Measuring and Analyzing Total Factor Productivity in West African Economies: The Trends, Shocks, and Policies

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Abstract

The purpose of this paper is to measure and analyze the total factor productivity (TFP) trend for fourteen countries in the Economic Community of West African states from 2016 to 2022. The Data Envelopment Analysis (DEA) method is used to measure TFP using the CCR model. Among all the countries surveyed, Nigeria has the highest TFP and is the only country within the bloc that is optimally using factors of production while Niger has the lowest total factor productivity in the ECOWAS region within the study period. The study shows that the ECOWAS as a bloc is not efficiently using its factors of production to produce output, a reason why the region's growth has been low and volatile for decades since independence. The inefficiency is coming from the slack use of land as a factor of production. The study also shows that the COVID-19 pandemic and the Russia-Ukraine war had a profound impact on the total factor productivity in the ECOWAS bloc. Thus, there is a need for a land reform policy to get optimal returns from land usage in the region.

Keywords: Total factor productivity, Data Envelopment Analysis, and CRR Model

Introduction

The Economic Community of West Africa States (ECOWAS) is a regional group consisting of 15 countries which was founded in 1975 to promote economic trade, national cooperation, and the creation of a monetary union throughout West Africa. Looking at economic growth since independence, the ECOWAS economy has not only been low, but it has also been volatile. The legacy of low and volatile growth has hindered inclusive and sustainable growth and this in turn affected the poverty alleviation in the region. The fundamental policy question that is occupying the mind of the policymakers is what factors are responsible for the low and volatile growth in the ECOWAS region?

The low and volatile growth is believed to be caused by low productivity in the productive sectors of the economy. ECOWAS has one of the lowest productivity levels as a bloc in Sub-Saharan Africa (SSA). The low productivity level is a concern because sustainable per capita income depends on the level of productivity of a country which in turn affects the level of poverty. Most research has shown that the difference in the level of income or even poverty level among countries can be attributed to differences in the level of total factor productivity (TFP). TFP is an important measure of efficiency and, hence, a significant gauge for policymakers. Using data from 14 countries from ECOWAS[1], this study measures TFP performance at the macro level. The objective of this research is to measure the productivity level of countries in the ECOWAS region. Moreover, the study also examines the effects of external shocks such as the COVID-19 pandemic and the Russia-Ukraine war on TFP.

There have been many studies on TFP but has been mainly limited to the use of conventional econometric or parametric methods. Limited studies are using non-parametric methods such as data envelopment analysis (DEA) to analyze TFP, especially in the ECOWAS region. This study aims to fill this gap using DEA. The advantage of non-parametric methods such as DEA over traditional econometric methods is that one does not need to make assumptions about the method use, hence, there is no issue of endogeneity problem in case when some assumptions do not hold. Moreover, the real sector of ECOWAS has undergone a major transformation in the past decade with two major shocks (COVID-19 and the Russia-Ukraine war), but there have been few studies analyzing how TFP changes during this period. Unlike

[1] Liberia was not used in the study due to lack of data.

many previous papers, the study analyzes how these shocks affect the ECOWAS's TFP using a non-parametric method.

To measure productivity, one may use stochastic frontiers analysis which is an econometric method or data envelopment analysis (DEA) which involves mathematical programming. The study used the DEA to measure productivity. The results show that among the countries surveyed in the study, Nigeria has the highest level of productivity and is the only country operating efficiently for the period under review. The second-best country in terms of productivity performance is Cabo Verde. Niger has the lowest productivity level in the region followed by The Gambia.

The remainder of the paper is organized as follows. Section 2 reviews the literature, especially on studies dedicated to nonparametric methods. Section 3 presents the methodology used to estimate TFP, section 4 reports the obtained results, and section 5 concludes the study and gives policy recommendations.

