Public Services, Welfare, and Growth under Baumol's Cost Disease

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Abstract

This study presents a three-sector growth model that consists of manufacturing, private services, and public services, and examines the relationship between sectoral compositions and the tax rate. We identify an optimal tax rate that maximizes instantaneous utility. The optimal tax rate increases as manufacturing productivity increases, though it converges to a certain level that is less than unity.

Keywords: public services; cost disease; optimal tax rate; growth disease

JEL Classification: H21; H40; H50; O14; O41

1 Introduction

The purpose of this study is to investigate structural change in industries, focusing particularly on the expansion of public services such as education and medical services. Why do public services expand? Does the tax burden increase alongside their expansion? How does the expansion of public services affect economic growth?

In developed economies, the service sector typically expands. However, current literature dealing with the relationship between service-sector expansion and economic growth mainly focuses on private services; the role of public services has been relatively overlooked. For a comprehensive understanding of this relationship, public services

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must be considered. Accordingly, in this study, we define the expansion of services as an expansion of the employment share of private and/or public services.

The structure of an economy is generally divided into three sectors, agriculture, manufacturing, and services. The employment share of agriculture in developed economies is very small, that of manufacturing decreases, and that of services increases. Why does the employment share of services expand? How does the expansion of service employment affect economic growth? The seminal work of Baumol (1967) answers these two related questions. He builds a two-sector model with manufacturing and private services, and shows that if the growth rate of manufacturing productivity is higher than that of private services and if demand for private services is price-inelastic, then the employment share of private services increases. Moreover, he reveals that the economic growth rate declines with the expansion of service employment share. Nordhaus (2008) calls this "Baumol's growth disease."[1] The productivity growth differential between the two sectors leads to an increase in the relative price of private services, which is usually called "Baumol's cost disease."[2] However, the explanation of the service economy in Baumol's model only includes the expansion of private services.

Thus, it ignores the role of government-supplied public services, such as education, medical services, and nursing care. If productivity growth of public services is stagnant, their employment share might increase.

Figure 1 displays the employment shares of manufacturing, private services, and public services in the Japanese economy (data used in this Introduction are explained in detail in Section 4.2). From 2000 to 2020, the employment share of manufacturing fell from about 20% to 15%, that of private services remained roughly constant, and that of public services rose from about 13% to 20%. Therefore, the overall increase in the employment share of services can be fully ascribed to the expansion of public services.

Figure 2 displays the employment shares of different public services. Medical services and health and hygiene remained roughly constant, public administration and education decreased, and social insurance and social welfare and nursing care increased.[3] Hence, the expansion of public services can be ascribed to the expansion of social insurance and social welfare and nursing care.

Government spending is decomposed into public investment such as building roads

1) Nordhaus (2008) conducts empirical tests to reveal that the US economy experiences Baumol's growth disease and cost disease. Hartwig (2011) also conducts empirical analysis for the EU economy.

2) For cost disease, see also Baumol (2012).

3) Jones (2002) builds a model to explain why US health expenditures as a share of GDP have risen. His explanation is that technological progress in medicine and a Medicare-like transfer program jointly increase the size of health expenditures.

