The concept of separate needs in cardinal utility theory: the leisure-consumption choice

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THE CONCEPT OF SEPARATE NEEDS IN CARDINAL UTILITY THEORY: THE LEISURE-CONSUMPTION CHOICE[1]

by

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ABSTRACT

Two propositions are required to introduce separate needs into utility theory. Firstly, the shape of the utility function must represent the different stages of fulfilment of a need as experienced by a consumer: deprivation, subsistence, sufficiency, satiation, surfeit. The second proposes weak separability for the utilities of commodities fulfilling the same need, and strong separability for different needs.

A utility function, formed from the addition of two leaning-S-shaped, bounded cardinal utilities with satiation at infinity, is used to create an indifference curve map. Functional forms for the leisure-consumption choice are derived and their diagrams drawn – labour supply, consumption demand and their Engels curves.

The main outcomes are:

- Concave- and convex-to-the-origin indifference curves, (the former defining 'dysfunctional poverty), are separated by a straight-line indifference curve, BA, (the slope of which is defined by relative-intensities-of-need), identifiable as an absolute poverty line. It leads to disequilibrium in the derived functional forms.
- Each commodity responds as superior, inferior and even Giffen, in different areas of the convex-to-the-origin indifference curves. Their boundaries are reflected in envelope curves in the derived functional form diagrams.
- An individual's labour supply responses vary markedly according to three levels of unearned consumption/income, representing dysfunctional poverty (involuntary unemployment), functional poverty (working, but deprived of either leisure or consumption) and sufficiency.
- The reservation wage is a U-shaped function of endowments of unearned consumption.

The functional form's parameters have meaningful psychological interpretations. The concept of separate needs in utility offers a new dimension in labour supply theory.

(248 words)

JEL classification: D11, J22.

Keywords: leaning-S-shaped utility, additive utilities, absolute poverty line, disequilibrium, Giffen good, envelope curve, involuntary unemployment, functional poverty, reservation wage.

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I. INTRODUCTION

This paper is the second in a trilogy exploring the concept of separate needs in cardinal utility theory. In the first, (MPRA, Miller, paper no. 121455), the addition of two 'leaning-S-shaped' bounded utilities, with satiation at infinite consumption, created a utility function for the general case, from which a functional form was derived. This paper examines the effect of separate needs when applied to the leisure-consumption choice. The method used is that of using the functional forms to create diagrams for the leisure-consumption choice, from which their theoretical and policy implications are explored. The third will comprise an empirical analysis, comparing this functional form and its parameter estimates with those of the Linear Expenditure System for cross-section data on labour and consumption.

Two propositions are required to introduce the concept of separate needs into cardinal utility theory. The first must capture the individual's potential experiences associated with the fulfilment of a need, through deprivation (increasing marginal utility (MU)), subsistence, sufficiency (diminishing MU), satiation at either finite consumption, with the possibility of surfeit, or at infinite consumption. This will be captured by the *shape* of a bounded cardinal utility function of a commodity (good, service or event) (Figure 1). The second proposition provides a *separability rule* – weak, based on multiplying the utilities of commodities fulfilling the same need, and strong separability, based on adding utilities, for commodities fulfilling different needs.

In section II, the paper gives a brief summary of the two propositions, and the new notation and changes that are anticipated for the leisure-consumption choice compared with the general case. It covers the creation of the utility function used to create the indifference curve map for leisure and consumption (Figure 2). In section III, the theoretical effects of the indifference curve map are explored. Section IV presents the two functional forms for consumption and labour, from which Figures 3 and 4 are created, with sets of diagrams illustrating labour supply, consumption demand and their Engels curves, which are then examined. 'The next steps' in section V suggest ways in which the theory can be tested empirically and indicate some areas for further theoretical exploration. Conclusions are drawn in section VI, summarising the main predictions from the introduction of the concept of separate needs into cardinal utility theory, as applied to the leisure-consumption choice, including its potential policy implications.

The Appendix provides the relevant equations from the first paper, using the leisure-consumption notation, together with the derivation of the U-shaped reservation wage and its minimum value.

II. TWO PROPOSITIONS, A UTILITY FUNCTION AND A DEMAND EQUATION

Two statements or propositions are required for the introduction of the concept of separate human needs into (currently undifferentiated) cardinal utility theory. In the first, the *shape* of the utility function must be able to express the different stages of fulfilment of a need experienced by an individual. The second must provide a *separability rule* with respect to the classification of human needs.

- The first proposes a leaning-S-shaped, bounded cardinal utility for a single commodity representing the different stages of fulfilment of a need that could be experienced by a consumer: deprivation (increasing MU), subsistence (a point of inflection), sufficiency (diminishing MU), and either satiation at finite consumption with the possibility of a surfeit, or satiation at infinite consumption (see Figure 1)[2].

This first proposition is based on the ground-breaking, seminal work of Bernard M S Van Praag (1968), which has been developed and applied by The Leyden School (Van Herwaarden and Kapteyn, 1981; Hagenaars, 1986; Van Praag and Kapteyn, 1994).

Van Praag further recognised an intermediate state between cardinal and ordinal utility in the form of bounded cardinal utility. Bounded cardinal utility functions, leading to both a minimum level of utility and a maximum (satiation – at either finite or infinite consumption), enable interpersonal welfare comparisons to be made, thus partially solving the non-measurability problem of utility.

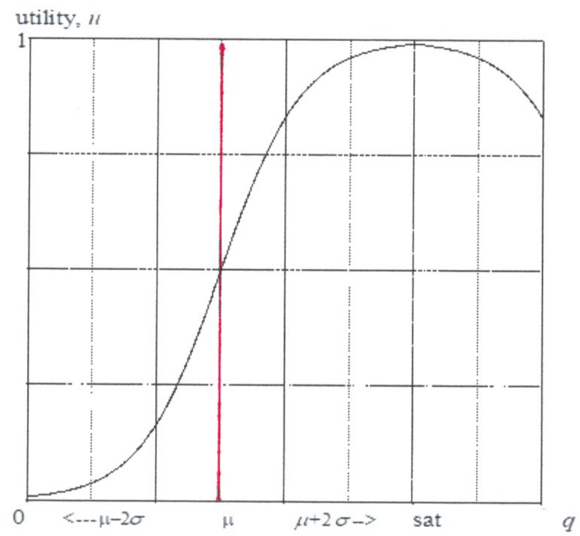
In the first paper, eight different models were identified for creating a leaning-S-shaped utility: a distribution function (DF) or a scaled down frequency function; with either a normal distribution (N) or a log normal (LN); together with either a 2-variable additive or an n-variable multiplicative model. Whereas Van Praag chose an 'n-Mult.LN-DF' for his work, the model chosen for this exercise is a '2-Add.N-DF', each incorporating satiation at infinity.

- The second proposes weak separability (multiplicativity) for the utilities of commodities fulfilling the same need, and strong separability (additivity) for the (currently undifferentiated) utilities fulfilling different needs.