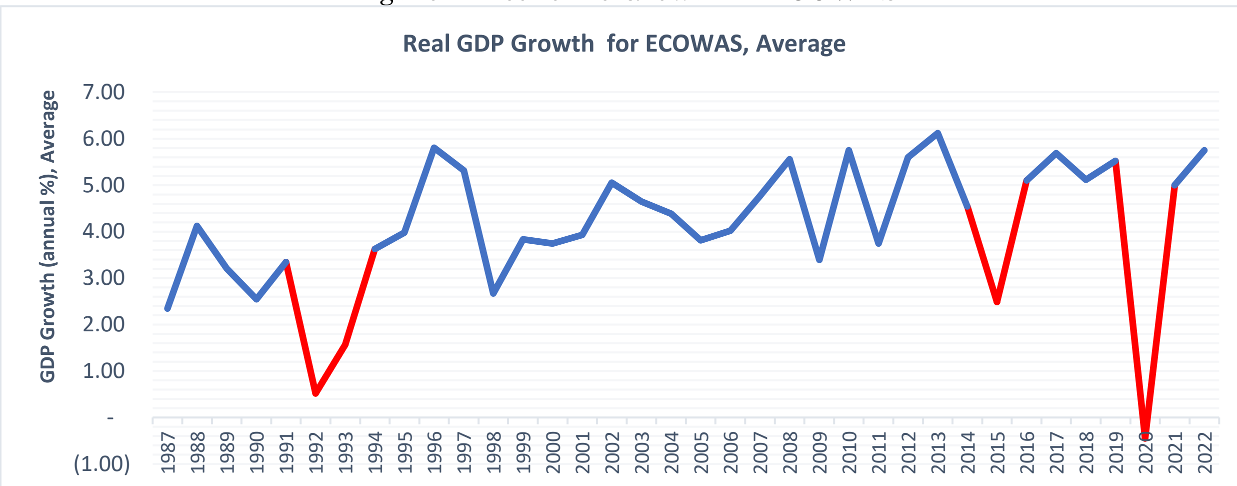
Literature Review

Figure 1 shows that the average economic growth in the ECOWAS region is low and volatile which is believed to be caused by low TFP in the region. There is a notion that the low TFP is caused by inefficient usage of factors of production such as capital, labour, and land to produce outputs. TFP is defined as the efficiency and intensity with which Decision-Making Units (firms, branches, countries, regions etc.) turn inputs into outputs. TFP has been pointed out as the major factor in generating and driving economic growth especially in the long run when factors accumulation is likely to go through diminishing returns. The graph shows that there was a huge contraction in growth in 1992, 2015, and 2020. The decline in growth in 2015 and 2020 was due to the Ebola outbreak and COVID-19 pandemic in the region, respectively.

Since the seminal work of Solow [1] there have been many studies that have examined the contribution of factors of production and TFP to economic growth of a country. Solow [1] has pointed out that in the long run, the growth of a country is dependent on the TFP since capital accumulation is subjected to diminishing returns in the long run.

Abekah-Koomson, Loon, Premaratne, & Yean [2] in their study titled "Total Factor Productivity Growth: Evidence from West African Economies", indicated that the most significant decrease in TFP in ECOWAS happened in the late 1990s and 2000s which they believed to be driven by spillover effect of the Asian and global financial crises. The study also used stochastic frontier modelling to show that the technical efficiency of the region is totally below the optimal level of production.

Figure 1: Economic Growth in ECOWAS



Source: World Development Indicators, World Bank, 2023.

Sissoko, Sloboda, and Kone [3] used the Malmquist Productivity Index to decompose the drivers of Economic growth in the ECOWAS region from 1981 to 2015. They have concluded that both factors of production and TFP are responsible for the increase in growth for the period under review, however, the main driver of growth was TFP. TFP has increased by 11.1 percent between 1981 and 2015. Sloboda and Sissoko [4] using the Arellano–Bover and Blundell–Bond estimator show that TFP has a significant impact on economic growth in the ECOWAS region.

Mendes and Arvanitis [5] in their study entitled 'Total factor productivity and growth strategy in Cabo-Verde' examined the evolution of Total Factor Productivity (TFP) for Cabo Verde. Their analysis shows that capital and labour contributed positively to growth, but the contribution of TFP to growth during 1980– 987 and 2007–2014 was negative. To enhance TFP they lamented the role of private sector competitiveness.

