Cheating in Second Price Auctions and Emotional Responses

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A very preliminary Version

Abstract

This paper aims to address a gap in literature at the intersection of cheating in auctions and emotional responses. In a second price auction with a cheating seller, we model the bidder's dislike for the possibility of cheating by drawing upon the idea of reference point-based utility. A symmetric increasing equilibrium strategy is characterised and comparative statics are analysed. A comparison of expected payoffs to honest and dishonest sellers is made. We find that if reference points are low enough then the cheating seller's payoff is lower than what a seller earns in a regular first-price auction. Our results show that even with bidders disliking cheating, honest sellers lose out due to bidders shading their bids to accommodate for the possibility of being cheated.

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1 Introduction

Auctions are probably as ancient as human civilisation itself. It's possible that even before the evolution of currency, people were engaged in barter auctions. The heart of this argument lies in the fundamental preferential hypothesis about decision making i.e. more is better. For a particular commodity to be exchanged with another the exchangers would have interacted with only those who offer the highest value/satisfaction to each other. With evolution of currency systems and property rights the process of auctions became more complex and refined. The first recorded auctions are known to be held in Greece where people used to auction women for marriage.(Zajicek, 2016). Romans too auctioned off the spoils of war, slaves and other loot material from captured territories. However, the popularity of auctions dipped after the collapse of the Roman empire. In the 17^{th} century candle auctions were in vogue where bids were accepted until the candle burnt out. Ancient Indian folklore too has references of auctions being held in ancient times[1]. It is not known whether auctions in ancient times were increasing price or decreasing price auctions but the word "auction" comes from the Latin word "auctus" which means increasing. Over time auctions evolved. The popular cinema image of an auction involves a wood finished spacious room with the classic hammer striking auctioneer and the participants yelling with auction fever is generally true for English auctions which are the most popular methods of auctions worldwide owing their popularity to transparent and simple rules.

Auctions are used worldwide for various purposes; for discovering the market prices of exotic items, recovery of debts, managing lost belongings at airports, most importantly in those situations where the market mechanism is unable to perform its most basic function, i.e. "price discovery"[2] (spectrum auctions, natural resource auctions, highly precious art forms etc.). Exotic Art usually sells at exorbitant prices because markets for such items do not exist. Art is emotive for collectors and enthusiasts though economic value of input

[1] King Harishchandra auctioned himself and his family to pay off his debt.

[2] Situations where objects sold are too unique that regular demand and supply mechanism cannot function properly. E.g. Unique objects such as the original "Monalisa" or "The Hope Diamond" cannot be reproduced, so neither the supply nor the demand curve at various quantities and price combinations can be derived

and labour used in artwork is low the value is derived from its "Art Appeal" and often as investment to avoid taxes.(Valle, 2018). In economic theory a lot of work has been done using game theoretic tools. Outside economics auctions are of great interest to computer scientists and legal scholars probably due to increasing popularity of online auctions.(McKay, staff, & Klein, 2020)

Malfeasance[3] in auctions is no hidden phenomena and it is an anomaly from standard models in auction theory, but what is cheating? Do cheaters consider themselves as cheats or do they have some beliefs that make their behaviour justified for themselves? There can be a lot of motivations for cheating, not just economic but also psychological, social etc. If so can such a behaviour can be problematic for the other agents interacting with cheaters? Economic theory suggests that players cheat if it benefits them but how do we define cheating ? Here we focus on a narrow aspect of cheating, i.e. given a mechanism, cheating is an act of deviating from the acceptable behaviour expected by the mechanism/auction with an intention to gain wrongfully[4].However there can be cases where people cheat not just for pecuniary gains but to harm others[5]. How is cheating different from corruption? Corruption can be understood as a subset of cheating, involving cheating by an agent in authority who decides the final result of the mechanism. He usually does so by arbitrary actions that are not acceptable or expected in the legal setting.

There has been a lot of work in modelling of cheating and corruption in auction theory. Corruption in procurement Auctions is a serious issue since it usually involves a lot of public money and essential utilities like Spectrum Bands, Strategic Mineral mines etc. Both bidders and sellers can cheat in auctions and they have different implications for the standard results in the theory. Often emotional bidding go hand in hand with auctions, the thrill it generates is sometimes sufficient for bidders to behave seemingly foolish, in open auctions bidders end

[3]Examples include shill bidders, bid rigging, favouritism in procurement auctions (E.g. 2G Spectrum Auction in India), collusion among bidders and many other forms of cheating

[4]Consider an example of a first price auction where bidders are only expected to bid or not but any other act such as adjusting bids after submitting them or preventing others from bidding etc would be called cheating. By wrongful gain we mean any gain that can be realised with positive probability by acts that would amount to cheating in that particular setting.

[5]More of this will be discussed in auctions with emotions.

up paying a higher price than they would have for the same article when sold in the market (Adam, Krämer, Jähnig, Seifert, & Weinhardt, 2011).This phenomenon is popularly known as Auction fever.[6]

Durant once said "Man is an emotional animal, occasionally rational; and through his feelings he can be deceived to his heart's content". In economic theory this has changed to "Man is an economic animal, acts rationally and always maximises his utility whenever possible." However, decisions are not just motivated by economic logic but also many non economic factors, one of them being "Emotions". Roughly speaking, Emotions are psychological phenomena that cause,mitigate or stimulate behavioural action, physical or mental. The complex emotional responses have developed over a long period of evolution and basic emotions like anger, fear, anxiety etc are common between all animals. With the development of the brain, stimuli triggering emotional responses expanded over time[7]. As the interaction with fellow humans evolved to include economic exchange, the set of stimuli that can trigger emotional responses also expanded. However the root of these responses can be traced back to basic forces of life i.e. fear, hunger and procreation. People collect and store articles to get rid of fear about future uncertainties or to just survive to see the next day and finally to dominate[8]. As society evolved the dimensions of survival expanded, a person want to survive not only physically but economically, socially, politically etc. So its important to look at models that study and axiomatise emotional behaviour in auction theory.

In this paper we consider cheating by a seller in Second price auction. The existing literature on cheating in second price auctions deals with rational bidders with no reactions on possibility of cheating in the environment, but usually people have a strong dislike for cheats and are willing to punish cheats for the same. Emotions in auctions is a separate literature and no work in our knowledge has incorporated emotional responses of bidders towards cheating. In this chapter we tackle this issue, we model bidders having preferences

[6] A situation where bidders are swayed away in emotions causing a bidder to deviate from an initially chosen bidding strategy.

[7] For instance absence of electricity may trigger anger and frustration in humans but not in lizards or pythons

[8] As domination over competing species ensure better chances of survival

that capture their dislike or emotional response towards possibility of cheating in auction. We draw on idea of reference dependence, where bidders have braced up themselves about possible cheating in auction and formed a reference point. They suffer additional utility(dis-utility) if the payments in worst case(when seller is a cheat) is less(more) than their reference point. The reference point can be thought of as tolerance towards possible cheating.

1.1. Related Literature

(Rothkopf & Harstad, 1995) model cheating in second price auctions by the bid taker/seller, they show that when the seller has an option to conduct a Vickery auction or a first price auction, only the seller with the highest possible cheating type chooses to organise a second price auction. In the dynamic model with reputations, the authors find that sellers who have not been caught cheating yet will use a second price auction and will cheat occasionally. However once a seller is caught cheating, then from that point onward it is unprofitable for the sellers to use a second price auction. (Porter & Shoham, 2005) examine cheating in a second price auction where the seller observes the bids and inserts his own fake bid and explicitly derive the equilibrium strategy. In a first price auction, authors examine cheating by a bidder who examines the other competing bids before deciding his own bid. The equilibrium strategy in a first price auction in a cheating setting is also explicitly derived. They find that honest sellers suffer a revenue loss from the possibility of cheating. In a second price auction an honest seller suffers a loss due to bidders shading their bids to account for the possibility of cheating. Extending the same model to account for interdependent values (Watanabe & Yamato, 2008a) investigate cheating in second price auctions when signals of bidders are affiliated. The method of cheating in second price auctions is the same as discussed above. They explicitly derive the equilibrium strategy for bidders.

