Hours, wages, and multipliers

Sztachera, Maciej

Goethe University Frankfurt

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Maciej Sztachera[†]

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Abstract

The quantitative HANK model, incorporating the coordination of hours worked in production, yields an improved empirical fit along two dimensions: a more concentrated steady-state distribution of hours worked and lower marginal propensities to earn (MPEs) with positive but moderate fiscal multipliers for separable preferences. In the model, failing to coordinate work hours with coworkers leads to wage penalties, and labor earnings display decreasing returns to hours. Consequently, households prefer working hours closer to the average and adjust their hours less in response to idiosyncratic shocks than in the standard model. Aggregate shocks increase optimal hours for all employees, and the coordination friction does not bind. The model matches the empirical estimates of the idiosyncratic and aggregate Frisch elasticities.

Keywords: Coordination, fiscal multipliers, HANK trilemma, hours

JEL codes: E24, E62, H31, H32

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[†]Goethe University Frankfurt, *email:* sztachera@econ.uni-frankfurt.de

1. Introduction

I establish that introducing coordinated work schedules (Yurdagul (2017)) into the canonical HANK model provides a better fit to the empirical evidence about consumption, labor earnings, and output dynamics. The coordination of hours worked in production can account for two facts about labor supply: most employees work around 40 hours a week, and the annual marginal propensities to earn (MPEs) are close to zero. A higher concentration of hours brings the model closer to the evidence in Bick et al. (2022). The model generates lower MPEs without increasing fiscal multipliers, offering a significantly improved fit to the targets set in the HANK Trilemma of Auclert et al. (2023).

The improvement derives from the way coordination affects labor earnings. Firms want to coordinate hours worked because modern production processes involving assembly lines, teamwork, and business-to-business interactions require simultaneous execution of multiple tasks. In the model, firms encourage coordination by conditioning wages on hours worked and pay the highest hourly wage at endogenously determined *optimal hours*. Deviations from *optimal hours* result in wage penalties, and the labor earnings display decreasing returns to hours. As a result, households find it optimal to supply hours closer to the mean than in the model without coordination, leading to a more concentrated distribution of hours worked. When households increase (decrease) hours worked, the marginal earnings decrease (increase), making households react less to idiosyncratic shocks. Household labor supply becomes less responsive to wealth shocks, leading to lower MPEs.

Moreover, *optimal hours* depend positively on all employees' hours worked, and individual marginal earnings depend positively on *optimal hours*. Since aggregate shocks affect all workers symmetrically and optimal hours adjust in the same direction, the coordination friction does not bind. In this way, coordination drives a wedge between the Frisch elasticity of hours to idiosyncratic and aggregate shocks. I target the Frisch elasticity of hours to aggregate shocks of 0.5, as in Battisti et al. (2024), and consider a range of values for the idiosyncratic Frisch elasticity.

The Frisch elasticity to idiosyncratic wage shocks declines as coordination in hours increases. Consequently, the steady-state distribution of hours worked becomes significantly more concentrated around optimal hours, bringing the model closer to the data (Bick et al. (2022)). Moreover, coordination makes agents adjust their hours *less* in response to idiosyncratic wealth shocks: the MPEs decline, bringing the model closer to the empirical targets of Auclert et al. (2023).

Empirical evidence on coordination of hours worked in production. The empirical literature provides ample suggestive evidence that firms coordinate hours in production. For example, Cubas et al. (2021) and Cubas et al. (2022) found that mothers earn lower hourly wages when they leave work at peak hours to spend time with their children. The wage penalties are higher in occupations with more coordinated work schedules. Similarly, Bick et al. (2022) find a robust hump-shaped relationship between hours and wages. They argue that coordination in hours worked explains the empirical relationship between hours and wages and provides a unifying explanation for bunching at mean hours across countries.

Coordination of hours worked in production provides a microfounded explanation for the prevalence of full-time contracts with uniform hours, such as 37.5 hours a week in Denmark. When firms coordinate work schedules, these contracts lower the dispersion of hours worked and, thus, improve productivity. Firms have incentives to impose uniform hours across workers regardless of whether the determination of hours worked is more decentralized, as in the US, or collectively bargained, as in Europe. Moreover, the optimum level of hours is an endogenous object that depends on the choices of all workers. Differences in productivity, wealth, and preferences can generate differences in optimal hours. Thus, coordination of hours worked can explain the bunching of usual weekly hours worked at the mean across countries.

The results documented by Bick et al. (2022) confirm and expand the previous empirical findings of positive returns to hours for part-time workers in Aaronson and French (2004) and decreasing returns to hours for long shifts in Pencavel (2015). Kuhn et al. (2023) find that the German firms that are more coordinated are more productive and pay higher wages but were also more vulnerable to worker absences following the pandemic shock. Yurdagul (2017) was the first to study a production function that features coordination of hours worked. In his model, the preference for flexible hours is a strong driver of entrepreneurship. The salaried workers produce in a coordinated manner, but the entrepreneurs do not. Individuals who prefer flexible hours self-select into the entrepreneurial sector. This finding is consistent with the relatively low income of entrepreneurs in the US data. I use his production function in the intermediate goods' production in the HANK model. Shao et al. (2023) estimated the Yurdagul (2017) production function using the Canadian firm-level data and found that individual hours worked are gross complements in production. Battisti et al. (2024) estimates a structural labor supply model with coordination in production using a matched employer-employee dataset from Italy. He finds that the Frisch elasticity of hours to firm-level shocks is 0.5, and the Frisch elasticity to idiosyncratic shock is substantially lower because firms coordinate hours strongly. These findings support my calibration strategy.

Related literature. Auclert et al. (2023) find that the sticky price HANK models with frictionless labor supply fail to simultaneously deliver the MPC, MPE, and fiscal multipliers consistent with empirical evidence: in the data the MPCs are high (Johnson et al. (2006), Jappelli and Pistaferri (2010), Fagereng et al. (2021)), annual MPEs are low (Cesarini et al. (2017), Golosov et al. (2023), Bartik et al. (2024)), and fiscal multipliers are positive but moderately sized (Ramey and Zubairy (2018), Ramey (2019)). Their solution to the puzzle relies on the assumption that unions impose uniform hours across workers. In a related paper, Gerke et al. (2024) relax the assumption that unions impose uniform hours across workers in the sticky wage model and show that the model with heterogeneous hours features even more attenuated impulse responses than a baseline sticky wage model. Ferraro and Valaitis (2024) provide an elegant solution to the wealth-hours puzzle. They argue that a data-consistent quality choice in consumption explains the low wealth sensitivity of hours worked.

In this paper, I study a sticky price model with flexible labor supply and coordinated work schedules. I use the model and methodology of Auclert et al. (2023), which relies on the Sequence-Space Jacobian algorithm introduced in Auclert et al. (2021) to efficiently solve for the first-order perfect foresight transitions of the aggregate variables. I show that the HANK-coordination model relaxes the MPE-fiscal multiplier tradeoff.

Road map. The remainder of the paper has the following structure. Section 2 derives an analytical relationship between the MPC and MPE in the partial equilibrium heterogeneous agent model extending the Proposition 1 of Auclert et al. (2023). Section 3 outlines the quantitative HANK model that extends the model of Auclert et al. (2023) to incorporate the coordination of hours worked in production. Section 4 presents the model calibration. Section 5 shows that coordination of hours worked in production substantially reduces the MPEs without raising the fiscal multipliers. Section 6 concludes.

2. The MPC-MPE relationship with non-linear labor earnings

In this section, I study a partial equilibrium heterogeneous agent model with frictionless labor supply, as in Aiyagari and McGrattan (1998), and Auclert et al. (2023), extended to nonlinear wage-hours dependence. I generalize Proposition 1 in Auclert et al. (2023) by allowing the hourly wage rate to depend on hours worked and show that decreasing returns to hours lead to lower idiosyncratic Frisch elasticities and lower MPEs.

Households maximize the discounted sum of expected utility flows from consumption and labor. They experience idiosyncratic labor productivity shocks x that follow a stationary Markov process, face a borrowing constraint, and solve the following Bellman equation.

$$V(a,x) = \max_{c,n,a'} U(c,n) + \beta \mathbb{E}\left[V(a',x') \mid x\right] \tag{1}$$

$$\text{s.t.} \quad c + a' = \tilde{w} x f(n) n + T + (1+r)a$$

$$a' \geq \underline{a}$$

To account for the empirical evidence on the hump-shaped relationship between hours worked and wages (Bick et al. (2022)), I allow the hourly wage $w(n) = \tilde{w} x f(n)$ to depend on hours worked through function $f(n)$. $\tilde{w}$ is a wage-level shifter that does not depend on hours worked. The literature usually assumes that income from labor is a linear function of the wage $Y^{labor} \equiv y(n) = \tilde{w} x f(n) n = \tilde{w} x n$, and the hourly wage does not depend on hours worked: $f(n) = 1$ and $f'(n) = 0$. The hump-shaped wage-hours relationship documented in the data (Bick et al. (2022)) corresponds to $f'(n) > 0$ for short hours and $f'(n) < 0$ for long hours worked. This specification nests the coordination of hours worked in production, which I will discuss in the next section.

I assume that the first-order conditions are necessary and sufficient.

$$U_c(c,n) = \lambda \tag{2}$$

$$U_n(c,n) = -\lambda \tilde{w} x \underbrace{\left(f(n) + f'(n) n\right)}_{=y'(n)} \tag{3}$$

The marginal benefit of working extra time depends on how labor earnings react to changes in hours. In the baseline model, marginal earnings $\tilde{w} x y'(n)$ are equal to the wage rate $w \equiv \tilde{w} x$. With the coordination of hours worked in production, workers face positive but decreasing returns to hours ($y'(n) > 0$, and $y''(n) < 0$).

The Frisch elasticity measures the sensitivity of hours to the wage level shifter $w = \tilde{w} x$, holding wealth constant. In this section, I only consider partial equilibrium effects and do not allow for aggregate shocks. Therefore, I will refer to the following measure as idiosyncratic Frisch elasticity. Applying the implicit function theorem to the optimal consumption-labor choice yields the following formula for the idiosyncratic Frisch elasticity of labor supply (see Appendix A.1).

$$\text{Frisch} \equiv \frac{\partial \log n(\lambda, w)}{\partial \log w} = \left(\frac{U_{nn} n}{U_n} - \frac{y''(n) n}{y'(n)} \right)^{-1} \tag{4}$$

The first term $\frac{U_{nn}n}{U_n}$ is equal to the elasticity of marginal disutility from labor to hours worked. The higher the curvature of disutility from labor, the less reactive households are to wage shocks. For the standard separable preferences $U(c,n) = \frac{c^{1-\sigma}-1}{1-\sigma} - \varphi\frac{n^{1+\nu}}{1+\nu} \implies \frac{U_{nn}n}{U_n} = \nu$. The second term $\frac{y''(n)n}{y'(n)}$ is new compared to the baseline model of endogenous labor supply, where marginal earnings are equal to the wage rate $y'(n) = w$ and do not change with hours worked $y''(n) = 0$. The term $\frac{y''(n)n}{y'(n)}$ equals the elasticity of marginal earnings to hours worked and measures whether there are increasing or decreasing returns to hours. With decreasing returns to hours, workers react less to idiosyncratic wage shocks.

In the next step, I extend proposition 1 in Auclert et al. (2023) that determines the partial equilibrium relationship between the MPC and MPE. In addition to the idiosyncratic Frisch elasticity, the MPE-MPC ratio depends on the elasticity of intertemporal substitution in consumption and the complementarity index between consumption and labor. The elasticity of intertemporal substitution is equal to the elasticity of consumption to changes in the marginal utility of wealth.

$$\text{EIS} \equiv -\frac{\partial \log c(\lambda, w)}{\partial \log \lambda}$$

The elasticity of intertemporal substitution regulates how quickly the marginal utility of consumption decreases in wealth. The lower the EIS, the faster the marginal utility of consumption declines with wealth, making leisure more attractive to households. They reduce their labor earnings more in response to unexpected wealth shocks, which results in a higher MPE.

The complementarity index measures the marginal propensity to consume out of wealth-compensated changes in the labor income.

$$\text{CI} \equiv \frac{\partial c(\lambda, w)}{\partial w} \Bigg/ \tilde{w}x\left(f(n) + f'(n)n\right)\frac{\partial n(\lambda, w)}{\partial w}$$

Lemma 1 from Auclert et al. (2023) holds (see Appendix B.1) and $\text{CI} = \frac{U_{cn}U_c}{U_{cc}U_n}$. Therefore, CI measures the degree of complementarity between consumption and labor. A preference for more positive comovement between consumption and labor reduces the wealth effect on labor supply, yielding a lower MPE.

This logic also holds in the extended version of Proposition 1 in Auclert et al. (2023).

Proposition 1. *For any individual in the state (a, x) at time t:*

$$\frac{MPE}{MPC} = \frac{\tilde{w}xf(n)n\left(1+\epsilon_{w,n}\right)}{c}\frac{Frisch}{EIS}(1 - CI) \tag{5}$$

where $\text{MPC} \equiv \frac{\partial c(a,w;T)}{\partial T}$, $\text{MPE} \equiv -\tilde{w}x\left(f(n) + f'(n)n\right)\frac{\partial n(a,w;T)}{\partial T}$ and $\epsilon_{w,n} \equiv \frac{f'(n)n}{f(n)}$ (proof in Appendix B.2). Low Frisch elasticity to idiosyncratic shocks is the best explanation for low

MPEs observed in the data. Households typically do not consume more than their income: $c < \tilde{w}xf(n)n$, and there is a broad consensus in the empirical literature that EIS is 0.5. Most employees work around mean hours, and $1 + \epsilon_{w,n}$ is close to one. In the next section, I show that high complementarity between consumption and labor leads to high fiscal multipliers, as in Auclert et al. (2023). Thus, these variables are unlikely to solve the puzzle. However, a mechanism that generates very low Frisch elasticities to idiosyncratic shocks and higher elasticities to aggregate shocks can explain low MPEs while respecting the empirical evidence about labor supply elasticities.

In the next section, I present a quantitative HANK model that features the coordination of hours worked in production, which endogenously generates a hump-shaped wage-hours schedule and the decreasing returns to hours $y''(n) < 0$. Consistent with Proposition 1, decreasing returns lead to lower idiosyncratic Frisch elasticities and lower MPEs, bringing the quantitative model closer to empirical targets. At the same time, the HANK-coordination model replicates the empirical aggregate Frisch elasticity of 0.5 as in Battisti et al. (2024).

