Capital Structure with Information about the Upside and the Downside

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Abstract

We introduce two dimensions of uncertainty, about the upside and the downside of an asset, in a model of asset valuation under asymmetric information. This justifies capital structures with equity and risky debt for information revelation purposes. However, a capital structure with only one information-sensitive security, equity, can be optimal when investors are less informed about the dimension that matters more for valuation. This is relevant for innovative firms with a large upside subject to strong information asymmetries, which often have abnormally low leverage, and for firms at an intermediate stage of their life cycle that don't issue risky debt.

Keywords: capital structure, downside risk, security design, tranching, upside potential.

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This paper considers capital structure design when assets differ along two dimensions of uncertainty, the upside and the downside, and when different investors are informed about these two dimensions. This is consistent with a recent literature that acknowledges that investors specialize and acquire limited information (e.g. Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), Glasserman and Mamaysky (2023), Maćkowiak, Matějka, and Wiederholt (2023)).

Considering two empirically relevant dimensions of uncertainty on which different investors are informed allows to generate new empirical implications. In particular, our capital structure model can explain the zero leverage puzzle (Strebulaev and Yang (2013)) for innovative firms with substantial "upside", and the propensity of firms at an intermediate stage of their life cycle to not issue risky debt.

We consider a simple model in which cash flows from an asset can take three different values, so that the cash flow distribution is fully described by two probabilities. Furthermore, these probabilities provide information about two different aspects of the distribution: the "upside" and the "downside". The firm has private information about these two probabilities, which describe its asset type, and about its discount factor, which represents its willingness to sell its asset and measures potential gains from trade. As in the seminal security design papers of Allen and Gale (1988) and Boot and Thakor (1993), the security design problem consists in splitting claims on the asset's cash flows to maximize asset valuation. An uninformed market maker offers to the firm a menu of security designs, which describe the asset's capital structure.[1] If the firm accepts the market maker's offer and sells its asset, investors submit orders for securities, which are motivated by either information or hedging motives. The price of each security, as set by the market maker, is equal to its expected value conditional on observed orders on all markets.

There are two possible capital structures, with different implications for information revelation and asset valuation.[2] First, there is a capital structure with only one information-sensitive security, equity, which is exposed to both dimensions of uncertainty. Second, there is a capital structure with two information-sensitive securities, leveraged equity and risky debt, each of which is exposed to a different dimension of uncertainty. In this second case, there are two market prices which can each reveal a different type of information. In any case, the firm can also use riskfree debt, which is information-insensitive and does not have implications for asset valuation. A priori, one might think that it will be optimal to create two security markets to maximize information revelation and minimize adverse selection.

The first step is to study how the asset's capital structure will affect the valuation effect of investors' decisions. Even though there is more information revelation with leveraged equity and risky debt, the expected market value of a high-quality asset on both dimensions can surprisingly be higher when equity is the only information-sensitive security. Intuitively, suppose that the upside matters more for asset valuation but investors are more informed about the downside of

[1] Despite the adverse selection problem, we show that the market maker will only offer one security design. Thus, as in Boot and Thakor (1993), the security design choice is not informative about the asset type in equilibrium.

[2] We analyze security designs with equity, risky debt, and riskfree debt, and we show in the Online Appendix that these securities are optimal in a strict sense under additional natural assumptions.

the asset. When all investors trade the same security, equity, all trades have the same impact on asset value. Then, when there is more informed trading on the less value relevant dimension of uncertainty (here, the downside), the asset's expected market value is more sensitive to the asset type on this dimension. This is value improving for a high-quality asset on both dimensions when trading on the more value relevant dimension of uncertainty is sufficiently informative. This is the *intra-subsidy* effect of pooling trading by using only equity.

There are two countervailing effects. First, still in the case when only equity is traded, when trading on the more value relevant dimension of uncertainty is barely informative, the price impact of any trade is small. This is the *intra-noisiness* effect of pooling trading. By contrast, when both leveraged equity and risky debt are traded, barely informative trading on one dimension only affects the sensitivity of market prices to trades on one market. As long as the other dimension is sufficiently value-relevant, a high-quality asset on both dimensions will achieve a higher expected valuation if its upside and its downside are traded separately via leveraged equity and risky debt.

Second, when all investors trade the same security and one dimension of uncertainty is much less value relevant than the other one, it becomes unprofitable for an investor informed about the less value relevant dimension of uncertainty to trade. Indeed, the small benefit he would derive from the trade would be outweighed by its price impact, which is the same for all trades in this case. By contrast, when leveraged equity and risky debt are traded separately, the price impact of a trade on the less value relevant dimension of uncertainty is lower, so that every investor participates. A capital structure with leveraged equity and risky debt thus ensures that all informed investors will (imperfectly) transmit their information via trading. When quantifying the magnitude of effects at play in section 4.1, we find that this effect is especially important. This suggests that when the upside is very small relative to the downside, or vice versa, it is especially valuable to create two separate markets to ensure that investors informed about these two different dimensions of asset value trade on their information.

The second step is to derive the equilibrium capital structure. There are potential gains from trade from the firm selling its asset to investors, as captured by the firm's lower discount factor as in DeMarzo and Duffie (1999). However, the market maker does not observe this discount factor, and the firm will decline to sell its asset if its expected market value is too low. As a result, we show that a market maker who derives a benefit from an accepted offer will offer the security design that minimizes the most extreme asset undervaluation as a fraction of asset value (the "relative adverse selection discount") across asset types.

The relative adverse selection discount is not always highest for a high-quality asset on both dimensions, so that the capital structure will not always be designed to cater to this type of asset. This is in contrast to the result that the preferences of a high-quality asset drive capital structure design with only one dimension of uncertainty (Boot and Thakor (1993)). To provide the intuition, suppose that there is much more informed trading on the downside than on the upside. In this case, an asset with high-quality upside but low-quality downside will be extremely undervalued in an equilibrium with leveraged equity and risky debt, since its low type on one dimension will be almost fully revealed, whereas its high type on the other dimension will remain largely unknown.

The relative adverse selection discount faced by this asset type, which is most severe, is minimized when all investors trade the same security – equity.

The results contrast to the case with one dimension of uncertainty, analyzed in the seminal paper on security design by Boot and Thakor (1993): when assets differ along one dimension only, the capital structure is designed to maximize information revelation via security prices, which is always beneficial to a high-quality asset – which also faces the highest adverse selection discount. In our paper, by contrast, designing the capital structure to maximize information revelation would not always maximize the valuation of a high-quality asset on both dimensions, nor would it always minimize the highest relative adverse selection discount across asset types. Thus, our results show that the compelling logic in Boot and Thakor (1993) cannot be straightforwardly extended to the case with several value-relevant dimensions of uncertainty.

The model predicts that the capital structure will involve a single information-sensitive security only when there is less informed trading on the more value relevant dimension of uncertainty. One could argue that this is an unreasonable assumption so that this case will not arise in practice. Yet, consider the example of an innovative firm for which the "upside", related to the development of new technologies, is more value-relevant, but with strong information asymmetries between firm insiders and investors on this dimension. In this case, investors might be better informed about the "downside", which in this application would reflect the value of the firm's existing assets, such as its real estate and its patents portfolio. Thus, it is possible for the intra-subsidy effect to dominate, which in turn justifies a capital structure with only one information-sensitive security, equity.

This prediction has cross-sectional and longitudinal implications. First, it is related to the finding in Strebulaev and Yang (2013) that about 10% of large public nonfinancial US firms have zero debt – the "zero leverage puzzle". They find that these firms tend to be innovative firms with intangible assets and high valuations. This prediction can also contribute to explain the negative correlation between profitability and leverage observed empirically, which is inconsistent with the static tradeoff theory of the capital structure. Second, the model predicts that both young and mature firms will issue risky debt, but that firms at an intermediate stage of their life cycle will not (see section 4.1). The implied nonmonotonic changes in the leverage target and debt ratio of a firm over time are consistent with longitudinal studies (DeAngelo, Gonçalves, and Stulz (2018)). We discuss these empirical implications of our results further in section 4.2.

The results in our paper have several potential applications. The first is a company establishing a market valuation for one of its assets that it intends to sell. The second is a corporation designing its capital structure to maximize its market valuation to allow existing investors to cash out on good terms (via a takeover bid, leveraged buyout, equity sale, etc.). The third is a financial institution with an originate-to-distribute business model.

We follow the security design approach of Allen and Gale (1988) and Boot and Thakor (1993) who study how to split asset cash flows into securities to maximize asset valuation. Our perspective based on asymmetric information is related to Chang and Yu (2010) and Machado and Pereira (2022), who study the effect of a firm's capital structure on the incentives of investors to acquire information or trade on their information in models with one source of uncertainty. In Allen and Gale (1988), information is symmetric, but investors have different risk preferences so that they value securities differently, and arbitrage is impossible because of short sale constraints. As a result, the firm splits its cash flows so that each security is bought by the investor who values it the most. Security design also matters when investors agree to disagree (Garmaise (2001)).

Another approach is to study the optimal security that the firm should issue. This includes models of financing (Myers and Majluf (1984), Nachman and Noe (1994)), which are especially relevant for young firms with inadequate cash generation. Papers in this branch of the literature consider variations in the information environment (Fulghieri and Lukin (2001), Axelson (2007), Yang and Zeng (2019), Yang (2020), Malenko and Tsoy (2020), Inostroza and Tsoy (2023)). Fulghieri, García, and Hackbarth (2020) show that the pecking order theory can be reversed when conditional stochastic dominance does not hold, or when asymmetric information is concentrated on the firm's assets in place rather than its growth opportunities. In their paper, all investors have the same information, and the firm issues a single security. By contrast, a crucial assumption in our paper is that different investors have information about different aspects of asset value, and the main question is whether the firm should issue one or several information-sensitive securities.

The financing model of Bertomeu, Beyer, and Dye (2011) also uses a model with three levels of output to study the design of the optimal security and voluntary disclosure when both the magnitude and the probability of the upside and the downside of the firm's cash flows are identical. By contrast, we focus on the optimality of using either one or multiple information-sensitive securities in a setting in which the asymmetry between the upside and the downside plays a crucial role. Models with three levels of output have also been used in papers with project choice in which each project is associated with two outcomes or less (John and John (1993), John, Saunders, and Senbet (2000)), and in models of risk shifting in which increasing risk increases the probability of extreme outcomes (Biais and Casamatta (1999)).

Finally, our results can provide insights into the securitization and tranching of cash flows from financial assets, for example via the originate-to-distribute model. Our results emphasize that tranching into equity and several tranches of debt (risky and riskfree) can be justified even without moral hazard or market illiquidity. Some related papers show how a financial intermediary can sell assets together ("pooling") to mitigate adverse selection (Subrahmanyam (1991), Axelson (1999), DeMarzo (2005), DeMarzo, Frankel, and Jin (2021)). The problem studied is different: they consider how to bundle different assets into securities, with an emphasis on creating an information-insensitive security, whereas we consider whether to split the cash flows produced by one asset (that can be interpreted as a pool of assets) into different tranches.[3]

[3]Asriyan and Vanasco (2022) study a similar problem with implications for mortgage-backed securities, but in their model all investors are uninformed, and security design is chosen by the asset's buyer to screen the seller.

1 The model

We consider a stylized model of security design and trading in which security prices partially but imperfectly reflect information about the asset type. The novel aspect of the model is that investors have differential information about two aspects of the asset value. The asset's capital structure matters because it determines how investors' information will be reflected in security prices and thus affect asset valuation.

1.1 Asset value

The firm's asset produces a contractible output $\tilde{y}$ at $t = 1$. This output is equal to H with probability $\frac{\theta_1}{2}$, to L with probability $\frac{1-\theta_2}{2}$, and to M with the complementary probability, with $0 \leq L < M < H$. The variables $\tilde{\theta}_1$ and $\tilde{\theta}_2$ are independent, realized at $t = 0$, and each follows a Bernoulli distribution with probability $\frac{1}{2}$: each is equal to 1 with probability $\frac{1}{2}$, and to 0 otherwise. An increase in θ_1 or θ_2 improves output in a first-order stochastic dominance sense. Figure 1 depicts the distribution of the asset's output.

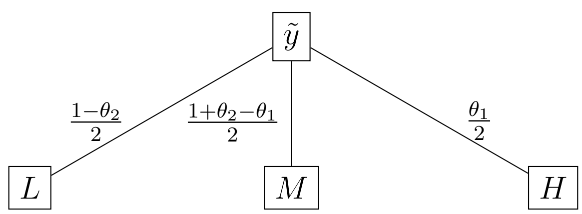

Figure 1: Distribution of output $\tilde{y}$ conditional on asset type $\{\theta_1, \theta_2\}$.

This parameterization allows us to distinguish between two aspects of the distribution of $\tilde{y}$. The realization of $\tilde{\theta}_1$ affects the "upside" of the distribution, whereas the realization of $\tilde{\theta}_2$ affects the "downside" of the distribution. We refer to the variable $\theta \equiv \{\theta_1, \theta_2\}$ as the "asset type". It is observed by the firm, and it is also potentially observed by different traders, as described below. Table 1 describes the distribution of $\tilde{y}$ for the various asset types.

	Pr(L)	Pr(M)	Pr(H)
$\theta_1 = 1, \theta_2 = 1$	0	$\frac{1}{2}$	$\frac{1}{2}$
$\theta_1 = 1, \theta_2 = 0$	$\frac{1}{2}$	0	$\frac{1}{2}$
$\theta_1 = 0, \theta_2 = 1$	0	1	0
$\theta_1 = 0, \theta_2 = 0$	$\frac{1}{2}$	$\frac{1}{2}$	0

Table 1: Distribution of output $\tilde{y}$ for different asset types.

As in Gorton and Pennacchi (1990), Boot and Thakor (1993), and Fulghieri and Lukin (2001), the asset's type along a dimension of uncertainty is binary (good or bad). In addition, the asset's type includes its quality along two dimensions of uncertainty, resulting in a total of four asset types (see Table 1). This modeling is in contrast to settings in which different risks have a similar effect on the distribution of asset value, such as Goldstein and Yang (2015) where firm value is additive in two normally distributed variables, and Goldstein and Yang (2019) where firm value is multiplicative in two lognormally distributed variables.

1.2 Traders

To take into account imperfectly informative trading on both dimensions of uncertainty, we assume that there are two traders, denoted by 1 and 2. Consider trader $i \in \{1, 2\}$. With probability $\mu_i \in (0, 1)$, he is an informed trader who observes θ_i; with probability $1 - \mu_i$, he has a hedging need described below. Thus, the parameters μ_1 and μ_2 are measures of trading informativeness with respect to the sources of uncertainty $\tilde{\theta}_1$ and $\tilde{\theta}_2$, respectively.

If trader 1 is a hedger, his stochastic endowment is 0 for $y \in \{L, M\}$, and either δ_H or $-\delta_H$, with probability $\frac{1}{2}$, for $y = H$, where $\delta_H = 1/(H - \mathbb{E}[\tilde{y}])$. If trader 2 is a hedger, his stochastic endowment is 0 for $y \in \{M, H\}$, and either δ_L or $-\delta_L$, with probability $\frac{1}{2}$, for $y = L$, where $\delta_L = 1/(\mathbb{E}[\tilde{y}] - L)$. A hedger minimizes the variance of his wealth. This modeling of hedgers is in the spirit of Dow and Gorton (1997).

The assumption that different traders are (potentially) endowed with different information is as in Goldstein and Yang (2015), who quote Paul (1993) to argue that this assumption "is in the spirit of Hayek's view that one of the most important functions of the price system is the decentralized aggregation of information and that no one person or institution can process all information relevant to pricing." For example, some traders may be better able to assess the liquidation value of assets, whereas others may be better able to assess the potential of a technology under development.[4]

1.3 Preferences and information

To generate gains from trade, we assume as in DeMarzo and Duffie (1999) that the firm is more impatient than traders.[5] Formally, its utility over consumption c_t at $t = 0$ and $t = 1$ is given by: $c_0 + \beta c_1$. The discount factor of the firm, β, is drawn at $t = 0$ from a known distribution with full support on $[0, \overline{\beta}]$, with CDF F and $\overline{\beta} < 1$, and the realization of $\tilde{\beta}$ is the firm's private information. This assumption is not crucial for the results: the results would still hold if $\tilde{\beta} = \overline{\beta}$ with probability 1. The discount factors of other agents are normalized to 1.

The market maker is an optimizing agent in our model. We simply assume that the market maker derives a benefit from the firm accepting its security design offer, as described below. This

[4]Likewise, Glasserman and Mamaysky (2023) argue that "Skilled investors specialize. Fund managers typically invest in stocks or bonds, but not both." This is related to a literature that has acknowledged that investors' information can be heterogenous (Van Nieuwerburgh and Veldkamp (2010), Kacperczyk, Van Nieuwerburgh, and Veldkamp (2016), Goldstein and Yang (2015, 2019)), possibly because of rational inattention (Maćkowiak, Matějka, and Wiederholt (2023)).

[5]This assumption is commonly used, e.g. Bessembinder, Hao, and Zheng (2015), Hébert (2018), Yang (2020), DeMarzo, Frankel, and Jin (2021).

benefit can be arbitrarily small. The purpose of this assumption is to avoid a market maker who is indifferent as to whether or not the firm accepts its offer.

As is standard in models of security design, there are information asymmetries between the firm and other agents: the firm privately observes its discount factor β and its asset type.

1.4 Security design and pricing

As in seminal security design papers (Allen and Gale (1988) and Boot and Thakor (1993)), designing securities is equivalent to splitting claims on the asset's output. In the main paper, we assume that the securities available to the firm include equity, risky debt, and riskfree debt – which are formally defined below. Note that markets can be complete with this set of available securities. In section 1 of the Online Appendix, we show that these securities (or composite securities also comprising riskfree debt which are equivalent for information revelation purposes) are optimal if we assume a small cost to the issuance of additional securities and a small trading cost.

As is standard, security prices must preclude arbitrage opportunities and be such that the market price of a security is equal to its expected payoff conditional on observed market orders on all markets. We refer to this condition as the "fair pricing" condition.

We use a stylized model of security design and market pricing with the following sequence of events. At $t = 0$, a market maker offers a menu of security designs such that the market maker can subsequently set security prices subject to fair pricing. The firm decides whether or not to sell the asset, and selects a security design offered by the market maker if it does so. Traders then submit observable market orders for the securities offered, and the market maker sets security prices. There is a riskfree security with a constant payoff in perfectly elastic supply, and the riskfree rate is zero.

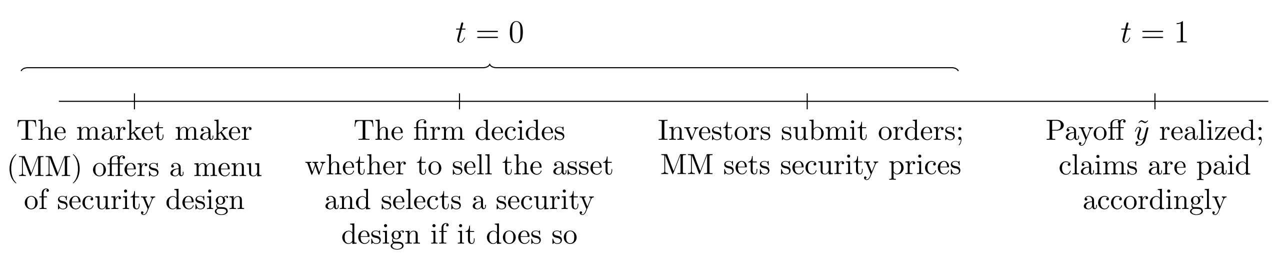

Figure 2: Timeline of the model.

2 Market valuation

As is standard in the literature, we consider perfect Bayesian equilibria (PBE):

Definition 1. *A PBE is a menu of security design offered by the market maker, the market maker's beliefs about the security design choice of every asset type, and a set of demand-contingent security prices such that: (i) the firm optimally decides whether or not to sell its asset; if it* *does, it chooses the optimal security design given its asset type; (ii) the market maker's beliefs about security design choices are verified, and the market maker rationally updates its beliefs after observing orders from traders;*[6] *(iii) the market maker makes zero expected profit and maximizes the probability that the firm sells its asset.*

Define v_{θ_1,θ_2} as expected output conditional on asset type $\{\theta_1, \theta_2\}$, which we refer to as "intrinsic asset value". From Figure 1, we have:

$$\begin{array}{ll} v_{1,1} = \frac{H+M}{2}, & v_{1,0} = \frac{H+L}{2}, \\ v_{0,1} = M, & v_{0,0} = \frac{L+M}{2}. \end{array} \tag{1}$$

If the asset type $\{\theta_1, \theta_2\}$ were public information, its market valuation would simply be equal to v_{θ_1,θ_2} as described in equation (1).

A security is defined by its state contingent payoff. Since there are three states, corresponding to the possible realizations of the firm's output, the payoff vector of security h is $(s_h(L), s_h(M), s_h(H))$, where $s_h(y)$ is its payoff in state y.

When the market price is imperfectly informative about the asset type, the expected market valuation of a given asset type is generally different from its intrinsic value. We define the adverse selection discount for a given asset type and security design i as follows:

$$\text{ASD}^i_{\theta_1,\theta_2} \equiv v_{\theta_1,\theta_2} - \mathbb{E}\left[P_i | \theta_1, \theta_2\right], \tag{2}$$

where P_i is a random variable that represents the market value of the asset under security design i. That is, a positive (negative) adverse selection discount means that the asset is expected to be undervalued (overvalued).

In sections 2.1 and 2.2 below, we analyze two potential equilibria. In section 3, we rule out other potential equilibria, and determine which of the two aforementioned equilibria is the unique equilibrium, depending on parameter values.

2.1 Equity and riskfree debt

In this subsection, we study the trading phase equilibrium with only one security which is information-sensitive, i.e. with a non-constant payoff vector. As further discussed in the Online Appendix (see the "first step" in the proof of Lemma 2), this implies that the only securities are riskfree debt, which by definition has a constant payoff vector and is therefore not information-sensitive, and equity.

To focus on properties of this equilibrium when the firm accepts the market maker's offer and sells its asset, in this subsection we simply assume that $\overline{\beta} = 0$, i.e. the firm is highly "impatient",

[6] Depending on the PBE considered, either one of these conditions is relevant, or both of them are. In a separating equilibrium in which each asset type chooses a different security design, only the former condition is relevant. In a pooling equilibrium in which the market maker only offers one security design, only the latter condition is relevant. In a semi-separating equilibrium, both conditions are relevant.

similar to the specification in Diamond and Dybvig (1983). In section 3, we will provide conditions under which this security design is offered and selected in equilibrium when $\overline{\beta} > 0$.

Proposition 1. *When securities offered by the market maker have payoff vectors* $(L-\gamma, M-\gamma, H-\gamma)$ *and* (γ,γ,γ) *for* $\gamma \in [0, L]$ *and* $\max\left\{\mu_2 + \mu_1 \frac{H-M}{M-L}, \mu_1 + \mu_2 \frac{M-L}{H-M}\right\} \leq 2$*:*

- *Hedgers and informed traders trade the security with payoff vector* $(L-\gamma, M-\gamma, H-\gamma)$*. Each trader submits a market order for a quantity* $\frac{1}{4}\frac{1}{var(\tilde{y})}$*. A hedger with a negative (positive) exposure to* $y = H$ *buys (sells) this amount; a hedger with a negative (positive) exposure to* $y = L$ *sells (buys) this amount. An informed trader who observes* $\theta_1 = 1$ *or* $\theta_2 = 1$ *buys this amount; an informed trader who observes* $\theta_1 = 0$ *or* $\theta_2 = 0$ *sells this amount.*

- *The asset's market value* P_U *conditional on observed orders is:*

$$P_U\,(buy, buy) = \frac{1+\mu_1}{2}\frac{H}{2} + \frac{2-\mu_1+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2} \tag{3}$$

$$P_U\,(buy, sell) = \frac{1}{2}\frac{H}{2} + \frac{M}{2} + \frac{1}{2}\frac{L}{2} \tag{4}$$

$$P_U\,(sell, sell) = \frac{1-\mu_1}{2}\frac{H}{2} + \frac{2+\mu_1-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2} \tag{5}$$

- *The expected market value of the asset conditional on asset type is:*

$$\mathbb{E}\left[P_U|\theta_1 = 1, \theta_2 = 1\right] = \frac{1+\frac{1}{2}\left(\mu_1^2+\mu_1\mu_2\right)}{4}H + \frac{2+\frac{1}{2}\left(-\mu_1^2+\mu_2^2\right)}{4}M + \frac{1+\frac{1}{2}\left(-\mu_2^2-\mu_1\mu_2\right)}{4}L \tag{6}$$

$$\mathbb{E}\left[P_U|\theta_1 = 1, \theta_2 = 0\right] = \frac{1+\frac{1}{2}\left(\mu_1^2-\mu_1\mu_2\right)}{4}H + \frac{2+\frac{1}{2}\left(-\mu_1^2-\mu_2^2+2\mu_1\mu_2\right)}{4}M + \frac{1+\frac{1}{2}\left(\mu_2^2-\mu_1\mu_2\right)}{4}L \tag{7}$$

$$\mathbb{E}\left[P_U|\theta_1 = 0, \theta_2 = 1\right] = \frac{1+\frac{1}{2}\left(-\mu_1^2+\mu_1\mu_2\right)}{4}H + \frac{2+\frac{1}{2}\left(\mu_1^2+\mu_2^2-2\mu_1\mu_2\right)}{4}M + \frac{1+\frac{1}{2}\left(-\mu_2^2+\mu_1\mu_2\right)}{4}L \tag{8}$$

$$\mathbb{E}\left[P_U|\theta_1 = 0, \theta_2 = 0\right] = \frac{1+\frac{1}{2}\left(-\mu_1^2-\mu_1\mu_2\right)}{4}H + \frac{2+\frac{1}{2}\left(\mu_1^2-\mu_2^2\right)}{4}M + \frac{1+\frac{1}{2}\left(\mu_2^2+\mu_1\mu_2\right)}{4}L \tag{9}$$

The securities considered in Proposition 1 are either unleveraged equity (for $\gamma = 0$), or riskfree debt and equity (for $\gamma \in (0, L]$). In any case, equity is the only information-sensitive security. A hedger uses securities to hedge his exposure to either $y = H$ or $y = L$, and will trade the appropriate amount of equity for this purpose. An informed trader, who has private information about the value of equity, will buy or sell the same amount as a hedger to not reveal its type to the market maker. The condition $\max\left\{\mu_2 + \mu_1 \frac{H-M}{M-L}, \mu_1 + \mu_2 \frac{M-L}{H-M}\right\} \leq 2$ ensures that the benefit that each informed trader derives from trading is sufficiently large for informed trading to be profitable once the price impact is taken into account (Proposition 2 below analyzes the outcome when this condition is not satisfied). Figure 3 depicts the probability distribution of the trades of traders 1 and 2 conditional on asset type.

