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30 July 2024

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July 30, 2024

Abstract

We consider the randomness of values and volumes of market deals as a major factor that describes lower bounds of uncertainty and upper limits on the accuracy of the forecasts of macroeconomic variables, prices, and returns. We introduce random macroeconomic variables, whose average values coincide with usual macroeconomic variables, and describe their uncertainty by coefficients of variation that depend on the volatilities, correlations, and coefficients of variation of random values or volumes of trades. The same approach describes bounds of uncertainty and limits on the accuracy of forecasts for growth rates, inflation, interest rates, etc. Limits on the accuracy of forecasts of macroeconomic variables depend on the certainty of predictions of their probabilities. The number of predicted statistical moments determines the veracity of macroeconomic probability. To quantify macroeconomic 2nd statistical moments, one needs additional econometric methodologies, data, and calculations of variables determined as sums of squares of values or volumes of market trades. Forecasting of macroeconomic 2nd statistical moments requires 2nd order economic theories. All of that is absent and for many years to come, the accuracy of forecasts of the probabilities of random macroeconomic variables, prices, and returns will be limited by the Gaussian approximations, which are determined by the first two statistical moments.

Keywords: bounds of uncertainty; limits on accuracy of forecasts; random macroeconomic variables; market deals; prices and returns

JEL : C01, E2, E3, E47, G1

This research received no support, specific grants, or financial assistance from funding agencies in the public, commercial, or nonprofit sectors. We welcome offers of grants, support, and positions.

1. INTRODUCTION

We consider the randomness of market trades as the factor that determines the lower bounds of the econometric uncertainty of macroeconomic variables and the upper limits on the accuracy of their forecasts. We describe the dependence of these limits on the volatilities and correlations of random values and volumes of market trades. To derive the quantitative assessments, one should develop econometric methodologies, collect data, and derive econometric approximations of these limits.

Assessments of the uncertainty of macroeconomic variables and the indeterminacy of their forecasts have been under research for decades. We do not give any review of the problem but indicate only some econometric issues. At least since Morgenstern (1950), investigations of the uncertainty of economic variables fill a long list of references. Cole (1969) studied the effect of the uncertainty of the initial source of economic data collection and the econometric errors on the accuracy of short- or long-term forecasts. Zarnowitz (1967; 1978) provided explicit analysis of the accuracy of short-term macroeconomic forecasts and described the measurements and errors in economists' predictions of changes in aggregate income, output, and the price level. Further, the accuracy of macroeconomic forecasts was studied by Diebold and Mariano (1994), Diebold (2012), Borovička and Hansen (2016), Barrero, Bloom, and Wright (2017), Reif, (2018) and many others. The assessments of the macroeconomic uncertainty using volatility of economic indicators (Jurado, Ludvigson, and Ng, 2015) ties up the uncertainty shocks with business cycles. The cyclical behavior of empirical measures of uncertainty along with business cycles was presented by (Cacciatore and Ravenna, 2020). Bloom (2013) gave a review of the uncertainty's fluctuations problem and discussed the change of uncertainty over time, the possible reasons for variations of uncertainty, the impact of uncertainty fluctuations on short-run investment and hiring, and the consequences of the uncertainty on the recession of 2007-2009. The effects of uncertainty on risk premia and business cycle fluctuations were discussed by Bianchi, Kung, and Tirskikh (2018). The impacts of uncertainty on firms' decisions were studied by Kumar, Gorodnichenko, and Coibion (2022).

Poor accuracy of macroeconomic predictions led Zarnowitz (1991) to discuss the obstacles that may prevent more reliable and precise macroeconomic forecasts. "There is much disenchantment with macroeconomic forecasts. The difficult question is how much of it is due to unacceptably poor performance and how much to unrealistically high expectations. I would argue that the latter is a major factor”. We support Zarnowitz’s conclusion, but for reasons that are completely different from his considerations.

Actually, the current studies of the uncertainty of macroeconomic variables and the low accuracy of their forecasts cover almost all possible econometric reasons and factors. However, the amazing feature of economics is that it always has some hidden factors that may play a major role in the uncertainty puzzle. We consider some of these missed issues and describe their impact on the uncertainty of macroeconomic variables and the accuracy of their forecasts.

We strongly believe that it is the duty of theoretical economics to develop general models and determine the meaning and definitions of macroeconomic variables required for the theoretical description and forecasting of economic processes. The goals of econometrics to provide measurements and valuations of macroeconomic variables, which are determined by theoretical economics. The testing of economic theories requires comparisons of the theoretical results with observable economic data. It is the duty of econometrics to measure and evaluate the macroeconomic variables that are determined by theoretical models in order to compare theoretical results and predictions based on the econometric assessments of these variables with observed economic processes.

Economic reality disturbs these simple rules. The description of highly irregular or even random economic processes requires the use of averaging procedures to derive regular, smooth macroeconomic variables. The choice of the averaging time interval, to a large extent, determines the uncertainty of macroeconomic variables estimated by econometrics and the accuracy of their forecasts. A great number of economic agents that have different economic variables, make various market transactions, and follow their own expectations and aims makes the problems of theoretical economics and the goals of econometrics really interesting but rather difficult.

In particular, economic complexity is manifested in the simple fact that some macroeconomic variables that are determined by theoretical economics cannot be directly measured using initial economic data because such data is absent. Indeed, the economic meaning of most additive macroeconomic variables is determined by the sums of the corresponding variables of economic agents. For example, macroeconomic assets, consumption, profits, etc. have the economic meaning of aggregates, of sums (without doubling) of the corresponding variables of all economic agents. In turn, the changes of additive variables of economic agents depend on the values and volumes of market trades made by agents during the averaging interval. Actually, econometrics doesn't have initial, direct data that describes the variables of all economic agents and the records of all market trades made by all agents during the averaging time interval. However, econometric methodologies permit the valuation of macroeconomic variables using other available, observable, and measurable economic data. This gap between the direct data that are required by the theoretical definitions of macroeconomic variables and the available econometric data that approximate values of these macroeconomic variables reveals the additional source of the uncertainty of macroeconomic variables.