3. HANK Model with the Hours' Coordination in Production

This section presents the quantitative sticky-price HANK model of Auclert et al. (2023) extended to include the coordination of hours worked in the intermediate goods' production. I find that empirically plausible levels of coordination in production generate a high concentration of hours worked around the mean and a significant decline in the MPE.

3.1. The Economic Environment

Firms. There are two types of firms: the final goods and intermediate goods producers. The final goods producers are standard and aggregate the intermediate varieties with the CES technology. Intermediate goods firms are monopolistically competitive and produce according to the Yurdagul (2017) production function. The production function is a CES aggregator over individual hours worked, which can capture the coordination of hours worked in production.

$$y(i) = z\left(\int_{x\in B_x}\int_0^\infty x\mu^i(n,x)n^\rho dndx\right)^{\frac{1}{\rho}}\left(\int_{x\in B_x}\int_0^\infty x\mu^i(n,x)dndx\right)^{1-\frac{1}{\rho}} - F \qquad (6)$$

The notation is the following: z is the aggregate productivity, $\mu^i(n,x)$ is the mass of agents working n hours with idiosyncratic productivity x, $\rho = \frac{\sigma-1}{\sigma}$ is a parameter that regulates the degree of coordination in hours and σ is the elasticity of substitution between individual hours worked. For $\rho = 1$, $\sigma \to \infty$, and the individual hours worked are perfect substitutes in production, a common assumption in the macro literature. As ρ declines, coordination in hours increases. For $\rho > 0$, hours are gross substitutes. For $\rho < 0$, hours are gross complements. As the parameter $\rho \to -\infty$ approaches negative infinity, hours worked become perfect complements. The available empirical evidence suggests values close to 0 but negative (Shao et al. (2023), Battisti et al. (2024)). However, even a mild complementarity of hours in production is sufficient to generate substantial wage penalties for deviating from optimal hours.

Firms choose the masses of workers $\mu^i(n, x)$ that they want to employ to minimize the total cost of producing a given quantity subject to the production function in equation (6). Solving the problem yields the following equilibrium wage function.

$$w(n, x) = \tilde{w}x\mathbb{E}\left[n^\rho\right]^{\frac{1}{\rho}}\left(\frac{n^{\rho-1}}{\rho\mathbb{E}\left[n^\rho\right]} + \frac{1}{n}\left(1 - \frac{1}{\rho}\right)\right) \tag{7}$$

where $\tilde{w} \equiv mc \cdot z$ and the expectation is over the distribution of individual labor inputs (see Appendix C.4 for more details). The expectation term is equal to the *power mean of hours* due to the CES specification.[1]

$$L \equiv \mathbb{E}[n^\rho]^{\frac{1}{\rho}} = \left(\int_{x\in B_x}\int_0^\infty n^\rho \frac{x\mu(n, x)}{\int_{x\in B_x}\int_0^\infty x\mu(n, x)dndx}dndx\right)^{\frac{1}{\rho}}$$

The remainder of the firm problem is standard and follows closely Auclert et al. (2023). Solving the Rotemberg pricing problem leads to a standard New Keynesian Philips Curve (see Appendix C.5 for a complete derivation).

$$\log(1 + \pi_t) = \kappa_p\left(mc_t - \frac{\varepsilon_p - 1}{\varepsilon_p}\right) + \frac{1}{1 + r_t^e}\frac{Y_{t+1}}{Y_t}\log\left(1 + \pi_{t+1}\right)$$

Aggregate firm dividends equal the total revenue minus the total wage bill and the price adjustment cost.

$$d_t = Y_t - \int_{x\in B_x}\int_0^\infty w(n, x)n\mu(n, x)dndx - \frac{\varepsilon_p}{2\kappa}\log\left(1 + \pi_t\right)^2 Y_t \tag{8}$$

where $\int_{x\in B_x}\int_0^\infty w(n, x)n\mu(n, x)dndx = \tilde{w}L$ (details in Appendix C.7).

Households choose their optimal consumption c, hours n, and savings a'. The utility function is GHH-Plus (Auclert et al. (2023)), nesting CRRA and GHH utility functions, allowing for

[1]Power means are increasing functions of the parameter ρ. In economic terms, this means that whenever there is dispersion in hours, the stronger the degree of complementarity in production, the lower the optimal hours.

an arbitrary complementarity between consumption and labor. They face a budget constraint, which includes the idiosyncratic risk term x and the equilibrium wage function derived from the intermediate firms' problem.

$$V_t(a,e) = \max_{c,n,a'} \frac{1}{1-\sigma}\left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu}\right)^{1-\sigma} - \varphi(1-\alpha)\frac{n^{1+\nu}}{1+\nu} + \beta\mathbb{E}\left[V_{t+1}(a',x') \mid x\right] \tag{9}$$

$$\text{s.t.} \quad c + a' = (1-\tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}}\left(\frac{n^\rho}{\rho\mathbb{E}[n^\rho]} + 1 - \frac{1}{\rho}\right) + T + (1+r)a$$

$$a' \geq \underline{a}$$

The choice of assets is subject to the borrowing constraint $a' > \underline{a}$. Households choose their hours worked, and there is no extensive margin. Therefore, households desiring to work very few hours might face negative labor earnings. In the calibrated model, the fiscal transfers T are sufficiently high to ensure that households have positive consumption, and the problem only affects a small fraction of agents.

$$U(c,n) = \frac{1}{1-\sigma}\left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu}\right)^{1-\sigma} - \varphi(1-\alpha)\frac{n^{1+\nu}}{1+\nu}$$

GHH-Plus preferences make it easy to calibrate the degree of complementarity between consumption and labor measured through the complementarity index $\text{CI} = \frac{\alpha U_c}{\alpha U_c + 1 - \alpha}$ by choosing the parameter α. The complementarity index directly affects the strength of the wealth effect on labor supply and increases the propagation of government shocks to consumption, as in Auclert et al. (2023).

The first-order conditions of the household problem are the following.

$$\left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu}\right)^{-\sigma} = \lambda \tag{10}$$

$$-\varphi n^\nu\left(\alpha\left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu}\right)^{-\sigma} + (1-\alpha)\right) + \lambda(1-\tau^w)\tilde{w}x\left(\frac{\mathbb{E}[n^\rho]^{\frac{1}{\rho}}}{n}\right)^{1-\rho} = 0 \tag{11}$$

$$\beta\mathbb{E}\left[V_a(a',x') \mid x\right] = \lambda \tag{12}$$

The envelope condition (derivative of the value function w.r.t. a).

$$V_a(a,x) = (1+r)\lambda \tag{13}$$

I follow Auclert et al. (2023) and solve the household problem using the endogenous gridpoints method of Carroll (2006) (details in Appendix C.1).

Asset market. There are two types of assets: bonds and equity. Both assets are perfectly liquid, and under certainty equivalence, both assets offer the same ex-ante rate of return.

$$1 + r_t^e = \mathbb{E}_t \left[\frac{d_{t+1} + p_{t+1}}{p_t} \right]$$

Bhandari et al. (2023) point out that the first-order MIT shock approach implicates certainty equivalence, and the optimal portfolio choice is indeterminate. Following Auclert et al. (2023), I assume that all households hold the same portfolio of equity and bonds. Thus, they solve their savings problem in terms of the whole portfolio of assets a', and the ex-post rate of return on the portfolio is equal to the weighted average of the realized rates of return to each asset.

$$1 + r_t = \underbrace{\frac{p_{t-1}}{p_{t-1} + B_{t-1}}}_{\text{equity share}} \frac{d_t + p_t}{p_{t-1}} + \underbrace{\frac{B_{t-1}}{p_{t-1} + B_{t-1}}}_{\text{bond share}} \left(1 + r_{t-1}^e\right)$$

Government. Linear labor taxation is the only source of government revenue. Debt issuance covers the remaining financing needs. Transfers T are rebated lump-sum and are constant in time. There is discretionary government spending G_t.

$$B_t + \tau_t^w \tilde{w}_t L_t = (1 + r_{t-1}^e) B_{t-1} + G_t + T$$

The deficit financing rule states that whenever government spending increases above its steady-state value, the fraction ρ_B of the extra spending is initially covered by debt issuance. In calibration, I set $\rho_B = 0.9$ as in Auclert et al. (2023). Thus, higher deficits cover most of the initial increase in spending, and debt is very persistent.

$$B_t - B_{ss} = \rho_B \left(B_{t-1} - B_{ss} + G_t - G_{ss}\right)$$

The tax rate τ_t^w varies in time, ensuring that the government budget holds even though T is constant, and the deficit financing rule pins down the debt level in each period. The variations in tax rates τ^w have important implications for the propagation of the government spending shock. The shock is typically expansionary, and total labor earnings $\tilde{w}_t L_t$ increase markedly. When higher debt covers most of the extra spending, the tax rate τ_t^w drops, equilibrating the government budget. A lower tax rate provides extra stimulus to the economy.

Equilibrium. An equilibrium in this economy consists of a set of decision paths for households $\{c_t, n_t, a_{t+1}\}_{t=0}^{\infty}$ and firms $\left\{\mu_t^i(n,x)\right\}_{t=0}^{\infty}$, the equilibrium wages $\{w_t(n,x)\}_{t=0}^{\infty}$ for each worker type (n,x), the rate of return on bonds $\{r_t^e\}_{t=0}^{\infty}$, equity price $\{p_t\}_{t=0}^{\infty}$, the fiscal variables $\{\tau_t^w, G_t, B_t, T_t\}_{t=0}^{\infty}$ and the distribution of households over assets and labor market states

$\{D_t(a,x)\}_{t=0}^{\infty}$ such that for every t:

(i) The policy functions of households and firms maximize their objective functions subject to the respective resource constraints, taking wage rates $\{w_t\}_{t=0}^{\infty} = \{\tilde{w}_t x_t\}_{t=0}^{\infty}$, equity prices, taxes, and transfers as given.

(ii) The household distributions are consistent with the individual policy functions.

(iii) The government budget constraint (3.1) holds in all periods.

(iv) The asset, labor, and goods markets clear for all t. In particular, the optimal hours L_t^d that households take as given are consistent with optimal hours implied by their choices L_t^s.

$$\mathcal{A}_t^d = \int_{x\in B_x}\int_{a\in B_a} a_{t+1}(a,x)D_t(a,x)dadx = \mathcal{A}_t^s = B_t + p_t \tag{14}$$

$$\underset{n,x}{\forall} \quad \mu_t(n,x) = \int_{a\in B_a} \mathbb{1}\left\{n(a,x)=n\right\} D_t(a,x)da \tag{15}$$

$$L_t^d = L_t^s = \left(\int_{x\in B_x}\int_{a\in B_a} x n_t(a,x)^{\rho} D_t(a,x)dadx\right)^{\frac{1}{\rho}} \tag{16}$$

$$Y_t^d = C_t + G_t + \frac{\varepsilon_p}{2\kappa_p}\log\left(1+\pi_t\right)^2 Y_t = Y_t^s = zL_t - F \tag{17}$$

3.2. Coordination and Hours Worked

Coordination of hours worked in production drives a wedge between the idiosyncratic and aggregate Frisch elasticities. The model generates low idiosyncratic Frisch elasticities necessary to explain low MPEs in the data while allowing for non-negligible aggregate and firm-level Frisch elasticities consistent with the empirical estimates.

The equilibrium wage function 7 delivers the hump-shaped relationship between wages and hours described in Bick et al. (2022). In the model, the maximum hourly wage is at optimal hours worked $L = \mathbb{E}[n^{\rho}]^{\frac{1}{\rho}}$, and there is a substantial wage penalty for deviating from the optimal hours when $\rho < 1$. The penalty is high for workers who wish to work fewer than optimal hours, and it is higher when there is more coordination in hours worked (parameter ρ is lower). Figure 1 shows that the wage function in the model is qualitatively similar to the cross-sectional estimates in the data.

The introduction of wage-hours dependence critically affects the optimal consumption-labor choice 11 through the multiplicative term $\frac{\mathbb{E}[n^{\rho}]^{\frac{1}{\rho}}}{n}$. In particular, the marginal benefit of working extra time depends positively on the ratio of optimal hours and individual hours worked $\frac{\mathbb{E}[n^{\rho}]^{\frac{1}{\rho}}}{n}$. The marginal benefit of increasing hours worked depends positively on firm-level optimal hours $\mathbb{E}[n^{\rho}]^{\frac{1}{\rho}}$. Coordination to higher hours increases all workers' incentives to work. On the other hand, the marginal benefit of working more decreases with hours worked n: for a given level of optimal hours, workers face diminishing returns to hours worked. Figure 2 provides a graphical explanation: conditional on optimal hours L, the slope of the earnings function declines with hours; conditional on hours worked n, the slope increases with optimal hours L.

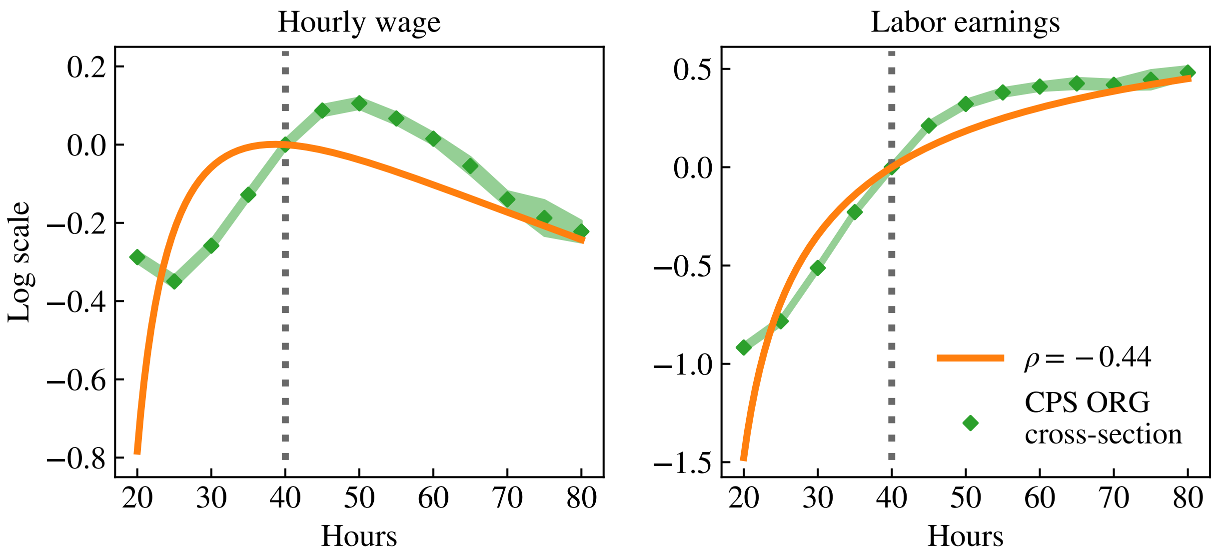

Figure 1: Wages and earnings in the model with $\rho = -0.44$, and estimated in CPS ORG August 1995-September 2007. Log hourly wages and labor earnings are scaled to 0 at 40 hours a week.

