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Abstract

Using a simple duopolistic trade model with demand uncertainty and an identical traded product, we show that we can view trade in goods as implicit exports/imports of risk and risk aversion. Specifically, we show that a relatively "risk-aversion abundant" country is more likely to be a net importer of the product – hence an importer of low risk-aversion. Similarly, a "relatively high-risk abundant" country is more likely to be the net exporter of the product - hence an importer of low risk.

We also show that market correlations and differences in risk aversion and risk are sources of implicit risk-sharing and diversification gains from trade. Consequently, the relatively high-risk or high-risk-aversion country always gains from trade, whereas the other country will most likely gain unless markets are highly positively correlated. Furthermore, we re-examine the (Brander-Spencer) strategic export subsidy game in the context of uncertainty and find that because both efficient risk management and rent shifting need to be considered, contrary to conventional wisdom, the equilibrium policy regime may be an export tax rather than a subsidy.

Keywords: Patterns of Trade, Gains from Trade, Risk, Risk Aversion, Exports.

JEL Classification: F12, F13, F15, D81.

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1 Introduction

Traditional trade theories explain trade patterns and gains from trade (GFT) by differences among countries. For example, Ricardian models focus on technology differences, whereas the Heckscher-Ohlin-Samuelson model focuses on differences in endowments. Relatively more modern trade theories focus on non-competitive markets, strategic behaviour, economies of scale and other imperfections, allowing them to explain also other phenomena, such as trade in similar commodities among similar countries, win-win outcomes, Etc.

Although these strands of international trade theory have generally been addressed under certainty, the importance of uncertainty has long been recognized in the literature. For example, Batra (1975) examined the validity of the Heckscher-Ohlin-Samuelson theorem in the context of uncertainty; Batra and Russell (1974) examined the GFT under uncertainty, and the effects of uncertainty within the context of the Ricardian model were studied by Ruffin (1974) and Turnovsky (1974). Other examples include Helpman and Razin (1978), where a comprehensive analysis of the impact of uncertainty on trade in the presence of stock markets is provided, and Appelbaum and Kohli (1997), who examined the effects of uncertainty on income distribution. Extensive literature also examines the condition under which the HO model holds or does not hold under uncertainty. For example, Hoff (1994) investigates when and why the Heckscher-Ohlin-Samuelson model does not hold under uncertainty; Anderson (1981) shows the conditions under which the Heckscher-Ohlin and Travis-Vanek theorems will hold under uncertainty, and Dumas (1980) extends trade theorems to a broader class of uncertainty models.[1]

More recently, the effects of uncertainty have been studied in the context of trade agreements (Limão and Maggi (2015), Appelbaum and Melatos (2016, 2020, 2024); trade policy (Handley (2014), Feng et al. (2017)); sourcing and export decisions ((Gervais (2018), Lewis (2014); trade generation in a general equilibrium framework (Baley et al. (2020)) and the effects of uncertainty on trade flows and income distribution (Novi and Taylor (2020)).

The implications of uncertainty in an identical product Cournot-duopolistic trade model (similar to Brander and Krugman (1983)) were first studied by Anam and Chiang (2003). Using a model where market demands in the two trading countries are stochastic and possibly correlated (but the countries themselves are identical in all respects), they demonstrate that when firms are risk averse, potential benefit from diversification becomes an extra motive for as well as a new source of social gains from trade. In this paper, we extend the role of uncertainty in the context of the Brander-Krugman (1983) model in several new directions. Specifically, we examine the role of risk, risk aversion and correlation (RRAC) in determining trade patterns, gains from trade, and hence motive for trade. We also reexamine the nature and role of Brander and Spencer's (1985) strategic trade policy in the stochastic environment of the model.

[1] Cheng (1987) examines the conditions under which self-sufficiency is optimal under uncertainty.

First, we show that differences in risk and risk aversion (RRA)provide (in themselves) an explanation of trade patterns. In the presence of RRA differences, in turn, market correlation (which is a non-difference-based characteristic) can exacerbate or mitigate the effects of RRA. More importantly, in the spirit of the "content of trade" models,[2] this paper shows that, indeed, the pattern of trade in goods also reflects the implicit RRA "contents of trade." Therefore, the flow of goods can be interpreted as a "flow" of risk and risk aversion.

Specifically, we show that, in general, if countries differ only in their attitudes toward risk, the less risk-averse country will be a net exporter of the product. We can interpret this result as saying that if a country is relatively "abundant in low risk-aversion," it will be a net exporter of low risk-aversion. We can think of this as the "risk aversion content of trade." We also show that if countries differ only in their risks, the country with lower risk will be a net importer of the product. Thus, we can interpret this result as saying that if a country is relatively "abundant in low risk," it will be an exporter of low risk.

We then consider a more general case where both risk and risk aversion are different. We show that a country is more likely to be an importer of the product (an importer of low risk-aversion) when its risk aversion is high. However, it is more likely to be an exporter of the product (an importer of low risk) when its risk is high. We find that the role of correlation is a bit more complicated: the likelihood that a country will become an importer decreases with correlation when the correlation is "sufficiently high" but increases with correlation when the correlation is "sufficiently low."[3]

Second, within the same model framework, we expand on the intuition developed in Anam and Chiang (2003) and show that RRAC introduce GFT due to implicit risk-sharing and diversification benefits that reduce the "cost of uncertainty." These difference-based and non-difference-based sources of GFT are in addition to the standard pro-competitive effect of trade (which is present with and without uncertainty). Differences in risk aversion generate GFT because they introduce indirect (non-cooperative) risk-sharing benefits, viewed as implicit partial insurance. To demonstrate this implicit insurance effect, consider the following. As is well known, bargaining or collusive equilibria always result in Pareto efficient risk-sharing, i.e., efficient insurance. We do not get this result, however, with non-cooperative interactions as in our duopoly model. Nevertheless, although there is no direct risk-sharing in our duopoly model, implicit risk-sharing, albeit not Pareto efficient, still occurs.[4] Moreover, the extent to which it occurs depends on the RRAC parameters.

As to the diversification benefits, the mere existence of a risky foreign market introduces diversification pos-

[2]See Vanek (1968), Helpman (1984), Davis and Weinstein (2002).

[3]Throughout the paper, when we talk about high and low correlation, we refer to the non-absolute values.

[4]For example, within a cooperative/bargaining framework, a risk-neutral agent always provides full insurance to a risk-averse agent, giving rise to Pareto efficient risk-sharing. However, in a non-cooperative duopoly model, the agents/firms do not collude or engage in cooperative bargaining. Therefore, although the resultant risk-sharing will not be Pareto efficient, it will not be entirely absent. See footnote 15 below.

sibilities. Market correlation can give rise to further diversification benefits. The intuition behind these sources of GFT is simple. Essentially, for two countries under autarky, trade is akin to introducing "new risky assets" to choose from when selecting an "optimal portfolio." Clearly, a larger feasible asset set expands an investor's (portfolio choice) efficiency frontier in the absence of strategic behaviour. An expanded efficiency frontier allows for better risk-management/diversification by taking into account the new assets' (in our case, the foreign country's) risks and market correlation. It is not just the mere introduction of a new asset that is beneficial; it is the new risk-management possibilities that the new asset brings. However, the picture in our model is more complicated because, unlike in portfolio choice theory, the countries behave strategically. Consequently, each country's ultimate portfolio value depends on its rival's actions and is determined by the game's Nash equilibrium. Nevertheless, the diversification benefits of an expanded choice set are still present.

We show that the country whose risk or risk-aversion is relatively high always gains from trade. The other country will, generally, also gain unless markets are highly positively correlated. However, the world always gains from trade, regardless of whether the other country gains. We also show that, in general, world gains from trade are likely to decrease with risk and the correlation, but the impact of risk aversion may be positive or negative (depending on whether the risk is low or high). We compare gains from trade with and without uncertainty and show that both world and country gains may, sometimes, be higher with uncertainty than without it. Finally, to assess the responsiveness of world gains from trade, we calculate local measures of the RRAC elasticities and find that world gains from trade are most responsive to changes in market correlation.

Finally, we revisit the role of strategic trade policy pioneered by Brander and Spencer (1985), in our stochastic model. Like the conventional model, each government has an incentive to intervene with an export subsidy or a tariff to confer strategic advantage to their respective firms, which the firms themselves are unable to achieve given the nature of the game they are locked in. Since the implications of export subsidies are more interesting in the context of our model, we confine our discussion of strategic trade policy to the subsidy case only. We set up a two-stage model, where in stage 1, each government non-cooperatively sets the rate of export subsidy, and in stage 2, the firms determine respective Cournot-duopoly outputs given these subsidies. The main departure from the conventional strategic trade policy models arises from the fact that in addition to rent shifting, risk-management benefits, which depend on the RRAC parameters, determine the level and direction of the subsidy. We show that for certain configurations of RRAC parameters, the equilibrium policy can be an export tax rather than a subsidy. In this case, the risk-sharing benefits of the tax outweigh the rents yielded to the rival firms in the export market.