Coming to modelling the effect of emotions in auctions, (Engelbrecht-Wiggans, 1989) analyse the effect of regret on bidding behaviour. (Morgan, Steiglitz, & Reis, 2003) model spiteful behaviour as a bidder's payoff being negatively affected by the surplus of a rival bidder. (Guha, 2018) explores the impact of malice on bidding behaviour in a second price auction where the seller makes a commitment to cancel the auction if any bidder drops out, in which case the object remains unsold. Prospect theory and behavioural economics have gained popularity in recent times, and have been applied to various fields of economics. These ideas have found applications in literature on auctions as well. People usually take decisions keeping some reference points in mind and evaluate their utility taking those reference points as base i.e., they assess utilities in comparison with reference points (Kahnemann, 1979); (Tversky & Kahneman, 1991). (Rosenkranz & Schmitz, 2007) model bidders having reference dependent utility and reference points depend on reserve prices set by the seller. They find that the optimal reserve price is increasing in the number of bidders if the reference points of bidders depend on reserve prices. When reserve prices are secret then the second price auction yields higher revenue than the first price auction. Interestingly auctions with secret reserve prices can fare better than public reserve prices, yielding higher revenue. Some auctions involve a "buy price" that is available as an option to close the auction by agreeing to buy the good at the quoted price the object is sold to him and the auction does not take place. (Shunda, 2009) analyses buy price as a reference point, he finds that the seller keeps a higher reserve price than in case he can affect the bidder's reference price through the auction's reserve price only. (Ahmad, 2015), endogenise reference points and use an extension of loss aversion equilibrium as defined by (Shalev, 2000) to characterise consistent reference points.

Emotions in auctions and reference based utility are separate literature and no work in our knowledge has incorporated emotional responses of bidders towards cheating. In this chapter we tackle this issue, we model bidders having preferences that capture their dislike or emotional response towards possibility of cheating in auction. We draw on idea of reference dependence, where bidders have braced up themselves about possible cheating in auction and formed a reference point. They suffer additional utility(dis-utility) if the payments in worst case(when seller is a cheat) is less(more) than their reference point. The reference point can be thought of as tolerance towards possible cheating.

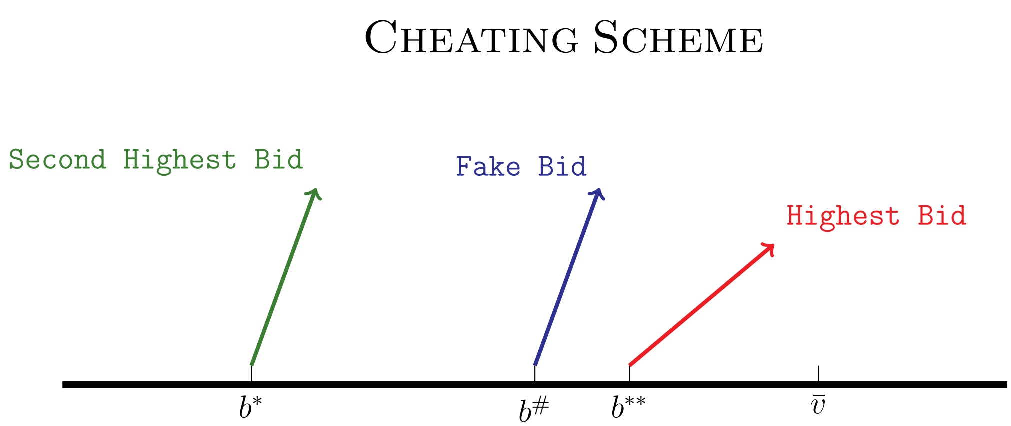
The realms of emotions and reference-based utility have conventionally been treated as distinct domains in auction literature. Remarkably, a conspicuous gap persists in the extant literature, where the fusion of emotional responses exhibited by bidders in response to fraudulent practices remains unexplored (Cheating by seller in our case). In this paper, we address this particular lacuna. Our approach encompasses the adept modeling of bidders' preferences, which encapsulate their aversions or emotional reactions engendered by the mere possibility of malfeasance within auctions.

In pursuit of this objective, we draw upon the foundation of reference dependence theory, wherein bidders proactively fortify their cognitive stance vis-à-vis potential instances of misconduct within auctions, thereby engendering the formation of an operative reference point. Consequently, these discerning bidders become subject to incremental utility fluctuations, manifesting as either disutility or utility, contingent upon whether the ensuing payments in a worst-case scenario—characterised by a deceitful seller—deviate in the direction of being lower or higher than the established reference point. This reference point, a pivotal construct in our framework, assumes the role of signifying the threshold of tolerance that bidders are willing to entertain vis-à-vis conceivable occurrences of cheating.

2 Model

There is a single object for sale and N potential buyers bidding for the object. A second price auction is being conducted. The seller has the value $x_0 = 0$ (reservation value) for the object. Bidder i has a value of X_i, the maximum amount a bidder is willing to pay for the object. Each X_i is independently and identically distributed on some interval $[0, \bar{v}]$ according to the distribution function F. It is assumed that F has a continuous density $f \equiv F'$ and has full support. It is also assumed $E[X_i] < \infty$ i.e the Expectation exists. The realisation x_i of X_i is known to the bidder and that other bidders values are independently distributed according to F. Bidders are assumed to be risk neutral. The distribution F is common knowledge and so are the number of bidders. There are no budget constraints on the bidders and all bidders are serious in the sense that they always honour their bids.

Seller cheats by inserting a fake bid between winning bid and the highest losing bid, therefore effectively a bidder who wins ends up paying a bid that is a convex combination of the highest bid among others and his own bid, if he is facing the cheating seller. Let α denote the parameter of seller's crookedness, if $\alpha = 1$, then seller inserts the fake bid so close to the winning bid that bidder ends up paying his own bid effectively, if $\alpha = 0$, then seller inserts bid very close to the highest losing bid, so effectively the auction operates like a second price auction. The figure below shows the scheme of cheating



A bidder's utility comprises of intrinsic utility as well as their emotional response towards the cheating environment they face. Bidders dislike the possibility of being cheated, so they have a fixed reference point bid in their mind when they enter the auction and they care about the worst possible payment that could be extracted from them in this setting. This means if the worst extracted payment from them is greater than their reference point then they suffer a negative utility, i.e. if payment to cheating seller is more than the reference point it reduces the overall utility for the bidders. We interpret the emotional response as deviation of the bidder's reference point from the payment that cheat can extract from the bidder if he wins. The reference point could be based on some previous experience of bidder, dislike for dishonesty(which may lead to reference points being very low) or buy prices for similar objects. Let η be such reference point of the bidder, bidder's utility is affected from deviation of payment extracted by a cheating seller from the reference point. Let k be the parameter of sensitivity that bidders place on emotional dislike for cheating, which we here model it as deviation from some reference point. We assume that $k \in (0, 1)$, Reference point of bidders is assumed to be exogenous and the utilities of the bidders depend upon the reference point as well in addition to the actual pecuniary payoff from the auction. The utility of a bidder in this scenario can be expressed as

$$U(x_i) = \begin{cases} \underbrace{(x - P(x))}_{\texttt{Intrinsic Utility}} + \underbrace{k(\eta - \text{Worst case payment})}_{\texttt{Emotional response towards cheating}} & \textit{if i wins} \\ 0 & \textit{otherwise} \end{cases}$$

where x is the valuation of bidder for the object, $P(x)$ is the average payment that a bidder makes in the auction. Worst case payment in our model would be the payment that the bidders have to make to a dishonest seller, as we assume that bidders dislike dishonesty.

Modelling cheating on lines of (Porter & Shoham, 2005), we assume seller cheats with some probability Φ and with complementary probability he does not cheat, or it can be interpreted as bidders do not know about the type of seller they are facing, they have a common prior over the type of seller they face. They think with probability Φ, they face a cheating seller and with complementary probability an honest one. Let Y_1 be the highest order statistic among other $N-1$ bidders and $W(\cdot)$ and $w(\cdot)$ denote its distribution and density function respectively. we can model cheating with triggered emotional response by introducing reference dependence in the utility of bidders, let $\beta(\cdot)$ be a strategy for a bidder is a function $\beta_i(\cdot) : [0, v] \longrightarrow \mathbb{R}+$, which dictates bid for him given his value for the object, in our case the utility to a bidder is

$$\begin{aligned} \mathbf{U}(\mathbf{x}) = \Phi\left[x - \alpha\beta(z) - (1-\alpha)[E(\beta(Y_1)) \mid Y_1 < z)]\right] \\ + (1-\Phi)\left[x - [E(\beta(Y_1)) \mid Y_1 < z)]\right] \\ + \underbrace{k(\eta - \alpha\beta(z) - (1-\alpha)\left[E(\beta(Y_1)) \mid Y_1 < z)\right])}_{\texttt{Emotional response towards cheating}} \end{aligned}$$

$W(\cdot)$ denotes the distribution function of the highest order statistic among other bidders values The expected payoff to a bidder in this setting can we expressed as

$$\begin{aligned} \mathbf{\Pi}(\mathbf{x}, \mathbf{x}) = \Phi W(x)\left[x - \alpha\beta(x) - (1-\alpha)[E(\beta(Y_1)) \mid Y_1 < x)]\right] \\ + (1-\Phi)W(x)\left[x - [E(\beta(Y_1)) \mid Y_1 < x)]\right] \\ + W(x)k(\eta - \alpha\beta(z) - (1-\alpha)\left[E(\beta(Y_1)) \mid Y_1 < x)\right]) \end{aligned}$$

where Φ is the probability that the seller cheats, when he does so then, the bidder ends up paying a bid between highest among others bid and their own bid if they win, i.e a convex combination of the highest among other bids and his own bid ; $\alpha \in [0, 1]$ and with probability $1 - \Phi$ the seller does not cheat, then bidder ends up paying the highest losing bid.