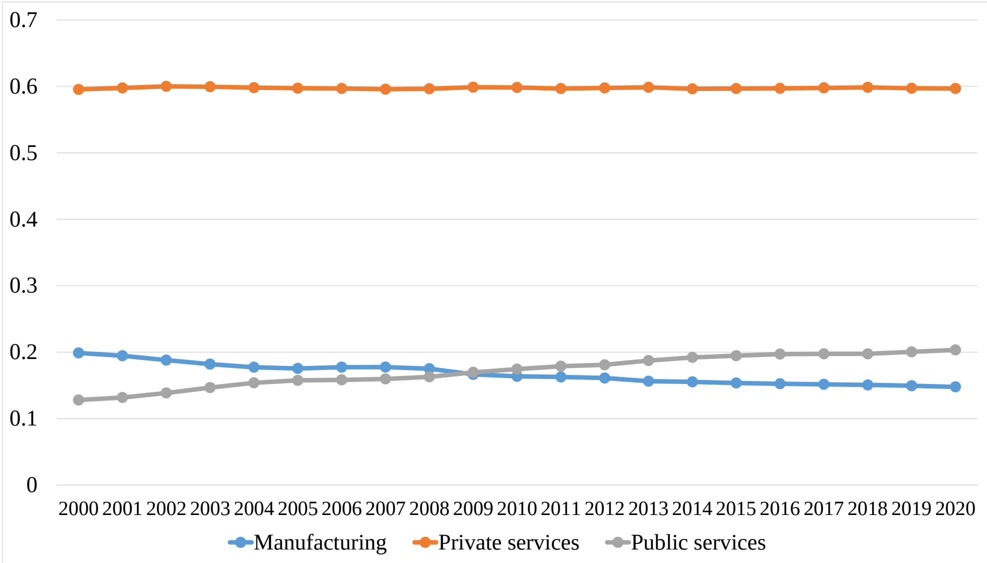

Figure 1: Employment share of each sector: Japan 2000-2020

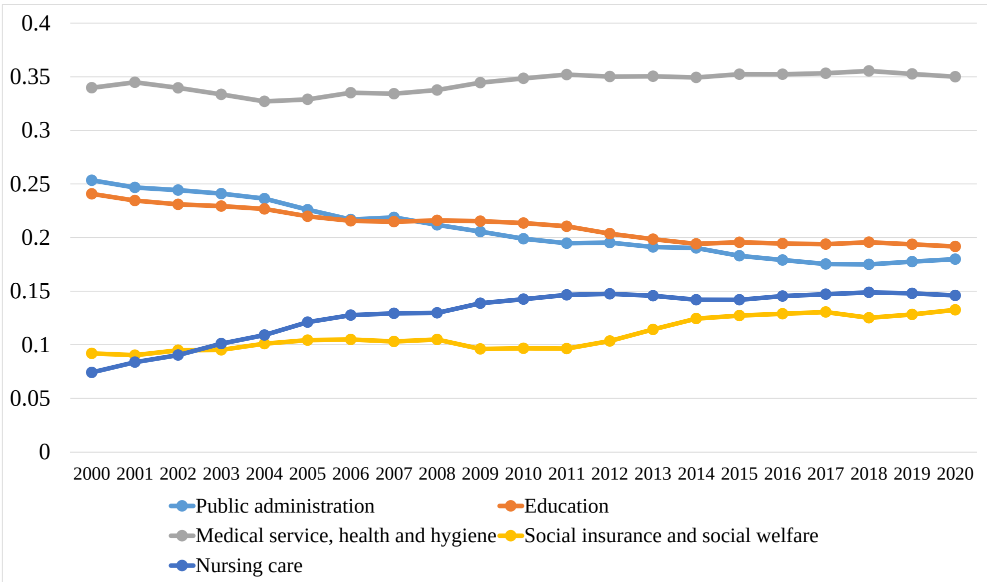

Figure 2: Employment share of different components of the public sector

and dams, changes in public inventories, and government consumption. Government consumption is further decomposed into individual and collective consumption expenditures. Social security benefits, such as medical services and nursing, account for a significant portion of individual consumption expenditure. Collective consumption expenditure mainly comprises public goods such as diplomacy, national defense, and police. Figure 3 shows that individual consumption expenditure increased while collective consumption expenditure remained roughly constant. This upward trend of individual consumption expenditure corresponds to the expansion of the employment shares of social insurance and social welfare and nursing care.