2 The Model

Consider two countries, each with one firm.[5] To isolate the effects of uncertainty, we assume that the two firms (countries) are identical in all respects except for those that pertain to risk or risk aversion. Furthermore, we make the common assumption that the markets in the two countries are physically distinct. Hence, we assume that the firms produce an identical product, x, using the same technology. Denote the amount of x that the firm in Country i (Firm i) sells in Country j as x_{ij}, $i, j = 1, 2$. The firms' cost functions are linear and given by

$$C_i = c_i(x_{i1} + x_{i2}),\ i = 1, 2$$

where C_i is the total cost in Country $i = 1, 2$, and c_i is the fixed marginal (and average) cost. To focus on the impact of uncertainty, we ignore transportation (and fixed) costs.[6]

We assume that both countries' firms face uncertain output prices, $p_j,\ j = 1, 2$. Prices are uncertain because some demand function parameters are uncertain. Specifically, we take the two demand functions as

$$p_j = a_j - (x_{1j} + x_{2j}),\ j = 1, 2, \tag{1}$$

where a_1 and a_2 are random variables with a joint distribution function $f(a_1, a_2)$, whose means and covariance matrix are given by $\mu = (\mu_1,\, \mu_2)$ and Σ (whose elements are σ_{ij}, where $\sigma_{ii} > 0$), respectively.[7] Demand curves are, therefore, uncertain because the intercepts are random.

The model is set up as a two-stage game. In stage one, each country sets its level of export subsidies, $s_i\ i, = 1..2,$ where s_i denotes the export subsidy given to Firm i by Country i and $s = (s_1, s_2)$. These subsidies may be negative, i.e., they may represent taxes. In stage two, given the chosen subsidies, firms set their Cournot-Nash output levels in each market, x_{ij}.

3 Stage 2: Output Choice

In general, uncertainty can be resolved at different points of the two-stage trade policy game. It may be resolved before all decisions (subsidies and outputs) are made ("early resolution"); after some but not all decisions are made ("intermediate resolution"), or after all decisions are made ("late resolution"). As demonstrated by Appelbaum and Melatos (2016), the ability to make at least some decisions after uncertainty is resolved introduces value-of-information considerations (i.e., option values) into the analysis. In the case of "early resolution,"however, such considerations do not arise.

[5] The first part of the model (the second stage of the game) is based on Appelbaum (2022). Some aspects of this model are also found in Appelbaum and Melatos (2024).

[6] See Brander and Krugman (1983).

[7] Since we are not interested in the other demand functions' parameters, we set the slope parameters to one.

Intermediate resolution of uncertainty is discussed, for example, in Appelbaum and Melatos (2020). In that case, both value-of-information and insurance considerations arise. In this paper, we only consider one policy instrument (the choice of tariffs, but no trade agreements exist), so, following Appelbaum (2022), we restrict ourselves to the case of "late resolution" of uncertainty, namely, both outputs and subsidies are chosen before uncertainty is resolved. Consequently, only insurance implications, but no value-of-information implications arise.

We assume that both outputs and subsidies are chosen before uncertainty is resolved.

The firms' profits are given by

$$\pi_1 = \sum_{j=1}^{2}([a_j - (x_{1j} + x_{2j})] - c_1)x_{1j} + s_1 x_{12}, \quad \pi_2 = \sum_{j=1}^{2}([a_j - (x_{2j} + x_{1j})] - c_2)x_{2j} + s_2 x_{21} \tag{2}$$

If we define the utility of the random variable π_i as $U_i[\pi_i]$ and if U_i has the expected utility form, then $U_i[\pi_i] = E[u_i(\pi_i)]$, where $u_i(\pi_i)$ is the utility of the realizations π_i (corresponding to the realization of a_1 and a_2). We assume that u_i is strictly monotonically increasing and (at least weakly) concave. In such a case, both firms maximize their expected utility of profits so that we can write their problems as:

$$\max_{x_{11},\ x_{12}} E[u_1(\pi_1)], \qquad \max_{x_{21},\ x_{22}} E[u_2(\pi_2)] \tag{3}$$

The first thing that is clear from these expected utility maximization problems is that while the two markets are physically distinct, they are no longer distinct for all nonlinear utility functions as far as the firms' decisions are concerned. For example, Firm 1 cannot choose its output in each market separately: x_{11} and x_{12} must be jointly chosen. We will pursue this point further later.

Unfortunately, using general (increasing and concave) utility functions substantially complicates the analysis. For example, even for a single decision-maker, properties of high-order derivatives of the utility function, as well as moments of an order higher than two, may be required. Furthermore, in this model, we need to find the Nash Equilibrium of a game with four best-reply functions. Since this paper aims to explain trade patterns and gains from trade, we choose the simplest possible framework that enables us to do so. Thus, defining the mean and variance of π_i as $E(\pi_i)$ and $Var(\pi_i)$, we use the standard approximation of the expected utility given by

$$E\{u_i[\pi_i]\} \approx u_i\{E(\pi_i) - \frac{1}{2}R_i Var(\pi_i)\} \equiv u_i\{E(\pi_i) - \theta_i\}, \ i = 1, 2, \tag{4}$$

where R_i is the measure of absolute risk aversion in Country i, which is assumed to be constant[8], and θ_i Country $i's$ risk premium, defined as:

$$\theta_i \equiv \frac{1}{2}R_i Var(\pi_i). \tag{5}$$

[8]In fact, it can be shown that if a_1 and a_2 are jointly elliptically distributed, expected utility is completely characterized by its mean and variance. Moreover, if we have a constant absolute risk aversion utility function, the expected utility of π_i is linear in the mean and variance of π_i.

Since the utility functions, u_i, are strictly monotonically increasing, the maximization of $u_i\{E(\pi_i) - \frac{1}{2}R_i Var(\pi_i)\}$ is equivalent to the maximization of $E(\pi_i) - \frac{1}{2}R_i Var(\pi_i)$ - in the sense that they yield the same solutions for the $x_{ij}'s$. Thus, the countries' maximization problems can be written as:

$$\max_{x_{11},\ x_{12}} \{E(\pi_1) - \frac{1}{2}R_1 Var(\pi_1)\}, \max_{x_{21},\ x_{22}} \{E(\pi_2) - \frac{1}{2}R_2 Var(\pi_2)\}, \tag{6}$$

where

$$E(\pi_1) = \sum_{j=1}^{2}([\mu_j - (x_{1j} + x_{2j})] - c_1)x_{1j} + s_1 x_{12}, \quad E(\pi_2) = \sum_{j=1}^{2}([\mu_j - (x_{2j} + x_{1j})] - c_2)x_{2j} + s_2 x_{21} \tag{7}$$

and,

$$Var(\pi_1) = x_{11}^2 v_1 + x_{12}^2 v_2 + 2x_{11}x_{12}\sigma_{12} = x_{11}^2 v_1 + x_{12}^2 v_2 + 2x_{11}x_{12}\rho\sqrt{v_1}\sqrt{v_2} \tag{8}$$

$$Var(\pi_2) = x_{21}^2 v_1 + x_{22}^2 v_2 + 2x_{21}x_{22}\sigma_{21} = x_{21}^2 v_1 + x_{22}^2 v_2 + 2x_{21}x_{22}\rho\sqrt{v_1}\sqrt{v_2},$$

where ρ, v_1 and v_2 are the correlation coefficient and variances, respectively.

As is shown in Appelbaum (2022), from equations (6) and the variances in (8), it is clear that, in general, the firms cannot treat the two markets as distinct, even though they are physically distinct. Specifically, we have the following proposition:

Proposition 1 *If demand functions are uncertain, the two markets can be treated as distinct if and only if* $R_i\rho = 0$.

Proof. Let $\lambda_{ij}^i \equiv \frac{\partial^2\{E(\pi_i) - \frac{1}{2}R_i Var(\pi_i)\}}{\partial x_{ii}\partial x_{ij}}$, $i \neq j$. The two markets can be treated as distinct if and only if $\lambda_{ij}^i = 0$. But, $\frac{\partial^2\{E(\pi_i) - \frac{1}{2}R_i Var(\pi_i)\}}{\partial x_{ii}\partial x_{ij}} = -R_i\rho\sqrt{v_i v_j}$, where v_i is the standard deviation of a_i. Thus, if uncertainty exists, the two markets can be treated as distinct if and only if $R_i\rho = 0$. In other words, we need either risk neutrality ($R_i = 0$) or no correlation ($\rho = 0$).[9] ■

Hence, although the markets are physically distinct, they will not be treated as distinct unless firms are risk-neutral or demand functions are uncorrelated. Consequently, each country must choose its outputs in the two markets simultaneously.