Table 1 Notation for leisure and consumption

GENERAL CASE		**LEISURE-CONSUMPTION**	
Variables		**Variables**	
q_1	Consumption of good 1	q_0	Leisure
q_2	Consumption of good 2	q	Consumption
p_1	Price of good 1	w	Wage rate
p_2	Price of good 2	p	Price of consumption
C_1	Endowment of good 1	T (constant)	Maximum endowment of leisure
C_2	Endowment of good 2	C	Endowment of unearned consumption
		$lab = \mathrm{T} - q_0$	Labour hours
Parameters		**Parameters**	
μ_1	Subsistence of need 1	γ_0	Subsistence leisure
μ_2	Subsistence of need 2	γ	Subsistence consumption
σ_1	Intensity-of-need 1	σ_0	Intensity-of-need for leisure
σ_2	Intensity-of-need 2	σ	Intensity-of-need for consumption

[2] Figures 1 – 5 were created using Seppo Mustonen's SURVO software (1992).
They were created with the following parameters, γ_0 = 112, γ = 168, σ_0 = 31.5, σ = 63, T = 168. Further effects of the functional form can be explored using other values for the parameters.



KEY

u	utility
q	consumption
μ	subsistence consumption
sat	satiation consumption
σ	intensity-of-need for consumption

FIGURE 1 LEANING-S-SHAPED U-FN

There are further differences in addition to notation. Leisure, q_0, is limited to a maximum of T in a given period. Since labour is defined as *lab* = T – q_0, any diagrams featuring labour will appear to be back-to-front or upside-down compared with the general case. Leisure and consumption each have a subsistence parameter, γ_0 *and* γ, and an intensity-of-need parameter, σ_0 *and* σ, respectively.

The other main difference is the fact that the individual has an endowment of time, T, valued at wage rate, *w*, and an endowment of unearned consumption, *C*, valued at price, *p*. Thus, full income = T.*w* + *C.p*. Endowment, *C*, is measured on the vertical right-hand axis of the indifference curve map, where q_0 = T on the horizontal axis. These endowments also alter the Engels curve diagrams compared with those illustrating the general case.

A utility function was created using the 2-Add.N-DF utilities, as given by equation (1) in the Appendix and an indifference curve map was drawn using equation (2).

III. THE INDIFFERENCE CURVE MAP

Two leaning-S-shaped, bounded cardinal utilities, representing the needs for leisure, (that is, unwaged time), q_0, and for consumption, q, with satiation at infinity, subsistence parameters, γ_0 and γ, and intensity-of-need parameters, σ_0 and σ, respectively, were added together to form a utility function (see equation (1) in the appendix). Equation (2) was used to create an indifference curve map (see Figure 2).

The theoretical effects noted from the indifference map are that:

- The horizontal axis represents the individual's leisure, q_0, constrained by a maximum endowment of time, T, ($0 \leq q_0 \leq$ T).
- The left-hand axis represents the individual's consumption, q.
- The map is divided into four quadrants by the two subsistence parameters, γ_0 and γ.
- The left-hand and lower quadrants represent deprivation with respect to leisure and consumption respectively.

- The map is further divided by a straight-line indifference curve, BA, through point E, (at co-ordinates γ_0, γ), with negative slope, σ/σ_0, creating an intercept on the right-hand vertical axis at A.
- σ/σ_0 provides a measure of the consumer's *relative intensities-of-need*, in this case of leisure over consumption. The smaller the value of σ_0, the greater the slope of the straight-line indifference curve (measured at corner A). If $\sigma/\sigma_0 > 1$, then leisure is valued more highly than consumption. The greater the intensity-of-need, the more highly valued leisure becomes, compared with consumption.
- The straight-line indifference curve BA separates the concave-to-the-origin indifference curves (defining 'dysfunctional poverty') closer to the origin in the rhomboid B0TA, from the convex-to-the-origin indifference curves.
- The rhomboid B0TA represents extreme deprivation in one or other dimensions of need. It is a non-solution space, except for non-tangential 'choices', (corner solutions) on the horizontal and right-hand axes.
- The straight-line indifference curve can be identified as an Absolute Poverty Line and point A on the right-hand axis as a survival endowment.

- The convex-to-the-origin indifference curves can be divided into areas of ultra-superior, superior normal, inferior normal and inferior-Giffen responses for each need.

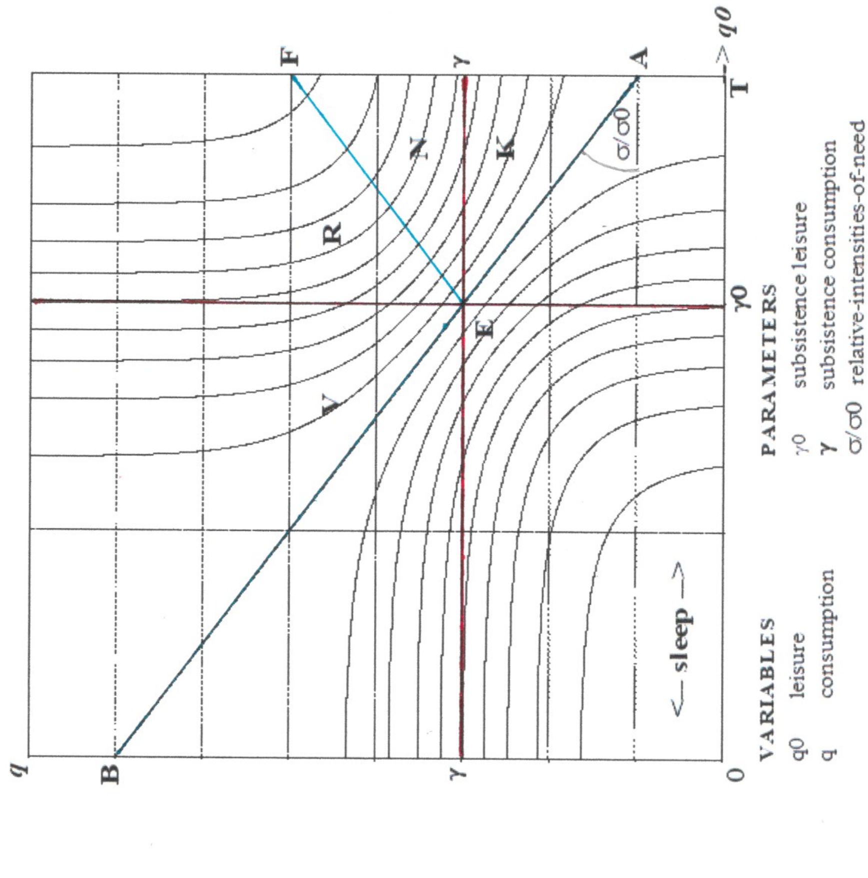


FIGURE 2 INDIFFERENCE CURVE MAP FOR LEISURE-CONSUMPTION CHOICE

It can be shown that in the top right-hand quadrant of Figure 2, both commodities are experienced as superior normal goods, (additivity and positive diminishing marginal utilities always yield superior normal characteristics). With additive utilities, the two goods are net substitutes for each other.

Inferior normal and inferior-Giffen responses occur for need that is experienced as sufficient but is combined with moderate deprivation in another, as anticipated by Berg (1987). Leisure responds as inferior in the triangular area marked as K in Figure 2, bounded by EA, the right-hand axis and the parameter γ, (Dougan, 1982; Silberberg *et al*,1984). That the Giffen experience is associated with a straight-line indifference curve, adjacent to a triangular non-solution space, was anticipated by Davies (1994).