3 Results

Proposition 1. *The symmetric increasing equilibrium strategy of the above auction is given by*

$$\beta(x) = \frac{1}{1+k}\left[k\eta + x - \frac{1}{H(x)}\int_0^x H(y)dy\right]$$

where $H(x) = [W(x)]^{\frac{1+k}{(k+\Phi)\alpha}}$

Proof. Suppose that all except bidder i follow the strategy $\beta \equiv \beta$ as given above. We will show that in that case it is optimal for bidder i to choose β as well. Let $z = \beta^{-1}(b)$ the value for which b is the equilibrium bid—that is, $\beta(z) = b$. Expected payoff of bidder i with value x who bids as if his value were z is x as follows:

$$\begin{aligned}\mathbf{\Pi}(\mathbf{x}, \mathbf{z}) = \Phi W(z)&\left[x - \alpha\beta(z) - (1-\alpha)\left[\int_0^z \frac{\beta(y_1)\, w(y_1)}{W(z)}dy\right]\right] \\ &+ (1-\Phi)W(z)\left[x - \left[\int_0^z \frac{\beta(y_1)\, w(y_1)}{W(z)}dy\right]\right] \\ &+ W(z)k\left[\eta - \alpha\beta(z) - (1-\alpha)\left[\int_0^z \frac{\beta(y_1)\, w(y_1)}{W(z)}dy\right]\right] \qquad (1)\end{aligned}$$

The bidder with valuation x who bids as if his valuation were z, chooses z to maximise the above equation, yields the following differential equation

$$\begin{aligned}\frac{\partial\Pi(x, z)}{\partial z} = \Phi &\left[w(z)(x - \alpha\beta(z)) - \alpha\beta'(z)W(z) - (1-\alpha)(\beta(z)w(z))\right] \\ &+ (1-\Phi)\left[xw(z) - \beta(z)w(z)\right] + k\left[w(z)(\eta - \beta(z)) - \alpha\beta'(z)W(z)\right]\end{aligned}$$

for a maximum, $\frac{\partial \Pi(x,z)}{\partial z} = 0$ at $z = x$; if $\beta(\cdot)$ is a symmetric increasing equilibrium strategy

$$\begin{aligned}
&\Phi w(x)x - \Phi\alpha w(x)\beta(x) - \Phi\alpha\beta'(x)W(x) - \Phi(1-\alpha)\beta(x)w(x)\\
&+ xw(x) - \beta(x)w(x) - \Phi xw(x) + \Phi\beta(x)w(x) + kw(x)(\eta - \beta(x)) - k\alpha\beta^{'}(x)W(x) = 0\\
&- \Phi\alpha\beta^{'}(x)W(x) + xw(x) - \beta(x)w(x) + k\left[w(x)(\eta - \beta(x)) - \alpha\beta^{'}(x)W(x)\right] = 0\\
&\implies k\alpha\beta^{'}(x)W(x) + \beta^{'}(x)\Phi\alpha W(x) = w(x)\left[(x - \beta(x)) + k(\eta - \beta(x))\right]
\end{aligned}$$

$$\begin{aligned}
&\beta^{'}(x)\left[k\alpha W(x) + \Phi\alpha W(x)\right] + w(x)\beta(x) + kw(x)\beta(x) = w(x)[x + k\eta]\\
&\beta^{'}(x) + \beta(x)\frac{w(x)(1+k)}{k\alpha W(x) + \Phi\alpha W(x)} = \frac{w(x)(x + k\eta)}{(k\alpha + \Phi\alpha)W(x)}
\end{aligned}$$

This is a linear differential equation with integrating factor $[W(x)]^{\frac{1+k}{k\alpha+\Phi\alpha}}$; multiplying the equation by the integrating factor and solving we get the equilibrium bid as

$$\beta(x) = \frac{1}{[W(x)]^{\frac{1+k}{k\alpha+\Phi\alpha}}}\int_0^x \frac{w(y)(k\eta + y)\left[W(y)\right]^{\frac{1+k}{k\alpha+\Phi\alpha}}}{W(y)(k\alpha + \Phi\alpha)}dy$$

Let $[W(x)]^{\frac{1+k}{k\alpha+\Phi\alpha}} = H(x)$ and $\psi = \frac{1+k}{\alpha(k+\Phi)}$, this implies $h(x) = \frac{1+k}{k\alpha+\Phi\alpha}w(x)[W(x)]^{\frac{1+k-(k+\Phi)\alpha}{k\alpha+\Phi\alpha}}$, therefore,

$$\beta(x) = \frac{1}{H(x)}\int_0^x \frac{[y + k\eta]h(y)}{1+k}dy$$

$$\begin{aligned}
\beta(x) &= \frac{1}{(1+k)H(x)}\int_0^x \left[yh(y) + k\eta h(y)\right]dy\\
&= \frac{1}{(1+k)H(x)}\int_0^x \left[yh(y)\right]dy + k\eta\cdot H(x)\\
&= \frac{k\eta}{1+k} + \frac{1}{(1+k)H(x)}\int_0^x \left[yh(y)\right]dy\\
&= \frac{k\eta}{1+k} + \frac{1}{(1+k)H(x)}\left[xH(x) - \int_0^x H(y)dy\right]\\
&= \frac{1}{1+k}\left[k\eta + x - \frac{1}{H(x)}\int_0^x H(y)dy\right]
\end{aligned}$$

Now, to show that this is indeed an equilibrium at $z = x$, we substitute the bidding function in the first order condition,

$$xw(z) - \beta(z)w(z) - \alpha\Phi\beta'(z)W(z) + w(z)k(\eta - \beta(z)) - k\alpha W(z)\beta'(z)$$

$$xw(z) - w(z)\frac{1}{1+k}\left[z + \eta k - \frac{1}{H(z)}\int_0^z H(y)y\right] - \frac{\Phi\alpha W(z)}{1+k}\cdot\frac{h(z)}{[H(z)]^2}\int_0^z H(y)dy$$
$$+w(z)k\left[\eta - \left(z + \eta k - \frac{1}{H(z)}\int_0^z H(y)dy\right)\frac{1}{1+k}\right] - \frac{k\alpha W(z)}{1+k}\cdot\frac{h(z)}{[H(z)]^2}\int_0^z H(y)dy$$

$$xw(z) - \frac{w(z)}{1+k}\left[z + \eta k - \frac{1}{H(z)}\int_0^z H(y)dy\right] - \frac{\alpha\Phi}{1+k}\cdot\frac{\psi W(z)^{\psi}w(z)}{W(z)^{2\psi}}\int_0^z H(y)dy$$
$$+w(z)k\left[\eta - \left(\frac{z+\eta k}{1+k}\right) + \frac{1}{(1+k)H(z)}\int_0^z H(y)dy\right] - \frac{k\alpha\psi W(z)^{\psi}w(z)}{W(z)^{2\psi}}\int_0^z H(y)dy$$

$$w(z)\left[x - \left[\frac{z+\eta k}{1+k}\right]\right] + \frac{w(z)}{(1+k)}\cdot\frac{1}{H(z)}\int_0^z H(y)dy - \frac{\alpha\Phi\psi}{1+k}\cdot\frac{w(z)}{H(z)}\int_0^z H(y)dy$$
$$+w(z)k\left[\eta - \left[\frac{z+\eta k}{1+k}\right] + \frac{1}{(1+k)H(z)}\int_0^z H(y)dy\right] - \frac{k\alpha\psi}{(1+k)}\cdot\frac{w(z)}{H(z)}\int_0^z H(y)dy$$