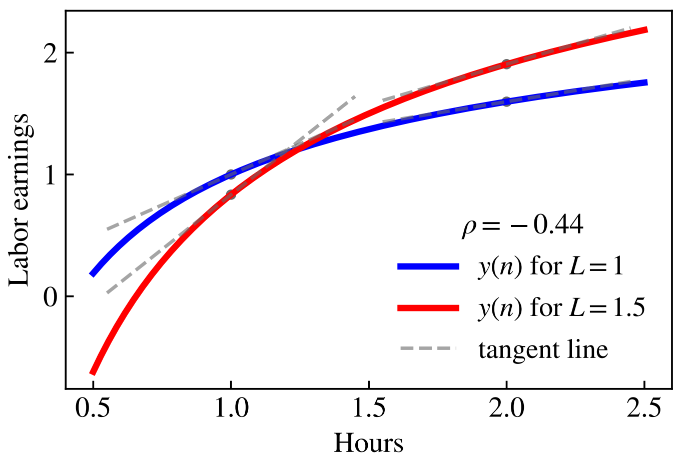

Figure 2: Marginal earnings in partial and general equilibrium: an illustrative plot.

Consequently, in the model with coordination, there are two distinct types of Frisch elasticity of labor supply: idiosyncratic and aggregate Frisch elasticity. Workers who draw an idiosyncratic shock recognize that other workers are unaffected and optimal hours stay constant. Therefore, on average, the stronger the coordination friction, the less workers adjust their hours to idiosyncratic shocks. The following expression determines the elasticity of hours to idiosyncratic shocks:

$$\left.\frac{\partial \log n}{\partial \log \tilde{w}}\right|_{\lambda=\bar{\lambda}} = \frac{1}{1-\rho+\nu} \tag{18}$$

If I keep ν constant and increase coordination in hours worked by decreasing the parameter ρ, the Frisch elasticity of labor supply to idiosyncratic shocks drops. Due to technological constraints, the preference-based willingness to substitute labor intertemporally is higher than the actual elasticity. The idiosyncratic Frisch elasticity with coordination in equation 18 is a special case of the general idiosyncratic Frisch elasticity in equation 4 derived in Section 2. In particular, the wage function 7 implies decreasing return to hours with $1-\rho$ measuring the elasticity of marginal earnings to hours.

$$\frac{y''(n)n}{y'(n)} = 1-\rho$$

On balance, the aggregate shocks affect all workers, and, as a result, optimal hours change in the same direction as individual choices. Therefore, the coordination friction does not affect the aggregate Frisch elasticity. The elasticity of hours to the aggregate productivity z and the taxation rate $1-\tau^w$ is solely governed by the curvature parameter ν:

$$\left.\frac{\partial \log n}{\partial \log z}\right|_{\lambda=\bar{\lambda}} = \left.\frac{\partial \log n}{\partial \log\left(1-\tau^w\right)}\right|_{\lambda=\bar{\lambda}} = \frac{1}{\nu} \tag{19}$$

All workers in the firm are affected symmetrically by aggregate shocks, optimal hours adjust in proportion to individual choices, and no coordination difficulties arise. The complete derivation in the spirit of Battisti et al. (2024) is available in Appendix A.2. Conversely, when a household experiences an idiosyncratic shock, it has to coordinate hours with unaffected households. Failing to do so would result in wage penalties. Therefore, the aggregate Frisch elasticity is higher than the idiosyncratic Frisch elasticity.

Divergent micro and macro elasticities of labor supply (Chetty et al. (2011)) have typically been reconciled by introducing extensive margins into the macro models (Keane and Rogerson (2015)). Coordination of hours worked in production introduces a wedge between the micro and macro labor supply elasticities in a continuous labor supply choice framework. Low idiosyncratic and higher aggregate and firm-level Frisch elasticities are consistent with the micro estimates (Battisti et al. (2024)) and help the model match other data moments, such as the MPE and distribution of hours worked, significantly better than the baseline sticky price HANK model without coordination.

4. Calibration

Calibration follows Auclert et al. (2023) and McKay et al. (2016). The model features no illiquid asset and thus belongs to the class of one-asset HANK models. To generate sufficiently high MPCs, I follow Auclert et al. (2023) and introduce discount factor heterogeneity. Half of the agents have a high, and the other half have a low discount factor. I calibrate the upper discount factor to match the aggregate supply of assets equal to 1.4 times the annual GDP (McKay et al. (2016)) and the lower discount factor to match the MPC of 0.25 (Johnson et al. (2006)). I calibrate the elasticity of substitution between the intermediate inputs to $\varepsilon_p = 7$, which implies a steady-state markup of $\mu = 1.1667$. The appropriate choice of fixed cost in production ensures that firm dividends yield the equity value of 0.85 times the annual GDP (Auclert et al. (2023)). I set the steady-state bond supply to 0.55 times the annual GDP (Auclert and Rognlie (2020)). I calibrate the disutility of labor parameter φ so that optimal hours $\mathbb{E}[n^\rho]^{\frac{1}{\rho}}$ on the household side are equal to the labor demand $L^d = 1$, and the labor market clears. Similarly, I choose the aggregate productivity z to normalize the steady-state output $Y = 1$. I summarize the calibration in Table 1; the subsequent sections describe the main aspects of the calibration in greater detail.

Production and dividends

Adjusting the fixed cost in production is a convenient tool to target equity value without compromising the markup calibration. In particular, steady-state firm dividends satisfy the following formula.

$$d = zL - \underbrace{\frac{z}{\mu}}_{\tilde{w}} L - F = \left(1 - \frac{1}{\mu}\right) zL - F$$

The per-period return to all equity holdings equals total dividends by no arbitrage.

$$pr = d = \left(1 - \frac{1}{\mu}\right) zL - F \tag{20}$$

Steady-state calibration sets $r = 0.005$, $\mu = 1.1667$, the scaling of hours L is pinned down by φ. The remaining two parameters, z and F, are responsible for output scaling and the return to equity. Substituting for the fixed cost from the production function $F = zL - 1$ (in steady state $Y = 1$) into the equation (20) yields a formula for z.

$$z = \frac{\mu\,(1 - pr)}{L}$$

The Frisch elasticities of labor supply

Battisti et al. (2024) estimates a structural model of labor supply with coordination in production using the matched employer-employee data from Italy. He finds that the curvature of disutility function parameter $\nu^{-1} = 0.483$, and the coordination in production parameter $\rho = -1.962$. I set $\nu = 2$ and consider a range of values for the parameter ρ. The baseline value is the estimate of Shao et al. (2023), who find $\rho = -0.44$ for the aggregate economy using the Canadian matched employer-employee dataset. The Frisch elasticity of labor supply to idiosyncratic wage shocks depends on the disutility of labor parameter ν and the coordination parameter ρ in equation (18).

Income process

I assume that idiosyncratic productivity x follows an AR(1) process with persistence ρ_x.

$$\log x' = \rho_x \log x + \varepsilon_x$$

I discretize the process as a Markov chain using the Rouwenhorst method. I target the persistence parameter $\rho_{\log x} = 0.966$ and choose the variance of idiosyncratic productivity $\sigma^2_{\log x} = \frac{\sigma^2_{\varepsilon x}}{1-\rho^2_x}$ to target a cross-sectional standard deviation of log labor earnings of 0.92 (Song et al. (2018)). I report the calibrated values of $\sigma_{\log x}$ in Appendix C.8. Coordination of hours worked leads to slightly lower $\sigma_{\log x}$ for separable preferences and to moderately higher $\sigma_{\log x}$ for nonseparable preferences. Coordination affects the dispersion in labor earnings in two opposite ways: it makes hours worked more concentrated and introduces wage penalties that can increase wage dispersion when there are more part-time than overtime workers, which is the case with separable preferences.

I adjust the calibration of the variance of the productivity process to account for the coordination of hours worked. In the standard linear wage case, the labor earnings equal $Y^{pretax}_{labor} = \tilde{w}xn$. However, in the model with coordination in hours, the following formula for the variance of log earnings applies.

$$\text{Var}\left[\log(Y^{pretax}_{labor})\right] = \text{Var}\left[\log\left(\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}}\left(\frac{n^\rho}{\rho\mathbb{E}[n^\rho]} + 1 - \frac{1}{\rho}\right)\right)\right]$$

When firms coordinate hours strongly, there are substantial penalties for deviating from optimal hours. Therefore, in rare cases, households with high wealth or low productivity work very few hours and thus receive negative labor earnings. Negative values are problematic for calibrating the variance of log labor earnings. Therefore, I calibrate the variance of log labor earnings only using households with positive labor earnings: Var $\left[\log(Y^{pretax}_{labor})|Y^{pretax}_{labor} > 0\right]$. In Appendix C.9, I report that fewer than 0.4% of households report negative labor earnings for all parametrizations. Therefore, negative labor earnings are unlikely to affect the results materially.

Parameter	Name	Value/Target	Source
ν	labor curvature	$\frac{\partial \log n}{\partial \log z}\|_{\lambda=\bar{\lambda}} = 0.5$	Battisti et al. (2024)
τ^w	wage tax rate	0.334	Auclert and Rognlie (2020)
T	transfer	$0.143 \cdot \tilde{w}L$	Auclert and Rognlie (2020)
ε_p	elasticity of substitution	7	Auclert et al. (2023)
κ_p	NKPC slope	0.01	Hazell et al. (2022)
B	government bonds	$0.55 \cdot 4Y$	Auclert et al. (2023)
E	steady-state equity value	$0.85 \cdot 4Y$	Auclert et al. (2023)
ρ_B	persistence of public debt	0.9	Auclert et al. (2023)
ρ_x	income shock persistence	0.966	McKay et al. (2016)
$\underline{a}$	borrowing constraint	0	
β_1	upper discount factor	$A = 5.6$	Auclert et al. (2023)
β_2	lower discount factor	MPC = 0.25	Johnson et al. (2006)
$1/\sigma$	U_c curvature	average EIS = 0.5	Havránek (2015)
φ	disutility of labor	$L = 1$	
Z	aggregate productivity	$Y = 1$	
σ_x	std of income shocks	$\text{Var}[\log(Y_{labor})] = 0.92^2$	Song et al. (2018)
F	fixed cost	$p = 0.85 \cdot 4Y$	McKay et al. (2016)

Table 1: Fixed and Calibrated Parameters

Government

I set $T = 0.143 \cdot Y^{labor}_{pretax}$, as in Auclert and Rognlie (2020), to match the empirical relationship between the pretax and post-tax-and-transfers labor income. I set the labor income tax to $\tau^w = 0.33$, as in Auclert et al. (2023). The labor tax rate affects the labor wedge and, thus, the fiscal multipliers with the GHH preferences. Therefore, for comparability, I keep this parameter unchanged. The calibration of the government implies the transfers to GDP ratio $\frac{T}{Y} = .140569$, and the government spending to GDP ratio $\frac{G}{Y} = .176753$. As Auclert et al. (2023), I calibrate the slope of the Philips curve $\kappa_p = 0.01$ in line with the recent empirical evidence (Hazell et al. (2022)).

Other calibration targets

I delegate the remaining calibration adjustments to the Appendix. In particular, in Appendix C.3, I show how the calculation of the elasticity of intertemporal substitution is affected by wage-hours dependence. Appendix C.5 outlines the procedure to calculate the model MPE and how it corresponds to empirical evidence. In Appendix C.7, I show that the firm total wage bill used to calculate dividends and tax revenues equals a product of the economy-wide wage rate and optimal hours $\tilde{w}L$. The remainder of the calibration is the same as in Auclert et al. (2023).

5. The HANK Trilemma with the Coordination of Hours Worked

Coordination of hours worked in production lowers MPEs without raising the fiscal multipliers. I use the HANK model with coordinated wage schedules described and calibrated in the previous two sections to repeat the exercise of Auclert et al. (2023). I investigate how changing the degree of coordination of hours worked in production affects the MPE-fiscal multiplier tradeoff. The connected blue dots in Figure 3 correspond to the baseline model of Auclert et al. (2023). Higher values of the complementarity index between consumption and labor deliver lower MPEs at the expense of unrealistically high fiscal multipliers. However, as I increase ρ and, thus, decrease the elasticity of substitution between individual hours worked in production, the MPEs decline substantially. Graphically, the improvement corresponds to a leftward shift of the orange, red, and brown dotted lines relative to the blue line.

The improvement in the MPE is consistent with Proposition 1. When firms coordinate hours more, the Frisch elasticity to idiosyncratic shocks declines, translating directly into lower MPEs. With coordinated work schedules, workers face sizable penalties for deviating from optimal hours worked, and there are decreasing returns to hours. Consequently, when an agent receives an unexpected idiosyncratic wealth shock, on average, he is reluctant to adjust his hours for fear of receiving a lower wage. Thus, the marginal propensity to earn (MPE) in Figure 3 declines substantially as the coordination of hours in production increases. Aggregate Frisch elasticity is the same in all calibrations; complementary in hours worked does not introduce complementarity between consumption and labor. Therefore, with separable preferences, coordination of hours worked delivers lower MPEs and moderate fiscal multipliers.

Calibrations $\rho = -0.44$, and $\rho = -1.962$ correspond to empirical estimates in Shao et al. (2023), and Battisti et al. (2024). These calibrations translate into the elasticities of substitution between hours worked of around 0.7 and 0.34, respectively. However, a more extreme calibration of $\rho = -7$, which gives an elasticity of substitution between hours of 0.125, is required to match the MPE in the empirical range. The three calibrations correspond to idiosyncratic Frisch elasticities of around 0.29, 0.2, and 0.1. Chetty et al. (2013) concludes that micro evidence suggests Frisch elasticities of around 0.5 at an intensive margin. Battisti et al. (2024) provide empirical evidence that Frisch elasticity to idiosyncratic shocks at about ≈ 0.2 is substantially lower than elasticity to firm-level and aggregate shocks ≈ 0.5. In the latter case, the coordination friction does not affect the labor supply adjustment, and households can adjust hours more freely.