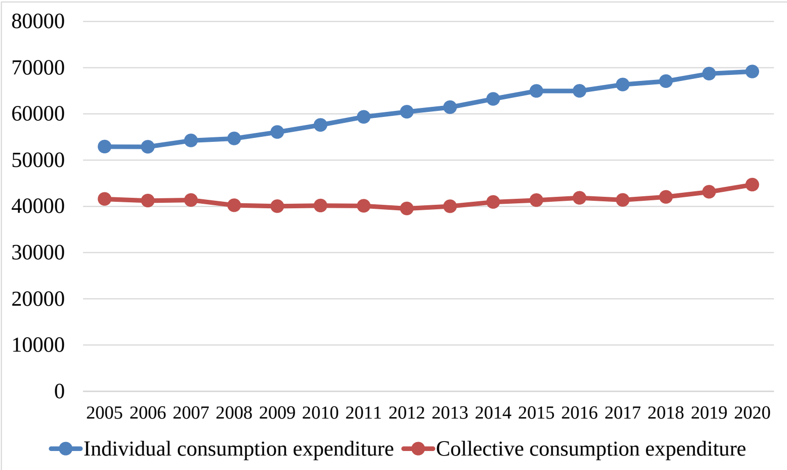

Figure 3: Government consumption expenditure

Productivity growth in public services is stagnant and demand for them is price-inelastic. Accordingly, some researchers adopt Baumol's model to public services based on the assumption that the employment share of public services increases endlessly. Furthermore, since supply of public services is financed by taxation, the expansion of public services necessarily causes a rise in the tax rate, in turn increasing citizens' tax burden and potentially threatening the welfare state.

However, we should be careful when applying Baumol's explanation to public services. Public services are supplied by the government, the source of revenue is tax income, and hence the expansion of public services is constrained by tax revenue. Therefore, no matter how high demand for public services becomes, public services will not expand unless the government raises the tax rate to increase their supply.

We take Lindbeck (2006), Van der Ploeg (2007), and Mann and Pecorino (2023) as examples of studies examining the expansion of public services. They state that the expansion of public services under Baumol's cost disease is problematic. Contrastingly, Andersen (2016) states that it is not so serious.

Specifically, Lindbeck (2006) and van der Ploeg (2007) argue that maintaining financial stability is difficult under Baumol's cost disease. Lindbeck (2006) states that increased consumption of tax-financed human services would be expected to require gradually higher tax rates—possibly until the top of the Laffer curve has been reached, when further tax financing becomes technically impossible. Mann and Pecorino (2023) build a two-sector model with private and government sectors, and conclude that the public sector will grow monotonically with the productivity differential between sectors and the tax rate will be pushed to the top of the Laffer curve over time.

Taking a similar stance to this study, Andersen (2016) builds a general equilibrium three-sector model with manufacturing, private services, and public services, and reveals that even under Baumol's cost disease, (i) the public sector does not necessarily increase endlessly, and (ii) the tax rate that maximizes instantaneous utility does not necessarily increase endlessly, because the optimal tax rate is not necessarily dependent on manufacturing productivity.[4] His analysis is also based on the key idea that tax revenue constrains the expansion of public services.

Based on the approach of Andersen (2016), this study presents a general equilibrium model and investigates the expansion of public services and the existence of a welfare-maximizing optimal tax rate.

Andersen (2016) obtains his results by conducting comparative statics. In contrast, we focus on the transitional dynamics under which all sectors' productivities grow at constant rates. We reveal that the employment share of public services increases but not endlessly. Moreover, the growth rate of per capita real GDP continues to decline with the expansion of service sectors, that is, Baumol's growth disease is present. Furthermore, we show the existence of an optimal tax rate that maximizes instantaneous utility; this optimal rate increases with time and approaches a certain level that is less than unity.

The remainder of this paper is organized as follows. Section 2 builds a two-good model with manufacturing and public services, and explains that the supply of public services is constrained by tax revenue. Section 3 presents a three-good model with manufacturing, public services, and private services, and reveals that the essence of the two-good model does not change if the number of sectors increases. It also clarifies that the utility-maximizing tax rate increases with an increase in manufacturing productivity though it converges to a certain level. Section 4 introduces leisure into the

4) Additionally, Andersen and Kreiner (2007) reveal that even under cost disease, the welfare state is sustainable and there is scope for Pareto improvement.

utility function, and reveals that even in this case, the expansions of the public sector and utility-maximizing tax rate have upper limits. Moreover, it provides some numerical examples using data from the Japanese economy. Finally, Section 5 concludes the study.

2 Two-good model with manufacturing and public services

This section presents a simple two-good model with manufacturing and public services, and examines the relationship between Baumol's cost disease and the expansion of public services employment share. We first introduce Lindbeck's (2006) argument before presenting the present study's position.