Wolassa L. Kumo [6] carried out a study to decompose economic growth to measure factor contribution to growth in Sierra Leone for the period 1980-2019 using the univariate HP filter and production function approaches. The results show that TFP declined significantly during the period of civil war causing real GDP and per capita to contract. The real GDP contracted by 62 percent in 2001 relative to the level in 1990. The economic and political reforms and macroeconomic stability helped the TFP to rebound a decade after the civil war by contributing to GDP by 50 percent from 2002 to 2011. However, the TFP started to contract two decades after the civil war due to two external shocks: the Ebola virus outbreak and a fall in the price of iron ore which is a main export commodity for Sierra Leone. This caused the TFP growth to decline during 1980-2019 causing the average real GDP growth to contract by 22 percent.

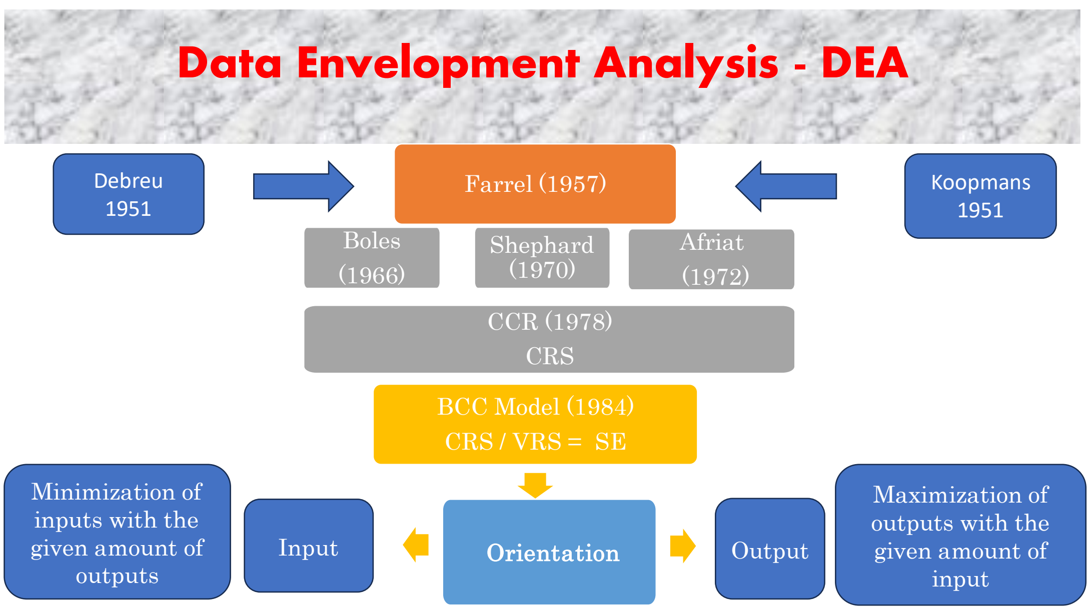
Shen and Valdmanis [7] examined the TFP across some African countries from 1989 to 2017 using a robust nonparametric method to solve the issue of heterogeneities in production technologies among African countries. The findings indicate that the yearly annual productivity growth rate in Africa ranges between 0.73 and 4.29 percent when convex technology and aggregate directional distance functions are applied. The result furthermore shows growth in Africa is mainly driven by improvement in scale efficiency and technological progress.

Method

To measure the Total Factor Productivity in ECOWAS, the study uses the Data Envelopment Analysis (DEA) method. DEA is a non-parametric technique used in operational research and economics to estimate production frontiers or curves [8]. DEA is a method of linear programming where a non-parametric production frontier or a piece-wise surface over data is constructed. The efficiency of each decision-making unit (DMU) is calculated relative to this frontier. Compared to parametric techniques that demand the specification of the production or cost function, the non-parametric methods link feasible input to output based on the data that is available without assuming the production or cost function [9]. The most non-parametric method used in the study of productivity analysis is DEA, this is due to its enveloping property of the dataset where the most efficient DMUs make up the production frontier against which all other DMUs are compared. The popularity of the DEA stemmed from not making assumptions about the production process and easy computation using linear programming [10].