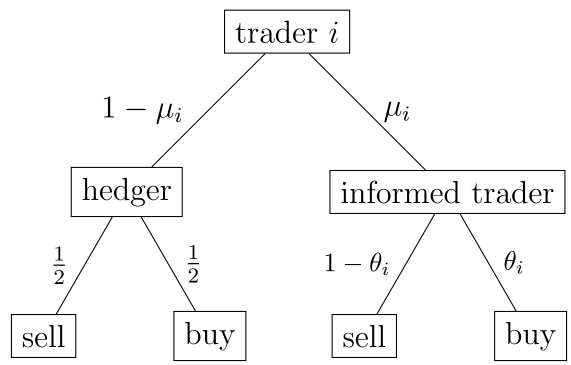

Figure 3: Equilibrium demand for the security with payoff vectors $(L-\gamma, M-\gamma, H-\gamma)$ of trader i ($i \in \{1,2\}$).

Since traders 1 and 2 trade the same security, the order flow is informative about θ_1 and θ_2. It is all the more informative that the probability of informed trading is high. An increase in the probability of informed trading μ_i increases the expected market value of a high θ_i-type asset.

The relative adverse selection discount in this equilibrium, defined as $\text{ASD}^1_{\theta_1,\theta_2}/v_{\theta_1,\theta_2}$, is not always highest for an asset with a high type on both dimensions ($\theta_i = 1$ for $i \in \{1,2\}$). This can be explained as follows. Consider the case when the upside is small relative to the downside ($\frac{H-M}{M-L}$ is low) but there is a lot of informed trading on the upside (μ_1 is very high). Then, a "steady" asset, which produces an output of M, faces a larger relative adverse selection discount than a "star" (with a high type on both dimensions), which produces an output of either H or M. Intuitively, these two asset types have a similar intrinsic value in this case since the upside is small. However, when all traders trade the same security (equity), all trades have the same effect on market value. Moreover, trader 1, who trades based on his information about the upside, will tend to buy a "star" asset and sell a "steady" asset. This improves the market value of a "star" asset, but worsens the market value of a "steady" asset. For example, with $H = 1.5$, $M = 1$, $L = 0$, $\mu_1 = 0.95$, and $\mu_2 = 0.5$, we have $\text{ASD}^1_{0,1}/v_{0,1} = 0.180 > 0.159 = \text{ASD}^1_{1,1}/v_{1,1}$.[7]

When the capital structure only involves one information-sensitive security, an informed trader will not always trade based on his information. This is because, since all traders trade the same security, all trades have the same impact on market value. A trader who has some information on a dimension that is barely value-relevant will not gain much from buying or selling equity, holding the price of equity constant, but his trade may move the price of equity substantially. Consequently, a trader who is informed about the upside will not trade when $\mu_1 + \mu_2 \frac{M-L}{H-M} > 2$,

[7] Likewise, in the case when the downside is small relative to the upside ($\frac{M-L}{H-M}$ is low) but there is a lot of informed trading on the downside (μ_2 is very high), a "high-risk" asset, which produces an output of either H or L, faces a larger relative adverse selection discount than a "star", which produces an output of either H or M. Intuitively, these two assets have a similar intrinsic value in this case since the downside is small. However, when all traders trade the same security (equity), all trades have the same effect on market value. Moreover, trader 2, who trades based on his information about the downside, will tend to buy a "star" asset and sell a "high-risk" asset. This improves the market value of a "star" asset, but worsens the market value of a "high-risk" asset. For example, with $H = 2$, $M = 1$, $L = 0.5$, $\mu_1 = 0.5$, and $\mu_2 = 0.95$, we have $\text{ASD}^1_{1,0}/v_{1,0} = 0.144 > 0.132 = \text{ASD}^1_{1,1}/v_{1,1}$. Finally, in the case when the upside and the downside have a similar magnitude ($H - M \approx M - L$), an asset with a high type on both dimensions (a "star") is subject to the highest relative adverse selection discount.

i.e. when the upside $H - M$ is sufficiently small compared to the downside $M - L$. Likewise, a trader who is informed about the downside will not trade when $\mu_2 + \mu_1 \frac{H-M}{M-L} > 2$, i.e. when the downside is sufficiently small compared to the upside. Proposition 2 summarizes the outcome in these two cases.

Proposition 2. *When securities offered by the market maker have payoff vectors $(L - \gamma, M - \gamma, H - \gamma)$ and (γ, γ, γ) for $\gamma \in [0, L]$ and $\max\left\{\mu_2 + \mu_1 \frac{H-M}{M-L}, \mu_1 + \mu_2 \frac{M-L}{H-M}\right\} > 2$, hedgers trade as in Proposition 1, and:*

- *When $\mu_1 + \mu_2 \frac{M-L}{H-M} > 2$, an informed trader who observes θ_1 does not trade, and the expected market value of the asset is:*

$$\mathbb{E}\left[P_U|\theta_1, \theta_2 = 1\right] = \frac{1}{2}\frac{H}{2} + \frac{2 + \mu_2^2 \frac{1+\mu_1}{2}}{2}\frac{M}{2} + \frac{1 - \mu_2^2 \frac{1+\mu_1}{2}}{2}\frac{L}{2} \quad (10)$$

$$\mathbb{E}\left[P_U|\theta_1, \theta_2 = 0\right] = \frac{1}{2}\frac{H}{2} + \frac{2 - \mu_2^2 \frac{1+\mu_1}{2}}{2}\frac{M}{2} + \frac{1 + \mu_2^2 \frac{1+\mu_1}{2}}{2}\frac{L}{2} \quad (11)$$

- *When $\mu_2 + \mu_1 \frac{H-M}{M-L} > 2$, an informed trader who observes θ_2 does not trade, and the expected market value of the asset is:*

$$\mathbb{E}\left[P_U|\theta_1 = 1, \theta_2\right] = \frac{1 + \mu_1^2 \frac{1+\mu_2}{2}}{2}\frac{H}{2} + \frac{2 - \mu_1^2 \frac{1+\mu_2}{2}}{2}\frac{M}{2} + \frac{1}{2}\frac{L}{2} \quad (12)$$

$$\mathbb{E}\left[P_U|\theta_1 = 0, \theta_2\right] = \frac{1 - \mu_1^2 \frac{1+\mu_2}{2}}{2}\frac{H}{2} + \frac{2 + \mu_1^2 \frac{1+\mu_2}{2}}{2}\frac{M}{2} + \frac{1}{2}\frac{L}{2} \quad (13)$$

When the only information-sensitive security traded is equity and there is no informed trading on a dimension, say the upside, there are two possible cases. First, with probability $1 - \mu_1$, there is still trading from a hedger exposed to the upside. Second, with probability μ_1, there is no trading from trader 1, i.e. only trader 2 trades the asset. The security price is more informative in this second case, since in the first case the market maker cannot disentangle the trade from the hedger from trader 2's trade. The second case is informationally equivalent to a case in which leveraged equity and risky debt are traded as in Proposition 3 but there is no informed trading of leveraged equity. As Proposition 4 will make clear, this implies that the outcome described in Proposition 2, which overall results in less information revelation than in the outcome described in Proposition 3, will not be an equilibrium of the model when we let $\overline{\beta} > 0$ as in section 3.

2.2 Equity and risky debt

In this subsection, we study the trading phase equilibrium with securities with payoff vectors (L, M, M) and $(0, 0, H - M)$. The first security is risky debt, the second security is leveraged equity. The expected payoffs conditional on asset type $\{\theta_1, \theta_2\}$ of leveraged equity (indexed by E)

and risky debt (indexed by D), respectively, are:

$$\begin{aligned} &v_E(\theta_1 = 1) = \tfrac{H-M}{2}, \quad v_E(\theta_1 = 0) = 0, \\ &v_D(\theta_2 = 1) = M, \qquad v_D(\theta_2 = 0) = \tfrac{L+M}{2}. \end{aligned} \tag{14}$$

The value of leveraged equity only depends on θ_1, and the value of risky debt only depends on θ_2. This security design is informationally equivalent to a more general security design that involves composite securities with payoff vectors $(L-\Gamma, M-\Gamma, M-\Gamma)$ and $(\Gamma, \Gamma, H-M+\Gamma)$ for $\Gamma \in [0, L]$ (see section 2 of the Online Appendix).

To focus on properties of this equilibrium when the firm accepts the market maker's offer and sells its asset, in this subsection we simply assume that $\overline{\beta} = 0$, i.e. the firm is highly "impatient". In section 3, we will provide conditions under which this security design is offered and selected in equilibrium when $\overline{\beta} > 0$.

Proposition 3. *When securities offered by the market maker have payoff vectors (L, M, M) and $(0, 0, H-M)$:*

- *Trader 1 trades leveraged equity by submitting a market order for a quantity $\frac{\delta_H}{H-M}$. A hedger with a negative (positive) exposure to $y = H$ buys (sells) this amount; an informed trader who observes $\theta_1 = 1$ buys this amount; an informed trader who observes $\theta_1 = 0$ sells this amount.*
  Trader 2 trades risky debt by submitting a market order for a quantity $\frac{\delta_L}{M-L}$. A hedger with a negative (positive) exposure to $y = L$ sells (buys) this amount; an informed trader who observes $\theta_2 = 1$ buys this amount; an informed trader who observes $\theta_2 = 0$ sells this amount.

- *The market price P_E of leveraged equity conditional on the observed order is:*

$$P_E(buy) = \frac{1+\mu_1}{2}\frac{H-M}{2} \tag{15}$$

$$P_E(sell) = \frac{1-\mu_1}{2}\frac{H-M}{2} \tag{16}$$

  The market price P_D of risky debt conditional on the observed order is:

$$P_D(buy) = \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} \tag{17}$$

$$P_D(sell) = \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} \tag{18}$$

- *The expected market value of the asset conditional on asset type is:*

$$\mathbb{E}\left[P_E + P_D|\theta_1 = 1, \theta_2 = 1\right] = \frac{1+\mu_1^2}{4}H + \frac{2-\mu_1^2+\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \quad (19)$$

$$\mathbb{E}\left[P_E + P_D|\theta_1 = 1, \theta_2 = 0\right] = \frac{1+\mu_1^2}{4}H + \frac{2-\mu_1^2-\mu_2^2}{4}M + \frac{1+\mu_2^2}{4}L \quad (20)$$

$$\mathbb{E}\left[P_E + P_D|\theta_1 = 0, \theta_2 = 1\right] = \frac{1-\mu_1^2}{4}H + \frac{2+\mu_1^2+\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \quad (21)$$

$$\mathbb{E}\left[P_E + P_D|\theta_1 = 0, \theta_2 = 0\right] = \frac{1-\mu_1^2}{4}H + \frac{2+\mu_1^2-\mu_2^2}{4}M + \frac{1+\mu_2^2}{4}L \quad (22)$$

The market value of the asset depends on trades by hedgers and informed traders. A hedger uses securities to hedge his exposure to either $y = H$ or $y = L$, and will trade the appropriate amount of the relevant security for this purpose. An informed trader, who has private information about the value of one of the two securities, will buy or sell the same amount as a hedger to not reveal its type to the market maker. Figures 4 and 5 depict the probability distribution of the trades by traders 1 and 2 conditional on asset type.

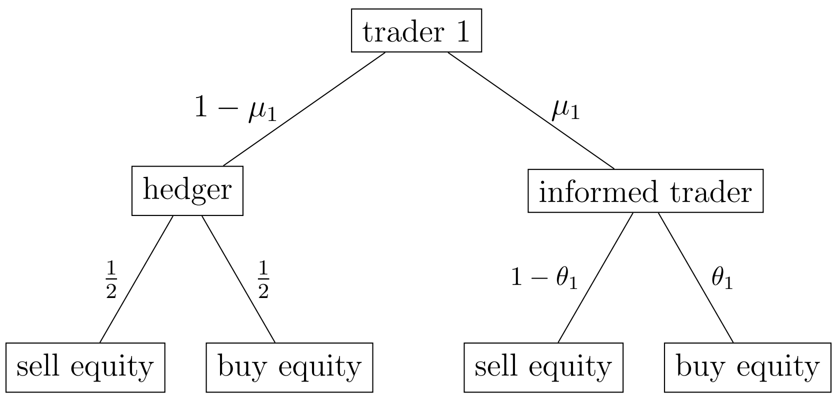

Figure 4: Equilibrium demand for leveraged equity of trader 1.

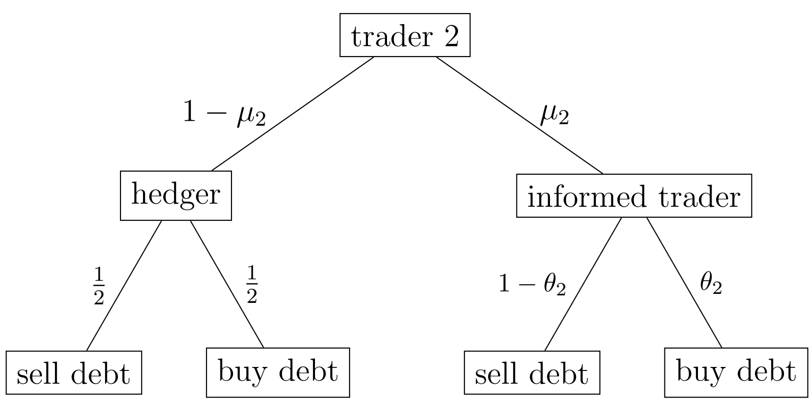

Figure 5: Equilibrium demand for risky debt of trader 2.

Since traders 1 and 2 trade two different securities, the order flow for leveraged equity is informative about θ_1, and the order flow for risky debt is informative about θ_2. In each case, it is all the more informative that the probability of informed trading is high: an increase in the probability of informed trading μ_i increases the expected market value of a high θ_i-type asset.

We show in the proof of Proposition 3 that the adverse selection discount in this equilibrium is always highest for an asset with a high type on both dimensions (a "star" asset). With two information-sensitive securities that are traded by different traders, the total adverse selection discount is simply the sum of the adverse selection discounts associated with each security (leveraged equity and risky debt), and in each case the adverse selection discount is highest for a firm with a high type on the relevant dimension.

3 Equilibrium capital structure

3.1 General results

This section determines the equilibrium security design. In a given equilibrium, the highest relative adverse selection discount (HRASD) plays an important role, as will soon be established. Let $\text{ASD}^1_{\theta_1,\theta_2}$ be the adverse selection discount, as defined in equation (2), when the security design is as in section 2.1. Let $\text{ASD}^2_{\theta_1,\theta_2}$ be the adverse selection discount when the security design is as in section 2.2. Let k_1 be an inverse measure of the highest HRASD across the two security designs described in sections 2.1 and 2.2, and let k_2 be an inverse measure of the lowest HRASD across these two security designs:

$$k_1 \equiv 1 - \max\left\{\max_{\theta_1,\theta_2} \frac{\text{ASD}^1_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}, \max_{\theta_1,\theta_2} \frac{\text{ASD}^2_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}\right\} \text{ and } k_2 \equiv 1 - \min\left\{\max_{\theta_1,\theta_2} \frac{\text{ASD}^1_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}, \max_{\theta_1,\theta_2} \frac{\text{ASD}^2_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}\right\}.$$

The gains from trade arise from the difference in impatience between traders and the firm. They can be measured by $1-\beta$, where β is distributed on $[0, \overline{\beta}]$ and is privately known to the firm. As already established, the pooling equilibria described in Propositions 1-3 are PBE for $\overline{\beta} = 0$, i.e. when gains from trade are very high and public knowledge. When gains from trade are not as large and not public information, however, a firm may decline to sell its asset depending on its asset type. We say that an asset type "participates" when a firm with this asset type accepts the market maker's offer and sells its asset. To focus on the interesting case when the security design offered matters and firms participate in equilibrium, we henceforth make the following assumption on the upper bound $\overline{\beta}$ of β's support.[8]

Assumption 1. $\overline{\beta} \in (k_1, k_2)$.

Assumption 1 implies that the gains from trade are sufficiently large for firms to participate in equilibrium, but potentially sufficiently small that security design matters. If instead $\overline{\beta}$ were lower than in Assumption 1, the market maker could offer the security design described in either

[8]Assumption 1 could equivalently be expressed in terms of model primitives: expressions for the adverse selection discounts are in the proofs of the Propositions as a function of the model's parameters and of intrinsic asset values, as specified in equation (1).

Proposition 1 or Proposition 3 such that a firm with any type of asset would participate for any realization of $\beta \in [0, \overline{\beta}]$. The marker maker would then be indifferent between these two security designs, i.e., there would be two equilibria. If instead $\overline{\beta}$ were higher than in Assumption 1, firms with at least one type of asset would not participate with a positive probability, regardless of the security design offered. Then, the security designs and pricing schedules described in Proposition 1-3 would not be equilibria. Moreover, as is common in models of adverse selection, a market breakdown in which only the worst type participates would be possible.[9] In summary, with Assumption 1, we focus on the case when the adverse selection discount is sufficiently serious to have an effect on security design, but, as is standard in models of security design, not sufficiently serious to prevent firm participation. This is illustrated in Figure 6.

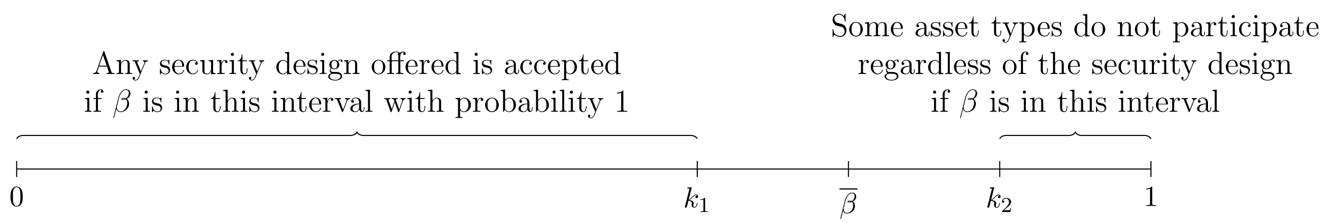

Figure 6: The firm's discount factor β.

We now establish that, despite the adverse selection problem that it faces, the market maker does not offer a menu of security designs.[10]

Lemma 1. *Separating or semi-separating equilibria in which all asset types participate are not PBE.*

Lemma 1 implies that only one security design can be offered in equilibrium, so that firms with different types of assets all use the same set of securities: the only possible equilibrium is a pooling equilibrium. This is a standard result in security design models. We show in the proof of Lemma 1 that there does not exist a separating equilibrium or a semi-separating equilibrium. This result can be explained as follows: if different security designs were offered, for any market maker's beliefs, there would always be at least one asset type with an incentive to deviate from the security design choice corresponding to the market maker's belief. We explain in more details the logic behind this argument in the paragraphs below (we only discuss a subset of possible cases for brevity).

1. Consider a potential equilibrium in which each asset type supposedly chooses a different security design.[11] Then the market valuation of this asset is equal to its intrinsic value in

[9] Indeed, there would be a market breakdown if the gains from trade, as measured by $1-\beta$, were sufficiently low. For example, suppose that $\beta \to 1$. Then in equilibrium, only an asset with the lowest type on both dimensions ($\theta_1 = 0, \theta_2 = 0$) is sold, and its market valuation is $v_{0,0}$ (see section 3 of the Online Appendix).

[10] In a pooling equilibrium, all types of assets are associated with same security design. In a separating equilibrium, each type of asset is associated with a different security design. In a semi-separating equilibrium, a security design is associated with several asset types but not all.

[11] The market maker can offer more than two security designs, by letting γ and/or Γ take different values.

equation (1), regardless of market orders. This gives incentives to the asset with a low type on both dimensions to deviate from its postulated choice, thus invalidating the equilibrium.

2. Consider a potential equilibrium in which assets with a high type on both dimensions and with a low type on both dimensions supposedly both choose a security design with unleveraged equity, whereas the two other asset types supposedly both choose another security design with risky debt and leveraged equity. Then one of these other asset types will be better off deviating. A "high-risk" asset with a high type on the upside but a low type on the downside will be better off deviating when the upside $(H - M)$ is small relative to the downside $(M - L)$, since it is better off using the pricing schedule calibrated for the former subset of asset types. On the contrary, a "steady" asset with a high type on the downside but a low type on the upside will be better off deviating when the downside is small relative to the upside.

3. Consider a potential equilibrium in which a "high-risk" asset supposedly chooses a security design with risky debt and leveraged equity, whereas other asset types supposedly all choose a security design with unleveraged equity. Then the choice of risky debt and leveraged equity leads to a market valuation of $\frac{H+L}{2}$ (see equation (1)). Moreover, if the upside is large relative to the downside, an asset with a low type on both dimensions will be better off deviating and choosing risky debt and leveraged equity to benefit from the pricing schedule calibrated for a "high risk" asset. Otherwise, a "high-risk" asset will be better off deviating and choosing unleveraged equity to benefit from the pricing schedule calibrated for the other asset types.

Lemma 1 implies that the only type of equilibrium in which the market maker can set security prices subject to the fair pricing condition is an equilibrium in which only one security design is offered, as in Propositions 1-3. Proposition 4 describes how the equilibrium of the model depends on the ratio of the maximum adverse selection discount to intrinsic asset value in the potential equilibria described in Propositions 1-3.

Proposition 4. *When* $\max_{\theta_1,\theta_2} \frac{ASD^1_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} \geq \max_{\theta_1,\theta_2} \frac{ASD^2_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}$*, the market maker offers the security design described in Proposition 3; in particular, this is the case when* $\max\{\mu_1 + \mu_2 \frac{M-L}{H-M}, \mu_2 + \mu_1 \frac{H-M}{M-L}\} > 2$*. Otherwise, the market maker offers the security design described in Proposition 1.*

The market maker offers the security design that minimizes the highest ratio of the adverse selection discount to intrinsic asset value across asset types, i.e., such that the most extreme undervaluation as a fraction of intrinsic asset value is least severe. This ensures that the firm will accept the security design offered in equilibrium regardless of its asset type. In particular, this necessitates that informed traders actually trade based on their information, which rules out the outcome described in Proposition 2. Using the expressions for intrinsic asset values in equation (1) and expressions for the adverse selection discounts derived in the proofs of Propositions 1-3 shows how the equilibrium depends on the exogenous parameters of the model. These exogenous parameters capture the relative amounts of "upside" $H-M$, "downside" $M-L$, and the intensity of informed trading on these two dimensions, as measured by μ_1 and μ_2.[12]

The equilibrium outcome is characterized by fair pricing and an efficient allocation in which the gains from trade are realized. Thus, allocative efficiency could not be improved by postulating alternative mechanisms.

3.2 Numerical illustrations

Figures 7 to 9 below illustrate how the equilibrium outcome depends on the exogenous parameters of the model. The first column in Figures 7 to 9 depicts the equilibrium security design as a function of μ_1 and μ_2 for several values of the upside and the downside. In section 4 of the Online Appendix, we provide three-dimensional depictions of the highest adverse selection discount to intrinsic asset value as a function of μ_1 and μ_2 for each of the two capital structures considered in sections 2.1 and 2.2, for several values of the upside and the downside.

