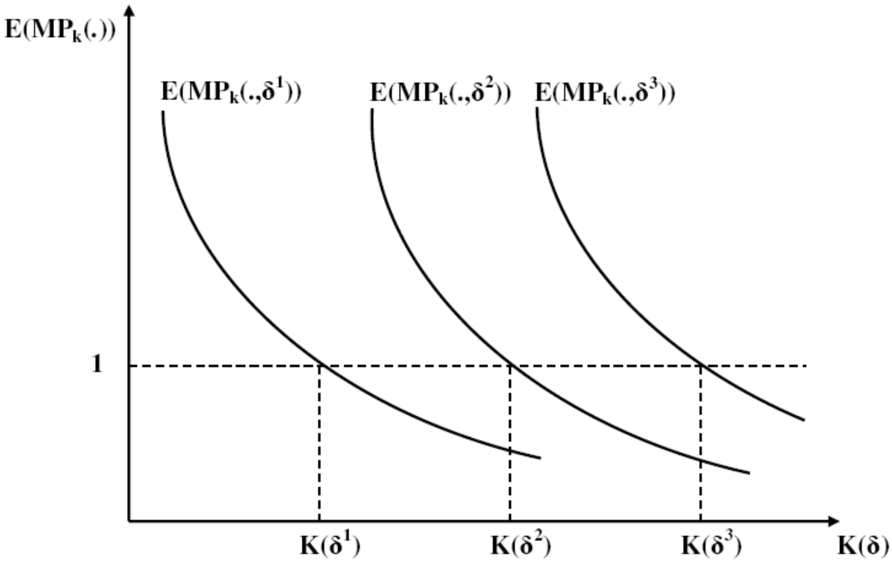


Figure A2

Figure A3 shows the difference between the expected utilities of an individual with innate ability $\delta$ with high or low skills. Each curve is associated with a different level of the threshold and takes into account the market clearing wages corresponding to the value of aggregate investments $K(\delta^t)$. The curves are drawn assuming $T = 0$. Consider, for instance, the one associated with $\delta^1$. We can clearly pick an appropriate value of $T$, $T^1$, such that $E(V^e(w^e(.); T = T^1); \delta^1) - E(V^{ne}(w^{ne}(.); \delta^1)) = 0$. As long as this function is increasing in $\delta$, it is easy to see that $\delta^1$, with associated $K(\delta^1)$, defines an equilibrium given $T^1$.

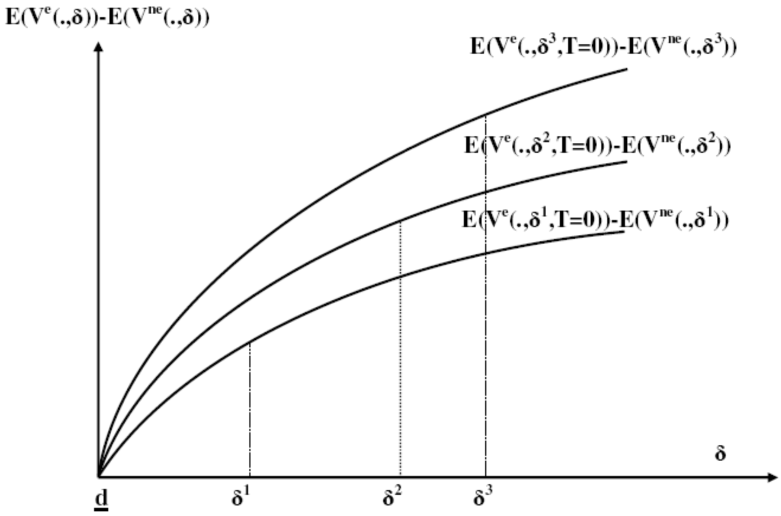


Figure A3

**Proposition A3:** In addition to the maintained assumptions, let all the inputs be *E-complements* and let $F_j(.) \equiv k_j^{\alpha}\left[\psi^{ne}\ell_j^{ne\theta} + \psi^{e}\ell_j^{e\theta}\right]^{\frac{1-\alpha}{\theta}}$, $\theta > 0$, $\alpha \in (0,1)$ for each $j$. Also, assume that the labor supply is perfectly inelastic and that $|\frac{\partial^2 F_j}{\partial \ell_j^{ne}\partial \ell_j^{e}}|$ is sufficiently small. Then, there exists an open set of economies such that an equilibrium exists.