$$w(z)\left[x - \left[\frac{z+\eta k}{1+k}\right]\right] + \frac{w(z)}{(1+k)H(z)}(1-\alpha\Phi\psi)\int_0^z H(y)dy$$
$$+w(z)k\left[\eta - \frac{(z+\eta k)}{1+k}\right] + \frac{w(z)k}{(1+k)H(z)}\int_0^z H(y)dy[1-\alpha\psi]$$

$$w(z)\left[x - \left[\frac{z+\eta k}{1+k}\right]\right] + \frac{w(z)}{(1+k)H(z)}\int_0^z H(y)dy\left[\frac{k(1-\Phi)}{k+\Phi}\right]$$
$$+w(z)k\left[\eta - \left[\frac{z+\eta k}{1+k}\right]\right] + \frac{w(z)k}{(1+k)H(z)}\int_0^z H(y)dy\left[\frac{k+\Phi-1-k}{k+\Phi}\right]$$

$$w(z)\left[x-\left[\frac{z+\eta k}{1+k}\right]\right]+\frac{w(z)k}{(1+k)H(z)}\int\limits_0^z H(y)dy\left[\frac{(1-\Phi)}{k+\Phi}\right]$$

$$+w(z)k\left[\eta-\left[\frac{z+\eta k}{1+k}\right]\right]+\frac{w(z)k}{(1+k)H(z)}\int\limits_0^z H(y)dy\left[\frac{\Phi-1}{k+\Phi}\right]$$

$$w(z)\left[x-\frac{(z+\eta k)}{1+k}\right]+w(z)k\left[\eta-\left[\frac{z+\eta k}{1+k}\right]\right]$$

$$w(z)\left[x-\frac{(z+\eta k)}{1+k}+k\eta-\frac{k(z+\eta k)}{1+k}\right]$$

$$w(z)\left[x-\frac{z}{1+k}-\frac{\eta k}{1+k}+k\eta-\frac{kz}{1+k}-\frac{(\eta k)k}{1+k}\right]$$

$$w(z)\left[x-z+k\eta-\frac{\eta k(1+k)}{1+k}\right]$$

$$w(z)[x-z]$$

and which is positive $\forall\, z < x$ negative $\forall\, z > x$. Therefore maximum at $z = x$ ■

Corollary 1.1. *Depending on value of signal for the bidder, bids without reference dependence can be more than with reference dependence.*

Bid without reference dependence($k = 0$), is

$$\tilde{\beta}(x)=\frac{1}{L(x)}\int\limits_0^x y\cdot l(y)\,dy$$

where $L(x) = [W(x)]^{\frac{1}{\Phi\alpha}}$ and $l(x) = \frac{1}{\Phi\alpha}[W(x)]^{\frac{1}{\Phi\alpha}-1}w(x)$, now its clear that $L(x)$ first order stochastically dominates $H(x)$, because $\frac{1+k}{k\alpha+\Phi\alpha} \leq \frac{1}{\Phi\alpha}$; let $\hat{\beta}(x) = \frac{1}{H(x)}\int_0^x y\cdot h(y)\,dy$ therefore

$$\beta(x)=\frac{1}{1+k}\left[\hat{\beta}(x)+k\eta\right]$$

and by $FOSD$, $\hat{\beta}(x) \leq \tilde{\beta}(x) \implies \frac{\hat{\beta}(x)}{1+k} \leq \tilde{\beta}(x)$ as $k > 0$, consider $M(x) = \tilde{\beta}(x) - \beta(x)$; $M(x) > 0$ if

$$\implies (1+k)\tilde{\beta}(x)-\hat{\beta}(x)\geq k\eta$$

$$\implies (1+k)\left[x-\int\limits_0^x\frac{L(y)}{L(x)}dy\right]-\left[x-\int\limits_0^x\frac{H(y)}{H(x)}dy\right]\geq k\eta$$

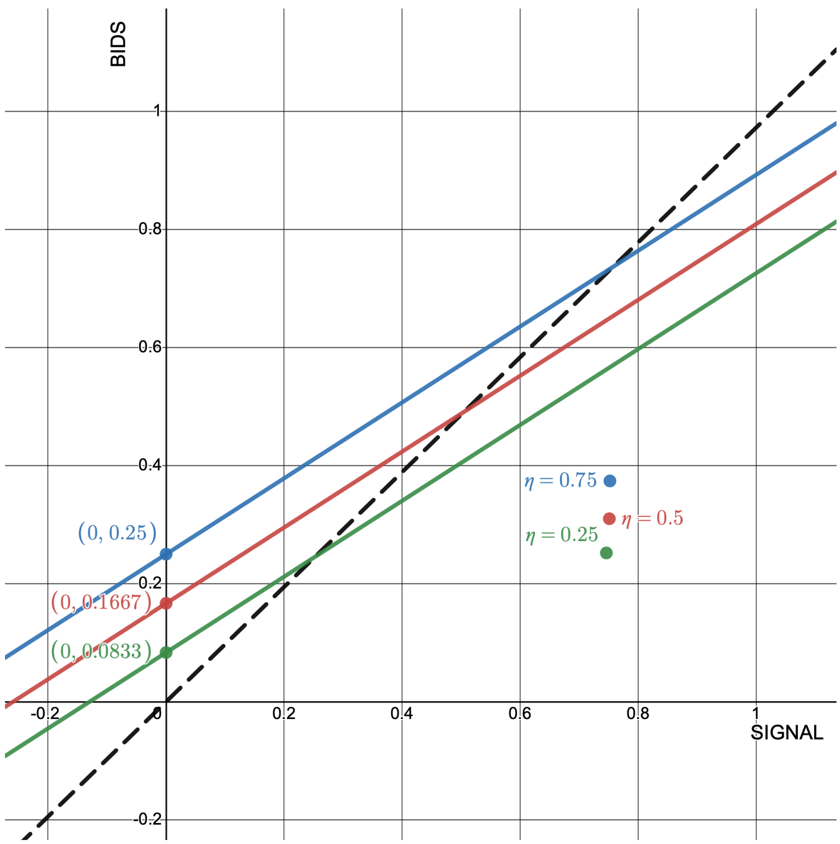
$$\implies \int\limits_0^x\frac{H(y)}{H(x)}dy-(1+k)\left[\int\limits_0^x\frac{L(y)}{L(x)}dy\right]\geq k(\eta-x)$$

EXAMPLE. If signals are drawn from $U[0,1]$ and let there be N bidders, then bidding strategy becomes

$$\beta(x) = \frac{1}{k+1}\left[\eta k + x - \frac{1}{x^{(N-1)\cdot\frac{k+1}{\alpha(\Phi+k)}}}\int_0^x \left(y^{(N-1)\cdot\frac{k+1}{\alpha(\Phi+k)}}\right)dy\right]$$
$$= \frac{1}{1+k}\left[\eta k + x - \frac{x\alpha(k+\Phi))}{(N-1)(1+k)+\alpha(k+\Phi)}\right]$$

Consider a case where $\{\Phi = 0.5, \alpha = 0.5, k = 0.5\}$; for $\eta \in \{0.25, 0.5, 0.75\}$ we draw the bidding functions associated with each case and also for the case when $k = 0$

