Optimal compensation schemes in organizations with interpersonal networks

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Abstract

I propose a mathematically tractable model to study the optimal compensation scheme in organizations with arbitrary interpersonal networks, in which agents connected in a network help neighbors and work collectively to produce a team's output, and payment is an equity share linked to the position in the network. The optimal compensation is shaped by: (a) "incentivizing the peripheral" mechanism: agents with a smaller (degree) centrality should be paid higher since they receive relatively less help; and (b) "incentivizing the central" mechanism: agents with a larger centrality should be paid higher to help others more. Due to these two conflicting mechanisms, the relationship between centrality and optimal pay may not be monotonic. When the externality of help is not pronounced, all agents should enjoy the ***same*** pay, as in a flat organization. I provide a necessary condition for monotonically increasing relationships and rule out monotonically decreasing relationships. I verify the theoretical predictions using numerical and empirical analyses.

JEL Classification: D23, D85, L23

Keywords: organization, network, team production, helping behavior, compensation scheme

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1 Introduction

The design of an optimal compensation scheme in an organization is a central research question in organizational economics. However, the existent literature is almost silent on this issue when considering complex interpersonal social networks that allow for multiple interactions and spillovers among connected agents. One of the most pressing reasons is that due to the complexity and arbitrariness of social networks, it is technically challenging to establish a mathematically tractable model to analyze this issue. In this paper, I provide the first tractable framework to investigate the relationship between the structure of social networks in an organization and its optimal compensation scheme, filling the gap in the literature.

In particular, as argued in Shi (2024) (*European Economic Review*), the structure of the social network within an organization is crucial for its performance, because network neighbors can help one another to create payoff spillovers.[1] In this paper, I ask how to design the optimal compensation scheme in which the pay of each agent is an equity share that may be linked to the position in the network, which provides a situation for strategic interactions among connected agents. To meet this end, I follow and extend the model setup of Shi (2024), but focus on providing theoretical insights on network-based optimal compensation schemes and empirical evidence. The major contribution of this paper is to provide a tractable framework to investigate the optimal compensation scheme associated with the network structure, which is also consistent with the empirical evidence.

In a networked organization,[2] each agent works to exert their own effort to jointly produce the team's output, and at the same time, connected agents can help each other to reduce the marginal disutility of working. Different from Shi (2024), I introduce heterogeneity in compensation and further introduce heterogeneity in ability.[3] Each agent receives a compensation which is a fixed share of the team's output, or an equity share as in Dasaratha, Golub, and Shah (2023). I assume that the output-sharing rule is employed for the compensation scheme to provide incentives for exerting efforts.[4] Moreover, since the principal cannot observe agents' efforts but can observe the network structure, the compensation has to depend on the share of the total output and the network position. Most of the results on the properties of the equilibrium in Shi (2024) still hold, including the existence and weak uniqueness of the equilibrium.

In a networked organization, where connected agents can help each other, the optimal compensation hinges upon the pattern of mutual help. For example, on an apple farm, workers have their friends and own social contacts within the farm. Each worker can do their own job sowing and watering apple trees that contribute to the output of the apples directly, and at the same time help their friends by handing over tools and sharing food that contributes to the output indirectly. In this setting, should more popular workers (workers who have more friends or social contacts within the farm) enjoy higher pay? My answer is surprising: no. Contrary to our intuition that more central agents should get higher pay, the relationship between centrality[5] and payment may not be strictly increasing, due to two competing forces. On the one hand, more pay should also go to peripheral

[1]In this paper, I assume that networks only allow for payoff spillovers but not information spillovers. I leave the latter case to future research.

[2]Networked organizations are defined as those with interpersonal social networks, which enable payoff spillovers through mutual help, as described in this paper.

[3]The heterogeneity in ability only applies to Proposition 5.

[4]If each agent gets a fixed share of the team's output, (s)he has incentives to work hard to increase the team's output, and, thus, his(her) payment.

[5]Centrality is the most commonly used measure to describe the position of a node in a network. In this paper, I focus on degree centrality, whereas the empirical analysis also holds for Bonacich centrality.

agents, since they do not enjoy access to much help from others, and have less incentive to work hard. Given a concave team production function, these peripheral agents are the weak spots of the organization. Therefore, this is the "incentivizing the peripheral" mechanism, whereby the peripheral agents should enjoy higher pay to be incentivized to work hard. On the other hand, more pay should go to agents with a large centrality, since they have the chance to help many other agents and thus increase the team's output. This is the "incentivizing the central" mechanism, whereby central agents should enjoy higher pay to be incentivized to help others.

The optimal compensation scheme should take into consideration the presence of these two mechanisms, which are contradicting. More specifically, I find that if the externality of helping effort on the team's output (characterized by β in the model part) is not salient enough, then these two effects neutralize each other perfectly. As a result, in this case, the same pay goes to the central and the peripheral agents.[6] This result is consistent with some real-world examples of organizations with flat organizational structures (despite uneven degree distribution of the social network) and payment schemes.[7]

A perfect example of such a flat organization is **Morning Star**, which is a food processing company located in California, USA.[8] Morning Star has implemented a self-management approach where each employee is responsible for managing their own work and making decisions that align with the company's mission. Therefore, the organization features a low level of β with a low level of the salience of helping efforts. It also features a flat organizational structure. There are no formal titles or supervisors, and employees enjoy a relatively flat payment scheme.

If helping effort matters a lot, or the externality of mutual help is salient, then the "incentivizing the central" effect dominates, since it is more important to incentivize the central agents to help others more. As a result, it is possible that higher pay may go to more central agents. This is a necessary (but not sufficient) condition for the monotonically increasing relationship between optimal pay and degree centrality. The intuition is that in this case, it is more important to incentivize the central agents since the payoff spillover or externalities from them are more salient. In particular, for star networks, I can provide sufficient and necessary conditions for a monotonically increasing relationship between optimal pay and degree centrality. I can further show that there can never be a monotonically decreasing relationship between optimal pay and degree centrality. When there is heterogeneity with respect to ability, there can be an arbitrary relationship between (degree) centrality and payment.[9] In all, the theory predicts that there should be a non-monotonic relationship between centrality and payment. In addition to the mathematical proof, I support such a prediction by numerical simulation and empirical analysis using the UK Workplace Employment Relations Study (WERS) data in Appendix D.

This paper is related to two strands of the literature. First, this paper contributes to research on incentive schemes in organizations (Groves, 1973; Holmstrom, 1982; Che and Yoo, 2001; Ichniowski and Shaw, 2003; Franco, Mitchell, and Vereshchagina, 2011). The main distinction of this paper compared to previous research is that this paper considers how compensation is related to the position in the network. In particular, this paper is related to Dasaratha, Golub, and Shah (2023), who study equity compensation in a networked team that

[6] The intuition relies on the concavity of the production function as a function of the compensation scheme and Jensen's inequality.

[7] In such organizations, the position in the social network does not matter much for the status of the agent in the organization and the payment received.

[8] See an introduction here: `https://www.morningstarco.com/exploring-morning-stars-mission-focused-self-management-mod`

[9] In the theoretical analysis, I focus on degree centrality, while in the empirical analysis, I use both the degree centrality and the Bonacich centrality. Moreover, the effect of "incentivizing the peripheral" cannot dominate in the setting of this paper because of the specific choice of functional forms. As a result, the relationship between optimal pay and centrality cannot be monotonically decreasing. However, under general functional form assumptions, it can be the dominant effect.

features exogenous and pre-determined complementarities between pairs of agents in production. The main difference between my paper and theirs is that the payoff spillovers among agents in my paper are endogenous decisions made by agents, not shaped by an exogenous structure as in their paper.

Second, this paper speaks to the literature on mutual help. This paper builds on the setting in Shi (2024), in which agents are connected through a fixed, undirected, and unweighted network, and work collectively to produce a team's output. Connected agents can help each other to reduce the marginal disutility of working. The optimal compensation scheme thus depends upon the pattern of connectedness in the network. This paper extends Kandel and Lazear (1992) in that they treat the compensation scheme as given and explore the pattern of mutual help, or more broadly, peer effects. Calvó-Armengol and Jackson (2010) study peer pressure in organizations that can be regarded as the opposite of mutual help. Different than these papers, I consider an organization with interpersonal networks, which are absent in these papers, and examine the optimal compensation in the presence of peer effects. Finally, Itoh (1991) concerns moral hazard problems in multi-agent situations where cooperation is an issue. This paper differs from it by introducing networks among agents.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium properties. Section 4 analyzes the optimal compensation scheme. Finally, Section 5 concludes.

2 Model

The setup of the model is heavily based on Shi (2024) except that the equity payment that goes to each agent can be different, and each agent can have different abilities in this paper.[10] As in Shi (2024), in the model, a principal is in charge of a team of agents who work collectively to produce a team's output. The principal cannot observe the effort of each agent but can observe the network structure, and hence the compensation of equity share given to each agent hinges only upon the team's output and the network position. I assume that the entire team's output is shared among all the agents. The output-sharing rule also provides incentives for each agent to exert more effort. The analysis is the same if a fixed share of less than one of the output is paid to the agents, and the rest of the output is paid to the principal. In the following analysis, I study the optimal compensation scheme in the sense that the total output is maximized.

Network Among Agents The model consists of a principal, denoted by P, and a team of $N \geq 2$ agents, denoted by $A = \{1, 2, ..., N\}$. Agents work together to produce a team's output, taking the compensation scheme set by the principal as given. Different from existent multi-agent models, here I assume that agents are inter-connected. Connections between agents could be seen as some interpersonal relationships such as acquaintance or friendship. Agents connected to i are deemed as her "neighbors," whose set is denoted by $\mathcal{N}(i) = \{j \in A : g_{ij} = 1\}$. The pattern of connections among agents is represented by a N by N matrix $\mathbf{G}$, which represents the adjacency of agents. Each entry of $\mathbf{G}$, $g_{ij} \in \{0, 1\}$, denotes whether there is a connection from i to j, and $g_{ij} = 1$ means that i and j are connected. By convention, I set $g_{i,i} = 0$.[11] I also denote the set of nodes of $\mathbf{G}$, $V(\mathbf{G})$, and the set of edges of $\mathbf{G}$, $E(\mathbf{G})$.[12]

Own Effort and Helping Effort In the model, the principal P observes the ability of each agent and

[10]Except for Proposition 5, I assume homogeneity of agents' abilities. However, different from Shi (2024), I assume throughout the paper that each agent can enjoy different payments.

[11]The network is undirected and connections are mutual, i.e., $g_{i,j} = g_{j,i}$ and $\mathbf{G}$ is a symmetric matrix.

[12]These notations are used in Definition 1.

sets the compensation scheme, which is the share of the output that is paid to each agent. Each agent $i \in A$ chooses her level of own effort which is denoted by e_i, as well as the level of helping effort to *each* of her neighbor $j \in \mathcal{N}(i)$, which is denoted by h_{ij}.[13] Agent i cannot afford the high psychological cost and hence is unwilling to help any other agent except her neighbors, who are more approachable to her.[14] The collection of own effort by each agent is denoted as $\mathbf{E} = \{e_1, ...e_N\}$, and the collection of the profile of helping effort of all agents is denoted as an N by N matrix $\mathbf{H}$, in which each entry, h_{ij}, is how much helping effort i exerts to j. In addition, denote the total received helping effort of each agent by $\mathbf{H}^r = \{\sum_{k \in N(1)} h_{k1}, ..., \sum_{k \in N(N)} h_{kN}\}$, and the total given helping effort of each agent is denoted by $\mathbf{H}^g = \{\sum_{k \in N(1)} h_{1k}, ..., \sum_{k \in N(N)} h_{Nk}\}$. Assume here that each agent has limited time and energy to expend on the two types of effort, i.e., both the own effort and the sum of helping effort have an upper bound, and hence $e_i \in [0, \bar{e}]$, and $\sum_{j \in \mathcal{N}(i)} h_{ij} \in [0, \bar{h}]$, where $\bar{e}, \bar{h} > 0$ are fixed constants and are sufficiently large so that in equilibrium each agent's strategy can never reach the upper bound. In such a setting, the strategy space for each agent is a convex and compact set.[15]

Each type of effort induces disutility or cost to agents. The cost of i's own effort is $(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji})^{-\beta} e_i$, where e_i is the level of i's own effort, and $(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji})^{-\beta} \equiv c(\theta, \sum_{j \in \mathcal{N}(i)} h_{ji})$ is the marginal disutility of exerting own effort, which is affected by i's "ability" to exert own effort, θ, and by helping effort received from i's neighbors, $\sum_{j \in \mathcal{N}(i)} h_{ji}$. Besides, in this setting, $c(\cdot)$ is a smooth function with $c \geq 0$, $c'_1 \leq 0$, $c'_2 \leq 0$, and $c''_{22} \geq 0$,[16] which means that the marginal effect of helping effort is decreasing.

β in the cost function $c(\theta, \sum_{j \in \mathcal{N}(i)} h_{ji})$ is the core parameter of the model. It reflects the salience of the effects of helping efforts on agents' own efforts. If β is large, then helping efforts play a quantitatively important role in the organization, since the effects of helping efforts on the marginal costs of agents' own efforts are large. This leads to the fact that more central agents should be more incentivized to help others more. If β is small, then helping efforts play a relatively trivial role in the organization. Thus, the same pay should go to all agents in the organization.

Disutility of helping effort is denoted by $\gamma v(\sum_{j \in \mathcal{N}(i)} h_{ij})$, where γ denotes the "ability" to help others, and v is a smooth function with $v \geq 0$, $v' \geq 0$, $v'' \geq 0$. Again, I assume for now that all agents have the same ability to help others, but I will relax this assumption later in Proposition 5. For simplicity, I assume that $v(x) = 0.5x^2$.[17] Hence the marginal disutility of helping effort at zero is zero, which guarantees that in (a nontrivial) equilibrium all agents are willing to help.

The efforts of agents, both individuals' own and helping efforts, can be illustrated as follows. Imagine a group of farm workers who are responsible for growing fruit. Each worker's own effort is focused on producing as much fruit as possible, which directly contributes to the farm's overall output. In addition, each worker can assist others, for example, by providing water to those who are thirsty or sharing their food with those who are hungry. This helping effort does not directly increase the amount of fruit produced or the farm's total output, but it does reduce the marginal disutility of working. In addition, in this example, agents' own effort and helping effort require different sets of skills, and, thus, can be regarded as independent. Therefore, the

[13] In the example of an apple farm, the agent's own effort is growing apple trees directly, and the helping effort is handing over tools and sharing food with friends.

[14] Please see Shi (2024) for a detailed discussion of the ability to help and the willingness to help.

[15] Note that there are no interactions between helping efforts and own efforts. They enter the cost function in an additively separable way. Such a setting is critical for the tractability of the model.

[16] $c'_1 = -\beta(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji})^{-\beta-1} \leq 0$, $c'_2 = -\beta(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji})^{-\beta-1} \leq 0$, and $c''_{22} = \beta(\beta+1)(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji})^{-\beta-2} \geq 0$.

[17] Hence, $v' = x$ and $v'' = 1$.

setting of the model is justified[18] in which agents' own effort and helping effort do not interact with each other, and, can face separate and independent constraints and take separate values from two different compact sets. I will revisit these assumptions in Section 4.3.

Production Technology, Compensation, and Payoff The team's output y is a function of own effort of all agents: $y = f(e_1, ..., e_N)$. For tractability, assume here that f takes the functional form of Cobb-Douglas: $f(\mathbf{e}) = \prod_{i=1}^{N} e_i^{w_0}$, where $Nw_0 < 1$, so that $f(\cdot)$ is strictly concave[19]. Hence each agent plays an identical role in production, and the production function exhibits a decreasing return to scale so that the scale of the organization is finite. In addition, we can see that f is a smooth function with strict concavity, $f \geq 0$, $f'_i \geq 0$ and $f''_{ij} \geq 0$[20] for any i and j, which means that the production technology exhibits the property of complementarity. Complementarity also implies the non-separability of each agent's contribution. It also provides a foundation for the proof of Lemmas 6 and 7.

The principal can observe the team's output, yet she cannot observe the agents' own efforts and helping efforts. Therefore, she sets the compensation scheme dependent only upon the team's output y. Moreover, the principal can observe each agent's network position, and, thus, she links the payment to each agent's network position. This is a novel setting in the literature of organizational economics. Consistent with the existing literature, here I only consider the simple output sharing rule and equity payment, where the principal pays a share of α_i in the output to agent i. Without loss of generality, I assume that $\sum_{i=1}^{N} \alpha_i = 1$, i.e. the principal does not retain any profit. In this way, I simplify the model by abstracting away the optimization of the principal's payoff but instead look at the optimal compensation scheme in the sense that the team's output is maximized. In other words, I first study the equilibrium outcomes given a fixed compensation scheme, and then how to design a compensation scheme to maximize the team's output.

The payoff of each agent is the compensation obtained minus disutility yielded from exerting their own effort and helping effort. Therefore the payoff of agent i given the above output sharing compensation and functional forms could be written as

$$\alpha_i \prod_{i=1}^{N} e_i^{w_0} - (\theta + \sum_{j \in \mathcal{N}(i)} h_{ji})^{-\beta} e_i - \frac{1}{2}\gamma(\sum_{j \in \mathcal{N}(i)} h_{ij})^2$$

.