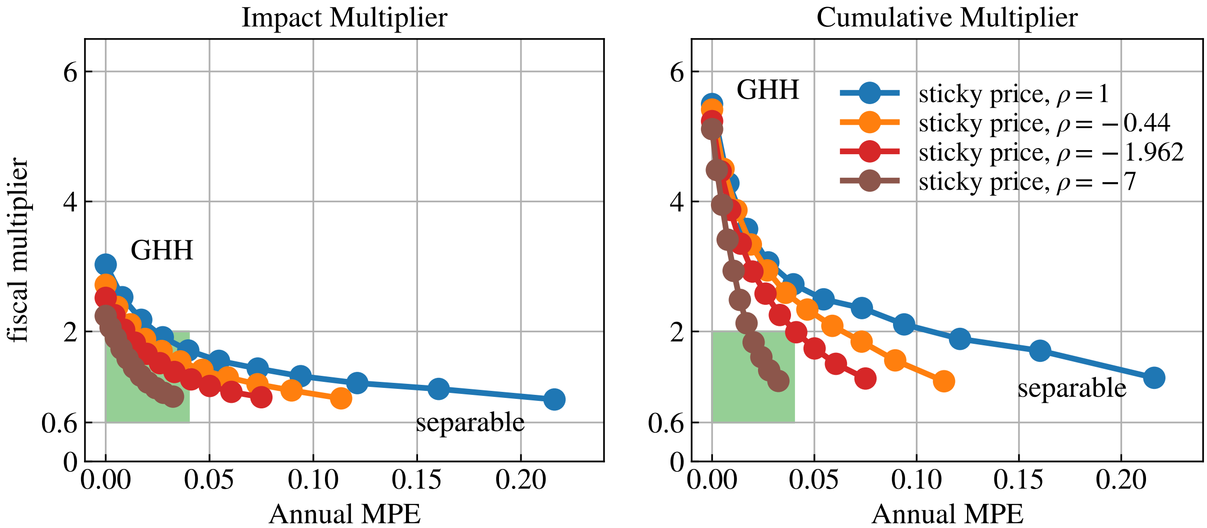

Figure 3: The impact of coordination of hours on the MPE-fiscal multiplier tradeoff

The takeaway is that low idiosyncratic Frisch elasticities not only do not contradict traditional micro estimates of the Frisch elasticity but also help explain a low responsiveness of household labor earnings to wealth shocks (low MPEs). Furthermore, as I argue in the next paragraph, low idiosyncratic Frisch elasticities are key to explaining the high concentration of hours worked in the US.

Coordination of hours worked in production yields a high concentration of hours worked around mean hours. I compare model and empirical hours by assuming that average hours in the model correspond to average hours in the data (further details in Appendix C.10). A lower Frisch elasticity to idiosyncratic shocks yields a more concentrated distribution of hours worked. The effect is apparent in Figure 4, where lower parameter ρ, corresponding to more coordination of hours in production, yields a steady-state distribution of hours worked closer to the empirical counterpart from Bick et al. (2022). In Appendix C.11, I show that this effect is present regardless of household preferences but tends to be more prominent for lower values of the complementarity index, for example, for separable preferences. In intuitive terms, household preferences are the same regardless of the degree of coordination of hours worked. However, when firms coordinate hours more, workers face higher wage penalties for deviating from optimal hours. The wage penalties are reflected in decreasing returns to hours (decreasing marginal earnings), which translate into lower idiosyncratic Frisch elasticities. When households decrease (increase) their hours, marginal earnings increase (decrease) rapidly. Therefore, as coordination increases, more and more households work around optimal hours.

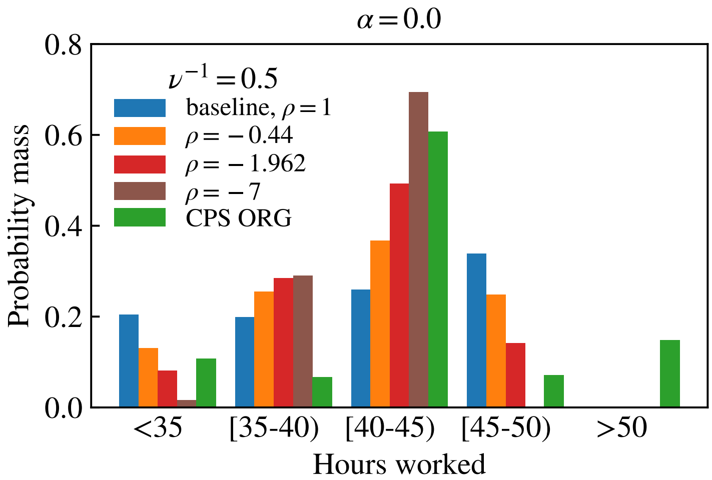

Figure 4: The steady-state distribution of hours worked in the model and the data. CPS ORG September 1995-August 2007 for women and men.

Even though coordination yields a substantial improvement in the empirical fit to the distribution of hours worked, the model fails to replicate a significant fraction of employees who usually work more than 50 hours a week in the data. As a side effect, coordination leads to a more equal distribution of wealth (see Figure 6 in Appendix C.12). Bick et al. (2022) show that correctly matching the empirical distribution of hours worked in the model requires introducing permanent differences in preferences and productivity and a negative correlation between the disutility from labor and productivity, which induces workers to work very long hours.

Coordination of hours worked does not increase fiscal multipliers under alternative monetary policy and debt regimes. In this section, I explore how coordinated work schedules affect fiscal multipliers in the canonical sticky price HANK model under alternative monetary policy and debt regimes. Empirical literature documents that fiscal multipliers are higher when monetary authority is at the zero lower bound (Ramey and Zubairy (2018), Ramey (2019)). Auclert et al. (2023) show that in a canonical HANK model with sticky prices, fiscal multipliers explode when the policy rate is pegged at zero, reaching values as high as 73.92. In the HANK-coordination model, multipliers are still very high but an order of magnitude lower than the canonical model and closer to the empirical multipliers at the ZLB of no more than 1.5. (Ramey and Zubairy (2018)). Table 2 presents the results.

	Monetary			Fiscal		
	Taylor rule	constant-r	3-year peg	$\rho_B = 0$	$\rho_B = 0.9$	$\rho_B = 0.95$
	$\rho = 1$ hours are perfect subtitutes					
separable	0.77	1.29	73.92	1.29	1.29	1.29
$\alpha = 0.5$	1.14	2.49	3.49	2.49	2.49	2.49
GHH	2.39	5.50	6.00	5.50	5.50	5.50
	$\rho = -0.44$ Shao et al. (2023)					
separable	0.77	1.23	2.53	1.23	1.23	1.23
$\alpha = 0.5$	1.20	2.59	3.69	2.59	2.59	2.59
GHH	2.27	5.41	6.39	5.41	5.41	5.41
	$\rho = -1.962$ Battisti et al. (2024)					
separable	0.80	1.28	2.40	1.28	1.28	1.28
$\alpha = 0.5$	1.22	2.58	3.69	2.58	2.58	2.58
GHH	2.20	5.24	6.65	5.24	5.24	5.24
	$\rho = -7$					
separable	0.81	1.24	1.80	1.24	1.24	1.24
$\alpha = 0.5$	1.22	2.49	3.59	2.49	2.49	2.49
GHH	2.20	5.11	8.06	5.11	5.11	5.11

Table 2: Cumulative multiplier under different monetary and fiscal policy regimes

6. Conclusion

Auclert et al. (2023) solve the HANK Trilemma using a sticky wage model where unions impose uniform hours across workers. However, in many countries, including the United States, union participation is low (Farber et al. (2021)), and there is plenty of heterogeneity in individual hours worked (Bick et al. (2022)). Therefore, I explore an alternative explanation: coordination of hours worked in production (Yurdagul (2017)). As reported in the literature review, ample empirical evidence supports the coordination of hours worked in production. I show that the HANK model with firms coordinating worker hours in production offers an improved fit to empirical evidence about MPEs and fiscal multipliers. Coordination drives a wedge between the Frisch elasticity to idiosyncratic and the Frisch elasticity to aggregate shocks. In line with empirical evidence in Battisti et al. (2024), I keep the Frisch elasticity to aggregate wage shocks constant at $\nu^{-1} = 0.5$. Therefore, increasing coordination in hours worked in production leads to lower Frisch elasticities to idiosyncratic shocks. Thus, consistent with Proposition 1, the model generates markedly lower MPEs without raising the multipliers and a much more concentrated distribution of hours worked: the former relaxes the MPE-fiscal multiplier tradeoff described by Auclert et al. (2023); the latter brings the model closer to the empirical distribution in Bick et al. (2022) for the US.

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Appendix

A. Labor supply elasticities

A.1. Idiosyncratic Frisch elasticity of labor supply

The optimal consumption-labor choice implicitly defines n and $\tilde{w}$.

$$H = U_n(c, n) + \lambda \tilde{w} x \left(f(n) + f'(n) n \right) = \lambda y'(n)$$

Differentiate the implicit function with respect to wages and hours.

$$\begin{aligned} H_{\tilde{w}} &= \lambda x y'(n) \\ H_n &= U_{nn}(c, n) + \lambda \tilde{w} x y''(n) \end{aligned}$$

Applying the implicit function theorem yields a formula for the Frisch elasticity.

$$\begin{aligned} \text{Frisch} = \left. \frac{\partial \log n}{\partial \log \tilde{w}} \right|_{\lambda = \bar{\lambda}} &= \left. \frac{\partial n}{\partial \tilde{w}} \right|_{\lambda = \bar{\lambda}} \frac{\tilde{w}}{n} = -\frac{H_{\tilde{w}}}{H_n} \frac{\tilde{w}}{n} \\ &= -\frac{\lambda y'(n)}{U_{nn}(c, n) + \lambda y''(n)} \frac{1}{n} \\ &= -\frac{1}{\frac{U_{nn} n}{\lambda} + \frac{y''(n) n}{y'(n)}} \end{aligned}$$

Substituting for $\lambda = -\frac{U_n}{y'(n)}$ from the optimum yields the final formula.

$$\text{Frisch} = \left(\frac{U_{nn} n}{U_n} - \frac{y''(n) n}{y'(n)} \right)^{-1}$$

A.2. Labor supply elasticity to aggregate shocks

In this section, I derive the elasticity of hours to aggregate productivity z and the taxation factor $1 - \tau^w$ in equation (19). The general idea is to show that optimal hours depend on the aggregate hours directly through the equilibrium wage function and indirectly through optimal hours L. The indirect effect captures the fact that coordination is not a friction when all agents are affected by a shock symmetrically. Thus, the elasticity of hours to the aggregate shocks is markedly higher than the idiosyncratic shocks.

The optimal consumption-leisure choice (11) implies the following optimal choice of hours worked:

$$n^* = \left((1 - \tau^w) \tilde{w} x \frac{\lambda}{\varphi \left(\alpha \lambda + 1 - \alpha \right)} \right)^{\frac{1}{1-\rho+\nu}} L^{\frac{1-\rho}{1-\rho+\nu}} \tag{21}$$

Now, aggregating the individual optimal choices of hours to obtain the wage-maximizing number of hours worked:

$$
\begin{aligned}
L = \mathbb{E}_x \left[n^{\rho}\right]^{\frac{1}{\rho}} &= \mathbb{E}_x \left[\left((1-\tau^w)\tilde{w}x \frac{\lambda}{\varphi\left(\alpha\lambda + 1 - \alpha\right)} \right)^{\frac{\rho}{1-\rho+\nu}} L^{\frac{1-\rho}{1-\rho+\nu}} \right]^{\frac{1}{\rho}} \\
&= L^{\frac{1-\rho}{1-\rho+\nu}} \left((1-\tau^w)\tilde{w}\right)^{\frac{1}{1-\rho+\nu}} \mathbb{E}_x \left[\left(\frac{x}{\varphi} \frac{\lambda}{\alpha\lambda + (1-\alpha)\lambda} \right)^{\frac{\rho}{1-\rho+\nu}} \right]^{\frac{1}{\rho}}
\end{aligned}
$$

Rearranging in terms of L:

$$
L = \left((1-\tau^w)\tilde{w}\right)^{\frac{1}{\nu}} \mathbb{E}_x \left[\left(\frac{x}{\varphi} \frac{\lambda}{\alpha\lambda + (1-\alpha)\lambda} \right)^{\frac{\rho}{1-\rho+\nu}} \right]^{\frac{1-\rho+\nu}{\nu\rho}} \tag{22}
$$

Substituting back into the optimal choice of individual hours:

$$
n^* = (1-\tau^w)^{\frac{1}{\nu}} \tilde{w}^{\frac{1}{\nu}} \left(\frac{x}{\varphi} \frac{\lambda}{\alpha\lambda + 1 - \alpha} \right)^{\frac{1}{1-\rho+\nu}} \mathbb{E}_x \left[\left(\frac{x}{\varphi} \frac{\lambda}{\alpha\lambda + (1-\alpha)\lambda} \right)^{\frac{\rho}{1-\rho+\nu}} \right]^{\frac{1-\rho}{\nu\rho}} \tag{23}
$$

Using that $\tilde{w} = mc \cdot z = \frac{z}{\mu}$, one directly obtains the desired result:

$$
\left.\frac{\partial \log n}{\partial \log z}\right|_{\lambda=\bar{\lambda}} = \left.\frac{\partial \log n}{\partial \log\left(1-\tau^w\right)}\right|_{\lambda=\bar{\lambda}} = \frac{1}{\nu}
$$

B. MPCs, MPEs, and coordination in hours

B.1. Proof of Lemma 1

The proof follows the same steps as in Auclert et al. (2023), accounting for the adjusted definition of the complementarity index and a slightly different function form of the first-order condition with respect to labor.