The modern efficiency measurements started with Farrell 1957 [11] who drew upon the work of Debreu [12] and Koopmans [13]. Farrell 1957 [11] proposed the piece-wise linear convex hull method for frontier estimation. The Farrel model assumed constant return to scale (CRS) and the method of measurement is input-orientation where the DMUs try to produce a given output by minimizing the inputs usage. The method used two-dimensional data where all the DMUs are graphed on a two-axis graph and the frontier is constructed. Those DMUs that lie on the frontier are considered efficient while those that do not lie on the frontier are considered inefficient. The efficiency scores of the DMUs are then calculated using the distance between their locations and the estimated frontiers. Thus, the frontier "envelops" the whole data.

Figure 2: Theoretical Development of DEA



Source: Author, Alfusainey Touray

Few authors adopted the Farrell model even twenty years after its initial publication. Boles 1966 [14], Shephard 1970 [15] and Afriat 1972 [16] tried to use mathematical programming to construct frontiers, but these works did not receive much attention until a paper by Charnes, Cooper, and Rhodes in 1978 (CCR model) [8] where the word DEA first appeared, and since then a lot of empirical studies have been carried out using CCR model or augmented versions. CCR model is built on the Farrell model by extending it to multiple inputs and multiple outputs method to measure productive efficiency using linear mathematical programming for the first time. The operationalization of the CCR method then became known as DEA. The CCR model is an input-oriented CRS model.

3.1. CCR (CRS) Model

The mathematical definition of efficiency for single input and single output is the ratio of the output to the input as given in equation (1):

$$Efficiency = \frac{Outputs}{Inputs} \qquad E[1]$$

The computation of efficiency becomes difficult when there are multiple inputs and multiple outputs, which is normally the case in real life. This is where DEA comes into play where it calculates efficiency as the weighted sum of output to the weighted sum of input, mathematically this is represented in equation (2):

$$Efficiency\ (E) = \frac{Weighted\ Sum\ of\ the\ Outputs}{Weighted\ Sum\ of\ the\ Inputs} \qquad E[2]$$

To understand equation (2) let's assume that we have a group of observed $N\,DMUs$ $\{DMU_j; j = 1, \ldots\ldots N\}$ with M *inputs* $\{X_{ij}; i = 1, \ldots\ldots M\}$ and S *outputs* $\{Y_{rj}; r = 1, \ldots\ldots S\}$. The efficiency of the jth DMU ($\boldsymbol{E_j}$) is defined as:

Fractional DEA Model **[3]**

$$\boldsymbol{E_j} = \boldsymbol{Max}\ \frac{Weighted\ Sum\ of\ the\ Outputs}{Weighted\ Sum\ of\ the\ Inputs} = Max\ \frac{\sum_{r=1}^{S} \boldsymbol{u_{rj}}\, \boldsymbol{Y_{rj}}}{\sum_{i}^{M} \boldsymbol{v_{ij}}\, \boldsymbol{X_{rj}}}$$

subject to:

$$E_j = \frac{\sum_{r=1}^{S} u_{rj}\, Y_{rj}}{\sum_{i}^{M} v_{ij}\, X_{ij}} \;\; \leq \; 1\,; \quad \forall j$$

$$u_r \quad v_i \;\geq\; 0 \;\forall r. \forall i$$

Where:

- $\boldsymbol{Y}-$is the output
- $\boldsymbol{X}-$is the input
- $\boldsymbol{Y_{rj}}-$ is the quantity of the rth output from DMU_j
- $\boldsymbol{u_{rj}}-$ is the weight of the rth output from DMU_j
- $\boldsymbol{X_{ij}}-$is the quantity of the ith input from DMU_j
- $\boldsymbol{v_{ij}}-$ is the weight of the ith input from DMU_j

The optimal weights ($\boldsymbol{u}*$, $\boldsymbol{v}*$) are obtained by solving the equation (3). This includes finding the optimal values for $\boldsymbol{u}$ and $\boldsymbol{v}$ such that the efficiency of j^{th} DMU is maximized subject to the constraints that the efficiency value of each DMU is less than or equal to zero. Moreover, the weights for input and output values range between zero and positive numbers.