2.1 Lindbeck's argument

Lindbeck (2006) states that the expansion of public services leads to an increase in the tax rate that supplies it, threatening the stability of public finances. Lindbeck's (2006) appendix briefly explains the mechanism. Let m and g denote manufacturing and public services, respectively. Labor is the sole factor of production. Each sector's production function is linear in labor inputs.

$$Y_m = A_m L_m, \tag{1}$$

$$Y_g = A_g L_g, \tag{2}$$

where Y_i denotes output, L_i is employment, and A_i is labor productivity.

Let ω_i denote the unit labor cost of each sector. Then, we obtain

$$\omega_m = \frac{w}{A_m}, \tag{3}$$

$$\omega_g = \frac{w}{A_g}, \tag{4}$$

where w denotes the wage rate. Suppose that labor is free to move between sectors. Then, w is equalized between the two sectors. From this, the relative cost of public services is given by

$$\frac{\omega_g}{\omega_m} = \frac{A_m}{A_g}. \tag{5}$$

As long as A_m increases faster than A_g, the relative cost of public services increases. This is known as Baumol's cost disease.

Supply of public services is financed by taxation. Let $\tau \in (0, 1)$ be the income tax rate and suppose that government's budget is always balanced. Then, we obtain

$$\underbrace{\omega_g Y_g}_{\text{Government expenditure}} = \underbrace{\tau(\omega_m Y_m + \omega_g Y_g)}_{\text{Government revenue}}$$
$$\Longrightarrow \tau = \frac{\omega_g Y_g}{\omega_m Y_m + \omega_g Y_g} = \frac{L_g}{L_m + L_g}. \tag{6}$$

This means that public services employment share is equal to the tax rate. Lindbeck (2006) interprets equation (6) as determining the tax rate, that is, the right-hand side (RHS) determines the left-hand side (LHS). If so, an increase in public services employment share increases the tax rate. Equation (6) can be rewritten as

$$\tau = \frac{1}{1 + \frac{Y_m}{Y_g} \cdot \frac{A_g}{A_m}}. \tag{7}$$

Baumol (1967) assumes that the ratio of the outputs of manufacturing and private services is constant. Following Baumol's assumption, Lindbeck (2006) assumes that Y_m/Y_g is constant. Then, as long as A_m increases faster than A_g, τ also increases. For this reason, under Baumol's cost disease, the tax rate inevitably increases, threatening the sustainability of the welfare state.

As stated above, Lindbeck (2006) interprets equation (6) as showing that public services employment share determines the tax rate. From this view, the RHS of equation (6) is already determined by some means. However, Lindbeck (2006) does not explain how the employment share is determined.

2.2 Our stance

Lindbeck's (2006) argument is that employment share is determined first and then the tax rate is determined. Contrastingly, our stance is that the tax rate is given first which then determines public services employment share.

Like Lindbeck (2006), we consider a two-good economy with manufacturing and public services; we also employ the same production functions and assume that government's budget is always balanced.

We consider labor market clearing. Let L be exogenous labor supply. The labor market clearing condition is given by

$$L_m + L_g = L. \tag{8}$$

Substituting equation (8) into equation (6), we obtain

$$\frac{L_g}{L} = \tau. \tag{9}$$

In this equation, RHS determines the LHS: public services employment share is determined by the tax rate. From this, the employment share of manufacturing is given by $L_m/L = 1-\tau$. Consequently, the expansion of public services is unrelated to Baumol's cost disease. Moreover, as long as the tax rate is constant, public services employment share never increases.

3 Three-good model with manufacturing, private services, and public services

As the preceding section shows, in a simple two-good model with manufacturing and public services, each sector's employment share is independent of productivities and consumer preferences. Therefore, we cannot obtain an expansion of services. This section extends the preceding model into a three-good model by introducing private services. Moreover, to capture structural change in industries, we introduce a non-homothetic consumer preference.