First-order Taylor approximation of the first-order condition with respect to consumption yields:

$$\begin{aligned} U_{c,t} &\approx U_c + U_{cc}(c_t - c) + U_{cn}(n_t - n) = U_c + U_{cc}c\hat{c}_t + U_{cn}n\hat{n}_t \\ \lambda_t &\approx \lambda + \lambda\frac{\lambda_t - \lambda}{\lambda} = \lambda(1 + \hat{\lambda}_t) \end{aligned}$$

Using the steady state relationship $\lambda = U_c$

$$\frac{cU_{cc}}{U_c}\hat{c}_t + \frac{nU_{cn}}{U_c}\hat{n}_t = \hat{\lambda}_t \tag{24}$$

Applying the first-order Taylor approximation to the first-order condition with respect to labor

$$\begin{aligned} U_{n,t} &\approx U_n + U_{nc}(c_t - c) + U_{nn}(n_t - n) = U_n + U_{nc}c\hat{c}_t + U_{nn}n\hat{n}_t \\ -\lambda_t\tilde{w}_t x\left(f(n_t) + f'(n_t)n_t\right) &\approx -\lambda\tilde{w}x\left(f(n) + f'(n)n\right) - \tilde{w}x\left(f(n) + f'(n)n\right)\lambda\hat{\lambda}_t \\ &\quad - \lambda\tilde{w}x\left(f(n) + f'(n)n\right)\hat{w}_t - \lambda\tilde{w}x\left(2f'(n) + f''(n)n\right)n\hat{n}_t \end{aligned}$$

The first-order approximations are exactly equal by the same argument. Using the steady state relationship $U_n = -\lambda\tilde{w}x\left(f(n) + f'(n)n\right)$

$$U_{nc}c\hat{c}_t + U_{nn}n\hat{n}_t = U_n\hat{\lambda}_t + U_n\hat{w}_t + U_n\frac{n\left(2f'(n) + f''(n)n\right)}{f(n) + f'(n)n}\hat{n}_t$$

Rearranging terms:

$$\frac{U_{nc}c}{U_n}\hat{c}_t + \left(\frac{U_{nn}n}{U_n} - \frac{n\left(2f'(n) + f''(n)n\right)}{f(n) + f'(n)n}\right)\hat{n}_t = \hat{\lambda}_t + \hat{w}_t \tag{25}$$

To shorten the notation, denote:

$$\tilde{\Psi} \equiv \frac{n\left(2f'(n) + f''(n)n\right)}{f(n) + f'(n)n}$$

Equations 24 and 25 form the following linear system

$$\begin{bmatrix} \frac{cU_{cc}}{U_c} & \frac{nU_{cn}}{U_c} \\ \frac{cU_{nc}}{U_n} & \frac{nU_{nn}}{U_n} - \tilde{\Psi} \end{bmatrix}\begin{bmatrix} \hat{c}_t \\ \hat{n}_t \end{bmatrix} = \begin{bmatrix} \hat{\lambda}_t \\ \hat{\lambda}_t + \hat{w}_t \end{bmatrix} \tag{26}$$

For as long as the matrix of the coefficients is invertible, the system has a unique solution

$$\begin{bmatrix} \hat{c}_t \\ \hat{n}_t \end{bmatrix} = \frac{U_c U_n}{cn} \frac{1}{U_{cc}\left(U_{nn} - \frac{U_n}{n}\tilde{\Psi}\right) - U_{cn}^2} \begin{bmatrix} \frac{nU_{nn}}{U_n} - \tilde{\Psi} & -\frac{nU_{cn}}{U_c} \\ -\frac{cU_{nc}}{U_n} & \frac{cU_{cc}}{U_c} \end{bmatrix} \begin{bmatrix} \hat{\lambda}_t \\ \hat{\lambda}_t + \hat{w}_t \end{bmatrix} \tag{27}$$

I use the result to calculate the complementarity index. From the linear system, it follows that

$$\frac{\partial \log c_t}{\partial \log w_t} = \frac{\partial \hat{c}_t}{\partial \hat{w}_t} = -\frac{U_c U_n}{cn} \frac{1}{U_{cc}\left(U_{nn} - \frac{U_n}{n}\tilde{\Psi}\right) - U_{cn}^2} \frac{nU_{cn}}{U_c}$$

$$\frac{\partial \log n_t}{\partial \log w_t} = \frac{\partial \hat{n}_t}{\partial \hat{w}_t} = \frac{U_c U_n}{cn} \frac{1}{U_{cc}\left(U_{nn} - \frac{U_n}{n}\tilde{\Psi}\right) - U_{cn}^2} \frac{cU_{cc}}{U_c}$$

Therefore, the ratio of the derivatives satisfies

$$\frac{\frac{\partial \log c_t}{\partial \log w_t}}{\frac{\partial \log n_t}{\partial \log w_t}} = \frac{\frac{\partial c(\lambda,w)}{\partial w}\frac{\tilde{w}}{c}}{\frac{\partial n(\lambda,w)}{\partial w}\frac{\tilde{w}}{n}} = \frac{\frac{\partial c(\lambda,w)}{\partial w}}{\frac{\partial n(\lambda,w)}{\partial w}}\frac{n}{c} = -\frac{U_{cn}}{U_{cc}}\frac{n}{c} \implies \frac{\frac{\partial c(\lambda,w)}{\partial w}}{\frac{\partial n(\lambda,w)}{\partial w}} = -\frac{U_{cn}}{U_{cc}}$$

Using the definition of the complementarity index

$$\mathrm{CI} \equiv \frac{\partial c(\lambda, w)}{\partial w} \Big/ \tilde{w}x\left(f(n) + f'(n)n\right)\frac{\partial n(\lambda, w)}{\partial w}$$

and from the first order condition with respect to labor

$$\tilde{w}x\left(f(n) + f'(n)n\right) = -\frac{U_n}{\lambda} = -\frac{U_n}{U_c}$$

Combining both yields a formula for the complementarity index

$$\mathrm{CI} = \frac{U_{cn}}{U_{cc}}\frac{U_c}{U_n}$$

which is unchanged from the baseline of Auclert et al. (2023).

B.2. Proof of Proposition 1

The proof closely follows Auclert et al. (2023). The derivations used to prove Lemma 1 will be useful. Using the linear system (27), calculate the derivative terms:

$$\frac{\partial \log n(\lambda, w)}{\partial \log \lambda} = \frac{\partial \hat{n}_t}{\partial \hat{\lambda}_t} = \frac{U_c U_n}{cn} \frac{1}{U_{cc}\left(U_{nn} - \frac{U_n}{n}\tilde{\Psi}\right) - U_{cn}^2} \left(\frac{cU_{cc}}{U_c} - \frac{cU_{nc}}{U_n}\right)$$

$$\frac{\partial \log n(\lambda, w)}{\partial \log w} = \frac{\partial \hat{n}_t}{\partial \hat{w}_t} = \frac{U_c U_n}{cn} \frac{1}{U_{cc}\left(U_{nn} - \frac{U_n}{n}\tilde{\Psi}\right) - U_{cn}^2} \frac{cU_{cc}}{U_c}$$

Therefore:

$$\frac{\frac{\partial \log n(\lambda,w)}{\partial \log \lambda}}{\frac{\partial \log n(\lambda,w)}{\partial \log x}} = \frac{\frac{cU_{cc}}{U_c} - \frac{cU_{nc}}{U_n}}{\frac{cU_{cc}}{U_c}} = 1 - \frac{U_{nc}U_c}{U_{cc}U_n} = 1 - \mathrm{CI}$$

It follows that the ratio of labor supply and consumption elasticities satisfies:

$$\frac{\frac{\partial \log n(\lambda,w)}{\partial \log \lambda}}{\frac{\partial \log c(\lambda,w)}{\partial \log \lambda}} = \frac{\frac{\partial \log n(\lambda,w)}{\partial \log \lambda}}{\frac{\partial \log n(\lambda,w)}{\partial \log x}} \frac{\frac{\partial \log n(\lambda,w)}{\partial \log x}}{\frac{\partial \log c(\lambda,w)}{\partial \log \lambda}} = (1 - \mathrm{CI}) \frac{\mathrm{Frisch}}{\mathrm{EIS}} \tag{28}$$

Then, using the definitions of the marginal propensities to consume and earn:

$$\mathrm{MPC} \equiv \frac{\partial c(a,x;T)}{\partial T}$$

$$\mathrm{MPE} \equiv -\tilde{w}x \left(f(n) + f'(n)n\right) \frac{\partial n(a,x;T)}{\partial T}$$

Therefore, it holds that:

$$\frac{\mathrm{MPE}}{\mathrm{MPC}} = -\frac{\tilde{w}x \left(f(n) + f'(n)n\right) \frac{\partial n(a,x;T)}{\partial T}}{\frac{\partial c(a,x;T)}{\partial T}} = -\frac{\tilde{w}x \left(f(n) + f'(n)n\right) \frac{\partial \log n(a,x;T)}{\partial \log \lambda} \frac{\partial \log \lambda}{\partial T} n}{\frac{\partial \log c(a,x;T)}{\partial \log \lambda} \frac{\partial \log \lambda}{\partial T} c}$$

Using equation (28) and simplifying yields the equation to be proved

$$\frac{\mathrm{MPE}}{\mathrm{MPC}} = \frac{\tilde{w}x f(n) n \left(1 + \frac{f'(n)n}{f(n)}\right)}{c} \frac{\mathrm{Frisch}}{\mathrm{EIS}} (1 - \mathrm{CI}) \tag{29}$$

C. Computational Appendix

C.1. Steady State of the Household Block

The solution uses the endogenous grid method of Carroll (2006) and is based on Auclert et al. (2021) and Auclert et al. (2023). The algorithm is adjusted to solve the household problem subject to the equilibrium earnings function derived from the model of coordinated work schedules.

I set up a grid for the future level of asset $a' \in \mathcal{G}_a$ and approximate the expected derivative of the value function tomorrow $W(a', x) \equiv \mathbb{E}\left[V_a(a', x') \mid x\right]$ starting from a guess that household consumes a constant fraction of its cash on hand. I use the Euler equation $W(a', x) = \lambda' = U_c(a', x)$ to recover $c(a(a'), x)$ and $n(a(a'), x)$ from the static first order conditions of the problem. That is, combining the first order condition w.r.t. c , a', and the envelope condition (equations 10, 12, and 13) yields:

$$\beta(1+r)W(a', x) = \beta(1+r)\mathbb{E}\left[V_a(a', x')|x\right] = \left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu}\right)^{-\sigma} = U_c(a', x) \tag{30}$$

Using the fact that we know $\beta(1+r)W(a', x) = U_c(a', x)$, one can recover the optimal labor supply from (11):

$$n(a(a'), x) = \left(\frac{U_c(a', x)(1-\tau^w)\tilde{w}\mathbb{E}[n^\rho]^{\frac{1}{\rho}-1}}{\varphi\left(\alpha U_c(a', x) + (1-\alpha)\right)}\right)^{\frac{1}{1-\rho+\nu}} \tag{31}$$

Then, from the first order condition with respect to consumption (10) and using the just calculated optimal labor supply, one can recover the optimal consumption:

$$c(a(a'), x) = U_c(a', x)^{-\frac{1}{\sigma}} + \varphi\alpha\frac{n(a', x)^{1+\nu}}{1+\nu} \tag{32}$$

The recovered policy functions c and n are defined on the endogenous grid for current assets $a(a') \notin \mathcal{G}^a$. I use linear interpolation to recover policy functions defined on the exogenous asset grid - $c(a, x)$ and $n(a, x)$ - such that $a \in \mathcal{G}^a$. Linear interpolation uses the fact that writing the budget constraint in the following fashion:

$$a'(a', x) + c(a', x) - (1-\tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}}\left(\frac{n(a', x)^\rho}{\rho\mathbb{E}[n^\rho]} + 1 - \frac{1}{\rho}\right) - T = (1+r)a(a') \tag{33}$$

This equation implicitly defines an endogenous grid for current assets $a(a')$, which is different from the exogenous grid $\mathcal{G}^a$. The iteration procedure requires that one updates $V_a(a)$ and that current assets a are on the exogenous grid $a \in \mathcal{G}^a$. Therefore, using linear interpolation, we look for current assets that lie on the exogenous grid $\mathcal{G}^a$, and, at the same time, obtain the future level of assets $a'(a)$ now defined on the exogenous grid $a \in \mathcal{G}^a$. Once assets today are on the exogenous grid $a \in \mathcal{G}^a$, we can recover assets tomorrow $a'(a,x)$ from the budget constraint:

$$a'(a,x) = (1-\tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}}\left(\frac{n(a,x)^\rho}{\rho\mathbb{E}[n^\rho]} + 1 - \frac{1}{\rho}\right) + T + (1+r)a - c(a,x) \tag{34}$$

Once we find the correct policy functions for the constrained agents as described in the subsection below, the iteration step closes with an update of the derivative of the value function using the envelope theorem $V_a(a) = (1+r)\lambda = (1+r)U_c$ using the policy function $c(a,x)$ and $n(a,x)$ where $a \in \mathcal{G}^a$.

C.2. The Constrained Households

Some agents would like to choose $a'(a,x) < 0$, which would violate the borrowing-constrained $a'(a,x) > 0$. For those agents, I set $a'(a,x)|_{bc} = 0$ and solve for their policy functions $c(a,x)|_{bc}$, $n(a,x)|_{bc}$ which maximize the following objective function

$$\begin{aligned}
&\max_{c,n} \frac{1}{1-\sigma}\left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu}\right)^{1-\sigma} - \varphi(1-\alpha)\frac{n^{1+\nu}}{1+\nu} \\
&\text{s.t.} \quad c = (1-\tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}}\left(\frac{n^\rho}{\rho\mathbb{E}[n^\rho]} + 1 - \frac{1}{\rho}\right) + T + (1+r)a
\end{aligned}$$

The first-order conditions of the static constrained optimization problem:

$$\left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu}\right)^{-\sigma} = \lambda$$

$$\varphi n^\nu\left(\alpha\left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu}\right)^{-\sigma} + (1-\alpha)\right) = \lambda(1-\tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}-1}n^{\rho-1}$$

The problem is a system of 2 equations in 2 unknowns:

$$\varphi n^\nu\left(\alpha\left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu}\right)^{-\sigma} + (1-\alpha)\right) = \left(c - \varphi\alpha\frac{n^{1+\nu}}{1+\nu}\right)^{-\sigma}(1-\tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}-1}n^{\rho-1}$$

$$(1-\tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}}\left(\frac{n^\rho}{\rho\mathbb{E}[n^\rho]} + 1 - \frac{1}{\rho}\right) + T + (1+r)a = c$$

I solve this system of equations using the algorithm of Auclert et al. (2023) because of its computational efficiency. The algorithm needs to be adjusted to handle the non-linear earnings properly; I provide the full algorithm description below.