Now, defining the firms' output vectors as $x_1 = (x_{11}, x_{12})$, $x_2 = (x_{21}, x_{22})$ and denoting the two countries' parameter vectors $\gamma_1 \equiv (v_1, v_2, \rho, R_1)$ and $\gamma_2 \equiv (v_1, v_2, \rho, R_2)$, we can write the two objective functions as:[10]

$$F^1(x_1, x_2, s_1; \gamma_1) \equiv E(\pi_1) - \frac{1}{2}R_1 Var(\pi_1) \tag{9}$$

$$F^2(x_1, x_2, s_2; \gamma_2) \equiv E(\pi_2) - \frac{1}{2}R_2 Var(\pi_2) \tag{10}$$

[9] See Appelbaum (2022).

[10] Remember that we have $\mu_1 = \mu_2 = 1$ and $c_1 = c_2 = .5$.

It is easy to verify that the functions $F^1(x_1, x_2, s_1; \gamma_1)$ and $F^2(x_1, x_2, s_2; \gamma_2)$ are strictly concave in x_1 and x_2, respectively.[11]

Defining the vector of all parameters as $\gamma \equiv (\gamma_1, \gamma_2) = (v_1, v_2, \rho, R_1, R_2)$ and $s = (s_1, s_2)$, the Nash equilibrium of this two-firm game is given by (the vectors) $x_1^*(\gamma, s) = \{x_{11}^*(\gamma, s), x_{12}^*(\gamma, s)\}$, and $x_2^*(\gamma, s) = \{x_{21}^*(\gamma, s), x_{22}^*(\gamma, s)\}$, such that:

$$\begin{aligned} F^1(x_1^*, x_2^*, s_1; \gamma_1) &\geq F^1(x_1, x_2^*, s_1; \gamma_1) \\ F^2(x_1^*, x_2^*, s_2; \gamma_2) &\geq F^2(x_1^*, x_2, s_2; \gamma_2) \end{aligned}$$

In other words, the pair (x_1^*, x_2^*) is the simultaneous solution to the two problems

$$\max_{x_1} \{ F^1(x_1, x_2, s_1; \gamma_1) \}$$

$$\max_{x_2} \{ F^2(x_1, x_2, s_2; \gamma_2) \}$$

The two countries' first-order conditions are given by the following equations, respectively,

$$\begin{aligned} \frac{\partial F^1(x_1, x_2, s_1; \gamma_1)}{\partial x_{11}} &= 0, \quad \frac{\partial F^1(x_1, x_2, s_1; \gamma_1)}{\partial x_{12}} = 0 \\ \frac{\partial F^2[x_1^*, x_2^*, s_2; \gamma_2]}{\partial x_{21}} &= 0, \quad \frac{\partial F^2[x_1^*, x_2^*, s_2; \gamma_2]}{\partial x_{22}} = 0 \end{aligned}$$

These conditions define a pair of best reply functions for each country, given by:[12]

$$\begin{aligned} x_{11} &= \frac{1 - (2R_1\rho\sqrt{v_1}\sqrt{v_2}x_{12} + 2x_{21})}{2(R_1 v_1 + 2)}, \; x_{12} = \frac{1 - (2R_1\rho\sqrt{v_1}\sqrt{v_2}x_{11} + 2x_{22} - 2s_1)}{2(R_1 v_2 + 2)} \\ x_{21} &= \frac{1 - (2R_2\rho\sqrt{v_1}\sqrt{v_2}x_{22} + 2x_{11} - 2s_2)}{2(R_2 v_1 + 2)}, \; x_{22} = \frac{1 - (2R_2\rho\rho\sqrt{v_1}\sqrt{v_2}x_{21} + 2x_{12})}{2(R_2 v_2 + 2)} \end{aligned} \tag{11}$$

As can be seen from these best reply functions, the pairs (x_{11}, x_{21}) and (x_{22}, x_{12}) are strategic substitutes, whereas the pairs (x_{11}, x_{22}) and (x_{21}, x_{12}) are strategic neutrals. Moreover, the pairs (x_{11}, x_{12}) and (x_{21}, x_{22}) are strategic substitutes (complements) if $\rho > (<) \, 0$. Given the solution to these four equations, we obtain the corresponding maximum functions, defined as[13]:

$$F^{*1}(s, \gamma) \equiv F^1[x_1^*(\gamma), x_2^*(\gamma), s_1; \gamma_1] \tag{12}$$

$$F^{*2}(s, \gamma) \equiv F^2[x_1^*(\gamma), x_2^*(\gamma), s_2; \gamma_2]. \tag{13}$$

[11] For example, in equation (9), the mean is concave in x_1 and, since the variance is a convex function, $-\theta_i$ is concave. The same is true for (10). Moreover, we can easily verify that the determinant of the corresponding Hessian matrix is strictly positive.

[12] Given that the $F^1(x_1, x_2; \gamma)$ and $F^1(x_1, x_2; \gamma)$ functions are strictly concave, the best reply functions are continuous.

[13] The expressions are rather long, so they are not included in the paper but are available upon request.

4 Stage 1: Subsidy Choice

In Stage 1, given Stage 2 output choices, the countries choose their subsidies. We define Country $i's$ welfare as the sum of consumer surplus and producer surplus net of subsidies. Using the Nash equilibrium quantities derived above, we can explicitly write Country $i's$ welfare in stage 2 as

$$w_i(t,\gamma) \equiv \frac{1}{2}(x_{1i}^* + x_{2i}^*)^2 + F^{*i}(s,\gamma) - x_{ij}^*(\gamma,s)s_i, \; i = 1,2, \; i \neq j \tag{14}$$

Note that since firms' outputs are functions of the taxes and the parameters, demand conditions are uncertain, so each $w_i(t;a,c)$ is also a random variable. In fact, it is straightforward to show that the countries' welfare functions in equations (14) do not depend on the random variables.

The two countries' welfare maximization problems are given by[14]

$$\max_{s_1}\{w_1(s,\gamma)\}$$
$$\max_{s_2}\{w_2(s,\gamma)\}.$$

Defining the Nash equilibrium subsidies in countries 1 and 2 as $s_1^*(\gamma)$ and $s_2^*(\gamma)$, respectively and letting $s^*(\gamma) \equiv \{s_1^*(\gamma), s_2^*(\gamma)\}.$, the Nash equilibrium welfare of the two countries is given by[15]

$$w_i^*(\gamma) \equiv w_i\{s^*(\gamma),\gamma\}, \; i = 1,2. \tag{15}$$

[14]We first examine the strategic relationship between s_1 and s_2. For the symmetric-uncertainty case where $R_1 = R_2 = R$ and $v_1 = v_2 = v$, we have

$$\frac{\partial^2 w_i}{\partial s_i \partial s_j} = \frac{N(S,\rho)}{(S\rho - 3 - S)^2(S\rho + 3 + S)^2(S\rho - 1 - S)^2(S\rho + 1 + S)^2} \; (i,j = 1,2, \; i \neq j)$$

where $S = Rv$ and $N(S,\rho) = -S\rho[(3+S)S^4\rho^4 - (2S^5 + 10S^4 + 18S^3 + 14S^2)\rho^2 + S^5 + 7S^4 + 14S^3 - 2S^2 - 31S - 21]$. Note that s_i and s_j are strategic substitutes (complements) if the cross partial $\frac{\partial^2 w_i^*(\gamma)}{\partial s_i \partial s_j}$ is negative (positive). Given that the denominator is positive, $\frac{\partial^2 w_i^*(\gamma)}{\partial s_i \partial s_j} > (<) \; 0$ if $N(S,\rho) > (<) \; 0$. It can be easily verified that (i) for $\rho > 0$, s_i and s_j are strategic substitutes (complements) if $S < (>) \; S_0(\rho)$, where $S_0(\rho)$ solves $N(S,\rho) = 0$; (ii) for $\rho < 0$, s_i and s_j are strategic complements (substitutes) if $S < (>) \; S_0(\rho)$. Moreover, t_i and t_j are strategic neutrals if $S\rho = 0$ or $S = S_0(\rho)$. For the general case, the strategic relationship between s_i and s_j depends on the relative magnitudes of parameter values $(R_1, R_2, v_1, v_2, \rho)$. Specifically, s_i and s_j are more likely to be strategic complements for country i if R_1 is sufficiently small relatively to R_j, v_i is sufficiently large relative to v_j, or ρ moves closer to the extreme values of $+1$ or -1.