The model in equation (3) is a fractional DEA model. One issue with the fractional DEA model is that it has infinitely many solutions. To solve this problem, one can impose a constraint $\boldsymbol{\sum_{i}^{M} v_{ij}\, X_{rj} \;=\; 1}$**,** which provides a linear programming (LP) model in equation (4):

CRS - Input-Oriented Model **[4]**

,

$$\boldsymbol{E_j} = \; Max \sum_{r=1}^{S} u_{rj}\, Y_{rj}$$

Subject to:

$$\sum_{r=1}^{S} u_{rj}\, Y_{rj} \; - \sum_{i}^{M} v_{ij}\, X_{ij} \;\leq\; 0; \qquad \forall j$$

$$\sum_{i}^{M} v_{ij}\, X_{ij} \;\; = \; 1$$

$$u_{rj}\,, v_{ij} \;\;\geq\; 0 \;;\; \forall r. \forall i$$

The duality in linear programming can be used to express equation (4) in terms of the minimization problem as in equation (5) below:

CRS - Output-Oriented Model **[5]**

$$\boldsymbol{E_j} = Min \sum_{i}^{M} v_{ij} X_{rj}$$

$$u_r, v_i$$

Subject to:

$$\sum_{r=1}^{S} u_{rj} Y_{rj} - \sum_{i}^{M} v_{ij} X_{ij} \leq 0; \quad \forall j$$

$$\sum_{r=1}^{S} u_{rj} Y_{rj} = 1$$

$$u_r, v_i \geq 0 \quad \forall r. \forall i$$

3.2. BCC Model

The assumption of the CRS or CCR model is appropriate when all the DMUs are operating at the optimal scale. However, in the presence of imperfect competition, government controls, limitations on finance etc. may prevent DMUs from operating optimally. To solve this short-coming of the CCR model, Afriat 1972 [16], Fare, Grosskopf and Logan 1983 [17], and Banker, Charnes, and Cooper 1984 (BCC model) [18] suggested augmenting the CCR model to accommodate for variable return to scale (VRS). The extension of the CCR model was given by Banker, Charnes, and Cooper in 1984 [16], and the method is known as the BCC or VRS model. Mathematically, the model can be stated as:

CRS - Input-Oriented Model **[6]**

,

$$\boldsymbol{E_j} = Max \sum_{r=1}^{S} u_{rj} Y_{rj} + w_0$$

Subject to:

$$\sum_{r=1}^{S} u_{rj}\, Y_{rj} - \sum_{i}^{M} v_{ij}\, X_{ij} + w_0 \leq 0; \qquad \forall j$$

$$\sum_{i}^{M} v_{ij}\, \mathrm{X_{ij}} = 1$$

$$u_{rj}, v_{ij} \geq 0 \;;\; \forall r. \forall i$$

The convexity constraint w_{qj} has an unrestricted sign and represents the difference between the CRR model and the BCC model. Once an optimal solution $(\boldsymbol{u}^*, \boldsymbol{v}^*, \boldsymbol{w_0}^*)$ of the model is derived efficiency through the BCC model (E_{BCC}) for the DMUs is obtained directly from the objective function.

The returns to scale (RTS) are determined using the optimal value of the free variable w_0* Given the point $(\boldsymbol{X}, \boldsymbol{Y})$ that lies on the efficient frontier, the RTS at this point are identified by the following three conditions:

- Increasing Returns to Scale (IRS) exist at $(\boldsymbol{X}, \boldsymbol{Y})$ if and only if $\sum w_0{}^* > \mathbf{0}$ for all optimal solutions. This means the increase in all production factors (inputs) resulted in more production (outputs).
- Decreasing Returns to Scale (DRS) exist at $(\boldsymbol{X}, \boldsymbol{Y})$ if and only if $\sum w_0{}^* < \mathbf{0}$ for all optimal solutions, meaning an equal increase in all production factors led to less production.
- Constant Return on Scale (CRS) exist at $(\boldsymbol{X}, \boldsymbol{Y})$ if and only if $\sum w_0{}^* = \mathbf{0}$ in any optimal solutions, where an equal increase in all production factors led to the same amount of increase in production.