Meanwhile, the theoretical definitions of macroeconomic variables on their own are the substantive source of the uncertainty and determine the limits on the accuracy of forecasts. In our paper, we describe the lower bounds of uncertainty that are determined by the theoretical definitions of additive macroeconomic variables as sums of corresponding variables of economic agents. The changes of agents' variables are determined by random market trades made by agents during the averaging time interval. We believe that the randomness of market trade is the major economic factor that defines the lower bounds of uncertainty of macroeconomic variables and the upper limits on the accuracy of their forecasts. We describe the dependence of the lower bounds of uncertainty and upper limits on the accuracy of forecasts on the volatilities and correlations of random values and volumes of market trades during the averaging interval. We highlight that the same approach describes the bounds of uncertainty and the limits on the accuracy of forecasts for all other variables, such as growth rates, inflation, interest rates, etc. The quantification of these limits requires the development of additional econometric methodologies, data collection, and approximations.

In Section 2, we discuss a theoretical framework. In Section 3 and in App. A, we describe the dependence of the lower bounds of uncertainties of macroeconomic variables, prices, and returns on the volatilities of market trades and consider coefficients of variation as the measure of uncertainty. In Section 4, we explain economic factors that limit the accuracy of the forecasts of prices, returns, and macroeconomic variables. In App. B., we consider the uncertainty of "complex" macroeconomic variables that depend on different market trades. We assume that all prices are adjusted to the current time t.

2. THEORETICAL FRAMEWORK

We consider macroeconomics as a system of agents that perform market deals with various assets, commodities, and services. As agents, we take banks and corporations, plants and factories, households, and shops—all participants in economic and financial transactions at various markets. Numerous economic and financial variables characterize each agent. The sums of similar additive variables of agents such as profits and investment, consumption and supply, etc. define additive macroeconomic variables. The ratios of additive macroeconomic variables define non-additive variables such as prices, inflation, bank rates, GDP rate, etc. We believe that the definitions of additive macroeconomic variables by sums of variables of agents reflect their economic essence. The changes of macroeconomic variables completely depend on the changes of agents' variables.

In turn, the changes of agents' variables depend on market trades made by agents. The changes of agents' investment, consumption, and supply depend on their market deals. We consider the economic and financial transactions of agents as the only origin of the change of their variables. That dependence - market deals, agents' variables, macroeconomic variables - determines the important problem - the selection of the averaging time interval. The duration of the averaging interval determines the uncertainty of market trades, agents' variables, and finally, the uncertainty of macroeconomic variables. Further, the duration of the averaging interval plays a core role in calculating the accuracy of the forecasts of macro variables. Indeed, current financial markets generate high-frequent trade time series. For simplicity, we assume that the interval ε between two consecutive market deals is a constant that can be equal to or even less than a second. Each market deal at time t_i, $t_i\text{-}t_{i-1}=\varepsilon$, $i=1,2,..,$ changes the corresponding variables of agents, and that can impact the values of macroeconomic variables and their forecasts. However, the direct use of market trade time series with periodicity $\varepsilon\leq 1\ sec$ is almost useless for the description of macroeconomic variables and their forecasts. To describe the evolution of macroeconomic and financial variables, one should consider the averaging intervals Δ, which could be equal to weeks, months, or years. Some financial variables, for example, the prices of major stocks, indices, and volatilities, can be considered on a daily basis or even on an hourly basis. The selection of the duration of the interval $\Delta>>\varepsilon$ raises the important economic problem of the averaging of market trade data. Indeed, during the averaging interval $\Delta >> \varepsilon$ market trade time series behave highly irregularly or randomly.

We consider the randomness of market trade time series during Δ as the only origin of the uncertainty of agents' variables and, respectively, the uncertainty of macroeconomic variables. We don't study how various factors impact the randomness of market trade. Instead, we describe how the random values and volumes of market deals during Δ determine the lower bounds of the uncertainty of corresponding prices, returns, and macroeconomic variables and how that limits the accuracy of their forecasts. The consideration of macroeconomic variables, prices, returns, and market trades as random variables during the averaging interval Δ establishes a uniform basis for the theoretical description of macroeconomics.

We consider the different time series of market deals with different assets, commodities, and services during Δ as different random variables. To derive regular estimates of the change of macroeconomic variables, prices, and returns during Δ, one should average market time series during Δ. The definition of the averaging procedure during the selected interval Δ determines the dependence of the uncertainty of macroeconomic variables, prices, and returns on the random properties of the corresponding time series of market trades. We describe the properties of random deals by the set of the *n-th* statistical moments of trade values and volumes. In App. A., we show how the volatilities and correlations of trade values and volumes determine the volatilities of market prices and returns and how their coefficients of variation define the lower bounds of their uncertainty. In Sec. 3, we describe the dependence of the lower bounds of uncertainty of macroeconomic variables on coefficients of variation and correlations of random values or volumes of market trades.

In Section 4, we describe how the direct dependence of the volatilities of prices, returns, and macroeconomic variables on the volatilities and correlations of market trade values and volumes define the upper limits on the accuracy of predictions of their probabilities by Gaussian distributions.

Our theoretical assessments of the lower bounds of the uncertainty of macroeconomic variables and the upper limits on the accuracy of macroeconomic forecasts should be supported by econometric estimates. That problem reveals the complementary roles of theoretical economics and econometrics. Actually, the direct, sufficient data that is required to calculate the values of macroeconomic variables doesn't exist. There is no sufficient data about all economic agents and their variables, and there is no sufficient data about all market transactions made by each agent during any averaging interval Δ. Direct assessments of macroeconomic variables as sums of all market trades made by all agents are impossible. At that point, our theoretical definitions of macroeconomic variables call for the help of econometrics as a way to estimate the values of macroeconomic variables in the absence of direct data. Econometrics solves these problems and gives the approximations of macroeconomic variables that are theoretically determined as sums of agents' variables. Econometrics provides estimates of macroeconomic variables using available, observable data according to econometric methodologies (Fox et al., 2019).