**Proof of Proposition A3.** Here, it is crucial to distinguish between investments of an individual firm, $k_j$, and average aggregate investments, $K$. Since at each equilibrium, $k_j = K$, we will just consider pairs satisfying this restriction. However, keep in mind that changes in $k_j$, for some $j$, do not have any effect on the endogenous variables (but the ones referred to firm $j$).

We start assuming that, at the equilibrium, workers are partitioned across skill according to some $\overline{\Delta} \equiv \left\{[\underline{d}, \delta^t), \left[\delta^t, \overline{d}\right]\right\}$. Later, we will show that this is actually the case.

Let $w(\delta^{ne}, \delta^{e}; K, \delta^t) \equiv \left\{w^{e}(\delta^{ne}, \delta^{e}; K, \delta^t), w^{ne}(\delta^{ne}, \delta^{e}; K, \delta^t)\right\}$ be the $\left(K, \delta^t\right)$ $-$conditional equilibrium pair in the sub-economy identified by $\left\{\delta^{ne}, \delta^{e}\right\}$. Under the maintained assumptions, it is easy to check that $w(\delta^{ne}, \delta^{e}; K, \delta^t)$ is a $C^1$ function of $(\delta^{ne}, \delta^{e}; K)$. Moreover, since all inputs are *E-complements,* $\nabla_K w >> 0$ at each $(\delta^{ne}, \delta^{e})$, as established in Lemma A1.

Given $\delta^t$, let $E(\Pi_j(w, k_j; K), \delta^t)$ be the expected profits, which depends directly on the investments of the single firm, $k_j$, and, indirectly, on the aggregate investments, $K$, because of their effects on the equilibrium wage map $w(\delta^{ne}, \delta^{e}; K, \delta^t)$.

At the market clearing wages, by the envelope theorem, $\frac{\partial E(\Pi_j(w,k_j;K,\delta^t))}{\partial k_j}|_{k_j=K} =$

$\frac{\partial E(F_j(k_j,\delta^{ne},\delta^e))}{\partial k_j} - 1$. Since, at each conditional equilibrium, labor markets clear,

$$\frac{\partial E(F_j(.))}{\partial k_j} - 1 = \frac{1}{\delta^t - \underline{d}} \int_{\underline{d}}^{\delta^t} \left( \frac{\int_{\delta^t}^{\overline{d}} \frac{\partial F_j(k_j,\delta^{ne},\delta^e)}{\partial k_j} d\delta^e}{\overline{d} - \delta^t} \right) d\delta^{ne} - 1.$$

Under the maintained assumptions on $F_j(.)$, and given that labor is inelastically supplied,

$$\lim_{k_j \to 0} \frac{1}{\frac{\partial E(F_j(.))}{\partial k_j}} = 0, \quad \text{and} \quad \lim_{k_j \to \infty} \frac{\partial E(F_j(.))}{\partial k_j} = 0.$$

Hence, for each $\delta^t \in (\underline{d}, \overline{d})$, there is a $k_j$ such that $\frac{\partial E(F(.))}{\partial k_j} = 1$. As returns to scale are constant, $E(\Pi_j(w, k_j; K, \delta^t))$ is a linear function of $k_j$. Therefore, $\frac{\partial E(F_j(.))}{\partial k_j}$ just depends upon aggregate investments, $K$, because of their effects on equilibrium wages. Since $w^s(\delta^{ne}, \delta^e; K, \delta^t)$ is increasing in $K$, for each $s$,

$$\frac{\partial^2 E(\Pi_j(.))}{\partial k_j \partial K} = \frac{\partial^2 E(\Pi_j(.))}{\partial k_j \partial w^e} \frac{\partial w^e}{\partial K} + \frac{\partial^2 E(\Pi_j(.))}{\partial k_j \partial w^{ne}} \frac{\partial w^{ne}}{\partial K} < 0.$$