In area V, in that part of the left-hand border where the indifference curves are convex-to-the-origin, the consumer is deprived of leisure, (with increasing MU), and, following Hirschleifer's terminology (1976, chap.4), leisure is here termed an ultra-superior good. Kohli (1985) calls this experience an 'anti-Giffen good', but 'anti-inferior' would be more accurate. The individual experiences *consumption* as inferior in area V.

- The line EF is the locus of points where the slope of the indifference curves is parallel to line BA, creating an intercept on the right-hand vertical axis at F. At point F, the individual's utility is close to his/her satiation utility.
- The line EF divides into two the rectangular area where both goods are experienced as superior normal, marked R and N on Figure 2. Areas N and K will yield a tangential point for a utility-maximising individual when facing a low real wage rate, that is, $w/p < \sigma/\sigma_0$.

- The right-hand vertical axis at q_0 = T represents an endowment of unearned consumption, *C*, granted to an individual by his/her family, community, education and other public welfare services, and via state benefits. A negative value of *C* represents a net debt.
- Faced with an endowment of unearned consumption which is less than his/her *survival level*, $0 \leq C < A$, an individual facing a low wage rate is unable to act as utility-maximising economic agent and will be trapped in dysfunctional poverty (corner solutions of involuntary unemployment[3]).
- Faced with an endowment of unearned consumption which is less than his/her *survival level*, $0 \leq C < A$, an individual can only act as a utility-maximising economic agent, if s/he faces very high real wage rates, in which case s/he can work long hours, attaining sufficient consumption, but being deprived of leisure. This combination of work-life balance can be described as 'functional poverty'.
- With an endowment of unearned consumption lying between survival level, A, and subsistence, $A < C < \gamma$, the individual can act as an economic agent and make choices but, if facing a low wage rate, could also experience functional poverty, deprived of consumption on a low wage.
- An endowment greater than the individual's subsistence level, $C > \gamma$, always leads to superior responses.

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[3] Keynes (1936) predicted the existence of involuntary unemployment.

The convexity assumption of neoclassical demand theory seems to be based on the statement that 'maximising utility will always yield a solution for an indifference curve that is convex to the origin (for positive prices), and thus indifference curves must be everywhere convex to the origin'. This is true for all multiplicative utility functions, and, in fact, the indifference curves for *both* additive and multiplicative utility functions that include only diminishing MU are, indeed, everywhere convex to the origin. Empirical evidence has rejected the assumption of additive utilities for leisure and consumption within the Stone-Geary (Linear Expenditure System) model (Blundell, 1988). This is to be expected. Being based only on diminishing marginal utilities, it is impossible to distinguish between multiplicative and additive separability. However, the introduction of a 'leaning-S-shaped' bounded cardinal utility function, together with separate explorations of multiplicative and additive utilities in the context of human needs, provides a broader understanding of utility.

IV. SETS OF FOUR DIAGRAMS FOR DEMAND, SUPPLY AND ENGELS CURVES

Let q_0 and q be leisure and consumption respectively.
w and p are the wage rate and the price of consumption.
γ_0 and γ, are the subsistence parameters for leisure and consumption respectively.
σ_0 and σ are their corresponding intensity-of-need parameters.
T is the individual's endowment of time, valued at wage rate, w, and
C is the (varying) endowment of unearned consumption, valued at price, p.
$C.p$ represents unearned income, including state benefits.
Negative values of $C.p$ represent a debt.
Labour, lab = T – q_0, represents hours worked for pay. Earnings = $lab.w$.
Labour can convert leisure (Sen's capabilities (1991)) into consumption at the rate of w/p per hour.

Full income is T.w + $C.p$. Survival income is $\gamma_0.w + \gamma.p$.
Supernumerary income Z = (T.w +c.p) – ($\gamma_0.w + \gamma.p$) = (T – γ_0).w + ($C - \gamma$).p.

Any budget constraint that passes through the co-ordinate (γ_0,γ) is a survival income, including the budget that is co-incidental with the straight-line indifference curve, BA.
The equation for BA is $q_2 = \mu_2 - (\sigma_2/\sigma_1).(q_1 - \mu_1)$.
An endowment of unearned consumption at point A on the right-hand axis is a *survival* endowment.

The linear budget is expressed as

$$q = (\mathrm{T} - q_0).w/p + C. \qquad (3a)$$

The utility function was maximised subject to the budget constraint, using the Lagrangian multiplier method, producing the optimality condition, equation (4a).

$$\left(\frac{q-\gamma}{\sigma}\right)^2 - \left(\frac{q_0-\gamma_0}{\sigma_0}\right)^2 = ln\left(\frac{\sigma_0.w}{\sigma.p}\right)^2 \qquad (4a)$$

Equation (4a) was then used to derive the functional forms, equations (9a) and (10a), for the demand and labour supply functions.

The consumption version for q, using $v = p/w$ and $h = \sigma_0/\sigma$, and substituting $Z/w = (T - \gamma_0) + (C - \gamma).p/w$ into equation (9) in the Appendix, is:

$$q = \gamma + \frac{[(T-\gamma_0)+(C-\gamma).v].v \; - h.\sqrt{\left[\left((T-\gamma_0)+(C-\gamma).v\right)^2 + (v^2-h^2).\left(\sigma^2.2.ln\left(\frac{v}{h}\right)\right)\right]}}{(v^2-h^2)} \qquad (9a)$$

The labour supply equation is obtained, using $x = w/p$ and $b = \sigma/\sigma_0$, and substituting $Z/p = (T - \gamma_0).w/p + (C - \gamma)$, and labour, $lab = T - q_0$ into equation (9) in the Appendix, yielding equation (10a).

$$lab = (T - \gamma_0) - \frac{[(T-\gamma_0).x+(C-\gamma)].x \; - b.\sqrt{\left[\left((T-\gamma_0).x+(C-\gamma)\right)^2 + (x^2-b^2).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{(x^2-b^2)}. \qquad (10a)$$

The two (very non-linear) derived functional forms for the demand equations, (labour supply, $lab = T - q_0$ and consumption, q, dependent on the real wage rate, w/p, and an endowment of unearned consumption, C), were used to create sets of four diagrams – labour supply curves, consumption demand and their associated Engels curves (see Figures 3 and 4). The straight-line indifference curve causes a disequilibrium[4] in each derived functional form, when both $w/p = \sigma/\sigma_0$, and $C = A$.

This disequilibrium can lead to an apparent instability in behaviour; that is, large reactions can occur in response to small changes in relative prices. If w/p were to waver slightly around σ/σ_0, then behaviour could appear to oscillate markedly.

In Figure 3, figures 3a to 3c and 3e to 3g are presented with a dependent variable on the vertical axis, and an independent variable on the horizontal axes, to aid visual comparisons. Figures 3d and 3h are diagrams 3c and 3g with their axes are reversed, giving the more familiar presentation of demand and supply curves.