That uncovers the duality of the problem of uncertainty of macroeconomic variables. The second part of the problem – the uncertainty of macroeconomic variables as a result of the uncertainty of observable econometric data and the uncertainty of the econometric calculations - has been studied deeply (Morgenstern, 1950; Cole,1969; Davidson and MacKinnon 2004; Mills and Patterson 2009; Hansen, 2014; Fox et al. 2019; Ilut and Schneider, 2022). However, the first part of the problem, which reveals the dependence of macroeconomic uncertainty on properties of random market trade during the averaging interval, was almost missed. Our article, at least partially, covers this gap and describes the theoretical bounds of uncertainty and the limits on the accuracy of forecasts, which are determined by the dependences of macroeconomic variables, prices, and returns on the randomness of market trades.

In Section 4, we discuss the dependence of the volatilities of macroeconomic variables on sums of squares of the values or volumes of corresponding market deals during the interval Δ. That dependence raises a new, tough challenge for econometric methodologies, data collection, and calculations. Indeed, current econometrics highly succeeds in estimating the averages of random macroeconomic variables, which are composed of *sums of 1st degrees* of market trade values or volumes during the averaging interval Δ. We call them the 1st order variables and denote economic models that describe their evolution as 1st order economic theories. To estimate volatilities of macroeconomic variables, prices, and returns that depend on ***sums of squares*** of trade values or volumes and determine the lower bounds of the uncertainty of prices, returns, and macroeconomic variables, one should develop new econometric methodologies and collect additional data. One should take into account the mutual dependence of 1st order variables and macro variables composed of ***sums of squares*** of trade values and volumes, that we call 2nd order variables. Almost each additive macroeconomic variable of 1st order should be complemented by its 2nd order pair. That at least doubles the number of variables that describe macroeconomic evolution. For convenience, we denote the modeling of 1st and 2nd order macroeconomic variables as 2nd order economic theories.

The forecasts of the volatilities prices, returns, and macroeconomic variables require the description of 2nd order variables composed of sums of squares of market trade values and volumes. Currently, 2nd order economic theories, the econometric methodologies, data, and calculations of 2nd order variables that complement macroeconomic variables of 1st order are absent. That means the lack of an economic basis for predictions of 2nd order variables. Simply speaking, current forecasting of volatilities of prices, returns, and macroeconomic variables have no economic foundation. As we show in Sec. 4, for many years to come, that limits the accuracy of predictions of probabilities of prices, returns, and macroeconomic variables by Gaussian distributions. One can find more details in Olkhov (2021-2024).

3. LOWER BOUNDS OF UNCERTAINTY

To estimate the uncertainty of macroeconomic variables as their volatility, one should consider them as certain random variables. To define such random variables, at the first step, we consider the changes of macroeconomic variables during Δ to be equal to the sums of the corresponding variables of economic agents. In turn, the changes of most additive variables of agents are equal to the sums of trade values or volumes made by agents during Δ. As examples, we consider agents' investment, credits, and consumption. The changes of these variables are equal to the sums of the investment, credit, and consumption market deals of agents during Δ. The sums of agents' investment, credits, and consumption (without repeating) determine the change of corresponding macro variables during Δ. The definitions of some additive variables require the use of sums of linear combinations of different market deals of agents. For example, the change of GDP during Δ equals the sum of the Value Added (VA) of all agents plus the net export trades (Fox, 2019). To calculate the VA of the agent one should sum the linear combinations of trade sales and purchases made by agent during Δ. The particular linear form that define agents' VA can vary due to different schemes of accounting, business specifics, and tax regulations. Anyway, the linear dependence of VA on market trades made by agents permits us to consider the uncertainty of GDP during Δ completely in the same way as the uncertainty of other macroeconomic variables such as investments, credits, or supply. Finally, the changes of different additive variables of agents during Δ can be presented by the linear forms of the sums of various market deals made by agents during Δ. As we show (App. B), the lower bounds of the uncertainties of macroeconomic variables during Δ are determined by the volatilities, correlations, and coefficients of variation of market trade values or volumes.

To illustrate that dependence, let us consider the lower bounds of the uncertainty of macroeconomic consumption.

We assume that during time interval Δ (3.1), each agent j, $j=1,\ldots M$, made $N(j)$ purchase deals that resulted in consumption, and there was no doubling. We denote $C(t_{ij})$ as the values of the consumption trades made by the agent j at time t_{ij}.

$$t-\frac{\Delta}{2} < t_{ij} < t+\frac{\Delta}{2} \quad ; \quad i = 1,\ldots N(j)\,;\; j = 1,..M \tag{3.1}$$

The total number K (3.2) of all consumption trades $C(t_{ij})$ during Δ (3.1) equals to:

$$K = \sum_{j=1}^{M} N(j) \tag{3.2}$$

The *n-th* statistical moments $C(n)$ of consumption trades during Δ take the form:

$$C(n) = E\left[C^n\left(t_{i,j}\right)\right] = \frac{1}{K}\sum_{j=1}^{M}\sum_{i=1}^{N(j)} C^n\left(t_{i,j}\right) \quad ; \quad n = 1,2,.. \tag{3.3}$$

$$C_\Delta(n) = \sum_{j=1}^{M}\sum_{i=1}^{N(j)} C^n\left(t_{i,j}\right) = K \cdot C(n) \tag{3.4}$$

Relations (3.3) give the approximations of the *n-th* statistical moments by the finite number K (3.2) of the consumption trades of all economic agents during Δ. The function $C(1)$ (3.3) denotes the average values of the consumption trades during Δ. The function $C_\Delta(1)$ in (3.4) equals the total value of all consumption deals of all economic agents during Δ, which exactly defines conventional macroeconomic consumption during Δ. However, we highlight that the econometric assessments of macroeconomic consumption $C_\Delta(1)$ in (3.4) could use completely different econometric methodology, data, and calculations to quantify the value of consumption $C_\Delta(1)$ in (3.4) during the time interval Δ. One should highlight and take into account the possible differences between the theoretical definition of a macroeconomic variable and the one that is given by econometric methodology to measure that variable using the available, observable data.

