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Wage Bargaining and Capital Accumulation: A Dynamic Version of the Monopoly Union Model*

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Abstract

In this paper, I explore the relationship between wage bargaining and capital accumulation by developing a differential game in which a monopolistic union sets the wage of its members by taking as given the optimal employment strategy of a representative firm and the way in which capital is evaluated over time. Under the assumption that investment amounts to a constant share of produced output, I show that a meaningful open-loop Stackelberg equilibrium requires the union to be more patient than the firm. Moreover, relying on some numerical simulations, I show that although adjustments towards the steady-state equilibrium occur through damped oscillations, after an initial period of decline the model predicts a stable union wage premium.

Keywords: Monopoly union model; Capital accumulation; Binding wage contracts; Differential games; Open-loop Stackelberg equilibrium.

JEL Classification: J31; J51; J52.

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1 Introduction

Wage bargaining and capital accumulation are critical economic processes that display a non-trivial relationship of strategic interdependence among the actions undertaken by the interested parties. On the one hand, the outcome of wage negotiations that usually involve trade unions and firms is crucial for the determination of the internal resources that entrepreneurs can directly spend to increase their productive capacity without relying on external finance (cf. Pischke, 2005; Chen and Chen, 2013; Ji et al., 2016). On the other hand, the level achieved by capital accumulation in the firms' plant is one of the essential drivers of the workers' productivity employed by entrepreneurs and such a productivity is paramount for determining the size of the cake to share during the wage negotiations carried out with the unions (cf. Kapadia, 2005; Karanassou et al., 2008).

In this paper, aiming at shedding lights on the bilateral relationship described above, I develop a differential game in which a monopolistic union sets the wage of its risk neutral members by taking as given the optimal employment strategy of a representative firm and the way in which it evaluates its productive capital over time (cf. van der Ploeg, 1987; Holm et al. 1994; Jafarey et al. 1998; Irmen and Wigger, 2002/2003). In other words, I consider a situation in which the union of workers behaves as a Stackelberg leader by pre-committing itself to a given wage strategy that cannot be reneged and that shapes the employment and the investment decisions of the firm that acts instead as the follower. As noticed by a number of authors, among the different non-Walrasian representations of the labour market, an institutional setting in which a monopolistic union has a strategic dominance in the wage bargaining process, although dynamically inconsistent, is the less costly in terms of the efficiency of realized allocations – especially in terms of employed capital – and is not too distant from the institutional setting of many different actual economies where long-term wage contracts are signed and capital is relatively malleable (cf. Grout, 1984, 1985; Anderson and Devereux, 1988; Devereux and Lockwood, 1991).

The novelty of the theoretical framework developed in this paper is twofold. First, regarding the modelling strategy, I limit to two the number of the control variables available for the two players. Specifically, assuming that investment is given by a constant share of produced output (e.g. Solow, 1956), I consider a situation in which the firm decides only about the level of employment and the union sets the wage rate. In this context, higher (lower) wages lead the firm to employ fewer (more) workers. For a given level of the capital stock, this means lower (higher) output and so lower (higher) investment by emphasizing a clear trade-off between the union's wage policy and capital accumulation in the firm's plant. Moreover, on a numerical perspective, such a simplification allows for a straightforward and parsimonious calibration of the model economy so that its findings can be easily compared with the actual data of the involved variables.

The results achieved in this theoretical exploration can be briefly summarized as follows.

First, under a binding wage agreement between the two parties, a meaningful open-loop Stackelberg equilibrium requires the union of workers to be more patient than the firm in which they are employed (cf. Cripps, 1997; Bressan and Jiang, 2020; Candido and Guerrazzi, 2023). Thereafter, relying on some numerical simulations tailored on the US economy, it is possible to show that although adjustments towards the steady-state equilibrium occur through damped oscillations, after an initial period of monotonic decline, the model under examination predicts a fairly stable union wage premium whose magnitude is in line with the available empirical evidence (cf. Açıkgöz and Kaymak, 2014; Gabriel and Schmitz, 2014).

The paper is arranged as follows. Section 2 develops the model economy. Section 3 explores its numerical properties. Finally, Section 4 concludes.

2 The model

I consider a deterministic model economy in which time (t) is continuous – so that $t \in \mathbb{R}_+$ – where a monopolistic union of identical workers sets the wage of its employed members by taking as given the optimal employment strategy and the evaluation of capital put forward over time by a representative firm endowed with a Cobb-Douglas technology whose choices are also constrained by the standard law of capital accumulation (cf. van der Ploeg, 1987; Jafarey et al., 1998).

On the one hand, the production function of the firm is assumed to be given by

$$Y(t) = (K(t))^{\alpha} (L(t))^{1-\alpha} \tag{1}$$

where $Y(t)$ is the flow of produced output, $K(t)$ is the predetermined and publicly observable stock of employed capital, $L(t)$ is the number of employed workers whereas $\alpha \in (0,1)$ is the elasticity of output with respect to capital (cf. Devereux and Lockwood, 1991).

On the other hand, following De Ménil (1971), I assume that the utilitarian union of workers maximizes the surplus of its labour income with respect to the wage bill that – on the whole – all members would obtain in a competitive spot labour market. Consequently, the objective function of the union can be written as

$$U(t) = L(t)(w(t) - w_0(t)) \tag{2}$$

where $U(t)$ is union's utility, $w(t)$ is the real wage rate whereas $w_0(t)$ is the time-varying competitive wage that would prevail in a spot labour market.

According to eq. (2), the two configurations of the wage enter $U(t)$ in a linear manner by revealing the risk attitude of unionized workers; indeed, a linear specification of the utility derived from the wage rate and the level of employment implies that union members are risk neutral regarding fluctuations in realized labour earnings (cf. Manning, 1991; Mezzetti, 1993; Jafarey et al. 1998).[1]

Normalizing the size of the union membership to 1, the competitive wage that the representative member would receive in a spot labour market is equal to the corresponding level of the marginal productivity of labour. Specifically, considering the production function in eq. (1), $w_0(t)$ is given by

$$w_0(t) = \left. \frac{\partial Y(t)}{\partial L(t)} \right|_{L(t)=1} = (1-\alpha)(K(t))^{\alpha} \tag{3}$$

The expression in eq. (3) shows that the remuneration that unionized workers can receive in a competitive labour market by leaving the bargaining table strictly depends on the level achieved by capital accumulation. Consequently, aware of the determinants of its outside option, the union of workers will set the wage not only by considering the impact of its choices on the employment decisions of the firm but also with a concern for the actual level of $K(t)$ owned by the firm and the way in which it is valued over time (cf. Pohjola, 1984; Manning, 1987; Anderson and Devereux, 1988; Irmen and Wigger, 2002/2003).

Relying on backward induction, I will solve first the problem of the firm and then the one of the union. Moreover, aiming at simplifying notation, all the times in which it does not detract from the clarity of the exposition, in the remainder of the paper I will omit the functional dependence of the involved variables on time.

2.1 The problem of the firm

For the sake of simplicity, I assume that the central bank is targeting a fixed value $r \gtreqless 0$ of the real interest rate and investment is boosted by a constant saved share of produced output (cf. Solow, 1956). Without any loss of generality, the latter hypothesis allows the model to have only two control variables, i.e., the level of employment chosen by the firm and the wage rate decided by the union.[2] Thereafter, taking the sequence of wages set by the union as given and considering the production function in eq. (1), the problem of the representative firm can be written as

$$\max_{L} \int_{t=0}^{\infty} \exp(-\rho_{\mathrm{F}} t) \left(K^{\alpha} L^{1-\alpha} - wL - rK\right) dt \tag{4}$$

[1] Risk aversion on the side of workers would deteriorate the bargaining position of the union. Its introduction would complicate the analytical treatment of the model without affecting its main findings.