Timing of the game The game proceeds as follows. In stage 0, P observes the ability $\{\theta, \gamma\}$ of each agent and the network structure $\mathbf{G}$, and sets the compensation scheme $\{\alpha_i\}_{i=1}^{N}$. The optimal compensation scheme denotes with a star as $\{\alpha_i^*\}_{i=1}^{N}$. I assume that the principal does not get any surplus, and instead leaves all surplus to the agents.[21] Thus, I do not study the profit/surplus maximization of the principal, but the maximization of the team's output. In stage 1, given the team structure $\mathbf{G}$ and the compensation scheme $\{\alpha_i\}_{i=1}^{N}$, each agent simultaneously chooses her helping effort $H_i = (h_{i1}, ..., h_{ij})$, $j \in \mathcal{N}(i)$. In stage 2, given team structure $\mathbf{G}$, compensation scheme $\{\alpha_i\}_{i=1}^{N}$, and helping effort $\mathbf{H}$ chosen in the former stage, each agent simultaneously decides her own effort e_i. This setting is the same as Calvó-Armengol and Jackson (2010), where agents first choose peer pressure exerted on other agents and then choose another action (such as whether to

[18] I discuss the implication of this setting in Section 4.3.

[19] The form of Cobb Douglas can make the model tractable. More general functional forms, like constant elasticity of substitution, cannot. Strict concavity is needed for several main theoretical results below.

[20] $f'_i = w_0 e_i^{-1} \prod_{i=1}^{N} e_i^{w_0} \geq 0$ and $f''_{ij} = w_0^2 e_i^{-1} e_j^{-1} \prod_{i=1}^{N} e_i^{w_0} \geq 0$.

[21] The analysis and the main results are the same if the principal retains a fixed proportion of the output.

participate) based on former choices of peer pressure.

In this paper, I focus on (pure strategy) subgame perfect Nash equilibria of the game, in which agents first decide the pattern of mutual help devoted to the neighbors, then decide their own efforts.

3 Equilibrium Properties

In this section, I analyze equilibrium properties given a fixed set of equity shares $\{\alpha_i\}$. I extend the results of the existence and generic uniqueness of equilibrium in Shi (2024). The only difference, for now, is that the equity compensation for each agent can be different, and in Proposition 5 below, I further introduce heterogeneity with respect to ability parameters. Thus, efforts given in equilibrium satisfy equations in which they are weighted by inverse equity share.[22] The details of model derivations and the proofs of this section are relegated to Appendix C.

Also, note that there is always a trivial equilibrium, which is a result of the functional form of the Cobb-Douglas production function: in stage 1 all agents exert zero helping effort, and in stage 2 all agents exert zero own effort. In this equilibrium, the output is zero. So this equilibrium is Pareto inferior to that where helping efforts and own efforts are strictly positive and thus the output is also strictly positive. I assume that there is some exogenous communication device to avoid such trivial and inefficient equilibrium.

3.1 Equilibrium Existence

First, I provide the proposition for the existence of a subgame perfect Nash equilibrium, for the specific functional forms of the model. We have a proposition regarding the existence of the equilibrium as follows:

Proposition 1. *Given the output sharing rule and the above-mentioned functional forms, a non-trivial subgame perfect Nash equilibrium of the game exists.*

This is a direct implication of Theorem 1 in Rosen (1965), which states that in an N-person concave game where the strategy space of each agent is convex and compact and the payoff function is concave in his own strategy, there always exists a Nash equilibrium. Note that the overall game has two classes of subgames of stages 1 and 2, and it is easy to verify that the existence condition stated by Theorem 1 in Rosen (1965) is satisfied in these two classes of subgames.

I use backward induction to solve for the equilibrium. In stage 2, given the collection of helping effort in stage 1 and the choices of own effort of others, each agent decides her level of own effort to maximize her payoff, taking the choices of other agents $\{e_j^*\}_{j\neq i}$ as given. Hence

$$e_i^* = \arg\max_{e_i} \alpha_i {e_1^*}^{w_0} {e_2^*}^{w_0} \cdots {e_i}^{w_0} \cdots {e_N^*}^{w_0} - (\theta_i + \sum_{j\in\mathcal{N}(i)} h_{ji})^{-\beta} e_i - \frac{1}{2}\gamma(\sum_{j\in\mathcal{N}(i)} h_{ij})^2. \tag{1}$$

The first-order condition for agent i with respect to e_i is that

$$\alpha_i w_0 {e_1^*}^{w_0} {e_2^*}^{w_0} \cdots {e_i}^{w_0-1} \cdots {e_N^*}^{w_0} - (\theta_i + \sum_{j\in\mathcal{N}(i)} h_{ji})^{-\beta} = 0,$$

[22] In a model with homogeneous equity shares, such weights all cancel out in the associated equations.

for each $i \in 1, 2, ..., N$.

Combining the N first-order conditions for each agent, we could solve for the equilibrium own effort of the subgame of stage 2. Note that given the specific functional form, the system of equations has a unique strictly positive solution $\{e_i^*\}$:

$$e_i^* = \alpha_i w_0^{\frac{1}{1-Nw_0}} (\theta + \sum_{k \in \mathcal{N}(i)} h_{ki})^\beta \prod_{j \geq 1} [\alpha_j (\theta + \sum_{k \in \mathcal{N}(j)} h_{kj})^\beta]^{\frac{w_0}{1-Nw_0}} \tag{2}$$

The details of derivations are relegated to Appendix C. By backward induction, substituting equation (2) to the payoff function, we could find that in stage 1 agent i's problem is to find $\{h_{ij}\}_{j \in \mathcal{N}(i)}$ given $\{h_{kl}^*\}_{k \neq i, l \in N(k)}$:

$$h_{ij}^* = \arg\max_{h_{ij}} \alpha_i \prod_{j \geq 1} \alpha_j^{\frac{w_0}{1-Nw_0}} w_0^{\frac{1}{1-Nw_0}} (\frac{1}{w_0} - 1)(\prod_{k \geq 1} (\theta + \sum_{l \in N(k)} h_{kl}^*)^\beta)^{\frac{w_0}{1-Nw_0}} - \frac{1}{2}\gamma(\sum_{j \in \mathcal{N}(i)} h_{ij})^2, \tag{3}$$

We could further derive the first-order condition with respect to h_{ij} as an inequality of a complementary slackness condition as follows

$$\alpha_i \prod_{j \geq 1} \alpha_j^{\frac{w_0}{1-Nw_0}} \frac{\beta w_0}{1 - Nw_0} \prod_{j \geq 1} (\theta + \sum_{k \in \mathcal{N}(j)} h_{kj}^*)^{\frac{\beta w_0}{1-Nw_0}} (\theta + \sum_{k \in \mathcal{N}(j)} h_{kj}^*)^{-1} (\alpha_i w_0)^{\frac{1}{1-Nw_0}} (\frac{1}{w_0} - 1) \leq \gamma(\sum_{j \in \mathcal{N}(i)} h_{ij}^*) \tag{4}$$

Note that the first-order condition (4) reflects the relation between the total helping effort exerted by i, $\sum_{j \in \mathcal{N}(i)} h_{ij}^*$, and the total effort received by j, $\sum_{k \in \mathcal{N}(j)} h_{kj}^*$. If h_{ij} takes a *corner* solution, then (4) takes a strict less equal sign. If h_{ij} takes a *interior* solution, then (4) takes an equal sign. Note that when $h_{ij} = 0$ (4) can take either a strict less equal sign or an equal sign. Also, $h_{ij} = 0$ can be an interior solution as well as a corner solution.

In equilibrium, if there exist agents $i, j, l, m \in A$ such that h_{ij}^* and h_{kl}^* take *interior* solutions, then $1/\alpha_i \times (\sum_{j \in \mathcal{N}(i)} h_{ij}^*)(\theta + \sum_{k \in \mathcal{N}(j)} h_{kj}^*) = 1/\alpha_k \times (\sum_{j \in N(k)} h_{kj}^*)(\theta + \sum_{m \in N(l)} h_{ml}^*)$, and thus provides an important equation to compute the equilibrium. It is also linked with the proof of Proposition 4, in which agents' own efforts are also weighed by the inverse equity share ($\frac{1}{\alpha_i}$). In addition, equation (4) directly implies Lemma 1 as follows.

Lemma 1. *In (any non-trivial) equilibrium and with non-zero equity shares, each non-isolated agent (the agent who has at least one neighbor) receives and gives a strictly positive amount of help. In the trivial equilibrium, each agent gives and receives zero help.*[23]

Since the marginal cost of helping effort is zero when the total helping effort is zero, all agents are willing to help for a strictly positive amount. On the other hand, in the network structure of the organization, all agents balance the marginal return of helping their different neighbors, and hence will not leave any of their neighbors without receiving any help. Below is another lemma that describes this aspect of equilibrium. On the other hand, however, there exists a trivial equilibrium in which all helping efforts and agents' own efforts are zero.

Lemma 2. *In (any non-trivial) equilibrium, each agent only helps her neighbors who receive the least amount of help (there would be more than one neighbor receiving the minimum help). Also, for each agent, only neighbors*

[23] There is no equilibrium that some of the efforts are strictly zero and others are strictly positive.

who give the least amount of help divided by the equity share would help her (there would be more than one neighbor giving minimum help).

Lemma 2 implies that in the network structure, in any equilibrium all agents simultaneously balance the marginal return and marginal cost for the whole organization: all agents will avoid helping their neighbors who already receive enough help, and all agents have the most efficient (least marginal cost) neighbors to help them. Lemma 2 in this paper is different from that in Shi (2024) since here the compensation for different agents can be different. Therefore, here, the helping efforts given should be weighted by inverse equity share.

Below is another lemma that describes the equilibrium condition derived by the first-order condition (4). Again, I need to adjust the argument for the given helping efforts by inverse equity shares, since they are related to the compensation scheme.

Lemma 3. *In (any non-trivial) equilibrium, for each agent i, if there are two neighbors j_1, j_2 such that $h^*_{j_1 i}$ and $h^*_{j_2 i}$ take interior solutions, then j_1 and j_2 give the same amount of help divided by the equity share (or weighted by inverse equity share α). Similarly, if $h^*_{ij_1}$ and $h^*_{ij_2}$ take interior solution, then they receive the same amount of help.*

Lemma 3 is established simply by comparing different first-order conditions, and it implies the equilibrium configuration in a connected network. The following Lemma 4 further shows that the equilibria can be characterized by bipartite graphs.[24]

Lemma 4. *For a non-trivial equilibrium: Suppose that $\boldsymbol{H}^*$ is an equilibrium profile of helping efforts in which the first-order conditions of all elements take interior solutions. Given $\boldsymbol{H}^*$, define a new matrix $\bar{\boldsymbol{H}}^* = (\bar{h}^*_{ij})$ whose entries are zeros or ones, with $\bar{h}^*_{ij} = 0$ if h^*_{ij} in $\boldsymbol{H}^*$ takes a corner zero solution, and $\bar{h}^*_{ij} = 1$ if h^*_{ij} in $\boldsymbol{H}^*$ takes an interior solution. Then, if $\bar{\boldsymbol{H}}^*$ is a* ***bipartite graph****, then there are no more than two different amounts of the total received helping efforts ($\sum_{k\in\mathcal{N}(j)} h^*_{kj}$) and two different amounts of the total given helping efforts ($\sum_{k\in\mathcal{N}(j)} h^*_{jk}$) weighted by inverse equity share, and agents in each of the two independent sets give and receive, respectively, the same amount of helping efforts. If $\bar{H}^*$ is not a* ***bipartite graph,*** *then there is only one amount of total received or given helping effort (weighted by inverse equity share).*

To rephrase Lemma 4 mathematically, the argument that "there are no more than two different amounts of the total received helping efforts ($\sum_{k\in\mathcal{N}(j)} h^*_{kj}$) and two different amounts of the total given helping efforts ($\sum_{k\in\mathcal{N}(j)} h^*_{jk}$) weighted by inverse equity share, and agents in each of the two independent sets give and receive, respectively, the same amount of helping efforts (weighted by inverse equity share)" is equivalent to saying that the elements in each of the sets $\{\sum_{k\in\mathcal{N}(j)} h^*_{kj}\}_{j\in A}$ and $\{\sum_{k\in\mathcal{N}(j)} h^*_{jk}\}_{j\in A}$ (weighted by $1/\alpha_j$) take at most two distinct values. Moreover, if the bipartite graph $A = A_1 \cup A_2$, where A_1 and A_2 are two independent sets, then for all $j_1, j_2 \in A_k$ ($k = 1, 2$), $\sum_{k\in\mathcal{N}(j_1)} h^*_{kj_1} = \sum_{k\in\mathcal{N}(j_2)} h^*_{kj_2}$, and $\sum_{k\in\mathcal{N}(j_1)} h^*_{j_1 k}/\alpha_{j_1} = \sum_{k\in\mathcal{N}(j_2)} h^*_{j_2 k}/\alpha_{j_2}$.

3.2 Equilibrium Uniqueness

In this section I characterize the most important property of subgame Nash equilibria: in the same fixed network, all equilibria lead to the same profile of own effort $\mathbf{E}^*$. Different networks can have different equilibria.

[24] A bipartite graph is a graph where the vertices can be divided into two disjoint sets such that all edges connect a vertex in one set to a vertex in another set. Such a set is an independent set of the bipartite graph.

In other words, in any different equilibria of the same fixed network, each of the agents gives (weighted by inverse equity share) and receives the same total amount of help. It is only the specific pattern of help, who gives help to whom or who receives help from whom, might be different across equilibria.

Proposition 2. *For any fixed network structure* $\boldsymbol{G}$*, all non-trivial subgame perfect Nash equilibria lead to the same profile of own effort* $\boldsymbol{E}^* = \{e_1^*, ..., e_N^*\}$. *in such a fixed network. For each agent, she exerts and receives the same total amount of helping effort in different equilibria. They also have the same level of payoff across equilibria.*

Proposition 2 establishes the weak uniqueness of equilibrium with respect to the *total* amount of given and received helping efforts, which is independent of the structure of the network.[25] On the other hand, however, the multiplicity of equilibria depends on the structure of the network: if there is no loop in the network, then the equilibrium helping effort can be uniquely determined. This is what Corollary 1 argues.

Corollary 1. *If the network contains loops, there are multiple equilibria; otherwise, there is a unique equilibrium.*

In Shi (2024), I also provide a mathematical interpretation and associate the loopy network structure with the rank of the auxiliary matrix defined therein. To be short, consider a ring network with N agents. There are two equilibria in which agents help one another in a clockwise or a counterclockwise fashion, but the equilibrium profile of agents' own efforts and payoffs in these two equilibria is the same.

4 Optimal Compensation Schemes

The above-mentioned game setting and functional forms produce an equilibrium that exists and is weakly unique. The properties are not essentially different than those in Shi (2024). I would further emphasize here that given the above setting, there exists an equilibrium (Proposition 1) in which the profile of own efforts and equilibrium payoffs is strictly unique given a fixed network structure (Proposition 2), if we do not consider the trivial equilibrium in which the equilibrium own and helping efforts are zero. In this section, I focus on presenting the theoretical results of the optimal compensation scheme.

4.1 Characterizing Optimal Compensation Schemes

In this paper, I assume that the total share of output paid to agents is 1. Thus, I do not examine the decision of the principal who may try to maximize his surplus gained from the organization of the agents. I study the optimal compensation scheme, in which "optimal" means that the maximum of the output of the organization is

[25] I can also use Theorem 3 in Kennan (2001) to establish the uniqueness result. It is based on the condition that $f(\cdot)$ (in that paper) is increasing and strictly concave and satisfies $f(0) > 0$ (can be further relaxed to $f(0) \geq 0$, since we can focus on an additional fixed point other than zero), $f(a) > a$, and $f(b) < b$ for some vector $b > a > 0$. In my setting, since the $f(\cdot)$ function is Cobb-Douglas, $c(\cdot)$ is linear, and $v(\cdot)$ function is quadratic, the weakly positive (at zero), increasing, and strictly concave (given $Nw_0 < 1$) conditions are satisfied. Moreover, given the Inada conditions of the Cobb-Douglas function, given the marginal return of own effort e_i goes to infinity as e_i goes to zero (the function is sufficiently "steep"), $f(a) > a$ is satisfied with a vector a that is close to 0, and given the marginal return of own effort e_i goes to 0 (the function is sufficiently "flat") as e_i goes to infinity, I can also find a vector b that is sufficiently large (but still finite). Therefore, I can show that all conditions required by Theorem 3 of Kennan (2001) are easily satisfied in my setting.

achieved. Thus, the optimal compensation scheme solves $\{\alpha_i^*\}_{i=1}^N = \arg\max_{\{\alpha_i\}_{i=1}^N} \prod_{i=1}^N e_i^*(\{\alpha_i\}_{i=1}^N)^{w_0}$, where $e_i^*(\{\alpha_i\}_{i=1}^N)$ is the optimal choice of own efforts given the compensation scheme.

The optimization problem to solve for the optimal compensation scheme is formally as follows:

$$\max_{\alpha_1,...,\alpha_N} \prod_{i=1}^N (e_i^*(\alpha_1,...,\alpha_N))^{w_0}, \tag{5}$$

The first-order condition is (for all $k \in \{1, 2, ..., N\}$):

$$\prod_{i=1}^N (e_i^*(\alpha_1,...,\alpha_N))^{w_0} w_0 \sum_{i=1}^N [(e_i^*)^{-1} \frac{\partial e_i^*}{\partial \alpha_k}] = \lambda. \tag{6}$$

where λ is the shadow price of the total equity shares, or equivalently, the Lagrangian multiplier associated with the constraint that all equity shares add up to one.

Thus, for all k:

$$\sum_{i=1}^N [(e_i^*)^{-1} \frac{\partial e_i^*}{\partial \alpha_k}] = \sum_{i=1}^N [(e_j^*)^{-1} \frac{\partial e_j^*}{\partial \alpha_k}]. \tag{7}$$

To start with, we have a lemma that rules out corner solutions of optimal compensation schemes.