To estimate the uncertainty of macroeconomic consumption $C_\Delta(1)$ during Δ, we define macroeconomic consumption as a random variable $x(t_{ij})$ (3.5):

$$x(t_{i,j}) = K \cdot C(t_{i,j}) = C(t_{i,j}) \cdot \sum_{j=1}^{M} N(j) \qquad (3.5)$$

We call $x(t_{ij})$ (3.5) a random macroeconomic consumption during Δ. The mathematical expectation $x(1)$ (3.6) of the random consumption $x(t_{ij})$ (3.5) takes the form:

$$x(1) = E[x(t_{i,j})] = \frac{1}{K}\sum_{j=1}^{M} x(t_{i,j}) = \sum_{j=1}^{M} C(t_{i,j}) = C_\Delta(1) \qquad (3.6)$$

Thus, the average $x(1)$ (3.6) of the random consumption $x(t_{ij})$ (3.5) equals conventional macroeconomic consumption $C_\Delta(1)$ (3.4) during Δ. The definition macroeconomic consumption as a random variable $x(t_{ij})$ (3.5) permits us to consider the volatility σ_x^2 (3.7) of the random variable $x(t_{ij})$ (3.5) as the assessment of the uncertainty of macroeconomic consumption $C_\Delta(1)$ (3.4) during Δ:

$$\sigma_x^2 = E[(x(i,j) - x(1))^2] \qquad (3.7)$$

$$E[x^2(i,j)] = \frac{1}{K}\sum_{j=1}^{M}\sum_{i=1}^{N(j)} x^2(t_{i,j}) = K\sum_{j=1}^{M}\sum_{i=1}^{N(j)} C^2(t_{i,j}) = K\, C_\Delta(2) \qquad (3.8)$$

From (3.3 - 3.8), obtain the dependence of volatility σ_x^2 (3.9) on volatility σ_C^2 (3.10):

$$\sigma_x^2 = K^2[\,C(2) - C^2(1)] = K^2\sigma_C^2 \qquad (3.9)$$

$$\sigma_C^2 = C(2) - C^2(1) \qquad (3.10)$$

In (3.9; 3.10), σ_C^2 denotes the volatility of consumption trade values $C(t_{ij})$. The square of the coefficient of variation χ_x^2 (3.11) of a random macroeconomic consumption $x(t_{ij})$ (3.5) equals the square of the coefficient of variation χ_C^2 (3.11) of consumption trade values during Δ:

$$\chi_x^2 = \frac{\sigma_x^2}{x^2(1)} = \frac{K^2\,\sigma_C^2}{K^2\,C^2(1)} = \frac{\sigma_C^2}{C^2(1)} = \chi_C^2 \qquad (3.11)$$

The coefficients of variation χ_x^2 and χ_C^2 (3.11) describe the volatilities of random variables with averages equal to one. We propose them as a good measure to determine the lower bounds of the uncertainty of random macroeconomic variables. The relations (3.11) demonstrate that lower bounds of the uncertainty of the consumption trade values $C(t_{ij})$, which we measure by χ_C^2 (3.11), coincide with the lower bounds of the uncertainty of macroeconomic consumption χ_x^2 (3.11) during Δ. We highlight that the quantitative assessments of the lower bounds of the uncertainty of consumption (3.11) need the development of econometric methodologies, data collections, and calculations to estimate the sums of squares of market trade values during Δ. All of that is absent now.

In App. B., we describe lower bounds of uncertainty of "complex" macroeconomic variables, like profits, GDP, etc., that are determined by different deals.

4. Upper limits on the accuracy of forecasts

We consider consumption as an example to describe the upper limits on the accuracy of macroeconomic forecasts. Above, we defined macroeconomic consumption as a random variable $x(t_{ij})$ (3.5), which is determined by the random values $C(t_{ij})$ of consumption deals. To predict a random variable one should forecast its probability. The accuracy of the forecasts of probability of a random consumption $x(t_{ij})$ (3.5) is determined by the accuracy of the predictions of the probability of random values $C(t_{ij})$ of consumption's deals. The more precise the predictions of probability of consumption's deals, the more accurate and reliable would be the forecasts of macroeconomic consumption during Δ.

It is well known that a random variable can be described equally by a probability measure, a characteristic function, or a set of the *n-th* statistical moments (Shiryaev, 1999; Shreve, 2004). The more statistical moments of a random variable are used for the approximation of the probability measure of a random variable, the higher the accuracy of the resulting approximation of probability. The forecasts of the first two statistical moments define the average and volatility of a random variable. The approximations of probability by the first two statistical moments take the form of Gaussian distributions.

The volatility σ_C^2 (3.3; 3.10) of the random values $C(t_{ij})$ of consumption's trades depends on the 2^{nd} statistical moment $C(2)$ (3.3), which is determined by the sum of squares of the values of consumption deals during Δ, and we call it a 2^{nd} order economic variable. As we already discussed, the predictions of $C(2)$ (3.3) require the development of 2^{nd} order economic theory. The quantitative assessments of $C(2)$ (3.3) require econometric methodologies, data, and calculations that are absent now. The assessments of the 3^{rd} or 4^{th} statistical moments require econometric methodologies, data, and calculations to quantify variables composed by the sums of the 3^{rd} and 4^{th} degrees of the values of consumption deals during Δ. The predictions of the 3^{rd} or 4^{th} statistical moments require economic theories that model the mutual evolution of variables up to the 3^{rd} or 4^{th} orders. All of that is absent now. That limits the accuracy of predictions of the probabilities of macroeconomic variables, in the best case, by Gaussian distributions.

The forecasting the probability of macroeconomic variables completely coincides with the problem of forecasting the probabilities of prices and returns, which also depend on predictions of the volatilities and correlations of the corresponding market trade values and volumes (Olkhov, 2021; 2023b; 2023c; 2024). The dependence of the volatilities of prices, returns, and macroeconomic variables on the sums of squares of different types of market deals ties up the predictions of their probabilities in a unified puzzle. The upper limits on the accuracy of the predictions of their probabilities are similar and are limited by Gaussian distributions. It should be agreed that there are no economic reasons to believe that one can predict the probabilities of a particular macroeconomic variable, price, or return with an accuracy that is higher than for others. The upper limits on the accuracy of forecasts depend on the development of 2nd order economic theories. In turn, the development of these theories depends on the creation of econometric methodology, data collections, and econometric valuations of the 2nd order variables. Until then, the upper limits on the accuracy of the forecast of macroeconomic variables, prices, and returns are limited by Gaussian approximations of their probabilities.