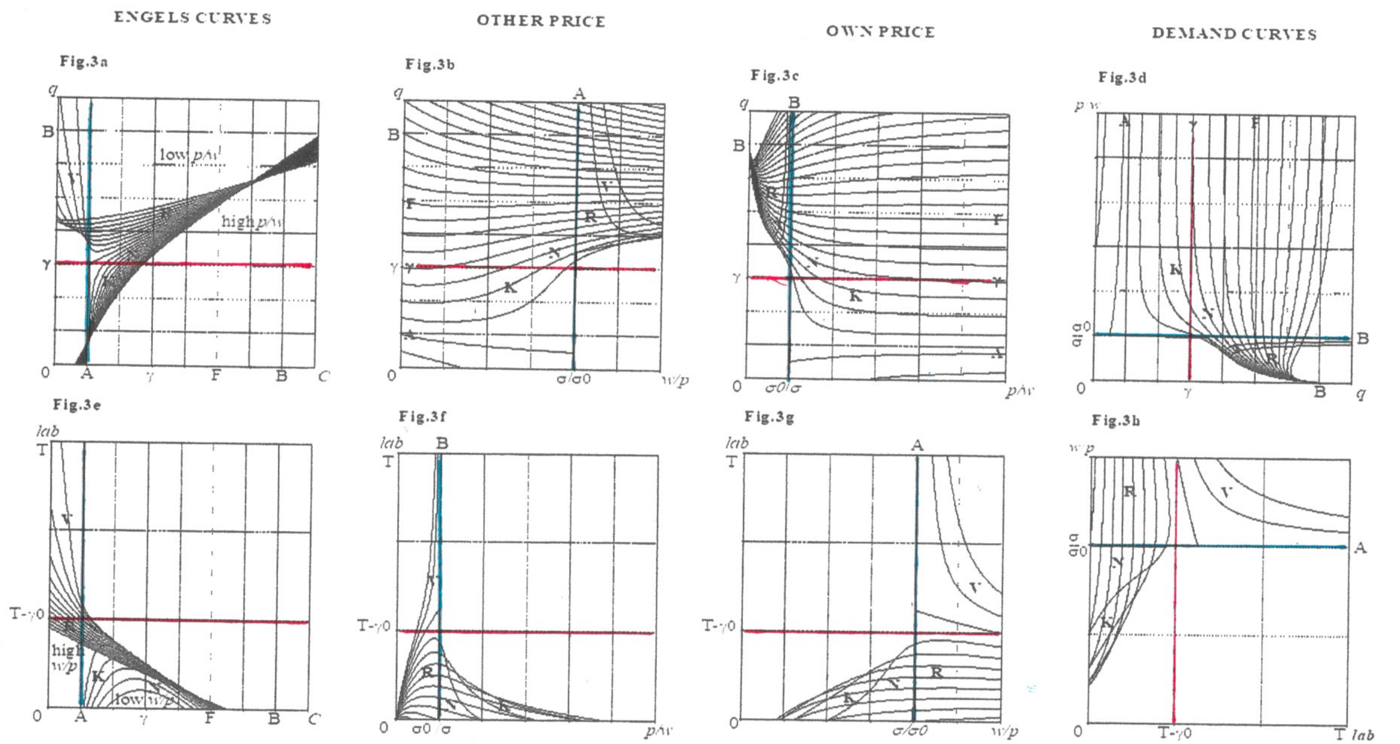
In Figure 4, the four derived functional form diagrams, consumption demand (CE), labour supply (LS), consumption Engels (CE) and labour Engels (LE) are combined by re-orientating their axes to show how q and lab relate to their two independent variables, w/p and C. Measured on the q-axis in the CD and CE diagrams, consumption, $q = lab.(w/p) + C$.

The four diagrams in Figure 4 are examined in turn to describe how an individual responds to his/her endowments, C, given his/her wage rates, w/p. Each diagram is divided into four quadrants by a subsistence parameter and by the disequilibrium when both $w/p = \sigma/\sigma_0$ and $C = A$.

Consumption Engels (CE) diagram in Figure 4.

- Consumption, q, is measured on the horizontal axis, and C on the vertical.
- At $C = 0$, s/he is unable to consume, ($q = 0$), represented by the origin.

[4] Keynes (1936) predicted the existence of disequilibrium in the labour market.



KEY dependent variables: *lab* labour; q consumption

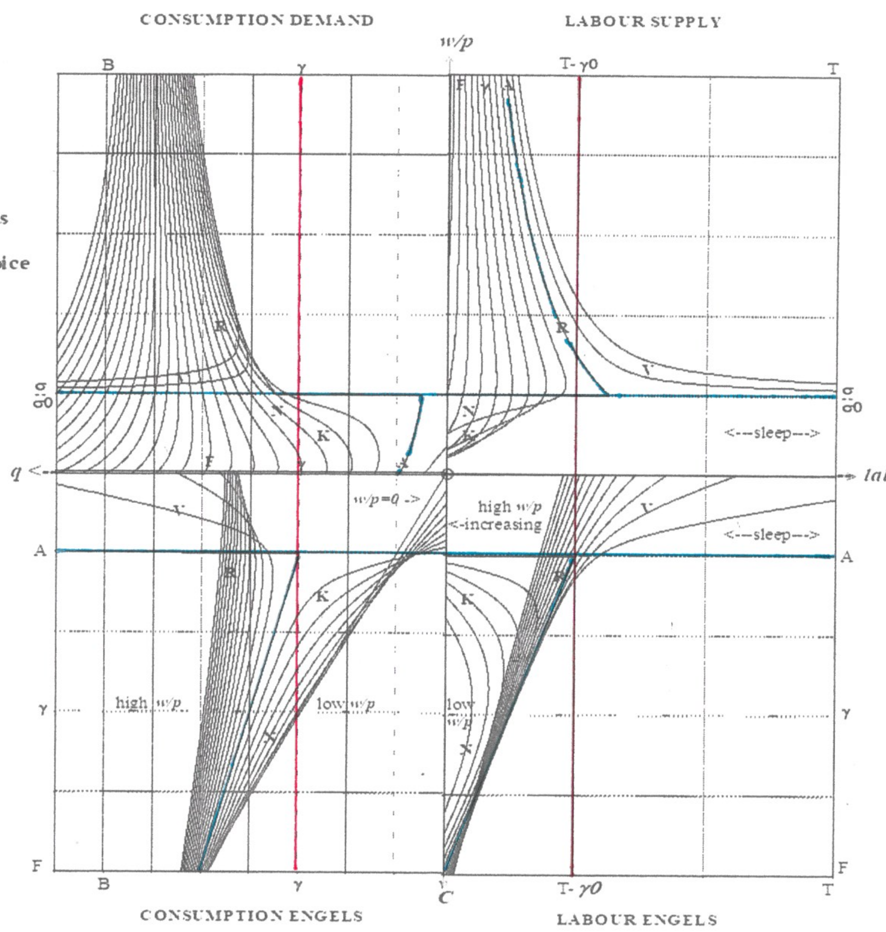
independent variables: w/p real wage rate; C endowments of unearned consumption

$\gamma 0$ subsistence leisure
γ subsistence consumption
σ $\sigma 0$ relative-intensity-of-need