[15]Note that $w_i^*(\gamma)$ solves the following first-order conditions:

$$\frac{\partial w_1(s,\gamma)}{\partial s_1} = (x_{1i}^* + x_{2i}^*)\frac{\partial(x_{11}^* + \partial x_{21}^*)}{\partial s_1} + \frac{\partial F^{*1}(s,\gamma)}{\partial s_1} + \frac{\partial x_{12}^*(\gamma,s)}{\partial s_1}s_1 = 0$$
$$\frac{\partial w_2(s,\gamma)}{\partial s_2} = (x_{12}^* + x_{22}^*)\frac{\partial(x_{12}^* + x_{22}^*)}{\partial s_2} + \frac{\partial F^{*2}(s,\gamma)}{\partial s_2} + \frac{\partial x_{21}^*(\gamma,s)}{\partial s_2}s_2 = 0.$$

5 Trade Patterns

5.1 Free Trade: $s_i = s_j = 0$

Given the Nash equilibrium values of $x_1^*(\gamma)$ and $x_2^*(\gamma)$, as defined in equations (11), we can now examine the effects of uncertainty on equilibrium trade patterns under free trade by setting $s_i = s_j = 0$. Define net exports of countries 1 and 2 as:

$$z_1^*(\gamma) = x_{12}^*(\gamma) - x_{21}^*(\gamma), \quad z_2^*(\gamma) = x_{21}^*(\gamma) - x_{12}^*(\gamma) = -z_1^*(\gamma) \tag{16}$$

First, note that there are eleven parameters in the model (given by the vector γ). However, we are interested in two identical countries in all respects except those related to risk and risk aversion. Thus, to isolate the impact of uncertainty, we assign specific, equal values to all parameters that are not directly related to uncertainty. We set the values of those "unrelated" parameters as follows: (i) marginal costs: $c_1 = c_2 = .5$, (ii) means of demand intercepts: $\mu_1 = \mu_2 = 1$.[16] Specifying these values leaves us with five "uncertainty-related parameters."Four are country-specific: $R_1,\ R_2,\ v_1,\ v_2$ and one is the common ρ.

From equations (11), it is clear that obtaining the effects of parameter changes on the patterns of trade or the level of net exports may not always be easy to obtain. First, all best reply functions are highly nonlinear in the parameters, so results may very well be "local" rather than "global.". Second, and more importantly, changes in $v_1,\ v_2$ and ρ shift *all* four best reply functions, and a change in each country's risk aversion shifts its *pair* of best reply functions. Consequently, changes in $v_1,\ v_2$ and ρ directly and indirectly affect the equilibrium levels of net exports. Such changes directly shift all best reply functions, hence changing each x_{ij} for any given values of the other outputs. But, because all other best reply functions also shift, the values of other outputs also change. Therefore, the equilibrium values of $x_{12}^*(v_1, R_1, \rho)$ and $x_{21}^*(v_1, R_1, \rho)$ change although x_{12} and x_{21} are strategic neutrals. A change in a country's measure of risk aversion directly shifts its two best reply functions, but it does not directly affect the other country's two best reply functions. However, a shift in Country 1's best reply functions, for example, indirectly affects the values of all other outputs, thus changing the equilibrium values of $x_{12}^*(v_1, R_1, \rho)$ and $x_{21}^*(v_1, R_1, \rho)$, even though they are strategic neutrals. For the remainder of this section, we look at the effects of parameter changes on (Country 1's) net export, the difference between $x_{12}^*(v_1, R_1, \rho)$ and $x_{21}^*(v_1, R_1, \rho)$ rather than $x_{12}^*(v_1, R_1, \rho)$ and $x_{21}^*(v_1, R_1, \rho)$ individually.

To separate the risk and risk aversion effects, as a first step, we consider two cases. In the first case, we assume that risks (variances) are the same, but the measures of risk aversion differ, whereas in the second case, we assume that the measures of risk aversion are the same, but risks differ. Since $U3c1$ is a common parameter,

[16] And, of course, the demand functions' slopes were already set to 1.

we do not examine its effects separately. Instead, we examine the extent to which it amplifies or mitigates the effects of differences in risk and risk aversion.

5.1.1 Differences in Measures of Risk Aversion

Setting $v_1 = v_2 = v$ and using the Nash equilibrium values of the firms' outputs, we calculate the countries' net exports. These are given by:

$$z_1^* = -\frac{1}{2}\frac{(1+\rho)(R_1 - R_2)v}{(1+\rho)^2 R_1 R_2 v^2 + 2v(1+\rho)(R_1 + R_2) + 3}, \quad z_2^* = -z_1^* \tag{17}$$

Thus, when $v_1 = v_2 = v > 0$, we have:

Proposition 2 *For all values of $v > 0$,*

(i) If $R_2 < R_1$ and $\rho > -1$, County 2 is a net exporter of x ($z_2^* > 0$), and Country 1 is a net importer of x.

(ii) If $R_2 < R_1$ but $\rho = -1$, neither country is a net importer ($z_1^* = z_2^* = 0$).[17]

Alternatively, Proposition 2 can be restated as:

Proposition 2a: For all values of $v > 0$, if Country 2 is relatively low-risk-aversion abundant ($R_2 < R_1$) and $\rho > -1$, then Country 2 is a net exporter of low risk-aversion, whereas Country 1 is a net importer of low risk-aversion.

Thus, although risk aversion is country-specific and immobile, effectively, it is imported/exported via net exports of the product. This result is reminiscent of the standard notion of the "factor content" of trade. In our context, we can think of it as the "risk aversion content of trade" (although its measurability is not clear). If $R_2 < R_1$, we can think of Country 2 as having a risk-aversion-driven uncertainty-cost comparative advantage. As a result, Country 2 is a net exporter of x. By importing x, Country 1 effectively imports lower risk aversion, mitigating its risk-aversion-driven uncertainty-cost comparative disadvantage. Alternatively, we can interpret the result as reflecting implicit risk-sharing.[18]

5.1.2 Differences in Risk

Setting $R_1 = R_2 = R > 0$ and using the Nash equilibrium values of the firms' outputs, we calculate Country 1's net export as:

$$z_1^* = \frac{1}{2}\frac{(v_1 - v_2)R}{(1-\rho^2)v_1 v_2 R^2 + 3R(v_1 + v_2) + 9} \tag{18}$$

[17] Proposition 2 follows immediately from equation (17) and the fact that if $\rho > -1$, both numerator and denominator are strictly positive, but when $\rho = -1$, the numerator is zero.

[18] It can be easily verified that with full collusion, with $R_1 = 0 < R_2$, the solution will always be $x_{11}^* = x_{12}^* = 0$, $x_{21}^* > 0$, $x_{22}^* > 0$. Namely, full insurance. On the other hand, without collusion, we have $x_{11}^* > 0$, $x_{12}^* > 0$. The countries' total (utility of) profits will be higher with full insurance than without it, and thus, assuming the firms share that total (with Country 1's share being fixed), they will each be better off.

Thus, when $R_1 = R_2 = R > 0$, we have:

Proposition 3 *For all values of ρ and R, if $v_1 > v_2$, Country 1 is a net exporter of x (i.e., $z_1^* > 0$), and Country 2 is a net importer of x.*[19]

Alternatively, Proposition 3 can be restated as:

Proposition 3a: If Country 1 is relatively high-risk abundant ($v_1 > v_2$), then, for all values of ρ and R, it is a net importer of low risk (through net exports of x), whereas Country 2 is a net exporter of low risk.

As in the case of differences in risk aversion, we can think of it as the "risk content of trade." Even though a country's risk is immobile, it is imported/exported via net exports/imports of the product. When $v_2 < v_1$, we can think of Country 2 as having a low-risk-driven uncertainty-cost comparative advantage. But Country 1 can "import" Country 2's low (risk-driven) uncertainty cost technology by diverting sales to Country 2 through exports. Doing so mitigates Country 1's uncertainty-cost comparative disadvantage. In part, this reflects the benefits of diversification.[20]

5.1.3 General Trade Patterns

In the two cases above, we compared the two countries by focusing on "one difference at a time." In the following, we carry out the comparison by allowing for differences in risk and risk aversion.

Since the countries' indexing is arbitrary, we assume that Country 1 is more risk-averse: $R_1 > R_2$. Moreover, since we are interested in the relative abundance of low risk aversion and low risk, we first fix "benchmark values" for Country 2's risk and risk aversion parameters. We then measure Country 1's corresponding parameters relative to those benchmark values. Specifically, we take $R_2 = 1 < R_1$ and $v_2 = 1$, but we do not make assumptions regarding the ranking of v_1 and v_2, and neither do we make assumptions regarding the common parameter, ρ. Thus, this leaves us with three parameters: v_1, R_1 and ρ. Furthermore, changes in v_1 and R_1 can now be viewed as changes relative to the benchmark values of v_2 and R_2.

Country 1's Nash equilibrium level of net exports is now given by:[21]

$$z_1^* = x_{12}^*(v_1, R_1, \rho) - x_{21}^*(v_1, R_1, \rho) \tag{19}$$

Note that to determine the effects of parameter changes on the *level* of net exports, we need to know the signs of the derivatives of $z_1^*(v_1, R_1, \rho)$. But, to determine *trade patterns,* it suffices to know the *sign* of $z_1^*(v_1, R_1)$. Although

[19]Since in equation (18) $v_1 > v_2$ and $(1 - \rho^2) \geq 0$.
[20]Which, by the way, would exist even with identical risks.
[21]The precise solution is given in Appendix 6.2.

these two questions are complementary and closely related, they require separate analyses. For example, if we find that over the whole parameter space, we have $\partial z_1^*(v_1, R_1, \rho)/\partial v_1 > 0$, it is likely that "*eventually*" we would have $z_1^*(v_1, R_1, \rho) > 0$. Locally, however, a marginal increase of v_1 may increase $z_1^*(v_1, R_1, \rho)$, but this may not be enough to make $z_1^*(v_1, R_1)$ strictly positive.