5. Conclusion

The consideration of macroeconomic variables, prices, and returns as random variables that are determined by the random values or volumes of market deals establishes the uniform basis for theoretical macroeconomics and for the studies of their lower bounds of uncertainty and the upper limits on the accuracy of forecasts. All these problems establish a unified economic puzzle that describes statistical moments of market trades and macroeconomic variables. There are no economic reasons to believe that one can predict the probabilities of a particular macroeconomic variable, price, or return with accuracy that is higher than the accuracy of others.

The economic roots of both limits are explained by the dependence on 2nd order variables on sums of squares of the values or volumes of market trades during Δ. The quantification of 2nd order economic variables raises tough problems for econometrics, data collection, and macroeconomic modeling. The current theories and econometrics describe the mutual evolution of the mean values of macroeconomic variables that depend on the sums of the 1st degrees of the values or volumes of market trades. The mutual description of 1st and 2nd order macroeconomic variables at least doubles the number of variables and the complexity of their modeling. In simple words, the accuracy of the description of mean values of macroeconomic variables, average prices, and returns depends on their 2nd statistical moments and volatilities. The more statistical moments that can be predicted, the higher the accuracy of the average macroeconomic variables, prices, and returns one could derive. The lack of direct initial data about the values and volumes of market trades made by all economic agents that is required for the valuation of their 2nd statistical moments makes direct quantitative assessments impossible. That raises the request for the development of econometric methodologies, data, approximations, and predictions of the 2nd statistical moments, volatilities, and correlations of the random values and volumes of market trades. The success of these econometric efforts will permit us to approximate and describe the volatilities of prices, returns, and macroeconomic variables and will support the development of macroeconomic models that describe the mutual evolution of the 1st and 2nd order variables. Only then will forecasts based on Gaussian approximations of probabilities of price, returns, and macroeconomic variables have an economic foundation.

The general origin of the economic complexity for valuation and modeling the 2nd order economic variables establishes common lower limits on the uncertainty of macroeconomic variables, prices, and returns and common upper limits on the accuracy of their forecast.

However, the investigations of the general problem could open a wide field for the development of approximations to "overcome" economic-based limits.

APPENDIX A: COEFFICIENTS OF VARIATION OF PRICE AND RETURN

In Appendix A, we briefly present the results by Olkhov (2022; 2023a; 2023b; 2024) and refer there for details. We consider the time series of market trade values $C(t_i)$ and volumes $U(t_i)$ during the averaging interval Δ (A.1):

$$t-\frac{\Delta}{2}< \ t_i < \ t+\frac{\Delta}{2} \quad ; \quad i=1,\dots N \tag{A.1}$$

We define price $p(t_i)$ and return $r(t_i,\tau)$ (A.2) for the constant time shift τ at time t_i:

$$C(t_i)=p(t_i)U(t_i) \quad ; \quad r(t_i,\tau)=\frac{p(t_i)}{p(t_i-\tau)} \tag{A.2}$$

One can convert (A.2) into equation (A.4) on return $r(t_i,\tau)$:

$$C(t_i)=p(t_i)U(t_i)=\frac{p(t_i)}{p(t_i-\tau)}p(t_i-\tau)U(t_i) \quad ; \quad C_o(t_i,\tau)=p(t_i-\tau)U(t_i) \tag{A.3}$$

$$C(t_i)=r(t_i,\tau)\,C_o(t_i,\tau) \tag{A.4}$$

In (A.3; A.4), $C_o(t_i,\tau)$ denotes the past market value of trade volume $U(t_i)$ at time t_i-τ. Equations (A.2) on price and (A.4) on return have the same forms, and that cause the similar forms of their volatilities. The *n-th* statistical moments $C(n)$ of market trade value, volume $U(n)$, and past market value $C_o(n,\tau)$ of irregular time series with N terms during Δ (A.1) are estimated similar to (3.3):

$$C(n)=E[C^n(t_i)]\sim\frac{1}{N}\sum_{i=1}^{N}C^n(t_i) \tag{A.5}$$

$$U(n)\sim\frac{1}{N}\sum_{i=1}^{N}U^n(t_i) \quad ; \quad C_o(n,\tau)\sim\frac{1}{N}\sum_{i=1}^{N}{C_o}^n(t_i,\tau) \tag{A.6}$$

Equations (A.2; A.4) mean that one can't define market-based statistical moments of price and return similar to (A.5; A.6). As market-based average price $a(1)$ (A.7), we take the well-known volume weighted average price (VWAP) (Berkowitz et al., 1989; Duffie and Dworczak, 2018). We denote $E_m[..]$ market-based mathematical expectation to differ it from frequency-based mathematical expectation $E[..]$ (3.3; A.5). The 1st price statistical moment or average price $a(1)$ takes the form:

$$a(1)=\frac{1}{\sum_{i=1}^{N}U(t_i)}\sum_{i=1}^{N}p(t_i)U(t_i)=\sum_{i=1}^{N}p(t_i)w(t_i;1)=\frac{C(1)}{U(1)} \tag{A.7}$$

$$w(t_i;1)=\frac{U(t_i)}{\sum_{i=1}^{N}U(t_i)} \quad ; \quad \sum_{i=1}^{N}w(t_i;1)=1 \tag{A.8}$$