[2] Assuming that the purchases of new productive capacity are given by a constant share of output straightforwardly implies that investment is procyclical, a business cycle regularity that the textbook q-model of investment with adjustment costs and exogenously given price of capital goods is unable to replicate (cf. Guerrazzi and Candido, 2023).

s.to

$$\dot{K} = sK^{\alpha}L^{1-\alpha} - \delta K \qquad K(0) = K_0 \tag{5}$$

where $\rho_{\mathrm{F}} > 0$ is the discount factor of the firm, $s \in (0,1)$ is the investment-output rate, $\delta > 0$ is the depreciation rate whereas $K_0 > 0$ is the initial level of the stock of capital (cf. Anderson and Devereux, 1988).

Given the time-path of w, the first-order conditions (FOCs) for the dynamic problem in eq.s (4) and (5) are given by

$$(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}(1+s\Lambda) - w = 0 \quad \text{for all } t \tag{6}$$

$$\dot{\Lambda} = \Lambda\left(\rho_{\mathrm{F}} + \delta - \alpha s\left(\frac{K}{L}\right)^{-(1-\alpha)}\right) + r - \alpha\left(\frac{K}{L}\right)^{-(1-\alpha)} \tag{7}$$

$$\lim_{t\to\infty} \exp\left(-r_{\mathrm{F}}t\right)\Lambda\left(t\right)K\left(t\right) = 0 \tag{8}$$

where Λ is the costate variable associated to the capital accumulation constraint in eq. (5) that in the present setting mirrors the shadow value of capital for the firm (cf. Hayashi, 1982).

The expression in eq. (6) is the FOC with respect to L and it provides the instantaneous demand for labour undertaken by the representative firm. Moreover, eq. (7) provides the optimal evolution of Λ and it implies that in each instant the marginal productivity of capital must be equal to the user cost of capital (cf. van der Ploeg, 1987). Furthermore, the expression in (8) is an endpoint limit on the value of the state variable and it represents the required transversality condition.

According to eq. (6), the optimal employment strategy of the firm can be written simply as

$$L = K\Phi^{\frac{1}{\alpha}} \tag{9}$$

where $\Phi \equiv (1-\alpha)(1+s\Lambda)/w$.

The expression in eq. (9) is the optimal open-loop strategy of the firm that supports the pre-know response to the control input set by the union, and it can be taken as a dynamic version of the demand for labour whose elasticity with respect to the wage is constant and equal to $-1/\alpha$. In detail, eq. (9) reveals that – given the level achieved by K – the optimal level of employment chosen by the firm in each instant is an increasing (decreasing) function of the value taken by the shadow value of capital (real wage rate set by the union).[3] Moreover, in

[3]The dependency of the optimal level of employment from the wage – the control variable of the union – reveals the controllability of the firm's problem (cf. Lambertini, 2018, Chapter 9).

terms of the involved parameters, employment is also boosted by the elasticity of the production function with respect to labour and by the share of invested output.[4]

2.2 The problem of the union

When it sets the wage, the union of workers is assumed to take into account the preferences of its members as well as the law of capital accumulation, the manner in which the firm makes its employment decisions and the way in which the latter evaluates its productive capacity over time. Therefore, considering the expressions in eq.s (2), (3), (5), (7) and (9), its dynamic problem can be written as

$$\max_{w} \int_{t=0}^{\infty} \exp\left(-\rho_{\mathrm{U}} t\right) \left(K \Phi^{\frac{1}{\alpha}} \left(w - \left(1-\alpha\right) K^{\alpha}\right)\right) dt \tag{10}$$

s.to

$$\dot{K} = K \left(s \Phi^{\frac{1-\alpha}{\alpha}} - \delta\right) \qquad K(0) = K_0 \tag{11}$$

$$\dot{\Lambda} = \Lambda \left(\rho_{\mathrm{F}} + \delta - \alpha s \Phi^{\frac{1-\alpha}{\alpha}}\right) + r - \alpha \Phi^{\frac{1-\alpha}{\alpha}} \tag{12}$$

where $\rho_{\mathrm{U}} > 0$ is the discount rate of the union that does not necessarily coincides with the one of the firm.

The FOCs for the problem in (10) – (12) are given by

$$\Phi - \frac{s\Gamma K + (1 + s\Lambda)\left(\left(1-\alpha\right) K - \alpha \Psi\right)}{K^{1+\alpha}} = 0 \qquad \text{for all } t \tag{13}$$

$$\dot{\Gamma} = \Gamma \left(\rho_{\mathrm{U}} + \delta - s \Phi^{\frac{1-\alpha}{\alpha}}\right) - \Phi^{\frac{1}{\alpha}} \left(w - \left(1-\alpha\right)\left(1+\alpha\right) K^{\alpha}\right) \tag{14}$$

$$\lim_{t \to \infty} \exp\left(-r_U t\right) \Gamma\left(t\right) K\left(t\right) = 0 \tag{15}$$

$$\dot{\Psi} = \Psi \left(\rho_{\mathrm{U}} - \rho_{\mathrm{F}} - \delta + s \Phi^{\frac{1-\alpha}{\alpha}}\right) - \frac{s(1-\alpha) K \Phi^{\frac{1-\alpha}{\alpha}} \left(1 + s\left(\Lambda + \Gamma\right) - \Phi K^{\alpha}\right)}{\alpha\left(1 + s\Lambda\right)} \qquad \Psi(0) = 0 \tag{16}$$

where Γ and Ψ are the costate variables associated, respectively, to the capital accumulation constraint in eq. (11) and to the optimal evolution of the shadow price of capital for the firm in eq. (12).

[4] Intuitively, the former is a proxy for the immediate contribution of labour to produced output, the latter a proxy for its deferred contribution.

Eq. (13) is the FOC with respect to w and it shows that in each instant the union sets the wage by balancing – at the margin – the increase in utility driven by higher wages with the corresponding reduction of employment driven higher values of w. Eq. (14) pins down the optimal dynamics of the costate variable associated to the capital accumulation constraint. Similarly to eq. (8), eq. (15) is the required transversality condition. Moreover, eq. (16) describes the optimal dynamics of the costate variable associated to the likewise optimal evolution of the shadow price of capital for the follower.

As far as eq. (16) is concerned, it is worth noticing that – at the beginning of the optimization horizon – the marginal union's utility triggered by variations of the shadow price of capital for the firm has to be equal to zero (cf. Başar and Olsder, 1999; Chapter 7; Dockner et al. 2000, Chapter 5). Such an additional initial condition for the model economy is required because the level of employment chosen by the firm (L) as well its marginal profit triggered by variations of the stock of capital (Λ) are free to jump in the initial stage of the game. Therefore, setting $\Psi(0) = 0$ allows the wage strategy of the union to be optimal independently of the choice of $\Lambda(0)$ (cf. Calvo, 1978; Lipton et al. 1982; van der Ploeg, 1987).