The Determinants of Trade Patterns For any level of net exports by Country 1, given by k, define Country 1's net exports iso-value curve (IVC), denoted by S_k^1, as:

$$S_k^1 \equiv \{(v_1, R_1, \rho) \ : z_1^*(v_1, R_1, \rho) = k\} \tag{20}$$

For example, S_0^1 is the zero net exports IVC, taking $k = 0$. The (strict) net import and net export sets (NIS and NES, respectively) are then defined by:

$$NIS \equiv \{(v_1, R_1, \rho) \ : z_1^*(v_1, R_1, \rho) < 0\} \tag{21}$$

$$NES \equiv \{(v_1, R_1, \rho) \ : z_1^*(v_1, R_1, \rho) > 0\} \tag{22}$$

In other words, NIS and NES are the sets of the parameters (v_1, R_1, ρ) that, respectively, lie on "opposite sides" of the iso-value curve S_0^1.

The S_0^1 IVC with the corresponding NIS and NES (in three-dimensional (v_1, R_1, ρ) space) are shown in Figure 1. The skewed "tent-like" blue surface shows the iso-value curve S_0^1. The corresponding NIS and NES are the areas below and above that blue surface, respectively.[22] As Figure 1 shows, the S_0^1 curve is continuous[23] and "well-behaved." For example, Figure 2 shows the contours of S_0^1 for different values of v_1 (higher values are farther away from the origin). As Figure 2 shows, NIS is a convex set, so that S_0^1 is quasi-concave relative to the (R_1, ρ) base.[24]

A single parameter change corresponds to cross-sectional movements in the three-dimensional (v_1, R_1, ρ) space. The effects of such parameter changes on the patterns of trade are shown in Figure 1. We do the following to determine whether a parameter change makes it more likely for Country 1 to be an importer of x. We pick any initial point strictly above S_0^1 (strictly inside the NES). For example, take a point h in Figure 2 that lies (strictly) above S_0^1. Since point h is above S_0^1, we know that, at h, Country 1 is a net exporter. Now, from point h, there are three possible cross-sectional movements: up/down (a change in v_1), forward/backward (a change in R_1) and left/right (a change in ρ). These three possible cross-sectional movements represent a change in one of the three parameters. For each of these cross-sectional movements, the direction we need to follow to "hit" the iso-value

[22]Notice that for all $0 < v_1 < 1$, Country 1 is always an importer of x.
[23]Its continuity can be verified from the explicit expression for equation (19).
[24]But NES is not convex.

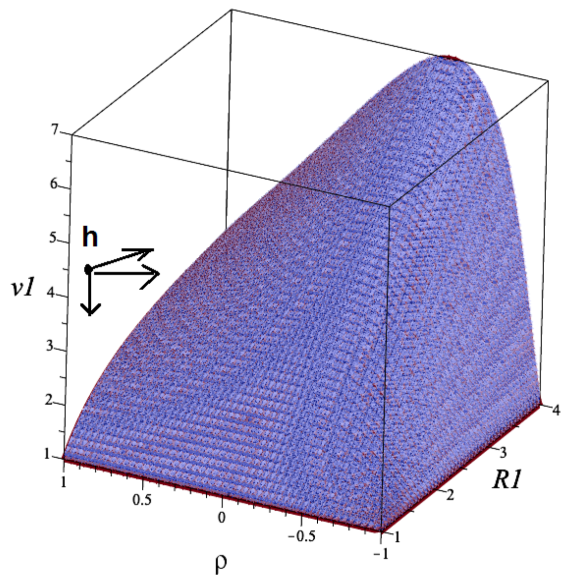

Figure 1: Zero Net Exports Iso-Surface

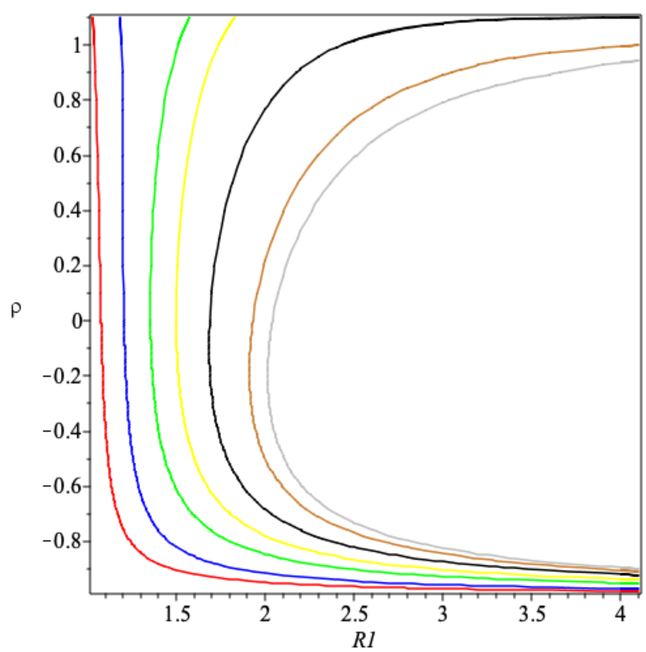

Figure 2: Top View of Iso-Value Curves

curve S_0^1 is the direction that increases the likelihood that Country 1 will become a net importer. On the other hand, if the initial point h is already (strictly) below S_0^1 (inside NIS), any cross-sectional movement toward the surface makes it less likely that Country 1 remains an importer of x. As mentioned above, given the nonlinearity of S_0^1, the result may depend on the location of point h.

Thus, Figure 1 shows that:

Proposition 4 *Other things being equal, (i) Country 1 is more likely to be an importer of x (an importer of low risk-aversion) when its measure of risk aversion is high (ii) Country 1 is more likely to be an exporter of x (hence an importer of low risk) when its risk is high.*

Proposition 4 and the intuition behind it are similar to and consistent with propositions 2 and 3 above.

Unlike the global results in Propositions 4, the effect of a change in correlation is local. As Figure 1 shows, starting from an initial position that lies (strictly) above S_0^1 (say, point h) where correlation is positive and high, Country 1 is more likely to be an importer as ρ decreases and moves to the right toward the "ridge/spine" of S_0^1. Beyond that "ridge," any further decrease in ρ makes Country 1 less likely to be an importer. Specifically, define the point where the effect of a decrease in ρ changes its sign as $\widetilde{\rho}(R_1)$.[25] Then, we have:

Proposition 5 *For all $\rho > \widetilde{\rho}$ (and given values of v_1 and R_1), Country 1 is more likely to become an importer as ρ decreases. But, for all $\rho < \widetilde{\rho}$, Country 1 is less likely to become an importer as ρ decreases.*

The effect of correlation is not global because, in general, diversification benefits are not monotonic in ρ over its whole domain. This result is related to the usual result in portfolio choice theory, where, generally, corner solutions for asset holdings are not optimal. For example, Country 1's "cost of uncertainty," as captured by its risk premium (and primarily determined by the variance of profits), is not monotonic in ρ for all $\rho \in [-1..1]$.[26]

5.2 Subsidies

We now analyze the Nash-equilibrium of the export-subsidy game and begin by considering the symmetric case where $R_1 = R_2 = R$ and $v_1 = v_2 = v$. Optimum subsidies from each government maximization problem in Stage 1 are given by

[25] The value for $\widetilde{\rho}$ can be obtained as follows. Solve the equation $z_1^*(v_1, R_1, \rho) = 0$ for v_1 to obtain $v_1 = v_1(R_1, \rho)$. Then solve the problem: $\max_\rho\{v_1(R_1, \rho)\}$. The solution to this problem is $\widetilde{\rho}(R_1)$. In other words, for any R_1, $\widetilde{\rho}(R_1)$ is the value of correlation that maximizes v_1, giving us the top of the ridge. It is easy to show that (as Figure 1 shows) $\widetilde{\rho}(R_1)$ decreases with R_1.