Lemma 5. *For any optimal compensation scheme $\{\alpha_i^*\}_{i=1}^N$, $0 < \alpha_i^* < 1$ for all $i = 1, 2, ..., N$, if $w_0, \beta > 0$.*

The proof is straightforward since under any equity share with $\alpha_i^* = 0$, we have $e_i^* = 0$ and $\frac{\partial e_j^*}{\partial \alpha_k} = 0$ given equation (2). Thus, the team's output is zero, which can never achieve the maximum since it is dominated by all profiles of equity shares with $0 < \alpha_i^* < 1$ for all $i = 1, 2, ..., N$ that produces a strictly positive team's output, given Lemma 1 that dictates that all agents' own efforts are strictly positive when $0 < \alpha_i^* < 1$. The role of Lemma 5 is to rule out all corner solutions for all theoretical results below.

Using the above equations (2), (4), (7), and the constraint on αs (adding up to one), we can first establish the increasing relationship between efforts and payment that provides incentives. The intuition is that given a higher level of reward incentives, each agent is willing to work harder. As agents' own efforts are complementary, each agent is also willing to work harder if others enjoy higher pay (when their own pay is fixed).

Lemma 6. *The equilibrium own efforts and helping efforts are strictly increasing functions of own and others' equity share given that γ is sufficiently small: $\frac{\partial e_i^*}{\partial \alpha_k}|_{\alpha_{j,j\neq k}} > 0$, for all $i \neq j, k \in \{1, 2, ..., N\}$. The partial derivatives are obtained by taking other $\alpha_{j,j\neq k}$ fixed.*

The proof is based on a system of equations (2) and (4) and exploits an implicit function theorem argument. The proof needs a parameter restriction that γ is sufficiently small (but still strictly positive). The intuition is that the marginal cost of helping effort should be small enough so that providing stronger incentives by a greater α indeed induces more (own) effort. Based on Lemma 6, we have the following important property for the production function: concavity with respect to $\{\alpha_i\}$.

Lemma 7. *Under parametric restrictions that (1) γ is sufficiently small; (2) $\frac{\beta w_0}{1-Nw_0}$ is sufficiently small; and (3) $\frac{Nw_0}{1-Nw_0} < 1$, the function*

$$g(\alpha_1, ..., \alpha_N) \equiv f(e_1^*(\alpha_1, ..., \alpha_N), ..., e_N^*(\alpha_1, ..., \alpha_N))$$

is strictly concave with respect to $\{\alpha_i\}$.

The intuition of the proof of Lemma 7 is to set an upper bound for the partial derivatives of one component. Next, we establish the proposition for the existence and uniqueness of the optimal payment scheme. It relies on the Weierstrass theorem and Lemma 7 since the production function is a continuous and strictly concave function defined on a compact set.

Proposition 3. *Given the parametric restrictions of Lemmas 6 and 7 so that* $g(\alpha_1, ..., \alpha_N)$ *is concave in* α*: For any network structure* $\boldsymbol{G}$*, there is a unique optimal compensation scheme that maximizes the team's output.*

Given the detailed network structure, we may also derive the following symmetry of the payment system. The results are also intuitive, since agents in the same environment of arbitrary order of network neighbors play exactly the same role in the organization and, thus, should enjoy the same level of pay in the optimal payment system. I provide the formal definition of symmetry below.

Definition 1. *Two nodes or agents* i *and* j *have symmetric positions in the network* $\boldsymbol{G}$*, if for any graph automorphism* ϕ *(a bijection* $\phi : V(\boldsymbol{G}) \to V(\boldsymbol{G})$ *such that* $uv \in E(\boldsymbol{G})$ *if and only if* $\phi(u)\phi(v) \in E(\boldsymbol{G})$*),* $i = \phi(j)$ *and* $j = \phi(i)$*.*

Recall that $V(\mathbf{G})$ is the set of nodes in $\mathbf{G}$, and $E(\mathbf{G})$ is the set of edges in $\mathbf{G}$. It is easy, then, to establish the following lemma regarding the payment of different agents who have a symmetric position in the optimal compensation scheme. The intuition is built on the definition of symmetry, which, in turn, is based on the definition of graph automorphism. Graph automorphism, by definition, preserves edge and node relationships. In particular, note that if two agents are regarded as having symmetric positions, then they have the same number of first-order network neighbors (denoted as $\mathcal{N}_1(i)$), second-order neighbors ($\mathcal{N}_2(i)$), ..., and kth-order neighbors ($\mathcal{N}_k(i)$), and their first-order neighbors have the same number of first-order network neighbors, second-order neighbors, ..., and kth-order neighbors, ..., and their kth-order neighbors have the same number of first-order network neighbors, second-order neighbors, ..., and kth-order neighbors, where k can be any arbitrarily large positive integer that tends to infinity. Thus, these symmetric agents face the same set of first-order conditions.

Lemma 8. *In an arbitrary network, given the parametric restrictions of Lemmas 6 and 7 so that Proposition 3 holds, the agents in the symmetric positions receive the same pay. As a result, they receive and give the same amount of helping efforts.*

The role of Lemma 8 is to simplify calculation to analyze some specific examples of networks (in Appendix B). All examples are consistent with the theoretical results. Given Lemma 8, we have the following corollary.

Corollary 2. *In any symmetric graph* $\boldsymbol{G}$ *(where given any two pairs of adjacent vertices* (u_1, v_1) *and* (u_2, v_2)*, there is an automorphism* $\phi : V(\boldsymbol{G}) \to V(\boldsymbol{G})$ *(defined in Definition 1), such that* $f(v_1) = v_2$ *and* $f(u_1) = u_2$*), all agents enjoy the same optimal pay for any parameter values, as long as these values are the same for each agent.*

Corollary 2 is a direct implication of Lemma 8, and its proof is straightforward. Corollary 2 also applies to specific symmetric graphs including ring networks and complete networks of an arbitrary number of agents.

Also, note that all agents should enjoy the same optimal pay given any set of parameter values, as long as these values are the same for each agent. However, we also have a result in which all agents enjoy the same optimal pay, but with **arbitrary network structures** and some **specific parameter values**. We thus have the following Proposition 4 below.

4.2 Payment and Position

In this subsection, I formalize two propositions to describe the relationship between network position, or (degree) centrality,[26] and optimal payment. I focus on degree centrality because it is easy to construct and, thus, can be naturally connected to the proofs of this paper. Also, in the theoretical results below, the first-order degree centrality is sufficient to characterize the monotonicity of the relationship between optimal pay and network position. The first proposition states that the relationship between centrality and payment can be non-monotonic, and it depends on the parameter β, which describes the salience of the externality of help. The proofs are relegated to Appendix A.

Proposition 4. *Given the parametric restrictions of Lemmas 6 and 7 so that $g(\alpha_1, ..., \alpha_N)$ is concave in α: For any network structure $\boldsymbol{G}$, and ability parameters $\{\theta, \gamma\}$, there is a cutoff $\bar{\beta} = \bar{\beta}(\boldsymbol{G}, \theta, \gamma)$, such that if $\beta \leq \bar{\beta}$, then in the optimal compensation scheme, same pay goes to agents with different positions; If $\beta > \bar{\beta}$, then the pay received by agents with different position may differ. The relationship between pay and degree centrality may not be monotonic. Finally, optimal pay may increase monotonically with (degree) centrality only when $\beta > \bar{\beta}$.*

The result in Proposition 4 is consistent with some real-world examples of organizations with flat organizational structures (despite potentially uneven degree distribution of the social network) and payment schemes. In such organizations, different agents possess different positions in the social network, but they have relatively equal status and play a relatively similar role in the organization, and, thus, enjoy similar pay.

In addition to the example of **Morning Star** in the Introduction, another valid example is **Semco Partners**,[27] a diversified manufacturing and services company located in Sao Paulo, Brazil. It is known for its democratic workplace where employees have significant and independent control over their work environment, schedules, and decision-making processes. Thus, the organization features a low β with a small salience of helping efforts. There is a minimal hierarchy, and employees often set their own goals, participate in their own decision-making, and enjoy a flat payment scheme.

Proposition 4 is also built on Lemma 7, and it indicates that $\beta > \bar{\beta}$ is the necessary condition for the monotonically increasing relationship between optimal pay and degree centrality, but it is not sufficient.[28] In particular, the parametric restrictions in Lemmas 6 and 7 can hold for both the cases of $\beta > \bar{\beta}$ and $\beta \leq \bar{\beta}$.

The intuition of the proof is that given the concavity (Lemma 7), a disturbance at equal pay should lead to a first-order reduction of the total output, especially given that the responses of own and helping efforts are bounded due to a small β. Given a larger β, the responses of own and helping efforts can be large enough to offset the reduction of the output due to concavity.

[26]Centrality, including degree centrality and Bonacich centrality, is the most important and widely used indicator to describe the network position of each node.

[27]See an introduction here: `https://hbr.org/1989/09/managing-without-managers`.

[28]Given my functional restrictions and as the examples suggest, it is infeasible to analytically derive for a sufficient condition for the monotonically increasing relationship between optimal pay and degree centrality. I can only show, numerically, that given some values of parameters, the relationship between optimal pay and degree centrality can be strictly increasing.

The reason to focus on degree centrality, not other forms of centrality, such as Bonacich centrality, as in the existing literature is that, in this paper, I do not use the traditional framework of quadratic payoff functions and linear best-replies, under which Bonacich centrality naturally pops out.[29] Moreover, the flat relationship between the degree centrality and optimal pay depends on the specific functional form restrictions. In the empirical analysis in Appendix D, however, I find that the results of non-monotonicity also apply to Bonacich centrality. Moreover, Proposition 4 is consistent with all numerical examples of a small number of agents (N) in Appendix B. For example, Figure B1(b) shows that when β is low, each agent should get the same pay, but when β is high, the central agent should get a higher pay.

More importantly, Proposition 4 only provides a necessary condition for the strictly increasing relationship between optimal pay and degree centrality. Unfortunately, due to the complexity of arbitrary network structures, it is infeasible to provide a sufficient condition under which the optimal pay must strictly increase with the degree centrality. For example, in a line network, the relationship between optimal pay and degree centrality can never be strictly increasing for any set of parameters' values. See a four-agent line network and Figure B2 for example. Thus, to establish a sufficient condition for monotonicity, I need to step back and focus on specific network structures. For example, I am able to establish a sufficient (and necessary) condition for monotonicity for star networks in the following corollary, which also requires the parametric restrictions of Lemma 7 so that Proposition 3 holds.

Corollary 3. *Given the parametric restrictions of Lemmas 6 and 7: In an N-agent star network $\boldsymbol{G}^{star,N}$, there is a cutoff $\bar{\beta}(N)$ (cutoff as a function of the number of nodes), such that all N agents get the same pay if and only if $\beta \leq \bar{\beta}(N)$, and the central agent gets a strictly higher pay than peripheral agents if and only if $\beta > \bar{\beta}(N)$.*

The proof is based on Lemma 8, which dictates that all peripheral agents should enjoy the same pay. Thus, the set of optimal pay of all agents only has two distinct values, and, thus, the proof follows. Numerical analyses and examples regarding star networks that are consistent with Corollary 3 are provided in Appendix B. For ring or circle networks, each agent must get the same pay regardless of β due to Lemma 8 and Corollary 2. For line networks, there is a non-monotonic relationship between degree centrality and optimal pay if and only if β is sufficiently large. In fact, the relationship can never be monotonically increasing for any set of parameters' values. Examples in Appendix B (a line with $N = 3, 4, 5$ agents) support this argument.

On the other hand, I can rule out the case that the relationship between optimal pay and degree centrality can be monotonically decreasing. Specifically, we have the following corollary, which also requires the parametric restrictions of Lemmas 6 and 7 so that Proposition 3 holds:

Corollary 4. *Given the parametric restrictions of Lemmas 6 and 7: For any network $\boldsymbol{G}$, any $i, j \in V(\boldsymbol{G})$ such that i has a strictly larger degree centrality than j ($\sum_{k \in \mathcal{N}(i)} g_{ik} > \sum_{k \in \mathcal{N}(j)} g_{jk}$), and any β, optimal pays $\alpha_i^* \leq \alpha_j^*$ does not always hold.*

The proof is completed by providing contradictions. The implication of Corollary 4 is that the mechanism of "incentivizing the peripheral" can never always dominate the mechanism of "incentivizing the central," resulting in a monotonically decreasing relationship between optimal pay and degree centrality. The intuition of the proof of Corollary 4 is to exploit contradictions resulting from Lemmas 6 and 7.

[29] The construction of degree centrality is simpler and more straightforward compared to Bonacich centrality, and, thus, can be better connected to my model, in which a simple and straightforward measure of network position is needed.

I further extend the discussion to the case with more heterogeneity that can match real-world data. To be more specific, I assume that the set of parameters $(w_0, \theta, \beta, \gamma)$ can be different for different agents. So far, I assume that these parameters are the same for all agents. For Proposition 5 whose proof is the only one that directly needs heterogeneity in ability parameters $(w_0, \theta, \beta, \gamma)$, agent i can have his own and different $(w_i, \theta_i, \beta_i, \gamma_i)$, but these parameters are common knowledge. Given such heterogeneity, the relationship between centrality and payment is also non-monotonic. The proof is relegated to Appendix A.

Proposition 5. *For any network structure* $\boldsymbol{G}$*, if the set of parameters for agent i,* $(w_i, \theta_i, \beta_i, \gamma_i)$*, are different for different agents, then there can be a non-monotonic relationship between degree centrality* $\{\sum_j 1(j \in \mathcal{N}(i))\}_{i=1}^N$ *and optimal pay* $\{\alpha_i^*\}_{i=i}^N$*.*

In Proposition 5, w_i corresponds to the parameter in the production function, θ_i is the ability to exert each agent's own effort, β_i is the externality of helping efforts, and γ_i is the ability to exert helping efforts. These parameters are heterogeneous only for Proposition 5.[30]

4.3 Model's Assumptions Revisited

In this section, I revisit the main assumptions, including the functional form assumptions, independence of agents' own efforts and helping efforts, perfect substitutability of helping efforts, and homogeneity assumption. I also discuss the extensions for relaxing such assumptions.

Functional form assumptions To get a closed-form solution for the equilibrium, I exploit three assumptions regarding the functional forms of production technology and costs: (1) Cobb-Douglas production function; (2) marginal cost of agents' own effort is $c(\theta, \sum_{j\in\mathcal{N}(i)} h_{ji}) = (\theta + \sum_{j\in\mathcal{N}(i)} h_{ji})^{-\beta}$; (3) cost function of helping effort is $v(\sum_{j\in\mathcal{N}(i)} h_{ij}) = \frac{1}{2}\gamma(\sum_{j\in\mathcal{N}(i)} h_{ij})^2$. These assumptions altogether construct a tractable framework for the game-theoretic setting of team production with network interactions, and, thus, can generate predictions for the relationship between optimal pay and degree centrality. The set of functional forms and the resulting tractable framework is one major contribution of this paper.

As the proof of Proposition 4 dictates, the flat relationship between optimal pay and degree centrality with a small β relies on Lemma 7, which in turn requires the concavity of the production function and the convexity of the cost function. Moreover, under alternative settings of functional forms (and associated curvatures), it is likely to generate monotonically increasing relationships between optimal pay and degree centrality. For example, when the marginal return of helping effort is strictly increasing and bounded below, the mechanism of "incentivizing the central" can always be strictly dominating, since the externality of helping effort plays a salient role. However, it is infeasible to find a specific analytical functional form that satisfies such criteria and makes the model tractable at the same time. Thus, I take a step back and still use the above-specified functional form in this paper.

Independence of agents' own efforts and helping efforts Another important assumption in the model is that agents provide helping effort and their own effort separately, so the cost of these two types of effort enters the payoff function separately. The costs are also subject to different and separate constraints. I make this assumption for the tractability of the model. If this assumption is not made, then the complexity of the relationship between own effort and helping effort makes the model highly untractable, since an agent's own

[30] Specifically, Proposition 5 does not rely on the uniqueness of the optimal compensation scheme, and, thus, does not require the parametric restrictions in Lemma 7 to hold.

effort and helping effort may be a complicated function of those of all other agents. As a result, it is infeasible to generate general theoretical predictions for the monotonicity of the relationships between optimal pay and degree centrality.

The independence between agents' own and helping efforts can also be justified using a real-world example mentioned previously: a group of farm workers who are responsible for growing fruit. Each worker's own effort is to grow as much fruit as (s)he can that contributes directly to the farm's outputs. At the same time, each agent can also help other agents, like handing over water to those who are thirsty or sharing his/her own food with those who are hungry. Such helping effort does not directly contribute to the amount of fruit, or the farm's output, but reduces the marginal disutility of working. In this example, agents' own effort and helping effort require different sets of skills and intellectual and physical resources, and, thus, can be regarded as independent. Therefore, the setting of the model is justified, in which agents' own effort and helping effort do not interact with each other, and, can take independent values from two separate compact sets.

Perfect substitutability of helping efforts In the model, I assume that the helping efforts given to and received from different neighbors are perfectly substitutable. In other words, conditional on the total amount of helping effort given and received, agents are indifferent regarding the source and target of the helping effort. Therefore, the marginal cost of agents' own effort and the cost of helping effort are both a function of the total amount of given and received helping effort. Such an assumption is used to establish Propositions 3 and 4. The reason is the same as above which requires some curvature restrictions of production function and cost functions.

Homogeneity assumption I assume throughout this article except for Proposition 5 that agents are the same in terms of abilities $(w_i, \theta_i, \beta_i, \gamma_i)$, their role in production. The only source of heterogeneity is the network position and the pay. Such homogeneity is heavily needed to establish Proposition 4, but is not needed to establish Lemmas 6, 7, and Proposition 3, since they only require conditions on functional curvatures.

5 Concluding Remarks

In this paper, I build on Shi (2024) to provide a mathematically tractable model to study the optimal compensation scheme in networked organizations, in which agents connected in a network work collectively to produce a team's output. Connected agents can help neighbors to reduce the disutility of working. The pay received by each agent is a fixed equity share of the team's output which is linked to the position in the network.