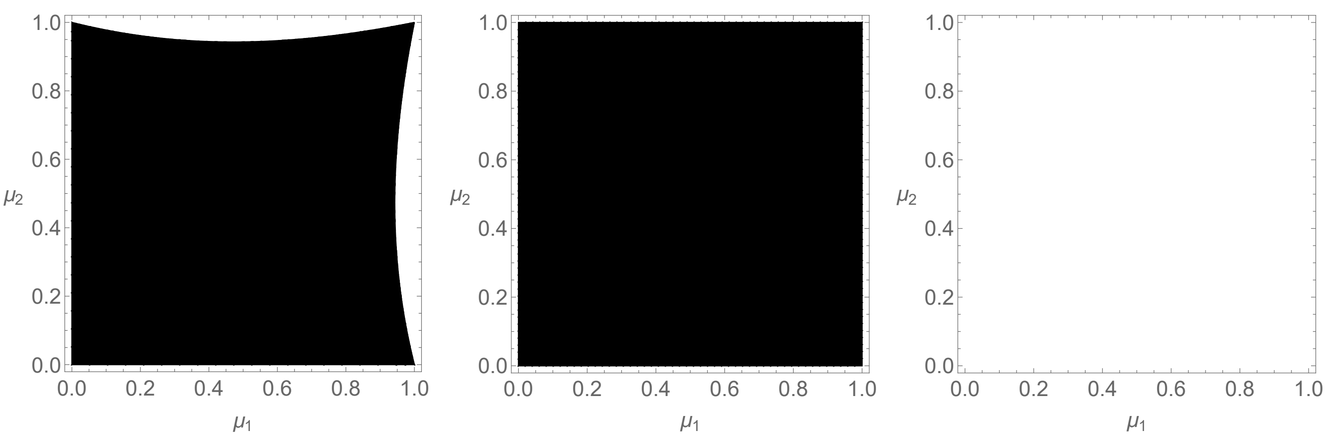

Figure 7: Balanced upside and downside.

In all cases: $H - M = 1$, $M - L = 1$. Panels are described from left to right. First column: the black (white) area is the subset of $\{\mu_1, \mu_2\}$ such that the equilibrium is as in Proposition 3 (Proposition 1). Second column: the black area is such that $\mathrm{ASD}^i_{1,1}/v_{1,1}$ is higher with $i = 1$ (unleveraged equity) than with $i = 2$ (leveraged equity and risky debt). Third column: the black area is such that $\max\left\{\mu_2 + \mu_1\frac{H-M}{M-L}, \mu_1 + \mu_2\frac{M-L}{H-M}\right\} > 2$.

[12]Our assumptions ensure that there are two dimensions of uncertainty in the model. To consider the outcome with only one dimension of uncertainty, consider an alternative case with $M - L = 0$, so that output realizations with a positive probability are M (with probability $1-\theta_1/2$) and H (with probability $\theta_1/2$). Any capital structure includes a security whose value is sensitive to θ_1, but no capital structure includes a security whose value is sensitive to θ_2. As a result, the investor informed about θ_1 will trade whereas the investor informed about θ_2 will not trade. In this case, we will have $k_1 = k_2$, and Assumption 1 is replaced by $\overline{\beta} = k_1$. Without additional assumptions on the cost of issuing additional securities, there are multiple equilibria. Indeed, any capital structure will elicit trading from the investor informed about θ_1, who is the only informed investor, and therefore will yield the same expected valuation. An equilibrium is for the firm to issue equity and riskfree debt, which is as in Boot and Thakor (1993).

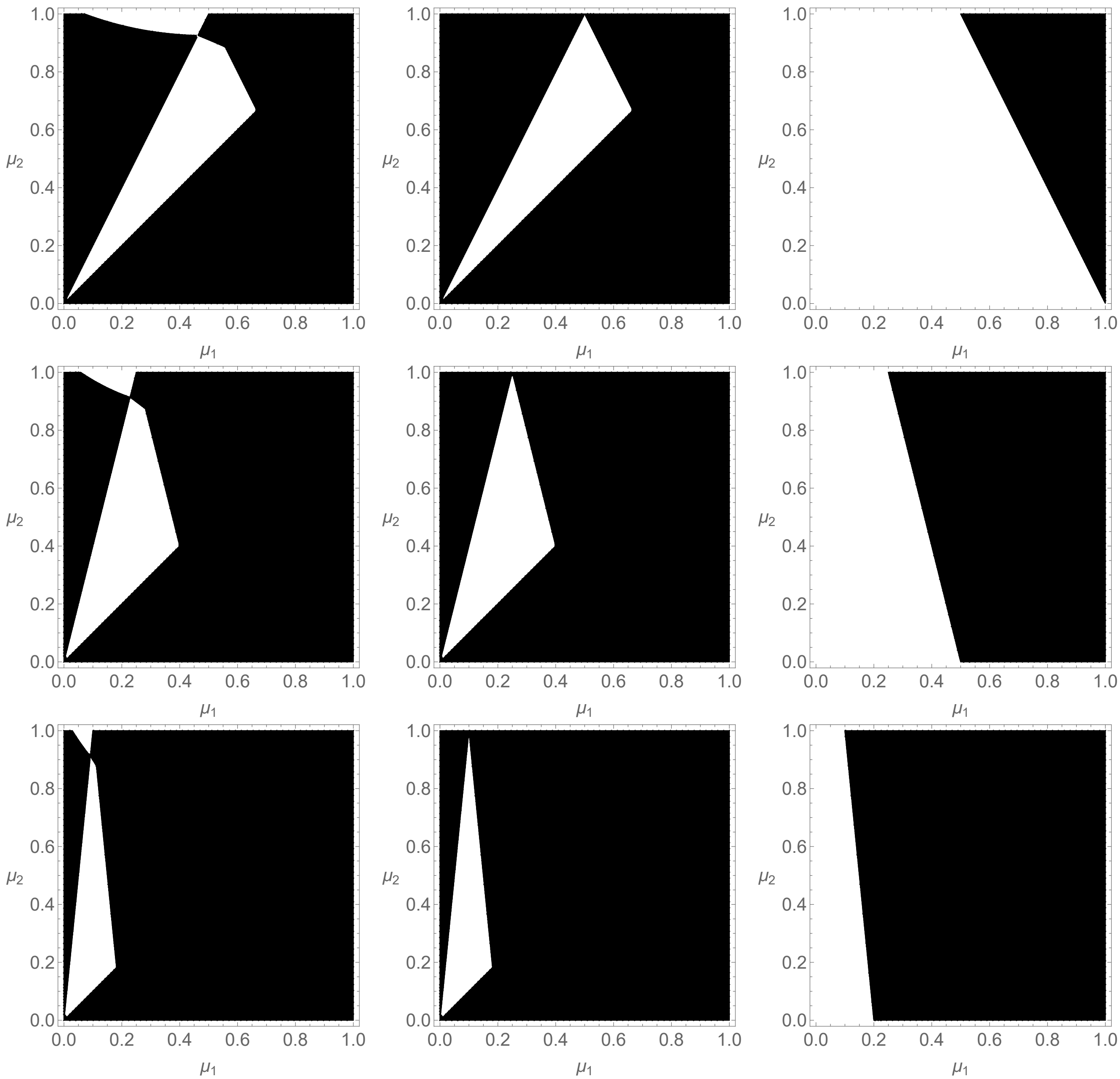

Figure 8: High upside, low downside.

Top row: $H - M = 1$, $M - L = 0.5$. Middle row: $H - M = 1$, $M - L = 0.25$. Bottom row: $H - M = 1$, $M - L = 0.1$. Panels are described from left to right. First column: the black (white) area is the subset of $\{\mu_1, \mu_2\}$ such that the equilibrium is as in Proposition 3 (Proposition 1). Second column: the black area is such that $\mathrm{ASD}^i_{1,1}/v_{1,1}$ is higher with $i = 1$ (unleveraged equity) than with $i = 2$ (leveraged equity and risky debt). Third column: the black area is such that $\max\left\{\mu_2 + \mu_1 \frac{H-M}{M-L}, \mu_1 + \mu_2 \frac{M-L}{H-M}\right\} > 2$.

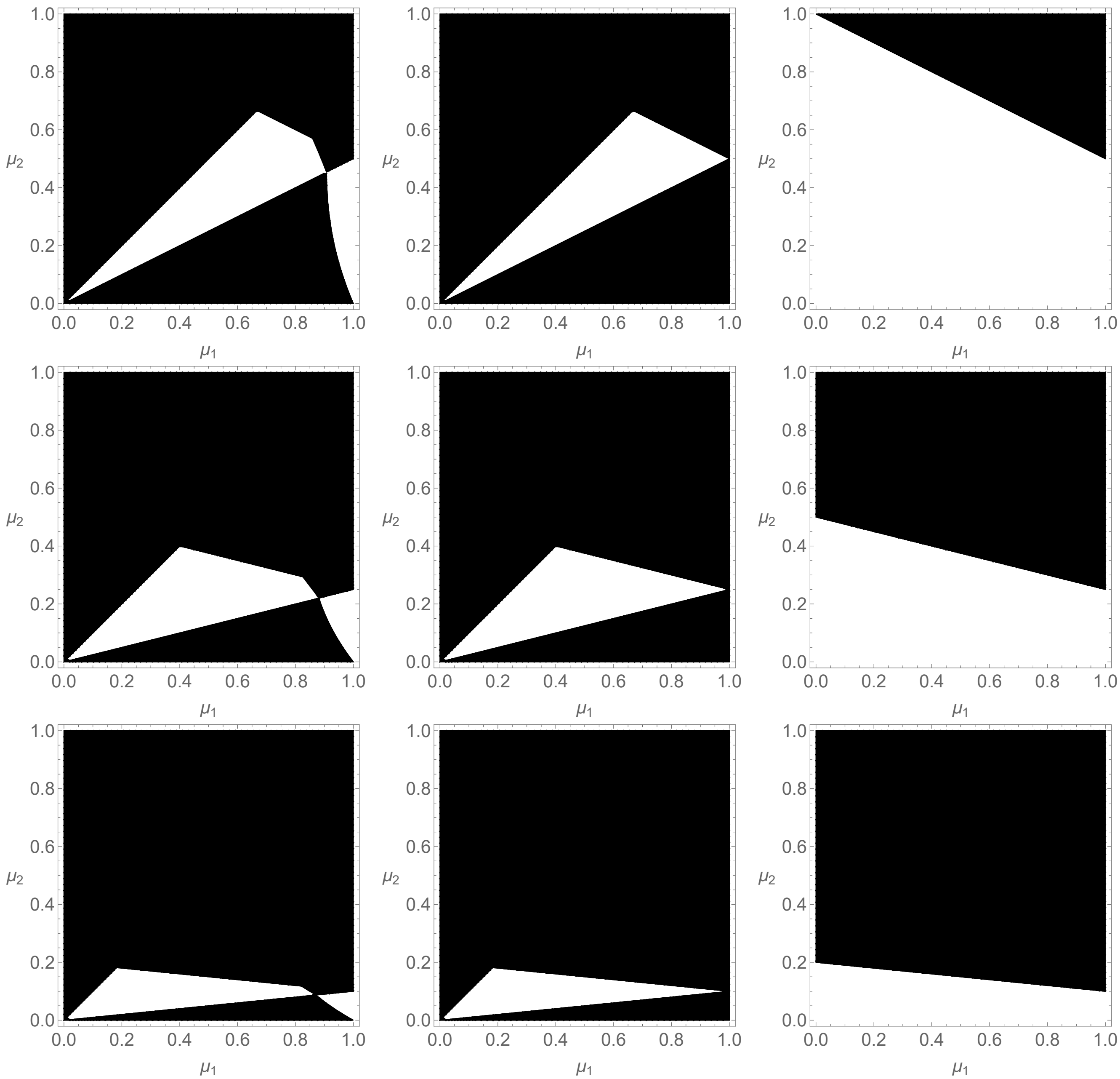

Figure 9: Low upside, high downside.

Top row: $H - M = 0.5$, $M - L = 1$. Middle row: $H - M = 0.25$, $M - L = 1$. Bottom row: $H - M = 0.1$, $M - L = 1$. Panels are described from left to right. First column: the black (white) area is the subset of $\{\mu_1, \mu_2\}$ such that the equilibrium is as in Proposition 3 (Proposition 1). Second column: the black area is such that $\text{ASD}^i_{1,1}/v_{1,1}$ is higher with $i = 1$ (unleveraged equity) than with $i = 2$ (leveraged equity and risky debt). Third column: the black area is such that $\max\left\{\mu_2 + \mu_1 \frac{H-M}{M-L}, \mu_1 + \mu_2 \frac{M-L}{H-M}\right\} > 2$.

3.3 Discussion of results

This section explains the effects that drive the equilibrium security design.

First, the equilibrium security design is such that all informed traders trade on their information (see the explanation that follows Proposition 2). This rules out a security design with only one information-sensitive security for a set of parameter values which is depicted as the black area in the third column of Figures 8-9. As the value relevance of the upside and the downside diverge more, this set of parameter values expands. This results in a security design with leveraged equity and risky debt for more values of $\{\mu_1, \mu_2\}$. In the case when either the upside or the downside is extremely small, this effect dominates for almost all values of $\{\mu_1, \mu_2\}$, and the security design is almost always leveraged equity and risky debt.

Second, the equilibrium security design is not always driven by a minimization of the undervaluation of an asset with a high type on both dimensions. Consider the case when the probability of informed trading on the less value relevant dimension of uncertainty is very high and the probability of informed trading on the more value relevant dimension of uncertainty is low. Then an asset with a low type on the less value relevant dimension of uncertainty and a high type on the more value relevant dimension of uncertainty faces a very high relative adverse selection discount when leveraged equity and risky debt are traded. Indeed, its high type on one dimension is barely revealed (because of very low informed trading on this dimension) whereas its low type on the other dimension is largely revealed (because of very high informed trading on this other dimension).[13] The adverse selection discount faced by this asset type in this case is lessened when all traders trade the same security – (unleveraged) equity. This explains the small white areas either at the top or to the right in the first column of Figures 7-9.

Third, for parameter values that do not correspond to the first two cases described above, the equilibrium security design is the one that minimizes the adverse selection discount for an asset with a high type on both dimensions (type $\{\theta_1, \theta_2\} = \{1, 1\}$), i.e., it is the one hypothetically preferred by a firm with this type of asset. There are three possible subcases.

First, when the probability of informed trading on the more value relevant dimension of uncertainty is higher than the probability of informed trading on the less value relevant dimension of uncertainty, the security design is leveraged equity and risky debt. Intuitively, in this case, having separate markets for information transmission on the upside and the downside maximizes the price impact of information on the more value relevant dimension of uncertainty. In Figure 8, this corresponds to the area below the 45 degrees line in the first and second columns. In Figure 9, this corresponds to the area above the 45 degrees line in the first and second columns.

Second, when the probability of informed trading on the less value relevant dimension of uncertainty is slightly higher than the probability of informed trading on the more value relevant dimension, an asset with a high type on both dimensions achieves a higher expected valuation in the equilibrium of Proposition 1 with only one information-sensitive security (equity). The

[13] Even though $\text{ASD}^2_{i,j}$ is always highest for $i = 1$ and $j = 1$ as noted in section 2.2, $\text{ASD}^2_{i,j}/v_{i,j}$ is not always highest for this asset type, as noted in section 2.1.

intuition for this result is as follows. Consider the case when the upside is substantially more value relevant than the downside, i.e. $H - M > M - L$ (as in Figure 8), for example because the asset has a high liquidation value, and there is slightly more informed trading about the downside than about the upside ($\mu_2 > \mu_1$). In the equilibrium of Proposition 3, the order from the trader potentially informed about the downside, while more informative, does not matter as much for overall asset valuation, whereas the order from the trader potentially informed about the upside, while less informative, matters more for asset valuation. In the equilibrium of Proposition 1, by contrast, orders from all traders are pooled in one market, so that any order is ambiguous in the sense that it could emanate from a trader informed about the upside or about the downside. Moreover, for an asset with a high type on both dimensions, when $\mu_2 > \mu_1$, buy orders from traders informed about the downside are more likely than from those informed about the upside. In this case, the expected market value of this asset type is larger in an equilibrium as in Proposition 1 with equity (and possibly riskfree debt) than in an equilibrium as in Proposition 3. This is the *intra-subsidy effect* of pooling trading, which is beneficial to the valuation of assets with a high type on the less value relevant dimension when there is more informed trading on this dimension. This effect is apparent in the triangular white areas in the second columns of Figure 8 and 9. It plays an important role in driving equilibrium selection as depicted in the first column of each Figure. This effect only arises when the upside and the downside differ (it does not arise in the case depicted in Figure 7), and when these two dimensions of uncertainty both matter substantially for asset valuation (it does not arise when $M - L \approx 0$ or $H - M \approx 0$).

Third, there is a countervailing effect. When the probability of informed trading on the less value relevant dimension of uncertainty is much higher than the probability of informed trading on the more value relevant dimension, an asset with a high type on both dimensions achieves a higher expected valuation with the security design of Proposition 3 with leveraged equity and risky debt (see the black area to the left of the white area in the first and second columns of Figures 8, and below it in Figure 9). Intuitively, in this case, the market maker knows that, with the security design of Proposition 1, orders are either largely random or much more likely to come from a trader informed about the less value relevant dimension of uncertainty. Thus, the market price of (unleveraged) equity is not very sensitive to market orders. This is the *intra-noisiness effect* of pooling trading, which is detrimental to the valuation of assets with a high type on the less value relevant dimension of uncertainty when there is much more informed trading on this dimension. In this case, the valuation of these assets is higher in an equilibrium as in Proposition 3 with leveraged equity and risky debt, since their market valuation better reflects their intrinsic value on the less value relevant dimension of uncertainty. This effect only arises when the upside and the downside differ (it does not arise in the case depicted in Figure 7), and when these two dimensions of uncertainty both matter substantially for asset valuation (it does not arise when $M - L \approx 0$ or $H - M \approx 0$).

4 Empirical implications

4.1 General implications

The model does not predict a unique capital structure: assets which differ either fundamentally because of the relative magnitude of their upside and downside, or informationally as reflected by varying trading intensities on these two dimensions of uncertainty, can be associated with different capital structures.[14] Moreover, when the capital structure involves different types of information-sensitive securities – risky debt and equity – the model predicts that these two securities will be traded by different investors with different information.

The most important and unique empirical implication of the model is that security design will involve a single information-sensitive security, (unleveraged) equity, only when there is less informed trading on the more value relevant dimension of uncertainty (upside or downside). On the contrary, when there is more informed trading on the more value relevant dimension of uncertainty, then the capital structure will generally involve two information-sensitive securities – leveraged equity and risky debt.

Figure 10 depicts the highest adverse selection discount to asset value across asset types (HRASD) when the capital structure involves either one or two information-sensitive securities.[15] Holding L and H constant at $L = 0$ and $H = 2$, we let M vary from $M = 0.01$ to $M = 1.99$. This allows to study cases with approximately no downside ($M = 0.01$), approximately no upside ($M = 1.99$), and anything in-between. The middle panel shows that, when the intensity of informed trading is the same for the upside and the downside ($\mu_1 = \mu_2$), it is always an equilibrium outcome to have equity and risky debt (the solid line is always below the dashed line in a weak sense). On the contrary, when the intensity of informed trading is different on the upside and the downside, the equilibrium outcome involves equity (and possibly riskfree debt) for some intermediate values of M. When there is more informed trading on the downside (left panel), this is true when the upside is a bit larger than the downside ($M \in [0.57, 0.80]$). When there is more informed trading on the upside (right panel), this is true when the downside is a bit larger than the upside ($M \in [1.20, 1.42]$).

[14] In our model, all types of assets that differ along unobservable vertical dimensions (high/low quality) use the same security design, but types of assets that differ along observable horizontal dimensions (relative magnitude of the upside potential and downside risk, and information of investors on these two dimensions) may use different security designs.

[15] The dashed line consists of three distinct discontinuous parts. The two parts on the left and right of each panel represent the HRASD when one informed trader does not participate (i.e. the condition in Proposition 2 is satisfied). As noted below Proposition 2, this case is never an equilibrium outcome, which is consistent with the solid line always being below these two parts of the dashed line.

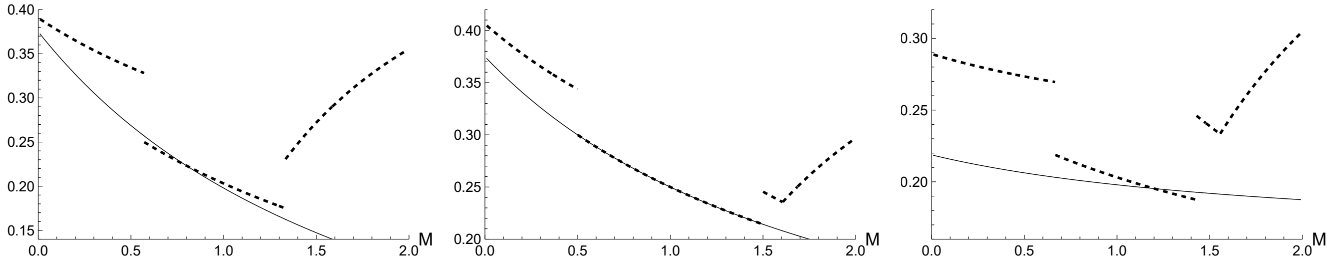

Figure 10: Highest adverse selection discount to asset value across asset types with two different capital structures.
Dashed line: HRASD as a function of M when the capital structure involves equity (and possibly riskfree debt). Solid line: HRASD as a function of M when the capital structure involves equity and risky debt. Left panel: $\mu_1 = 0.50$, $\mu_2 = 0.75$. Middle panel: $\mu_1 = 0.50$, $\mu_2 = 0.50$. Right panel: $\mu_1 = 0.75$, $\mu_2 = 0.50$. In all cases, $L = 0$ and $H = 2$.

Figure 10 also emphasizes that when the upside is very small relative to the downside, or conversely when the downside is very small relative to the upside, it is especially important to issue equity and risky debt. This ensures that all informed investors trade on their information. Otherwise, if the firm were to only issue equity and riskfree debt, some informed investors (either those informed about the upside or those informed about the downside) would not trade.

4.2 Implications for firm leverage

This section discusses the relation between our results and empirical findings on firms' capital structures.

First, our results have implications for the use of risky debt through the life cycle of a firm. The typical life cycle of a growing firm can be captured by an increase in the parameter M, holding other parameters (L, H, μ_1, μ_2) constant. A small young firm with barely any assets has a very limited downside: M is close to L. At the other end of the spectrum, a very large and mature firm has barely any upside: M is close to H. As illustrated in Figure 10, the model predicts that both young and mature firms will have a capital structure that involves risky debt. In our model, "risky debt" refers to debt that might not be fully repaid (e.g. "speculative" or "high yield" debt), so that information about the firm's "downside" is valuable for investors. By contrast, firms at an intermediate stage of their life cycle will not issue risky debt, although they might still issue riskfree debt (e.g. highly rated bonds). When there is more informed trading on the downside than on the upside, this intermediate stage occurs earlier in the life cycle than when there is more informed trading on the upside. Although more research is needed, this predicted nonnomotonic relation between risky debt issuance and firm age or size is consistent with the empirical evidence that leverage is high for young firms, then declines, then rises again as the firm becomes larger.[16]

[16] Fluck et al. (1997) find that external finance declines over the early stages of a business' life cycle, then increases thereafter. Berger and Udell (1998) find that "debt held by the principal owner through loans and credit debt as well as debt held by other individuals (mostly family and friends) decline as the firm matures and retires these insider loans that were needed in the early stages." DeAngelo, Gonçalves, and Stulz (2018) show that after the leverage ratio reaches a peak, the median firm reduces its leverage to near zero in about six years. Hovakimian,

It could also explain why the sign of regression coefficients in empirical studies will vary depending on the sample of firms (e.g. small private firms or large publicly listed firms) and the variables used, notably whether they are backward looking book values or forward looking market values (see Frank and Goyal (2009)).

Second, our results can contribute to explain the finding by Strebulaev and Yang (2013) that about 10% of large public nonfinancial US firms have zero debt – the "zero leverage puzzle" which they argue has yet to be explained.[17] They find that large firms with zero leverage tend to be more R&D intensive, with a higher market-to-book ratio, less capex and less tangible assets, which is suggestive of innovative firms which are subject to especially strong information asymmetries about their upside potential.[18]

In our model, the equilibrium capital structure can be unleveraged equity when the probability of informed trading is lower for the more value-relevant dimension of uncertainty. This case is arguably relevant for innovative firms in which information about the downside, which may involve an assessment of the liquidation value of assets in place, is more readily available than information about the upside, which may instead involve an assessment of new technologies developed by the firm. Thus, our model can explain why a capital structure with zero leverage would be more prevalent in innovative firms, which is consistent with the findings in Strebulaev and Yang (2013).