The algorithm solves the budget constraint using Newton's updates incorporating the first-order conditions. The total derivative of the expenditure function takes into account the first-order conditions. To make the function more linear, use the logarithm of marginal utility $\log U_c$ as an argument of the expenditure function.

The general structure of Newton's method is as follows: Approximate a function $f(x)$ to first order at a point x^*

$$f(x) \approx f(x^*) + f'(x^*)(x - x^*)$$

Then, in approximation, it holds that:

$$x \approx x^* + \frac{f(x) - f(x^*)}{f'(x^*)}$$

We want the target point x to be the root of the function f such that $f(x) = 0$, then:

$$x \approx x^* - \frac{f(x^*)}{f'(x^*)}$$

The approximation motivates the iteration procedure in Newton's method.

$$x^{(n+1)} = x^{(n)} - \frac{f(x^n)}{f'(x^n)}$$

Function f is the budget constraint, and the argument $x = \log U_c$ is equal to the logarithm of the marginal utility. The budget constraint is evaluated using the policy functions $c = c(\log U_c)$ and $n = n(\log U_c)$, which are both functions of $\log U_c$.

$$f(\log U_c) = c - (1 - \tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}} \left(\frac{n^\rho}{\rho\mathbb{E}[n^\rho]} + 1 - \frac{1}{\rho} \right) - T$$

and the derivative of the expenditure function with respect to the $\log U_c$

$$f'(\log U_c) = \frac{\partial f}{\partial \log U_c} = \frac{\partial c}{\partial \log U_c} - (1 - \tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}-1} n^{\rho-1} \frac{\partial n}{\partial \log U_c}$$

It follows that Newton's iteration, in our case, is the following

$$\log U_c^{(n+1)} = \log U_c^{(n)} - \frac{c - (1 - \tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}} \left(\frac{n^\rho}{\rho\mathbb{E}[n^\rho]} + 1 - \frac{1}{\rho} \right) - T}{\frac{\partial c(\log U_c)}{\partial \log U_c} - (1 - \tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}-1} n^{\rho-1} \frac{\partial n(\log U_c)}{\partial \log U_c}} \tag{35}$$

Taking the first order condition w.r.t. labor and substituting out $\lambda = U_c$, one obtains

$$\varphi n^\nu (\alpha U_c + 1 - \alpha) = U_c (1 - \tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}-1} n^{\rho-1}$$

Taking logs, I get an implicit function

$$H = (1 - \rho + \nu)\log n - \log\left((1 - \tau^w)\tilde{w}x\mathbb{E}[n^\rho]^{\frac{1}{\rho}-1} \right) + \log\varphi - \log U_c + \log(1 - \alpha + \alpha U_c)$$

Applying an implicit function theorem to recover $\frac{\partial n}{\partial \log U_c}$ one obtains.

$$H_{\log n} = \frac{\partial H}{\partial \log n} = 1 - \rho + \nu$$

$$H_{\log U_c} = \frac{\partial H}{\partial \log U_c} = -1 + \frac{\partial \log(1 - \alpha + \alpha U_c)}{\partial U_c} U_c = -1 + \frac{\alpha U_c}{1 - \alpha + \alpha U_c} = -\frac{1 - \alpha}{1 - \alpha + \alpha U_c}$$

$$\frac{\partial \log n}{\partial \log U_c} = -\frac{\partial H_{\log U_c}}{\partial H_{\log n}} = \frac{1}{1 - \rho + \nu} \frac{1 - \alpha}{1 - \alpha + \alpha U_c}$$

$$\frac{\partial n}{\partial \log U_c} = n \frac{\partial \log n}{\partial \log U_c} = \frac{n}{1 - \rho + \nu} \frac{1 - \alpha}{1 - \alpha + \alpha U_c} \tag{36}$$

The derivative $\frac{\partial c}{\partial \log U_c}$ can be recovered from the marginal utility function.

$$U_c = \left(c - \varphi \alpha \frac{n^{1+\nu}}{1 + \nu} \right)^{-\sigma}$$

Taking logs, one obtains the following implicit function

$$H = \log U_c + \sigma \log \left(c - \varphi \alpha \frac{n^{1+\nu}}{1 + \nu} \right)$$

Using the same approach as before

$$H_{\log U_c} = \frac{\partial H}{\partial \log U_c} = 1 - \frac{\sigma}{c - \varphi \alpha \frac{n^{1+\nu}}{1+\nu}} \varphi \alpha n^{\nu} \frac{\partial n}{\partial \log U_c}$$

$$H_c = \frac{\partial H}{\partial c} = \frac{\sigma}{c - \varphi \alpha \frac{n^{1+\nu}}{1+\nu}}$$

$$\frac{\partial c}{\partial \log U_c} = -\frac{H_{\log U_c}}{H_c} = \varphi \alpha n^{\nu} \frac{\partial n}{\partial \log U_c} - \frac{1}{\sigma} \left(c - \varphi \alpha \frac{n^{1+\nu}}{1 + \nu} \right) = \varphi \alpha n^{\nu} \frac{\partial n}{\partial \log U_c} - \frac{1}{\sigma} U_c^{-\frac{1}{\sigma}} \tag{37}$$

After providing an initial guess for the marginal utility, using, for example, the GHH utility case, all that remains is to bring the iteration (35) to convergence using the partial derivatives (36) and (37).

C.3. Elasticity of intertemporal substitution with GHH Plus Preferences

Note that the formula (37) provides a direct way to compute the elasticity of intertemporal substitution for the GHH Plus preferences.

$$\text{EIS} = -\frac{\partial \log c(\lambda, w)}{\partial \log \lambda} = -\frac{1}{c} \frac{\partial c}{\partial \log U_c}$$

C.4. The Cost Minimization and the Equilibrium Wage

A representative monopolistically competitive intermediate good firm i solves the following cost minimization problem.

$$\min_{\mu(n,x)} \int_{x\in B_x}\int_0^\infty w(n,x)n\mu^i(n,x)dndx \text{ s.t.}$$

$$z\left(\int_{x\in B_x}\int_0^\infty x\mu^i(n,x)n^\rho dndx\right)^{\frac{1}{\rho}}\left(\int_{x\in B_x}\int_0^\infty x\mu^i(n,x)dndx\right)^{1-\frac{1}{\rho}} - F = y_t(i) \quad (mc_t^i)$$

where mc_t^i is the Lagrange multiplier on the technological constraint and measures the marginal cost of production.

$$\mathcal{L}(\mu^i(n,x);mc_t^i) = \int_{x\in B_x}\int_0^\infty w(n,x)n\mu^i(n,x)dndx$$

$$+mc_t^i\left(y_t(i) - z\left(\int_{x\in B_x}\int_0^\infty x\mu^i(n,x)n^\rho dndx\right)^{\frac{1}{\rho}}\left(\int_{x\in B_x}\int_0^\infty x\mu^i(n,x)dndx\right)^{1-\frac{1}{\rho}} + F\right)$$

The first-order condition of the Lagrangian defines the optimal wage function.

$$\frac{\partial\mathcal{L}}{\partial\mu^i(n,x)}: \quad w(n,x)n = mc_t^i z\left(\int_{x\in B_x}\int_0^\infty n^\rho\frac{x\mu^i(n,x)}{\int_{x\in B_x}\int_0^\infty x\mu^i(n,x)dndx}dndx\right)^{\frac{1}{\rho}}$$

$$\left[\frac{n^\rho}{\rho\int_{x\in B_x}\int_0^\infty n^\rho\frac{x\mu^i(n,x)}{\int_{x\in B_x}\int_0^\infty x\mu^i(n,x)dndx}dndx} + 1 - \frac{1}{\rho}\right]$$

Since the equilibrium is symmetric, all firms choose the same mass of workers of each type to hire $\mu^i(n,x) = \mu^j(n,x) \equiv \mu(n,x)$ for all i,j and n,x. Therefore, integrating over all firms $\int \mu^i(n,x)di = \mu(n,x)$. Moreover, we know that the total mass of agents is unity. Thus, in equilibrium, $\int_{x\in B_x}\int_0^\infty \mu^i(n,x)dndx = 1$. The masses $\mu^i(n,x)$ need to be consistent with the distribution of agents over (n,x) from the household block. In the model, the expected productivity equals one: $\mathbb{E}[x] = 1$. Therefore, in equilibrium $\int_{x\in B_x}\int_0^\infty x\mu^i(n,x)dndx = 1$, allowing me to write an equilibrium wage.

$$w(n,x)n = mc_t z\left(\int_{x\in B_x}\int_0^\infty xn^\rho\ \mu(n,x)dndx\right)^{\frac{1}{\rho}}\left[\frac{n^\rho}{\rho\int_{x\in B_x}\int_0^\infty xn^\rho\ \mu(n,x)dndx} + 1 - \frac{1}{\rho}\right]$$

Since $\mu(x,n)$ integrates to unity, denote $\mathbb{E}[n^\rho] \equiv \int_{x\in B_x}\int_0^\infty xn^\rho\ \mu(n,x)dndx$. Furthermore, denote $\tilde{w} = mc\cdot z$ that is common across all workers. Therefore, the equilibrium wage function takes the following form.

$$w(n,x) = \tilde{w}x\mathbb{E}\left[n^\rho\right]^{\frac{1}{\rho}}\left(\frac{n^{\rho-1}}{\rho\mathbb{E}\left[n^\rho\right]} + \frac{1}{n}\left(1-\frac{1}{\rho}\right)\right)$$

C.5. The New Keynesian Philips Curve

I derive the New Keynesian Philips Curve as in Auclert et al. (2023). The firm maximizes the present value of its profits using the ex-ante real rate of return on bonds r_t^e as its discount rate. The derivation assumes certainty equivalence, which holds in transitions after an MIT shock, and thus, I omit the expectations operator. The firm chooses its price P_t subject to the quadratic price adjustment cost proportional to its production and defined in the log domain.

Using the result from Appendix C.7 that the total cost is equal to the product of the wage shifter term $\tilde{w}$ and aggregate effective hours L

$$TC = \int_{x \in B_x} \int_0^\infty w(n,x) n \mu(n,x) dn dx = mc_t z \mathbb{E}[n^\rho]^{\frac{1}{\rho}} \equiv \tilde{w} L$$

the total cost depends linearly on the marginal cost $mc_t = \frac{\tilde{w}_t}{z_t}$.

$$TC = \tilde{w} L = \frac{\tilde{w}}{z} z L = mc_t Y_t$$

Therefore, defining the objective function in real terms, in every period t, the firm solves the following maximization problem by choosing the optimal price level P_t^*.

$$\max_{P_t^*} \sum_{s=0}^{\infty} \left(\frac{1}{1+r_t^e} \right)^s \frac{P_t}{P_{t+s}} Y_{t+s} \left[P_{t+s}^* \left(\frac{P_{t+s}^*}{P_{t+s}} \right)^{-\varepsilon_p} - P_{t+s}^* mc_{t+s} \left(\frac{P_t^*}{P_{t+s}^*} \right)^{-\varepsilon_p} - \frac{\psi}{2} \left(\log\left(1+\pi_t^*\right) \right)^2 P_{t+s} \right]$$

P denotes the aggregate price level, and the firm-level inflation rate is $\pi_t^* \equiv \frac{P_t^* - P_{t-1}^*}{P_{t-1}^*}$. The per-period profits inside the square brackets are in nominal terms. The $\frac{P_t}{P_{t+s}}$ transforms the objective function from nominal to real values. The first-order condition of this problem is the following.

$$\frac{\partial}{\partial P_t^*}: \quad (1-\varepsilon_p) \left(\frac{P_t^*}{P_t} \right)^{-\varepsilon_p} Y_t + \varepsilon_p mc_t \left(\frac{P_t^*}{P_t} \right)^{-\varepsilon_p} Y_t - \psi \log\left(1+\pi_t\right) \frac{P_t}{P_t^*} Y_t + \frac{\psi}{1+r_t^e} \log\left(1+\pi_{t+1}\right) \frac{P_t}{P_t^*} Y_{t+1} = 0$$

Assuming a symmetric equilibrium where all firms set the same price $P_t = P_t^*$ (thus $\pi_t = \pi_t^*$), the first-order condition simplifies to.

$$\frac{\varepsilon_p}{\psi} Y_t \left(mc_t - \frac{\varepsilon_p - 1}{\varepsilon_p} \right) + \frac{\psi}{1+r_t^e} \log\left(1+\pi_{t+1}\right) Y_{t+1} = \psi \log\left(1+\pi_t\right) Y_t$$

Rearranging terms and defining the slope of the Philips curve as $\kappa_p \equiv \frac{\varepsilon_p}{\psi}$.

$$\log\left(1+\pi_t\right) = \kappa_p \left(mc_t - \frac{\varepsilon_p - 1}{\varepsilon_p} \right) + \frac{1}{1+r_t^e} \frac{Y_{t+1}}{Y_t} \log\left(1+\pi_{t+1}\right)$$

Steady-state MPC

The model is quarterly, and the steady-state calibration targets MPC = 0.25 broadly consistent with empirical evidence in Johnson et al. (2006) and Fagereng et al. (2021) among many others. As in Auclert et al. (2023), I use discount factor heterogeneity to target aggregate savings and MPCs consistent with empirical evidence. A sufficiently high discount factor ensures that households generate savings consistent with the data. A sufficiently low smaller discount factor yields the steady-state MPC of 0.25.

In the steady state, I calculate the MPC and the MPE using the finite differences method to approximate the derivatives. In particular, for each individual characterized by a pair of states (a_i, x), I calculate his MPC and MPE in the following fashion:

$$\text{MPC}(l, a, x) = \frac{c(a_{i+1}, x) - c(a_{i-1}, x)}{(1+r)\,(a_{i+1} - a_{i-1})}$$
$$\text{MPE}(l, a, x) = \frac{Y^{lab}(a_{i+1}, x) - Y^{lab}(a_{i-1}, x)}{(1+r)\,(a_{i+1} - a_{i-1})}$$

For individuals at the maximum gridpoint, I take a finite difference between point n_a and $n_a - 1$. For households that are borrowing-constrained, back out the MPE from an analytical relationship derived in equation 5 in the main text.

$$\frac{\text{MPE}(\underline{a}, x)}{\text{MPC}(\underline{a}, x)} = \frac{Y^{lab}(\underline{a}, x)(1 + \epsilon_{w,n}(\underline{a}, x))}{c(\underline{a}, x)} \frac{\text{Frisch}}{\text{EIS}(\underline{a}, x)} \left(1 - \text{CI}(\underline{a}, x)\right)$$

The formula yields a ratio of the MPE and MPC. Combining the formula above with the differentiated budget constraint: MPC + MPE = 1 yields the MPC and MPE for each borrowing-constrained agent. Most variables needed to calculate the MPE-MPC ratio are available from the solution of the household problem.