Functions $w(t_i;1)$ (A.7; A.8) play the role of weight functions. Markowitz (1952), in his famous work on portfolio choice 37 years earlier than Berkowitz et al. (1989), proposed the average portfolio return $h(1,\tau)$ (A.9) in the same form as VWAP (A.7):

$$h(1,\tau)=\frac{1}{\sum_{i=1}^{N}C_o(t_i,\tau)}\sum_{i=1}^{N}r(t_i,\tau)\,C_o(t_i,\tau)=\sum_{i=1}^{N}r(t_i,\tau)\,z(t_i;\tau,1)=\frac{C(1)}{C_o(1,\tau)} \tag{A.9}$$

$$z(t_i,\tau,1)=\frac{C_o(t_i,\tau)}{\sum_{i=1}^{N}C_o(t_i,\tau)}\quad ;\quad \sum_{i=1}^{N}z(t_i,\tau,1)=1 \tag{A.10}$$

Functions $z(t_i;\tau,1)$ (A.9; A.10) play the role of weight functions similar to $w(t_i;1)$ (A.7; A.8). We take the average return (A.9) of the portfolio as the market-based average of return's time series $r(t_i,\tau)$ (A.2) during Δ (A.1). To justify such a choice, we highlight that one can consider the set of returns $r(t_i,\tau)$ (A.2) during Δ (A.1) as the returns of the selected portfolio. The identities of the forms of equations (A.2) and (A.4) and the forms of the average price (A.7) and average return (A.9) result in the same forms of their volatilities. For brevity, we present a derivation of market-based volatility of price only. Let us consider the 2^{nd} degrees of the trade price equation (A.2):

$$C^2(t_i)=p^2(t_i)U^2(t_i) \tag{A.11}$$

For $m=1,2$ we define the m-th statistical moments $p(t;m,2)$ of price in a form similar to (A.7):

$$p(m,2)=\sum_{i=1}^{N}p^m(t_i)\,w(t_i;2)\quad ;\quad w(t_i;2)=\frac{U^2(t_i)}{\sum_{i=1}^{N}U^2(t_i)}\quad ;\quad \sum_{i=1}^{N}w(t_i;2)=1 \tag{A.12}$$

Functions $w(t_i;2)$ (A.12) play the role of weight functions similar to (A.8). To define market-based 2nd statistical moment $a(2)$ of price we consider market-based volatility σ_p^2 of price:

$$a(2)=E_m[p^2(t_i)]\quad ;\quad \sigma_p^2=E_m\left[\left(p(t_i)-a(1)\right)^2\right]=a(2)-a^2(1)\geq 0 \tag{A.13}$$

We derive volatility σ_p^2 of price by averaging over weight functions $w(t_i;2)$ (A.12) and from (A.12; A.13) obtain market-based 2nd statistical moment of price $a(2)$:

$$\sigma_p^2=\sum_{i=1}^{N}\left(p(t_i)-a(1)\right)^2 w(t_i;2)=p(2,2)-2p(1,2)a(1)+a^2(1) \tag{A.14}$$

Simple transformations of (A.14) derive volatility σ_p^2 and the 2nd statistical moment $a(2)$ of price (Olkhov, 2022; 2023a; 2023b; 2024):

$$\sigma_p^2=\frac{\Omega_C^2+a^2(1)\Omega_U^2-2a(1)corr[CU]}{U(2)}\quad ;\quad a(2)=\frac{C(2)+2a^2(1)\Omega_U^2-2a(1)corr[CU]}{U(2)} \tag{A.15}$$

In (A.15) we use (A.6) and denote volatilities of market trade value Ω_C^2 and volume Ω_U^2:

$$\Omega_C^2=C(2)-C^2(1)\quad ;\quad \Omega_U^2=U(2)-U^2(1) \tag{A.16}$$

The correlation $corr[CU]$ (A.17) of trade values and volumes during Δ takes the form:

$$corr[CU]=E\left[\left(C(t_i)-C(t;1)\right)\left(U(t_i)-U(t;1)\right)\right]=E[C(t_i)U(t_i)]-C(1)U(1) \tag{A.17}$$

The joint average $E[C(t_i)U(t_i)]$ (A.17) of the product of trade value and volume equals:

$$E[C(t_i)U(t_i)]=\frac{1}{N}\sum_{i=1}^{N}C(t_i)U(t_i) \tag{A.18}$$

Market-based volatility $\sigma_r^2(\tau)$ (A.20) and the 2nd statistical moment $h(2,\tau)$ (A.21) of return (A.2) take the form similar to (A.15), but past market values substitute trade volumes:

$$h(2,\tau) = E_m[r^2(t_i,\tau)] \quad \text{(A.19)}$$

$$\sigma_r^2(\tau) = E_m\left[\left(r(t_i,\tau) - h(1,\tau)\right)^2\right] = h(2,\tau) - h^2(1,\tau) \geq 0 \quad \text{(A.20)}$$

$$\sigma_r^2(\tau) = \frac{\Omega_C^2 + h^2(1,\tau)\Omega_{Co}^2(\tau) - 2h(1,\tau)corr[CC_o(\tau)]}{C_o(2,\tau)} \quad \text{(A.21)}$$

$$h(2,\tau) = \frac{C(2) + 2h^2(1,\tau)\Omega_{Co}^2(\tau) - 2h(1,\tau)corr[CC_o(\tau)]}{C_o(2,\tau)} \quad \text{(A.22)}$$

$$\Omega_{Co}^2(\tau) = C_o(2,\tau) - C_o^2(1,\tau) \quad ; \quad E[C(t_i)C_o(t_i,\tau)] = \frac{1}{N}\sum_{i=1}^{N} C(t_i)C_o(t_i,\tau) \quad \text{(A.23)}$$

$$corr[CC_o(\tau)] = E[C(t_i)C_o(t_i,\tau)] - C(1)C_o(1,\tau) \quad \text{(A.24)}$$