T is the maximum time endowment
K, N, R and V relate to indifference map
BA is the straight-line indifference curve

FIGURE 3 DERIVED FUNCTIONAL FORMS

FIGURE 4.
Derived functional forms for the consumption-leisure choice



KEY:

VARIABLES:

$q0$	leisure
lab = T- $q0$	labour
q	consumption
w/p	real wage rate
C	unearned endowments

PARAMETERS:

$\gamma 0$	subsistence leisure
γ	subsistence consumption
$\sigma 0$	leisure intensity-of-need
σ	consumption intensity-of-need

T is maximum time endowment
A is survival endowment
F: utility is close to satiation level
K, N, R and V relate to indifference map

- For $0 < C <$ A, an individual facing a low wage rate ($w/p < \sigma/\sigma_0$) is unable to enter the labour market (*lab = 0*). S/he is in a state of dire dysfunctional poverty, surviving on inadequate unearned endowments, $q = C$, illustrated in the top right-hand quadrant.
- As C increases further, his/her consumption curve for $w/p = 0$ is represented by a straight line, but s/he will be in a state of deprivation until $C = \gamma$.

- At $C = 0$, an individual will only be able to consume if s/he commands a high wage rate, $w/p > \sigma/\sigma_0$.
- Similarly for 0 < C < A, but it leaves him/her deprived of leisure. (illustrated in the top, left-hand quadrant here, and marked as area V in all the diagrams).
- An **envelope curve** can be identified in the top-left-hand quadrant as C increases to survival level A, which can be identified as the boundary between inferior normal and inferior-Giffen responses for consumption, all within area V.

- When C = 0, **the polarisation of consumption in society** is at its greatest**.**
- **Disequilibrium** occurs when both C = A and $w/p = \sigma/\sigma_0$.
- By C = A, the difference in consumption between a high-waged worker and a non-worker is reduced.
- The difference in consumption between high- and low-waged individuals reduces further as C increases.

Labour Engels (LE) diagram in Figure 4.

- The LE diagram shows most clearly that, for $C <$ A, an individual will work only if s/he commands high wages, and s/he will work long hours, deprived of leisure.

- For $C <$ A, an individual facing a low wage, ($w/p < \sigma/\sigma_0$), is unable to work for pay.
- His/her **involuntary unemployment**, *lab* = 0, is represented on the vertical axis.

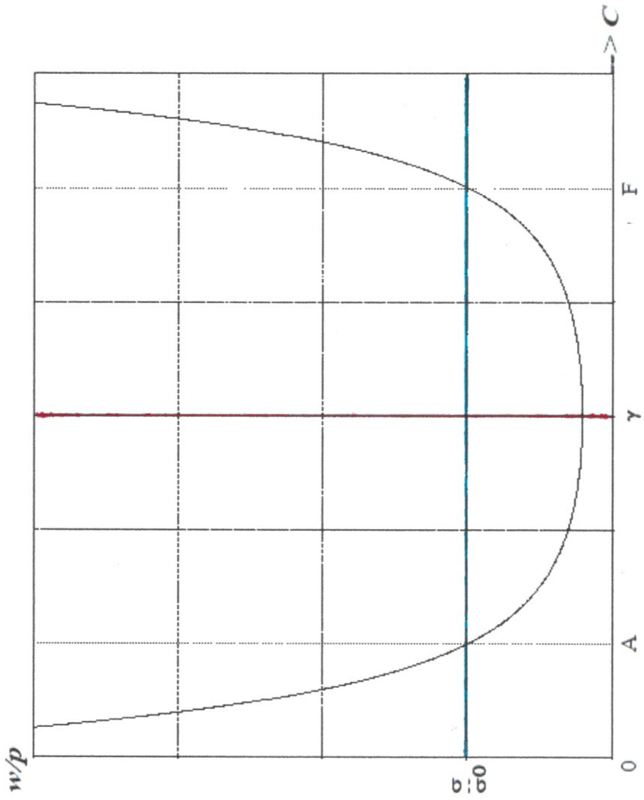
- However, if an individual faces both $w/p = \sigma/\sigma_0$ and C = A, the dramatic impact of the **disequilibrium,** caused by the straight-line indifference curve, could result in his/her offering a range of hours of work, but s/he is most likely to offer his/her maximum labour supply while avoiding being deprived of leisure, $lab = T - \gamma_0$.

- For $C \geq$ A, the **reservation wage** (RW) (below which it is not worth working for pay, (*lab* = 0), because s/he would be worse off in terms of utility), plays a clear role with respect to low pay. It can be located as the intercepts of the low-pay curves on the vertical axis, C.

- For C = A and $w/p = \sigma/\sigma_0$, the labour Engels curve is initially horizontal and then becomes a straight line sloping downwards ending with *lab* = 0 at C = F.

- For $C >$ A and $w/p < \sigma/\sigma_0$, each labour Engels curve is bow-shaped, beginning and ending with *lab* = 0, illustrating equation (17) in the Appendix, which shows that RW is a U-shaped function of endowments, C, (see Figure 5), with a minimum when $C = \gamma$, after which RW increases as C continues to increase.

- The reservation wage, *RW*, is a U-shaped function of unearned consumption, C, and is at its lowest when $C = \gamma$, and $w/p = (\sigma/\sigma_0).\sqrt{[\exp(-(T - \gamma_0)/\sigma_0)]}$, (with *lab* = 0).



w/p real wage-rate
C endowments of unearned consumption
γ subsistence consumption
σ/σ0 relative-intensity-of-need parameter
F at point F, utility is near satiation level
A survival endowment

FIGURE 5. RESERVATION WAGE

- Further Increases in endowments, such that *C* > F, fail to have any further significant effect on labour supply.

- The **boundary** between leisure's inferior normal and inferior-Giffen responses is not easily identifiable in the lower left-hand quadrant of the LE diagram, but it runs in a bow-shaped curve concave to the origin, from close to the intersection of *lab* = T – γ_0, and *C* = A, to the intersection of *lab* = 0 and *C* = γ.

Labour Supply (LS) diagram in Figure 4.

- For the individual facing a high wage, *C* = 0 is represented by the right-hand-most labour supply curve, stretching over the upper quadrants, with deprivation of leisure indicated in the right-hand quadrant.
- As *C* increases to A, the labour supply curves shift to the left.

- For 0 < *C* < A, an individual facing a low wage, (*w/p* < σ/σ_0), who thus is unable to enter the labour market, is in a state of dire dysfunctional poverty.
- His/her **involuntary unemployment**, (*lab* = 0), is represented on the vertical axis of the LS diagram.

- It is not until C = A that an individual facing a low wage rate, (*w/p* < σ/σ_0), is able to offer hours of work, but the effect can be dramatic, as illustrated in the lower, left-hand quadrant.
- When C = A, a **disequilibrium** occurs for any individual facing *w/p* = σ/σ_0, and the amount of labour to be offered could vary such that 0 < *lab* < T hours.

- For *C* > A, the most dramatic effect occurs for an individual facing a wage rate just below *w/p* = σ/σ_0, who wishes to offer his/her maximum hours without being deprived of leisure, (or just above that number of hours). But his/her labour supply curve quickly becomes very elastic (flat) representing inferior responses when s/he becomes deprived of consumption, as in area K of the indifference curve map.