For any given network structure, the optimal compensation scheme exists and is unique. There are two effects determining the optimal compensation: (a) "incentivizing the peripheral" effect: the nodes with a smaller centrality have to enjoy a relatively higher pay to be incentivized to work harder since they receive relatively less help; and (b) "incentivizing the central" effect: the nodes with a larger centrality have to enjoy a relatively higher pay to be incentivized to help others more, due to the externality of help. When the externalities of helping efforts are relatively unimportant for the team's output, the same pay goes to agents in different network positions because the two effects neutralize each other perfectly. When the externalities of helping efforts are relatively important, the pay varies with the position, because the second effect dominates. Due to these two conflicting effects, the relationship between centrality and optimal pay may not be monotonically increasing. On the other hand, it cannot be monotonically decreasing. Such a non-monotonic relationship is supported by the numerical and empirical analyses in Appendix D.

This paper, together with Shi (2024), gives theoretical predictions regarding the performance and optimal compensation system in organizations featuring social networks and mutual help. Future research could focus on another form of spillover—information spillover—which is mentioned in Shi (2023b), and its impact on optimal network structures and compensation schemes.

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Appendix For Publication

Appendix A Proof of Main Results

Proof of Lemma 6: Here, I show that $\frac{\partial e_i^*}{\partial \alpha_k}|_{\alpha_{j,j\neq k}} > 0$ for all i and k. The partial derivative is obtained taking $\alpha_{j,j\neq k}$ fixed. The system of equations to determine e^*s and h^*s at the equilibrium level includes equations (2) and (4). We can rewrite equation (2) as

$$e_i^*(\{\alpha_i\}) = m_i(\{h_{ij}^*(\{\alpha_i\}), \{\alpha_i\}\}), \tag{A1}$$

where $m_i(\cdot)$ is some function that is defined to fit the formulation of equation (2), $\{h_{ij}^*\}$ is a collection of h^*s, and $\{\alpha_i\}$ is a collection of αs. We can rewrite equation (4) as

$$n_i(\{h_{ij}^*(\{\alpha_i\})\}, \{\alpha_i\}) = 0, \tag{A2}$$

where $n_i(\cdot)$ is some function that is defined to fit the formulation of equation (4).

Thus, given equations (A1) and (A2), and an implicit function theorem or chain rules, we have, for all i, k:

$$\frac{\partial e_i^*}{\partial \alpha_k} = \sum_{i,j} \frac{\partial m_i(\cdot)}{\partial h_{ij}^*} \frac{\partial h_{ij}^*}{\partial \alpha_k} + \frac{\partial m_i(\cdot)}{\partial \alpha_k}, \tag{A3}$$

and,

$$\sum_{i,j} \frac{\partial n_i(\cdot)}{\partial h_{ij}^*} \frac{\partial h_{ij}^*}{\partial \alpha_k} + \frac{\partial n_i(\cdot)}{\partial \alpha_k} = 0. \tag{A4}$$

From the algebraic formula of equation (2), we have $\frac{\partial m_i(\cdot)}{\partial h_{ij}^*} > 0$ and $\frac{\partial m_i(\cdot)}{\partial \alpha_k} > 0$ for all i, j, k. From the algebraic formula of equation (4), we have $\frac{\partial n_i(\cdot)}{\partial h_{ij}^*} < 0$ and $\frac{\partial n_i(\cdot)}{\partial \alpha_k} > 0$ for all i, j, k.

Plugging in specific functional forms, we have $\frac{\partial m_i(\cdot)}{\partial h_{ij}^*} = \beta \frac{e_i^*}{(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji}^*)}$, $\frac{\partial n_i(\cdot)}{\partial h_{ij}^*} = -K_1 \frac{e_i^*}{(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji}^*)^2} - K_2 \gamma \times \frac{e_i^*}{\sum_{j \in \mathcal{N}(i)} h_{ij}^*}$, and $\frac{\partial n_i(\cdot)}{\partial \alpha_k} = K_3 \frac{e_i^*}{(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji}^*)}$, where $K_1, K_2, K_3 > 0$ are constants (functions of parameters), and K_3 is not a function of parameter γ. When $\gamma > 0$ is sufficiently small, then with the above results (and with simple manipulation with equations (A3) and (A4) and plugging in specific functional forms in the associated partial derivatives), we have $\sum_{i,j} \frac{\partial m_i(\cdot)}{\partial h_{ij}^*} \frac{\partial h_{ij}^*}{\partial \alpha_k} > 0$. Therefore, $\frac{\partial e_i^*}{\partial \alpha_k} = \sum_{i,j} \frac{\partial m_i(\cdot)}{\partial h_{ij}^*} \frac{\partial h_{ij}^*}{\partial \alpha_k} + \frac{\partial m_i(\cdot)}{\partial \alpha_k} > \sum_{i,j} (\min_{i,j}(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji}^*) \times (K_3 \frac{e_i^*}{(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji}^*)}) - \max_{i,j}(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji}^*) \times (K_2 \gamma \times \frac{e_i^*}{\sum_{j \in \mathcal{N}(i)} h_{ij}^*})) > 0$. □

Proof of Lemma 7: First, by the uniqueness of equilibrium established by Proposition 2, for any $\{\alpha_1, ..., \alpha_N\}$ there is a unique $\{e_1^*, ..., e_N^*\}$. By Lemma 6, since $\frac{\partial e_i^*}{\partial \alpha_k} > 0$ and $\frac{\partial e_i^*}{\partial \alpha_k}$ is a smooth function, all $\frac{\partial e_i^*}{\partial \alpha_k}$ can be uniquely determined given any $\{\alpha_1, ..., \alpha_N\}$. To further establish the proof, I exploit the fact that $f(x)$: $R^n \to R$ is a strictly concave function if and only if the associated Hessian matrix $H(x) \equiv \frac{\partial^2 f(x)}{\partial x^2}$ is negative definite. Moreover, for a negative definite matrix H, it is equivalent to saying that all the kth leading principal minors, $det(H_k)$, satisfy $(-1)^k det(H_k) > 0$. Now, I show that this case is indeed under some parametric restrictions. For the function of interest, $g(\alpha_1, ..., \alpha_N)$, I can express it as a function of $\{\alpha_i\}$ and $\{h^*_{ij}\}$, the latter of which is in turn a function of $\{\alpha_i\}$. The mathematical formula is the following:

$$g(\alpha_1, ..., \alpha_N) = \prod_{i=1}^{N} \alpha_i^{\frac{w_0}{1-Nw_0}} (\frac{1}{w_0} - 1) w_0^{\frac{1}{1-Nw_0}} \prod_{i=1}^{N} (\theta + \sum_{j \in \mathcal{N}(i)} h^*_{ji})^{\frac{\beta w_0}{1-Nw_0}}, \tag{A5}$$

in which, again, h^*_{ij} is a function of $\{\alpha_i\}$. Thus, $g(\cdot)$ contains two parts that are related to $\{\alpha_i\}$: (1) $g_1(\cdot) \equiv \prod_{i=1}^{N} \alpha_i^{\frac{w_0}{1-Nw_0}}$; and (2) $g_2(\cdot) \equiv \prod_{i=1}^{N} (\theta + \sum_{j \in \mathcal{N}(i)} h^*_{ji})^{\frac{\beta w_0}{1-Nw_0}}$. Thus, $g(\alpha_1, ..., \alpha_N) \equiv (\frac{1}{w_0} - 1) w_0^{\frac{1}{1-Nw_0}} \times g_1(\cdot) \times g_2(\cdot)$. Given that $\frac{Nw_0}{1-Nw_0} < 1$, $g_1(\cdot)$ is strictly concave (given the property of a Cobb-Douglas function). If we restrict that $\frac{\beta w_0}{1-Nw_0}$ is sufficiently small, so that $A \equiv \frac{\beta w_0}{1-Nw_0} \max_{i,j,k} |(\theta + \sum_{j \in \mathcal{N}(i)} h^*_{ji})^{\frac{\beta w_0}{1-Nw_0} - 1} \frac{\partial h^*_{ji}}{\partial \alpha_k}|$ (the maximum of the norm of the derivative of $g_2(\cdot)$) and its first-order derivatives are sufficiently small, weakly smaller than those of $B \equiv \min_{i,j,k} |\frac{\partial g_1(\cdot)}{\partial \alpha_k}|$ (the minimum of the norm of the derivative of $g_1(\cdot)$), then the curvature of $g_2(\cdot)$ is not salient enough that can change the strict concavity of $g_1(\cdot) \times g_2(\cdot)$. $\beta = 0$ or $w_0 = 0$ is a set of parameters that satisfies these inequalities. Using an argument of continuity, I can find another set of parameters $\beta, w_0 > 0$ that also satisfies these inequalities.

Alternatively, the reason to use the above restrictions can be established by the following argument: consider the second-order derivatives, $\frac{\partial^2 g}{\partial \alpha_i^2}$ and $\frac{\partial^2 g}{\partial \alpha_i \partial \alpha_j}$, if the required restrictions hold, then the derivatives are dominated by the second-order derivatives of $g_1(\cdot)$, and the sign of the derivatives are the same as those of $g_1(\cdot)$, which in turn satisfy the conditions required by all the kth leading principal minors. If the above restrictions can be satisfied, then the requirements on all the kth leading principal minors of $g(\cdot)$ can also be satisfied.

Therefore, the parametric restrictions to be satisfied to establish the concavity are (1) A and its first-order derivatives are weakly smaller than those of B, respectively, which in turn requires that $\frac{\beta w_0}{1-Nw_0}$ is sufficiently small; and (2) $\frac{Nw_0}{1-Nw_0} < 1$. Moreover, we need (3) γ to be small enough so that Lemma 6 holds. They also serve as sufficient conditions for which the function $g(\cdot)$ is strictly concave. □

Proof of Proposition 3: Suppose $\{\alpha_1, ..., \alpha_N\}$ and $\{\alpha'_1, ..., \alpha'_N\}$ both solve the optimality condition for the optimal compensation scheme and without loss of generality assume that $\alpha_1 > \alpha'_1$ and $\alpha_2 < \alpha'_2$. Then there exist α''_1 and α''_2 such that $\alpha_1 > \alpha''_1 > \alpha'_1$ and $\alpha_2 < \alpha''_2 < \alpha'_2$ and the team's output attains a larger level at $\{\alpha''_1, \alpha''_2, \alpha_3, ..., \alpha_N\}$, due to the concavity (and Jensen's inequality) of the production function (a function $g(\cdot)$ of $\{\alpha_i\}$, Lemma 7). This leads to a contradiction. Note that parameter restrictions are needed since the result is built on Lemma 7. □

Proof of Lemma 8: This is a direct implication from the definition of symmetry (Definition 1) and Propositions 2 and 3, which imply that $\{e_1^*, ..., e_N^*\}$ and $\{\alpha_1^*, ..., \alpha_N^*\}$ are both uniquely determined. Therefore, it must be the case that symmetric positions (i_1, i_2) are related to the same set of first-order conditions (2) and (4), since $i_1 = \phi(i_2)$ and $i_2 = \phi(i_1)$ with graph automorphism ϕ, and graph automorphism preserves all edge and node relationships. The first-order conditions of arbitrary kth order neighbors are also the same. As a result, we have $\alpha_{i_1}^* = \alpha_{i_2}^*$, and according to equations (2) and (4), the own efforts should satisfy $e_{i_1}^* = e_{i_2}^*$, $\sum_{j\in N(i_1)} h_{i_1 j}^* = \sum_{j\in N(i_2)} h_{i_2 j}^*$, and $\sum_{j\in N(i_1)} h_{j i_1}^* = \sum_{j\in N(i_2)} h_{j i_2}^*$. In other words, the total amount of helping efforts given and received is also the same for these agents, with the same equity share. □

Proof of Proposition 4: By equation (2) and Lemma 6, e_i^* is an increasing and continuous function of α_k. In other words, a higher α_k leads to a larger e_i^*, and the relationship is continuous. Given that $\frac{\partial h_{ij}^*}{\partial \alpha_k}$ is continuous almost everywhere in a compact set $[0, \bar{h}]$, and does not reach infinity, $\frac{\partial h_{ij}^*}{\partial \alpha_k}$ is bounded above for all i, j, k. Denote the upper bound by $\bar{H}$, which is a strictly increasing function of β, since $\frac{\partial h_{ij}^*}{\partial \alpha_k}$ cannot be negative everywhere. I now argue that $y^* = \prod_{i=1}^N (e_i^*)^{w_0}$ attains the maximum at $\alpha_1^* = \alpha_2^* = ... = \alpha_N^* = \frac{1}{N}$ if β is smaller than a threshold, say $\bar{\beta}(\mathbf{G}, \theta, \gamma)$, by the concavity of the production function ($g(\cdot)$ function, Lemma 7) and the property that e_i^* is increasing in α_i. To be more specific, I try to argue that given any small disturbance at the solution $\alpha_1^* = \alpha_2^* = ... = \alpha_N^*$, without any loss of generality, suppose we have a new maximum $\alpha_1^{**} > \alpha_2^{**}, \alpha_3^{**} = ... = \alpha_N^{**} = \frac{1}{N}$. This disturbance creates a first-order effect of a reduction of the output due to concavity.

To be more specific, recall that

$$g(\alpha_1, ..., \alpha_N) \equiv (\frac{1}{w_0} - 1) w_0^{\frac{1}{1-Nw_0}} \times g_1(\cdot) \times g_2(\cdot) = \prod_{i=1}^N \alpha_i^{\frac{w_0}{1-Nw_0}} (\frac{1}{w_0} - 1) w_0^{\frac{1}{1-Nw_0}} \prod_{i=1}^N (\theta + \sum_{j\in\mathbb{N}(i)} h_{ji}^*)^{\frac{\beta w_0}{1-Nw_0}}, \tag{A6}$$

where (1) $g_1(\cdot) \equiv \prod_{i=1}^N \alpha_i^{\frac{w_0}{1-Nw_0}}$; and (2) $g_2(\cdot) \equiv \prod_{i=1}^N (\theta + \sum_{j\in\mathbb{N}(i)} h_{ji}^*)^{\frac{\beta w_0}{1-Nw_0}}$. $g_1(\cdot)$ is a symmetric and concave function of $\{\alpha_i\}$, so that it reaches the maximum when $\alpha_1^* = \alpha_2^* = ... = \alpha_N^* = 1/N$. A deviation around the maximum point yields a reduction (denoted as $\Delta^*(g_1)$) of the team's output. Now, I argue that it "creates a first-order effect of a reduction of the output" as argued above. $g_2(\cdot)$ is a function of h^*s, which are in turn a function of $\{\alpha_i\}$. However, a deviation around the same point only yields a small change (denoted as $\Delta^*(g_2)$) to the team's output given $g_2(\cdot)$ when β is very small, since $g_2(\cdot)$ is a power function of β. Specifically, as $\beta \to 0$, the change $\Delta^*(g_2) \to 0$. Given continuity, $\Delta^*(g_2)$ must be sufficiently small under a small $\beta > 0$. Therefore, the disturbance around the previous maximum point creates a first-order effect of a reduction of the output due to concavity when β is small enough. Or, mathematically, $\Delta^*(g_1) > \Delta^*(g_2)$.

Furthermore and under closer scrutiny, we can reinforce the above argument that $\Delta^*(g_2) \to 0$ when $\beta \to 0$, taking into account the response of agents' efforts. Since $\frac{\partial e_1^*}{\partial \alpha_k}$ $\frac{\partial e_2^*}{\partial \alpha_k}$ are both bounded by a small number $\tilde{E}$, and $\frac{\partial h_{1j}^*}{\partial \alpha_k}$, $\frac{\partial h_{i1}^*}{\partial \alpha_k}$, $\frac{\partial h_{2j}^*}{\partial \alpha_k}$ and $\frac{\partial h_{i2}^*}{\partial \alpha_k}$ $(i, j, k = 1, 2, ..., N$, and $i, j \neq 1, 2)$ are all bounded by a small number $\tilde{H}$ (they are all a continuous function defined on a compact set), and given a sufficiently small β, the improvement in $\{e_i^*\}$, if any, is also sufficiently small. Therefore, as β approaches 0, the improvement $\Delta^*(g_2)$ also approaches 0.

Thus, given a cutoff value $\bar{\beta}(\mathbf{G}, \theta, \gamma)$, if $\beta < \bar{\beta}$, then $\alpha_1^{**} > \alpha_2^{**}, \alpha_3^{**} = ... = \alpha_N^{**} = \frac{1}{N}$ is inferior to $\alpha_1^* = \alpha_2^* = ... = \alpha_N^*$, which is the optimal solution, driven by concavity (and hence Jensen's inequality), and a continuity argument (all involved functions are continuous so that the inequality is preserved in an open interval). In addition, the argument that $\Delta^*(g_1) > \Delta^*(g_2)$ rules out the case that α_i can be distinct when $\beta \to 0$, as the change around the disturbance resulted from $g_2(\cdot)$ is always strictly smaller than that from $g_1(\cdot)$, or $\Delta^*(g_1) > \Delta^*(g_2)$, provided that β is small.