Summing up, the solution of the union problem conveyed by eq.s (13) $-$ (16) – together with definition of Φ – provides the following non-linear 4×4 system of differential equations:

$$\begin{aligned}
\dot{\Lambda} &= \Lambda \left(\rho_{\mathrm{F}} + \delta - \alpha s \Omega^{\frac{1-\alpha}{\alpha}} \right) + r - \alpha \Omega^{\frac{1-\alpha}{\alpha}} \\
\dot{\Gamma} &= \Gamma \left(\rho_{\mathrm{U}} + \delta - s \Omega^{\frac{1-\alpha}{\alpha}} \right) - (1-\alpha)\, \Omega^{\frac{1-\alpha}{\alpha}} \left(1 + s\Lambda - (1+\alpha)\, \frac{s\Gamma K + (1+s\Lambda)((1-\alpha)K - \alpha\Psi)}{K} \right) \\
\dot{K} &= K \left(s \Omega^{\frac{1-\alpha}{\alpha}} - \delta \right) \\
\dot{\Psi} &= \Psi \left(\rho_{\mathrm{U}} - \rho_{\mathrm{F}} - \delta + s \Omega^{\frac{1-\alpha}{\alpha}} \right) - s(1-\alpha) \Omega^{\frac{1-\alpha}{\alpha}} \left(K + \Psi \right)
\end{aligned} \tag{17}$$

where $\Omega \equiv \left(s\Gamma K + (1 + s\Lambda) \left((1-\alpha) K - \alpha \Psi \right) \right) / K^{1+\alpha}$.

2.3 Steady state

In the model economy described above, steady-state allocations are defined as the set of quadruplets $\mathcal{S} := \left\{ \overline{\Lambda}, \overline{\Gamma}, \overline{K}, \overline{\Psi} \right\} \in \mathbb{R}^4_+$ such that $\dot{\Lambda} \left(\overline{\Lambda}, \overline{\Gamma}, \overline{K}, \overline{\Psi} \right) = \dot{\Gamma} \left(\overline{\Lambda}, \overline{\Gamma}, \overline{K}, \overline{\Psi} \right) = \dot{K} \left(\overline{\Lambda}, \overline{\Gamma}, \overline{K}, \overline{\Psi} \right) = \dot{\Psi} \left(\overline{\Lambda}, \overline{\Gamma}, \overline{K}, \overline{\Psi} \right) = 0$. Obviously, in order to have economically meaningful stationary allocations, the eligible values of $\overline{K}$ have to be strictly positive. Moreover, in case of asymptotic stability, some elements of that set will be also characterized by the fact that $\lim_{t \to \infty} \Lambda(t) = \overline{\Lambda} \wedge \lim_{t \to \infty} \Gamma(t) = \overline{\Gamma} \wedge \lim_{t \to \infty} K(t) = \overline{K} \wedge \lim_{t \to \infty} \Psi(t) = \overline{\Psi}$ by leading also to the convergence of the level of employed set by the firm and the wage rate chosen by the union.

The elements of the set $\mathcal{S}$ can be derived as follows. First, setting $\dot{K} = 0$ in the third row of the dynamic system in (17), allows us to find that

$$\overline{\Omega} = \left(\frac{\delta}{s}\right)^{\frac{\alpha}{1-\alpha}} \tag{18}$$

Second, setting $\dot{\Lambda} = 0$ in the first row of (17) by considering the result in eq. (18) provides the following expression:

$$\overline{\Lambda} = \frac{\alpha\delta - sr}{s\left(\rho_{\mathrm{F}} + \delta\left(1-\alpha\right)\right)} \tag{19}$$

A steady-state equilibrium in which investment offsets the exogenous depreciation of capital requires a positive value of $\overline{\Lambda}$. Interestingly, this is always the case when the real interest rate is negative so that the productive capacity of the firm is actually subsidized (cf. Agarwal and Kimball, 2019). Since negative real interest rates are not the rule but only sporadic exceptions, however, the baseline calibration of the model economy exploited for the numerical analysis will be grounded on a positive value of r and will take into account situation in which $\overline{\Lambda}$ is always positive.[5]

Third, the results in eq.s (18) and (19) and the definition of Φ allows us to find the long-run value of the real wage ($\overline{w}$) set by the union whose analytical expression is given by

$$\overline{w} = \frac{\left(1-\alpha\right)\left(\rho_{\mathrm{F}} + \delta - sr\right)}{\overline{\Omega}\left(\rho_{\mathrm{F}} + \delta\left(1-\alpha\right)\right)} \tag{20}$$

Notice that if $\overline{\Lambda}$ is positive, $\overline{w}$ is also positive. Obviously, this means that whenever the firm finds permanently profitable to increase its productive capacity in order to counteract depreciation, employed workers persistently receive a positive equilibrium wage. In addition, the positivity of $\overline{\Lambda}$ implies that $\partial\overline{w}/\partial\rho_{\mathrm{F}}$ is strictly negative. Unsurprisingly, in a dynamic context, the degree of impatience of the firm is negatively related to its actual bargaining power (cf. Rubinstein, 1982; Guerrazzi, 2012).

Third, $\dot{\Gamma} = 0$ in the second row of (17), eq.s (18) and (20) allows us to find the following expression:

$$\overline{\Gamma} = \mathcal{A}_1\left(\mathcal{A}_2 - \left(1+\alpha\right)\overline{\Omega}\left(\overline{K}\right)^{\alpha}\right) \tag{21}$$

where $\mathcal{A}_1 \equiv \left(1-\alpha\right)\delta/s\rho_{\mathrm{U}} > 0$ and $\mathcal{A}_2 \equiv \left(\rho_{\mathrm{F}} + \delta - sr\right)/\left(\rho_{\mathrm{F}} + \delta\left(1-\alpha\right)\right) > 0$.

Fourth, setting $\dot{\Psi} = 0$ in the fourth row of (17) and using the results in eq.s (18), (19) and (21) allows us to find the following expression:

$$\overline{\Psi} = \frac{\left(1-\alpha\right)\left(\mathcal{A}_2\left(1+s\mathcal{A}_1\right)\delta\overline{K} - \mathcal{A}_3\left(\overline{K}\right)^{\alpha}\right)}{\alpha\mathcal{A}_2\left(\rho_{\mathrm{U}} - \rho_{\mathrm{F}}\right)} \tag{22}$$

[5] As it will be apparent later on, the historical average values of the involved variables are consistent with that hypothesis.

where $\mathcal{A}_3 \equiv (1 + (1 + \alpha)\, s\mathcal{A}_1)\, \overline{\Omega} > 0$.

Eq. (22) reveals that a finite equilibrium value of the shadow value for the union of the marginal contribution of capital to the stream of profit of the firm requires the two players to have different discount rates.

Finally, the definition of Ω together with the results in eq.s (13), (18), (19), (21) and (22) allows us to convey the equilibrium stock of capital as the positive solution of the following non-linear expression:

$$\overline{\Omega}\left(\overline{K}\right)^{\alpha} = \mathcal{A}_5\left(\mathcal{A}_2 - (1+\alpha)\,\overline{\Omega}\left(\overline{K}\right)^{\alpha}\right) + \mathcal{A}_6\left(1 - \frac{\delta\left(\mathcal{A}_2\,(1 + s\mathcal{A}_1) - \mathcal{A}_3\left(\overline{K}\right)^{\alpha}\right)}{\mathcal{A}_2\,(\rho_{\mathrm{U}} - \rho_{\mathrm{F}})}\right) \tag{23}$$

where $\mathcal{A}_5 \equiv s\mathcal{A}_1$ and $\mathcal{A}_6 \equiv (1-\alpha)\,\mathcal{A}_2$.