- For a slightly higher level of C, as the wage rate falls, an individual will offer some hours of work with a superior response initially, until s/he hits the deprivation of consumption threshold, when very elastic inferior responses can be observed again.

- Shifts in the labour supply curves in response to the increases in endowments, *C*, reduce the maximum amount of labour offered.

- The **reservation wage**, *RW*, is represented by the intercepts of the labour supply curves on the vertical *w/p* axis, (where *lab* = 0).
- *RW* is at its lowest when *C* = γ.
- An **envelope curve** can be identified in the lower left-hand quadrant, representing the boundary between leisure's inferior and superior responses in areas K and N of the indifference curve map respectively.

- The top left-hand quadrant represents the superior normal responses observed in area R of the indifference curve map.

Consumption Demand (CD) diagram of Figure 4.

- A ‘consumption demand’ diagram features own price, *p/w*, (see Figs 3c and 3d in Figure 3 above). This ‘consumption demand’ in Figure 4 features ‘other price’, *w/p*, (see figure 3b in Figure 3 above). When *w/p* is low, *p/w* is high.
- Consumption, *q*, is measured on the horizontal axis, with its ‘other price’, *w/p*, on the vertical axis.
- His/her consumption for different levels of *C* is represented by the intercepts of the consumption demand curves on the horizontal axis, but they are likely to be affected by real price levels, *p/w*, which will be relatively high when *w/p* is low. S/he is only released from deprivation when $C = q = \gamma$.

- For C < A and $w/p < \sigma/\sigma_0$, an individual is unable to enter the labour market and is in a state of dire dysfunctional poverty.

- In contrast, the consumption demand when $C = 0$ and $w/p > \sigma/\sigma_0$ is represented by the upper-most of the two backward bending curves.

- An **envelope curve** can be identified in the upper left-hand quadrant, as wage rates decrease to level $w/p = \sigma/\sigma_0$, and this forms the boundary between inferior and superior responses for consumption, that is, between areas V and R of the indifference curve map respectively.

V. THE NEXT STEPS

The first step will be an empirical test of the two (very non-linear) functional forms derived in the Appendix (based on equations (9a) and (10a)) using cross-section consumption and labour data, and non-linear regression analysis. Ideally, the separability assumption would be tested for pairs of commodities by comparing both additive and multiplicative versions of the same functional forms, and testing between them, when suitable functional forms have been developed.

In the meantime, the first stage will be to compare the results, including estimates of the four parameters, for this functional form, with those of the Linear Expenditure System (Stern, 1986). The strong separability assumption allows the demand equations for any two commodities fulfilling separate needs to be estimated independently of any other commodity fulfilling another need.

The second stage will be to estimate the four parameters, with their realistic psychological interpretations, for different groups of people by age, gender, cohabitation status, number of dependents, etc.

The functional form could also be used to test whether either housing and/or insurance, (representing satisfiers of the need for protection and security), is additive with other types of consumption. It could also help to explain health inequalities and wellbeing. Similarly, education might be regarded as the most appropriate satisfier of the need for understanding.

Are different types of addictions additively or multiplicatively separable? If suitable data were available, could estimates of the relative intensity of need parameters be used to test Maslow’s ‘Hierarchy of Needs’ hypothesis (1943) without resorting to assumptions about lexicographic orderings of preferences? How many ‘needs’ can be identified?

Clearly, there is also scope for theoretical developments based on the concept of needs, including the following:

- What are its implications for the secondary worker hypothesis, where her endowment of unearned income includes a proportion of the primary worker's income?
- What would be its implications, if any, for general equilibrium analysis or optimal taxation theory?
- What are the properties of the contract curves derived from Edgeworth boxes, when one party is deprived of one or other of the needs for which commodities are being traded? Might it help to define exploitation?
- What are the implications for individuals experiencing satiation at finite consumption in at least one, but not all, needs?
- Could this functional form provide useful insights if used as a production function?

This functional form could also be useful in poverty and inequality studies, and in tax and benefit policy analysis.

VI. CONCLUSIONS

General conclusions

The indifference curve map is divided into four by the subsistence parameters, while the convex-to-the-origin part of the indifference curve map can be further divided into four. The derived functional form diagrams are also divided into four quadrants by the dependent variable's own subsistence parameter and the conjunction of C = A and $w/p = \sigma/\sigma_0$.

The straight-line indifference curve causes a **disequilibrium** in each of the four derived functional forms, at C = A, when $w/p = \sigma/\sigma_0$.

When an individual is sufficiently fulfilled in one need, but moderately deprived in another, s/he could experience commodities fulfilling that fulfilled need as **inferior normal**, or even **inferior-Giffen**, in response to changes in income and prices. Both leisure and consumption can be experienced as inferior.

When an individual is deprived of either leisure or consumption, his/her labour and consumption responses are much **more elastic**, compared with when s/he is not deprived.

Envelope curves on the demand/supply diagrams of the derived functional forms indicate the boundaries between superior and inferior responses, and those on the Engels curve diagrams indicate the boundaries between inferior-normal and inferior-Giffen responses.

The needs approach can be tested empirically by comparing this derived functional form with other labour supply functions using cross-section data. Its four parameters are both estimable using non-linear estimation and have realistic psychological interpretations as committed leisure and consumption, (subsistence parameters), and relative-intensity-of-need parameters. The parameters are likely to vary for different groups of individuals within and between populations. The strong separability allows the parameters of each pair of needs to be estimated independently of other needs.

The concept of separate needs in cardinal utility theory could have implications for the second worker hypothesis, for optimal taxation theory and for general equilibrium analysis, in addition to being useful in poverty and inequality studies and for tax and benefit policy analysis.

Policy implications

With zero endowments, society is extremely **polarised** into high-waged individuals (who work long hours and are deprived of leisure but not of consumption), and the rest who suffer **involuntary unemployment**, being unable to enter the labour market, who are not just deprived of consumption, but have nothing, trapped in dysfunctional poverty, with all its physical and mental ill-health outcomes.

The straight-line indifference curve, BA, is the ultimate **Absolute Poverty Line**, between an individual being trapped in dysfunctional poverty and being able to make choices as an economic agent.

The **reservation wage** is a U-shaped function of endowments of unearned consumption, C, with a minimum at $C = \gamma$, the consumption subsistence level, and has a significant effect on the supply of labour by low-paid workers. The reservation wage is created by a series of combinations of w/p when *lab* = 0, with endowments, C. The RW can be likened to a gateway for lower-waged workers, but with a high step that prevents all but the highest of lower-waged workers, to join the labour market. An increase in endowments would appear to reduce this step incrementally until $C = \gamma$.

An endowment equivalent to, or greater than, ***survival level**, C $\geq$ A, could reduce the* difference in consumption experienced by workers and non-workers. At the disequilibrium caused by the straight-line indifference curve, when $w/p = \sigma/\sigma_0$ and C = A, a potential low-waged worker is not only enabled to join the labour market but is able to offer his/her maximum labour supply response. A greater than survival level endowment, $C \geq$ A, could have a dramatic beneficial effect on the wellbeing of the population.