[26] Neither "corner" represents minimum profits variance, which is similar to what standard mean-variance portfolio choice theory suggests.

$$s_i = \frac{-(1/2)[(2\rho + 2\rho^2)S^2 + (3\rho - 1)S - 3](S\rho - 1 - S)^2}{D},\ i = 1, 2$$

where

$$\begin{aligned} D &= (-4\rho^2 + 2 + 2\rho^4)S^5 + (-26\rho^2 - 2\rho + 20 + 2\rho^3 + 6\rho^4)S^4 \\ &\quad +(-54\rho^2 - 18\rho + 74 + 6\rho^3)S^3 + (124 - 42\rho - 38\rho^2)S^2 + (-30 + 92)S\rho + 24 > 0 \\ S &= Rv \geq 0. \end{aligned}$$

Since $D > 0$, s_i $(i = 1, 2)$ is positive (negative) if $L(S, \rho) = (2\rho+2\rho^2)S^2+(3\rho-1)S-3 < (>)\ 0$. Clearly, if $v = 0$ or $R = 0$, then $S = 0$, yielding $L(S, \rho) = -3 < 0$. In this case, $s_1 = s_2 > 0$, confirming the conventional result that optimum subsidies are positive under no uncertainty or risk neutrality. For a given ρ (where $-1 \leq \rho \leq 1$), s_i remains positive as long as S is sufficiently small. It is easily seen that as S increases and crosses the point where $(2\rho + 2\rho^2)S^2 + (3\rho - 1)S - 3 > 0$, s_i turns negative. That is, when risk and/or risk aversion is sufficiently high, Nash-equilibrium policy is to tax rather than subsidize exports. For a given S, a negative subsidy is more likely to occur when the market correlation ρ becomes sufficiently positive. In general, $s_i < 0$ $(i = 1, 2)$ for all combinations of (S,ρ) above the green line in Figure 3. The intuition here is that with high risk, risk aversion and positive correlation, efficient risk management policy involves inducing firms to pull back from export markets with an export tax. Given that, the pairs of (S, ρ) on the $L(S, \rho) = 0$ locus (the green line), $s_i = 0$. This result can be illustrated in

Proposition 6 *If* $R_1 = R_2 = R$ *and* $v_1 = v_2 = v$, *then* $s_1 = s_2 \geq (<)$ *0 if* $L(S, \rho) = (2\rho+2\rho^2)S^2+(3\rho-1)S-3 \leq (>)\ 0$.

Using Proposition 6, it is possible to show that for asymmetric countries, Nash equilibrium may be such that one country subsidizes while the other taxes its exports. To show this possibility, assume that $R_2 = 1, v_2 = 1, R_1 = 0.5, v_1 = \sqrt{2}$. In Figure 4, s_1 is represented by the red line, whereas s_2 is represented by the green line. Clearly, both s_1 and s_2 are positive when the correlation is sufficiently negative. As the correlation increases toward +1, Country 2 will begin to impose an export tax (see the green line) as the net benefit from diversification turns negative when the market correlation becomes sufficiently positive for a low-risk country (i.e., Country 2 in this example).

Next, we examine the impact of subsidies on the pattern of trade, denoted by the next export status z_i^{SB*}, relative to free trade z_i^*. When $\rho = 0$[27] **and** $\mathbf{v}_1 = \mathbf{v}_2 = v$, calculate

[27] Note that the result remains intact when ρ takes a different value other than zero.

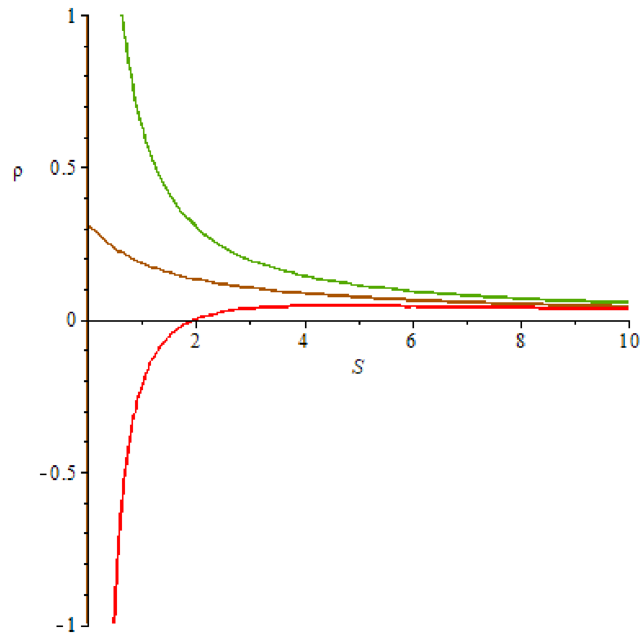

Figure 3: Locus of $(2+2)S+(3-1)S-3=0$ (Green Line)

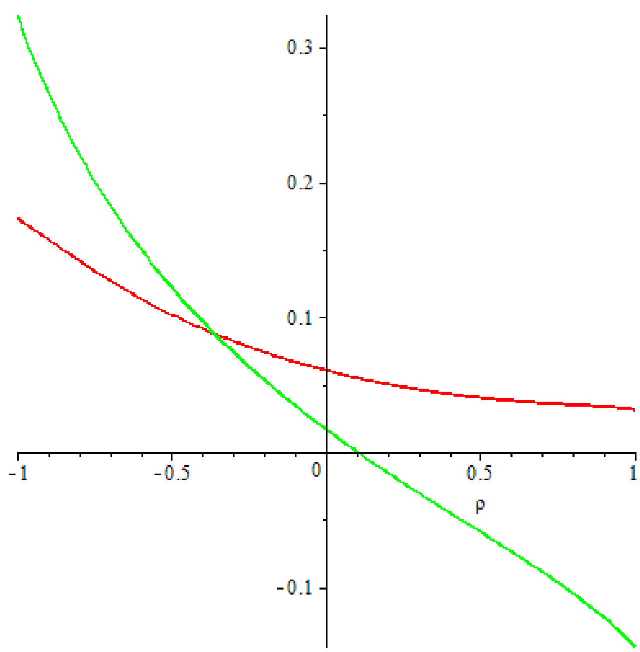

Figure 4: s_1 (Red) vs s_2 (Green) , assuming that $R_2 = 1, v_2 = 1, R_1 = 0.5, v_1 = \sqrt{2}$

$$z_i^{SB*} = \frac{-v(R_i - R_j)}{2(R_j v^2 R_i + 2R_i v + 2R_j v + 2)} \gtreqless 0 \text{ if } R_i \lesseqgtr R_j$$

$$z_i^{SB*} - z_i^* = \frac{-v(R_i - R_j)}{2(R_j v^2 R_i + 2R_i v + 2R_j v + 2)(R_j v^2 R_i + 2R_i v + 2R_j v + 3)} \gtreqless 0 \text{ if } R_i \lesseqgtr R_j$$

Similarly, when $\rho = 0$ and $R_1 = R_2 = R$, we obtain

$$z_i^{B*} = \frac{R(\sqrt{v_i} - \sqrt{v_j})(\sqrt{v_i} + \sqrt{v_j})(R^2 v_i v_j + Rv_i + Rv_j)}{2(R^2 v_j^2 + 4Rv_j + 2)(R^2 v_i^2 + 49Rv_i + 2)} \gtreqless 0 \text{ if } v_i \gtreqless v_j$$

$$z_i^{SB*} - z_i^* = \frac{R(\sqrt{v_i} - \sqrt{v_j})(\sqrt{v_i} + \sqrt{v_j})(R^2 v_i v_j + R^2 v_i^2 + 7Rv_j + R^2 v_i^2 + 7Rv_i + 14)}{2(R^2 v_j^2 + 4Rv_j + 2)(R^2 v_i^2 + 49Rv_i + 2)(Rv_j + 3)(Rv_i + 3)} \gtreqless 0 \text{ if } v_i \gtreqless v_j.$$

As related to Propositions 2 and 2a, this gives

Proposition 2b: (i) If $\rho = 0$ and $v_1 = v_2 = v$, then Country i under subsidy is a net exporter (importer) (i.e., $z_i^{SB*} > (<)\ 0$) if $R_i < (>)\ R_j$. Moreover, subsidy will further enhance Country i's volume of net exports when $R_i < (>)\ R_j$.; (ii) If $\rho = 0$ and $R_1 = R_2 = R$, then County i under subsidy is a net exporter (importer) (i.e., $z_i^{SB*} > (<)\ 0$) if $v_i > (<)\ v_j$. Similarly, a subsidy will further increase Country i's volume of net exports when $v_i > (<) v_j$.

Propositions 2 and 2a summarize the relationship between subsidies and net exports as a function of the relative levels of risk aversion and uncertainty. Proposition 2b shows that when markets are equally risky, a more risk-averse country's net exports must increase with equilibrium subsidies. Similarly, $w$hen risk aversion is the same, subsidies boost net exports of the relatively high-risk country. In sum, the difference in net exports under subsidy vis-à-vis free trade (i.e., $z_i^{S*} - z_i^*$) is proportional to the difference in the measure of risk aversion $(R_j - R_i)$ and degree of uncertainty $(v_i - v_j)$.

Interestingly, if countries differ with respect to both risk and risk aversion, subsidies may reverse the pattern of net exports. Figure 5 shows the net trade balance z_1^{SB*} under subsidy (represented by the red line) and the net trade balance z_1^* under free trade (represented by the green line). A comparison between these two functions shows that a country which is a net importer under free trade, may become a net exporter in the Nash equilibrium of the subsidy game. As an example, when $R_1 = 2, R_2 = 0.1, v_1 = \sqrt{3}, v_2 = 1$, and $\rho = -0.85$, then $z_1^{SB*} = 0.027 > 0$ and $z_1^* = -0.024 < 0$.