The labor earnings and consumption for the borrowing-constrained household are readily available from the solution of the household problem. The Frisch elasticity to idiosyncratic shocks equals $\frac{1}{1-\rho+\nu}$. One can calculate the elasticity of intertemporal substitution from the formula 37.

$$\text{EIS} = \frac{\partial \log c}{\partial \log U_c} = \frac{1}{c}\left(\varphi \alpha n^{\nu} \frac{\partial n}{\partial \log U_c} - \frac{1}{\sigma} U_c^{-\frac{1}{\sigma}}\right)$$

The complementarity index is defined in the main text as $\text{CI} = \frac{\alpha U_c}{1-\alpha+\alpha U_c}$. The remaining object that I need to derive is the wage elasticity to hours worked $\epsilon_{w,n} = \frac{f'(n)n}{f(n)}$. $f(n)$ is the wage function. In the quantitative model, I define it in the following fashion.

$$f(n) = (1 - \tau^w)\tilde{w} x L \left(\frac{n^{\rho-1}}{\rho L^{\rho}} + \frac{1}{n}\left(1 - \frac{1}{\rho}\right)\right)$$

Differentiating with respect to hours yields a formula for the elasticity $\epsilon_{w,n}$ of wages to hours worked.

$$f'(n) = (1-\tau^w)\tilde{w}xL\left(\frac{\rho-1}{\rho}\frac{n^{\rho-2}}{L^\rho} - \frac{1}{n^2}\left(1-\frac{1}{\rho}\right)\right)$$
$$= \left(1-\frac{1}{\rho}\right)(1-\tau^w)\tilde{w}xL\frac{1}{n^2}\left(\left(\frac{n}{L}\right)^\rho - 1\right)$$
$$\epsilon_{w,n} = \left(1-\frac{1}{\rho}\right)\frac{\frac{1}{n}L\left(\frac{n^\rho}{L^\rho}-1\right)}{L\left(\frac{n^{\rho-1}}{\rho L^\rho}+\frac{1}{n}\left(1-\frac{1}{\rho}\right)\right)} = \left(1-\frac{1}{\rho}\right)\frac{\left(\left(\frac{n}{L}\right)^{\rho-1}-\frac{L}{n}\right)}{\frac{1}{\rho}\left(\frac{n}{L}\right)^{\rho-1}+\frac{L}{n}\left(1-\frac{1}{\rho}\right)}$$
$$1+\epsilon_{w,n} = \frac{\left(\frac{n}{L}\right)^{\rho-1}}{L\left(\frac{n^{\rho-1}}{\rho L^\rho}+\frac{1}{n}\left(1-\frac{1}{\rho}\right)\right)}$$

The last formula for $1+\epsilon_{w,n}$ corresponds directly to the implementation in the code.

MPE in the model and in the data

The model is quarterly, as in Auclert et al. (2023), but almost all empirical evidence about the MPEs is at annual frequencies. Therefore, to match the data, it is necessary to simulate a transition path of individual household choices in response to a one-time wealth shock. In particular, I calculate the model MPE as a mean partial equilibrium cumulative response of labor earnings over the 4-quarter horizon to a small transfer shock.

$$\text{MPE} = \sum_{s=0}^{3}\frac{\partial Y_s^{labor}}{\partial T_0}$$

Moreover, it is necessary to determine what MPE values are compatible with empirical evidence. Auclert et al. (2023) argue that the direct MPE measure from lottery winnings in Swedish administrative data in Cesarini et al. (2017) was the best available evidence when their paper was published. Cesarini et al. (2017) find an annual average MPE of 0.01. On the other hand, Imbens et al. (2001) study the impact of 20-year-long annuity payments on labor supply. The MPE in Imbens et al. (2001) measures a change in labor earnings per annual payment from a 20-year annuity. Auclert et al. (2023) note that in the permanent income model with unit discount factor and gross interest rate, receiving an annuity over 20 years is equivalent to receiving the whole amount immediately. Thus, the model MPE equals 1/20 of the Imbens et al. (2001) MPE. However, the more borrowing-constrained an agent is, the more he will use the one-time payment to reduce his hours in the same period. Thus, the ratio of the two MPE measures is model-dependent and reflects the prevalence of borrowing constraints. In the baseline model of Auclert et al. (2023), the ratio of the Imbens et al. (2001) MPE to the model MPE is 3.6. Combined with the maximum MPE estimate of 0.122 in Imbens et al. (2001), Auclert et al. (2023) consider MPE values between 0 and $0.122/3.6 \approx 0.04$ an acceptable range.

The most recent estimates of the MPE based on high-quality administrative data from the US in Golosov et al. (2023) are within the target range put forward by Auclert et al. (2023). Golosov et al. (2023) find an annual average MPE = 0.023 at a household level. This value is very close to the middle of the original interval in Auclert et al. (2023).

Moreover, I redo the calculation of the MPE ratio for the 20-year annuity and one-time winning in Auclert et al. (2023) for the model with coordination in hours worked. For separable preferences, hours as gross complements in production with $\rho = -0.44$, the ratio of the Imbens et al. (2001) and Auclert et al. (2023) MPEs is 3.4, little changed from the baseline model. Therefore, I use the original MPE interval from Auclert et al. (2023).

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1	0.55	0.49	0.43	0.38	0.33	0.28	0.23	0.17	0.12	0.06	0.0
0.5	0.48	0.42	0.38	0.33	0.28	0.23	0.19	0.14	0.09	0.05	0.0
0.02	0.43	0.37	0.32	0.28	0.24	0.20	0.16	0.12	0.08	0.04	0.0
-0.44	0.38	0.33	0.29	0.25	0.21	0.18	0.14	0.10	0.07	0.03	0.0
-1.962	0.28	0.24	0.21	0.18	0.15	0.12	0.10	0.07	0.05	0.02	0.0
-7	0.15	0.13	0.11	0.10	0.08	0.06	0.05	0.03	0.02	0.01	0.0

Table 3: Golosov et al. (2023) MPE in the model

Additional considerations. A prominent feature of the labor earnings responses to unexpected lottery winnings is their high persistence. Very high *MPEs out of changes in unearned income* estimated by Golosov et al. (2023) suggest that the baseline one-year MPE measure does not fully capture the nature of labor supply adjustment. In particular, the labor earnings responses in Cesarini et al. (2017) and Golosov et al. (2023) are moderate compared to the size of the wealth shock. However, the response is persistent and builds up to a substantial fraction of the total lottery size over the lifetime. To further compare the model's performance with the estimates in Golosov et al. (2023), I calculate the lifetime MPE as in Golosov et al. (2023). In particular, I calculate the following measure:

$$\text{MPE}_{Golosov} = \frac{\partial\left(\sum_{h=0}^{20} Y_{t+h}^{lab}\right)}{\partial\left(\sum_{h=0}^{20}(1+r)a_{t+h} - a_{t+h+1}\right)}$$

In other words, I calculate transitional dynamics to a one-time wealth shock over a 5-year horizon and calculate the ratio of the average response of labor earnings to the average change in an unearned income. I use the fact that the response of $(1+r)a_t$ in the initial period is equal to the ϵ-sized wealth shock. The wealth shock is idiosyncratic, and I keep the interest rate at its steady-state level r.

C.6. Aggregation

The labor market encompasses submarkets for worker types (n, x). Therefore, the market clearing condition on each market requires that the demanded measures $\mu(n, x)$ are equal to the measure of household supplying this combination of hours and productivity.

$$\forall_{n,x} \quad \mu(n,x) = \int_{B_a} D(a,x)\mathbb{1}\left\{n(a,x) = n\right\} da$$

The model features a unit mass of workers, and both sides of the equation integrate to 1.

$$\int_{x\in B_x}\int_0^\infty \mu(n,x)dndx = \int_{x\in B_x}\int_{a\in B_a} D(a,x)\left(\int_0^\infty \mathbb{1}\left\{n(a,x)=n\right\}dn\right)dadx = 1$$

Integration over the entire state space $\mathcal{A}\times\mathcal{X}$ must also yield a unit mass of households. Therefore, integration over hours worked n, and x equals the integration over a and x.

$$\int_{x\in B_x}\int_0^\infty \mu(n,x)dndx = \int_{x\in B_x}\int_{a\in B_a} D(a,x)dadx = 1$$

In the next steps, I provide a discrete approximation to the labor market variables from the main text. First, calculate the expected worker productivity over the measure of workers $\mu(n, x)$.

$$\mathbb{E}[x] \equiv \int_{x\in B_x}\int_0^\infty x\mu(n,x)dndx = \int_{x\in B_x}\int_{a\in B_a} xD(a,x)dadx = 1 \tag{38}$$

Calculate an analogous integral for hours n^ρ.

$$\int_{x\in B_x}\int_0^\infty xn^\rho\mu(n,x)dndx = \int_{x\in B_x}\int_{a\in B_a} xn(a,x)^\rho D(a,x)dadx \tag{39}$$

Introduce a notation for the expected efficiency hours.

$$\mathbb{E}_x\left[n^\rho\right] \equiv \int_{x\in B_x}\int_{a\in B_a} n(a,x)^\rho \frac{xD(a,x)}{\int_{x\in B_x}\int_{a\in B_a} xD(a,x)dadx}dadx \tag{40}$$

Aggregation requires the approximation of the integrals from the main text. I approximate the aggregate distribution on the discrete grid for assets $\mathcal{G}^a$ and for the idiosyncratic income states $\mathcal{G}^x$. The distribution affects the labor market where firms optimized over worker masses $\mu^i(n,x)$ (and $\mu^i(n,x) = \mu(n,x)$ due to symmetry). Approximate the integral over the idiosyncratic state x using a discretized grid $\mathcal{G}^x$.

$$\int_{x\in B_x}\int_0^\infty xn^\rho \mu(n,x)dndx \approx \sum_{x\in\mathcal{G}^x}\int_0^\infty xn^\rho\mu(n,x)dn \tag{41}$$

The total mass of agents is unity regardless of whether we sum over masses of agents working a specific number of hours or having a specific amount of assets.

$$\sum_{x\in\mathcal{G}^x}\int_0^\infty \mu(n,x)dn = 1 \approx \sum_{x\in\mathcal{G}^x}\sum_{a\in\mathcal{G}^a} D(a,x) \tag{42}$$

Similarly, use that integration over assets and hours are equivalent and approximate to obtain a fraction of agents at the idiosyncratic state x.

$$\int_0^\infty \mu(n,x)dn = \int_0^\infty \mu(a,x)da \approx \sum_{a\in\mathcal{G}^a} D(a,x) \equiv D(x) \tag{43}$$

The expectation of the income state in the model is equal to unity, such that:

$$1 = \mathbb{E}[x] \approx \sum_{x\in\mathcal{G}^x} xD(x) \tag{44}$$

Using these results, one can derive optimal hours L as a function of assets and income states.

$$\begin{aligned}
L = \mathbb{E}[n^\rho]^{\frac{1}{\rho}} &= \left(\int_{x\in B_x}\int_0^\infty x\mu(n,x)n^\rho dndx\right)^{\frac{1}{\rho}}\left(\int_{x\in B_x}\int_0^\infty x\mu(n,x)dndx\right)^{1-\frac{1}{\rho}} \\
&\approx \left(\sum_{x\in\mathcal{G}^x}\int_0^\infty xn^\rho \frac{\mu(n,x)}{\sum_{x\in\mathcal{G}^x}\int_0^\infty x\mu(n,x)dn}dn\right)^{\frac{1}{\rho}}\left(\sum_{x\in\mathcal{G}^x}\int_0^\infty x\mu(n,x)dn\right) \\
&\overset{(41)}{\approx} \left(\sum_{x\in\mathcal{G}^x} x\int_0^\infty n^\rho \frac{\mu(n,x)}{\sum_{x\in\mathcal{G}^x} x\sum_{a\in\mathcal{G}^a} D(a,x)}dn\right)^{\frac{1}{\rho}}\left(\sum_{x\in\mathcal{G}^x} x\sum_{a\in\mathcal{G}^a} D(a,x)\right) \\
&\overset{(43)}{=} \left(\sum_{x\in\mathcal{G}^x} x\int_0^\infty n^\rho\mu(n,x)dn\right)^{\frac{1}{\rho}}\left(\sum_{x\in\mathcal{G}^x} xD(x)\right) \\
&\overset{(44)}{\approx} \left(\sum_{x\in\mathcal{G}^x} x\int_0^\infty n^\rho\mu(n,x)dn\right)^{\frac{1}{\rho}} \\
&\approx \left(\sum_{x\in\mathcal{G}^x}\sum_{a\in\mathcal{G}^a} xn(a,x)^\rho D(a,x)\right)^{\frac{1}{\rho}}
\end{aligned}$$

Using this approximation, one recovers the aggregate labor input (optimal hours) L and, by extension, the total production Y. The total firm wage bill is also a function of L.