Third, the prediction that some firms do not issue risky debt has implications for firms' adjustment toward a target capital structure. A simple tradeoff theory model would predict an inverse relation between existing leverage and net debt issuance. By contrast, our model predicts that some firms will not issue risky debt. They may still issue approximately riskfree debt, notably to benefit from tax savings, and therefore adjust toward a low level of indebtedness. On the contrary, other firms will issue substantial amounts of risky debt to maintain their high leverage. This is consistent with a nonmonotonic empirical relation between existing leverage and subsequent debt

Opler, and Titman (2001) find that firms with better growth opportunities are more likely to issue equity rather than debt and to repurchase debt rather than equity. Dinlersoz et al. (2019) find that, for publicly listed firms, the relation between leverage and firm size is ∪-shaped, whereas Rajan and Zingales (1995) and Frank and Goyal (2009) document a positive relation between leverage and firm size among publicly listed firms.

[17] They write: "The way theoretical work has typically addressed the low-leverage puzzle is by considering plausible economic forces that would drive the optimal average leverage ratio down (e.g., Goldstein, Ju, and Leland, 2001). However, this reconciles empirical facts with theory only insofar as average leverage ratios are equated. What we show is that to explain the low-leverage puzzle one needs to explain why some firms tend not to have debt at all instead of why firms on average have lower outstanding debt than expected, and most of extant models fail on this dimension."

[18] Alternative explanations for the zero leverage puzzle generate different empirical implications. Equity can be the optimal security in theories of security design in which the firm only sells part of the cash flows generated by the asset for funding purposes. In particular, the reverse pecking order theory model of Fulghieri, García, and Hackbarth (2020) predicts that firms with stronger information asymmetries on the upside ("growth options") than on the downside ("assets in place") will issue risky debt rather than equity, i.e. they will not have zero leverage. Our model generates the opposite prediction that this type of firm may issue (unleveraged) equity. Other papers in this literature do not consider the upside and the downside aspects of firm valuation. They find that firms will issue equity when firm insiders such as entrepreneurs are less informed than professional investors (Axelson (2007), Yang and Zeng (2019)) or in young firms subject to strong uncertainty (Malenko and Tsoy (2020)). In Fulghieri and Lukin (2001), selling equity to outside investors stimulates information acquisition by investors, which is especially relevant for private firms which do not have as much equity research coverage as public firms. The information environment that we consider can rationalize zero leverage in more established public firms.

issuance: Dangl and Zechner (2021) find a change in net debt which is ∪-shaped when moving from low to high leverage buckets.

Fourth, our results can shed light on an important limitation of the tradeoff theory of the capital structure. According to Myers (1993), "The most telling evidence against the static trade-off theory is the strong inverse correlation between profitability and leverage." This limitation has inspired a large literature (e.g. Frank and Goyal (2009), Danis, Rettl, and Whited (2014), Eckbo and Kisser (2021)). Our model can explain this negative correlation, in a setting where the capital structure is not affected by funding requirements or investment decisions.[19] Indeed, when the upside is more relevant for firm valuation but is subject to strong information asymmetries, the capital structure may not involve any risky debt, i.e. leverage will be comparatively low. There is some evidence that firms with a valuable upside which is subject to strong information asymmetries are highly innovative and profitable (Geroski, Machin, and Van Reenen (1993), Healy and Palepu (2001), Cefis and Ciccarelli (2005)).

We conclude this section by providing suggestions and guidance for future empirical work. An ideal empirical setup would distinguish between the following groups of firms, in decreasing order of the upside/downside ratio: (i) small innovative firms with a plausible growth trajectory (as opposed to small firms that will remain small by design) and an inconsequential amount of existing assets; (ii) young but rapidly growing innovative firms with some assets; (iii) mature firms with valuable assets that can still grow; (iv) very large and dominant firms with little upside potential but substantial downside. In addition, an ideal empirical setup would have measures of information asymmetry about the upside and the downside of an asset. For example, the upside of a firm can be viewed as its growth opportunities, whereas its downside can be viewed as the value of its assets in liquidation. This is in contrast to empirical studies that developed measures of information asymmetry about an asset or a firm as a whole, notably to test the applicability and relevance of the pecking order theory. Finally, some predictions require a distinction between riskfree debt and risky debt, which generally cannot be boiled down to a measure of firm leverage. For example, some firms with high leverage may also have valuable non-firm-specific assets that can be used as collateral, so that their debt may still be approximately risk-free.

5 Conclusion

This paper studies the standard problem of maximizing the market valuation of an asset by partitioning its cash flows into securities. It is well-known that, when assets differ along one dimension of uncertainty, the capital structure is designed to maximize information revelation via security prices: informed investors trade information-sensitive equity, whereas uninformed investors trade information-insensitive debt (Boot and Thakor (1993)). This is beneficial to a high-quality asset, which faces the highest adverse selection discount. In this paper, we have

[19] The pecking order theory already provides an explanation for this relation when security issuance is dictated by funding needs: more profitable firms generate more cash internally, and therefore do not need to issue debt to fund investment.

analyzed the case in which assets differ along two dimensions of uncertainty, the upside and the downside, and different investors are informed about different dimensions. Even though it is possible to partition the asset's cash flows so that each security is exposed to a different dimension of uncertainty, we have shown that the compelling logic of Boot and Thakor (1993) cannot be straightforwardly extended to this case. Indeed, tranching the asset's cash flows into different securities to maximize information revelation does not always maximize the valuation of an asset with a high quality on both dimensions, nor does it always minimize the highest relative adverse selection discount across asset types.

A novel empirical implication is that capital structures will involve a single information-sensitive security (equity) only when there is more informed trading on the less value relevant dimension of uncertainty. We have argued that this case may be relevant for innovative firms subject to strong information asymmetries about their upside, and that it can contribute to explain the zero leverage puzzle. It can also help explain the negative correlation between firm profitability and leverage observed empirically, and the cessation of risky debt issuance by firms at an intermediate stage of their life cycle. In other cases, including for young and mature firms, the model predicts that capital structures will generally involve an information-sensitive security exposed to the upside – leveraged equity – and another one exposed to the downside – risky debt.

The model presented in this paper could be extended along several dimensions. The modelization of uncertainty about the upside and the downside could be used to study other research questions, including those that involve moral hazard. It would also be interesting to consider the case with multiple assets and take into consideration that correlations across assets are different with respect to the upside and the downside. Future research could also consider the security design implications of other dimensions of uncertainty, such as those considered in Smith (2019).

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A Proofs

Proof of Proposition 1:

Suppose that the market maker offers securities with payoff vectors (γ, γ, γ) and $(L-\gamma, M-\gamma, H-\gamma)$ for $\gamma \in [0, L]$, associated with subscripts γ and y, respectively. Denote by p_γ the $t=0$ price of the former, and by p_y the $t=0$ price of the latter. Denote by x_γ a trader's demand for the former, and by x_y a trader's demand for the latter.

Consider the optimization problem of a hedger with negative exposure to $y = H$:

$$\min_{x_y, x_\gamma} var\left(\tilde{w}_H^-(x_y, x_\gamma)\right), \text{ where } w_H^-(x_y, x_\gamma) = \begin{cases} x_y(L-\gamma-\tilde{p}_y) + x_\gamma(\gamma - p_\gamma) & \text{for } y = L \\ x_y(M-\gamma-\tilde{p}_y) + x_\gamma(\gamma - p_\gamma) & \text{for } y = M \\ -\delta_H + x_y(H-\gamma-\tilde{p}_y) + x_\gamma(\gamma - p_\gamma) & \text{for } y = H \end{cases}, \quad (23)$$

where $\tilde{p}_y = \overline{p}_y$ with probability $\frac{1}{2}$ (the unconditional probability with which the other trader is buying equity), and $\tilde{p}_y = \underline{p}_y$ with probability $\frac{1}{2}$ (the unconditional probability with which the other trader is selling equity), with $\overline{p}_y > \underline{p}_y$. Given the zero riskfree rate we have $p_\gamma = \gamma$, so that x_γ is irrelevant; let $x \equiv x_y$ to alleviate notations. Using $var(\tilde{w}) = \mathbb{E}[\tilde{w}^2] - (\mathbb{E}[\tilde{w}])^2$, and using notations $l_1 \equiv L-\gamma-\underline{p}_y$, $m_1 \equiv M-\gamma-\underline{p}_y$, $h_1 \equiv H-\gamma-\underline{p}_y$, $l_2 \equiv L-\gamma-\overline{p}_y$, $m_2 \equiv M-\gamma-\overline{p}_y$, $h_2 \equiv H-\gamma-\overline{p}_y$, the optimization problem rewrites as:

$$\min_x \sum_{i=1,2} \frac{1}{2}\left(\frac{1}{4}x^2 l_i^2 + \frac{1}{2}x^2 m_i^2 + \frac{1}{4}(-\delta_H + x h_i)^2 - \left(\frac{1}{4}x l_i + \frac{1}{2}x m_i + \frac{1}{4}(-\delta_H + x h_i)\right)^2\right) \quad (24)$$

The first-order condition (FOC) is:

$$\sum_{i=1,2} \frac{1}{2}\left(\frac{1}{2}x l_i^2 + x m_i^2 + \frac{1}{2}(x h_i^2 - h_i\delta_H) - 2\left(\frac{1}{4}l_i + \frac{1}{2}m_i + \frac{1}{4}h_i\right)\left(\frac{1}{4}x l_i + \frac{1}{2}x m_i + \frac{1}{4}(-\delta_H + x h_i)\right)\right) = 0$$

The second-order condition is:

$$\sum_{i=1,2} \frac{1}{2}\left(\frac{1}{2}l_i^2 + m_i^2 + \frac{1}{2}h_i^2 - 2\left(\frac{1}{4}l_i + \frac{1}{2}m_i + \frac{1}{4}h_i\right)^2\right) > 0, \quad (25)$$

where the inequality can be explained as follows. Let the random variable $\tilde{Y}_i$ be equal to l_i with probability $\frac{1}{4}$, to m_i with probability $\frac{1}{2}$, and to h_i with probability $\frac{1}{4}$; the term in parentheses on the LHS of equation (25) is equal to twice the variance of $\tilde{Y}_i$ with the formula $var(\tilde{Y}_i) = \mathbb{E}[\tilde{Y}_i^2] - (\mathbb{E}[\tilde{Y}_i])^2$, and its variance is strictly positive and independent from i, so that we can write $var(\tilde{Y}_i) = var(\tilde{Y})$. In the minimization problem in equation (24), the objective function is convex for any x, so that the FOC is necessary and sufficient for a minimum. Solving the FOC for x, the optimum x^* is such that:

$$\sum_{i=1,2}\left(x^*\left[\frac{1}{2}l_i^2+m_i^2+\frac{1}{2}h_i^2\right]-\frac{1}{2}h_i\delta_H-2x^*\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)-\frac{1}{2}\delta_H\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)\right)=0$$

$$\Leftrightarrow \quad x^*=\frac{\sum_{i=1,2}\left(\frac{1}{2}h_i\delta_H+\frac{1}{2}\delta_H\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)\right)}{\sum_{i=1,2}\left(\frac{1}{2}l_i^2+m_i^2+\frac{1}{2}h_i^2-2\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)^2\right)}=\frac{\delta_H}{4}\frac{\sum_{i=1,2}\left(h_i-\mathbb{E}[\tilde{Y}_i]\right)}{\sum_{i=1,2}var(\tilde{Y}_i)}=\frac{\delta_H}{4}\frac{H-\mathbb{E}[\tilde{y}]}{var(\tilde{y})}>0.$$

Likewise, for a hedger with positive exposure to $y = H$, the optimal market order for the security with payoff vector described in equation (74) is:

$$x^*=-\frac{\delta_H}{4}\frac{H-\mathbb{E}[\tilde{y}]}{var(\tilde{y})}<0. \tag{26}$$

Consider the optimization problem of a hedger with negative exposure to $y = L$:

$$\min_{x_y,x_\gamma} var\left(\tilde{w}_L^-(x_y,x_\gamma)\right), \text{ where } w_L^-(x_y,x_\gamma)=\begin{cases}-\delta_L+x_y(L-\gamma-\tilde{p}_y)+x_\gamma(\gamma-p_\gamma) & \text{for } y=L\\ x_y(M-\gamma-\tilde{p}_y)+x_\gamma(\gamma-p_\gamma) & \text{for } y=M\\ x_y(H-\gamma-\tilde{p}_y)+x_\gamma(\gamma-p_\gamma) & \text{for } y=H\end{cases}, \tag{27}$$

where $\tilde{p}_y = \hat{p}_y$ with probability $\frac{1}{2}$ (the unconditional probability with which the other trader is buying equity), and $\tilde{p}_y = \check{p}_y$ with probability $\frac{1}{2}$ (the unconditional probability with which the other trader is selling equity), with $\hat{p}_y > \check{p}_y$. Given the zero riskfree rate we have $p_\gamma = \gamma$, so that x_γ is irrelevant; let $x \equiv x_y$ to alleviate notations. Using $var(\tilde{w}) = \mathbb{E}[\tilde{w}^2] - (\mathbb{E}[\tilde{w}])^2$, and using notations $l_1 \equiv L-\gamma-\check{p}_y$, $m_1 \equiv M-\gamma-\check{p}_y$, $h_1 \equiv H-\gamma-\check{p}_y$, $l_2 \equiv L-\gamma-\hat{p}_y$, $m_2 \equiv M-\gamma-\hat{p}_y$, $h_2 \equiv H-\gamma-\hat{p}_y$, the optimization problem rewrites as:

$$\min_x \sum_{i=1,2}\frac{1}{2}\left(\frac{1}{4}(-\delta_L+xl_i)^2+\frac{1}{2}x^2m_i^2+\frac{1}{4}x^2h_i^2-\left(\frac{1}{4}(-\delta_L+xl_i)+\frac{1}{2}xm_i+\frac{1}{4}xh_i\right)^2\right) \tag{28}$$

The first-order condition (FOC) is:

$$\sum_{i=1,2}\frac{1}{2}\left(\frac{1}{2}(xl_i^2-l_i\delta_L)+xm_i^2+\frac{1}{2}xh_i^2-2\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)\left(\frac{1}{4}(-\delta_L+xl_i)+\frac{1}{2}xm_i+\frac{1}{4}xh_i\right)\right)=0$$

The second-order condition is:

$$\sum_{i=1,2}\frac{1}{2}\left(\frac{1}{2}l_i^2+m_i^2+\frac{1}{2}h_i^2-2\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)^2\right)>0, \tag{29}$$

where the reasoning is the same as for equation (25), and the random variable $\tilde{Y}_i$ is defined similarly. Solving the FOC for x, the optimum x^* is such that:

$$\sum_{i=1,2}\left(x^*\left[\frac{1}{2}l_i^2+m_i^2+\frac{1}{2}h_i^2\right]-\frac{1}{2}l_i\delta_L-2x^*\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)-\frac{1}{2}\delta_L\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)\right)=0$$

$$\Leftrightarrow \quad x^*=\frac{\sum_{i=1,2}\left(\frac{1}{2}l_i\delta_L+\frac{1}{2}\delta_L\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)\right)}{\sum_{i=1,2}\left(\frac{1}{2}l_i^2+m_i^2+\frac{1}{2}h_i^2-2\left(\frac{1}{4}l_i+\frac{1}{2}m_i+\frac{1}{4}h_i\right)^2\right)}=\frac{\delta_L}{4}\frac{\sum_{i=1,2}\left(l_i-\mathbb{E}[\tilde{Y}_i]\right)}{\sum_{i=1,2}var(\tilde{Y}_i)}=\frac{\delta_L}{4}\frac{L-\mathbb{E}[\tilde{y}]}{var(\tilde{y})}<0.$$

Likewise, for a hedger with positive exposure to $y=L$, the optimal market order for the security with payoff vector described in equation (74) is:

$$x^*=-\frac{\delta_L}{4}\frac{L-\mathbb{E}[\tilde{y}]}{var(\tilde{y})}>0. \tag{30}$$

With $\delta_H=1/(H-\mathbb{E}[\tilde{y}])$ and $\delta_L=1/(\mathbb{E}[\tilde{y}]-L)$, we have:

$$\frac{\delta_H}{4}\frac{H-\mathbb{E}[\tilde{y}]}{var(\tilde{y})}=-\frac{\delta_L}{4}\frac{L-\mathbb{E}[\tilde{y}]}{var(\tilde{y})}, \qquad \text{and} \qquad -\frac{\delta_H}{4}\frac{H-\mathbb{E}[\tilde{y}]}{var(\tilde{y})}=\frac{\delta_L}{4}\frac{L-\mathbb{E}[\tilde{y}]}{var(\tilde{y})}. \tag{31}$$

That is, hedgers 1 and 2 submit the same market order $x^*=\frac{1}{4}\frac{1}{var(\tilde{y})}$ when they buy the security with payoff vector $(L-\gamma, M-\gamma, H-\gamma)$ ("equity"), and the same market order $x^*=-\frac{1}{4}\frac{1}{var(\tilde{y})}$ when they sell it. Denote these two orders by 'buy', and 'sell', respectively.

Using standard arguments, any informed trader who trades will buy or sell the same quantity as a hedger (see 'buy' and 'sell' orders defined above), otherwise the order would reveal his information to the market maker. An informed trader who observes either $\theta_1=1$ or $\theta_2=1$ will buy, and an informed trader who observes either $\theta_1=0$ or $\theta_2=0$ will sell.

We now take the market maker's perspective. For brevity, we relegate some detailed calculations including Bayesian updating to section 6 of the Online Appendix. For now, we postulate that an informed trader trades based on his information (see below). Then, the asset's market price (the sum of security prices) conditional on observed orders is:

$$P_U(\text{buy},\text{buy}) = \frac{1+\mu_1}{2}\frac{H}{2}+\frac{2-\mu_1+\mu_2}{2}\frac{M}{2}+\frac{1-\mu_2}{2}\frac{L}{2} \tag{32}$$

$$P_U(\text{buy},\text{sell}) = \frac{1}{2}\frac{H}{2}+\frac{M}{2}+\frac{1}{2}\frac{L}{2} \tag{33}$$

$$P_U(\text{sell},\text{sell}) = \frac{1-\mu_1}{2}\frac{H}{2}+\frac{2+\mu_1-\mu_2}{2}\frac{M}{2}+\frac{1+\mu_2}{2}\frac{L}{2} \tag{34}$$

We take firm's perspective, still postulating that an informed trader trades based on his information. The expected market value of asset type $\theta_1 = 1, \theta_2 = 1$ is:

$$\begin{aligned}
& \mathbb{E}\left[P_U|\theta_1 = 1, \theta_2 = 1\right] = Pr\left(\text{buy}, \text{buy}|\theta_1 = 1, \theta_2 = 1\right) P_U\left(\text{buy}, \text{buy}\right) \\
& + Pr\left(\text{buy}, \text{sell}|\theta_1 = 1, \theta_2 = 1\right) P_U\left(\text{buy}, \text{sell}\right) + Pr\left(\text{sell}, \text{sell}|\theta_1 = 1, \theta_2 = 1\right) P_U\left(\text{sell}, \text{sell}\right) \\
= \;& \frac{1+\mu_1}{2}\frac{1+\mu_2}{2} P_U\left(\text{buy}, \text{buy}\right) + \left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right) P_U\left(\text{buy}, \text{sell}\right) \\
& + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2} P_U\left(\text{sell}, \text{sell}\right) \\
= \;& \frac{1+\frac{1}{2}\left(\mu_1^2+\mu_1\mu_2\right)}{4} H + \frac{2+\frac{1}{2}\left(-\mu_1^2+\mu_2^2\right)}{4} M + \frac{1+\frac{1}{2}\left(-\mu_2^2-\mu_1\mu_2\right)}{4} L \qquad (35)
\end{aligned}$$

The expected market value of asset type $\theta_1 = 1, \theta_2 = 0$ is:

$$\begin{aligned}
& \mathbb{E}\left[P_U|\theta_1 = 1, \theta_2 = 0\right] = Pr\left(\text{buy}, \text{buy}|\theta_1 = 1, \theta_2 = 0\right) P_U\left(\text{buy}, \text{buy}\right) \\
& + Pr\left(\text{buy}, \text{sell}|\theta_1 = 1, \theta_2 = 0\right) P_U\left(\text{buy}, \text{sell}\right) + Pr\left(\text{sell}, \text{sell}|\theta_1 = 1, \theta_2 = 0\right) P_U\left(\text{sell}, \text{sell}\right) \\
= \;& \frac{1+\frac{1}{2}\left(\mu_1^2-\mu_1\mu_2\right)}{4} H + \frac{2+\frac{1}{2}\left(-\mu_1^2-\mu_2^2+2\mu_1\mu_2\right)}{4} M + \frac{1+\frac{1}{2}\left(\mu_2^2-\mu_1\mu_2\right)}{4} L \qquad (36)
\end{aligned}$$

The expected market value of asset type $\theta_1 = 0, \theta_2 = 1$ is:

$$\begin{aligned}
& \mathbb{E}\left[P_U|\theta_1 = 0, \theta_2 = 1\right] = Pr\left(\text{buy}, \text{buy}|\theta_1 = 0, \theta_2 = 1\right) P_U\left(\text{buy}, \text{buy}\right) \\
& + Pr\left(\text{buy}, \text{sell}|\theta_1 = 0, \theta_2 = 1\right) P_U\left(\text{buy}, \text{sell}\right) + Pr\left(\text{sell}, \text{sell}|\theta_1 = 0, \theta_2 = 1\right) P_U\left(\text{sell}, \text{sell}\right) \\
= \;& \frac{1+\frac{1}{2}\left(-\mu_1^2+\mu_1\mu_2\right)}{4} H + \frac{2+\frac{1}{2}\left(\mu_1^2+\mu_2^2-2\mu_1\mu_2\right)}{4} M + \frac{1+\frac{1}{2}\left(-\mu_2^2+\mu_1\mu_2\right)}{4} L \qquad (37)
\end{aligned}$$

The expected market value of asset type $\theta_1 = 0, \theta_2 = 0$ is:

$$\begin{aligned}
& \mathbb{E}\left[P_U|\theta_1 = 0, \theta_2 = 0\right] = Pr\left(\text{buy}, \text{buy}|\theta_1 = 0, \theta_2 = 0\right) P_U\left(\text{buy}, \text{buy}\right) \\
& + Pr\left(\text{buy}, \text{sell}|\theta_1 = 0, \theta_2 = 0\right) P_U\left(\text{buy}, \text{sell}\right) + Pr\left(\text{sell}, \text{sell}|\theta_1 = 0, \theta_2 = 0\right) P_U\left(\text{sell}, \text{sell}\right) \\
= \;& \frac{1+\frac{1}{2}\left(-\mu_1^2-\mu_1\mu_2\right)}{4} H + \frac{2+\frac{1}{2}\left(\mu_1^2-\mu_2^2\right)}{4} M + \frac{1+\frac{1}{2}\left(\mu_2^2+\mu_1\mu_2\right)}{4} L \qquad (38)
\end{aligned}$$

The adverse selection discount $\text{ASD}^1_{\theta_1,\theta_2}$, which for any given asset type $\{\theta_1, \theta_2\}$ is equal to $v_{\theta_1,\theta_2} - \mathbb{E}\left[P_U|\theta_1, \theta_2\right]$, is:

$$\begin{aligned}
\text{ASD}^1_{1,1} &= \frac{1-\frac{1}{2}\left(\mu_1^2+\mu_1\mu_2\right)}{4} H - \frac{\frac{1}{2}\left(-\mu_1^2+\mu_2^2\right)}{4} M - \frac{1+\frac{1}{2}\left(-\mu_2^2-\mu_1\mu_2\right)}{4} L \qquad (39) \\
\text{ASD}^1_{1,0} &= \frac{1-\frac{1}{2}\left(\mu_1^2-\mu_1\mu_2\right)}{4} H - \frac{2+\frac{1}{2}\left(-\mu_1^2-\mu_2^2+2\mu_1\mu_2\right)}{4} M + \frac{1-\frac{1}{2}\left(\mu_2^2-\mu_1\mu_2\right)}{4} L \qquad (40) \\
\text{ASD}^1_{0,1} &= -\frac{1+\frac{1}{2}\left(-\mu_1^2+\mu_1\mu_2\right)}{4} H + \frac{2-\frac{1}{2}\left(\mu_1^2+\mu_2^2-2\mu_1\mu_2\right)}{4} M - \frac{1+\frac{1}{2}\left(-\mu_2^2+\mu_1\mu_2\right)}{4} L \qquad (41) \\
\text{ASD}^1_{0,0} &= -\frac{1+\frac{1}{2}\left(-\mu_1^2-\mu_1\mu_2\right)}{4} H - \frac{\frac{1}{2}\left(\mu_1^2-\mu_2^2\right)}{4} M + \frac{1-\frac{1}{2}\left(\mu_2^2+\mu_1\mu_2\right)}{4} L \qquad (42)
\end{aligned}$$

We now verify when an informed trader will trade based on his information. He will do so whenever it is profitable once the price impact is accounted for. A trader 1 who observes $\theta_1 = 1$ will buy based on his information if and only if:

$$\frac{1}{2}H + \frac{1}{4}M + \frac{1}{4}L \geq \frac{1}{2}P_U(\text{buy}, \text{buy}) + \frac{1}{2}P_U(\text{buy}, \text{sell}) \quad \Leftrightarrow \quad 2 \geq \mu_1 + \mu_2 \frac{M - L}{H - M} \tag{43}$$

A trader 1 who observes $\theta_1 = 0$ will sell based on his information if and only if:

$$\frac{3}{4}M + \frac{1}{4}L \leq \frac{1}{2}P_U(\text{buy}, \text{sell}) + \frac{1}{2}P_U(\text{sell}, \text{sell}) \quad \Leftrightarrow \quad 2 \geq \mu_1 + \mu_2 \frac{M - L}{H - M} \tag{44}$$

Likewise, an informed trader 2 will trade based on his information if and only if:

$$2 \geq \mu_2 + \mu_1 \frac{H - M}{M - L} \tag{45}$$

When both equations (43) and (45) hold, any informed trader trades on his information. ■

Proof of Proposition 2:

We analyze two cases: when equation (43) does not hold (and equation (45) holds); when equation (45) does not hold (and equation (43) holds). For brevity, we relegate some detailed calculations including Bayesian updating to section 6 of the Online Appendix.