Market-based volatility σ_p^2 (A.15) of price and volatility $\sigma_r^2(\tau)$ (A.20; A.21) of return define the lower bounds of their uncertainty, which are determined by the volatilities and correlations of market trade values, volumes, and past market values. To compare the lower bounds of uncertainty of different variables, prices, and returns, one should consider the volatilities of random variables with averages equal to one. That is the reason to consider the squares of coefficients of variation of price and return as their lower bounds of uncertainty. Simple transformations of (A.15; A.21) give squares of coefficients of variation of price χ_p^2, return $\chi_r^2(\tau)$, market trade value χ_C^2, volume χ_U^2, past market trade value χ_{Co}^2, and their correlations:

$$\chi_p^2 = \frac{\sigma_p^2}{a^2(1)} \quad ; \quad \chi_p^2 \cdot (1 + \chi_U^2) = \chi_C^2 + \chi_U^2 - 2\frac{corr[CU]}{C(1)U(1)} \quad \text{(A.25)}$$

$$\chi_r^2(\tau) = \frac{\sigma_r^2(\tau)}{h^2(1,\tau)} \quad ; \quad \chi_r^2(\tau) \cdot (1 + \chi_{Co}^2(\tau)) = \chi_C^2 + \chi_{Co}^2(\tau) - 2\frac{corr[CC_o(\tau)]}{C(1)C_o(1,\tau)} \quad \text{(A.26)}$$

$$\chi_C^2 = \frac{\sigma_C^2}{C^2(1)} \quad ; \quad \chi_U^2 = \frac{\sigma_U^2}{U^2(1)} \quad ; \quad \chi_{Co}^2(\tau) = \frac{\sigma_r^2(\tau)}{C_o^2(1,\tau)} \quad \text{(A.27)}$$

The squares of coefficients of variation (A.25-A.27) describe the uncertainty of normalized random variables with an average equal one. They are very convenient for mutual comparisons of the uncertainty of the market trades, prices, and returns.

A similar approach describes the lower bounds of the uncertainty of other non-additive macroeconomic variables such as bank rates, inflation, GDP growth rates, etc. However, the quantitative assessments of (A.25-A.27) require the development of econometric methodologies, data, and calculations that are absent now.

APPENDIX B: COEFFICIENTS OF VARIATION OF "COMPLEX" MACROECONOMIC VARIABLES

The derivation of the average and volatility of a random variable composed by the sum of Q random variables is presented in most manuals on probability (Shiryaev, 1999; Shreve, 2004). Let us assume that a random variable a takes the form:

$$a = \sum_{q=1}^{Q} \beta(q) a(q) \quad \text{(B.1)}$$

and different random variables $a(q)$, $q=1,..Q$ have their averages $A(q)$, volatilities $\sigma^2(q)$, and correlations $corr(q;k)$:

$$A(q) = E[a(q)] \quad ; \quad \sigma^2(q) = E\left[\left(a(q) - A(q)\right)^2\right] \quad \text{(B2.)}$$

$$corr(q;k) = E[(a(q) - A(q))(a(k) - A(k))] = E[a(q)a(k)] - A(q)A(k) \quad \text{(B.3)}$$

The average A and volatility σ_A^2 of a random variable a (B.1) equal:

$$A = E[a] = \sum_{q=1}^{Q} \beta(q) A(q) \quad \text{(B.4)}$$

$$\sigma_A^2 = E[(a - A)^2] = \sum_{q=1}^{Q} \beta^2(q)\sigma^2(q) + 2\sum_{q=1;k>q}^{Q-1} \beta(q)\,\beta(k) corr(q;k) \quad \text{(B.5)}$$

The square of the coefficient of variation χ_A^2 (B.6) of a random variable a (B.1) depends on the squares of the coefficients of variation $\chi^2(q)$ (B.7) of the random variables $a(q)$, their averages $A(q)$ and correlations $corr(q;k)$:

$$\chi_A^2(t) = \frac{\sigma_A^2}{A^2} = \sum_{q=1}^{Q} \Theta(q) \cdot \chi^2(q) + 2\sum_{q=1;k>q}^{Q-1} \Phi(q,k)\ \Psi(t;q) \quad \text{(B.6)}$$

$$\chi^2(q) = \frac{\sigma^2(q)}{A^2(q)} \quad ; \quad \Theta(q) = \frac{\beta^2(q)A^2(q)}{A^2} \quad \text{(B.7)}$$

$$\Phi(q,k) = \frac{\beta(q)\beta(k)A(q)A(k)}{A^2} \quad ; \quad \Psi(q,k) = \frac{corr(q,k)}{A(q)A(k)} \quad \text{(B.8)}$$

$$\sum_{q=1}^{Q} \Theta(q) + 2\sum_{q=1;k>q}^{Q-1} \Phi(q,k) = 1 \quad \text{(B.9)}$$

One should take into account that (B.9) doesn't play the role of an averaging because some coefficients $\beta(q)$ and $\Phi(q,k)$ can be negative. Relations (B.4-B.9) present the volatility and coefficient of variation for any "complex" macroeconomic variable that is determined by a linear form of different market trades through the volatilities and coefficients of variations of these trades.

In particular, macroeconomic profits Pr (B.10) during Δ in the most simplified form can be determined as a difference (B.10) between revenues determined as the sums of sales $Sa(t_{ij})$ and expenses determined as the sums of purchases $Ex(t_{ij})$. The square of the coefficient of variation χ_{Pr}^2 (B.11) of profits (B.10), takes the form:

$$Pr = \sum_{j=1}^{M} \sum_{i=1}^{N(j,Sa)} Sa(t_{i,j}) - \sum_{j=1}^{M} \sum_{i=1}^{N(j,Ex)} Ex(t_{i,j}) = K_S \cdot Sa(1) - K_E \cdot Ex(1) \quad \text{(B.10)}$$

In (B.10), $N(j,Sa)$ and $N(j,Ex)$ denote the numbers of trades of sales $Sa(t_{ij})$ and purchases $Ex(t_{ij})$ made by agent j during Δ. K_S and K_E – total numbers of sales and purchases in economy during Δ. $Sa(1)$ and $Ex(1)$ (B.13) denote the average values of single sale and purchase during Δ.

$$\chi_{Pr}^2 = \chi_{Sa}^2 \cdot \Theta_{Sa} + \chi_{Ex}^2 \cdot \Theta_{Ex} + 2 \cdot \Phi \cdot \Psi \quad \text{(B.11)}$$

$$\chi_{Sa}^2 = \frac{\sigma_{Sa}^2}{Sa^2(1)} \ ; \ \chi_{Ex}^2 = \frac{\sigma_{Ex}^2}{Ex^2(1)} \ ; \ \Theta_{Sa} = \frac{K_S^2\, Sa^2(1)}{Pr^2} \ ; \ \Theta_{Ex} = \frac{K_E^2\, Ex^2(1)}{Pr^2} \quad \text{(B.12)}$$