If we intend to follow Baumol (1967) and examine structural change, we need to introduce price-inelastic demand or non-homothetic preference.[5] For ease of analysis, we use a Stone–Geary type utility function, a typical example of non-homothetic preference (Geary, 1950; Stone, 1954).

$$\max\ u = (c_m - \bar{c}_m)^{\alpha}(c_s + \bar{c}_s)^{\beta} c_g^{1-\alpha-\beta}. \tag{10}$$

$$\text{s.t. } p_m c_1 + p_s c_2 = (1-\tau)w, \tag{11}$$

where c_i denotes consumption, and $\bar{c}_i$ is a minimum level of consumption of c_i. For simplicity, we assume the labor supply is normalized as $L = 1$. Many previous studies assume that $\bar{c}_m > 0$ and $\bar{c}_s > 0$ (e.g., Isçan, 2010).

5) Baumol (1967) assumes that the output (i.e., consumption) ratio between manufacturing and private services is constant. This means that the utility function of consumers takes the Leontief form, in which case derived demand functions are unrelated to price changes.

By solving the utility maximization problem, we obtain each demand function as follows:

$$c_m = \frac{\alpha}{\alpha+\beta}\left[(1-\tau)\frac{w}{p_m} - \bar{c}_m + \frac{p_s}{p_m}\bar{c}_s\right] + \bar{c}_m, \tag{12}$$

$$c_s = \frac{\beta}{\alpha+\beta}\left[(1-\tau)\frac{w}{p_s} - \frac{p_m}{p_s}\bar{c}_m + \bar{c}_s\right] - \bar{c}_s, \tag{13}$$

$$c_g = A_g\tau. \tag{14}$$

Solving the goods market clearing condition $Y_i = c_i$ $(i = m, s, g)$, we obtain each sector's employment share.

$$L_m = \frac{\alpha}{\alpha+\beta}\left[(1-\tau) + \frac{\bar{c}_s}{A_s}\right] + \frac{\beta}{\alpha+\beta}\frac{\bar{c}_m}{A_m}, \tag{15}$$

$$L_s = \frac{\beta}{\alpha+\beta}\left[(1-\tau) - \frac{\bar{c}_m}{A_m}\right] - \frac{\beta}{\alpha+\beta}\frac{\bar{c}_s}{A_s}, \tag{16}$$

$$L_g = \tau. \tag{17}$$

From equation (17), similarly to the two-good model, public services employment share is equal to the income tax rate. As long as labor demand is equal to labor supply, that is, we impose the full employment condition $\sum_i L_i = 1$, this property does not change in the three-good model.

From equations (15) and (16), under Baumol's cost disease, L_m decreases while L_s increases; this mirrors Baumol's (1967) explanation. We assume the case of Baumol's cost disease: A_m grows faster thatn A_s. Then, the employment shares of manufacturing and private services approach the following values, respectively:

$$\lim_{t\to\infty} L_m = \frac{\alpha}{\alpha+\beta}(1-\tau), \tag{18}$$

$$\lim_{t\to\infty} L_s = \frac{\beta}{\alpha+\beta}(1-\tau). \tag{19}$$

We investigate the existence of a tax rate that maximizes instantaneous utility. Taking logarithms of both sides, that is, $\log u$, we examine whether there exists a τ such that $\partial \log u/\partial\tau = 0$.

$$\frac{\partial \log u}{\partial \tau} = -\frac{\alpha}{(1-\tau) - \bar{c}_m + \frac{A_m}{A_s}\bar{c}_s} - \frac{\beta}{(1-\tau) - \frac{A_s}{A_m}\bar{c}_m + \bar{c}_s} + \frac{1-\alpha-\beta}{\tau} = 0 \tag{20}$$

Since we have $\partial^2 \log u/\partial\tau^2 < 0$, there exists a τ^* that maximizes instantaneous utility.