On the one hand, the expression on the LHS of eq. (23) is an increasing concave function of the stock of capital that starts from the origin. On the other hand, the shape of the expression on the RHS of eq. (23) strictly depends on difference between the degree of impatience of the involved players. Considering a situation in which the leader (the union) is less impatient than the follower (the firm), i.e., considering a situation in which ρ_{U} is lower than ρ_{F}, the expression on the RHS of eq. (23) is a decreasing convex function of the capital stock characterized by a positive vertical intercept given by $\Theta \equiv \mathcal{A}_5\mathcal{A}_2 + \mathcal{A}_6\left(1 - \delta\,(1 + s\mathcal{A}_1)\,/\,(\rho_{\mathrm{U}} - \rho_{\mathrm{F}})\right)$. Consequently, as illustrated in Figure 1, when the union discounts its utility streams at a lower rate with respect to the one used by the firm to discount realized profits, there will be a unique economically meaningful value of $\overline{K}$ that fulfils eq. (23) and it will represent the steady-state level of capital accumulation.

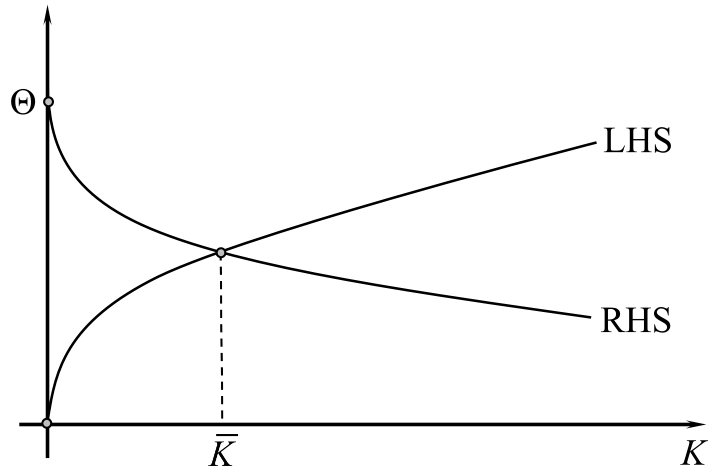

Figure 1: Steady-state level of capital

Given the level of $\overline{K}$ depicted in Figure 1 – that can be easily retrieved numerically after a proper model calibration – the other components of the unique quadruplet of $\mathcal{S}$, namely, $\overline{\Lambda}$, $\overline{\Gamma}$

and $\overline{\Psi}$, can be derived, respectively, form eq.s (19), (21) and (22). Thereafter, given the values of $\overline{K}$, $\overline{\Lambda}$ and $\overline{w}$, the value of $\overline{L}$ follows instead from eq. (9), and it is given simply by $\overline{K}\left(\overline{\Omega}\right)^{1/\alpha}$.

The requirement for the union to be more patient than the firm is a feature that mirrors the role of leader that it is assumed to play in the game (cf. Bressan and Jiang, 2020; Candido and Guerrazzi, 2023). Relying on a stochastic model of alternating wage offers in which binding labour contracts are signed, Cripps (1997) shows that when the worker involved in the negotiation is less impatient than the firm in which she/he is employed, the managers of the latter – given the outcome of the bargaining process – tend to delay the activation of new investment projects. As it will become apparent later on, in the model economy under scrutiny the occurrence of this kind of behaviour on the side of the firm, i.e., the reduction of purchases of new productive capacity induced by high wages, may ends up in phases of capital decumulation during which the actual flow of investment does not offset the depreciation of the employed productive capacity.

2.4 Local dynamics

The local dynamics of the model economy around the unique economically meaningful element of $\mathcal{S}$ is described by the following 4×4 linear system:

$$\begin{pmatrix} \dot{\Lambda} \\ \dot{\Gamma} \\ \dot{K} \\ \dot{\Psi} \end{pmatrix} = \begin{bmatrix} \rho_{\mathrm{F}} + \mathcal{B}_6 & -\frac{\mathcal{B}_1\mathcal{B}_2}{\mathcal{B}_3} & -\frac{\mathcal{B}_1\mathcal{B}_2\mathcal{B}_5}{s\mathcal{B}_3\overline{K}} & \frac{\alpha\mathcal{B}_1\mathcal{B}_2^2}{s\mathcal{B}_3\overline{K}} \\ -\mathcal{B}_1\left(1+\frac{\mathcal{B}_4\mathcal{B}_5}{\alpha\mathcal{B}_3\overline{K}}\right) & \rho_{\mathrm{U}} - \frac{\mathcal{B}_1\mathcal{B}_5}{\alpha\mathcal{B}_3} & \mathcal{B}_1\left(\frac{\alpha^2(1+\alpha)\mathcal{B}_2\overline{\Psi}-s\mathcal{B}_4\mathcal{B}_5\overline{K}}{\alpha s\mathcal{B}_3\left(\overline{K}\right)^2}\right) & \frac{\mathcal{B}_1\mathcal{B}_2\mathcal{B}_5}{s\mathcal{B}_3\overline{K}} \\ \frac{s\mathcal{B}_1\mathcal{B}_4}{\alpha\mathcal{B}_3} & \frac{s\mathcal{B}_1\overline{K}}{\alpha\mathcal{B}_3} & \frac{\mathcal{B}_1\mathcal{B}_5}{\alpha\mathcal{B}_3} & -\frac{\mathcal{B}_1\mathcal{B}_2}{\mathcal{B}_3} \\ -\frac{s\mathcal{B}_1\mathcal{B}_4^2}{\alpha\mathcal{B}_3\overline{K}} & -\frac{s\mathcal{B}_1\mathcal{B}_4}{\alpha\mathcal{B}_3} & -\mathcal{B}_1\left(1+\frac{\mathcal{B}_4\mathcal{B}_5}{\alpha\mathcal{B}_3\overline{K}}\right) & \rho_{\mathrm{U}} - \rho_{\mathrm{F}} - \mathcal{B}_6 \end{bmatrix} \begin{pmatrix} \Lambda - \overline{\Lambda} \\ \Gamma - \overline{\Gamma} \\ K - \overline{K} \\ \Psi - \overline{\Psi} \end{pmatrix} \tag{24}$$

where $\mathcal{B}_1 \equiv (1-\alpha)\,\delta > 0$, $\mathcal{B}_2 = \mathcal{A}_2$, $\mathcal{B}_3 \equiv \overline{\Omega}\left(\overline{K}\right)^{\alpha} > 0$, $\mathcal{B}_4 \equiv (1-\alpha)\,\overline{K} - \alpha\overline{\Psi} > 0$, $\mathcal{B}_5 \equiv s\overline{\Gamma} + (1-\alpha)\,\mathcal{B}_2 - (1+\alpha)\,\mathcal{B}_3$ whereas $\mathcal{B}_6 \equiv \mathcal{B}_1\left(1 - \mathcal{B}_2\mathcal{B}_4/\mathcal{B}_3\overline{K}\right)$.