6 The Gains from Trade

6.1 Autarky

Under autarky, each country has a single monopolist. If we set $x_{ij} = 0$ for $i \neq j$, then the mean and variance of profits of monopolist $i's$ profits are given by equations (7) and (8). Specifically, under autarky, we have:

$$E(\pi_i) = (\mu_i - x_{ii} - c_i)x_{ii}, \quad i = 1, 2 \tag{23}$$

$$Var(\pi_i) = x_{ii}^2 v_i, \quad i = 1, 2 \tag{24}$$

The corresponding maximization problem is, then, given by (equation (6) with $x_{ij} = 0$ for $i \neq j$):

$$\max_{x_{ii}} \{(\mu_i - x_{ii} - c_i)x_{ii} - \frac{1}{2} R_i x_{ii}^2 v_i\}, \; i = 1, 2 \tag{25}$$

Or, using the notation in equations (9) and (10) above, this can be written as:

$$\max_{x_{ii}} \{F^{ai}(x_{ii}; \gamma_i^a)\}$$

where $\gamma_i^a = (\mu_i, v_i, R_i, c_i)$ is the vector of parameters in Country i and $F^{ai}(x_{ii}; \gamma_i^a)$ is the objective function of Country i under autarky.

Define the solution for this problem as: $x_{ii}^{*a}(\gamma_i^a), \; i = 1, 2$, and let the corresponding maximum function be defined as: $F^{*ai}(\gamma_i^a) \equiv F^{ai}(x_{ii}^{a*}(\gamma_i^a); \gamma_i^a)$. The explicit solution for $F^{*ai}(\gamma_i^a)$ is easily obtained as:

$$F^{*ai}(\gamma_i^a) = \frac{1}{2} \frac{(\mu_i - c_i)^2}{R_i v_i + 2} \tag{26}$$

We set the values of the "uncertainty-unrelated" parameters in line with the normalizations applied in earlier sections: (i) marginal costs: $c_1 = c_2 = .5$, (ii) means of demand intercepts: $\mu_1 = \mu_2 = 1$. As a result, we are left with two parameters in each country, so $\gamma_i^a = (v_i, R_i)$, and the solution becomes:

$$F^{*ai}(R_i, v_i) = \frac{1}{8} \frac{1}{R_i v_i + 2} \tag{27}$$

where the Nash equilibrium value of x_{ii} is:

$$x_{ii}^{*a}(R_i, v_i) = \frac{1}{2} \frac{1}{R_i v_i + 2} \tag{28}$$

6.2 Gains from Free Trade

We take the sum of consumer and producer surplus to measure welfare in both the autarky and trade cases. Since we use linear demand functions, the calculation of consumer surplus is quite simple. Specifically, using Country

$i's$ Nash equilibrium solutions, with and without trade ($x_i^*(\gamma)$ as defined in equations (11) and $x_{ii}^{*a}(\gamma_i^a)$, as defined in equations (28)), consumer surplus in Country i, with and without trade, denoted as, cs^i and cs^{ai}, respectively, is given by:

$$cs^i = \frac{1}{2}[x_{ii}^*(\gamma) + x_{ji}^*(\gamma)]^2, \quad cs^{ai} = \frac{1}{2}[x_{ii}^{*a}(\gamma_i^a)]^2 \tag{29}$$

Country $i's$ welfare, with and without trade, is, therefore, given by:

$$w^i(\gamma) = F^{*i}(\gamma) + cs^i(\gamma), \quad w^{ai}(\gamma_i^a) = F^{*ai}(\gamma_i^a) + cs^{ai}(\gamma_i^a) \tag{30}$$

Given our normalizations of Country 2's RRA parameters ($v_2 = 1,\ R_2 = 1$), the only remaining (three) parameters are $v_1,\ R_1$ and ρ. Thus, we can write the countries' gains from trade as,

$$G^1(v_1, R_1, \rho) \equiv w^1(v_1, R_1, \rho) - w^{a1}(R_1, v_1), \quad G^2(v_1, R_1, \rho) \equiv w^2(v_1, R_1, \rho) - w^{a2}(1, 1) \tag{31}$$

Two points are worth mentioning before we consider the effects of uncertainty on the countries' GFT. First, it is important to remember that, as is well known, one source of GFT is the pro-competitive effect of trade, which results in lower markups, thus reducing prices. This pro-competitive effect is, of course, present with and without uncertainty. Therefore, in addition to measuring the GFT (comparing autarky with trade), it is also important to compare GFT with and without uncertainty. Such a comparison will provide what can be viewed as the net gain from trade (NGFT), disentangling gains due to the pro-competitive effect from uncertainty-related ones. Since all parameters that are not directly related to uncertainty were taken as constants, the gains from trade with no uncertainty are also constant and equal.[28] We can calculate them by taking $R_i = 0$ (thus making the risk premia in the two countries equal to zero) and define them as $G_{nu}^1 = G_{nu}^1 = G_{nu} > 0$.[29] Then, Country $i's$ NGFT is given by $G^i(\gamma) - G_{nu}$.

Second, given our normalizations, it follows immediately from (27) and (28) that for all $R_1 > 1$ and $v_1 > 1$, Country 2's welfare, under autarky, is higher than Country 1's: $w^{a2} > w^{a1}$.

Now, for any GFT level given by q_i, define Country $i's$ corresponding gains from trade iso-value curve, B_q^i, as:

$$B_{q_i}^i \equiv \{(v_1, R_1, \rho) \ : G^i(v_1, R_1, \rho) = q_i\}, \quad i = 1, 2 \tag{32}$$

It is easy to verify that for Country 1, for all parameters values, we have $G^1(v_1, R_1, \rho) > 0$. In other words, the set of parameters B_0^1 is empty, so Country 1 always gains from trade.[30] However, although Country 1's GFT are always positive, it is unclear whether its gains without uncertainty are lower or higher than those with uncertainty.

[28] Since the countries only differ in terms of RRA, with no uncertainty, they become identical, so their GFT are the same.

[29] Where $G_{nu} = 5/288$.

[30] If we break up Country 1's GFT into their two components, we can show that, as a result of trade, consumer surplus always increases and, generally (but not always), producer surplus also increases (especially when v_1 is high, and the non-absolute value of ρ is low - but for $\rho < 0$ it always increases). The sum of these two components is, however, always positive.

The reason for this result is that although uncertainty is costly with and without trade, its adverse impact will, in general, not be the same in the two cases. Thus, if uncertainty's impact on Country 1 is more severe under autarky than with trade, Country 1's GFT may be higher with uncertainty.

To compare the gains with and without uncertainty, we need to consider Country 1's IVC that corresponds to $q_1 = G^1_{nu}$, the level of Country $1's$ GFT without uncertainty. This IVC, defined by,

$$B^1_{G^1_{nu}} \equiv \{(v_1, R_1, \rho) \ : G^1(v_1, R_1, \rho) = G^1_{nu}\}, \tag{33}$$

is shown by the red curve in Figure 6. For all combinations of parameters on the red surface, Country 1's GFT will have the same (positive value) with and without uncertainty. All points to the left of the red surface represent strictly positive NGFT. As is clear from the figure, Country 1's NGFT will be strictly positive for a wide range of parameter values. How likely is such a case to occur? Since G^i_{nu} is constant, all we need to know is which parameter combinations are likely to place us to the left of the red surface in Figure 6. As Figure 6 shows, this is more likely to happen when RRAC are low.[31]

For Country 2, we define the iso-value curve for the autarky value of GFT when there is no uncertainty (G^2_{nu}). To easily compare the countries' GFT, we place G^2_{nu} (green curve) side by side with G^1_{nu} (red curve) in Figure 6.[32] As Figure 6 shows, Country 1's GFT is likely to be higher than Country 2's when v_1 is low and R_1 is high. Moreover, Figure 6 also shows that Country 1's NGFT will be strictly positive over a "larger" set of parameter values than Country 2. Thus, even though Country 2's risk and risk aversion are lower than Country 1's, and its consumer and producer surpluses (before and after the trade) are higher than Country 1's, its GFT may not always be positive. In contrast, Country 1's GFT is always positive.

Hence, we have:

Proposition 7 *(i) Country 1's gains from trade are always positive, and Country 2's gains from trade are positive unless the correlation is positive and very high. (ii) For a wide range of parameters, both countries' NGFT can be strictly positive. (iii) Country 1's NGFT are more likely to be strictly positive than Country 2's.*

Proposition 7 is based on the intuition behind the diversification and implicit insurance sources of GFT. It disentangles the pro-competitive effects from uncertainty-related ones and shows the RRAC parameters' role in

[31] The non-absolute values in the case of correlation.

[32] It can be easily verified that Country 2's GFT are positive for a wide range of parameter values (except when the correlation is positive and very high). As for the two components of Country 2's GFT, change in producer surplus (going from autarky to trade) is positive over a smaller parameter set compared to country 1 (it increases when the non-absolute value of the correlation is low, R_1 is high and v_1 is low). Usually, the change in consumer surplus is positive. For example, consumer surplus decreases when the correlation is positive and very high (with a high R_1 and low v_1). Furthermore, Country 2's NGFT will also be strictly positive for a wide range of parameter values. Specifically, its NGFT are likely to be positive when ρ and v_1 are low and when R_1 is high. Moreover, its GFT decreases with ρ (unless ρ is near -1), decreases with v_1 (when ρ is sufficiently high) and increases with R_1 (over the whole domain shown in the diagram).