Equation (20) is a quadratic equation of τ, which leads to

$$\tau^2 - \underbrace{\left[(1-\beta)(1+\bar{c}_s) + (1-\alpha)(1-\bar{c}_m) + (1-\alpha)\,\frac{A_m}{A_s}\,\bar{c}_s\right]}_{\equiv b>0}\tau + \underbrace{(1-\alpha-\beta)(1+\bar{c}_s)\left(1-\bar{c}_m+\frac{A_m}{A_s}\,\bar{c}_s\right)}_{\equiv c>0} = 0. \tag{21}$$

Solving equation (21), we obtain two real positive roots. One root is larger than unity while the other is less than unity. Accordingly, τ^* corresponds to the smaller one.

$$\tau^* = \frac{b-\sqrt{b^2-4c}}{2}. \tag{22}$$

Notably, from (21), we see that A_m and A_s do not affect τ^* if $\bar{c}_s = 0$. This suggests that whether consumer preference is non-homothetic affects the result.[6)]

The optimal tax rate τ^* depends on A_m; it is an increasing function of A_m. Suppose that A_m increases at a constant rate $g_{A_m} > 0$ and that the other productivities are constant, that is, $g_{A_s} = g_{A_g} = 0$. Then, τ^* increases as A_m increases. Nevertheless, τ^* does not necessarily increase endlessly. Let $A_m(t) = A_m(0)e^{g_{A_m}t}$ be manufacturing productivity at t. Substituting $A_m(t)$ into τ^* and increasing time t, we find that τ^* increases and then approaches a constant value.[7)] This is shown in Figure 4.

We note that τ^* depends on the relative productivity $A_m(t)/A_s(t)$. Accordingly, even if we let $A_s(t)$ increase over time, that is, $A_s(t) = A_s(0)e^{g_{A_s}t}$, we also obtain the above property as long as $g_{A_s} < g_{A_m}$. Indeed, Figure 4 is generated with $g_{A_m} = 0.03$ and $g_{A_s} = 0.01$.[8)]

Proposition 1. *The optimal tax rate that maximizes instantaneous utility depends on the relative productivity between manufacturing and private services; it increases with an increase in relative productivity. Nevertheless, it approaches a certain level that is less than unity.*

Andersen (2016) proves that an increase in manufacturing productivity increases the optimal tax rate if goods in the utility function are complements while their relationship

6) The restriction $\bar{c}_s > 0$ affects the result more than the restriction $\bar{c}_m > 0$. For this reason, the literature dealing with structural change often assumes that $\bar{c}_m = 0$ and $\bar{c}_s > 0$ (Kongsamut *et al.*, 2001; Buera and Kaboski, 2009)

7) In equation (22), in the long run, b and $\sqrt{b^2-4c}$ grow at a common rate. Moreover, we have $b > \sqrt{b^2-4c}$. Consequently, τ^* approaches a constant value.

8) The other parameters are chosen as $\alpha = 0.2$, $\beta = 0.7$, $\bar{c}_m = 0.1$, $\bar{c}_s = 0.1$, $A_m(0) = 2$, and $A_s(0) = 1$.

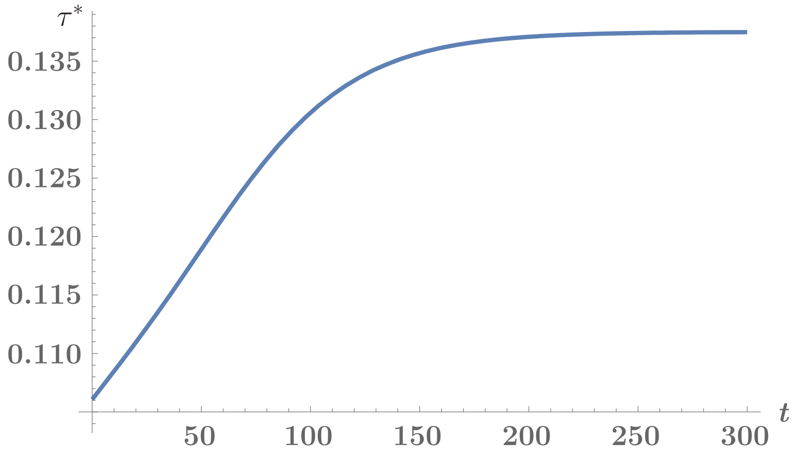

Figure 4: Convergence of optimal tax rate

becomes ambiguous if goods are substitutes. Since we use the Stone–Geary utility function, our three goods are substitutes. Accordingly, our analysis corresponds to the case where an increase in manufacturing productivity has an ambiguous effect on the optimal tax rate in Andersen's (2016) argument. Notably, unlike our study where the utility function is specified, Andersen (2016) examines a general case without specifying this function. Furthermore, Andersen (2016) does not discuss the limit of τ^* as his results are obtained through the use of comparative statics.