On the other hand, if $\beta > \bar{\beta}$, then α_i should be larger for larger $\frac{\partial h_{ij}^*}{\partial \alpha_k}$. As $\bar{H}$ and β approach infinity, $\frac{\partial h_{ij}^*}{\partial \alpha_k}$ can be sufficiently large. Thus, $\{\frac{e_i^*}{\alpha_i}\}_{i=1}^N$ (e_i^* weighted by $\frac{1}{\alpha_i}$), shaped by $\frac{\partial h_{ij}^*}{\partial \alpha_k}$ can be sufficiently large. In fact, when β approaches infinity, these partial derivatives also approach infinity (they are all a multiplicative function of β), and the associated agents' own efforts become sufficiently large ($e^* \to \infty$ as $\beta \to \infty$). When we revisit the small disturbance at $\alpha_1^{**} > \alpha_2^{**}, \alpha_3^{**} = ... = \alpha_N^{**} = \frac{1}{N}$, we have the result that an increase in α_1^{**} leads to a very large increase in helping efforts and own efforts of agent 1 (and his neighbors), and thus there can be an improvement of the team's output under such a disturbance. Therefore, we have $\Delta^*(g_1) < \Delta^*(g_2)$. Finally, I can show that there is a cutoff because I can use a continuity argument. For any $\beta \in [0, \bar{\beta}(\mathbf{G}, \theta, \gamma))$, there exists a small neighborhood $B(\beta, \epsilon) = (\beta - \epsilon, \beta + \epsilon) \subseteq [0, \bar{\beta}(\mathbf{G}, \theta, \gamma))$, such that for any $\beta' \in B(\beta, \epsilon)$, the optimal pay associated with β' has the same property as that associated with β. The proof for $\beta \in (\bar{\beta}(\mathbf{G}, \theta, \gamma), \infty)$ is similar. Moreover, the parametric restrictions in Lemma 7 that $\frac{\beta w_0}{1 - N w_0}$ is sufficiently small does not create a contradiction with the requirement on β in this proof, since we can find proper values of w_0 to satisfy both. Finally, to show the non-monotonicity of the relationship between payment and degree centrality, we only need one example of Figure B3. □

Proof of Corollary 3: First, using Lemma 8, we have that all peripheral agents enjoy the same pay, since their positions are symmetric. Thus, the system of optimal pays only contains two values, α_1 for peripheral agents, and α_2 for the central agent. $(N-1)\alpha_1 + \alpha_2 = 1$. Thus, using the second half of the proof of Proposition 4, we have that $\alpha_1 = \alpha_2$ if and only if $\beta \leq \bar{\beta}(N)$, and $\alpha_1 < \alpha_2$ if and only if $\beta > \bar{\beta}(N)$. Specifically, if β approaches infinity, then α_2 approaches 1. A similar continuity argument used in the proof of Proposition 4 can be applied here to show $\alpha_2 > \alpha_1$. We can also use the results in Corollary 4 below: since there can never be a strictly decreasing relationship between optimal pay and degree centrality, we must have $\alpha_2 > \alpha_1$. □

Proof of Corollary 4: Proof by contradiction. By Proposition 4, we only need to focus on the case in which β is larger than the cutoff value, or otherwise all agents should enjoy the same pay. Relabel agents in ascending order of degree centrality, such that $\sum_{k \in \mathcal{N}(1)} g_{k1} \leq \sum_{k \in \mathcal{N}(2)} g_{k2} \leq ... \leq \sum_{k \in \mathcal{N}(N)} g_{kN}$. Suppose a contradiction exists, i.e., for all $i, j = 1, ..., N$ and $i > j$, $\alpha_i^* \leq \alpha_j^*$.

I now show that there exist $i', j' = 1, ..., N$ and $i' > j'$, so that switching $\alpha^*_{i'}$ and $\alpha^*_{j'}$ between i' and j' (so that the new $\alpha^*_{i'}$ is strictly greater than the new $\alpha^*_{j'}$) while fixing other pays strictly increases team's output. Let $i' = i^*$ such that i^* has neighbors, e^*_{i*} is the smallest among e^*s, and $j' \in \mathcal{N}(i')$ and $\alpha^*_{j'} = \max_{k\in\mathcal{N}(i')} \alpha^*_k$.[A1] After switching, $e^*_{i'}$ strictly increases due to higher pay (and his neighbors are more willing to help him), and it leads to an increase in $e^*_{k'_1}$ for all $k'_1 \in \mathcal{N}(i')$ due to complementarity of own effort (equation (2) and thus their neighbors are more willing to help them). $e^*_{j'}$ decreases, and for the same reason it leads to a decrease in $e^*_{k'_2}$ for all $k'_2 \in \mathcal{N}(j')$. However, due to the facts that (1) $j' \in \mathcal{N}(i')$, (2) $\sum_{k\in\mathcal{N}(j')} g_{kj'} \leq \sum_{k\in\mathcal{N}(N)} g_{ki'}$, (3) the old $e^*_{i'} \leq e^*_{j'}$, and agents in ascending order of degree centrality and descending order of payment level, and (4) the positive marginal effect of helping effort on own effort is decreasing in agents' own effort (implied by the cost function $(\theta + \sum_{j\in\mathcal{N}(i)} h_{ji})^{-\beta}$, so that agents' own effort responds to incentives α_i less positively when it is larger), and (5) the marginal product of agents' own effort is decreasing in itself (implied by the Cobb-Douglas functional form, so that the increase in the team's output is larger when we add an improvement on agents' own effort at a lower level), we must have that the increase in $e^*_{i'}$ (driven by a higher $\alpha^*_{i'}$ and thus i's neighbors are more willing to help i') and $e^*_{k'_1}$ for all $k'_1 \in \mathcal{N}(i')$ (driven by complementarity of agents' own efforts) outweighs the decrease in $e^*_{j'}$ (driven by a lower $\alpha^*_{j'}$ and thus j's neighbors are less willing to help j') and $e^*_{k'_2}$ for all $k'_2 \in \mathcal{N}(j')$ (driven by complementarity of agents' own efforts) in the contribution to the team's output.

To be more specific, Fact (1) implies that the increase in $e^*_{i'}$ (due to the increase in $\alpha^*_{i'}$) also leads to the increase in $e^*_{j'}$ and the decrease in $e^*_{j'}$ (due to the decrease in $\alpha^*_{j'}$) also leads to the decrease in $e^*_{j'}$, since helping efforts also respond to the switch of $\alpha^*_{i'}$ and $\alpha^*_{j'}$ and the associated changes in returns of helping efforts; combined with Facts (4), this implies that the resulting increase in $e^*_{j'}$ outweighs the decrease in $e^*_{i'}$ (given decreasing marginal effects of helping efforts). Therefore, since the new $e^*_{i'} > e^*_{j'}$, combining with Fact (5) implies that switching $\alpha^*_{i'}$ and $\alpha^*_{j'}$ leads to an increase in the team's output due to the change in $e^*_{i'}$ and $e^*_{j'}$ (in response to switching $\alpha^*_{i'}$ and $\alpha^*_{j'}$). Denote this as Fact (6). Also, the increase in $e^*_{i'}$ also leads to the increase in $e^*_{k'_1}$ (where $k'_1 \in \mathcal{N}(i')$) and the decrease in $e^*_{j'}$ also leads to the decrease in $e^*_{k'_2}$ (where $k'_2 \in \mathcal{N}(j')$), and Facts (2) and (3) imply that more neighbors of i' are affected than those of j'. Further combined with Fact (4), we have that for each neighbor of j', say $k'_2 \in \mathcal{N}(j')$, we can find a distinct neighbor of i', say $k'_1 \in \mathcal{N}(i')$, such that the decrease in k'_2's own effort is outweighed by the increase in k'_1's own effort (also given decreasing marginal effects of helping efforts). Denote this as Fact (7). Combining Facts (3), (5), (6), and (7) implies that the responses of relevant agents' own efforts lead to an improvement in the team's output. Thus, a contradiction is established.

□

Proof of Proposition 5: I provide examples to establish the proof. For brevity, I only discuss

[A1] We can find the second-smallest, and so forth, if i^* does not have any neighbor.

the proof for w_i, but the proof for $\theta_i, \beta_i, \gamma_i$ can be analogously obtained. With a low level of β, each agent enjoys a similar level of pay. Consider two different agents 1 and 2 with $w_1 = 0$ and $w_2 > 0$. Then, by first-order conditions (equation (7) and (1)), $\alpha_1^* = 0$ and $\alpha_2^* > 0$. Since agents 1 and 2 can interchangeably have arbitrary network positions, we can establish the non-monotonic relationship between degree centrality and optimal payment share. For even a positive $w_1 = \epsilon$, the results still hold due to a continuity argument, since all involved functions are smooth almost everywhere. □

Appendices Not For Publication

Appendix B Examples for Calculation with Small N

In this section, I provide some examples for an illustration of the lemmas and propositions in the main text. In particular, these examples are all consistent with Proposition 4, which indicates the non-monotonicity of the relationship between compensation and degree centrality.

B.1 Cases with $N \leq 5$

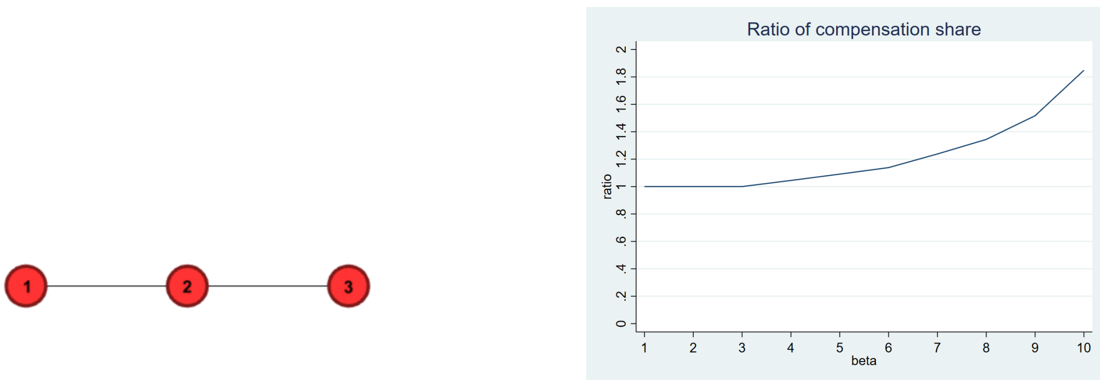
Cases with $N = 3$ First consider a star network with $N = 3$, shown in Figure B1. By Lemma 8, in optimal compensation scheme, $\alpha_1 = \alpha_3 = \bar{\alpha}$. By Lemma 1, $h^*_{12}, h^*_{21}, h^*_{23}, h^*_{32} > 0$. By Lemma 3, we have $h^*_{12} = h^*_{32} \equiv h_1$, $h^*_{21} = h^*_{23} \equiv h_2$. Hence h^*_1, h^*_2 are the solution to the following equations

$$\bar{\alpha}(\bar{\alpha}^{\frac{2w_0}{1-3w_0}}(1-2\bar{\alpha})^{\frac{w_0}{1-3w_0}})\frac{\beta w_0}{1-3w_0}w_0^{\frac{1}{1-3w_0}}(\theta+2h^*_1)^{2*\frac{\beta w_0}{1-3w_0}-1}(\theta+h^*_2)^{\frac{\beta w_0}{1-3w_0}}(\frac{\alpha}{3}w_0)^{\frac{1}{1-3w_0}}(\frac{1}{w_0}-1) = \gamma h^*_1, \quad \text{(B1)}$$

$$(1-2\bar{\alpha})(\bar{\alpha}^{\frac{2w_0}{1-3w_0}}(1-2\bar{\alpha})^{\frac{w_0}{1-3w_0}})\frac{\beta w_0}{1-3w_0}w_0^{\frac{1}{1-3w_0}}(\theta+2h^*_1)^{2*\frac{\beta w_0}{1-3w_0}}(\theta+h^*_2)^{\frac{\beta w_0}{1-3w_0}-1}(\frac{\alpha}{3}w_0)^{\frac{1}{1-3w_0}}(\frac{1}{w_0}-1) = 2\gamma h^*_2 \quad \text{(B2)}$$

These equations have a unique solution h^*_1, h^*_2. Plugging h^*_1, h^*_2 into equation (2), we can get e^*_1, e^*_2, e^*_3, and thus the team's output $y = \prod_{i=1}^{3}(e^*_i)^{w_0}$. For varying βs, we can find the related optimal $\bar{\alpha}$. The relation between $\frac{\alpha_2}{\alpha_1} = \frac{1-2\bar{\alpha}}{\bar{\alpha}}$ and β is shown in Figure B1(b). We can see that for a small value of β, $\frac{\alpha_2}{\alpha_1} = 1$, which implies that the same pay should go to the central node and the peripheral nodes. For a large value of β, the central node should enjoy a higher level of pay compared to the peripheral nodes, and the gap of the pay increases as β increases.

Figure B1



(a) (b)

Notes: The figure illustrates the relationship between the ratio of the equity share $\frac{\alpha_2}{\alpha_1}$ and β in a 3-agent star network. The values of the parameters are $w_0 = 0.1$, $\theta = 1$, $\gamma = 1$.

Cases with $N = 4$ First consider a line network with $N = 4$, shown in Figure B2. By Lemma 8, in optimal compensation scheme, $\alpha_1 = \alpha_4 = \bar{\alpha}_1$, and $\alpha_2 = \alpha_3 = \bar{\alpha}_2$. By Lemma 1,

$h^*_{12}, h^*_{34}, h^*_{21}, h^*_{43} > 0$. Again, by Lemma 3, we have $h^*_{21} = h^*_{34} \equiv h_1$, $h^*_{12} = h^*_{43} \equiv h_2$, $h^*_{23} = h^*_{32} \equiv h_3$. First, suppose $h_3 > 0$, then by Lemma 3, we have $h_2 = h_1 + h_3$, and $\frac{h_2+h_3}{\bar{\alpha}_2} = \frac{h_1}{\bar{\alpha}_1}$. Then we have $h_2 = \frac{\bar{\alpha}_1+\bar{\alpha}_2}{2\bar{\alpha}_1} h_1$, and $h_2 = \frac{\bar{\alpha}_2-\bar{\alpha}_1}{2\bar{\alpha}_1} h_1$, which requires that $\bar{\alpha}_2 > \bar{\alpha}_1$. Hence h_1 is the solution to the following equation:

$$\bar{\alpha}_2(\bar{\alpha}_1\bar{\alpha}_2)^{\frac{2w_0}{1-4w_0}} \frac{\beta w_0}{1-4w_0} w_0^{\frac{1}{1-4w_0}} (\theta + \frac{\bar{\alpha}_1+\bar{\alpha}_2}{2\alpha_1} h_1)^{4*\frac{\beta w_0}{1-4w_0}-1} = \gamma \frac{\bar{\alpha}_2}{\bar{\alpha}_1} h_1. \tag{B3}$$

This equation has a unique solution h^*_1. Plugging h^*_1 into equation (2), we can get $e^*_1, e^*_2, e^*_3, e^*_4$, and thus the team's output $y = \prod_{i=1}^4 (e^*_i)^{w_0}$. If $\alpha_2 \leq \alpha_1$, then $h_3 = 0$, and h_1, h_2 are the solution to the following equations:

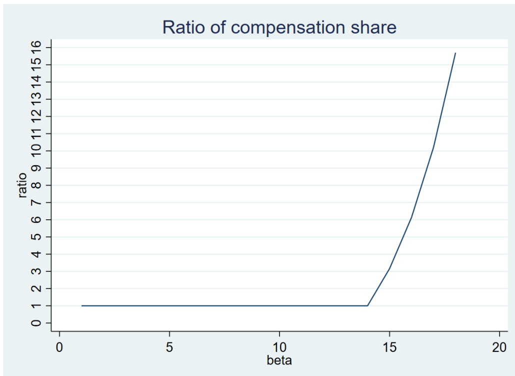
$$\alpha_1(\bar{\alpha}_1\bar{\alpha}_2)^{\frac{2w_0}{1-4w_0}} \frac{\beta w_0}{1-4w_0} w_0^{\frac{1}{1-4w_0}} (\theta + h_1)^{2*\frac{\beta w_0}{1-4w_0}-1} (\theta + h_2)^{2*\frac{\beta w_0}{1-4w_0}} (\frac{1}{w_0} - 1) = \gamma h_1, \tag{B4}$$

$$\bar{\alpha}_2(\bar{\alpha}_1\bar{\alpha}_2)^{\frac{2w_0}{1-4w_0}} \frac{\beta w_0}{1-4w_0} w_0^{\frac{1}{1-4w_0}} (\theta + h_1)^{2*\frac{\beta w_0}{1-4w_0}} (\theta + h_2)^{2*\frac{\beta w_0}{1-4w_0}-1} (\frac{1}{w_0} - 1) = \gamma h_2 \tag{B5}$$

These equations have a unique solution h^*_1, h^*_2. Again, we can solve for the team production as a function for $\bar{\alpha}_1, \bar{\alpha}_2$. We can plot the relationship between the maximized team's output and compensation scheme α_1. It is shown in Figure B3. For $\beta = 6$, the team's output reaches the maximum at $\bar{\alpha}_1 = 0.25, \bar{\alpha}_2 = 0.25$. For $\beta = 16$, the team's output reaches the maximum at $\bar{\alpha}_1 = 0.073, \bar{\alpha}_2 = 0.427$.

Figure B2

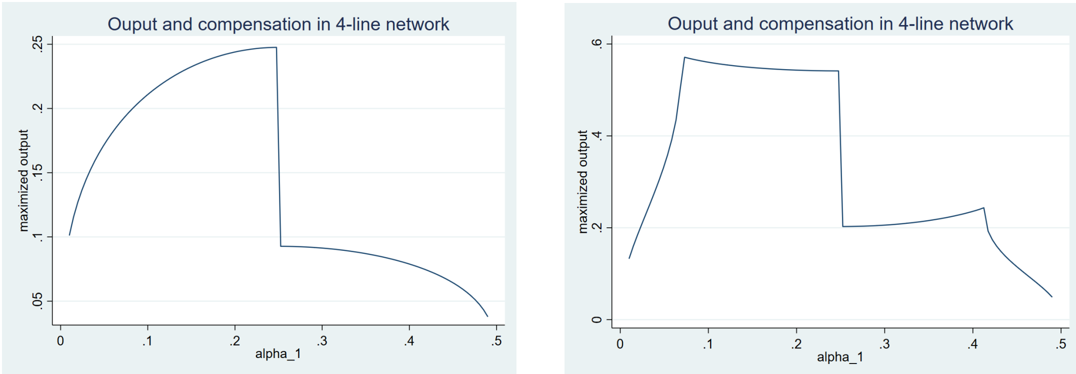
(a)



(b)

Notes: The figure illustrates the relationship between the ratio of the optimal equity share $\frac{\bar{\alpha}_2}{\bar{\alpha}_1}$ *and* β *in a 4-agent line network. The values of the parameters are* $w_0 = 0.1$, $\theta = 1$, $\gamma = 1$.