In (B.12) χ_{Sa}^2 and χ_{Ex}^2 denote the squares of the coefficients of variation and volatilities σ_{Sa}^2, σ_{Ex}^2 of sales Sa and expenses Ex respectively.

$$Sa(1) = \frac{1}{K_S} \sum_{j=1}^{M} \sum_{i=1}^{N(j,Sa)} Sa(t_{i,j}) \ ; \ Ex(1) = \frac{1}{K_E} \sum_{j=1}^{M} \sum_{i=1}^{N(j,Ex)} Ex(t_{i,j}) \quad \text{(B.13)}$$

$$\Phi = \frac{K_S\, K_E\, Sa(1) Ex(1)}{Pr^2} \quad ; \quad \Psi = \frac{corr(S,E)}{Sa(1)\ Ex(1)} \quad \text{(B.14)}$$

$$corr(S,E) = E\left[Sa(t_{i,j}) Ex(t_{i,j})\right] - Sa(1) Ex(1) \quad \text{(B.15)}$$

We propose the square of the coefficient of variation χ_{Pr}^2 (B.11) as a lower bound of the uncertainty of macroeconomic profits Pr. However, the lack of direct data for calculations of (B.11-B.15) raises a problem of econometric approximations of the squares of the coefficients of variation χ_{Pr}^2, χ_{Sa}^2, χ_{Ex}^2, volatilities σ_{Sa}^2, σ_{Ex}^2, and correlations $corr(S,E)$ using available data.

References

Barrero, J.M., Bloom, N. and I. Wright, (2017). Short And Long Run Uncertainty, NBER WP23676, 1-46

Berkowitz, S.A., Dennis E. Logue, D.E. and E. A. Noser, Jr., (1988). The Total Cost of Transactions on the NYSE, The Journal Of Finance, 43, (1), 97-112

Bianchi, F., Kung, H. and M. Tirskikh, (2018). The Origins And Effects Of Macroeconomic Uncertainty, NBER WP25386, 1-56

Bloom, N. (2013). Fluctuations In Uncertainty, NBER WP19714, 1-29

Borovička, J. and L.P. Hansen, (2016). Term Structure Of Uncertainty In The Macroeconomy, NBER WP22364, 1-59

Cacciatore, M. and F. Ravenna, (2020). Uncertainty, Wages, And The Business Cycle, NBER WP27951, 1-37

Cole, R. (1969). Data Errors and Forecasting Accuracy, 47-82, in Ed. Mincer, J.A., Economic Forecasts and Expectations: Analysis of Forecasting Behavior and Performance, NBER,

Davidson, R. and J.G. MacKinnon, (2004). Econometric Theory and Methods, Oxford Univ. Press, NY, 766

Diebold, F.X. and R.S. Mariano, (1994). Comparing predictive accuracy, NBER TWP 169, 1-33

Diebold, F.X. (2012). Comparing Predictive Accuracy, Twenty Years Later: A Personal Perspective On The Use And Abuse Of Diebold-Mariano Tests, NBER, WP 18391, 1-16

Duffie, D. and P. Dworczak, (2018). Robust Benchmark Design, NBER WP 20540, 1-56

Fox, D.R., et al. (2019). Concepts and Methods of the U.S. National Income and Product Accounts. BEA, Dep. Commerce, US, Chapters 1-13, 1- 449

Ilut, C.L. and M. Schneider, (2022). Modeling Uncertainty As Ambiguity: A Review, NBER WP 29915, 1-42

Jurado, K, Ludvigson, S.C. and S. Ng, (2015). Measuring Uncertainty, Am. Ec. Rev., 105, 3, 1177–1216

Hansen, L.P. (2014). Uncertainty Outside And Inside Economic Models, NBER WP 20394, 1-56

Kumar, S., Gorodnichenko, Y. and O. Coibion, (2022). The Effect Of Macroeconomic Uncertainty On Firm Decisions, NBER WP30288, 1-71

Markowitz, H. (1952). Portfolio Selection, J. Finance, 7(1), 77-91

Mills, T.C and K. Patterson (Ed), (2009). Palgrave Handbook of Econometrics, v.2, Applied Econometrics, Palgrave MacMillan, 1377 p

Morgenstern, O. (1950). On the Accuracy of Economic Observations, Princeton Univ., 322

Olkhov, V., (2021). Theoretical Economics and the Second-Order Economic Theory. What is it?, MPRA WP120536, 1-13

Olkhov, V. (2022). Market-Based Asset Price Probability, MPRA WP120026, 1-18

Olkhov, V.(2023a). Market-Based Probability of Stock Returns, MPRA WP120117, 1-17

Olkhov, V. (2023b). Theoretical Economics as Successive Approximations of Statistical Moments, MPRA WP120763, 1-17

Olkhov, V. (2023c). Economic Complexity Limits Accuracy of Price Probability Predictions by Gaussian Distributions, MPRA WP120636, 1-23

Olkhov, V. (2024). Volatility Depends on Market Trade and Macro Theory, MPRA WP121221, 1-16

Reif, M. (2018). Macroeconomic Uncertainty and Forecasting Macroeconomic Aggregates, Leibniz Institute for Economic Research, Univ. Munich, WP 265, 1-36

Shiryaev, A.N. (1999). Essentials Of Stochastic Finance: Facts, Models, Theory. World Sc. Pub., Singapore. 1-852

Shreve, S. E. (2004). Stochastic calculus for finance, Springer finance series, NY, USA

Zarnowitz, V. (1967). An Appraisal of Short-Term Economic Forecasts, NBER, Occasional Paper 104, 1-140

Zarnowitz, V. (1978). On The Accuracy And Properties Of Recent Macroeconomic Forecasts, NBER, WP 229, 1-39

Zarnowitz, V. (1991). Has Macro-Forecasting Failed? NBER, WP3867. 1-38