The reservation wage can also help to explain why, for an endowment of $C \geq$ A, the introduction of a **National Minimum Wage**, (if greater than the wage when $C = \gamma$), can lead to an increase in employment, contrary to expectations.

If endowments were to increase to consumption **subsistence** level, $C = \gamma$, labour supply would reduce, but the divisions in society would also be further reduced. Although those who do not enter the labour market will never be as well-off in terms of consumption as workers, at least they could achieve their subsistence level, $q = \gamma$, and would no longer be deprived. As endowments increase, $C > \gamma$, labour supply reduces, and at C > F, the effect of C become insignificant.

The introduction of the concept of separate needs with respect to leisure and consumption opens a new dimension in labour supply theory.

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APPENDIX

The functional form for the ‘2-Add.N-DF’ utility function for the leisure-consumption choice.

The 2-Add.N-DF utility function is defined as the sum of two distribution functions for the normal distribution (which have no statistical connotations in the present context), representing consumption, q_i, $-\infty < q_i < +\infty$, i = 1, 2, where the i’th commodity fulfils the i’th need. The sum is scaled equally such that utility, u, lies between 0 and 1.

q_0 is leisure (unwaged time) ($0 \leq q_0 \leq T$, where T is the individual’s maximum endowment of leisure).
q is consumption, ($q \geq 0$).
$\gamma_0, \gamma \geq 0$ are subsistence parameters representing ’survival level’ thresholds, and
$\sigma_0, \sigma > 0$ are the intensity-of-need parameters for commodities q_0 and q.

The ‘2.Add.N-DF’ **utility function** is given as:

$$u(q_0, q) = \tfrac{1}{2} F_1(q_0) + \tfrac{1}{2} F_2(q)$$

$$u(q_0, q) = \frac{1}{2}\int_{-\infty}^{q_0} \frac{\exp\left[-(R_1-\mu_1)^2/2\sigma_1^2\right]}{\sigma_1.\sqrt{2\pi}}\, dR_1 + \frac{1}{2}\int_{-\infty}^{q} \frac{\exp\left[-(R_2-\mu_2)^2/2\sigma_2^2\right]}{\sigma_2.\sqrt{2\pi}}\, dR_2 \qquad (1)$$

where u, $0 \leq u \leq 1$, is utility,

The indifference curves

Using the logistic distribution, which is similar to the normal distribution (Johnson and Kotz, 1970; 244),

$$P(t) = \frac{e^t}{[1 + e^t]^2} = \frac{e^{-t}}{[1 + e^{-t}]^2}.$$

an indifference curve map (Figure 2) was created, adjusted for location and scale, using equation (2) to draw the indifference curves:

$$q = \gamma - \{\log [(0.5 * \text{bracket}) / (u * \text{bracket} - 0.5) - 1] \} / (1.82/\sigma_2), \qquad (2)$$

where u is utility and bracket = $(1 + \exp (- (1.82/\sigma_1) * (q_0 - \gamma_0)))$.

The budget equation

Let T and C be endowments of leisure and unearned consumption, of q_0 and q, valued at the wage rate w and price of consumption p respectively.

Full income, $M = T.w + C.p$, where M, w and $p \geq 0$
Survival income = $\gamma_0.w + \gamma_0.p$.
Supernumerary income, $Z = M$ – survival income

$$Z = (T.w + C.p) - (\gamma_0.w + \gamma.p) = (T - \gamma_0).w + (C - \gamma).p.$$

The linear budget constraint is $M = T.w + C.p = q_0.w + q.p$.

$$q = (T - q_0).w/p + C. \tag{3a}$$

Any budget constraint that passes through the co-ordinates (γ_0, γ) is a survival income, (including the budget that is co-incidental with the straight-line indifference curve, BA). The equation for the straight-line indifference curve BA, through point E, (γ_0, γ), with negative slope, σ/σ_0, and intersecting the right-hand axis at A, is $q = \gamma - (\sigma/\sigma_0).(q_0 - \gamma_0)$.

The utility function, together with the budget constraint, represents the structural form of the model.

The optimality condition

Maximising u(q_0,q) in equation (1), subject to the budget constraint, equation (3a), using the Lagrangian multiplier method, leads to the optimality condition:

$$\left(\frac{q-\gamma}{\sigma}\right)^2 - \left(\frac{q_0-\gamma_0}{\sigma_0}\right)^2 = ln\left(\frac{\sigma_0.w}{\sigma.p}\right)^2 \tag{4a}$$

The optimality condition describes the *income-consumption locus* for a given price ratio, w/p, on the indifference curve map.

The boundary between superior and inferior responses for q_0

By expressing equation (4) in terms of q_0, and differentiating with respect to q, the boundary between q_0 being superior or inferior is expressed as equation (5a):

$$\frac{dq_0}{dq} = \frac{\left(\frac{q-\gamma}{\sigma}\right).\left(\frac{\sigma_0}{\sigma}\right)}{\sqrt{\left[\left(\frac{q-\gamma}{\sigma}\right)^2 - 2.ln\left(\frac{\sigma_0.w}{\sigma.p}\right)\right]}} = 0. \tag{5a}$$

By setting dq_0/dq = 0, in equation (5a), the locus for the threshold between q_0 being superior and its being inferior on the indifference curve map, is found to be coincidental with $q = \gamma$, for $q_0 > \gamma_0$. This is the boundary between areas K and N on the indifference curve map. Thus, in area K, q_0 will react to price changes as an inferior good.

Similarly, the boundary for q is $q_0 = \gamma_0$, for $q > \gamma$, which is the boundary between areas labelled V and R on the indifference curve map. Thus, in area V, q will react to price changes as an inferior good in area V.

An equation for the boundary between q_0 being inferior normal and inferior-Giffen is not yet available.

The demand and labour supply curve equations

Let $x = w/p$ (relative prices). b = σ/σ_0 (relative intensities-of-need).

Supernumerary income, $Z = (T.w + C.p) - (\gamma_0.w + \gamma.p) = (T - \gamma_0).w + (C - \gamma).p$.

The budget equation is: $T.w + C.p = q_0.w + q.p$.

$$q = (T - q_0).x + C.$$

Substituting for $q = (Tw + Cp - q_0.w)/p$, from the budget constraint,
and for $Tw + Cp = Z + \gamma_0.w + \gamma.p$ from the supernumerary expenditure equation,
into optimality condition, equation (4a), yields an '**implicit demand equation**' (8a):

$$\left(\frac{q-\gamma}{\sigma}\right)^2 - \left(\frac{q_0-\gamma_0}{\sigma_0}\right)^2 = ln\left(\frac{\sigma_0.w}{\sigma.p}\right)^2 \quad (4a)$$

$$\left[\frac{\frac{Z}{p} - (q_0-\gamma_0).x}{\sigma}\right]^2 = \left[\frac{(q_0-\gamma_0)}{\sigma_0}\right]^2 + ln\left[\frac{x}{b}\right]^2 \quad (8a)$$

which is a quadratic equation in $(q_0 - \gamma_0)$, which is solved using the negative square root, yielding demand equation (9) for commodity, q_0:

$$q_0 = \gamma_0 + \frac{\left(\frac{Z}{p}\right).x \ - b.\sqrt{\left[\left(\frac{Z}{p}\right)^2 + (x^2-b^2).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{(x^2-b^2)} \quad (9)$$