Plot of bidding function with 10 bidders



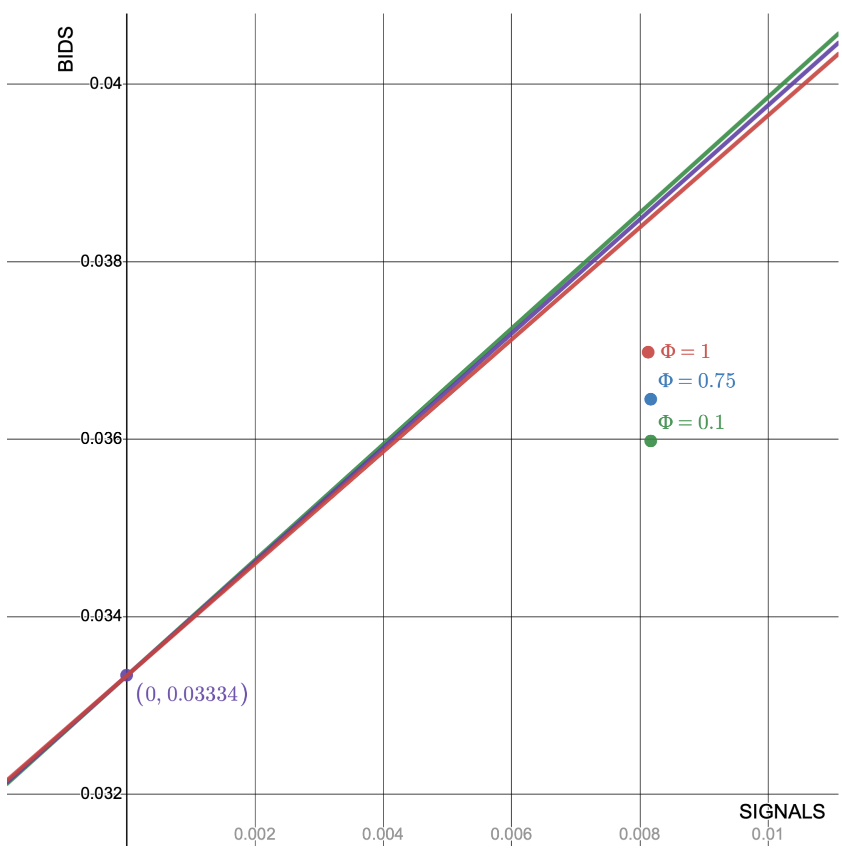
In the above figure the black dotted line is bidding function when $k = 0$, i.e no emotional responses (no reference based utility) and the green, red and blue lines represent the bidding function when reference points are $\{\eta = 0.25, \eta = 0.5, \eta = 0.75\}$ respectively. Notice that higher reference points lead to a higher intercept for the bidding function and the range of signals for the bidder for which the bids are more than the case without reference dependence increases.

Proposition 2. *An increase in probability of cheating(Φ) or seller's crookedness (α), decreases the equilibrium bids.*

Proof. Taking the derivative of the bidding strategy w.r.t Φ and α respectively gives $\frac{\partial \beta(x)}{\partial \Phi} = \frac{1}{\alpha(k+\Phi\alpha)^2} \int\limits_0^x \left[\frac{W(y)}{W(x)}\right]^{\frac{1+k}{(k+\Phi)\alpha}} \cdot \log\left[\frac{W(y)}{W(x)}\right] dy < 0$ and similarly, $\frac{\partial \beta(x)}{\partial \alpha} = \frac{1}{\alpha^2(k+\Phi\alpha)} \int\limits_0^x \left[\frac{W(y)}{W(x)}\right]^{\frac{1+k}{(k+\Phi)\alpha}} \cdot \log\left[\frac{W(y)}{W(x)}\right] dy < 0$ ■

This happens because an increase in probability of cheating pushes the auction towards the first price auction, which leads to shading of bids by the bidders, similarity in case when sellers crookedness increased, the bidders now have to pay bids even closer to their own bids, therefore bidders shade their bids.

EXAMPLE. Bidding functions with 10 bidders and signals from U[0,1]



The above graph depicts a case when signal are drawn from $U[0,1]$ and $[\eta = 0.1, \Phi = 0.5, \alpha = 0.5]$, as shown in the graph increase in Φ decreases the slope of the bidding function, i.e. bidders shade their bids as probability of cheating increases.

Proposition 3. *An increase in the sensitivity parameter* (k) *of the bidders, increases the intercept and decreases the slope of the bidding function.*

Proof. The equilibrium bid can be written as $\beta(x) = \underbrace{\frac{k\eta}{1+k}}_{\text{intercept(I)}} + \underbrace{\frac{\hat{\beta}(x)}{1+k}}_{slope\,part(II)}$ notice that the intercept here is increasing in k as $\frac{\partial I}{\partial k} = \frac{\eta}{(1+k)^2} > 0$ and the slope coefficient is decreasing in k as

$$\frac{\partial II}{\partial k} = -\frac{1}{k+1}\left[\int_0^x \left(\frac{W(y)}{W(x)}\right)^{\frac{k+1}{\alpha(\Phi+k)}} \left(\frac{1}{\alpha(\Phi+k)} - \frac{k+1}{\alpha(\Phi+k)^2}\right) \log\left(\frac{W(y)}{W(x)}\right) dy\right]$$

$$- \frac{1}{(k+1)^2}\left[x - \int_0^x \left(\frac{W(y)}{W(x)}\right)^{\frac{k+1}{\alpha(\Phi+k)}dy}\right]$$

$$= \frac{1}{(k+1)^2}\left[-x + \int_0^x \left(\frac{W(y)}{W(x)}\right)^{\frac{k+1}{\alpha(\Phi+k)}} dy - \frac{(\Phi-1)(k+1)\int_0^x \left(\frac{W(y)}{W(x)}\right)^{\frac{k+1}{\alpha(\Phi+k)}} \log\left(\frac{W(y)}{W(x)}\right) dy}{\alpha(\Phi+k)^2}\right]$$

$$= -\frac{1}{(k+1)^2\alpha(\Phi+k)^2}\left[(k+1)(\Phi-1)\int_0^x \left(\frac{W(y)}{W(x)}\right)^{\frac{k+1}{\alpha(\Phi+k)}} \log\left(\frac{W(y)}{W(x)}\right) dy\right]$$

$$+ \frac{1}{(k+1)^2\alpha(\Phi+k)^2}\left[\left[-x + \int_0^x \left(\frac{W(y)}{W(x)}\right)^{\frac{k+1}{\alpha(\Phi+k)}} dy\right]\alpha(\Phi+k)^2\right]$$

$$= \frac{1}{(k+1)^2\alpha(\Phi+k)^2}\left[(k+1)(1-\Phi)\int_0^x \left(\frac{W(y)}{W(x)}\right)^{\frac{k+1}{\alpha(\Phi+k)}} \log\left(\frac{W(y)}{W(x)}\right) dy\right]$$

$$- \frac{1}{(k+1)^2\alpha(\Phi+k)^2}\left[\left[x - \int_0^x \left(\frac{W(y)}{W(x)}\right)^{\frac{k+1}{\alpha(\Phi+k)}} dy\right]\alpha(\Phi+k)^2\right] < 0$$

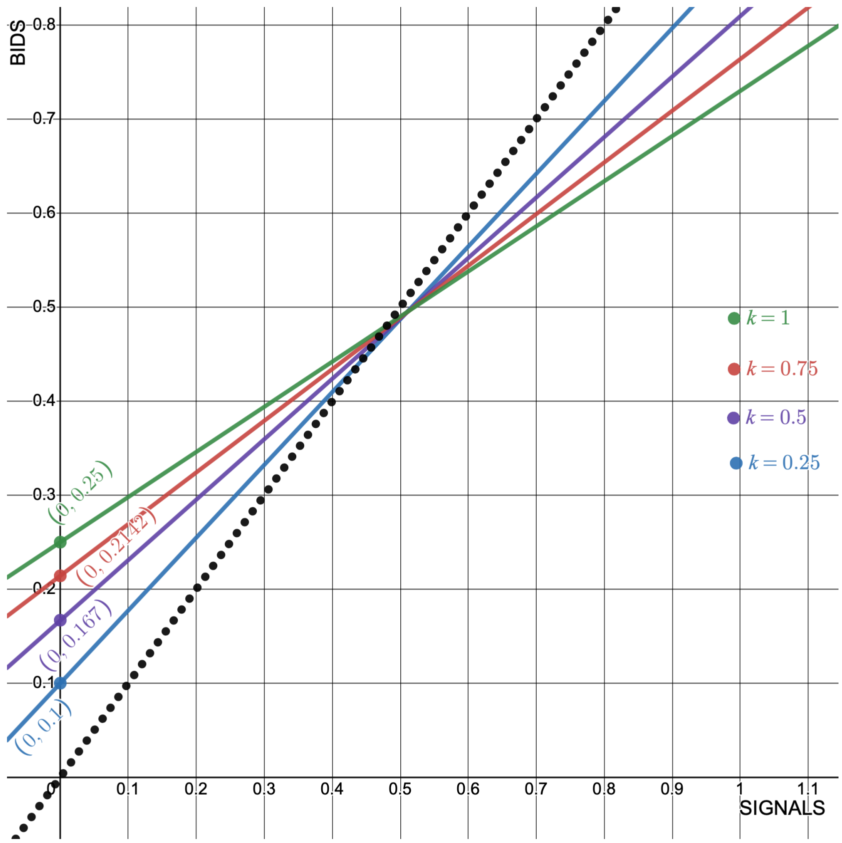
as for $y < x$, $\log(\frac{W(y)}{W(x)})$ is negative and $\left[x - \int_0^x \left(\frac{W(y)}{W(x)}\right)^{\frac{k+1}{\alpha(\Phi+k)}} dy\right]$ is positive[9] ■

Increase in sensitivity parameter increases the effective reference point for the bidders on one hand and on the other hand bidders get more sensitive of their reference point utility.