The duality in linear programming can be used to express equation (4) in terms of the minimization problem as in equation (7) below:

CRS - Output-Oriented Model [7]

$$E_j = Min \sum_{i}^{M} v_{ij} X_{ij} + w_0$$

$$u_r, v_i$$

Subject to:

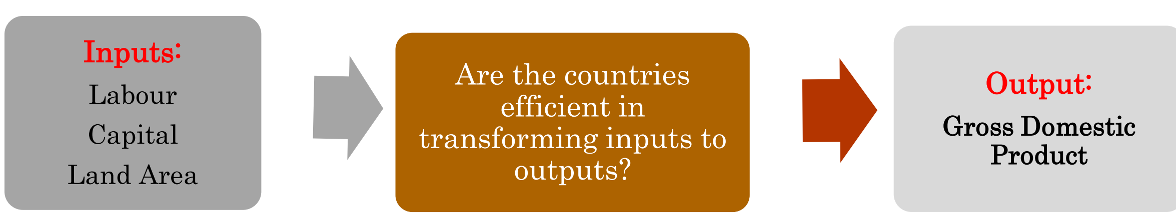
$$\sum_{r=1}^{S} u_{rj} Y_{rj} - \sum_{i}^{M} v_{ij} X_{rj} + w_0 \leq 0; \quad \forall j$$

$$\sum_{r=1}^{S} u_{rj} Y_{rj} = 1$$

$$u_r, v_i \geq 0 \quad \forall r. \forall i$$

This study uses the CCR (CRS) model to compute the TFP of 14 ECOWAS countries[2] using labour, capital, and land area as inputs and gross domestic product as output as shown in Figure 3:

Figure 3: The Input and Output



[2] Liberia was not included because of lack of data.

Table 1: Variable used in the Study.

Variables	Variable	Measurement Unit	**Source**
Input	Land Area	Land area (sq. km)	WDI, World Bank, 2023
	Capital	Gross capital formation (constant 2015 U$)	WDI, World Bank, 2023
	Labour	Labor force, total	WDI, World Bank, 2023
Output	GDP	GDP (constant 2015 US$)	WDI, World Bank, 2023

Source: World Development Indicators, World Bank, 2023.

Table 2: Decision Making Units, ECOWAS

DMU	Code
Benin	BEN
Burkina Faso	BFA
Cabo Verde	CPV
Cote d'Ivoire	CIV
Gambia, the	GMB
Ghana	GHA
Guinea	GIN
Guinea-Bissau	GNB
Mali	MLI
Niger	NER
Nigeria	NGA
Senegal	SEN
Sierra Leone	SLE
Togo	TGO

Note: Liberia was not included because of lack of data.

Results

Table 3 shows the descriptive statistics of the variable used in the study. The output (GDP) has an average value of $52,250,081,072 with a minimum and maximum value of $1,113,878,218 and $535,336,034,389, respectively. The average value of land area (sq. km), capital ($), and labour (people) are 352,618, $9, 658, 666, 593, and 9, 074, 971, respectively.

Table 3: Descriptive Statistics for ECOWAS Region from 2016 to 2022, Average

Variables	**Variables**	**Average**	**Minimum**	**Maximum**	**Std Deviation**
Input	Land Area	352,618	4,030	1,266,700	425,033
	Capital	$9, 658, 666, 593	$84, 755, 163	$83, 121, 870, 898	18, 964, 202, 574
	Labour	9, 074, 971	228,928	73,272,344	16, 015, 844
Output	GDP	$52, 250, 081, 072	$1,113,878,218	$535,336,034,389	4,269,180,039

Source: Authors Calculation

Table 4 shows the TFP level scores for each country from 2016 to 2022 and each country's average efficiency from 2016 to 2022. On average, Nigeria has the highest efficiency level (1.00), and it is the only country that has been operating at an efficient level in using land, labour, and capital in the producing output (GDP) from 2016 to 2022. The second country is Cabo Verde with an average efficiency level of 0.994. The country with the lowest average efficiency level from 2016 to 2022 is Niger (0.439) followed by the Gambia (0.547). Figure 4 also shows this information.