The consumption version for q, using $v = p/w$ and $h = \sigma_0/\sigma$, and substituting $Z/w = (T - \gamma_0) + (C - \gamma).p/w$ into equation (9), is:

$$q = \gamma + \frac{[(T-\gamma_0)+(C-\gamma).v].v \ - h.\sqrt{\left[\left((T-\gamma_0)+(C-\gamma).v\right)^2 + (v^2-h^2).\left(\sigma^2.2.ln\left(\frac{v}{h}\right)\right)\right]}}{(v^2-h^2)} \quad (9a)$$

The labour supply equation can be obtained by substituting $Z/p = (T - \gamma_0).w/p + (C - \gamma)$, and labour, $lab = T - q_0$ into equation (9), yielding equation (10a).

$$lab = T - \gamma_0 - \frac{[(T-\gamma_0).x+(C-\gamma)].x \ - b.\sqrt{\left[\left((T-\gamma_0).x+(C-\gamma)\right)^2 + (x^2-b^2).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{(x^2-b^2)} \quad (10a)$$

This demonstrates that the dependent variables, q and lab, are non-linear functions of the independent variables, 'own' relative price, ($x = w/p$ or $v = p/w$), and an endowment of unearned consumption, C, with parameters, γ_0, γ, σ_0 and σ.

Equation (9) is the negative root to the solution to a quadratic equation (8a) in ($q_0 - \gamma_0$) and gives two solutions. The equations for q_0 and q are symmetric and homogeneous of degree zero in w, p and Z. The two demand equations, for q_0 and q_2, represent the reduced form of the model.

When both $(T.w + C.p) \geq Z$, and the budget line is parallel to the straight-line indifference curve, and thus $x^2 = b^2$, and using the negative root, equation (9) simplifies to

$$q_0 = \gamma_0 + \frac{\left(\frac{Z}{p}\right).x - \mathrm{b}.\sqrt{\left(\frac{Z}{p}\right)^2}}{(\mathrm{x}-\mathrm{b}).(x+b)}.$$

$$q_0 = \gamma_0 + \frac{Z/p}{(x+b)} \qquad (11a)$$

The **envelope curve on the labour supply curves,** (clearly visible on Figs 3g and 3h and Figure 4), is given by:

$$lab = (\mathrm{T} - \gamma_0) - \sigma_0\sqrt{\left[+2.\,ln\left(\frac{b}{x}\right)\right]}\text{, for x < b, } (w/p < \sigma/\sigma_0). \qquad (13a)$$

An equation for the envelope curve on the Engels curves, associated with the border between an inferior normal and an inferior-Giffen response is not yet available.

The envelope curve on the consumption demand equation, (clearly visible on Figs 3c and 3d), is given by:

$$q = \gamma + \sigma\sqrt{\left[+2.\,ln\left(\frac{h}{v}\right)\right]}\text{, for v < h, } (p/w < \sigma_0/\sigma). \qquad (13b)$$

The reservation wage

Let $x = w/p$ and $b = \sigma/\sigma_0$. Let $a = (T - \gamma_0)$ and $g = (C - \gamma)$. $Z/p = (a.x + g)$
The reservation wage, $x = w/p$, is a function of unearned endowments, C. It can be obtained by setting $lab = 0$ in equation (10a) and rearranging it in terms of $x = w/p$, as follows:

$$lab = (\mathrm{T} - \gamma_0) - \frac{[(T-\gamma_0).x+(C-\gamma)].x \; - b.\sqrt{\left[\left((T-\gamma_0).x+(C-\gamma)\right)^2 + (x^2-b^2).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{(x^2-b^2)} = 0. \qquad (10a)$$

$$\frac{a.(x^2-b^2)-[a.x+g].x}{(x^2-b^2)} = \frac{+\,b.\sqrt{\left[(a.x+g)^2 + (x^2-b^2).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}}{(x^2-b^2)} \qquad (10a)$$

Square both sides of the equation.

$$\left[a.(x^2 - b^2) - [a.x^2 + g.x]\right]^2 = b^2.(a.x + g)^2 + b^2.(x^2 - b^2).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)$$

$$a^2.(x^2 - b^2)^2 - 2.a.(x^2 - b^2).x.[a.x + g] + x^2.[a.x + g]^2 - b^2(a.x + g)^2$$
$$-b^2.(x^2 - b^2).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right) = 0$$

$$a^2.(x^2 - b^2)^2 - 2.a.(x^2 - b^2).x.[a.x + g] + (x^2 - b^2).[a.x + g]^2$$
$$-b^2.(x^2 - b^2).\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right) = 0$$

Divide through by $(x^2 – b^2)$

$$a^2.(x^2 - b^2) - 2.a.x.[a.x + g] + [a.x + g]^2 - b^2.\left(\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right) = 0$$

This is a quadratic in (a.x + g), where (a.x + g) = $(–b \pm \sqrt{(b^2 – 4ac)})/2a$

and a = 1; b = –2.a.x; c = $a^2.(x^2 – b^2) – b^2.\ \sigma_0^2.2.\ln(x/b)$.

$$(a.x + g) = \left[2.a.x - \sqrt{\left[4.a^2.x^2 - 4\left(a^2.(x^2 - b^2) - b^2.\sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}\right]/2$$

$$(a.x + g) = a.x - b\sqrt{\left[\left(a^2 + \sigma_0^2.2.ln\left(\frac{x}{b}\right)\right)\right]}$$

$$(a.x + g) = a.x - \sigma\sqrt{\left[\left(\left(\frac{a}{\sigma_0}\right)^2 + 2.ln\left(\frac{x}{b}\right)\right)\right]}$$

$$\frac{g}{\sigma} = -\sqrt{\left[\left(\left(\frac{a}{\sigma_0}\right)^2 + 2.ln\left(\frac{x}{b}\right)\right)\right]}$$

Square both sides:

$$\left(\frac{g}{\sigma}\right)^2 = \left(\frac{a}{\sigma_0}\right)^2 + 2.ln.\left(\frac{x}{b}\right)$$

$$\left(\frac{g}{\sigma}\right)^2 - \left(\frac{a}{\sigma_0}\right)^2 = \ln\left(\frac{x}{b}\right)^2.$$

$$\left(\frac{C-\gamma}{\sigma}\right)^2 - \left(\frac{T-\gamma_0}{\sigma_0}\right)^2 = \ln\left(\frac{x}{b}\right)^2. \quad (16)$$

Rearranging (16) in terms of x gives:

$$x = b.\sqrt{exp\left[\left(\frac{C-\gamma}{\sigma}\right)^2 - \left(\frac{T-\gamma_0}{\sigma_0}\right)^2\right]}\ . \qquad (17)$$

The reservation wage, x = w/p, is a U-shaped function of C, symmetric about C = γ for the '2-Add.N-DF' model. See Figure 5.

When C = γ, $$x = b.\sqrt{exp\left[-\left(\frac{T-\gamma_0}{\sigma_0}\right)^2\right]}\ . \qquad (18)$$

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