[9] To see that consider $\mu(x) = \left[x - \int_0^x \left(\frac{W(y)}{W(x)}\right)^{\frac{k+1}{\alpha(\Phi+k)}} dy\right]$, now as $\mu(0) = 0$ and $\mu'(x) > 0$ therefore $\mu(x) > 0 \;\; \forall \;\; x > 0$

Corollary 3.1. *The total effect of increase in k on the bids depends on the the relative strength of the two opposing effects (increase in intercept and decrease in slope). For bidders with higher signal, increase in k would reduce the bid.*

BIDDING FUNCTIONS WITH 10 BIDDERS AND SIGNALS FROM U[0,1]



Above graphs depicts a case where $[\eta = 0, 5, \Phi = 0.5, \alpha = 0.5]$ and signals of bidders are drawn from $U[0, 1]$ the black dotted line is 45° line from origin. Consider $k = 0.25$ as the initial case, notice that as k increases the intercept of bidding function increases but slope decreases and there is a cutoff value of signal after which slope effect starts dominating the intercept increase effect.

EXAMPLE. Suppose there are 10 bidders and signals are drawn from $U[0, 1]$, then
$\beta(x) = \frac{1}{1+k}\left[\eta k + x - \frac{x(k+\Phi)\alpha)}{9(1+k)+(k+\Phi)\alpha)}\right]$; $\frac{\partial\beta(x)}{\partial k} = \frac{\partial I}{\partial k} + \frac{\partial II}{\partial k}$;

$$\frac{\partial I}{\partial k} = \frac{\eta}{(1+k)^2} > 0$$

$$\frac{\partial II}{\partial k} = \frac{1}{k+1}\left[\frac{9\alpha x\left(\Phi+k\right)}{\left(\alpha\left(\Phi+\alpha\right)+9k+9\right)^{2}} - \frac{\alpha x}{\alpha\left(\Phi+\alpha\right)+9k+9}\right] - \frac{1}{\left(k+1\right)^{2}}\left[\frac{-\alpha x\left(\Phi+k\right)}{\alpha\left(\Phi+\alpha\right)+9k+9} + x\right]$$

$$= \frac{-x}{\left(k+1\right)^{2}\left(\alpha\left(\Phi+\alpha\right)+9k+9\right)^{2}}\Big[\alpha\left(k+1\right)\left(-9\Phi+\alpha\left(\Phi+\alpha\right)+9\right) + \left(\alpha\left(\Phi+\alpha\right)+9k+9\right)\left(\alpha\left(\Phi+\alpha\right)-\alpha\left(\Phi+k\right)+9k+9\right)\Big] \quad < 0 \ \forall \ x$$

Consider a case where $\{\Phi = 0.5, \alpha = 0.5, \eta = 0.5, k = 0.5\}$; $\frac{\partial \beta(x)}{\partial k} < 0 \quad \forall\ x > 0.508$

Proposition 4. *The expected revenue for the cheating seller is*

$$\mathbf{\Gamma}^{Cheat} = \frac{1}{1+k}\Bigg[k\eta - \frac{(1-\alpha)N(N-1)\psi}{(1-\psi)}\int\limits_{0}^{\bar{v}} y\cdot f(y)(1-F(y))F^{N-2}(y)dy + \frac{\psi(1-\alpha\psi)}{(1-\psi)}\cdot\frac{N(N-1)}{(N-1)(1-\psi)+1}\int\limits_{0}^{\bar{v}}\Big[[F(y)]^{(N-1)\psi-1} - [F(y)]^{N-1}\Big]yf(y)\ dy\Bigg]$$

where $\psi = \frac{1+k}{\alpha(k+\Phi)}$

Proof. The expected revenue to the seller is number of bidders times the ex-ante expectation of Expected payment by a bidder in the auction, the expected payment to the cheating seller is $W(x)[\underbrace{\alpha\beta(x)}_{\text{A}} + \underbrace{(1-\alpha)E[\beta(y) \mid Y_1 < x]}_{\text{B}}]$, now consider the term

$$\begin{aligned}
A =& \frac{\alpha}{1+k}\left[k\eta W(x) + \frac{W(x)}{H(x)}\int\limits_{0}^{x} yh(y)dy\right] \\
=& \frac{\alpha}{1+k}\left[k\eta W(x) + \frac{W(x)}{[W(x)]^{\psi}}\int\limits_{0}^{x} yh(y)dy\right] \\
=& \frac{\alpha}{1+k}\left[k\eta W(x) + W(x)^{1-\psi}\int\limits_{0}^{x} yh(y)dy\right] \\
=& \frac{\alpha}{1+k}\left[k\eta W(x) + W(x)^{1-\psi}\psi\int\limits_{0}^{x} y[W(y)]^{\psi-1}w(y)dy\right] \qquad (2)
\end{aligned}$$

$$B = W(x)(1-\alpha)\frac{1}{W(x)}\int_0^x \beta(y)w(y)dy$$

$$= (1-\alpha)\int_0^x \beta(y)w(y)dy$$

$$= \frac{1-\alpha}{1+k}\int_0^x \left[k\eta + \int_0^y \frac{th(t)}{H(y)}dt\right] w(y)dy$$

$$= \frac{1-\alpha}{1+k}\left[k\eta W(x) + \int_0^x\int_0^y \frac{th(t)}{H(y)}w(y)dtdy\right]$$

$$= \frac{1-\alpha}{1+k}\left[k\eta W(x) + \int_0^x\int_0^y \frac{th(t)}{[W(y)]^{\psi}}w(y)dtdy\right]$$

$$= \frac{1-\alpha}{1+k}\left[k\eta W(x) + \int_0^x\int_t^x \frac{w(y)}{[W(y)^{\psi}]}t\cdot h(t)dydt\right]$$

$$= \frac{1-\alpha}{1+k}\left[k\eta W(x) + \int_0^x \frac{[W(y)]^{-\psi+1}}{-\psi+1}\Bigg|_t^x th(t)dt\right]$$

$$= \frac{1-\alpha}{1+k}\left[k\eta W(x) + \int_0^x \left[\frac{W(x)^{1-\psi} - W(t)^{1-\psi}}{1-\psi}\right] th(t)dt\right]$$

$$= \frac{1-\alpha}{1+k}\left[kTW(x) + \int_0^x \left[\frac{W(x)^{1-\psi}W(t)^{\psi-1} - 1}{1-\psi}\right] t\psi w(t)dt\right]$$

$$= \frac{1-\alpha}{1+k}\left[k\eta W(x) + \frac{\psi}{1-\psi}\int_0^x tw(t)W(t)^{\psi-1}W(x)^{1-\psi}dt - \frac{\psi}{1-\psi}\int_0^x tw(t)dt\right]$$

Because definite integral is independent of change of name of variable, therefore the expected payment to a cheating seller is $A+B$

$$\mathbb{E}^{Cheat} = \frac{\alpha}{1+k}\left[k\eta W(x) + \psi\cdot[W(x)]^{1-\psi}\int_0^x y\cdot w(y)[W(y)]^{\psi-1}dy\right] + \frac{(1-\alpha)}{1+k}\Bigg[k\eta W(x)$$

$$+\frac{\psi}{1-\psi}\int_0^x [W(x)]^{1-\psi}[W(y)]^{\psi-1}yw(y)dy - \frac{\psi}{1-\psi}\int_0^x yw(y)dy\Bigg]$$

$$= \frac{k\eta W(x)}{1+k} + \frac{1}{1+k}\Bigg[\frac{\psi(1-\alpha\psi)}{1-\psi} \cdot [W(x)]^{1-\psi} \int_0^x y \cdot w(y)[W(y)]^{\psi-1} dy - \frac{(1-\alpha)\psi}{(1-\psi)} \int_0^x y \cdot w(y) dy\Bigg] \tag{3}$$