C.7. Total wage bill

First, using the definition of the wage bill $\int_{x \in B_x} \int_0^\infty w(n,x) n \mu(n,x) dndx$ and the optimal wage schedule $w(n,x) = \tilde{w} x \mathbb{E}[n^\rho]^{\frac{1}{\rho}} \left[\frac{1}{\rho} \frac{n^{\rho-1}}{\mathbb{E}[n^\rho]} + \frac{1}{n}\left(1 - \frac{1}{\rho}\right)\right]$, one derives the following.

$$\int_{x \in B_x} \int_0^\infty w(n,x) n \mu(n,x) dndx = \int_{x \in B_x} \int_0^\infty \tilde{w} x \mathbb{E}[n^\rho]^{\frac{1}{\rho}} \left(\frac{n^\rho}{\rho \mathbb{E}[n^\rho]} + 1 - \frac{1}{\rho}\right) \mu(n,x) dndx =$$

$$\tilde{w} \mathbb{E}[n^\rho]^{\frac{1}{\rho}} \left(\frac{1}{\rho \mathbb{E}[n^\rho]} \int_{x \in B_x} \int_0^\infty x n^\rho \mu(n,x) dndx + 1 - \frac{1}{\rho}\right)$$

Using the definition of the expectation term $\mathbb{E}[n^\rho] \equiv \int_{x \in B_x} \int_0^\infty x n^\rho \frac{\mu(n,x)}{\int_{x \in B_x} \int_0^\infty x \mu(n,x) dndx} dndx$ from the main text, and that $\mathbb{E}[x] = \int_{x \in B_x} \int_0^\infty x \mu(n,x) dndx = 1$, the total wage bill can be expressed in terms of $\tilde{w}$ and L.

$$\int_{x \in B_x} \int_0^\infty w(n,x) n \mu(n,x) dndx = \tilde{w} \mathbb{E}[n^\rho]^{\frac{1}{\rho}} = \tilde{w} L$$

Thus, one can calculate the total wage bill using the expression for L from above. That is.

$$\tilde{w} L \approx \tilde{w} \left(\sum_{x \in \mathcal{G}^x} \sum_{a \in \mathcal{G}^a} x n(a,x)^\rho D(a,x)\right)^{\frac{1}{\rho}}$$

C.8. Variance of log idiosyncratic productivity

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	0.95	0.85	0.80	0.76	0.73	0.70	0.68	0.66	0.64	0.63	0.61
0.5	0.95	0.86	0.81	0.77	0.74	0.72	0.70	0.68	0.66	0.64	0.63
0.02	0.94	0.87	0.82	0.79	0.76	0.73	0.71	0.69	0.67	0.66	0.65
-0.44	0.94	0.87	0.83	0.80	0.77	0.74	0.72	0.70	0.69	0.67	0.66
-1.962	0.93	0.89	0.85	0.82	0.80	0.77	0.76	0.74	0.72	0.71	0.70
-7	0.93	0.90	0.88	0.86	0.84	0.83	0.82	0.80	0.79	0.78	0.77

Table 4: $\sigma_{\log x}$

This section reports the calibrated value of the log idiosyncratic productivity term $\log x$. For separable preferences $\alpha = 0$, the wage penalties incurred by part-time workers more than offset a higher concentration of hours worked and a lower standard deviation of log idiosyncratic income terms $\sigma_{\log x}$ is required to match the empirical variance of log labor earnings.

For non-separable preferences, the choice of hours depends more on productivity, and there are relatively more workers working high hours (see Figure 5). The impact of more concentrated hours prevails, and a higher standard deviation of $\log x$ is required to match the empirical variance of log labor earnings.

I follow Auclert et al. (2023) and do not separately model the transitory and persistent shock components, contrary to Krueger et al. (2016). For comparison, the corresponding estimate of $\sigma_{\log x}$ in Krueger et al. (2016) derives from the following formula.

$$\sigma_{\log x} = \sqrt{\frac{\sigma_\eta^2}{1-\phi^2} + \sigma_\epsilon^2}$$

Krueger et al. (2016) estimate the persistence of the permanent shock component $\phi = 0.9695$, variance of the persistent shock $\sigma_\eta^2 = 0.0384$, and variance of the transitory shock $\sigma_\epsilon^2 = 0.0522$. Substituting into the formula yields $\sigma_{\log x} = 0.8315$.

C.9. Negative labor earnings in the model of coordination

	α										
ρ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00	0.0e+00
0.5	6.5e-05	1.3e-05	3.9e-06	1.2e-06	3.7e-07	1.1e-07	3.7e-08	6.7e-08	6.1e-08	6.0e-08	6.0e-08
0.02	2.1e-04	7.2e-05	3.1e-05	1.4e-05	7.6e-06	4.6e-06	3.3e-06	3.2e-06	1.8e-05	1.8e-05	1.8e-05
-0.44	3.1e-04	1.3e-04	6.4e-05	3.4e-05	2.0e-05	2.7e-05	2.3e-05	2.1e-05	2.1e-05	2.0e-05	1.8e-05
-1.962	3.8e-04	2.1e-04	1.2e-04	8.4e-05	6.0e-05	5.3e-05	1.5e-04	1.5e-04	1.4e-04	1.4e-04	1.4e-04
-7	1.9e-04	1.2e-04	7.4e-05	4.7e-05	4.6e-05	4.2e-05	1.5e-04	1.4e-04	1.4e-04	1.4e-04	1.4e-04

Table 5: Fraction of households reporting negative labor earnings

If optimal hours L are positive and $\rho < 1$, labor earnings become negative for sufficiently low hours.

$$\lim_{n \longrightarrow 0} w(n)n = \tilde{w} x \mathbb{E}_{x,l}\left[n^\rho\right]^{\frac{1}{\rho}} \left(\lim_{n \longrightarrow 0} \frac{n^\rho}{\rho \mathbb{E}_{x,l}\left[n^\rho\right]} + 1 - \frac{1}{\rho} \right) < 0$$

In the model with coordination, choosing $n = 0$ has a severe adverse impact on firm production, and the equilibrium wage is negative. Therefore, in contrast to the standard model with perfectly substitutable hours, $n = 0$ cannot be interpreted as non-participation.

Table 5 reports the fraction of households with negative labor earnings for each model parametrization. The share of households with negative labor earnings increases with the degree of coordination of hours worked in production and is relatively insensitive to preferences. The highest reported share is for separable preferences $\alpha = 0$, and $\rho = -1.962$: around 3.8‰, a negligible fraction of households.

C.10. Distribution of hours worked in the model and the data

Data. I calculate the distribution of hours worked in the data using the usual hours worked variable (*uhrswork1*) from the CPS Outgoing Rotation Group. The sample includes working-age women and men between 25 and 64 years old. Otherwise, the sample selection criteria are the same as in the baseline sample of Bick et al. (2022). I focus on individuals holding a single job, not enrolled in school, not self-employed, reporting positive earnings, and earning more than half a federal minimum wage. I discard observations with imputed or missing hours worked or earnings. I calculate group-specific mean usual weekly hours worked using CPS weights *wtfinl*.

Model. To translate the hours' distribution in the model into the data, I assume that the model mean hours correspond to the mean hours in the data. More concretely, in the model, I calculate the following object:

$$N = \int_{x \in B_x} \int_{a \in B_a} n(a,x) D(a,x)$$

N typically takes values around 1, and I assume that N corresponds to the mean usual hours worked at the main job in CPS: 40.72 hours a week. Based on this assumption, for each point on the state space, I calculate hours worked $n_{emp} = \frac{n_{model}}{N} \cdot 40.72$. Next, I assign individuals to bins as in Bick et al. (2022). I calculate the probability masses of agents within each bin and create histograms as in Figure 4.

C.11. Steady-state hours with GHH plus preferences

Figure 5 shows that introducing coordination of hours worked in production leads to a distribution of hours worked more concentrated around the mean. The effect is present regardless of the utility function assumed but tends to be stronger for low α calibrations, for example, for separable preferences.

C.12. Coordination and wealth distribution

Higher coordination of hours worked in production discourages households from working high hours. As a result, the wealth distribution becomes more equal with a smaller fraction of wealth held by the top 5% (see Figure 6).

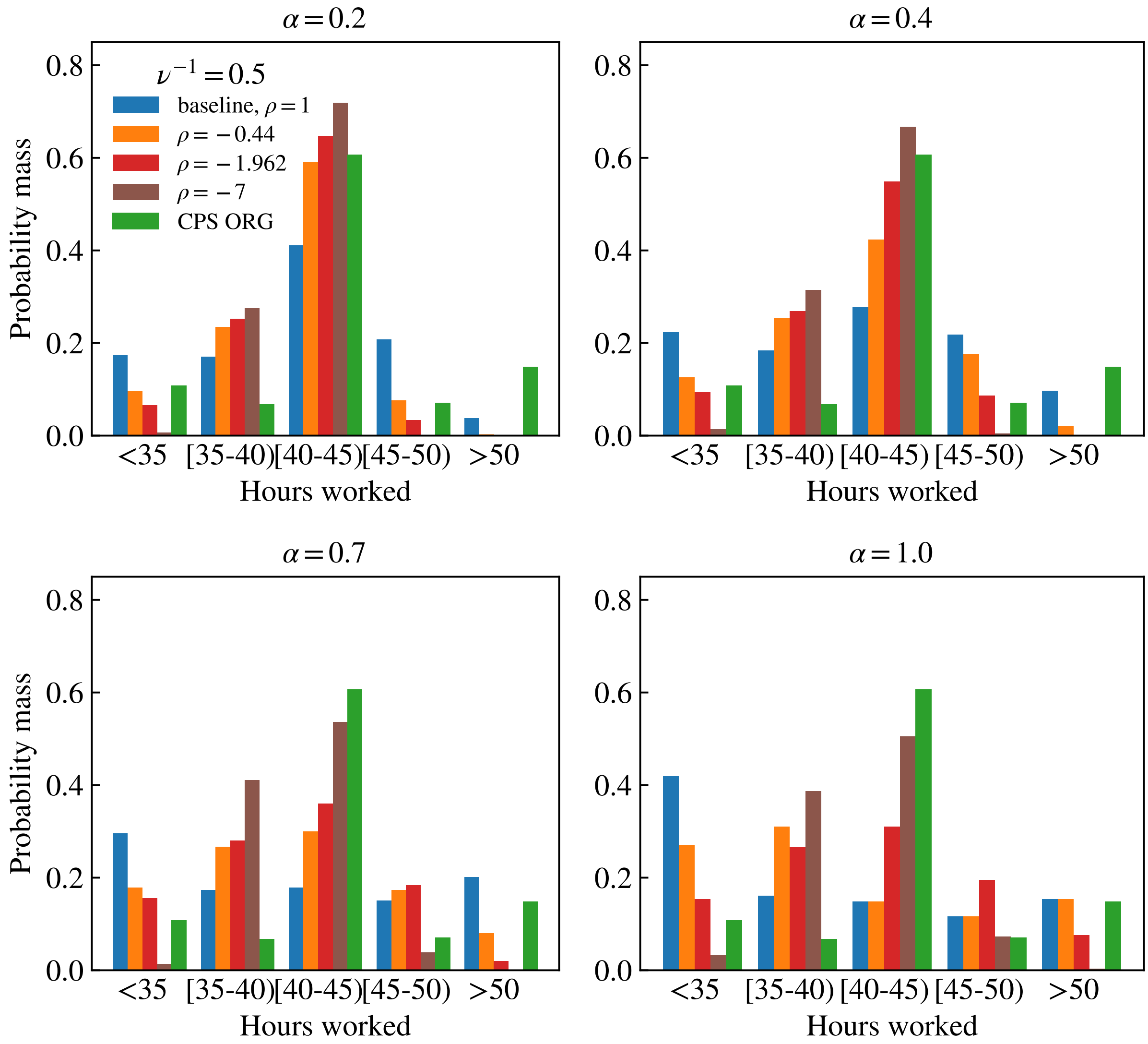

Figure 5: Steady-state distribution of hours worked for different values of α

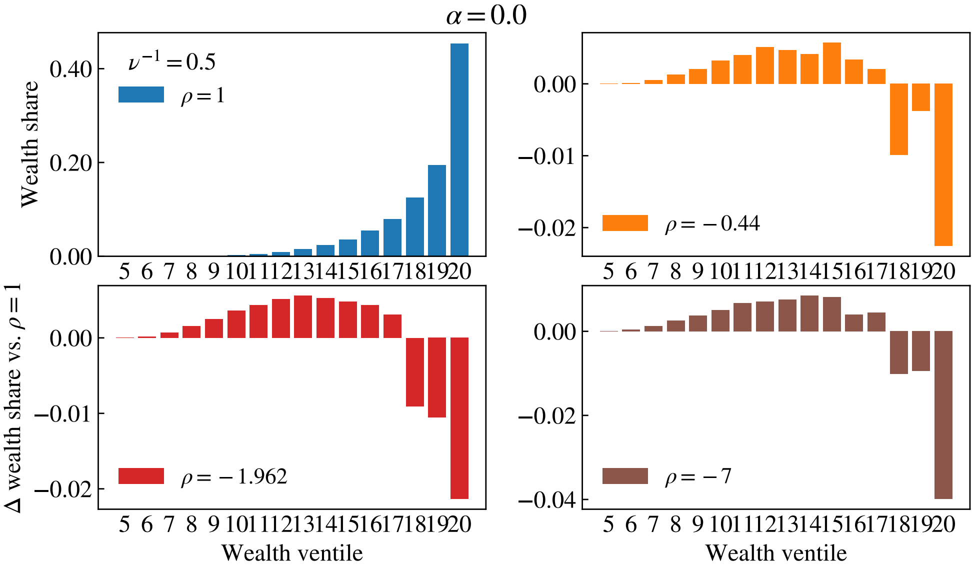

Figure 6: Hours' coordination and the steady state wealth distribution with separable preferences

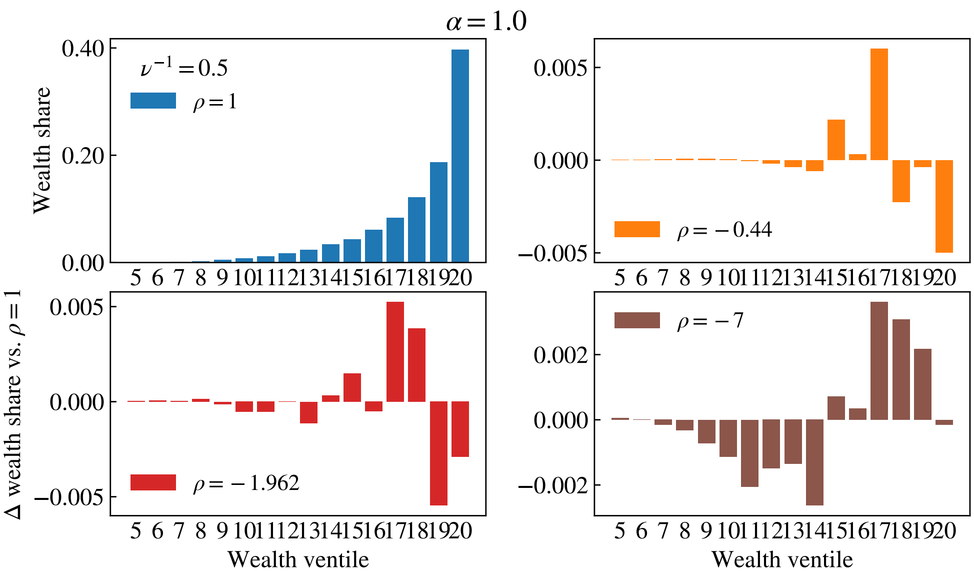

Figure 7: Hours' coordination and the steady state wealth distribution with GHH preferences