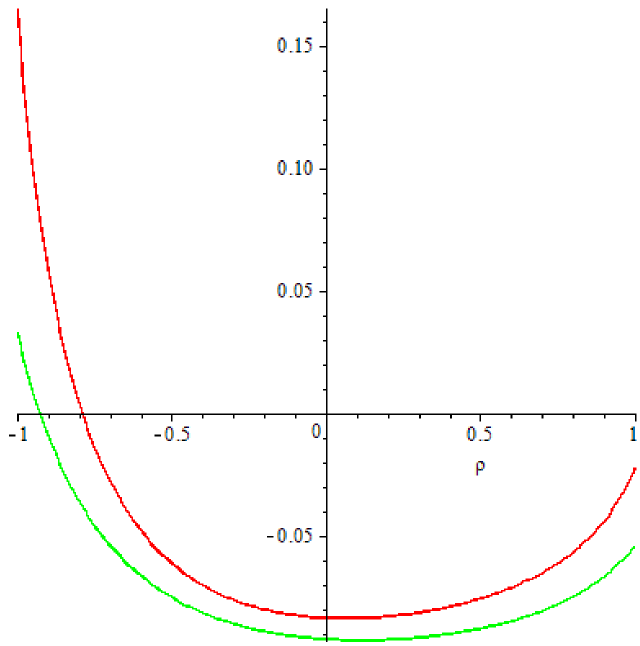

Figure 5: z_1^{SB*} and z_1^*, assuming that $R_1 = 2, R_2 = 0.1, v_1 = \sqrt{3}, v_2 = 1,$

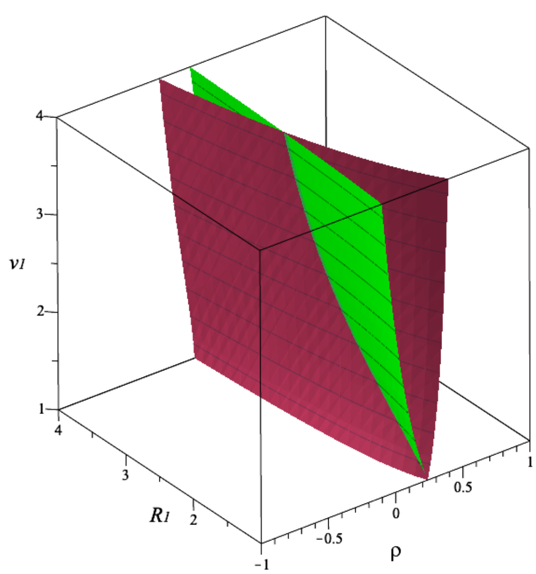

Figure 6: $G^1 = G^1_{nu}$ (Red) and $G^2 = G^2_{nu}$ (Green) Iso-Value Curves

determining the GFT.

6.3 Gains from Trade Under Subsidy

In this sub-section, we examine social welfare under Nash-equilibrium subsidies, denoted by $w^{SB*}(\gamma)$, compared to free trade $w^*(\gamma)$. To fix ideas, consider the symmetric case where $R_1 = R_2 = R$ and $v_1 = v_2 = v$. It can be shown that the difference in social welfare between subsidy and free trade is given by

$$w_1^{SB*} - w_1^* = \frac{N}{M}$$

where

$$\begin{aligned}
N &= [(2\rho + 2\rho^2)S^2 + (3\rho - 1)S - 3][-21 + (2\rho^3 + 2\rho^2 - 2\rho - 2)S^5 + (11\rho^3 + 19\rho^2 - 11\rho - 19)S^4 \\
&\quad +(15\rho^3 + 52\rho^2 - 13\rho - 66)S^3 + (7\rho - 102 + 45\rho^2)S^2 + (-70 + 9\rho)S)] \\
M &= (1/8)[(-2\rho^2 + 1 + \rho^4)S^5 + (-13\rho^2 - \rho + 10 + \rho^3 + 3\rho^4)S^4 + (-27\rho^2 - 9\rho + 37 + 3\rho^3)S^3 \\
&\quad +(62 - 21\rho - 19\rho^2)S^2 + (-15\rho + 46)S + 12)]^2(S\rho + 3 + S)^2 > 0 \\
S &= Rv
\end{aligned}$$

Since $M > 0$ and the terms in the second bracket of N is negative, it is evident that $w_1^{SB*} - w_1^* > \ (<)\ 0$ if $L(S, \rho) = (2\rho + 2\rho^2)S^2 + (3\rho - 1)S - 3 < (>)\ 0$. Note that $L(S, \rho)$ here is the same condition that determines the sign of s_1 and s_2 (see Proposition 6). Thus, for a given S $(= Rv)$, subsidies raise the welfare of both countries above the free-trade level ($w_1^{SB*} - w_1^* > 0$) when ρ is small or negative. Conversely, $w_1^{SB*} - w_1^* < 0$ for points above the green locus, which occurs when, given S, ρ is sufficiently positive. [33]

This is summarized in

Proposition 8 *If $R_1 = R_2 = R$ and $v_1 = v_2 = v$, both countries are better (worse) off with Nash-equilibrium subsidies, relative to free trade, if the pair of (S, ρ) is such that* $(2\rho + 2\rho^2)S^2 + (3\rho - 1)S - 3 < \ (>)\ \ 0$.

As we deviate from the symmetric case, Nash-equilibrium subsidies may raise the welfare of one country and reduce that of the other. For example, assume that two countries have the same risk aversion (i.e., $R_1 = R_2$) and $v_2 = 4$, each country's welfare under export subsidy relative to free trade (i.e., $w_i^{SB*} - w_i^*$) is shown in Figure 7, where the red (green) surface represents $w_1^{SB*} - w_1^*$ ($w_2^{SB*} - w_2^*$). As seen, $w_1^{SB*} - w_1^* > 0$ but $w_2^{SB*} - w_2^* < 0$ when v_1 is low. The relative gains are reversed when v_1 is high.[34]

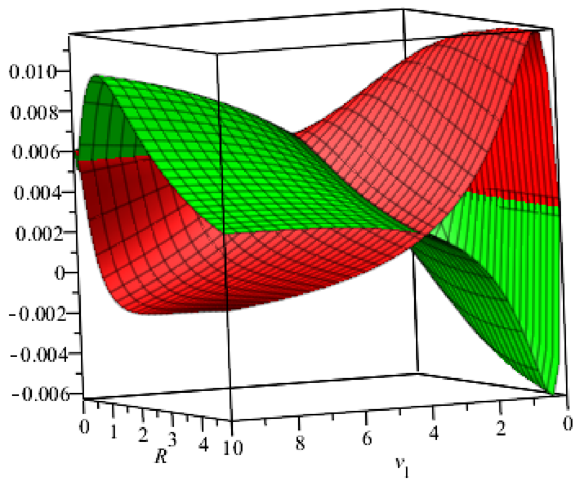

Figure 7: $w_1^{SB*} - w_1^*$ (Red) vs $w_2^{SB*} - w_2^*$ (Green), assuming that $\rho = -0.9$, $v_2 = 4$

7 Conclusion

We show that market correlation and RAR differences can explain trade patterns by using a simple partial equilibrium duopolistic trade model with an identical product and demand uncertainty. We demonstrate that we can view export/import patterns as reflecting trade's implicit risk and risk aversion content based on their relative abundance. Specifically, a relatively "risk-aversion abundant" country is likely to be a net importer of the product - hence an importer of low risk-aversion; and a relatively high-risk abundant country is more likely to be a net exporter of the product - hence an importer of low risk.

We also show that RAR differences and market correlation are sources of GFT due to implicit insurance and diversification benefits. The GFT are generally positive for both countries and consequently, world gains from trade are always positive. Furthermore, GFT may, sometimes, be higher with uncertainty than without it.

On the policy front, we show that, as in the traditional non-stochastic model, governments have an incentive to engage in strategic trade policy games. We reexamine the well-known Brander-Spencer (1985) export subsidy game. We show that the benefits from rent shifting and risk management may work in opposite directions. Hence, unlike the traditional model, Nash policy equilibrium may be an export tax rather than a subsidy.

[33] By allowing S $(=Rv)$ and ρ to vary, one can easily verify that $w_1^{SB*} - w_1^* > 0$ for most pairs of (S, ρ).

[34] A similar comparison between subsidy and autarky can also be made. If $R_1 = R_2 = R$, $v_1 = v_2 = v$, and $\rho = 0$, one obtains $w_1^{SB*} - w_1^* = \frac{(Rv+1)(R^4v^4+8R^3v^3+18R^2v^2+12Rv+3)}{8(2+Rv)^2(R^2v^2+4Rv+2)^2} > 0$, implying that subsidy is superior to autarky.

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