Now the expected revenue for the cheating seller is the expectation of (3) over types multiplied by the number of bidders in the auction

$$\mathbf{\Gamma}^{\text{Cheat}} = \frac{N}{1+k}\Bigg[\underbrace{\int_0^{\bar{v}} k\eta W(x) f(x) dx}_{\text{I}} - \frac{\psi(1-\alpha)}{(1+k)(1-\psi)} \int_0^{\bar{v}}\int_0^x \underbrace{yw(y)f(x)\,dydx}_{\text{II}} + \frac{\psi(1-\alpha\psi)}{(1-\psi)(1+k)} \int_0^{\bar{v}}\int_0^x \underbrace{[W(x)]^{1-\psi}[W(y)]^{\psi-1} y \cdot w(y) f(x)\,dydx}_{\text{III}}\Bigg] \tag{4}$$

$$I = \left[k\eta \int_0^{\bar{v}} F^{N-1}(x) f(x) dx\right] = \frac{k\eta}{N} \tag{5}$$

$$\begin{aligned} II &= \int_0^{\bar{v}}\int_0^x yw(y)f(x)dydx \\ &= \int_0^{\bar{v}}\int_y^{\bar{v}} f(x)yw(y)dxdy \\ &= \int_0^{\bar{v}}\int_y^{\bar{v}} f(x)y(N-1)F^{N-2}(y)f(y)dxdy \\ &= (N-1)\int_0^{\bar{v}} (1-F(y))\, y \cdot F^{N-2}(y) f(y) dy \end{aligned} \tag{6}$$

$$\begin{aligned} III &= \int_0^{\bar{v}}\int_0^x [W(x)]^{1-\psi}[W(y)]^{\psi-1} y \cdot w(y) f(x)\,dydx \\ &= \int_0^{\bar{v}}\int_0^x (N-1)[F(x)]^{(N-1)(1-\psi)}[F(y)]^{(N-1)(\psi-1)} y [F(y)]^{(N-2)} f(y) f(x)\,dydx \end{aligned}$$

$$= \int_0^{\bar{v}} \int_y^{\bar{v}} (N-1)[F(x)]^{(N-1)(1-\psi)} f(x)\, y \cdot f(y)[F(y)]^{(N-1)(\psi-1)}[F(y)]^{(N-2)}\, dxdy$$

$$= \int_0^{\bar{v}} \frac{[F(x)]^{(N-1)(1-\psi)+1}}{(N-1)(1-\psi)+1}\Bigg|_y^{\bar{v}} (N-1)[F(y)]^{(N-1)\psi-1} y f(y) dy$$

$$= \int_0^{\bar{v}} \left[\frac{1-[F(y)]^{(N-1)(1-\psi)+1}}{(N-1)(1-\psi)+1}\right](N-1)[F(y)]^{(N-1)\psi-1} y f(y) dy$$

$$= \int_0^{\bar{v}} \left[\frac{[F(y)]^{(N-1)\psi-1}-[F(y)]^{(N-1)}}{(N-1)(1-\psi)+1}\right](N-1) y f(y) dy \tag{7}$$

substituting (5), (7) and (6) in (4) and multiplying by N gives

$$\mathbf{\Gamma}^{Cheat} = \frac{1}{1+k}\Bigg[k\eta + \frac{\psi(1-\alpha\psi)}{(1-\psi)} \cdot \frac{N(N-1)}{(N-1)(1-\psi)+1}\int_0^{\bar{v}} \Big[[F(y)]^{(N-1)\psi-1} - [F(y)]^{N-1}\Big] y f(y)\, dy$$
$$- \frac{(1-\alpha)N(N-1)\psi}{(1-\psi)} \int_0^{\bar{v}} y \cdot f(y)(1-F(y))F^{N-2}(y) dy\Bigg]$$

■

Remark. Notice if $\psi = 1$ (which happens if $k = 0$, $\Phi = 1$ and $\alpha = 1$), then $\Gamma^{Cheat} = E(Y^{N-1})$, which is expectation of second highest order statistic.

Corollary 4.1. *If signals are drawn from the U[0,1], then*

$\Gamma = \frac{1}{1+k}\left(k\eta \ + \frac{\psi(1-\alpha\psi)}{1-\psi} \cdot \frac{N(N-1)}{(N+1)((N-1)\psi+1)} - \frac{\psi(1-\alpha)(N-1)}{(1-\psi)(N+1)}\right)$ *and if there are no reference point effects then* $\Gamma_{k=0} = \frac{\xi(1-\alpha\xi)}{1-\xi} \cdot \frac{N(N-1)}{(N+1)((N-1)\xi+1)} - \frac{\xi(1-\alpha)(N-1)}{(1-\xi)(N+1)}$, *where* $\xi = \frac{1}{\Phi\alpha}$ *Notice if* $\psi = 1$ *(which happens if* $k = 0, \Phi = 1, \alpha = 1$*) then the expected revenue in this case is* $\frac{N-1}{N+1}$ *which is same as in case of a regular first price or second price auction with N bidders.*

Proposition 5. *The expected revenue for the honest seller is*

$$\mathbf{\Gamma}^{Honest} = \frac{1}{1+k}\Bigg[k\eta + \frac{\psi N(N-1)}{1-\psi}\Bigg[\int_0^{\bar{v}} \frac{[F(y)]^{(N-1)\psi-1} - [F(y)]^{N-1}}{(N-1)(1-\psi)+1} \cdot y f(y) dy$$
$$- \int_0^{\bar{v}} y \cdot f(y) F^{N-2}(y)(1-F(y)) dy\Bigg]\Bigg]$$

Proof. As the honest seller expects to receive the expectation of second highest bid in the auction, The expected revenue to an honest seller is given by

$$\Gamma^{Honest} = N \int_0^x \underbrace{W(x)E[\beta(y) \mid Y_1 < x]]}_{\text{I}} f(x)dx$$

$$\begin{aligned}
I &= W(x)\frac{1}{W(x)}\int_0^x \beta(y)w(y)dy \\
&= \int_0^x \beta(y)w(y)dy \\
&= \frac{1}{1+k}\int_0^x \left[k\eta + \int_0^y \frac{th(t)}{H(y)}dt\right] w(y)dy \\
&= \frac{1}{1+k}\left[k\eta W(x) + \int_0^x\int_0^y \frac{th(t)}{H(y)}w(y)dtdy\right] \\
&= \frac{1}{1+k}\left[k\eta W(x) + \int_0^x\int_0^y \frac{th(t)}{[W(y)]^{\psi}}w(y)dtdy\right] \\
&= \frac{1}{1+k}\left[k\eta W(x) + \int_0^x\int_t^x \frac{w(y)}{[W(y)]\psi}t\cdot h(t)dydt\right] \\
&= \frac{1}{1+k}\left[k\eta W(x) + \int_0^x \frac{[W(y)]^{-\psi+1}}{-\psi+1}\bigg|_t^x th(t)dt\right] \\
&= \frac{1}{1+k}\left[k\eta W(x) + \int_0^x \left[\frac{W(x)^{1-\psi} - W(t)^{1-\psi}}{1-\psi}\right] th(t)dt\right] \\
&= \frac{1}{1+k}\left[kW(x) + \int_0^x \left[\frac{W(x)^{1-\psi}W(t)^{\psi-1} - 1}{1-\psi}\right] t\psi w(t)dt\right] \\
&= \frac{1}{1+k}\left[k\eta W(x) + \frac{\psi}{1-\psi}\int_0^x tw(t)W(t)^{\psi-1}W(x)^{1-\psi}dt - \frac{\psi}{1-\psi}\int_0^x tw(t)dt\right]
\end{aligned}$$

Taking the expectation over types as done in previous proof and multiplying by number of bidders in the auction gives us the desired result ■

Proposition 6. *Expected revenue of cheating seller is always larger than the honest seller*

Proof. It is easy to check that $\Gamma^{Cheat} - \Gamma^{Honest}$ is always positive, as $\psi > 1$ ■

Possibility of cheating harms that expected revenue of an honest seller because bidders shade their bids to accommodate for the possibility of cheating in the auction. This result is in line with that (Porter & Shoham, 2005) find, dislike for cheating by bidders does not turn tables in favour of honest sellers.

Proposition 7. *Expected revenue of cheating seller with bidders having reference dependent preferences can be more or less than the case without reference dependence.*

Proof. Without reference dependence ($k = 0$), the. expected revenue to the seller is

$$\mathbf{\Gamma}_{k=0}^{Cheat} = \left[\frac{\xi(1-\alpha\xi)}{(1-\xi)} \cdot \frac{N(N-1)}{(N-1)(1-\xi)+1} \int_0^{\bar{v}} \left[[F(y)]^{(N-1)\xi-1} - [F(y)]^{N-1}\right] y f(y)\, dy \right.$$
$$\left. - \frac{(1-\alpha)N(N-1)\xi}{(1-\xi)} \int_0^{\bar{v}} y \cdot f(y)(1-F(y))F^{N-2}(y) dy \right]$$

where $\xi = \frac{1}{\Phi\alpha}$.