In our model, if $\bar{c}_m = \bar{c}_s = 0$, then the optimal tax rate is given by $\tau^* = 1 - \alpha - \beta$. Accordingly, the optimal tax rate does not depend on productivities. This property is also explained in Andersen (2016): if the utility function is homothetic and weakly separable, the optimal tax rate is independent of manufacturing productivity. He also states that the property whereby the optimal tax rate is dependent on manufacturing productivity depends on the shape of the utility function and that this property does not necessarily arise from the productivity growth differential per se. When the utility function takes the Cobb–Douglas form (i.e., $\bar{c}_m = \bar{c}_s = 0$), it satisfies homotheticity and weak separability; hence, the optimal tax rate does not depend on manufacturing productivity.

Next, we examine Baumol's growth disease. Nominal GDP in this model economy is given by

$$\begin{aligned}\text{GDP} &= p_m Y_m + p_s Y_s + \omega_g Y_g \\ &= wL_m + wL_s + wL_g = w. \end{aligned} \tag{23}$$

Since government expenditure is a component of GDP, $\omega_g Y_g$ is included in this calculation. Each sector's value added share leads to

$$\frac{p_m Y_m}{\text{GDP}} = L_m, \tag{24}$$

$$\frac{p_s Y_s}{\text{GDP}} = L_s, \tag{25}$$

$$\frac{\omega_g Y_g}{\text{GDP}} = L_g = \tau. \tag{26}$$

This means that each value added share is equal to each employment share.[9)]

We want to obtain the growth rate of per capita real GDP. To consider economic growth in real terms, we must eliminate the effect of price variations. The per capita real GDP growth rate g_y can be defined as follows:

$$\begin{aligned} g_y &= \frac{p_m \dot{Y}_m + p_s \dot{Y}_s + \omega_g \dot{Y}_g}{p_m Y_m + p_s Y_s + \omega_g Y_g} \\ &= L_m g_{A_m} + L_s g_{A_s} + L_g g_{A_g} \\ &= (1 - \tau - L_s) g_{A_m} + L_s g_{A_s} + \tau g_{A_g}. \end{aligned} \tag{27}$$

Note that we assume $L = 1$. Accordingly, the growth rate of per capita GDP equals that of GDP. As already analyzed, under Baumol's cost disease, L_s increases while L_m decreases, which makes the coefficients of g_{A_m} and g_{A_s} decrease and increase, respectively. Then, as long as $g_{A_m} > g_{A_s}$, the per capita real GDP continues to decrease as the private services employment share increases. Nordhaus (2008) calls this result Baumol's growth disease.

The limit of the growth rate of per capita real GDP is given by

$$\lim_{t \to \infty} g_y = \frac{1}{\alpha + \beta} \Big[(1 - \tau)(\alpha g_{A_m} + \beta g_{A_s}) + \tau(\alpha + \beta) g_{A_g} \Big]. \tag{28}$$

This is a decreasing function of τ. If we assume $\tau = 0$ and $L_s = 1$, as in Baumol (1967), we obtain $g_y = g_{A_s}$. Therefore, to increase the growth rate of per capita real GDP in the long run, we need to increase the growth rate of private services productivity. In contrast, if we consider public services, from equation (28), an increase in the growth rate of the productivity of any sector increases the long-run growth rate of per capita real GDP.

9) In general, in models without intermediate inputs, value added shares are equal to employment shares. This property also holds when considering capital stock as a factor input (Buera and Kaboski, 2009). In contrast, if we consider intermediate inputs, value added shares are not equal to employment shares even in a model where labor is the sole production factor (Sasaki, 2020). For the role of services as intermediate inputs in growth models, see Oulton (2001) and Sasaki (2007, 2020).