Figure B3



(a) (b)

Notes: The figure illustrates the relationship between the team's output and the equity share $\bar{\alpha}_1$ in a 4-agent line network. The values of the parameters for (a) are $w_0 = 0.1$, $\theta = 1$, $\beta = 6$, $\gamma = 1$, and those for (b) are $w_0 = 0.1$, $\theta = 1$, $\beta = 16$, $\gamma = 1$.

For varying βs, we can find the related optimal $\bar{\alpha}_1, \bar{\alpha}_2$. The relation between optimal $\frac{\bar{\alpha}_2}{\bar{\alpha}_1} = \frac{1-3\bar{\alpha}}{\bar{\alpha}}$ is shown in Figure B2(b). We can see that for a small value of β, $\frac{\bar{\alpha}_2}{\bar{\alpha}_1} = 1$, which implies that the same pay should go to the central node and the peripheral nodes. For a large value of β, the central node should enjoy a higher level of pay compared to the peripheral nodes, and the gap of the pay increases as β increases.

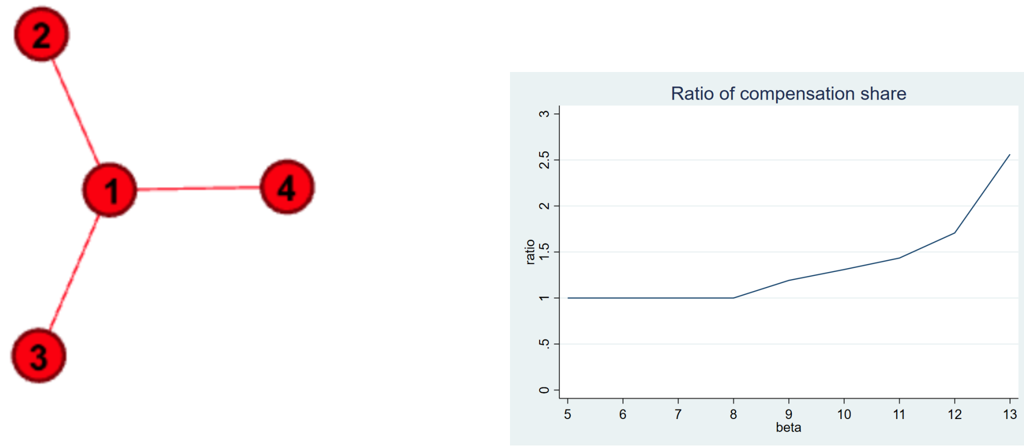
Then consider a star network with $N = 4$, shown in Figure B4(a). By Lemma 8, in optimal compensation scheme, $\alpha_2 = \alpha_3 = \alpha_4 = \bar{\alpha}$. By Lemma 1, $h^*_{i1}, h^*_{1i} > 0$ for all $i \in \{2, 3, 4\}$. By Lemma 3, we have $h^*_{21} = h^*_{31} = h^*_{41} \equiv h_1$, $h^*_{12} = h^*_{13} = h^*_{14} \equiv h_2$. Hence h^*_1, h^*_2 are the solution to the following equations

$$\bar{\alpha}(\bar{\alpha}^{\frac{3w_0}{1-4w_0}}(1-3\bar{\alpha})^{\frac{w_0}{1-4w_0}})\frac{\beta w_0}{1-4w_0}w_0^{\frac{1}{1-4w_0}}(\theta+3h^*_1)^{3*\frac{\beta w_0}{1-4w_0}-1}(\theta+h^*_2)^{\frac{\beta w_0}{1-4w_0}}(\frac{\alpha}{4}w_0)^{\frac{1}{1-4w_0}}(\frac{1}{w_0}-1) = \gamma h^*_1, \tag{B6}$$

$$(1-3\bar{\alpha})\alpha(\bar{\alpha}^{\frac{3w_0}{1-4w_0}}(1-3\bar{\alpha})^{\frac{w_0}{1-4w_0}})\frac{\beta w_0}{1-4w_0}w_0^{\frac{1}{1-4w_0}}(\theta+3h^*_1)^{3*\frac{\beta w_0}{1-4w_0}}(\theta+h^*_2)^{\frac{\beta w_0}{1-4w_0}-1}(\frac{\alpha}{4}w_0)^{\frac{1}{1-4w_0}}(\frac{1}{w_0}-1) = 3\gamma h^*_2 \tag{B7}$$

These equations have a unique solution h^*_1, h^*_2. Plugging h^*_1, h^*_2 into equation (2), we can get $e^*_1, e^*_2, e^*_3, e^*_4$, and thus the team's output $y = \prod_{i=1}^{4}(e^*_i)^{w_0}$. For varying βs, we can find the related optimal $\bar{\alpha}$. The relation between $\frac{\alpha_2}{\alpha_1} \equiv \frac{1-3\bar{\alpha}}{\bar{\alpha}}$ is shown in Figure B4(b). We can see that for a small value of β, $\frac{\bar{\alpha}}{1-3\bar{\alpha}} = 1$, which implies that the same pay should go to the central node and the peripheral nodes. For a large value of β, the central node should enjoy a higher level of pay compared to the peripheral nodes, and the gap of the pay increases as β increases.

Figure B4



(a) (b)

Notes: The figure illustrates the relationship between the ratio of the equity share $\frac{\bar{\alpha}_2}{\bar{\alpha}_1}$ *and* β *in a 4-agent star network. The values of the parameters are* $w_0 = 0.1$, $\theta = 1$, $\gamma = 1$.

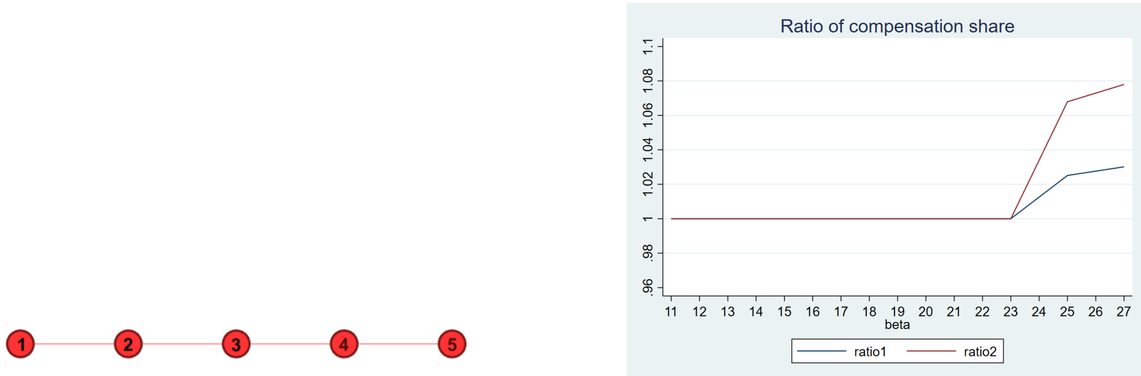
Cases with $N = 5$ Consider a line of 5 agents. Let $\alpha_1 = \alpha_5 = \bar{\alpha}_1$, $\alpha_2 = \alpha_4 = \bar{\alpha}_2$. By Lemma 3: $h_{12} = h_{54} \equiv h_1$, $h_{21} = h_{45} \equiv h_2$, $h_{23} = h_{43} \equiv h_3$, and $h_{32} = h_{34} \equiv h_4$. By Lemma 1: $h_1, h_2, h_3, h_4 > 0$. Thus by Lemma 3: $h_2 = 2h_3$, and $\frac{h_1}{\alpha_1} = \frac{2h_4}{1-2\alpha_1-2\alpha_2}$. Then h_2, h_4 are the solution to the following equations:

$$\begin{aligned}&\bar{\alpha_1}(\bar{\alpha_1}^2\bar{\alpha}_2^2(1-2\alpha_1-2\alpha_2))^{\frac{w_0}{1-5w_0}}\frac{\beta w_0}{1-5w_0}w_0^{\frac{1}{1-5w_0}}\\ &\times(\theta+h_2)^{3*\frac{\beta w_0}{1-5w_0}-1}(\theta+\frac{1-2\bar{\alpha}_2}{1-2\bar{\alpha}_1-2\bar{\alpha}_2}h_4)^{2\frac{\beta w_0}{1-5w_0}}(\frac{\alpha}{5}w_0)^{\frac{1}{1-5w_0}}(\frac{1}{w_0}-1)=\gamma\frac{2\bar{\alpha}_1}{1-2\bar{\alpha}_1-2\bar{\alpha}_2}h_4,\end{aligned} \tag{B8}$$

$$\begin{aligned}&\bar{\alpha}_2(\bar{\alpha_1}^2\bar{\alpha}_2^2(1-2\bar{\alpha}_1-2\bar{\alpha}_2))^{\frac{w_0}{1-5w_0}}\frac{\beta w_0}{1-5w_0}w_0^{\frac{1}{1-5w_0}}\\ &\times(\theta+h_2)^{3*\frac{\beta w_0}{1-5w_0}}(\theta+\frac{1-2\bar{\alpha}_2}{1-2\bar{\alpha}_1-2\bar{\alpha}_2}h_4)^{2\frac{\beta w_0}{1-5w_0}-1}(\frac{\alpha}{5}w_0)^{\frac{1}{1-5w_0}}(\frac{1}{w_0}-1)=\gamma\frac{3}{2}h_2.\end{aligned} \tag{B9}$$

These equations have a unique solution h_2^*, h_4^*. Plugging h_2^*, h_4^* into equation (2), we can get $e_1^*, e_2^*, e_3^*, e_4^*, e_5^*$, and thus the team's output $y = \prod_{i=1}^5 (e_i^*)^{w_0}$. The relationship between compensation and output is shown in Figure B6. In Figure B6(a), the value of β is relatively small, and thus output attains the maximum when $\alpha_1 = \alpha_2 = 1-2\alpha_1-2\alpha_2 = 0.2$. In Figure B6(b), the value of β is relatively large, and the output attain maximum when $\alpha_1 = 0.201, \alpha_2 = 0.198$.

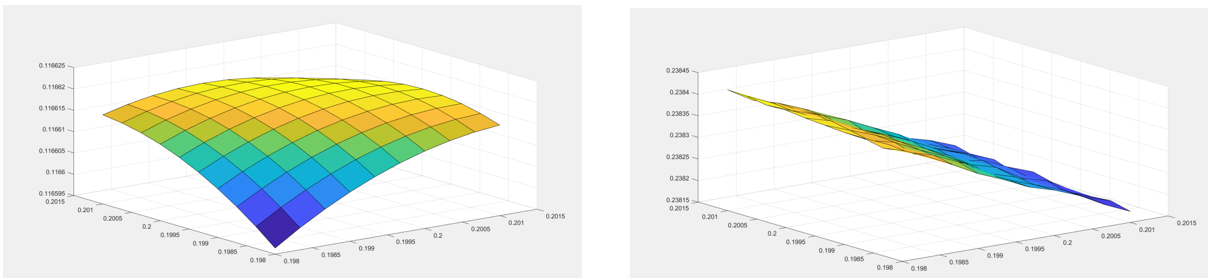
Figure B5



(a) (b)

Notes: The figure illustrates the relationship between the ratio of the equity share $\frac{\alpha_2}{\alpha_1}$ *and* β *in a 5-agent line network. The values of the parameters are* $w_0 = 0.1$, $\theta = 1$, $\gamma = 1$.

Figure B6



(a) (b)

Notes: The figure illustrates the relationship between the team's output and the equity share α_1, α_2 *in a 5-agent line network. The values of the parameters for (a) are* $w_0 = 0.1$, $\theta = 1$, $\beta = 7$, $\gamma = 1$, *and those for (b) are* $w_0 = 0.1$, $\theta = 1$, $\beta = 27$, $\gamma = 1$.

For varying βs, we can find the related optimal $\bar{\alpha}_1, \bar{\alpha}_2$. The relation between $\frac{\alpha_1}{\alpha_2}, \frac{1-2\bar{\alpha}_1-2\bar{\alpha}_2}{\bar{\alpha}_2}$ and β is shown in Figure B5(b). We can see that for a small value of β, $\frac{\alpha_2}{\alpha_1} = 1$, which implies that the same pay should go to the central node and the peripheral nodes. For a large value of β, the central node should enjoy a higher level of pay compared to the peripheral nodes, and the gap of the pay increases as β increases.

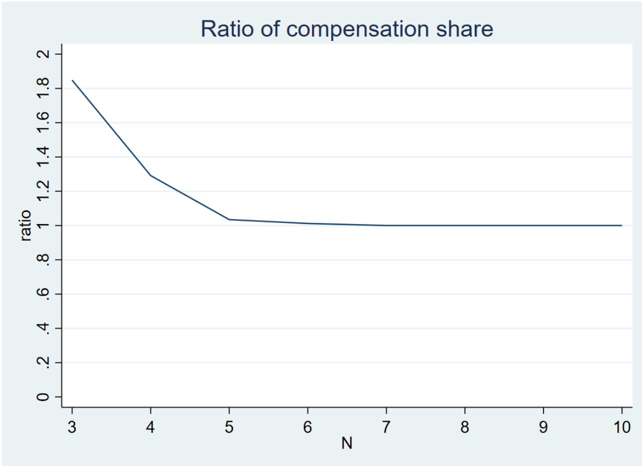
B.2 Generalization of N agents

Now, I analyze the case of three stylized networks with an arbitrary number of agents. These examples are all consistent with Proposition 4, which indicates the non-monotonicity of the relationship between compensation and degree centrality.

Star of N agents Consider a star with N agents. Let the central agent be agent 1, and $\{2, 3, ..., N\}$ are the peripheral nodes. The following paragraph and Figure B7 describe the structure of compensation for a star network. This is also consistent with Corollary 3.

Example 1. *(Optimal compensation in a star) In a star with N agents, in the optimal compensation scheme, when β is small, the same pay goes to all agents. When β is large, higher pay goes to the central nodes, and all peripheral nodes get the same, and lower, pay.*

Figure B7



Notes: The figure illustrates the relationship between the ratio of the equity share $\frac{\alpha_1}{\alpha_N}$ and N. The values of the parameters are $w_0 = 0.1$, $\theta = 1$, $\beta = 10$, $\gamma = 1$.

Line of N agents Consider a line with N agents as shown in Figure 5(b). The equilibrium of the network structure is characterized as follows, which can be obtained using induction (from line networks of small even and odd numbers of nodes):

Example 2. *(Optimal compensation in a line) In a line network with N agents, all agents get the same pay if β is small. When β is large, there is a non-monotonic relationship between optimal pay and degree centrality.*

Circle of N agents Consider a circle with N agents. The following argument can be obtained using the symmetry argument in Lemma 8, since all nodes in the circle have symmetric positions.

Example 3. *(Optimal compensation in a circle) In a circle with N agents, the optimal compensation system is one in which the same pay goes to each agent.*

Appendix C Additional Proofs

Details of deriving the best-reply functions: From equation (1), we take a first-order condition, and thus the power of e_i should be $w_0 - 1$, and the first term in equation (1) should be multiplied by w_0. The second term in equation (1) is linear in e_i, and thus taking a first-order derivative yields $(\theta + \sum_{j \in \mathcal{N}(i)} h_{ji})^{-\beta}$. Therefore, we have the following equation:

$$\alpha_i w_0 {e_1^*}^{w_0} {e_2^*}^{w_0} \cdots {e_i}^{w_0 - 1} \cdots {e_N^*}^{w_0} - (\theta + \sum_{j \in \mathcal{N}(i)} h_{ji})^{-\beta} = 0. \tag{C1}$$

Rearrange equation (C1) by expressing e_i as a function of e_j^* $(j \neq i)$ yields

$$e_i = ((\theta + \sum_{j \in \mathcal{N}(i)} h_{ji})^{\frac{-\beta}{1-w_0}}) \prod_{j \neq i} (e_j^*)^{\frac{w_0}{1-w_0}}. \tag{C2}$$

Since equation (C2) holds for all $i = 1, 2, ..., N$, comparing the equation for i and j yields

$$e_j^* = \frac{\alpha_i}{\alpha_j} e_i \times \frac{(\theta + \sum_{k \in \mathcal{N}(i)} h_{ki})^{-\beta}}{(\theta + \sum_{k \in \mathcal{N}(j) h_{kj}})^{-\beta}}. \tag{C3}$$

Express all e_j^* as a function of e_i by equation (C3), and plug back to equation (C2), we have

$$e_i = (\alpha_i w_0) \times e_i^{w_0 N} \times ((\theta + \sum_{j \in \mathcal{N}(i)} h_{ji})^{\beta}) \prod_{j \neq i} [(\theta + \sum_{k \in \mathcal{N}(i)} h_{ki})^{-\beta w_0 N} (\prod_{j=1}^{N} (\theta + \sum_{k \in \mathcal{N}(j)} h_{kj})^{\beta w_0})]. \tag{C4}$$

Rearranging equation (C4) yields equation (2). Thus, we can express the payoff function of all helping efforts h_{ij}, which is the right-hand side of the equation (3). Taking a first-order derivative of the first term of the equation (3) yields the left-hand side of the inequality (4). Taking a first-order derivative of the second term of the equation (3) yields the right-hand side of the inequality (4). Since h_{ij} should be weakly positive, the direction of the inequality (4) should be "less than or equal to." Standard complementary slackness conditions follow.