It suffices to show that for a there exists a case in which for a small enough reference point, the expected revenue without reference dependence ($k = 0$) can be more than the case with reference dependence. Suppose signals of bidders are drawn from $U[0,1]$, then $\Gamma_{k=0}^{Cheat} = \frac{\xi(1-\alpha\xi)}{1-\xi} \cdot \frac{N(N-1)}{(N+1)((N-1)\xi+1)} - \frac{\xi(1-\alpha)(N-1)}{(1-\xi)(N+1)}$, now $\Gamma_{k=0} - \Gamma > 0$ if

$$\begin{aligned}
\Longleftrightarrow & \frac{\xi(1-\alpha\xi)}{1-\xi} \cdot \frac{N(N-1)}{(N+1)((N-1)\xi+1)} - \frac{\xi(1-\alpha)(N-1)}{(1-\xi)(N+1)} - \frac{1}{1+k}\left[k\eta \right. \\
& \left. + \frac{\psi(1-\alpha\psi)}{1-\psi} \cdot \frac{N(N-1)}{(N+1)((N-1)\psi+1)} - \frac{\psi(1-\alpha)(N-1)}{(1-\psi)(N+1)}\right] > 0 \\
\Longleftrightarrow & \frac{k\eta}{1+k} < \frac{\xi(1-\alpha\xi)}{1-\xi} \cdot \frac{N(N-1)}{(N+1)((N-1)\xi+1)} - \frac{\xi(1-\alpha)(N-1)}{(1-\xi)(N+1)} \\
& - \frac{1}{1+k}\left[-\frac{\psi(1-\alpha)(N-1)}{(1-\psi)(N+1)} + \frac{\psi(1-\alpha\psi)}{1-\psi} \cdot \frac{N(N-1)}{(N+1)((N-1)\psi+1)}\right] \\
\Longleftrightarrow \eta & < \frac{(1+k)}{k}\left[\frac{\xi(1-\alpha\xi)}{1-\xi} \cdot \frac{N(N-1)}{(N+1)((N-1)\xi+1)} - \frac{\xi(1-\alpha)(N-1)}{(1-\xi)(N+1)} \right. \\
& \left. - \frac{1}{1+k}\left[-\frac{\psi(1-\alpha)(N-1)}{(1-\psi)(N+1)} + \frac{\psi(1-\alpha\psi)}{1-\psi} \cdot \frac{N(N-1)}{(N+1)((N-1)\psi+1)}\right]\right] = \bar{\eta}
\end{aligned}$$

This means for all $\eta < \bar{\eta}$, revenue without reference effects is greater than that with reference point ■

Higher reference points serve as a cushion and increase the effective valuation for them regardless of their intrinsic valuation, therefore for large enough reference points expected revenue with the reference dependence is significantly higher than otherwise, but as bidders dislike cheating, we can expect reference points to be lower than what they would from otherwise in a regular auction so the

EXAMPLE. Suppose there are 10 bidders, let $[k = 0.5, \Phi = 0.5, \alpha = 0.5]$, then its is easy to check if $\eta < \frac{9747}{11396}$; $\Gamma_{k=0} > \Gamma$ and if $\eta > \frac{9747}{11396}$ then $\Gamma_{k=0} < \Gamma$

Proposition 8. *If reference points are small enough then a cheating seller's expected revenue can be less than the revenue in a regular first price auction.*

Proof. It suffices to show that there exists a case where the above proposition holds true, consider a case where signals are drawn from $U[0, 1]$ then expected revenue in a first price auction is given by $R^I = \frac{N-1}{N+1}$, now consider the expression, $R^I - \Gamma^{Cheat}$ this is greater than zero if

$$\frac{N-1}{N+1} - \frac{1}{1+k}\left(k\eta + \frac{\psi(1-\alpha\psi)}{1-\psi} \cdot \frac{N(N-1)}{(N+1)((N-1)\psi+1)} - \frac{\psi(1-\alpha)(N-1)}{(1-\psi)(N+1)}\right) > 0$$

$$\frac{k\eta}{1+k} < \frac{N-1}{N+1} - \frac{1}{1+k}\left(\frac{\psi(1-\alpha\psi)}{1-\psi} \cdot \frac{N(N-1)}{(N+1)((N-1)\psi+1)} - \frac{\psi(1-\alpha)(N-1)}{(1-\psi)(N+1)}\right)$$

$$\eta < \frac{1+k}{k}\left(\frac{N-1}{N+1}\right) - \frac{1}{k}\left(\frac{\psi(1-\alpha\psi)}{1-\psi} \cdot \frac{N(N-1)}{(N+1)((N-1)\psi+1)} - \frac{\psi(1-\alpha)(N-1)}{(1-\psi)(N+1)}\right) = \bar{\eta}$$

■

Now for all $\eta < \bar{\eta}$, the revenue in a regular first price auction is greater than the cheater's revenue. This shows if bidders have a great dislike for cheating(low reference points then cheating sellers are hurt)

EXAMPLE. Suppose there are 10 bidders, let $[k = 0.5, \Phi = 0.5, \alpha = 0.5]$, then its is easy to check if $\eta < \frac{243}{308}$; $R^I > \Gamma^{Cheat}$ and if $\eta > \frac{243}{308}$ then $R^I < \Gamma^{Cheat}$

4 Discussion

This paper extended the literature on cheating in second price auctions by including the dimension of reference dependent preferences. In our knowledge no other work till date has explored the impact of modelling dislike for cheating in second price auctions, we used the idea of reference based utility to model the same, our paper fills that gap. While (Rothkopf & Harstad, 1995) model seller's type as the probability that he will cheat in second price auction whereas in our model we have assumed that seller's type is whether or not he will cheat in a second price auction and bidders have prior over the two possible types of sellers. We modelled cheating on lines of (Porter & Shoham, 2005), but our formulation is different in two aspects, firstly it is more general as when seller cheats winner ends up paying a convex combination of his bid and the highest losing bid, secondly it introduces emotional response towards cheating by seller in the auction where bidders an additional dis-utility if they end up paying anything more than their reference bid . Our results characterise the symmetric increasing equilibrium strategy in the auction, expected revenue to the seller and effects of cheating on the bids in equilibrium. While (Porter & Shoham, 2005) for the case when signals are drawn from $U[0, 1]$ find that honest sellers always lose out to cheating sellers and calculate the extent of loss when $\Phi = 0.5$; However in their case $\alpha = 1$, in this paper we have extended the result for any $(\alpha, \Phi) \in (0, 1)$ and even with bidders disliking cheating, honest seller suffers. We have defined expected revenues to cheating and honest types as defined by (Watanabe & Yamato, 2008b), but their model is with interdependent valuations in a result they find expected revenues of honest seller are less than that of a cheating seller, our model has the same conclusion but with independent values and emotional responses. Mathematically our bidding strategy looks similar to case of first price auction with reference dependent preferences as derived by (Rosenkranz & Schmitz, 2007), bidding function has intercept that indicates effect of reference points and sensitivity of bidders towards the reference dependent utility, there are a few differences however, firstly the probability of winning $[W(x)]$ does not directly come in our bidding function, instead it gets entangled with probability of cheating (Φ) and level of sellers crookedness (α). In contrast to their model, probability of cheating and level of seller's crookedness affect equilibrium bids negatively in our model.

Our model has interesting results, A cheating seller do not always do better than a seller conducting a first price auction, if reference points are small enough then cheating seller's expected revenue is lower than what a seller in a regular first price auction gets. Its feasible that bidders have a low reference point if there is a possibility of being cheated by seller then in such cases cheating sellers are worse off than sellers in a first price auction. Secondly if reference points are small then bidder's dislike for cheating hurt cheating sellers more as compared to cheating sellers dealing with bidders without emotional responses towards cheating. In terms of our example, when signals of bidders are drawn from $U[0,1]$, notice that $\bar{\eta} > \tilde{\eta}$ this means dishonest seller dealing with bidders without reference dependence does better than honest seller for a wider range of reference points. An implication of the model is that honest sellers can afford to signal themselves as honest up to the loss in the expected revenue they suffer due to bidders not being able to distinguish between honest and dishonest ones.

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