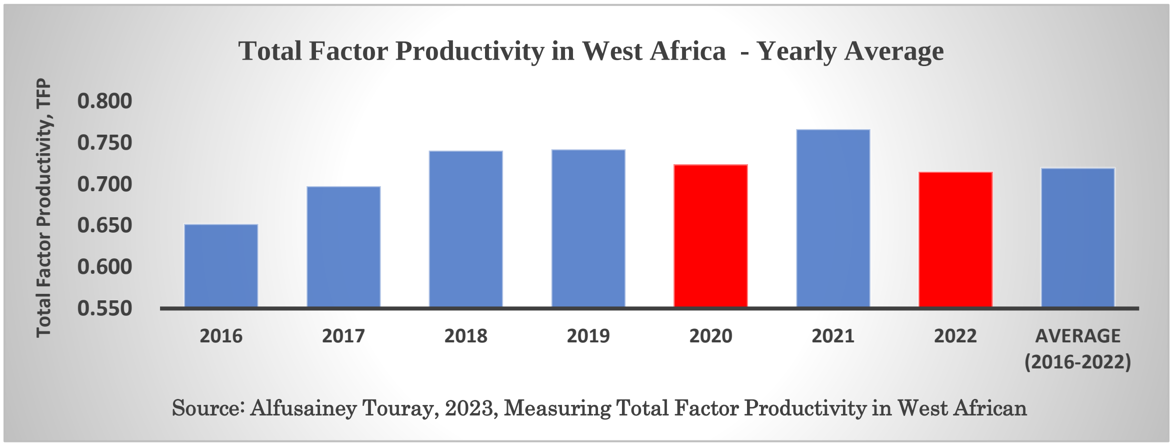
Table 4: Total Factor Productivity of ECOWAS and ECOWAS Countries

DMU	2016	2017	2018	2019	2020	2021	2022	Average: 2016-2022
Benin	0.670	0.565	0.567	0.576	0.563	0.629	0.530	0.586
Burkina Faso	0.563	0.528	0.558	0.527	0.611	0.561	0.605	0.565
Cabo Verde	0.958	1.000	1.000	1.000	1.000	1.000	1.000	0.994
Cote d'Ivoire	0.685	0.733	0.748	0.786	0.851	0.840	0.823	0.781
Gambia, the	0.541	0.557	0.619	0.604	0.446	0.622	0.437	0.547
Ghana	0.504	0.547	0.568	0.608	0.627	0.636	0.626	0.588
Guinea	0.323	0.612	0.712	0.786	0.731	0.841	0.720	0.675
Guinea-Bissau	1.000	1.000	1.000	0.901	0.792	0.892	0.793	0.911
Mali	0.542	0.654	0.792	0.780	0.842	0.781	0.847	0.749
Niger	0.424	0.431	0.440	0.439	0.455	0.450	0.432	0.439
Nigeria	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Senegal	0.555	0.646	0.610	0.609	0.644	0.657	0.652	0.625
Sierra Leone	0.761	0.737	1.000	1.000	1.000	1.000	1.000	0.928
Togo	0.600	0.754	0.755	0.772	0.571	0.819	0.546	0.688
ECOWAS-Average	0.652	0.697	0.741	0.742	0.724	0.766	0.715	0.720

Source: Author's Computation

Figure 4 shows the TFP level of the ECOWAS region has been increasing from 2016 to 2019 from 0.652 to 0.742 then it declined to 0.724 in 2020. The decline in the productivity or efficiency level in 2020 can be attributed to the impact of the COVID-19 pandemic. The slack in economic resources in 2020 came from a decline in the productivity level of capital and labour due to restrictions and lockdown in 2020. There was a recovery in the productivity level in 2021, however, it declined in 2022. The decline in productivity in 2022 was caused by the Russia-Ukraine conflict in 2022. This shows that the productivity in the ECOWAS region is vulnerable to shocks.