Let λ_1, λ_2, λ_3 and λ_4 be the four eigenvalues of the Jacobian matrix – say, $\mathcal{J} \in \mathbb{R}^{4\times 4}$ – in (24) whose trace is simply $2\rho_{\mathrm{U}}$. Thereafter, the corresponding characteristic equation is given by

$$\lambda^4 - 2\rho_{\mathrm{U}}\lambda^3 + \mathcal{C}_2\lambda^2 - \mathcal{C}_3\lambda + \det\left[\mathcal{J}\right] = 0 \tag{25}$$

where $\mathcal{C}_2$ and $\mathcal{C}_3$ are, respectively, the sum of all diagonal second and third order minors of $\mathcal{J}$ that according to the decomposition suggested by Bosi and Desmarchelier (2019) amount, respectively, to $\lambda_1\left(\lambda_2+\lambda_3+\lambda_4\right) + \lambda_2\left(\lambda_3+\lambda_4\right) + \lambda_3\lambda_4$ and $\lambda_1\lambda_2\left(\lambda_3+\lambda_4\right) + \lambda_3\lambda_4\left(\lambda_1+\lambda_2\right)$.

As suggested by Dockner (1985), the four roots of eq. (25) can be written as

$$\frac{\rho_{\mathrm{U}}}{2} \pm \sqrt{\left(\frac{\rho_{\mathrm{U}}}{2}\right)^2 - \frac{1}{2}\left(\mathcal{C}_2 - \rho_{\mathrm{U}}^2 \pm \sqrt{\left(\mathcal{C}_2 - \rho_{\mathrm{U}}^2\right)^2 - 4\det\left[\mathcal{J}\right]}\right)} \tag{26}$$

The expression in eq. (26) may lead to different outcomes which are directly responsible for the out-of-equilibrium adjustments of the 4 variables that enter the system in (24). An intriguing option whose actual realization – for the moment – we can only guess because of the dimensionality of the problem is the one that arises when eq. (25) has a quasi-palindromic structure. Specifically, whenever $\det[\mathcal{J}]$ is higher that $\left(\left(\mathcal{C}_2-\rho_{\mathrm{U}}^2\right)/2\right)^2$ and $\det[\mathcal{J}]-\left(\left(\mathcal{C}_2-\rho_{\mathrm{U}}^2\right)/2\right)^2-\rho_{\mathrm{U}}^2\left(\mathcal{C}_2-\rho_{\mathrm{U}}^2\right)/2$ is positive, it is possible to show that there are four complex-conjugate eigenvalues, two having negative real parts and two having positive real parts (cf. Pelgin and Venditti, 2022). Whenever such a condition is fulfilled, there is only one trajectory that satisfies the dynamic system in (17) by converging to the steady-state equilibrium whereas all the others diverge. In other words, in the model under examination the equilibrium path is locally determinate, i.e., taking a given initial value of the capital stock (K_0) and the supplementary initial condition $\Psi(0)=0$ conveyed by (16), there is a unique vector $\begin{pmatrix}\Lambda(0) & \Gamma(0)\end{pmatrix}\in\mathbb{R}^{1\times 2}$ in the neighbourhood of $\{\overline{\Lambda},\overline{\Gamma}\}$ that generates a trajectory converging towards the unique element of $\mathcal{S}$ by means of damped oscillations. Specifically, the values of $\Lambda(0)$ and $\Gamma(0)$ should be selected to satisfy the transversality conditions in (8) and (15) by placing the system in (17) on the stable branch of the saddle point $\{\overline{K},\overline{\Lambda},\overline{\Gamma},\overline{\Psi}\}$.

In the remainder of the paper, the stable saddle path followed by K, Λ, Γ, Ψ, and, implicitly, by L and w, will be taken as the perfect-foresight path of the model economy and it will be explored numerically relying on the following analytical specification. Let $\mathbb{V}(\lambda_1)$ and $\mathbb{V}(\lambda_2)$ be the row eigenvectors associated with the convergent complex-conjugate roots λ_1 and λ_2. Thereafter, as suggested by Stemp and Herbert (2006), the solution of the linearized dynamic system in (24) can be written as

$$\begin{pmatrix}\Lambda(t)\\ \Gamma(t)\\ K(t)\\ \Psi(t)\end{pmatrix}=\begin{pmatrix}\overline{\Lambda}\\ \overline{\Gamma}\\ \overline{K}\\ \overline{\Psi}\end{pmatrix}+\begin{bmatrix}V_1(\lambda_1) & V_1(\lambda_2)\\ V_2(\lambda_1) & V_2(\lambda_2)\\ V_3(\lambda_1) & V_3(\lambda_2)\\ V_4(\lambda_1) & V_4(\lambda_2)\end{bmatrix}\begin{pmatrix}\mathcal{D}_1\\ \mathcal{D}_2\end{pmatrix} \tag{27}$$

where the 2×1 vector on the RHS is defined as

$$\begin{pmatrix}\mathcal{D}_1\\ \mathcal{D}_2\end{pmatrix}\equiv\begin{pmatrix}(\mathcal{E}_1+i\mathcal{E}_2)\exp(\mathrm{Re}(\lambda_1))(\cos(\mathrm{Im}(\lambda_1)t)+i\sin(\mathrm{Im}(\lambda_1)t))\\ (\mathcal{E}_1-i\mathcal{E}_2)\exp(\mathrm{Re}(\lambda_2))(\cos(\mathrm{Im}(\lambda_2)t)-i\sin(\mathrm{Im}(\lambda_2)t))\end{pmatrix}\qquad \mathcal{E}_1,\mathcal{E}_2\in\mathbb{R} \tag{28}$$

Given the properties of the trigonometric functions collected in (28) and the FOC in eq. (16), the values $\mathcal{E}_1$ and $\mathcal{E}_2$ consistent with the required initial conditions are given by the solution of the following system:

$$\begin{pmatrix}K_0\\ 0\end{pmatrix}=\begin{pmatrix}\overline{K}\\ \overline{\Psi}\end{pmatrix}+\begin{bmatrix}V_3(\lambda_1) & V_3(\lambda_2)\\ V_4(\lambda_1) & V_4(\lambda_2)\end{bmatrix}\begin{pmatrix}(\mathcal{E}_1+i\mathcal{E}_2)\\ (\mathcal{E}_1-i\mathcal{E}_2)\end{pmatrix} \tag{29}$$

3 Numerical properties

In this section, I explore the numerical properties of the bargaining model with capital accumulation developed above. In this direction, the model economy is calibrated on an annual basis by taking as reference the US economy, a context in which multi-annual labour contracts are quite recurring (cf. Grout, 1984, 1985; Anderson and Devereux, 1988). Specifically, the elasticity of output with respect to capital (α) and its depreciation rate (δ) are set at the values suggested by Kydland and Prescott (1982). Thereafter, the real interest rate (r) as well as the investment-output rate (s) are set at their corresponding average historical values retrieved from FRED.[6] Considering the expression in eq. (19), the combination of these four parameter values straightforwardly implies a positive value of $\overline{\Lambda}$.[7] Furthermore, the two values of the discount rates (ρ_{F} and ρ_{U}, respectively) are taken from Itskhoki and Moll (2019). All the parameter values together with their respective description are collected in Table 1.

Parameter	Description	Value
α	Elasticity of output with respect to capital	0.360
δ	Depreciation rate of capital	0.100
r	Real interest rate	0.013
s	Investment-output rate	0.173
ρ_{F}	Discount rate of the firm	0.050
ρ_{U}	Discount rate of the union	0.030

Table 1: Calibration

As implied by the figures in Table 2, the baseline calibration reported in Table 1 has some interesting implications for the differential game under examination. First, as argued by van der Ploeg (1987), the negative values of $\overline{\Gamma}$ and $\overline{\Psi}$ mirror the non-cooperative nature of the strategic situation in which the firm and union are involved. In other words, the two players have conflicting objectives so that their respective payoffs will tend to move in opposite directions by confirming the trade-off mentioned in the introduction. Second, the expressions in eq.s (9), (19) and (20), imply that the long-run equilibrium values of the two control variables, namely $\overline{L}$ and $\overline{w}$, are given respectively by 1.0350 and 1.1290. Since I assumed that union membership is normalized to 1, the former figure implies that in the steady-state equilibrium a share of the employed workers are not members of the union.[8] Although the retrieved figure underestimates

[6] Historical US data can be downloaded at https://fred.stlouisfed.org.