Suppose that equation (43) does not hold, so that trader 1 does not trade when informed. The market maker is aware that one trade emanates from trader 2 (which occurs with probability μ_1), and that two trades emanate from trader 2 and hedger 1 (which occurs with probability $1 - \mu_1$). Thus:

$$\begin{aligned}
P_U(\text{buy}) &= \frac{H - M}{4} + \frac{1 + \mu_2}{2}M + \frac{1 - \mu_2}{2}\frac{L + M}{2} \\
P_U(\text{sell}) &= \frac{H - M}{4} + \frac{1 - \mu_2}{2}M + \frac{1 + \mu_2}{2}\frac{L + M}{2} \\
P_U(\text{buy}, \text{buy}) &= Pr(\theta_1 = 1, \theta_2 = 1 | \text{buy}, \text{buy})\, v_U(\theta_1 = 1, \theta_2 = 1) \\
&\quad + Pr(\theta_1 = 1, \theta_2 = 0 | \text{buy}, \text{buy})\, v_U(\theta_1 = 1, \theta_2 = 0) \\
&\quad + Pr(\theta_1 = 0, \theta_2 = 1 | \text{buy}, \text{buy})\, v_U(\theta_1 = 0, \theta_2 = 1) \\
&\quad + Pr(\theta_1 = 0, \theta_2 = 0 | \text{buy}, \text{buy})\, v_U(\theta_1 = 0, \theta_2 = 0) \\
&= \frac{1}{2}\frac{H}{2} + \frac{2 + \mu_2}{2}\frac{M}{2} + \frac{1 - \mu_2}{2}\frac{L}{2} && (46) \\
P_U(\text{buy}, \text{sell}) &= \frac{1}{2}\frac{H}{2} + \frac{M}{2} + \frac{1}{2}\frac{L}{2} && (47) \\
P_U(\text{sell}, \text{sell}) &= \frac{1}{2}\frac{H}{2} + \frac{2 - \mu_2}{2}\frac{M}{2} + \frac{1 + \mu_2}{2}\frac{L}{2} && (48)
\end{aligned}$$

Using equations (64) and (65), and equation (35) with $\mu_1 = 0$, the expected market value of an asset with type $\theta_2 = 1$ is:

$$\begin{aligned}\mathbb{E}\left[P_U|\theta_1,\theta_2=1\right] &= \mu_1\left(\frac{H-M}{4}+\frac{3+\mu_2^2}{2}\frac{M}{2}+\frac{1-\mu_2^2}{2}\frac{L}{2}\right)+(1-\mu_1)\left(\frac{1}{4}H+\frac{2+\frac{1}{2}\mu_2^2}{4}M+\frac{1-\frac{1}{2}\mu_2^2}{4}L\right)\\ &= \frac{1}{2}\frac{H}{2}+\frac{2+\mu_2^2\frac{1+\mu_1}{2}}{2}\frac{M}{2}+\frac{1-\mu_2^2\frac{1+\mu_1}{2}}{2}\frac{L}{2} \qquad (49)\end{aligned}$$

Using equations (64) and (65), and equation (36) with $\mu_1 = 0$, the expected market value of an asset with type $\theta_2 = 0$ is:

$$\begin{aligned}\mathbb{E}\left[P_U|\theta_1,\theta_2=0\right] &= \mu_1\left(\frac{H-M}{4}+\frac{3-\mu_2^2}{2}\frac{M}{2}+\frac{1+\mu_2^2}{2}\frac{L}{2}\right)+(1-\mu_1)\left(\frac{1}{4}H+\frac{2-\frac{1}{2}\mu_2^2}{4}M+\frac{1+\frac{1}{2}\mu_2^2}{4}L\right)\\ &= \frac{1}{2}\frac{H}{2}+\frac{2+\mu_2^2\frac{1+\mu_1}{2}}{2}\frac{M}{2}+\frac{1-\mu_2^2\frac{1+\mu_1}{2}}{2}\frac{L}{2} \qquad (50)\end{aligned}$$

Suppose that equation (45) does not hold, so that trader 2 does not trade when informed. The market maker is aware that one trade emanates from trader 1 (which occurs with probability μ_2), and that two trades emanate from trader 1 and hedger 2 (which occurs with probability $1-\mu_2$). Thus:

$$\begin{aligned}P_U(\text{buy}) &= \frac{1+\mu_1}{2}\frac{H-M}{2}+\frac{3}{4}M+\frac{L}{4}\\ P_U(\text{sell}) &= \frac{1-\mu_1}{2}\frac{H-M}{2}+\frac{3}{4}M+\frac{L}{4}\\ P_U(\text{buy},\text{buy}) &= Pr\left(\theta_1=1,\theta_2=1|\text{buy},\text{buy}\right)v_U(\theta_1=1,\theta_2=1)\\ &\quad +Pr\left(\theta_1=1,\theta_2=0|\text{buy},\text{buy}\right)v_U(\theta_1=1,\theta_2=0)\\ &\quad +Pr\left(\theta_1=0,\theta_2=1|\text{buy},\text{buy}\right)v_U(\theta_1=0,\theta_2=1)\\ &\quad +Pr\left(\theta_1=0,\theta_2=0|\text{buy},\text{buy}\right)v_U(\theta_1=0,\theta_2=0)\\ &= \frac{1+\mu_1}{2}\frac{H}{2}+\frac{2-\mu_1}{2}\frac{M}{2}+\frac{1}{2}\frac{L}{2} \qquad (51)\\ P_U(\text{buy},\text{sell}) &= \frac{1}{2}\frac{H}{2}+\frac{M}{2}+\frac{1}{2}\frac{L}{2} \qquad (52)\\ P_U(\text{sell},\text{sell}) &= \frac{1-\mu_1}{2}\frac{H}{2}+\frac{2+\mu_1}{2}\frac{M}{2}+\frac{1}{2}\frac{L}{2} \qquad (53)\end{aligned}$$

Using equations (62) and (63), and equation (35) with $\mu_2 = 0$, the expected market value of an asset with type $\theta_1 = 1$ is:

$$\begin{aligned}\mathbb{E}\left[P_U|\theta_1=1,\theta_2\right] &= \mu_2\left(\frac{1+\mu_1^2}{2}\frac{H-M}{2}+\frac{3}{4}M+\frac{1}{4}L\right)+(1-\mu_2)\left(\frac{1+\frac{1}{2}\mu_1^2}{4}H+\frac{2-\frac{1}{2}\mu_1^2}{4}M+\frac{1}{4}L\right)\\ &= \frac{1+\mu_1^2\frac{1+\mu_2}{2}}{4}H+\frac{2-\mu_1^2\frac{1+\mu_2}{2}}{4}M+\frac{1}{4}L \qquad (54)\end{aligned}$$

Using equations (62) and (63), and equation (37) with $\mu_2 = 0$, the expected market value of an asset with type $\theta_1 = 0$ is:

$$\begin{aligned}\mathbb{E}\left[P_U|\theta_1=0,\theta_2\right] &= \mu_2\left(\frac{1-\mu_1^2}{2}\frac{H-M}{2}+\frac{3}{4}M+\frac{1}{4}L\right)+(1-\mu_2)\left(\frac{1-\frac{1}{2}\mu_1^2}{4}H+\frac{2+\frac{1}{2}\mu_1^2}{4}M+\frac{1}{4}L\right)\\ &= \frac{1-\mu_1^2\frac{1+\mu_2}{2}}{4}H+\frac{2+\mu_1^2\frac{1+\mu_2}{2}}{4}M+\frac{1}{4}L \qquad (55)\end{aligned}$$

■

Proof of Proposition 3:

Suppose that the market maker offers securities with payoff vectors (L, M, M) (risky debt) and $(0, 0, H - M)$ (leveraged equity), associated with subscripts D and E, respectively. Denote by p_D the $t = 0$ price of the former, and by p_E the $t = 0$ price of the latter. Denote by x_D a trader's demand for the former, and by x_E a trader's demand for the latter.

Consider the optimization problem of a hedger with negative exposure to $y = H$:

$$\min_{x_D,x_E} var\left(\tilde{w}_H^-(x_D,x_E)\right), \text{ where } w_H^-(x_D,x_E)=\begin{cases} x_D(L-p_D)+x_E(-p_E) & \text{for } y=L\\ x_D(M-p_D)+x_E(-p_E) & \text{for } y=M\\ -\delta_H+x_D(M-p_D)+x_E(H-M-p_E) & \text{for } y=H\end{cases}$$

A variance of zero is achieved if and only if:

$$\begin{cases} x_D(L-p_D)+x_E(-p_E)=x_D(M-p_D)+x_E(-p_E)\\ x_D(M-p_D)+x_E(-p_E)=-\delta_H+x_D(M-p_D)+x_E(H-M-p_E)\end{cases} \qquad (56)$$

The first equality gives $x_D = 0$, and the second equality gives $x_E = \frac{\delta_H}{H-M}$. Likewise, for a hedger with positive exposure to $y = H$, $x_D = 0$ and $x_E = -\frac{\delta_H}{H-M}$.

Consider the optimization problem of a hedger with negative exposure to $y = L$:

$$\min_{x_D,x_E} var\left(\tilde{w}_L^-(x_D,x_E)\right), \text{ where } w_L^-(x_D,x_E)=\begin{cases} -\delta_L+x_D(L-p_D)+x_E(-p_E) & \text{for } y=L\\ x_D(M-p_D)+x_E(-p_E) & \text{for } y=M\\ x_D(M-p_D)+x_E(H-M-p_E) & \text{for } y=H\end{cases}$$

A variance of zero is achieved if and only if:

$$\begin{cases} -\delta_L+x_D(L-p_D)+x_E(-p_E)=x_D(M-p_D)+x_E(-p_E)\\ x_D(M-p_D)+x_E(-p_E)=x_D(M-p_D)+x_E(H-M-p_E)\end{cases} \qquad (57)$$

The second equality gives $x_E = 0$, and the first equality gives $x_D = -\frac{\delta_L}{M-L}$. Likewise, for a hedger with positive exposure to $y = L$, $x_E = 0$ and $x_D = \frac{\delta_L}{M-L}$.

Using standard arguments, if trader 1 is informed, he will buy or sell the same quantity of leveraged equity as a hedger – either 'buy', corresponding to $x_E = \frac{\delta_H}{H-M}$, or 'sell', corresponding to $x_E = -\frac{\delta_H}{H-M}$ – otherwise the order would reveal his information to the market maker. Likewise, if trader 2 is informed, he will buy or sell the same quantity of risky debt as a hedger – either 'buy', corresponding to $x_D = \frac{\delta_L}{M-L}$, or 'sell', corresponding to $x_D = -\frac{\delta_L}{M-L}$ – otherwise the order would reveal his information to the market maker.

We now take the market maker's perspective. For brevity, we relegate some detailed calculations including Bayesian updating to section 6 of the Online Appendix. The market price of leveraged equity conditional on a buy or sell order is:

$$\begin{aligned} P_E(\text{buy}) &= Pr\left(\theta_1 = 1|\text{buy}\right) v_E(\theta_1 = 1) + Pr\left(\theta_1 = 0|\text{buy}\right) v_E(\theta_1 = 0) = \frac{1+\mu_1}{2}\frac{H-M}{2} \\ P_E(\text{sell}) &= Pr\left(\theta_1 = 1|\text{sell}\right) v_E(\theta_1 = 1) + Pr\left(\theta_1 = 0|\text{sell}\right) v_E(\theta_1 = 0) = \frac{1-\mu_1}{2}\frac{H-M}{2} \end{aligned}$$

The market price of risky debt conditional on a buy or sell order is:

$$\begin{aligned} P_D(\text{buy}) &= Pr\left(\theta_2 = 1|\text{buy}\right) v_D(\theta_2 = 1) + Pr\left(\theta_2 = 0|\text{buy}\right) v_D(\theta_2 = 0) = \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} \\ P_D(\text{sell}) &= Pr\left(\theta_2 = 1|\text{sell}\right) v_D(\theta_2 = 1) + Pr\left(\theta_2 = 0|\text{sell}\right) v_D(\theta_2 = 0) = \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} \end{aligned}$$

Total expected asset value (the value of leveraged equity plus risky debt) conditional on market orders is:

$$P_E(\text{buy}) + P_D(\text{buy}) = \frac{1+\mu_1}{2}\frac{H}{2} + \frac{2-\mu_1+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2} \tag{58}$$

$$P_E(\text{buy}) + P_D(\text{sell}) = \frac{1+\mu_1}{2}\frac{H}{2} + \frac{2-\mu_1-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2} \tag{59}$$

$$P_E(\text{sell}) + P_D(\text{buy}) = \frac{1-\mu_1}{2}\frac{H}{2} + \frac{2+\mu_1+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2} \tag{60}$$

$$P_E(\text{sell}) + P_D(\text{sell}) = \frac{1-\mu_1}{2}\frac{H}{2} + \frac{2+\mu_1-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2} \tag{61}$$

Finally, we take the firm's perspective. Expected security prices depend on the asset type as follows:

$$\mathbb{E}\left[P_E|\theta_1 = 1\right] = \frac{1+\mu_1^2}{2}\frac{H-M}{2} \tag{62}$$

$$\mathbb{E}\left[P_E|\theta_1 = 0\right] = \frac{1-\mu_1^2}{2}\frac{H-M}{2} \tag{63}$$

$$\mathbb{E}\left[P_D|\theta_2 = 1\right] = \frac{3+\mu_2^2}{2}\frac{M}{2} + \frac{1-\mu_2^2}{2}\frac{L}{2} \tag{64}$$

$$\mathbb{E}\left[P_D|\theta_2 = 0\right] = \frac{3-\mu_2^2}{2}\frac{M}{2} + \frac{1+\mu_2^2}{2}\frac{L}{2} \tag{65}$$

Thus, total expected asset value for each asset type is described as in equations (19)-(22).The adverse selection discount $\text{ASD}^2_{\theta_1,\theta_2}$, which for any given asset type $\{\theta_1, \theta_2\}$ is equal to $v_{\theta_1,\theta_2} -$

$\mathbb{E}\left[P_E + P_D|\theta_1, \theta_2\right]$, is:

$$\text{ASD}^2_{1,1} = \frac{1-\mu_1^2}{4}H + \frac{\mu_1^2 - \mu_2^2}{4}M + \frac{-1+\mu_2^2}{4}L \quad (66)$$

$$\text{ASD}^2_{1,0} = \frac{1-\mu_1^2}{4}H + \frac{-2+\mu_1^2+\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \quad (67)$$

$$\text{ASD}^2_{0,1} = \frac{-1+\mu_1^2}{4}H + \frac{2-\mu_1^2-\mu_2^2}{4}M + \frac{-1+\mu_2^2}{4}L \quad (68)$$

$$\text{ASD}^2_{0,0} = \frac{-1+\mu_1^2}{4}H - \frac{\mu_1^2-\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \quad (69)$$

With $\mu_1 \in (0,1)$, $\mu_2 \in (0,1)$, and $H > M > L$, the adverse selection discount for an asset type $\{1,1\}$, in equation (66), is the highest. ■

Proof of Lemma 1: The proof, which involves straightforward calculations, is relegated to section 6.4 of the Online Appendix.

Proof of Proposition 4:

Given Lemma 1, the only potential PBE are pooling equilibria in which all asset types are associated with the same security design. Thus, the market market will offer either the security design described in Propositions 1 and 2, or the security design described in Proposition 3.

To start, consider the case with $\max\{\mu_1+\mu_2\frac{M-L}{H-M}, \mu_2+\mu_1\frac{H-M}{M-L}\} > 2$, so that potential equilibria are described in Proposition 2 and 3. Suppose that $\mu_1 + \mu_2\frac{M-L}{H-M} > 2$. Then the adverse selection discount in the equilibrium of Proposition 2 is highest either for asset type $\{\theta_1, \theta_2\} = \{1,1\}$ or for asset type $\{\theta_1, \theta_2\} = \{1,0\}$. When equity is the only information-sensitive security:

1. The expected market value of asset type $\{1,1\}$ is as in equation (10) in Proposition 2. This is lower than the expected market value of the same asset type when leveraged equity and risky debt are traded, as described in equation (19) in Proposition 3.

2. The expected market value of asset type $\{1,0\}$ is as in equation (11) in Proposition 2. This is lower than the expected market value of the same asset type when leveraged equity and risky debt are traded, as described in equation (20) in Proposition 3.

In sum, for each of the two relevant asset types, the adverse selection discount is highest in the equilibrium of Proposition 2. This implies that, for each of the two relevant asset types, the relative adverse selection discount is highest in the equilibrium of Proposition 2 (dividing both sides of an inequality by the same positive number does not change the inequality). It follows that the highest relative adverse selection discount across asset types is also highest in this equilibrium. Likewise, in the case with $\mu_2+\mu_1\frac{H-M}{M-L} > 2$, which involves asset types $\{\theta_1, \theta_2\} = \{1,1\}$ and $\{\theta_1, \theta_2\} = \{0,1\}$, the relative adverse selection discount is highest in the equilibrium of Proposition 2.

Let P_i be a random variable that represents the market value of an asset under security design $i \in \{1,2\}$, where security design 1 is the one described in Propositions 1 and 2, and security design

2 is the one described in Proposition 3. By definition of the adverse selection discount in equation (2) and the firm's utility function with discount factor β, a firm with asset type $\{\theta_1, \theta_2\}$ sells its asset given security design i and the associated pricing schedule (described in Propositions 1-3) if and only if:

$$\mathbb{E}\left[P_i|\theta_1, \theta_2\right] \geq \beta \times v_{\theta_1,\theta_2} \qquad \Leftrightarrow \qquad \frac{\text{ASD}^i_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} \leq 1-\beta. \tag{70}$$

With $\beta = 0$, this condition is always satisfied since $v_{\theta_1,\theta_2} > 0$ and $\mathbb{E}\left[P_i|\theta_1, \theta_2\right] > 0$ for any $\{\theta_1, \theta_2\}$.

Proposition 1 or 2 describes the pooling equilibrium when all firms participate and use only one information-sensitive security (equity), and Proposition 3 describes the pooling equilibrium when all firms participate and use two information-sensitive securities (equity and risky debt). We now show that only one of these potential equilibria is a PBE when $\overline{\beta}$ is as in Assumption 1.

Consider the case $\max_{\theta_1,\theta_2} \frac{\text{ASD}^1_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} < \max_{\theta_1,\theta_2} \frac{\text{ASD}^2_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}$, so that $k_1 = 1 - \max_{\theta_1,\theta_2} \frac{\text{ASD}^2_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}$ by definition of k_1. First, because of Assumption 1, this implies that $\max_{\theta_1,\theta_2} \frac{\text{ASD}^2_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} > 1 - \overline{\beta}$. Moreover, β has full support on $[0, \overline{\beta}]$ and, given security design i, a given asset type participates if and only if $\frac{\text{ASD}^i_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} \leq 1 - \beta$ (see equation (70)). Therefore, at least one asset type does not participate with a strictly positive probability if the market maker offers the security design $i = 2$ described in Proposition 3. Second, by definition of k_2 and because of Assumption 1, this case with $k_2 = 1 - \max_{\theta_1,\theta_2} \frac{\text{ASD}^1_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}$ also implies that $\max_{\theta_1,\theta_2} \frac{\text{ASD}^1_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} < 1 - \overline{\beta}$, so that all asset types participate if the market maker offers the security design $i = 1$ described in Proposition 1. Thus, since the MM derives a benefit from the firm accepting its offer, it optimally offers the security design described in Proposition 1 in this case.

Consider the case $\max_{\theta_1,\theta_2} \frac{\text{ASD}^2_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} < \max_{\theta_1,\theta_2} \frac{\text{ASD}^1_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}$, so that $k_1 = 1 - \max_{\theta_1,\theta_2} \frac{\text{ASD}^1_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}$ by definition of k_1. First, because of Assumption 1, this implies that $\max_{\theta_1,\theta_2} \frac{\text{ASD}^1_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} > 1 - \overline{\beta}$. Moreover, β has full support on $[0, \overline{\beta}]$ and, given security design i, a given asset type participates if and only if $\frac{\text{ASD}^i_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} \leq 1 - \beta$ (see equation (70)). Therefore, at least one asset type does not participate with a strictly positive probability if the market maker offers the security design $i = 1$ described in Proposition 1. Second, by definition of k_2 and because of Assumption 1, this case with $k_2 = 1 - \max_{\theta_1,\theta_2} \frac{\text{ASD}^2_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}$ also implies that $\max_{\theta_1,\theta_2} \frac{\text{ASD}^2_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} < 1 - \overline{\beta}$, so that all asset types participate if the market maker offers the security design $i = 2$ described in Proposition 3. Thus, since the MM derives a benefit from the firm accepting its offer, it optimally offers the security design described in Proposition 3 in this case.

When $\max\{\mu_1 + \mu_2 \frac{M-L}{H-M}, \mu_2 + \mu_1 \frac{H-M}{M-L}\} > 2$, we have $\max_{\theta_1,\theta_2} \frac{\text{ASD}^2_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}} < \max_{\theta_1,\theta_2} \frac{\text{ASD}^1_{\theta_1,\theta_2}}{v_{\theta_1,\theta_2}}$ as already established, so the market maker optimally offers the security design described in Proposition 3. ■

Online Appendix

Capital Structure with Information about the Upside and the Downside

1 Optimal securities

This section considers general securities, without restricting attention to debt and equity. Securities are such that the payoff of security h at $t = 1$ can be a function of asset output y, and is denoted by $s_h(y)$. The payoff vector of security h is $(s_h(L), s_h(M), s_h(H))$. For any number $N \in \{1, 2, \dots\}$ of securities used, securities' payoffs must satisfy:

$$s_h(y) \geq 0 \qquad \text{for } y \in \{L, M, H\} \tag{71}$$

$$\sum_{h=1}^{N} s_h(y) = y \qquad \text{for } y \in \{L, M, H\} \tag{72}$$

First, any security must have a nonnegative payoff in any state (equation (71)). Second, as in seminal security design papers (Allen and Gale (1988) and Boot and Thakor (1993)), designing securities is equivalent to splitting claims on the asset's output (equation (72)). Securities with linearly dependent payoffs (a payoff vector of (s^L, s^M, s^H) for one security and of (ks^L, ks^M, ks^H) with $k > 0$ for another) are counted as one security.

In principle, there is an infinity of optimal security designs, including security designs with redundant securities, and security designs that are equivalent for asset valuation when trading is costless. Assumptions 2 and 3 allow us to narrow down the set of potentially optimal security designs by postulating small costs of security issuance and trading.

Assumption 2. *A security design with n securities, with $n > 2$, involves an incremental cost $c(n) > 0$ for the market maker.*

Assumption 3. *the market maker incurs an additional cost for any market transaction above the second.*[20]

We now specify how these costs enter the market maker's objective function. In this section, we assume that the market maker has lexicographic preferences whereby it first maximizes the probability that the firm accepts the offer, and second it minimizes the administrative costs in Assumptions 2 and 3. This captures the notion that these costs are small compared to the adverse selection problem. Assumption 2 allows to rule out security designs with redundant securities. Assumption 3 allows to rule out security designs which result in unnecessary transactions.

[20] If the cost of two market transactions were equal to $C > 0$, the fair pricing condition would need to be modified to incorporate that cost, and so would market pricing and firm valuation. To simplify the exposition, since there will be at minimum two transactions, and since it is the relative cost of additional transactions that matters for security design, we set this cost to zero.

The next Lemma uses Assumptions 2 and 3 to reduce the set of potentially optimal security designs to the ones described in Propositions 1 and 5 (the latter, in section 2 of this Online Appendix, is a generalization of the security design described in Proposition 3).