Proof of Proposition 1: This proposition directly derives from the Theorem 1 in Rosen (1965), which states that in an n-person concave game where the strategy space of each agent is convex and compact and the payoff function is concave in his own strategy, there always exists a Nash equilibrium. To prove that there exists a subgame perfect equilibrium of the subgame of stages 1 and 2, we need to show that the existence condition stated in Theorem 1 in Rosen (1965) is satisfied in the subgames of stage 2 and stage 1.

In stage 2, each agent i decides her level of own effort e_i^* to maximize her payoff, taking the choices of other agents $\{e_j^*\}_{j \neq i}$ as given. Thus she solves

$$e_i^* = \arg\max_{e_i} \alpha_i {e_1^*}^{w_0} {e_2^*}^{w_0} \cdots {e_i}^{w_0} \cdots {e_N^*}^{w_0} - (\theta_i + \sum_{j \in \mathcal{N}(i)} h_{ji})^{-\beta} e_i - \frac{1}{2}\gamma (\sum_{j \in \mathcal{N}(i)} h_{ij})^2. \tag{C5}$$

Notice that if e_i approaches $+\infty$, then the above equality cannot be satisfied. Thus, we can impose an upper limit to the range of e_i. Then, we can observe that the strategy space for e_i is convex and compact and that the objective function is concave in e_i, therefore there exists a Nash equilibrium of this subgame. Actually, the profile of $\mathbf{E}^* = (e_1^*, .., e_N^*)$ is given by equation (2).

By backward induction, substituting equation (2) to the payoff function, we could find that in stage 1 agent i's problem is to find $\{h_{ij}\}_{j\in\mathcal{N}(i)}$ given $\{h^*_{kl}\}_{k\neq i, l\in N(k)}$:

$$h^*_{ij} = \arg\max_{h_{ij}} (\alpha_i w_0)^{\frac{1}{1-Nw_0}} (\frac{1}{w_0} - 1)(\prod_{k=1}^{N} (\theta + \sum_{l\in N(k)} h^*_{kl})^{\beta})^{\frac{w_0}{1-Nw_0}} - \gamma(\sum_{j\in\mathcal{N}(i)} h_{ij})^2, \tag{C6}$$

First, note that if h_{ij} goes to $+\infty$, then the complimentary slackness condition (4) may fail. So the solution $\mathbf{H}^* = (h^*_{ij})$ is bounded above. Again, observe that the strategy space for h_{ij} is convex and compact and that the objective function is concave in h_{ij}, given $\beta \in (0, \frac{1-Nw_0}{w_0})$. Therefore there exists a Nash equilibrium of the subgame of stage 1, and there exists a subgame perfect Nash equilibrium of the overall game. □

Proof of Lemma 1: Prove by contradiction. First show that every non-isolated agent gives a strictly positive amount of help. For such agent i, let $j \in \mathcal{N}(i)$ denote one of her neighbors. If in equilibrium the total amount of help i gives is zero, then the first-order condition for h_{ij} implies that

$$0 < \frac{\beta w_0}{1-Nw_0} \prod_{j\geq 1} (\theta + \sum_{k\in\mathcal{N}(j)} h^*_{kj})^{\frac{\beta w_0}{1-Nw_0}} (\theta + \sum_{k\in\mathcal{N}(j)} h^*_{kj})^{-1} (\alpha_i w_0)^{\frac{1}{1-Nw_0}} (\frac{1}{w_0} - 1) \leq \gamma(\sum_{j\in\mathcal{N}(i)} h^*_{ij}) = 0,$$

and hence renders a contradiction.

Then show that every non-isolated agent receives a strictly positive amount of help. For non-isolated agent i, assume that all of her neighbors do not give her any help, i.e., for any $j \in \mathcal{N}(i)$, $h^*_{ji} = 0$. There are two cases. (1) If one of i's neighbors say j, has no other neighbors except for i, then since all non-isolated agents give a strictly positive amount of help, we have $h^*_{ji} > 0$, which is a contradiction. (2) If all i's neighbors have other neighbors except i, i.e. for any $j \in \mathcal{N}(i)$, $\mathcal{N}(j)$ has no less than two elements, then since all non-isolated agents give a strictly positive amount of help, there exists $k \in \mathcal{N}(j)(k \neq i)$, such that $h^*_{jk} > 0$. Then the first-order conditions of h_{ji} and h_{jk} implies

$$\frac{\beta w_0}{1-Nw_0} \prod_{j\geq 1} (\theta + \sum_{k\in\mathcal{N}(j)} h^*_{kj})^{\frac{\beta w_0}{1-Nw_0}} \theta^{-1} (\alpha_i w_0)^{\frac{1}{1-Nw_0}} (\frac{1}{w_0} - 1) \leq \gamma(\sum_{k\in\mathcal{N}(j)} h^*_{jk}), \tag{C7}$$

and

$$\frac{\beta w_0}{1-Nw_0} \prod_{j\geq 1} (\theta + \sum_{k\in\mathcal{N}(j)} h^*_{kj})^{\frac{\beta w_0}{1-Nw_0}} (\theta + \sum_{k\in\mathcal{N}(j)} h^*_{jk})^{-1} (\alpha_i w_0)^{\frac{1}{1-Nw_0}} (\frac{1}{w_0} - 1) = \gamma(\sum_{k\in\mathcal{N}(j)} h^*_{jk}). \tag{C8}$$

Since $h^*_{jk} > 0$, $\theta < \theta + \sum_{k\in\mathcal{N}(j)} h^*_{jk}$. Hence, compare (C7) and (C8) we have $\sum_{k\in\mathcal{N}(j)} h^*_{jk} > \sum_{k\in\mathcal{N}(j)} h^*_{jk}$, and hence a contradiction. □

Proof of Lemma 2: The proof is straightforward noting that for any i and j, $(\sum_{j\in\mathcal{N}(i)} h^*_{ij})(\theta +$

$\sum_{k\in\mathcal{N}(j)} h^*_{kj}) \geq M(\mathbf{H}^*)$. $h^*_{ij} > 0$ implies that the first-order condition takes an equality sign, and thus for any $j' \in \mathcal{N}(i)/\{j\}$, $(\sum_{j\in\mathcal{N}(i)} h^*_{ij})(\theta + \sum_{k\in N(j')} h^*_{kj'}) \geq (\sum_{j\in\mathcal{N}(i)} h^*_{ij})(\theta + \sum_{k\in\mathcal{N}(j)} h^*_{kj})$. Since $\sum_{j\in\mathcal{N}(i)} h^*_{ij} > 0$, we have $\sum_{k\in\mathcal{N}(j)} h^*_{kj} \leq \sum_{k\in\mathcal{N}(j)} h^*_{kj'}$ for any $j' \in \mathcal{N}(i)/\{j\}$. By the same reasoning, if $h^*_{ij} > 0$, we have for any $i' \in \mathcal{N}(j)/\{i\}$, $\sum_{j\in\mathcal{N}(i)} h^*_{ij} \leq \sum_{j\in N(i')} h^*_{i'j}$. □

Proof of Lemma 3: This is a direct implication of the first-order condition (4). For $j_1, j_2 \in \mathcal{N}(i)$ such that $h^*_{ij_1}, h^*_{ij_2} > 0$, we have the following to equations:

$$\frac{\beta w_0}{1 - Nw_0} \prod_{j\geq 1} (\theta + \sum_{k\in\mathcal{N}(j)} h^*_{kj})^{\frac{\beta w_0}{1-Nw_0}} (\theta + \sum_{k\in N(j_1)} h^*_{kj_1})^{-1} (\alpha_i w_0)^{\frac{1}{1-Nw_0}} (\frac{1}{w_0} - 1) = \gamma(\sum_{k\in\mathcal{N}(j)} h^*_{jk}), \quad \text{(C9)}$$

$$\frac{\beta w_0}{1 - Nw_0} \prod_{j\geq 1} (\theta + \sum_{k\in\mathcal{N}(j)} h^*_{kj})^{\frac{\beta w_0}{1-Nw_0}} (\theta + \sum_{k\in N(j_2)} h^*_{kj_2})^{-1} (\alpha_i w_0)^{\frac{1}{1-Nw_0}} (\frac{1}{w_0} - 1) = \gamma(\sum_{k\in\mathcal{N}(j)} h^*_{jk}), \quad \text{(C10)}$$

Compare (C9) and (C10) we have $\theta + \sum_{k\in N(j_1)} h^*_{kj_1} = \theta + \sum_{k\in N(j_2)} h^*_{kj_2}$, and hence j_1, j_2 receive the same amount of help. We could prove the second part of this lemma in the same way. □

Proof of Lemma 4: Suppose the underlying network $\mathbf{G}$ has n agents and l links, where $n \geq 2$, and each node has at least degree 1 (thus the network is a connected one). Thus $n - 1 \leq l \leq \frac{n(n-1)}{2}$. There are $2l$ unknown helping efforts (each link is associated with 2 helping efforts) in total. The sum of degrees is also $2l$. Since all elements in H^* are interior solutions, we can use Lemma 3 to determine which helping efforts are equal (adjusted by compensation), and this may yield $2(2l - n) = 4l - 2n$ equations[B1]. Also observe that the number of nodes n and the number of edges l satisfy the following inequality: $n - 1 \leq l \leq \frac{n(n-1)}{2}$, and thus $2l - 2 \leq 4l - 2n \leq 4l - 2\sqrt{2l}$. This implies that there are at most 2 unknowns remaining. For a bipartite network, there are two independent sets, and each node in the same independent set gives and receives the same amount of helping efforts, respectively, and each amount of given (received) helping efforts is associated with one unknown. For a non-bipartite network, it then has to be the case that all nodes must receive and give the same amount of helping efforts, as a result of Lemma 3. □

Proof of Proposition 2: By Lemma 4, if each h_{ij} $(g_{ij} = 1)$ in the profile of helping efforts $\mathbf{H}$ takes interior solution, then there are at most two unknowns, and there are at most two amounts of helping efforts given and received (adjusted by compensation), namely ($\mathbf{H}^g_1$, $\mathbf{H}^g_2$) and ($\mathbf{H}^r_1$, $\mathbf{H}^r_2$). If there are exactly two unknowns, then we can express $\mathbf{H}^g_i$ and $\mathbf{H}^r_i$ as a (strictly increasing) linear transformation of the two unknowns (namely $\bar{h}_1$ and $\bar{h}_2$), and thus $\mathbf{H}^r_i = \mathbf{H}^r_i(\bar{h}_1, \bar{h}_2)$, and $\mathbf{H}^g_i = \mathbf{H}^g_i(\bar{h}_1, \bar{h}_2)$, where $i \in \{1, 2\}$. The original set of first-order conditions is reduced to two and can be

[B1]With n nodes and l links, there are $2l - n$ pairs of neighbors, and thus $2(2l - n) = 4l - 2n$ equations on total helping efforts.

rewritten as

$$M(\mathbf{H}^*)(\theta + \mathbf{H}_1^r(\bar{h}_1, \bar{h}_2))^{-1} = \mathbf{H}_1^g(\bar{h}_1, \bar{h}_2), \tag{C11}$$

and

$$M(\mathbf{H}^*)(\theta + \mathbf{H}_2^r(\bar{h}_1, \bar{h}_2))^{-1} = \mathbf{H}_2^g(\bar{h}_1, \bar{h}_2), \tag{C12}$$

Since the system of first-order conditions only has two degrees of freedom, and the LHS is strictly decreasing in $(\bar{h}_1, \bar{h}_2)$ and the RHS is strictly increasing in $(\bar{h}_1, \bar{h}_2)$, and at least one solution exists by Proposition 1. Then we have that $(\bar{h}_1, \bar{h}_2)$ and hence $\mathbf{H}_i^r$ and $\mathbf{H}_i^g$ $(i \in \{1, 2\})$ can be uniquely determined.

If there is exactly one unknown, then there is one amount of helping efforts given and received, namely $\mathbf{H}^g$ and $\mathbf{H}^r$. Then, we can express $\mathbf{H}^g$ and $\mathbf{H}^r$ as a (strictly increasing) linear transformation of the only unknown (namely $\bar{h}$), and thus $\mathbf{H}^r = \mathbf{H}^r(\bar{h})$, and $\mathbf{H}^g = \mathbf{H}^g(\bar{h})$. There is only one distinct first-order condition:

$$M(\mathbf{H}^*) - \mathbf{H}^g(\bar{h})(\theta + \mathbf{H}^r(\bar{h})) = 0. \tag{C13}$$

Again, since the first-order condition only has one degree of freedom, and by the same reasoning as above, we have that $\bar{h}$ and hence $\mathbf{H}^r$ and $\mathbf{H}^g$ can be uniquely determined.

On the other hand, if some h_{ij} $(g_{ij} = 1)$ in the profile of helping efforts $\mathbf{H}$ take a corner solution, then in (4) equality does not hold, and instead h_{ij} is directly pinned down by $h_{ij} = 0$. Therefore, the total degree of freedom for $\mathbf{H}$ is the same when some h_{ij} take a corner solution, as in the case where all h_{ij} take an interior solution.

In addition, we can use a special version of the fixed-point theorem as in Kennan (2001) to provide another approach and intuitions for the proof, as argued in Shi (2024). □

Proof of Corollary 1: For a network without any loop (i.e., a *tree*), we can find its *leaf* nodes, whose degrees are equal to 1. For these leaf nodes, the total amount of helping effort (both given and received) is equal to the helping effort on the single link between the nodes and their neighbor (Suppose a leaf node i and its neighbor j, then h_{ij}^* and h_{ji}^*, adjusted by compensation, are pinned down by $M(\mathbf{H}^*) = h_{ij}^*(\theta + \sum_{k \in \mathcal{N}(j)} h_{kj}^*)$, and $M(\mathbf{H}^*) = \sum_{k \in \mathcal{N}(j)} h_{jk}^*(\theta + h_{ji}^*))$. Given all h_{ij}^* and h_{ji}^* of the leaf nodes, we can pin down the helping efforts for all nodes with degree 2, with the same method. Given these helping efforts, we can apply the above reasoning to the rest of the nodes, until all helping efforts are determined. Such is the unique way in which $\mathbf{H}^*$ is determined. For any network with loops, the proof is obvious because it can be directly derived from the example of a circle with $N = 3$. As in Shi (2024), I also have an argument that involves an auxiliary matrix defined therein. □

Appendix D Empirical Analysis

In this section, I first validate model predictions using simulation data, and then test them using real-world observational data.

D.1 Empirical Strategy

The main empirical strategy for estimating the relationship between network position, especially centrality, and optimal payment is a nonparametric specification as follows:

$$Payment_i = f(Centrality_i) + X_i\beta + u_i, \tag{D1}$$

where $Payment_i$ is the optimal payment (share) in the numerical analysis and the observed payment in the empirical analysis of agent i, $Centrality_i$ is the degree centrality or Bonacich centrality,[B2] $f(\cdot)$ is an unspecified function to be nonparametrically estimated (can be parameterized as a linear or quadratic function when estimating with observational data, and this function is different from that in the Model section), X_i is a vector of control variables, including personal characteristics and fixed effects when using the real-world observational data. Finally, u_i is the error term.[B3] For robustness checks and to establish plausible causality and not just correlation, I also exploit a Hausman-style instrumental variable estimation. The results are qualitatively similar.

D.2 Numerical Analysis

I generate random Poisson graphs with different levels of mean degree—5, 10, 15, 20—and $N = 1000$ nodes. I also assign different values of β and calculate the optimal compensation scheme for each case. In the network, each agent has the same production and cost parameters: $(w_0, \theta, \beta, \gamma)$. Thus, there is no additional heterogeneity besides the payment share α. I compute the optimal compensation scheme given the first-order conditions and the degree and Bonacich centrality. I exploit a nonparametric specification (D1) to gauge the relationship between optimal payment share and degree and Bonacich centrality of each agent.

I first present the relationship for different network densities with a low β, $\beta = 0.05$, in Figures D1 and D2. In Figure D3 (a), as the degree increases, optimal pay first increases, then decreases, and then increases again. In Figure D1 (b)-(d), the optimal pay first remains roughly constant and then

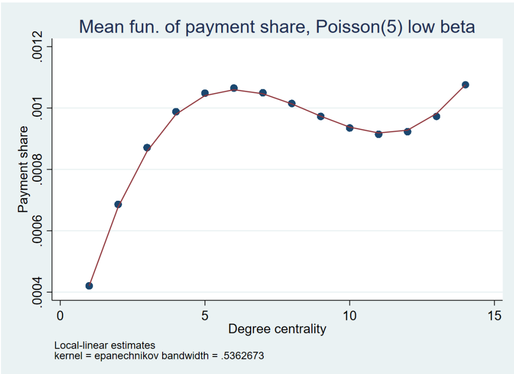
[B2]According to Section 3, the theoretical results are closer connected to degree centrality, but empirically they can also be applied to Bonacich centrality, as mentioned below.

[B3]I use robustness standard errors, or equivalently, cluster standard errors at the individual level.

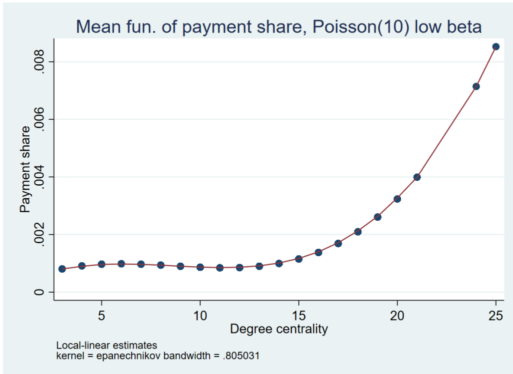
increases. This is consistent with Proposition 4 that optimal pay can be position-invariant, at least for a subset of agents. Figures D2 (a)-(d) with Bonacich centrality have similar patterns.

Next, I present the relationship for a high β, $\beta = 0.5$, in Figures D3 and D4. All figures indicate a strictly increasing relationship between the optimal payment share and the degree or Bonacich centrality. This is also consistent with Proposition 4, which suggests that a monotonic relationship can appear with a high β. This is the necessary condition for the strictly increasing relationship between optimal pay and degree centrality.