[7] A fortiori, a positive value of $\overline{\Lambda}$ would be obtained also by measuring s with the average historical value of the saving rate which amounts to 8.8%.

[8] All the employed workers, however, are covered by the union wage agreements which is above of the implied value of the marginal productivity of labour. According to the figures provided by Budd and Na (2000), between 1983 and 1993, 10% of private sector US employees covered by collective bargaining agreement were not union members.

the phenomenon, this finding is consistent with the declining unionization rate observed in the US over the last 40 years (cf. Farber and Krueger, 1992; Solomon, 2003; Blanchflower and Bryson, 2004). Third, even if the production technology displays constant returns to scale, the profits of the firm are always positive by revealing that the imperfection of the labour market spills over into the market for goods (cf. Guerrazzi, 2023; Devereux and Lockwood, 1988). In addition, since eq. (3) implies that $\overline{w}_0 = 0.8820$, the retrieved value of $\overline{w}$ reveals that the long-run union wage premium, i.e., the relative difference between the wage chosen by the union and the competitive wage, amounts to 28% a figure which is not too distant from the available estimations of that premium that suggest values slightly above 20% (cf. Hirsch and Schumacher, 2004; Blackburn, 2008; Açıkgöz and Kaymak, 2014; Gabriel and Schmitz, 2014). Furthermore, the existence of two complex-conjugate eigenevalues with negative real part corroborates the conjecture on local asymptotic stability put forward in Section 2 by revealing that the convergence towards the stationary solution actually occurs through damped oscillations.

VARIABLE	DESCRIPTION	VALUE
$\overline{\Lambda}$	Shadow value of capital for the firm	1.7113
$\overline{\Gamma}$	Shadow value of capital for the union	-0.9972
$\overline{K}$	Capital stock	2.4371
$\overline{\Psi}$	Shadow value of Λ for the union	-1.8568
λ_1	Eigenvalue associated to capital adjustments	$-0.1049 + 0.0075i$
λ_2	Eigenvalue associated to the adjustments of Ψ	$-0.1049 - 0.0075i$

Table 2: Steady-state values and convergent eigenvalues

Relaying on the baseline calibration in Table 1 and setting the initial level of the capital stock 1% below its steady-state value, a suitable discretization of (27) allows us to simulate the model economy by generating artificial series for all the involved variables.[9] In order to test the empirical performance of the theoretical framework under examination, I begin by tracking the implied trajectories followed by the Λ, Γ, K and Ψ. Specifically, their implied saddle path-dynamics are illustrated in the four panels of Figure 2.

[9]Recall that the initial condition for Ψ is given by the FOCs in (16). Moreover, the baseline calibration is integrated by setting to 1 the time-step of simulation and simulating the model for 120 periods in order to allow the full convergence of all the variables. MAT LAB codes are available from the author upon reasonable request.

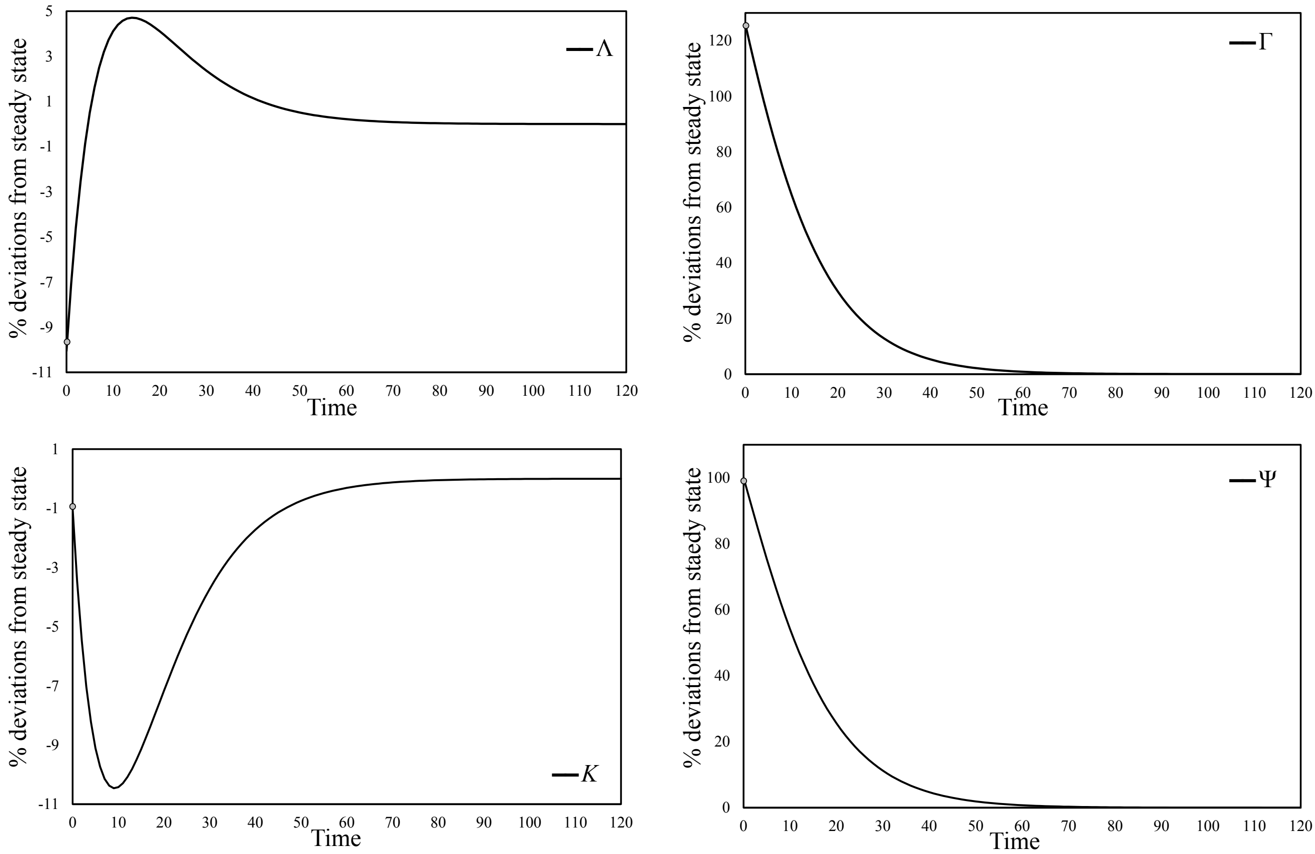

Figure 2: Saddle-path dynamics

On the one hand, the first and the third panels of Figure 2 show that the stock of capital (K) and its shadow price for the firm (Λ) tend to move in opposite directions during their damped adjustment processes toward their respective steady-state value. This pattern is consistent with the textbook q-model of investment with adjustment costs in which capital accumulation (decumulation) occurs whenever the value of the marginal contribution of an additional unit of capital to the stream of discounted profits of the firm, i.e., the so-called Tobin' q, is above (below) its long-run equilibrium value (cf. Guerrazzi and Candido, 2023). On the other hands, the second and the fourth panels shows that the two costate variables of the union's problem jump above their respective steady-state value and then display a monotonic decline until they reach their long-run references.