Lemma 2. *The market maker will only offer security designs of the following types:*

- *The securities have payoff vectors of the following form, with* $\gamma \in [0, L]$*:*

$$(\gamma, \gamma, \gamma) \tag{73}$$

$$(L - \gamma, M - \gamma, H - \gamma) = (L, M, H) - (\gamma, \gamma, \gamma). \tag{74}$$

- *The securities have payoff vectors of the following form, with* $\Gamma \in [0, L]$*:*

$$(L - \Gamma, M - \Gamma, M - \Gamma) = (L, M, M) - (\Gamma, \Gamma, \Gamma) \tag{75}$$

$$(\Gamma, \Gamma, H - M + \Gamma) = (0, 0, H - M) + (\Gamma, \Gamma, \Gamma). \tag{76}$$

We now explain the logic behind Lemma 2. In the first case, security issuance is such that markets are incomplete. Then there is only one information-sensitive security, so that the security design is either unleveraged equity (for $\gamma = 0$) or riskfree debt and leveraged equity (for $\gamma \in (0, L])$. In the second case of Lemma 2, since a riskfree asset already exists, the markets are complete with two assets with nonconstant and linearly independent payoff vectors. In the absence of arbitrage opportunities, the overall value of the asset (the sum of securities' values) is independent of security design as long as markets are complete. Furthermore, payoff vectors in equations (75) and (76) are the only ones such that only two securities are used, and a hedger optimally trades only one of these two securities, which minimizes the security issuance costs and transaction costs borne by the market maker.

Proof of Lemma 2:

First step. Consider for now a security design offered by the market maker that does not involve two or more securities with nonconstant and linearly independent payoffs. There are two possible cases. First, suppose that the security design involves only one security. Because of the assumption in equation (72), this security must be unleveraged equity with a payoff vector (L, M, H). Second, suppose that the security design involves two or more securities – but not two securities with nonconstant and linearly independent payoffs (this case is studied below in the second step). Unless the security design involves a riskfree security with constant payoff vector and another security, this is a contradiction, since by assumption two securities with linearly dependent payoffs are counted as one security. Thus, the only remaining possibility is that the security design involves a riskfree security and another security which must be such that the assumption in equation (72) holds. These two securities therefore have payoff vectors: (γ, γ, γ) and $(L - \gamma, M - \gamma, H - \gamma)$, with $\gamma \in (0, L]$ ($\gamma = 0$ corresponds to the first case mentioned in this paragraph). This concludes the first step.

Second step. Consider for now a security design offered by the market maker that involves two or more securities with nonconstant and linearly independent payoffs. Consider a postulated equilibrium with a security design different from the one described in equations (75) and (76) in Lemma 2. The proof will show that there exists another equilibrium with a security design as described in equations (75) and (76) instead which is strictly preferred by the market maker (MM).

With a security design with two securities with nonconstant and linearly independent payoffs, the assumed existence of the riskfree asset then implies that there are three assets with linearly independent payoffs. Given that the economy has three possible states of the world (L, M, and H), this in turn implies that markets are complete. When markets are complete, portfolio allocations and prices can equivalently be analyzed with existing securities or with contingent claims. We now use the latter approach.

First, consider market orders. Let $\tilde{w}_H^-(\mathbf{x})$ denote the stochastic wealth of a hedger with negative exposure δ_H to $y = H$ with portfolio $\mathbf{x} \equiv \{x_L, x_M, x_H\}$. The problem of this hedger is to choose a portfolio of contingent claims such that:

$$\min_{\mathbf{x}} var\left(\tilde{w}_H^-(\mathbf{x})\right), \text{ where } w_H^-(\mathbf{x}) = \begin{cases} x_L - \sum_{k=L,M,H} x_k p_k & \text{for } y = L \\ x_M - \sum_{k=L,M,H} x_k p_k & \text{for } y = M \\ -\delta_H + x_H - \sum_{k=L,M,H} x_k p_k & \text{for } y = H \end{cases} \tag{77}$$

The solution to this optimization problem is simply: $\mathbf{x} = \{0, 0, \delta_H\}$, which achieves a zero variance. Likewise, a hedger with positive exposure δ_H to $y = H$ chooses the portfolio of contingent claims $\mathbf{x} = \{0, 0, -\delta_H\}$, and a hedger with positive (negative) exposure δ_L to $y = L$ chooses the portfolio of contingent claims $\mathbf{x} = \{-\delta_L, 0, 0\}$ ($\mathbf{x} = \{\delta_L, 0, 0\}$).

Second, consider asset valuation. When markets are complete, in the absence of arbitrage the vector of state prices exists and is unique. Denote state prices by $\{p_L, p_M, p_H\}$. The price of any security h with payoff vector $(s_h(L), s_h(M), s_h(H))$ is then: $P_h \equiv \sum_{y=L,M,H} p_y s_h(y)$. Firm value is the sum of security prices:

$$\sum_{h=1}^{N} P_h = \sum_{h=1}^{N} \sum_{y=L,M,H} p_y s_h(y) = \sum_{y=L,M,H} p_y \sum_{h=1}^{N} s_h(y) = \sum_{y=L,M,H} p_y y, \tag{78}$$

where the last equality uses equation (72). Thus, asset valuation only depends on state prices, which are independent of security design as long as markets are complete.

In sum, any security design with strictly more than two securities with nonconstant and linearly independent payoffs yields equivalent market orders and asset valuation as a security design as in equations (75) and (76), but the latter is strictly preferred by the MM due to the incremental security design cost that would be incurred by the MM with the former.

In the last part of the proof, we will consider the subset of security designs with two securities with nonconstant and linearly independent payoffs. We will describe the structure of security payoffs such that a hedger can achieve a variance of zero by trading only one security. This type of security design, if it exists, will be strictly preferred by the MM to other security designs in the aforementioned subset, since it minimizes the transaction costs incurred by the MM.

Consider the optimization problem of a trader with negative exposure δ_H to $y = H$, and consider a given security with payoff vector $(s_F(L), s_F(M), s_F(H))$, denoted as security F with price p_F:

$$\min_{x_F} var\left(\tilde{w}_H^-(x_F)\right), \text{ where } w_H^-(x_F) = \begin{cases} x_F s_F(L) - x_F p_F & \text{for } y = L \\ x_F s_F(M) - x_F p_F & \text{for } y = M \\ -\delta_H + x_F s_F(H) - x_F p_F & \text{for } y = H \end{cases} \tag{79}$$

A variance of zero is achieved by trading only this security if and only if:

$$x_F s_F(L) - x_F p_F = x_F s_F(M) - x_F p_F = -\delta_H + x_F s_F(H) - x_F p_F \tag{80}$$

$$\Leftrightarrow \quad s_F(L) = s_F(M), \quad s_F(H) \neq s_F(M), \text{ and } \quad x_F = \frac{\delta_H}{s_F(H) - s_F(M)}, \tag{81}$$

i.e., a variance of zero can be achieved if and only if the payoff vector of the security takes the form (Γ, Γ, F), with $F \neq \Gamma$. In this case, and only in this case, the hedger with negative exposure to $y = H$ can achieve a variance of zero by trading only one security, with $x_F = \frac{\delta_H}{F-\Gamma}$. The demonstration for the case of a hedger with positive exposure to $y = H$ is similar (he achieves a variance of zero with $x_F = -\frac{\delta_H}{F-\Gamma}$).

Now consider the optimization problem of a trader with negative exposure δ_L to $y = L$, and consider a given security with payoff vector $(s_f(L), s_f(M), s_f(H))$, denoted as security f with price p_f:

$$\min_{x_f} var\left(\tilde{w}_L^-(x_f)\right), \text{ where } w_L^-(x_f) = \begin{cases} -\delta_L + x_f s_f(L) - x_f p_f & \text{for } y = L \\ x_f s_f(M) - x_f p_f & \text{for } y = M \\ x_f s_f(H) - x_f p_f & \text{for } y = H \end{cases} \tag{82}$$

A variance of zero is achieved by trading only this security if and only if:

$$-\delta_L + x_f s_f(L) - x_f p_f = x_f s_f(M) - x_f p_f = x_f s_f(H) - x_f p_f \tag{83}$$

$$\Leftrightarrow \quad s_F(H) = s_F(M), \quad s_F(L) \neq s_F(M), \text{ and } \quad x_f = -\frac{\delta_L}{s_f(M) - s_f(L)}, \tag{84}$$

i.e., a variance of zero can be achieved if and only if the payoff vector of the security takes the form (f, k, k), with $f \neq k$. In this case, and only in this case, the hedger with negative exposure to $y = L$ can achieve a variance of zero by trading only one security, with $x_f = -\frac{\delta_L}{k-f}$. The demonstration for the case of a hedger with positive exposure to $y = L$ is similar (he achieves a variance of zero with $x_f = \frac{\delta_L}{k-f}$).

We have shown that a hedger with exposure to $y = H$ can achieve a variance of zero by trading only one security if and only if this security has a payoff vector of the form (Γ, Γ, F) with $\Gamma \neq F$.

Likewise, a hedger with exposure to $y = L$ can achieve a variance of zero by trading only one security if and only if this security has a payoff vector of the form (f, k, k) with $f \neq k$.

Now consider a security with payoff vector (Γ, Γ, F), and another security with payoff vector (f, k, k). These parameter values must be such that assumption (72) on security design is satisfied:

$$\begin{cases} \Gamma + f = L \\ \Gamma + k = M \\ k + F = H \end{cases} \tag{85}$$

This is a system of three linear equations with four unknowns. This system has an infinity of solutions, which must satisfy:

$$\begin{cases} f = L - \Gamma \\ k = M - \Gamma \\ F = H - M + \Gamma \end{cases} \tag{86}$$

where, according to assumption (71), the free parameter Γ is such that $\Gamma \in [0, L]$. That is, for $\Gamma \in [0, L]$, the securities with payoffs (Γ, Γ, F) and (f, k, k) have payoff vectors such that:

$$(f, k, k) = (L - \Gamma, M - \Gamma, M - \Gamma) = (L, M, M) - (\Gamma, \Gamma, \Gamma) \tag{87}$$

$$(\Gamma, \Gamma, F) = (\Gamma, \Gamma, H - M + \Gamma) = (0, 0, H - M) + (\Gamma, \Gamma, \Gamma) \tag{88}$$

In sum, the securities described in equations (87) and (88), for $\Gamma \in [0, L]$, make up the only security design consisting of two securities with nonconstant and linearly independent payoffs such that a hedger trades only one security. As is standard, an informed trader will submit the same trades as a hedger, otherwise he would be identified as an informed trader and make zero profit. This concludes the second step. ■

2 Composite securities

In this subsection, we revisit the trading phase equilibrium in section 2.2 by considering securities with payoff vectors $(L - \Gamma, M - \Gamma, M - \Gamma)$ and $(\Gamma, \Gamma, H - M + \Gamma)$ for $\Gamma \in [0, L]$. These are composite securities which were derived in section 1 of this Online Appendix. The first security has the payoff of debt with face value M, with payoff vector (L, M, M), minus a riskfree security with payoff vector (Γ, Γ, Γ) where $\Gamma \in [0, L]$. The second security has the payoff of leveraged equity, with payoff vector $(0, 0, H - M)$, plus the same riskfree security with payoff vector (Γ, Γ, Γ). For example, in the polar cases $\Gamma = 0$ and $\Gamma = L$, the payoff vectors of the securities used are:

$$\text{With } \Gamma = 0 : \begin{cases} (L, M, M) \\ (0, 0, H - M) \end{cases} \qquad \text{With } \Gamma = L : \begin{cases} (0, M - L, M - L) \\ (L, L, H - M + L) \end{cases} \tag{89}$$

With $\Gamma = 0$, the securities are debt with face value M and leveraged equity, as in Proposition 3. With $\Gamma = L$, there is a composite security which combines senior debt with face value L and equity, and the other security is junior debt with face value $M - L$. Intuitively, the asset generates an output of L in the worst case scenario, and this baseline level of output can be allocated either to the risky security exposed to the downside of the distribution ("risky debt"), with $\Gamma = 0$, or to the risky security exposed to the upside of the distribution ("leveraged equity"), with $\Gamma = L$, or it can be allocated partially to both, with $\Gamma \in (0, L)$.

The expected payoffs conditional on asset type $\{\theta_1, \theta_2\}$ of "leveraged equity" (indexed by E) and "risky debt" (indexed by D), respectively, are:

$$\begin{array}{ll} v_E(\theta_1 = 1) = \frac{H-M}{2} + \Gamma, & v_E(\theta_1 = 0) = \Gamma, \\ v_D(\theta_2 = 1) = M - \Gamma, & v_D(\theta_2 = 0) = \frac{L+M}{2} - \Gamma. \end{array} \tag{90}$$

The value of leveraged equity only depends on θ_1, and the value of risky debt only depends on θ_2.

To focus on properties of this equilibrium when the firm accepts the market maker's offer and sells its asset, we simply assume that $\overline{\beta} = 0$, i.e. the firm is highly "impatient". Proposition 3 generalizes as follows.

Proposition 5. *When securities offered by the market maker have payoff vectors* $(\Gamma, \Gamma, H - M + \Gamma)$ *and* $(L - \Gamma, M - \Gamma, M - \Gamma)$ *for* $\Gamma \in [0, L]$*:*

- *Trader 1 trades leveraged equity by submitting a market order for a quantity* $\frac{\delta_H}{H-M}$*. A hedger with a negative (positive) exposure to* $y = H$ *buys (sells) this amount; an informed trader who observes* $\theta_1 = 1$ *buys this amount; an informed trader who observes* $\theta_1 = 0$ *sells this amount.*

  Trader 2 trades risky debt by submitting a market order for a quantity $\frac{\delta_L}{M-L}$*. A hedger with a negative (positive) exposure to* $y = L$ *sells (buys) this amount; an informed trader who observes* $\theta_2 = 1$ *buys this amount; an informed trader who observes* $\theta_2 = 0$ *sells this amount.*

- *The market price* P_E *of leveraged equity conditional on the observed order is:*

$$P_E(buy) = \frac{1+\mu_1}{2}\frac{H-M}{2} + \Gamma \tag{91}$$

$$P_E(sell) = \frac{1-\mu_1}{2}\frac{H-M}{2} + \Gamma \tag{92}$$

  The market price P_D *of risky debt conditional on the observed order is:*

$$P_D(buy) = \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} - \Gamma \tag{93}$$

$$P_D(sell) = \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} - \Gamma \tag{94}$$

- *The expected market value of the asset conditional on asset type is:*

$$\mathbb{E}\left[P_E + P_D|\theta_1 = 1, \theta_2 = 1\right] = \frac{1+\mu_1^2}{4}H + \frac{2-\mu_1^2+\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \tag{95}$$

$$\mathbb{E}\left[P_E + P_D|\theta_1 = 1, \theta_2 = 0\right] = \frac{1+\mu_1^2}{4}H + \frac{2-\mu_1^2-\mu_2^2}{4}M + \frac{1+\mu_2^2}{4}L \tag{96}$$

$$\mathbb{E}\left[P_E + P_D|\theta_1 = 0, \theta_2 = 1\right] = \frac{1-\mu_1^2}{4}H + \frac{2+\mu_1^2+\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \tag{97}$$

$$\mathbb{E}\left[P_E + P_D|\theta_1 = 0, \theta_2 = 0\right] = \frac{1-\mu_1^2}{4}H + \frac{2+\mu_1^2-\mu_2^2}{4}M + \frac{1+\mu_2^2}{4}L \tag{98}$$

Even though the prices of each of the two securities depend on the parameter Γ, the market value of the asset does not, since changes in Γ merely reallocate a constant payoff across securities. Proposition 5 and its proof below are therefore very similar to Proposition 3 and its proof, and some parts are abbreviated.

Proof of Proposition 5:

Suppose that the market offers offers securities with payoff vectors $(\Gamma, \Gamma, H - M + \Gamma)$ and $(L-\Gamma, M-\Gamma, M-\Gamma)$ for $\Gamma \in [0, L]$, for $\Gamma \in [0, L]$, associated with subscripts D and E, respectively. Consider the optimization problem of a hedger with negative exposure to $y = H$:

$$\min_{x_D, x_E} var\left(\tilde{w}_H^-(x_D, x_E)\right), \text{ where } w_H^-(x_D, x_E) = \begin{cases} x_D(L-\Gamma-p_D) + x_E(\Gamma - p_E) & \text{for } y = L \\ x_D(M-\Gamma-p_D) + x_E(\Gamma - p_E) & \text{for } y = M \\ -\delta_H + x_D(M-\Gamma-p_D) + x_E(H-M+\Gamma-p_E) & \text{for } y = H \end{cases}$$

A variance of zero is achieved if and only if:

$$\begin{cases} x_D(L-\Gamma-p_D) + x_E(\Gamma-p_E) = x_D(M-\Gamma-p_D) + x_E(\Gamma-p_E) \\ x_D(M-\Gamma-p_D) + x_E(\Gamma-p_E) = -\delta_H + x_D(M-\Gamma-p_D) + x_E(H-M+\Gamma-p_E) \end{cases} \tag{99}$$

The first equality gives $x_D = 0$, and the second equality gives $x_E = \frac{\delta_H}{H-M}$. Likewise, for a hedger with positive exposure to $y = H$, $x_D = 0$ and $x_E = -\frac{\delta_H}{H-M}$.

Consider the optimization problem of a hedger with negative exposure to $y = L$:

$$\min_{x_D, x_E} var\left(\tilde{w}_L^-(x_D, x_E)\right), \text{ where } w_L^-(x_D, x_E) = \begin{cases} -\delta_L + x_D(L-\Gamma-p_D) + x_E(\Gamma - p_E) & \text{for } y = L \\ x_D(M-\Gamma-p_D) + x_E(\Gamma - p_E) & \text{for } y = M \\ x_D(M-\Gamma-p_D) + x_E(H-M+\Gamma-p_E) & \text{for } y = H \end{cases}$$

A variance of zero is achieved if and only if:

$$\begin{cases} -\delta_L + x_D(L-\Gamma-p_D) + x_E(\Gamma-p_E) = x_D(M-\Gamma-p_D) + x_E(\Gamma-p_E) \\ x_D(M-\Gamma-p_D) + x_E(\Gamma-p_E) = x_D(M-\Gamma-p_D) + x_E(H-M+\Gamma-p_E) \end{cases} \tag{100}$$

The second equality gives $x_E = 0$, and the second equality gives $x_D = -\frac{\delta_L}{M-L}$. Likewise, for a hedger with positive exposure to $y = L$, $x_E = 0$ and $x_D = \frac{\delta_L}{M-L}$.

Using standard arguments, if trader 1 is informed, he will buy or sell the same quantity of leveraged equity as a hedger – either 'buy', corresponding to $x_E = \frac{\delta_H}{H-M}$, or 'sell', corresponding to $x_E = -\frac{\delta_H}{H-M}$ – otherwise the order would reveal his information to the market maker. Likewise, if trader 2 is informed, he will buy or sell the same quantity of risky debt as a hedger – either 'buy', corresponding to $x_D = \frac{\delta_L}{M-L}$, or 'sell', corresponding to $x_D = -\frac{\delta_L}{M-L}$ – otherwise the order would reveal his information to the market maker.

We now take the market maker's perspective. The market price of leveraged equity conditional on a buy or sell order is:

$$\begin{aligned} P_E(\text{buy}) &= Pr\left(\theta_1 = 1|\text{buy}\right) v_E(\theta_1 = 1) + Pr\left(\theta_1 = 0|\text{buy}\right) v_E(\theta_1 = 0) = \frac{1+\mu_1}{2}\frac{H-M}{2} + \Gamma \\ P_E(\text{sell}) &= Pr\left(\theta_1 = 1|\text{sell}\right) v_E(\theta_1 = 1) + Pr\left(\theta_1 = 0|\text{sell}\right) v_E(\theta_1 = 0) = \frac{1-\mu_1}{2}\frac{H-M}{2} + \Gamma \end{aligned}$$

The market price of risky debt conditional on a buy or sell order is:

$$\begin{aligned} P_D(\text{buy}) &= Pr\left(\theta_2 = 1|\text{buy}\right) v_D(\theta_2 = 1) + Pr\left(\theta_2 = 0|\text{buy}\right) v_D(\theta_2 = 0) \\ &= \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} - \Gamma \\ P_D(\text{sell}) &= Pr\left(\theta_2 = 1|\text{sell}\right) v_D(\theta_2 = 1) + Pr\left(\theta_2 = 0|\text{sell}\right) v_D(\theta_2 = 0) \\ &= \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} - \Gamma \end{aligned}$$

Total expected asset value conditional on market orders is:

$$\begin{aligned} P_E(\text{buy}) + P_D(\text{buy}) &= \frac{1+\mu_1}{2}\frac{H-M}{2} + \Gamma + \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} - \Gamma \\ &= \frac{1+\mu_1}{2}\frac{H}{2} + \frac{2-\mu_1+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2} \qquad (101) \\ P_E(\text{buy}) + P_D(\text{sell}) &= \frac{1+\mu_1}{2}\frac{H-M}{2} + \Gamma + \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} - \Gamma \\ &= \frac{1+\mu_1}{2}\frac{H}{2} + \frac{2-\mu_1-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2} \qquad (102) \\ P_E(\text{sell}) + P_D(\text{buy}) &= \frac{1-\mu_1}{2}\frac{H-M}{2} + \Gamma + \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} - \Gamma \\ &= \frac{1-\mu_1}{2}\frac{H}{2} + \frac{2+\mu_1+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2} \qquad (103) \\ P_E(\text{sell}) + P_D(\text{sell}) &= \frac{1-\mu_1}{2}\frac{H-M}{2} + \Gamma + \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} - \Gamma \\ &= \frac{1-\mu_1}{2}\frac{H}{2} + \frac{2+\mu_1-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2} \qquad (104) \end{aligned}$$

Finally, we take the firm's perspective. Expected security prices depend on the asset type as follows:

$$\mathbb{E}[P_E|\theta_1 = 1] = \frac{1+\mu_1^2}{2}\frac{H-M}{2} + \Gamma \quad (105)$$
$$\mathbb{E}[P_E|\theta_1 = 0] = \frac{1-\mu_1^2}{2}\frac{H-M}{2} + \Gamma \quad (106)$$
$$\mathbb{E}[P_D|\theta_2 = 1] = \frac{3+\mu_2^2}{2}\frac{M}{2} + \frac{1-\mu_2^2}{2}\frac{L}{2} - \Gamma \quad (107)$$
$$\mathbb{E}[P_D|\theta_2 = 0] = \frac{3-\mu_2^2}{2}\frac{M}{2} + \frac{1+\mu_2^2}{2}\frac{L}{2} - \Gamma \quad (108)$$

Total expected asset value for different asset types are:

$$\mathbb{E}[P_E + P_D|\theta_1 = 1, \theta_2 = 1] = \frac{1+\mu_1^2}{4}H + \frac{2-\mu_1^2+\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \quad (109)$$
$$\mathbb{E}[P_E + P_D|\theta_1 = 1, \theta_2 = 0] = \frac{1+\mu_1^2}{4}H + \frac{2-\mu_1^2-\mu_2^2}{4}M + \frac{1+\mu_2^2}{4}L \quad (110)$$
$$\mathbb{E}[P_E + P_D|\theta_1 = 0, \theta_2 = 1] = \frac{1-\mu_1^2}{4}H + \frac{2+\mu_1^2+\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \quad (111)$$
$$\mathbb{E}[P_E + P_D|\theta_1 = 0, \theta_2 = 0] = \frac{1-\mu_1^2}{4}H + \frac{2+\mu_1^2-\mu_2^2}{4}M + \frac{1+\mu_2^2}{4}L \quad (112)$$

The adverse selection discount $\text{ASD}^2_{\theta_1,\theta_2}$, which for any given asset type $\{\theta_1, \theta_2\}$ is equal to $v_{\theta_1,\theta_2} - \mathbb{E}[P_E + P_D|\theta_1, \theta_2]$, is:

$$\text{ASD}^2_{1,1} = \frac{1-\mu_1^2}{4}H + \frac{\mu_1^2-\mu_2^2}{4}M + \frac{-1+\mu_2^2}{4}L \quad (113)$$
$$\text{ASD}^2_{1,0} = \frac{1-\mu_1^2}{4}H + \frac{-2+\mu_1^2+\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \quad (114)$$
$$\text{ASD}^2_{0,1} = \frac{-1+\mu_1^2}{4}H + \frac{2-\mu_1^2-\mu_2^2}{4}M + \frac{-1+\mu_2^2}{4}L \quad (115)$$
$$\text{ASD}^2_{0,0} = \frac{-1+\mu_1^2}{4}H - \frac{\mu_1^2-\mu_2^2}{4}M + \frac{1-\mu_2^2}{4}L \quad (116)$$

With $\mu_1 \in (0,1)$, $\mu_2 \in (0,1)$, and $H > M > L$, the adverse selection discount for an asset type $\{1, 1\}$ in equation (113) is the highest. ■

3 High discount factor

Let $\beta \to 1$. By construction, in any equilibrium that involves the participation of more than one asset type, the adverse selection discount will be strictly positive for some asset types, and strictly negative for other asset types (according to the zero profit condition for the market maker, the adverse selection discount must by construction be zero on average across participating asset types in any potential equilibrium). As $\beta \to 1$, this is a contradiction, because an asset type with a strictly positive adverse selection discount will optimally not participate.