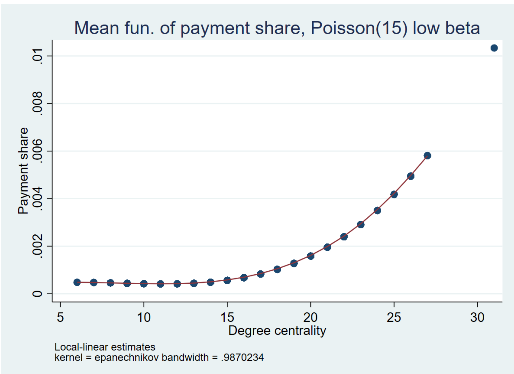
Figure D1: Relationship between optimal pay and degree centrality, low β



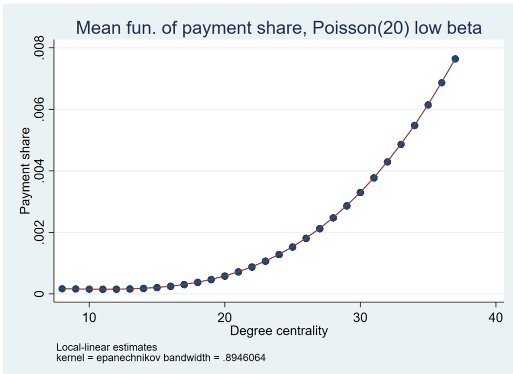
(a)



(b)



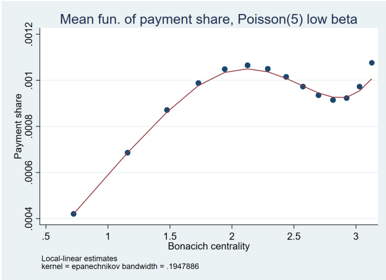
(c)



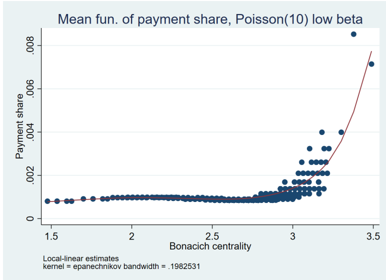
(d)

Notes: These figures illustrate the nonparametric relationship between the mean of optimal compensation α^* and degree centrality of each agent in different random Poisson networks, all with $N = 1000$ nodes. A local-linear kernel estimator is exploited, using the STATA command ***npregress kernel****. There is no heterogeneity with respect to the production and cost parameters of different agents. The value of β is low, 0.05. These figures show that the relationship between optimal pay and centrality can be non-increasing or flat with a low β.*

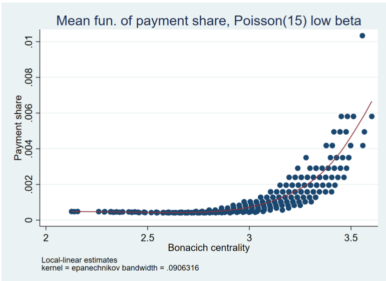
Figure D2: Relationship between optimal pay and Bonacich centrality, low β



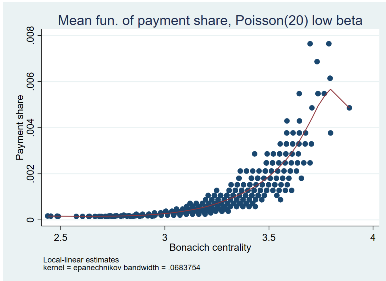
(a)



(b)



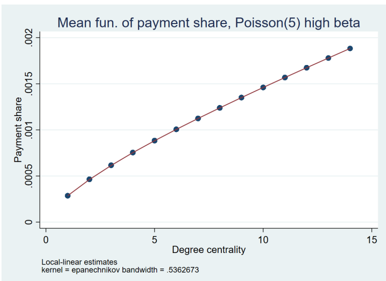
(c)



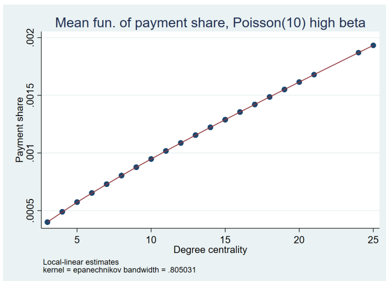
(d)

*Notes: These figures illustrate the nonparametric relationship between the mean of optimal compensation α^* and Bonacich centrality of each agent in different random Poisson networks, all with $N = 1000$ nodes. A local-linear kernel estimator is exploited, using the STATA command **npregress kernel**. There is no heterogeneity with respect to the production and cost parameters of different agents. The value of β is low, 0.05. These figures show that the relationship between optimal pay and centrality can be non-increasing or flat with a low β.*

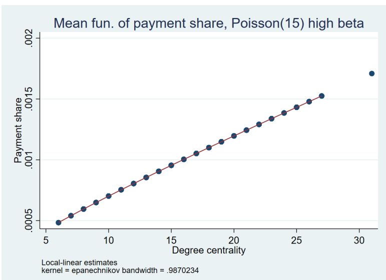
Figure D3: Relationship between optimal pay and degree centrality, high β



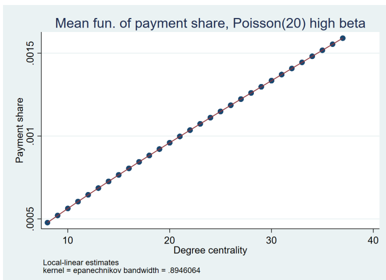
(a)



(b)



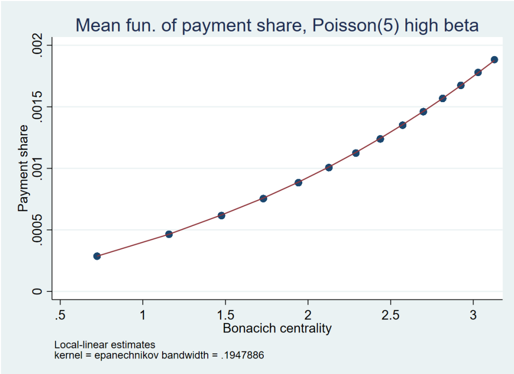
(c)



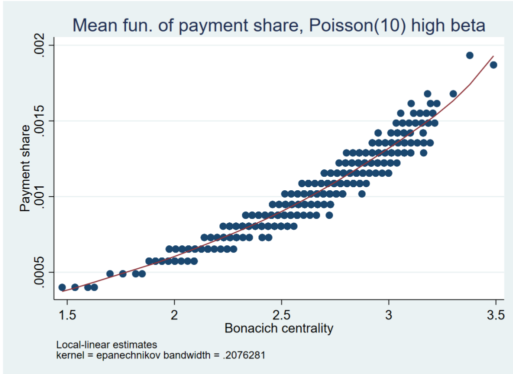
(d)

*Notes: These figures illustrate the nonparametric relationship between the mean of optimal compensation α^* and degree centrality of each agent in different random Poisson networks, all with $N = 1000$ nodes. A local-linear kernel estimator is exploited, using the STATA command **npregress kernel**. There is no heterogeneity with respect to the production and cost parameters of different agents. The value of β is high, 0.5. These figures show that the relationship between optimal pay and centrality can be strictly increasing with a high β.*

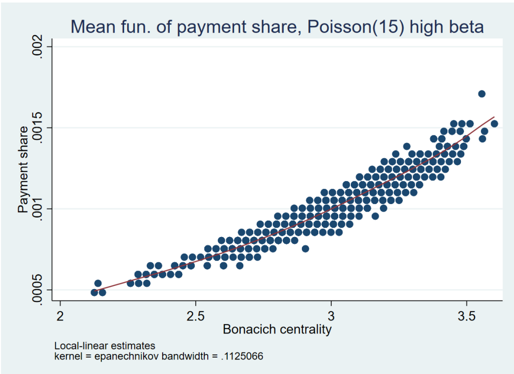
Figure D4: Relationship between optimal pay and Bonacich centrality, high β



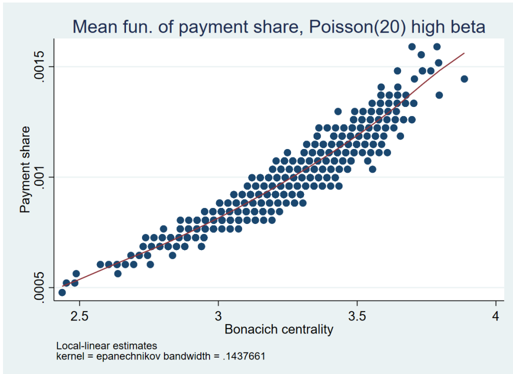
(a)



(b)



(c)



(d)

Notes: These figures illustrate the nonparametric relationship between the mean of optimal compensation α^* and Bonacich centrality of each agent in different random Poisson networks, all with $N = 1000$ nodes. A local-linear kernel estimator is exploited, using the STATA command ***npregress kernel****. There is no heterogeneity with respect to the production and cost parameters of different agents. The value of β is high, 0.5. These figures show that the relationship between optimal pay and centrality can be strictly increasing with a high β.*

D.3 Analysis Using Observational Data

Next, I use the Workplace Employment Relations Study (WERS) 2011 survey data from the United Kingdom to provide empirical support for the theoretical results. The data set provides information on the weekly pay and the social networks, including the number of workers with similar educational and social class (thus a high level of homophily), same gender and family background (another measure of homophily), same religion, and same ethnicity. Therefore, I construct the associated measures of degree centrality. Also, note that the data does not provide enough information to construct Bonacich centrality, so I cannot use this centrality measure for empirical analysis. I construct a degree centrality measure for each of the social connections. The details of the data can be found in Appendix D. I use log weekly payments to proxy the compensation scheme for each agent. Note that since I have controlled for workplace fixed effects, it is equivalent to using log equity share as the dependent variable. Under this log form, the regression coefficients can be interpreted as (semi-)elasticities.

In order to control for high-dimensional fixed effects, I use parametric regressions for ease of computation. The $f(\cdot)$ function in equation (D1) is specified as a linear or a quadratic function. In the regression, I control for age dummies, work experience dummies, a gender dummy, and workplace and survey date fixed effects.

Moreover, the empirical setting corresponds to the theoretical setting under the assumption that each workplace employs an "optimal" compensation scheme. This assumption is plausibly valid since I only include private firms in competitive sectors that maximize total efficiency.

I first use the full sample for regression analysis. I report the results in Table D1. In columns (1) through (4), in which the $f(\cdot)$ function is linear, none of the degree centrality measures is positively and significantly associated with the weekly pay. As I use a quadratic $f(\cdot)$ function in columns (5) through (8), the relationships become statistically insignificant.

Table D1: Network position and payment: Full sample results

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Dependent variable: log Weekly Pay Full Sample							
Deg. Educational Backgd.	-0.0177***				-0.0327***			
	(0.00200)				(0.0102)			
Deg. Gender Family Backgd.		-0.00366*				-0.00135		
		(0.00216)				(0.00977)		
Deg. Religious Backgd.			0.000855				0.00646	
			(0.00235)				(0.00947)	
Deg. Ethnicity Backgd.				0.00381				0.0162
				(0.00233)				(0.0112)
Deg. Educational Backgd.^{2}					0.000817			
					(0.000522)			
Deg. Gender Family Backgd.^{2}						-0.000129		
						(0.000533)		
Deg. Religious Backgd.^{2}							-0.000352	
							(0.000574)	
Deg. Ethnicity Backgd.^{2}								-0.000685
								(0.000612)
Workplace FE	Y	Y	Y	Y	Y	Y	Y	Y
Survey Date FE	Y	Y	Y	Y	Y	Y	Y	Y
Controls	Y	Y	Y	Y	Y	Y	Y	Y
Observations	20,902	20,902	20,902	20,902	20,902	20,902	20,902	20,902
R-squared	0.288	0.286	0.286	0.286	0.288	0.286	0.286	0.286

Notes: The sample is constructed using WERS 2011 data. Controls include age dummies, work experience dummies, a gender dummy, and workplace and survey date fixed effects. "Deg." is an abbreviation for "degree," and "Backgd." is an abbreviation for "Background." * Significant at 10%, ** 5%, *** 1%. The standard errors are clustered at the workplace level.

Next, I examine the heterogeneity underlying the main results. In particular, I look at the situations for a high β and a low β. According to Bandiera, Barankay, and Rasul (2004), I define offices for "white-collar" workers as workplaces with a high β—a high level of externality of mutual help or cooperation. Examples include a financial institution or a law firm that needs frequent interactions among members of the organization. A factory assembly line, a supermarket, or a farm is not such a case, and it corresponds to the case with a low β. Table D2 presents the results. Columns (1) through (4) correspond to the high β case, and all degree centrality measures are positively correlated with the weekly pay. Columns (5) through (8) correspond to the low β case, and the positive correlations disappear. Thus, the empirical findings support Proposition 4.

Table D2: Heterogeneous results based on cooperation externality

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Dependent variable: log Weekly Pay							
	High Cooperation Externality Workplace				Low Cooperation Externality Workplace			
Deg. Educational Backgd.	0.0255***				-0.0163***			
	(0.00291)				(0.00251)			
Deg. Gender Family Backgd.		0.0159***				-0.00309		
		(0.00243)				(0.00258)		
Deg. Religious Backgd.			0.0306***				0.00257	
			(0.00251)				(0.00274)	
Deg. Ethnicity Backgd.				0.0215***				0.00428
				(0.00235)				(0.00280)
Workplace FE	Y	Y	Y	Y	Y	Y	Y	Y
Survey Date FE	Y	Y	Y	Y	Y	Y	Y	Y
Controls	Y	Y	Y	Y	Y	Y	Y	Y
Observations	7,417	7,417	7,417	7,417	13,813	13,813	13,813	13,813
R-squared	0.828	0.984	0.831	0.828	0.297	0.295	0.295	0.295

Notes: The sample is constructed using WERS 2011 data. Controls include age dummies, work experience dummies, a gender dummy, and workplace and survey date fixed effects. "Deg." is an abbreviation for "degree," and "Backgd." is an abbreviation for "Background." * Significant at 10%, ** 5%, *** 1%. The standard errors are clustered at the workplace level.

Finally, I employ an instrumental variable estimation strategy for robustness checks, where the instrument is a Hausman-style instrument which is obtained by calculating the average in the same sector-location pair, excluding its own.[B4] The results reported in Appendix Table E2 are still robust.

Appendix E Data Compilation and Robustness Checks

The Workplace Employment Relations Study (WERS) is a national survey of people at work in Britain. It collects data from employers, employee representatives, and employees in a representative sample of workplaces. I use the wave of 2011 survey for empirical analysis. The information collected in WERS comes from 3 distinct sources: (1) a random probability sample of workplaces in which face-to-face structured interviews are conducted with the most senior manager responsible for employment relations and personnel issues—in each workplace a self-completion questionnaire is distributed before the interview to collate information on the basic characteristics of the workforce, and a second questionnaire is left at the end of the interview to assess the financial performance of the workplace; (2) survey interviews are undertaken in the same workplaces, with 1 trade union employee representative and 1 non-trade union representative where present; and (3) a self-completion survey with a representative group of up to 25 employees, randomly selected from each workplace participating in the survey.

Some of the information that has been produced by the survey includes: how workplaces are managed and organized; individual and collective representation at work; trade union recognition and membership, etc. In particular, the data provides information on social networks and the wages of surveyed employees. I construct 4 degree centrality measures based on such information: (1) degree of ethnicity background, which is the number of coworkers with the same ethnicity; (2) degree of

[B4]The instrument exploits the same idea as those in Hausman, Leonard, and Zona (1994) and Shi (2023a).

gender and family background, which is the number of coworkers with the same gender and family background (marital status and the number of children); (3) degree of educational background, which is the number of coworkers with the same educational attainment and social class; and (4) degree of religious background, which is the number of coworkers with the same religion. The summary statistics are provided in Table E1

Table E1: Summary statistics

Variable	Obs	Mean	Std. Dev.	Min	Max
log Weekly Pay	20,988	3.343	0.992	0	4.511
Degree Ethnicity Background	21,981	11.745	5.741	1	25
Degree Gender Family Background	21,981	9.783	4.911	1	23
Degree Educational Background	21,981	12.547	5.615	1	25
Degree Religious Background	21,981	7.848	4.354	1	21

Table E2: Robustness checks: Hausman instrumental variable regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Dependent variable: log Weekly Pay							
	High Cooperation Externality Workplace				Low Cooperation Externality Workplace			
	IV-2SLS							
Deg. Educational Backgd.	0.0254***				-0.0155***			
	(0.00291)				(0.00252)			
Deg. Gender Family Backgd.		0.159***				-0.00274		
		(0.00243)				(0.00257)		
Deg. Religious Backgd.			0.0306***				0.00297	
			(0.00250)				(0.00274)	
Deg. Ethnicity Backgd.				0.0217***				-0.00107
				(0.00235)				(0.00280)
Workplace FE	Y	Y	Y	Y	Y	Y	Y	Y
Survey Date FE	Y	Y	Y	Y	Y	Y	Y	Y
Controls	Y	Y	Y	Y	Y	Y	Y	Y
Observations	7,417	7,417	7,417	7,417	13,813	13,813	13,813	13,813
R-squared	0.828	0.984	0.830	0.827	0.295	0.294	0.294	0.294

Notes: The sample is constructed using WERS 2011 data. Controls include age dummies, work experience dummies, a gender dummy, and workplace and survey date fixed effects. "Deg." is an abbreviation for "degree," and "Backgd." is an abbreviation for "Background." * Significant at 10%, ** 5%, *** 1%. The instrumental variable is a Hausman instrument, which is constructed by calculating the average of the same variable in the same sector-location pair, excluding its own. The standard errors are clustered at the workplace level.