Hence, for each $\delta^t$, there is, at most, a unique value $K(\delta^t)$ such that $\frac{\partial E(F_j(.))}{\partial k_j} = 1$. This implies that $K(\delta^t)$ is a function on its domain of definition. By the *IFT* applied to the eq. $\frac{\partial E(F_j(.))}{\partial k_j} = 1$, $\frac{\partial K(\delta^t)}{\partial \delta^t} = -\frac{\frac{\partial^2 E(F_j(.))}{\partial k_j \partial \delta^t}}{\frac{\partial^2 E(F_j(.))}{\partial k_j \partial K}}$. By direct computation,

$$\frac{\partial^2 E(F_j(.))}{\partial k_j \partial \delta^t} = \frac{\frac{\int_{\delta^t}^{\overline{d}} \frac{\partial F(k_j,\delta^{ne}=\delta^t,\delta^e)}{\partial k_j} d\delta^e}{\overline{d}-\delta^t} - \frac{\partial E(F_j(.))}{\partial k_j}}{\delta^t - \underline{d}} - \frac{\frac{\int_{\underline{d}}^{\delta^t} \frac{\partial F_j(k_j,\delta^{ne},\delta^e=\delta^t)}{\partial k_j} d\delta^{ne}}{\delta^t - \underline{d}} - \frac{\partial E(F_j(.))}{\partial k_j}}{\overline{d} - \delta^t} > 0.$$

This inequality holds because, under *E-complementarity*, $\frac{\partial^2 F_j(k_j,\delta^{ne}=\delta^t,\delta^e)}{\partial k_j \partial \ell^s} > 0$, for each $s$, which implies

$$\frac{\int_{\delta^t}^{\overline{d}} \frac{\partial F_j(k_j,\delta^{ne}=\delta^t,\delta^e)}{\partial k_j} d\delta^e}{\overline{d} - \delta^t} > \frac{\partial E(F_j(.))}{\partial k_j},$$

and

$$\int_{\underline{d}}^{\delta^t} \frac{\partial F_j(k_j, \delta^{ne}, \delta^e = \delta^t)}{\partial k_j} d\delta^e < \frac{\partial E(F_j(.))}{\partial k_j}.$$

As already established, $\frac{\partial^2 E(F_j(.))}{\partial k_j \partial K} < 0$. Thus, $\frac{\partial K(\delta^t)}{\partial \delta^t} > 0$. These results basically translate into Figure A2 above.

Consider any $\delta^t$−conditional equilibrium. The strategy of our proof is to construct explicitly an equilibrium for an arbitrarily selected value of $T$. Define the map $M(T) : \mathbb{R}_+ \to \mathbb{R}$,

$$M(T) = E(u(\delta^e = \delta^t, w^e(\delta^{ne}, K(\delta^t)); \delta^t)) - E(u(w^{ne}(\delta^{ne} = \delta^t, \delta^e, K(\delta^t)); \delta^t)).$$

Given the production function specified above, for each $K>0$, $w^e(\delta^{ne},\delta^t,K(\delta^t))\geq w^e(\underline{d},\delta^t,K(\delta^t))$ and $w^{ne}(\delta^t,\overline{d},K(\delta^t))\geq w^{ne}(\delta^t,\delta^e,K(\delta^t))$. Clearly,

$$\frac{w^e(\underline{d},\delta^t,K(\delta^t))\delta^t}{w^{ne}(\delta^t,\overline{d},K(\delta^t))\delta^t}=\left(\frac{\psi^{ne}\underline{d}^{\theta}+\psi^e\delta^{t\theta}}{\psi^{ne}\delta^{t\theta}+\psi^e\overline{d}^{\theta}}\right)^{\frac{1-\alpha-\theta}{\theta}}\frac{\psi^e}{\psi^{ne}}>1 \qquad (B)$$

for $\frac{\psi^e}{\psi^{ne}}$ large enough. Therefore, for appropriate values of the parameters, $w^e(\delta^{ne},\delta^t,K(\delta^t))\delta^t>w^{ne}(\delta^t,\delta^e,K(\delta^t))\delta^t$ for each pair $(\delta^{ne},\delta^e)$. This immediately implies that $M(0)>0$. By continuity of $M(T)$, there is some value $\overline{T}$ such that $M(\overline{T})=0$. Therefore, the selected $\delta^t$ is a threshold for $T=\overline{T}$.