Consequently, the only possible equilibria involves the participation of either zero or one asset type, whose adverse selection discount is by construction zero (since its type is known). Since the market maker maximizes the probability of asset type participation, the selected equilibrium involves the participation of one asset type. Suppose that this asset type is other than the low type on both dimensions ($\{0,0\}$), with intrinsic value v_{θ_1,θ_2} as defined in equation (1), with $\{\theta_1,\theta_2\} \neq \{0,0\}$. Then by fair pricing, market valuation is v_{θ_1,θ_2}, which is strictly higher than the intrinsic value $v_{0,0}$ of a $\{0,0\}$ type. This implies that asset type $\{0,0\}$ will participate as well, a contradiction. In sum, the only possible equilibrium involves the participation of a $\{0,0\}$ type which is priced as $v_{0,0}$ (see equation (1)) regardless of market orders.

4 Plots of the highest relative adverse selection discount

Figure 11 depicts the ratio of the adverse selection discount (ASD) to intrinsic asset value as a function of the probabilities of informed trading μ_1 and μ_2 for the security designs described in sections 2.1 and 2.2 and several values of the upside $H - M$ and downside $M - L$. Demarcations in the surfaces correspond to changes in the expression for the highest relative ASD, due either to changes in the asset type with the highest relative ASD or to changes in the relevant set of parameter values (cf. the condition that differs in Propositions 1 and 2). The discontinuities in the orange surface correspond to points such that highest ASD with the security design described in section 2.1 switches from the one described in Proposition 1 to the one described in Proposition 2.

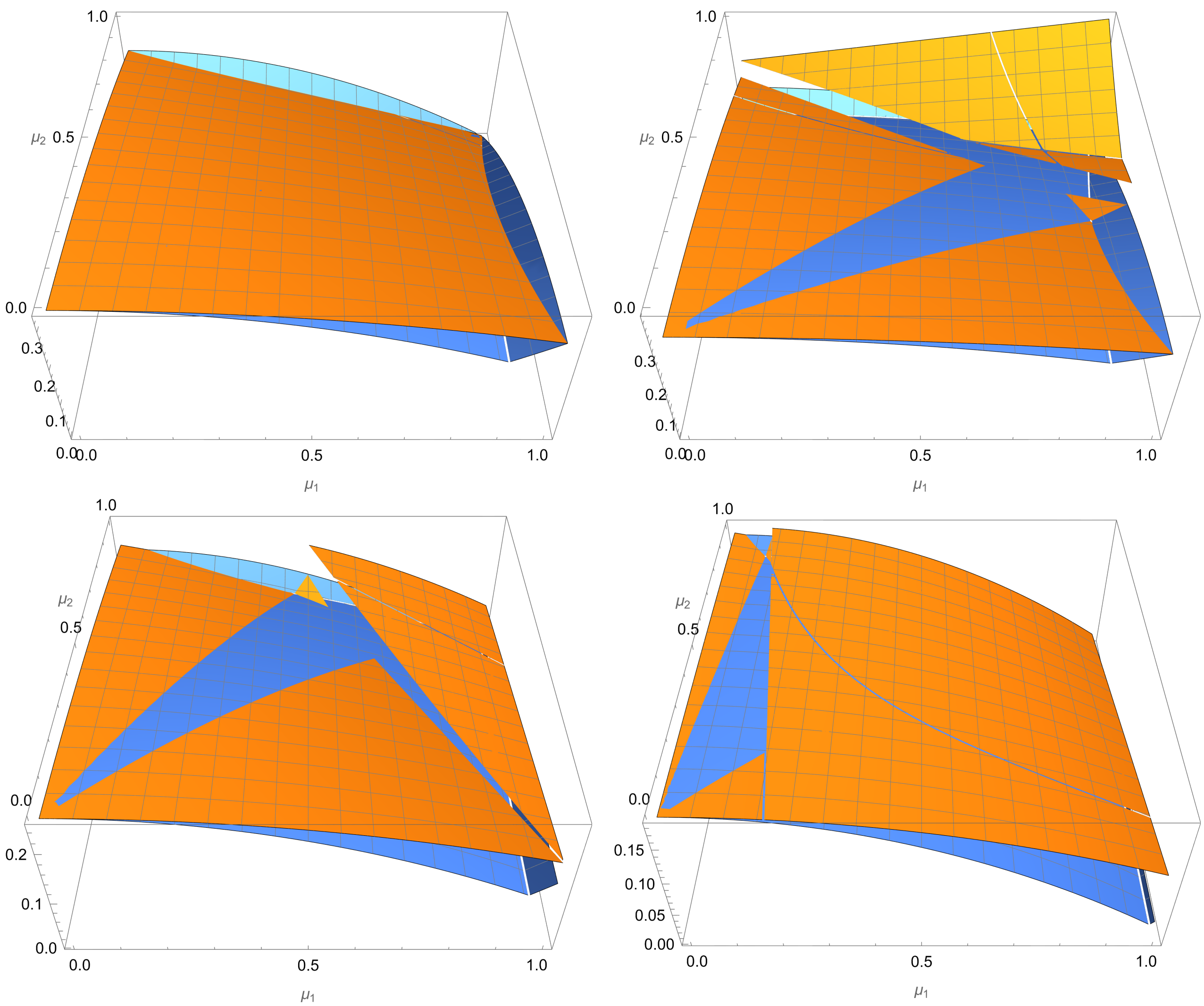

Figure 11: Highest ratio of the adverse selection discount to intrinsic asset value for two capital structures.

In all panels, as a function of μ_1 and μ_2, the orange surface is the highest value of $\text{ASD}^1_{i,j}/v_{i,j}$ across asset types, and the blue surface is the highest value of $\text{ASD}^2_{i,j}/v_{i,j}$ across asset types. Top row left: $H - M = 1$, $M - L = 1$. Top row right: $H - M = 0.5$, $M - L = 1$. Bottom row left: $H - M = 1$, $M - L = 0.5$. Bottom row right: $H - M = 1$, $M - L = 0.1$.

5 Probability of informed trading and value relevance

This section studies numerically the outcome when the probability of informed trading is increasing in the value-relevance of each dimension of uncertainty. We assume that the greater the upside $H - M$ (respectively downside $M - L$), the higher the probability of informed trading on the upside (resp. downside). Specifically, letting X be the upside $H-M$ or downside $M-L$, we let the probability μ of informed trading on this dimension be $\mu = a \times X^b$, for $a \in (0, 1)$, $b \in (0, 2)$, and several values of $X \in (0, 1]$, which ensures that $\mu_1 \in (0, 1)$ and $\mu_2 \in (0, 1)$. The parameter a parameterizes the sensitivity of the probability of informed trading to the value-relevance of each dimension of uncertainty. The parameter b parameterizes the curvature of this probability with respect to the value-relevance of each dimension of uncertainty: it is concave for $b < 1$, linear for $b = 1$, and convex for $b > 1$.

Numerical results in Figure 12 have the following implications. First, if both dimensions of uncertainty have the same value relevance ($H - M = M - L$), the highest ASD is the same for both types of capital structures, so that the market maker is indifferent with regards to the capital structure offered. Second, if the probability of informed trading is higher for the more value relevant dimension of uncertainty, then the equilibrium involves two information-sensitive securities – leveraged equity and risky debt. Indeed, in this case, the intra-subsidy effect does not exist.

Demarcations in the surfaces correspond to changes in the expression for the highest relative ASD, due either to changes in the asset type with the highest relative ASD or to changes in the relevant set of parameter values (cf. the condition that differs in Propositions 1 and 2). Discontinuities in the orange surface correspond to points such that the highest relative adverse selection discount with the security design described in section 2.1 switches from the one described in Proposition 1 to the one described in Proposition 2.

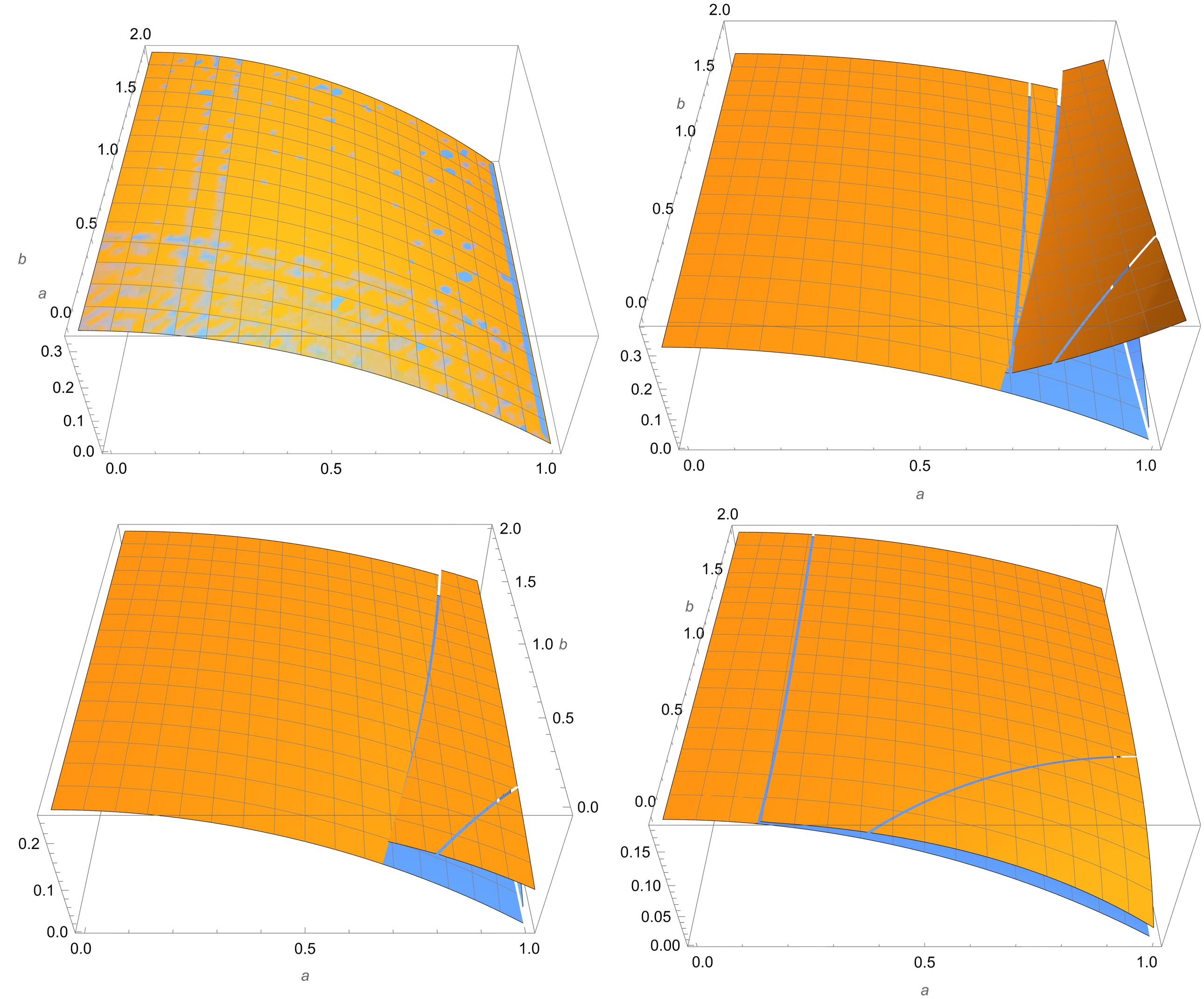

Figure 12: Highest ratio of the adverse selection discount to intrinsic asset value for two capital structures.

In all panels, the orange surface is the highest value of $\text{ASD}^1_{i,j}/v_{i,j}$ across asset types, and the blue surface is the highest value of $\text{ASD}^2_{i,j}/v_{i,j}$ across asset types. In all cases, we have $\mu_1 = a \times (H - M)^b$, and $\mu_2 = a \times (M - L)^b$. Top row left: $H = 2$, $M = 1$, $L = 0$. Top row right: $H = 1.5$, $M = 1$, $L = 0$. Bottom row left: $H = 2$, $M = 1$, $L = 0.5$. Bottom row right: $H = 2$, $M = 1$, $L = 0.9$.

6 Bayesian updating: additional calculations

6.1 Additional calculations for the proof of Proposition 1:

The probability of buy or sell orders (non-ordered) conditional on asset type is:

$$
\begin{aligned}
Pr\left(\text{buy},\text{buy}|\theta_1=1,\theta_2=1\right) &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\\
Pr\left(\text{buy},\text{sell}|\theta_1=1,\theta_2=1\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\\
Pr\left(\text{sell},\text{sell}|\theta_1=1,\theta_2=1\right) &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\\
Pr\left(\text{buy},\text{buy}|\theta_1=1,\theta_2=0\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\\
Pr\left(\text{buy},\text{sell}|\theta_1=1,\theta_2=0\right) &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\\
Pr\left(\text{sell},\text{sell}|\theta_1=1,\theta_2=0\right) &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\\
Pr\left(\text{buy},\text{buy}|\theta_1=0,\theta_2=1\right) &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\\
Pr\left(\text{buy},\text{sell}|\theta_1=0,\theta_2=1\right) &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\\
Pr\left(\text{sell},\text{sell}|\theta_1=0,\theta_2=1\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\\
Pr\left(\text{buy},\text{buy}|\theta_1=0,\theta_2=0\right) &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\\
Pr\left(\text{buy},\text{sell}|\theta_1=0,\theta_2=0\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\\
Pr\left(\text{sell},\text{sell}|\theta_1=0,\theta_2=0\right) &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}
\end{aligned}
$$

Using Bayesian updating, the distribution of $\tilde{\theta}_1, \tilde{\theta}_2$ conditional on observed orders is given by:

$$
\begin{aligned}
Pr\left(\theta_1=1,\theta_2=1|\text{buy},\text{buy}\right) &= \frac{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\frac{1}{4}}{\frac{1}{4}}=\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\\
Pr\left(\theta_1=1,\theta_2=1|\text{buy},\text{sell}\right) &= \frac{\left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1+\mu_2}{2}\frac{1-\mu_1}{2}\right)\frac{1}{4}}{\frac{1}{2}}=\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1+\mu_2}{2}\frac{1-\mu_1}{2}\right)\\
Pr\left(\theta_1=1,\theta_2=1|\text{sell},\text{sell}\right) &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\\
Pr\left(\theta_1=1,\theta_2=0|\text{buy},\text{buy}\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\\
Pr\left(\theta_1=1,\theta_2=0|\text{buy},\text{sell}\right) &= \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right)\\
Pr\left(\theta_1=1,\theta_2=0|\text{sell},\text{sell}\right) &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\\
Pr\left(\theta_1=0,\theta_2=1|\text{buy},\text{buy}\right) &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\\
Pr\left(\theta_1=0,\theta_2=1|\text{buy},\text{sell}\right) &= \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right)\\
Pr\left(\theta_1=0,\theta_2=1|\text{sell},\text{sell}\right) &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\\
Pr\left(\theta_1=0,\theta_2=0|\text{buy},\text{buy}\right) &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\\
Pr\left(\theta_1=0,\theta_2=0|\text{buy},\text{sell}\right) &= \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1+\mu_2}{2}\frac{1-\mu_1}{2}\right)\\
Pr\left(\theta_1=0,\theta_2=0|\text{sell},\text{sell}\right) &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}
\end{aligned}
$$

The asset's market price (the sum of security prices) conditional on observed orders is:

$$
\begin{aligned}
P_U\left(\text{buy},\text{buy}\right) &= Pr\left(\theta_1=1,\theta_2=1|\text{buy},\text{buy}\right)v_U(\theta_1=1,\theta_2=1) \\
&\quad +Pr\left(\theta_1=1,\theta_2=0|\text{buy},\text{buy}\right)v_U(\theta_1=1,\theta_2=0) \\
&\quad +Pr\left(\theta_1=0,\theta_2=1|\text{buy},\text{buy}\right)v_U(\theta_1=0,\theta_2=1) \\
&\quad +Pr\left(\theta_1=0,\theta_2=0|\text{buy},\text{buy}\right)v_U(\theta_1=0,\theta_2=0) \\
&= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\frac{H+M}{2}+\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\frac{H+L}{2} \\
&\quad +\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}M+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\frac{L+M}{2} \\
&= \frac{1+\mu_1}{2}\frac{H}{2}+\frac{2-\mu_1+\mu_2}{2}\frac{M}{2}+\frac{1-\mu_2}{2}\frac{L}{2} \qquad (117)\\
P_U\left(\text{buy},\text{sell}\right) &= \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1+\mu_2}{2}\frac{1-\mu_1}{2}\right)\frac{H+M}{2}+\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right)\frac{H+L}{2} \\
&\quad +\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right)M+\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1+\mu_2}{2}\frac{1-\mu_1}{2}\right)\frac{L+M}{2} \\
&= \frac{1}{2}\frac{H}{2}+\frac{M}{2}+\frac{1}{2}\frac{L}{2} \qquad (118)\\
P_U\left(\text{sell},\text{sell}\right) &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\frac{H+M}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\frac{H+L}{2} \\
&\quad +\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}M+\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\frac{L+M}{2} \\
&= \frac{1-\mu_1}{2}\frac{H}{2}+\frac{2+\mu_1-\mu_2}{2}\frac{M}{2}+\frac{1+\mu_2}{2}\frac{L}{2} \qquad (119)
\end{aligned}
$$

6.2 Additional calculations for the proof of Proposition 2:

Suppose that equation (43) does not hold, so that trader 1 does not trade when informed. The market maker is aware that one trade emanates from trader 2 (which occurs with probability μ_1), and that two trades emanate from trader 2 and hedger 1 (which occurs with probability $1-\mu_1$).

Thus:

$$\begin{aligned}
P_U\,(\text{buy}) &= \frac{H-M}{4} + \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} \\
P_U\,(\text{sell}) &= \frac{H-M}{4} + \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} \\
P_U\,(\text{buy},\text{buy}) &= Pr\,(\theta_1=1,\theta_2=1|\text{buy},\text{buy})\,v_U(\theta_1=1,\theta_2=1) \\
&\quad +Pr\,(\theta_1=1,\theta_2=0|\text{buy},\text{buy})\,v_U(\theta_1=1,\theta_2=0) \\
&\quad +Pr\,(\theta_1=0,\theta_2=1|\text{buy},\text{buy})\,v_U(\theta_1=0,\theta_2=1) \\
&\quad +Pr\,(\theta_1=0,\theta_2=0|\text{buy},\text{buy})\,v_U(\theta_1=0,\theta_2=0) \\
&= \frac{1}{2}\frac{1+\mu_2}{2}\frac{H+M}{2} + \frac{1}{2}\frac{1-\mu_2}{2}\frac{H+L}{2} + \frac{1}{2}\frac{1+\mu_2}{2}M + \frac{1}{2}\frac{1-\mu_2}{2}\frac{L+M}{2} \\
&= \frac{1}{2}\frac{H}{2} + \frac{2+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2} \qquad (120) \\
P_U\,(\text{buy},\text{sell}) &= \frac{1}{2}\left(\frac{1}{2}\frac{1-\mu_2}{2} + \frac{1+\mu_2}{2}\frac{1}{2}\right)\frac{H+M}{2} + \frac{1}{2}\left(\frac{1}{2}\frac{1+\mu_2}{2} + \frac{1}{2}\frac{1-\mu_2}{2}\right)\frac{H+L}{2} \\
&\quad +\frac{1}{2}\left(\frac{1}{2}\frac{1+\mu_2}{2} + \frac{1}{2}\frac{1-\mu_2}{2}\right)M + \frac{1}{2}\left(\frac{1}{2}\frac{1-\mu_2}{2} + \frac{1+\mu_2}{2}\frac{1}{2}\right)\frac{L+M}{2} \\
&= \frac{1}{2}\frac{H}{2} + \frac{M}{2} + \frac{1}{2}\frac{L}{2} \qquad (121) \\
P_U\,(\text{sell},\text{sell}) &= \frac{1}{2}\frac{1-\mu_2}{2}\frac{H+M}{2} + \frac{1}{2}\frac{1+\mu_2}{2}\frac{H+L}{2} + \frac{1}{2}\frac{1-\mu_2}{2}M + \frac{1}{2}\frac{1+\mu_2}{2}\frac{L+M}{2} \\
&= \frac{1}{2}\frac{H}{2} + \frac{2-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2} \qquad (122)
\end{aligned}$$

Suppose that equation (45) does not hold, so that trader 2 does not trade when informed. The market maker is aware that one trade emanates from trader 1 (which occurs with probability μ_2), and that two trades emanate from trader 1 and hedger 2 (which occurs with probability $1-\mu_2$).

Thus:

$$
\begin{aligned}
P_U\left(\text{buy}\right) &= \frac{1+\mu_1}{2}\frac{H-M}{2}+\frac{3}{4}M+\frac{L}{4} \\
P_U\left(\text{sell}\right) &= \frac{1-\mu_1}{2}\frac{H-M}{2}+\frac{3}{4}M+\frac{L}{4} \\
P_U\left(\text{buy},\text{buy}\right) &= Pr\left(\theta_1=1,\theta_2=1|\text{buy},\text{buy}\right)v_U(\theta_1=1,\theta_2=1) \\
&\quad +Pr\left(\theta_1=1,\theta_2=0|\text{buy},\text{buy}\right)v_U(\theta_1=1,\theta_2=0) \\
&\quad +Pr\left(\theta_1=0,\theta_2=1|\text{buy},\text{buy}\right)v_U(\theta_1=0,\theta_2=1) \\
&\quad +Pr\left(\theta_1=0,\theta_2=0|\text{buy},\text{buy}\right)v_U(\theta_1=0,\theta_2=0) \\
&= \frac{1+\mu_1}{2}\frac{1}{2}\frac{H+M}{2}+\frac{1+\mu_1}{2}\frac{1}{2}\frac{H+L}{2} \\
&\quad +\frac{1-\mu_1}{2}\frac{1}{2}M+\frac{1-\mu_1}{2}\frac{1}{2}\frac{L+M}{2} \\
&= \frac{1+\mu_1}{2}\frac{H}{2}+\frac{2-\mu_1}{2}\frac{M}{2}+\frac{1}{2}\frac{L}{2} \qquad (123) \\
P_U\left(\text{buy},\text{sell}\right) &= \frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1}{2}\frac{1-\mu_1}{2}\right)\frac{H+M}{2}+\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1-\mu_1}{2}\frac{1}{2}\right)\frac{H+L}{2} \\
&\quad +\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1-\mu_1}{2}\frac{1}{2}\right)M+\frac{1}{2}\left(\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1}{2}\frac{1-\mu_1}{2}\right)\frac{L+M}{2} \\
&= \frac{1}{2}\frac{H}{2}+\frac{M}{2}+\frac{1}{2}\frac{L}{2} \qquad (124) \\
P_U\left(\text{sell},\text{sell}\right) &= \frac{1-\mu_1}{2}\frac{1}{2}\frac{H+M}{2}+\frac{1-\mu_1}{2}\frac{1}{2}\frac{H+L}{2} \\
&\quad +\frac{1+\mu_1}{2}\frac{1}{2}M+\frac{1+\mu_1}{2}\frac{1}{2}\frac{L+M}{2} \\
&= \frac{1-\mu_1}{2}\frac{H}{2}+\frac{2+\mu_1}{2}\frac{M}{2}+\frac{1}{2}\frac{L}{2} \qquad (125)
\end{aligned}
$$

6.3 Additional calculations for the proof of Proposition 3:

The probability of buy or sell orders conditional on asset type for leveraged equity is:

$$
\begin{aligned}
Pr\left(\text{buy}|\theta_1=1\right) &= \mu_1+(1-\mu_1)\frac{1}{2}=\frac{1+\mu_1}{2} \\
Pr\left(\text{sell}|\theta_1=1\right) &= \frac{1-\mu_1}{2}
\end{aligned}
$$

Using Bayesian updating, the distribution of $\tilde{\theta}_1$ conditional on observed orders is given by:

$$\begin{aligned}
Pr\left(\theta_1 = 1|\text{order}\right) &= \frac{Pr\left(\theta_1 \cap \text{order}\right)}{Pr\left(\text{order}\right)} = \frac{Pr\left(\text{order}|\theta_1\right) Pr\left(\theta_1 = 1\right)}{Pr\left(\text{order}\right)} \\
Pr\left(\theta_1 = 1|\text{buy}\right) &= \frac{\frac{1+\mu_1}{2}\frac{1}{2}}{\frac{1}{2}} = \frac{1+\mu_1}{2} \\
Pr\left(\theta_1 = 0|\text{buy}\right) &= \frac{1-\mu_1}{2} \\
Pr\left(\theta_1 = 1|\text{sell}\right) &= \frac{1-\mu_1}{2} \\
Pr\left(\theta_1 = 0|\text{sell}\right) &= \frac{1+\mu_1}{2}
\end{aligned}$$

The probability of a buy or sell order conditional on asset type for risky debt is:

$$\begin{aligned}
Pr\left(\text{buy}|\theta_2 = 1\right) &= \mu_2 + (1-\mu_2)\frac{1}{2} = \frac{1+\mu_2}{2} \\
Pr\left(\text{sell}|\theta_2 = 1\right) &= \frac{1-\mu_2}{2}
\end{aligned}$$

Using Bayesian updating, the distribution of $\tilde{\theta}_2$ conditional on observed orders is given by:

$$\begin{aligned}
Pr\left(\theta_2 = 1|\text{order}\right) &= \frac{Pr\left(\theta_2 \cap \text{order}\right)}{Pr\left(\text{order}\right)} = \frac{Pr\left(\text{order}|\theta_2\right) Pr\left(\theta_2 = 1\right)}{Pr\left(\text{order}\right)} \\
Pr\left(\theta_2 = 1|\text{buy}\right) &= \frac{\frac{1+\mu_2}{2}\frac{1}{2}}{\frac{1}{2}} = \frac{1+\mu_2}{2} \\
Pr\left(\theta_2 = 0|\text{buy}\right) &= \frac{1-\mu_2}{2} \\
Pr\left(\theta_2 = 1|\text{sell}\right) &= \frac{1-\mu_2}{2} \\
Pr\left(\theta_2 = 0|\text{sell}\right) &= \frac{1+\mu_2}{2}
\end{aligned}$$

6.4 Proof of Lemma 1

We rule out "separating" and "semi-separating" equilibria such that a market maker (MM) offers a menu of security designs (instead of only one security design) such that firms with different asset types make different choices. For brevity, below we refer to "asset types" making choices.