The trajectories plotted in Figure 2 allows us to retrieve the dynamics followed by the control variables of the two players which are given by the level of employment (L) chosen by the firm and the wage (w) decided by the union. The out-of-equilibrium adjustments of both variables are illustrated in the two panels of Figure 3.

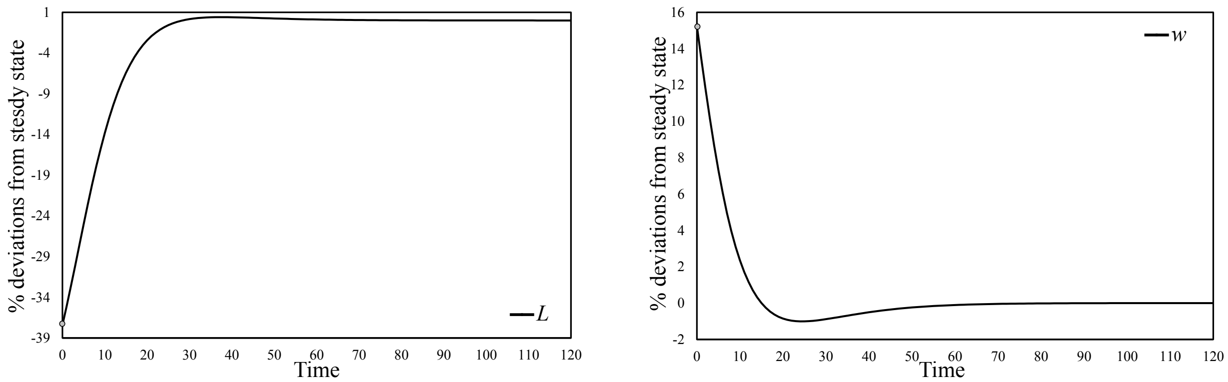

Figure 3: The implied dynamics of the controls of the two players

The diagrams in Figure 3 reveal that at the beginning of the game the union tends to restrict the labour supply to the firm by asking high wages. This pattern is clearly responsible for the initial phase of capital decumulation documented by the plot on the third panel of Figure 2. Specifically, at the outset of the game, the stock of capital and the level of employment are quite low – the former for the selected value of the initial condition, the latter because of the wage strategy adopted by the union. Consequently, the share of produced output invested by the firm falls short of the depreciation of employed capital by leading to a contraction of K. Thereafter, as the union broadens labour supply by moderating its wage demand, i.e., when it starts to trade off lower wages for a larger workforce, the level of employment decided by the firm – together with the existing stock of capital – allows to produce and to invest an amount of resources that exceeds the depreciation of the installed productive capacity by triggering the increase of K which is required to achieve $\overline{K}$.[10]

I close my numerical exploration with the plot of Figure 4 that tracks the path followed by the union wage premium.

[10]The implied negative relationship between the wage and the shadow value of capital for the firm is also found in the initial phase of the end game developed by Lawrence and Lawrence (1985).

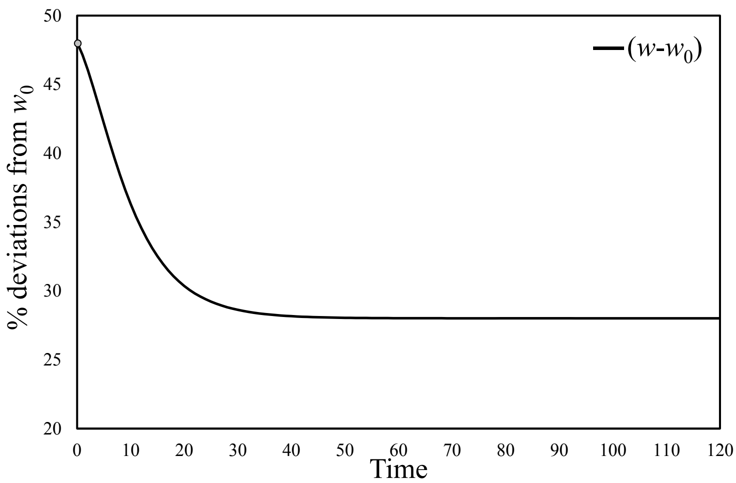

Figure 4: The implied dynamics of the union wage premium

Despite the out-of-equilibrium adjustments of w and K – recall that given eq. (3) the stock of capital is the only determinant of w_0 – are non-linear, the plot of Figure 4 displays an initial monotonic decline of the union wage premium and then a rapid convergence of that indicator towards its steady-state value. This finding is consistent with the longitudinal analysis of the union wage premium for US workers carried out by Gabriel and Schmitz (2014) who find that – after the decline observed during the 70s and the 80s – the long-term private-sector union wage premium for men has remained fairly constant over the period 1990 to 2010. As it will become apparent when I will deal with the effects of variations in the investment-output ratio and in the conduct of monetary policy, in the model economy under scrutiny employed workers who are covered by the long-term union contract are somehow insulated from a variety of swings that can affect the surroundings in which they interact with the firm.

3.1 Comparative statics

The baseline calibration in Table 1 is build relying on the historical averages of the observed values of the involved parameters. Consequently, the numerical findings described above convey the long-run implications of the theoretical framework developed in Section 2. In order to grasp the robustness of these findings, in what follows I sketch out what happens when there is a modification in the point value of one critical parameter value whereas the others remain the same. Specifically, I consider the consequences triggered by (*i*) an increase in the investment-output ratio; (*ii*) a lower value of the real interest rate; (*iii*) an increase in the gap between the discount rate of the firm and the one of the union; and (*iv*) an increase in the depreciation rate of capital.

Case 1: An increase in the investment-output ratio. Setting s to 20%, a figure which is consistent with many European countries, the model loses its complex dynamics, but it preserves its saddle-path stability; indeed, the convergent eigenvalues become, respectively, $\lambda_1 = -0.1278$ and $\lambda_2 = -0.0947$. Consequently, with that value of the investment-output ratio it will hold that $\mathcal{C}_2 - \rho_{\mathrm{U}}^2 < 0$ and $0 < \det[\mathcal{J}] \leqslant \left(\left(\mathcal{C}_2 - \rho_{\mathrm{U}}^2\right)/2\right)^2$ (cf. Dockner, 1985).[11] The increase in s leads also to an increase in the equilibrium values of the capital stock and the wage rate and to a reduction of the equilibrium level of employment with respect to the values reported in Table 2. Specifically, with an investment-output ratio of 20%, $\overline{K}$, $\overline{w}$, and $\overline{L}$ amount, respectively, to 3.0368, 1.2221 and 1.0281. These figures imply that a higher (lower) value of s leads the firm to substitute labour (capital) for capital (labour), but it leaves unaltered the equilibrium union wage premium, because the consequent increase (reduction) in the equilibrium wage is of the same order of magnitude of the increase (reduction) of the competitive wage.