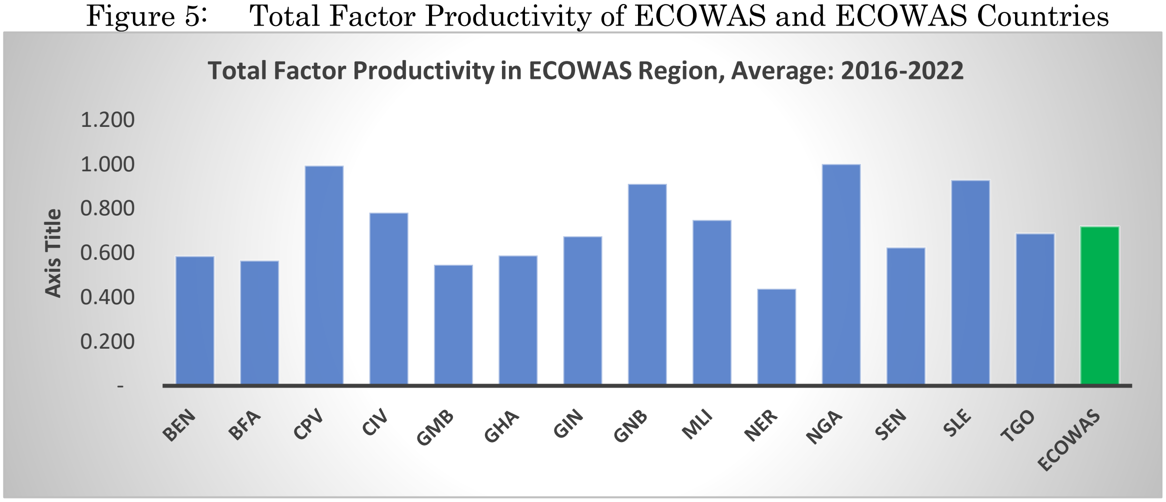
Figure 4: Total Factor Productivity of ECOWAS and ECOWAS Countries



Source: Author

Figure 5 shows the average TFP level from 2016 to 2020 of the ECOWAS bloc and its countries. The figure shows that out of the fourteen countries, on average, Nigeria is the only country that is operating at the optimal level, the rest of the countries are not optimally using the inputs to produce output. The inefficiency in most of the countries is due to low labour and land productivity.

Figure 5: Total Factor Productivity of ECOWAS and ECOWAS Countries



Source: Author

Conclusion

The study uses the CRR model with a constant return-to-scale assumption. The empirical results show that other than Nigeria all the other 13 countries are inefficient in using inputs to produce output from 2016 to 2022, this is mainly due to low productivity in land and labour as inputs. Moreover, the study shows that the ECOWAS region's productivity is vulnerable to shocks such as COVID-19 and the Russia-Ukraine war.

The cause of the inefficiency is the slack in the usage of land and labour as inputs. Most countries are not efficiently using land and labour to produce goods and services. This can be attributed to the land tenure system and the low quality of labour. The land tenure system in the ECOWAS bloc does not promote commercial agriculture, hence, productivity from the agricultural sector in the region is low which in turn leads to lower TFP. Moreover, the labour force has been expanding, however, the quality of the labour force has not been improved. Hence, output per work has been in decline causing TFP to also decline in the region. The policy implication is that the policymakers need to change the land tenure system that will encourage the commercialization of agriculture on a land scale. Moreover, the education system needs to be reformed to meet the needs of the job market.

The study uses the CRR model with constant return-to-scale assumptions. This does not hold in a situation where the market is not competitive or when countries face resource constraints. Thus, future research can use a model where it has both constant and variable return to scales.

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