The MM sets market prices after observing the asset type's choice of security design, by using its beliefs about asset types' choices and the fair pricing condition. For any postulated separating or semi-separating equilibrium, we will show that there is a profitable deviation for at least one asset type: at least one asset type will not choose the security design that the MM believes. This in turn implies that the MM's beliefs are not verified, which invalidates the postulated equilibrium.

The proof is in four parts. The first part rules out separating equilibria, the next three parts rule out semi-separating equilibria.

Part (i). We rule out equilibria in which each asset type chooses a different set of securities (in principle, there can be more than two sets of securities on the menu offered by the MM; for example, let γ take different values in the type of security design described in Proposition 1). If this were an equilibrium, then an asset would be valued according to its type $\{\theta_1, \theta_2\}$, which would be revealed by security issuance decisions, so that asset market valuations would be as in equation (1) in the absence of arbitrage. In particular, with securities associated with type $\{0, 0\}$, the asset would be assigned a valuation $\frac{L+M}{2}$, and could achieve a strictly higher valuation by issuing a set of securities associated with another asset type, which contradicts that asset type $\{0, 0\}$ will issue a different set of securities (relative to other asset types) in equilibrium.

Part (ii). We rule out equilibria in which two asset types issue the same set of securities, a third type chooses another set of securities, and the fourth type chooses yet another set of securities. There are several possibilities:

1. If $\{0, 0\}$ is the only asset type issuing a given set of securities, then in equilibrium its type is revealed by security issuance decisions. With this choice of securities, the asset will be assigned a valuation $\frac{L+M}{2}$ according to equation (1) in the absence of arbitrage. However, it could achieve a strictly higher valuation by issuing a set of securities associated with another asset type, which contradicts that an asset type $\{0, 0\}$ will be the only asset type issuing a given set of securities in equilibrium.

2. If $\{1, 1\}$ is the only asset type issuing a given set of securities, then in equilibrium its type is revealed by security issuance decisions. With this choice of securities, the asset will be assigned a valuation $\frac{H+M}{2}$ according to equation (1) in the absence of arbitrage. However, any other asset type could then achieve a strictly higher valuation compared to its equilibrium valuation by issuing this set of securities, which contradicts that an asset type $\{1, 1\}$ will be the only asset type issuing a given set of securities in equilibrium.

3. If $\{1, 1\}$ and $\{0, 0\}$ are issuing the same set of securities, and $\{1, 0\}$ and $\{0, 1\}$ are each issuing a different set of securities, then issuing a set of securities that is in equilibrium issued by asset type $\{1, 0\}$ (asset type $\{0, 1\}$) will give the asset a valuation of $\frac{H+L}{2}$ (M) according to equation (1) in the absence of arbitrage. There are two non-trivial cases (the case when all asset types choose the same type of security design – either of the type described in Proposition 1 or of the type described in Proposition 3 – is trivial). First, suppose that asset types $\{1, 1\}$ and $\{0, 0\}$ issue only one security with nonconstant payoff. In this equilibrium, when an asset type chooses one security with nonconstant payoff:

$$
\begin{aligned}
Pr(\theta_1 = 1, \theta_2 = 1|\text{buy}, \text{buy}) &= \frac{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1 = 0, \theta_2 = 0|\text{buy}, \text{buy}) &= \frac{\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1 = 1, \theta_2 = 1|\text{buy}, \text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1}{2} \\
Pr(\theta_1 = 0, \theta_2 = 0|\text{buy}, \text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1}{2} \\
Pr(\theta_1 = 1, \theta_2 = 1|\text{sell}, \text{sell}) &= \frac{\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1 = 0, \theta_2 = 0|\text{sell}, \text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}
\end{aligned}
$$

The price schedule for one security with nonconstant payoff is:

$$
\begin{aligned}
P_U(\text{buy}, \text{buy}) &= \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H+M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L+M}{2} \\
&= \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2} \\
P_U(\text{buy}, \text{sell}) &= \frac{1}{2}\frac{H+M}{2} + \frac{1}{2}\frac{L+M}{2} = \frac{H}{4} + \frac{M}{2} + \frac{L}{4} \\
P_U(\text{sell}, \text{sell}) &= \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H+M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L+M}{2} \\
&= \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}
\end{aligned}
$$

In this equilibrium, expected asset value conditional on asset type is:

$$\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=1,\theta_2=1\right] &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&\quad +\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\left(\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
&\quad +\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]\left(\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right) \\
&\quad +\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\left(\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
\mathbb{E}\left[P_U|\theta_1=0,\theta_2=0\right] &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&\quad +\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\left(\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
&\quad +\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]\left(\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right) \\
&\quad +\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\left(\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right)
\end{aligned}$$

If an asset type $\{1,0\}$ deviates and chooses one security, its expected value is:

$$\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=1,\theta_2=0\right] &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&\quad +\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\left[\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&\quad +\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]\left[\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right] \\
&\quad +\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\left[\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&= \frac{2+\mu_1^2-\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{H}{8}+\frac{M}{2}+\frac{2-\mu_1^2+\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{L}{8} \\
&= \frac{1}{4}H+\frac{1}{2}M+\frac{1}{4}L+\frac{\mu_1^2-\mu_2^2}{8+8\mu_1\mu_2}[H-L] \qquad (126)
\end{aligned}$$

If an asset type $\{0,1\}$ deviates and chooses one security, its expected value is:

$$
\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=0,\theta_2=1\right] &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&\quad +\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\left[\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&\quad +\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]\left[\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right] \\
&\quad +\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\left[\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&= \frac{2-\mu_1^2+\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{H}{8}+\frac{M}{2}+\frac{2+\mu_1^2-\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{L}{8} \\
&= \frac{1}{4}H+\frac{1}{2}M+\frac{1}{4}L-\frac{\mu_1^2-\mu_2^2}{8+8\mu_1\mu_2}[H-L] \qquad (127)
\end{aligned}
$$

An asset type $\{1,0\}$ had rather deviate if and only if the expression in (126) is larger than $\frac{H+L}{2}$, that is:

$$
\frac{H+L}{2}-\frac{\mu_1^2-\mu_2^2}{4+4\mu_1\mu_2}[H-L]<M \qquad (128)
$$

An asset type $\{0,1\}$ had rather deviate if and only if the expression in (127) is larger than M, that is:

$$
\frac{H+L}{2}H-\frac{\mu_1^2-\mu_2^2}{4+4\mu_1\mu_2}[H-L]>M \qquad (129)
$$

Generically, either equation (128) holds or equation (129) holds. That is, either type $\{1,0\}$ or type $\{0,1\}$ deviates, which invalidates the equilibrium. Second, suppose that an asset types $\{1,0\}$ and $\{0,1\}$ each issue only one security with nonconstant payoff, and asset types $\{1,1\}$ or $\{0,0\}$ choose the same security design with at least two securities with nonconstant and linearly independent payoffs. The proof that such an equilibrium does not exist is similar to point 3. above.

Part (iii). We rule out equilibria in which one asset type chooses a given set of securities, and the other three types choose another set of securities.

1. Suppose that asset type $\{0,0\}$ chooses a given set of securities, and other asset types choose another set of securities. Then any asset type issuing the former set of securities is identified as a type $\{0,0\}$ and assigned a valuation $\frac{L+M}{2}$. An asset type $\{0,0\}$ could achieve a strictly higher valuation by issuing the same set of securities as other asset types, which contradicts that an asset type $\{0,0\}$ chooses the former set of securities in equilibrium.

2. Suppose that asset type $\{1,1\}$ chooses a given set of securities, and other asset types choose another set of securities. Then any asset type issuing the former set of securities is identified as a type $\{1,1\}$ and assigned a valuation $\frac{H+M}{2}$. An asset type $\{1,0\}$, $\{0,1\}$, or $\{0,0\}$ could achieve a strictly higher valuation by issuing the same set of securities as a $\{1,1\}$ asset type and achieve the same high valuation, which contradicts that only $\{1,1\}$ chooses the former set of securities in equilibrium.

3. Suppose that asset type $\{1,0\}$ chooses a given set of securities, and other asset types choose another set of securities. Then any asset type issuing the former set of securities is identified as a type $\{1,0\}$ and assigned a valuation $\frac{H+L}{2}$. First, suppose that asset types other than $\{1,0\}$ choose only one security with nonconstant payoff whereas asset type $\{1,0\}$ chooses another security design. In this equilibrium, when an asset type chooses one security with nonconstant payoff, Bayesian updating gives:

$$
\begin{aligned}
Pr(\theta_1=1,\theta_2=1|\text{buy},\text{buy}) &= \frac{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=1|\text{buy},\text{buy}) &= \frac{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1+\mu_2-\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=0|\text{buy},\text{buy}) &= \frac{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}\\
Pr(\theta_1=1,\theta_2=1|\text{buy},\text{sell}) &= \frac{\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]}{\frac{2}{3}\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]+\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]} = \frac{2-2\mu_1\mu_2}{6-2\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=1|\text{buy},\text{sell}) &= \frac{\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]}{\frac{2}{3}\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]+\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]} = \frac{2+2\mu_1\mu_2}{6-2\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=0|\text{buy},\text{sell}) &= \frac{\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]}{\frac{2}{3}\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]+\frac{1}{3}\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]} = \frac{2-2\mu_1\mu_2}{6-2\mu_1\mu_2}\\
Pr(\theta_1=1,\theta_2=1|\text{sell},\text{sell}) &= \frac{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=1|\text{sell},\text{sell}) &= \frac{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}} = \frac{1+\mu_1-\mu_2-\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}\\
Pr(\theta_1=0,\theta_2=0|\text{sell},\text{sell}) &= \frac{\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1}{3}\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1}{3}\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}
\end{aligned}
$$

The price schedule for an asset type issuing one security with nonconstant payoff is:

$$
\begin{aligned}
P_U(\text{buy},\text{buy}) &= \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}\frac{H+M}{2}+\frac{1-\mu_1+\mu_2-\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}M+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{3-\mu_1+\mu_2+\mu_1\mu_2}\frac{L+M}{2}\\
P_U(\text{buy},\text{sell}) &= \frac{2-2\mu_1\mu_2}{6-2\mu_1\mu_2}\frac{H+M}{2}+\frac{2+2\mu_1\mu_2}{6-2\mu_1\mu_2}M+\frac{2-2\mu_1\mu_2}{6-2\mu_1\mu_2}\frac{L+M}{2}\\
P_U(\text{sell},\text{sell}) &= \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}\frac{H+M}{2}+\frac{1+\mu_1-\mu_2-\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}M+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{3+\mu_1-\mu_2+\mu_1\mu_2}\frac{L+M}{2}
\end{aligned}
$$

Consider the problem of an asset type $\{0,0\}$. If it deviates and chooses two securities with nonconstant and linearly independent payoffs as does asset type $\{1,0\}$, then its assigned value is $\frac{H+L}{2}$. If it chooses one security with nonconstant payoff, its expected value is:

$$\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]P_U(\text{buy},\text{sell})+\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell},\text{sell}) \tag{130}$$

If the expression in equation (130) is smaller than $\frac{H+L}{2}$, then asset type $\{0,0\}$ deviates, which is inconsistent with the postulated equilibrium. If the expression in equation (130) is larger than $\frac{H+L}{2}$, then asset type $\{0,0\}$ does not deviate. However, this implies that the following expression:

$$\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]P_U(\text{buy},\text{sell})+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell},\text{sell}) \tag{131}$$

is larger than $\frac{H+L}{2}$, so that it is in the interests of asset type $\{1,0\}$ to deviate by issuing only one security with nonconstant payoff, which is again inconsistent with the postulated equilibrium. The proof that there is no equilibrium in which asset types other than $\{1,0\}$ choose two securities with nonconstant and linearly independent payoffs whereas asset type $\{1,0\}$ chooses another security design is similar and is omitted.

4. Suppose that asset type $\{0,1\}$ chooses a given set of securities, and other asset types choose another set of securities. The proof that such an equilibrium does not exist is similar to point 3. above.

Part (iv). We rule out equilibria in which two asset types choose one set of securities, and the other two asset types choose another set of securities.

1. Suppose that asset types $\{0,1\}$ and $\{0,0\}$ choose a given set of securities, and asset types $\{1,1\}$ and $\{1,0\}$ choose another set of securities. It is then easy to show that an asset type $\{0,0\}$ would be better off deviating and issuing the latter set of security(ies) to pool with types $\{1,1\}$ and $\{1,0\}$ (indeed, on the market for the latter set of securities, prices are a weighted average of the market values of asset types $\{1,1\}$ and $\{1,0\}$, whereas on the market for the former set of securities, prices are a weighted average of the values of asset types $\{0,1\}$ and $\{0,0\}$), a contradiction.

2. Suppose that asset types $\{1,0\}$ and $\{0,0\}$ choose a given set of securities, and asset types $\{1,1\}$ and $\{0,1\}$ choose another set of securities. It is then easy to show that asset type $\{0,0\}$ would be better off deviating and issuing the latter set of securities to pool with types $\{1,1\}$ and $\{0,1\}$ (indeed, on the market for the latter set of securities, prices are a weighted average of the market values of asset types $\{1,1\}$ and $\{0,1\}$, whereas on the market for the former set of securities, prices are a weighted average of the values of asset types $\{1,0\}$ and $\{0,0\}$), a contradiction.

3. There are two non-trivial cases (the case when all asset types choose the same type of security design – either of the type described in Proposition 1 or of the type described in Proposition 3 – is trivial). First, suppose that asset types $\{1,0\}$ and $\{0,1\}$ choose two securities with nonconstant and linearly independent payoffs, and asset types $\{1,1\}$ and $\{0,0\}$ choose only one security with nonconstant payoff. In this equilibrium, when an asset type chooses one security with nonconstant payoff:

$$\begin{aligned}
Pr(\theta_1 = 1, \theta_2 = 1 | \text{buy}, \text{buy}) &= \frac{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1 = 0, \theta_2 = 0 | \text{buy}, \text{buy}) &= \frac{\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1 = 1, \theta_2 = 1 | \text{buy}, \text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1}{2} \\
Pr(\theta_1 = 0, \theta_2 = 0 | \text{buy}, \text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1-\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1}{2} \\
Pr(\theta_1 = 1, \theta_2 = 1 | \text{sell}, \text{sell}) &= \frac{\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2} \\
Pr(\theta_1 = 0, \theta_2 = 0 | \text{sell}, \text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}}{\frac{1+\mu_1}{2}\frac{1+\mu_2}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}} = \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}
\end{aligned}$$

The price schedule for an asset type issuing one security with nonconstant payoff is:

$$\begin{aligned}
P_U(\text{buy}, \text{buy}) &= \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H+M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L+M}{2} \\
&= \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2} \\
P_U(\text{buy}, \text{sell}) &= \frac{1}{2}\frac{H+M}{2} + \frac{1}{2}\frac{L+M}{2} = \frac{H}{4} + \frac{M}{2} + \frac{L}{4} \\
P_U(\text{sell}, \text{sell}) &= \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H+M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L+M}{2} \\
&= \frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2} + \frac{M}{2} + \frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}
\end{aligned}$$

In this equilibrium, expected asset value conditional on asset type is:

$$
\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=1,\theta_2=1\right] &= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\left(\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
&+\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]\left(\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right) \\
&+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\left(\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
\mathbb{E}\left[P_U|\theta_1=1,\theta_2=1\right] &= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&+\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\left(\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right) \\
&+\left[\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\right]\left(\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right) \\
&+\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}\left(\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right)
\end{aligned}
$$

In this equilibrium, when an asset type chooses two securities with nonconstant and linearly independent payoffs, on the market for the security with payoff vector of the type (Γ,Γ,F):

$$
\begin{aligned}
Pr(\theta_1=1|\text{buy}) &= \frac{\frac{1+\mu_1}{2}\frac{1}{2}}{\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1-\mu_1}{2}\frac{1}{2}}=\frac{1+\mu_1}{2} \\
Pr(\theta_1=1|\text{sell}) &= \frac{\frac{1-\mu_1}{2}\frac{1}{2}}{\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1-\mu_1}{2}\frac{1}{2}}=\frac{1-\mu_1}{2} \\
Pr(\theta_1=0|\text{buy}) &= \frac{\frac{1-\mu_1}{2}\frac{1}{2}}{\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1-\mu_1}{2}\frac{1}{2}}=\frac{1-\mu_1}{2} \\
Pr(\theta_1=0|\text{sell}) &= \frac{\frac{1+\mu_1}{2}\frac{1}{2}}{\frac{1+\mu_1}{2}\frac{1}{2}+\frac{1-\mu_1}{2}\frac{1}{2}}=\frac{1+\mu_1}{2}
\end{aligned}
$$

The price schedule for the security with payoff vector of the type (Γ,Γ,F) is:

$$P_E(\text{buy}) = \frac{1+\mu_1}{2}\frac{H-M}{2}+\Gamma \tag{132}$$

$$P_E(\text{sell}) = \frac{1-\mu_1}{2}\frac{H-M}{2}+\Gamma \tag{133}$$

In this equilibrium, when an asset type chooses two securities with nonconstant and linearly independent payoffs, on the market for the security with payoff vector of the type (f, k, k):

$$\begin{aligned}
Pr(\theta_2 = 1|\text{buy}) &= \frac{\frac{1+\mu_2}{2}\frac{1}{2}}{\frac{1+\mu_2}{2}\frac{1}{2} + \frac{1-\mu_2}{2}\frac{1}{2}} = \frac{1+\mu_2}{2} \\
Pr(\theta_2 = 1|\text{sell}) &= \frac{\frac{1-\mu_2}{2}\frac{1}{2}}{\frac{1+\mu_2}{2}\frac{1}{2} + \frac{1-\mu_2}{2}\frac{1}{2}} = \frac{1-\mu_2}{2} \\
Pr(\theta_2 = 0|\text{buy}) &= \frac{\frac{1-\mu_2}{2}\frac{1}{2}}{\frac{1+\mu_2}{2}\frac{1}{2} + \frac{1-\mu_2}{2}\frac{1}{2}} = \frac{1-\mu_2}{2} \\
Pr(\theta_2 = 0|\text{sell}) &= \frac{\frac{1+\mu_2}{2}\frac{1}{2}}{\frac{1+\mu_2}{2}\frac{1}{2} + \frac{1-\mu_2}{2}\frac{1}{2}} = \frac{1+\mu_2}{2}
\end{aligned}$$

The price schedule for the security with payoff vector of the type (f, k, k) is:

$$P_D(\text{buy}) = \frac{1+\mu_2}{2}M + \frac{1-\mu_2}{2}\frac{L+M}{2} - \Gamma = \frac{3+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2} - \Gamma \quad (134)$$

$$P_D(\text{sell}) = \frac{1-\mu_2}{2}M + \frac{1+\mu_2}{2}\frac{L+M}{2} - \Gamma = \frac{3-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2} - \Gamma \quad (135)$$

In this equilibrium, expected asset value conditional on asset type is:

$$\begin{aligned}
\mathbb{E}\left[P_E + P_D|\theta_1 = 1, \theta_2 = 0\right] &= \frac{1+\mu_1}{2}\frac{1+\mu_1}{2}\frac{H-M}{2} + \frac{1-\mu_1}{2}\frac{1-\mu_1}{2}\frac{H-M}{2} \\
&\quad + \frac{1-\mu_2}{2}\left[\frac{3+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2}\right] + \frac{1+\mu_2}{2}\left[\frac{3-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2}\right] \\
&= \frac{1+\mu_1^2}{2}\frac{H}{2} + \frac{2-\mu_1^2-\mu_2^2}{2}\frac{M}{2} + \frac{1+\mu_2^2}{2}\frac{L}{2} \\
&= \frac{1}{4}H + \frac{1}{2}M + \frac{1}{4}L + \frac{\mu_1^2}{4}[H-M] - \frac{\mu_2^2}{4}[M-L] \quad (136) \\
\mathbb{E}\left[P_E + P_D|\theta_1 = 0, \theta_2 = 1\right] &= \frac{1-\mu_1}{2}\frac{1+\mu_1}{2}\frac{H-M}{2} + \frac{1+\mu_1}{2}\frac{1-\mu_1}{2}\frac{H-M}{2} \\
&\quad + \frac{1+\mu_2}{2}\left[\frac{3+\mu_2}{2}\frac{M}{2} + \frac{1-\mu_2}{2}\frac{L}{2}\right] + \frac{1-\mu_2}{2}\left[\frac{3-\mu_2}{2}\frac{M}{2} + \frac{1+\mu_2}{2}\frac{L}{2}\right] \\
&= \frac{1-\mu_1^2}{2}\frac{H}{2} + \frac{2+\mu_1^2+\mu_2^2}{2}\frac{M}{2} + \frac{1-\mu_2^2}{2}\frac{L}{2} \\
&= \frac{1}{4}H + \frac{1}{2}M + \frac{1}{4}L - \frac{\mu_1^2}{4}[H-M] + \frac{\mu_2^2}{4}[M-L] \quad (137)
\end{aligned}$$

If an asset type $\{1, 0\}$ deviates and chooses only one security with nonconstant payoff, its expected value is:

$$\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=1,\theta_2=0\right] &= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\left[\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&+\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]\left[\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right] \\
&+\frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\left[\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&= \frac{2+\mu_1^2-\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{H}{8}+\frac{M}{2}+\frac{2-\mu_1^2+\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{L}{8} \\
&= \frac{1}{4}H+\frac{1}{2}M+\frac{1}{4}L+\frac{\mu_1^2-\mu_2^2}{8+8\mu_1\mu_2}[H-L] \qquad (138)
\end{aligned}$$

If an asset type $\{0,1\}$ deviates and chooses only one security with nonconstant payoff, its expected value is:

$$\begin{aligned}
\mathbb{E}\left[P_U|\theta_1=0,\theta_2=1\right] &= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}P_U(\text{buy},\text{buy})+\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]P_U(\text{buy},\text{sell}) \\
&+\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}P_U(\text{sell},\text{sell}) \\
&= \frac{1-\mu_1}{2}\frac{1+\mu_2}{2}\left[\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&+\left[\frac{1+\mu_1}{2}\frac{1+\mu_2}{2}+\frac{1-\mu_1}{2}\frac{1-\mu_2}{2}\right]\left[\frac{H}{4}+\frac{M}{2}+\frac{L}{4}\right] \\
&+\frac{1+\mu_1}{2}\frac{1-\mu_2}{2}\left[\frac{1-\mu_1-\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{H}{2}+\frac{M}{2}+\frac{1+\mu_1+\mu_2+\mu_1\mu_2}{2+2\mu_1\mu_2}\frac{L}{2}\right] \\
&= \frac{2-\mu_1^2+\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{H}{8}+\frac{M}{2}+\frac{2+\mu_1^2-\mu_2^2+2\mu_1\mu_2}{1+\mu_1\mu_2}\frac{L}{8} \\
&= \frac{1}{4}H+\frac{1}{2}M+\frac{1}{4}L-\frac{\mu_1^2-\mu_2^2}{8+8\mu_1\mu_2}[H-L] \qquad (139)
\end{aligned}$$

An asset type $\{1,0\}$ had rather deviate if and only if the expression in (138) is larger than the expression in (136). An asset type $\{0,1\}$ had rather deviate if and only if the expression in (139) is larger than the expression in (137). Generically, one of these two asset types deviates, which invalidates the equilibrium.
Second, suppose that asset types $\{1,0\}$ and $\{0,1\}$ choose only one security with nonconstant payoff, whereas asset types $\{1,1\}$ and $\{0,0\}$ choose at least two securities with nonconstant and linearly independent payoffs. The proof that such an equilibrium does not exist is similar to point 3. above.

This concludes the proof. ■