Case 2: A lower real interest rate. Whenever the central bank is able to set a nominal interest rate equal to expected inflation, the real interest rate goes to zero, so that $r = 0$. In this case, it would be possible to show that the model preserves its oscillatory dynamics; indeed, a zero-interest rate policy implies that $\lambda_1 = -0.1055 + 0.0055i$ and $\lambda_2 = -0.1055 - 0.0055i$. However, the equilibrium values of the capital stock, the level of employment and the wage increase by amounting, respectively, to 2.5415, 1.0793 and 1.1464. These figures imply that an expansionary (restrictive) monetary policy is beneficial (harmful) both for the firm and the union, but it leaves unaltered the union wage exactly as it does a variation of the investment-output ratio.[12] The invariance of union wage premium with respect to the conduct of the monetary policy and to the level of the investment-output ratio may represent a rationale for its observed stability recalled above (cf. Gabriel and Schmitz, 2014).

Case 3: A less impatient union. In a growth model à la Ramsey, the maximum equilibrium level of consumption is achieved when consumers discounts utility streams at the rental rate of capital (cf. Guerrazzi, 2023). In our model, setting $\rho_{\mathrm{U}} = 0.013$ does not alter the complex dynamics of out-of-equilibrium adjustments; indeed, such a value of the union's discount rate implies that $\lambda_1 = -0.1079 + 0.0306i$ and $\lambda_2 = -0.1079 - 0.0306$. However, it would be possible to show that the equilibrium values of the capital stock and the level of employment decreases – by amounting, respectively, to 2.1971 and 0.9330 – whereas the equilibrium wage remains the same.[13] Obviously, these figures lead to an increase in the equilibrium value of the union wage premium. Unsurprisingly, this means that when the union becomes less (more) impatient it also strengthens (weakens) its bargaining position (cf. Rubinstein, 1982).

[11] The analytical specification of the solution when the convergent eigenvalues lose their complex part is given in Appendix.

[12] Qualitatively similar findings are also triggered by variations of the elasticity of output with respect to capital which is the only determinant of the elasticity of the labour demand function in eq. (9). The only difference is that an increase (decrease) of α does not only lead to an increase (decrease) in $\overline{K}$, $\overline{L}$ and $\overline{w}$, but it also leads to an increase (decrease) in the union wage premium. This kind of variation can be taken as a proxy of the effects induced by a productivity shocks.

[13] Recall that according to eq. (20) $\overline{w}$ is unaffected by the value of ρ_{U}.

Case 4: A higher depreciation rate. According to Gomme and Lkhagvasuren (2013) the depreciation rate for equipment and software amounts to 14.6%. Relying on that value of δ, the model loses its complex dynamics, but it preserves its saddle-path stability exactly as it does when there is an increase in the investment-output ratio; indeed, the convergent eigenvalues become, respectively, $\lambda_1 = -0.1654$ and $\lambda_2 = -0.1266$. Moreover, it would be possible to show that in this case the equilibrium values of the stock of capital and the wage rate decrease whereas the equilibrium value of employment increases. Specifically, with a depreciation rate equal to 14.6%, $\overline{K}$, $\overline{w}$, and $\overline{L}$ amount, respectively, to 1.4736, 0.9511 and 1.1304. These figures imply that an increase (decrease) in δ leads the firm to substitute capital (labour) for labour (capital) and they also lead to an increase (reduction) of the equilibrium union wage premium, because the relative reduction (increase) of the competitive wage is higher (lower) than the corresponding reduction (increase) of the bargained wage. This finding reveals that a production technology characterized by a faster speed of senescence of physical capital may be beneficial for unions and harmful for firms.

4 Concluding remarks

In this paper, I developed a dynamic version of the monopoly union model in which capital accumulation at the firms' sites evolves over time under a binding wage agreement. Thereafter, assuming that investment is given by a constant fraction of produced output, I showed that an open-loop Stackelberg equilibrium with a positive long-run value of the stock of capital requires the firm to be more impatient than the union (cf. Cripps, 1997; Bressan and Jiang, 2020; Candido and Guerrazzi, 2023). Moreover, relying on some numerical simulations calibrated on the US economy, I showed that despite convergence towards the steady-state equilibrium occurs through damped oscillations, after an initial period of decline the model predicts a fairly stable union wage premium whose point value is consistent with the available empirical evidence (cf. Açıkgöz and Kaymak, 2014; Gabriel and Schmitz, 2014). Furthermore, I emphasised that the model under scrutiny offers a rationale for the observed stability of the union wage premium because it implies that such an indicator is not affected by variations in the investment-output ratio and in the conduct of monetary policy.

Throughout the analysis carried out in the paper I took for granted the existence of a legally binding labour contract between the firm and the union to bypass the problem of dynamic inconsistency that usually affects controllable Stackelberg games. As argued by van der Ploeg (1987), however, the empirical relevance of that institutional setting may be questionable because in the absence of contract committing the union has incentive to renege on the announced wage strategies once the capital stock has been accumulated (cf. Lambertini, 2018, Chapter 9). Consequently, this may somehow question the reliability of the stable pattern followed by the union wage premium documented above.

The time inconsistency that involves differential games with a hierarchical structure is certainly a delicate matter and the possibility of deviations from the strategies announced at the outset of the game deserves to be taken in serious consideration. In the real world, however, there may be forces that make it difficult to renege on former commitments. For instance, whenever the firm is in the position to punish the union of its workers by stopping, reducing or delaying hiring, the incentives to deviate from the starting wage strategy may be very feeble, especially when the latter is not too impatient as actually required by our model to have a meaningful stationary solution (cf. van der Ploeg, 1987; Guerrazzi and Candido, 2023). In addition, the decline in union membership observed over the last years and the likely reduction of union power may well discourage too frequent renegotiations of labour contracts for fear of obtaining less than what was obtained in the past.[14]

The analysis carried out in this paper could be developed in different directions. For example, the problem of the firm could be enriched in order to consider internal and external adjustment costs of investment (cf. Hayashi, 1982; Mino, 1987). Moreover, the problem of the union (firm) could be solved relying on different specifications of its utility (production or revenue) function by looking for a time-consistent equilibrium with no controllability (cf. Lawrence and Lawrence, 1985; Faber, 1987; Mezzetti, 1993). Furthermore, adding taxes to wages and profits, the model could be exploited to address the effects of fiscal policy (cf. Holm et al. 1994). These will be the promising avenues for further developments.

Appendix: Saddle-path dynamics with real eigenvalues

Whenever λ_1 and λ_2 have no complex part, the solution of the linearized dynamic system in (24) can be written as

$$\begin{pmatrix} \Lambda(t) \\ \Gamma(t) \\ K(t) \\ \Psi(t) \end{pmatrix} = \begin{pmatrix} \overline{\Lambda} \\ \overline{\Gamma} \\ \overline{K} \\ \overline{\Psi} \end{pmatrix} + \begin{bmatrix} \frac{v_{1,1}}{v_{1,3}} & \frac{v_{2,1}}{v_{2,4}} \\ \frac{v_{1,2}}{v_{1,3}} & \frac{v_{2,2}}{v_{2,4}} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \exp(\lambda_1 t)\left(K_0 - \overline{K}\right) \\ -\exp(\lambda_2 t)\,\overline{\Psi} \end{pmatrix} \tag{A1}$$

where $v_{i,j}$ is the j-th element of $\mathbb{V}_i(\lambda_i)$ (cf. Guerrazzi, 2023).

[14] According to Lawrence and Lawrence (1985), in an industry characterized by a stable demand for its product, the union always finds profitable to behave as a Stackelberg leader; indeed, by committing to a certain wage strategy the union is able to avoid the depletion of the firm's rents by allowing the increase of the productive capacity which is required for a sustained hiring process.

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