To verify that $\delta^t$ is the unique equilibrium threshold, given $\overline{T}$, it has to be proved that, given $K(\delta^t)$, $M(\delta,\overline{T})\equiv E(u(w^e(.))-E(u(w^{ne}(.)))>0$ if and only if $\delta>\delta(T)$. This immediately follows if $\frac{\partial M}{\partial\delta}>0$, where

$$\frac{\partial M}{\partial\delta}=\int_{\underline{d}}^{\delta^t}\frac{\frac{\partial v}{\partial c^e}\left(w^e(.)+\frac{\partial w^e(.)}{\partial\delta}\delta\right)d\delta^{ne}}{\delta^t-\underline{d}}-\int_{\delta^t}^{\overline{d}}\frac{\frac{\partial v}{\partial c^{ne}}\left(w^{ne}(.)+\frac{\partial w^{ne}(.)}{\partial\delta}\delta\right)}{\overline{d}-\delta^t}d\delta^e$$

Using the FOCs of the firms' optimization problems and the market clearing conditions, and remembering that $u(c^s,h^s)=v(c^s)$, one obtains

$$\begin{aligned}\frac{\partial M}{\partial\delta} &= \int_{\underline{d}}^{\delta^t}\frac{\frac{\partial v}{\partial c^e}w^e(\delta^{ne},\delta,K(\delta^t))}{\delta^t-\underline{d}}\frac{\theta\psi^{ne}\delta^{ne\theta}+(1-\alpha)\psi^e\delta^{\theta}}{\psi^{ne}\delta^{ne\theta}+\psi^e\delta^{\theta}}d\delta^{ne}\\ &\quad-\int_{\delta^t}^{\overline{d}}\frac{\frac{\partial v}{\partial c^{ne}}w^{ne}(\delta,\delta^e,K(\delta^t))}{\overline{d}-\delta^t}\frac{\theta\psi^e\delta^{e\theta}+(1-\alpha)\psi^{ne}\delta^{\theta}}{\psi^{ne}\delta^{\theta}+\psi^e\delta^{e\theta}}d\delta^e.\end{aligned}$$

For $\frac{\partial^2 v}{\partial c^2}$ sufficiently close to 0, $\frac{\partial v}{\partial c^e}\sim\frac{\partial v}{\partial c^{ne}}$. Then, given that $(1-\alpha)\geq\theta$, we can write

$$\begin{aligned}\frac{\partial M}{\partial\delta} &> \theta\int_{\underline{d}}^{\delta^t}\frac{w^e(\delta^{ne},\delta,K(\delta^t))d\delta^{ne}}{\delta^t-\underline{d}}-(1-\alpha)\int_{\delta^t}^{\overline{d}}\frac{w^{ne}(\delta,\delta^e,K(\delta^t))d\delta^e}{\overline{d}-\delta^t}\\ &\Longrightarrow\\ \frac{\partial M}{\partial\delta} &> \theta\int_{\underline{d}}^{\delta^t}\frac{w^e(\delta^{ne},\delta,K(\delta^t))d\delta^{ne}}{\delta^t-\underline{d}}-(1-\alpha)\int_{\delta^t}^{\overline{d}}\frac{w^{ne}(\delta,\delta^e,K(\delta^t))d\delta^e}{\overline{d}-\delta^t}\\ &\Longrightarrow\\ \frac{\partial M}{\partial\delta} &> \theta w^e(\delta,\underline{d},K(\delta^t))\delta^t-(1-\alpha)w^{ne}(\overline{d},\delta,K(\delta^t))\delta^t>0,\end{aligned}$$

where the last inequality follows, for $\frac{\psi^e}{\psi^{ne}}$ large enough, by the same argument used to establish $(B)$ above. This concludes the proof.

To summarize, provided that the two types of labor are *E-complements*, that the second order derivative of the utility function is sufficiently small (in absolute value), and that $\frac{\psi^e}{\psi^{ne}}$ is large enough, an equilibrium exists. Since it is, locally, defined by a collection of $C^1$ functions, it is easy to check that the set of economies such that an equilibrium exists is open. ■

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