4 Four-good model: introduction of leisure

In the preceding three-good model, the expansion of public services employment share is unrelated to Baumol's cost disease. Thus, according to Andersen (2016), we introduce leisure as follows:

$$\max u = (c_m - \bar{c}_m)^\alpha (c_s + \bar{c}_s)^\beta \ell^\gamma c_g^{1-\alpha-\beta-\gamma}. \tag{29}$$

$$\text{s.t. } p_m c_m + p_s c_s + (1-\tau) w \ell = (1-\tau) w T, \tag{30}$$

where T denotes total available time and ℓ is leisure time. Accordingly, total labor supply is given by $T - \ell$. For simplicity, we assume $T = 1$.

Solving the utility maximization problem, we obtain

$$c_m = \frac{\alpha}{\alpha+\beta+\gamma}\left[A_m(1-\tau) - \bar{c}_m + \frac{A_m}{A_s}\bar{c}_s\right] + \bar{c}_m, \tag{31}$$

$$c_s = \frac{\beta}{\alpha+\beta+\gamma}\left[A_s(1-\tau) - \frac{A_s}{A_m}\bar{c}_m + \bar{c}_s\right] - \bar{c}_s, \tag{32}$$

$$\ell = \frac{\gamma}{\alpha+\beta+\gamma}\left[1 - \frac{\bar{c}_m}{A_m(1-\tau)} + \frac{\bar{c}_s}{A_s(1-\tau)}\right]. \tag{33}$$

By using the market clearing conditions $Y_i = C_i$ $(i = m, s, g)$, we obtain each sector's employment share as follows:

$$L_m = \frac{\alpha}{\alpha+\beta+\gamma}\left[(1-\tau) + \frac{\bar{c}_s}{A_s}\right] + \frac{\beta+\gamma}{\alpha+\beta+\gamma}\frac{\bar{c}_m}{A_m}, \tag{34}$$

$$L_s = \frac{\beta}{\alpha+\beta+\gamma}\left[(1-\tau)1 - \frac{\bar{c}_m}{A_m}\right] - \frac{\alpha+\gamma}{\alpha+\beta+\gamma}\frac{\bar{c}_s}{A_s}, \tag{35}$$

$$L_g = \frac{\gamma}{\alpha+\beta+\gamma}\frac{\tau}{1-\tau}\left(\frac{\bar{c}_m}{A_m} - \frac{\bar{c}_s}{A_s}\right) + \frac{\alpha+\beta}{\alpha+\beta+\gamma}\tau. \tag{36}$$

Introduction of leisure makes the public services employment share L_g dependent on productivities. This property depends on the non-homotheticity of the utility function; when $\bar{c}_m = \bar{c}_s = 0$, L_g does not depend on productivities.

When each sector's productivity increases, it is difficult to investigate the time path of L_i analytically. However, the limit of each sector's employment share is easy to obtain.

$$\lim_{t\to\infty} L_m = \frac{\alpha}{\alpha+\beta+\gamma}(1-\tau), \tag{37}$$

$$\lim_{t\to\infty} L_s = \frac{\beta}{\alpha+\beta+\gamma}(1-\tau), \tag{38}$$

$$\lim_{t\to\infty} L_g = \frac{\alpha+\beta}{\alpha+\beta+\gamma}\tau, \tag{39}$$

$$\lim_{t\to\infty} \ell = \frac{\gamma}{\alpha+\beta+\gamma}. \tag{40}$$

Therefore, these values are determined by the tax rate and parameters of the utility function. The next section investigates these time paths using numerical simulations.

From the above analysis, we obtain the indirect utility function. Thus, we can examine whether an optimal tax rate exists. However, in the four-good model, we cannot find analytical solutions. If $\bar{c}_m = \bar{c}_s = 0$, the optimal tax rate is given by $\tau^* = 1 - \alpha - \beta$ as in the three-good model.

Let us analyze Baumol's growth disease. As in the three-good model, GDP is equal to w, and each value added share is equal to each employment share. Using this property, we obtain the growth rate of per capita real GDP, $g_y = L_m g_{A_m} + L_s g_{A_s} + L_g g_{A_g}$. The limit of g_y is as follows:

$$\lim_{t\to\infty} g_y = \frac{1}{\alpha+\beta+\gamma}\left[(1-\tau)(\alpha g_{A_m} + \beta g_{A_s}) + \tau(\alpha+\beta) g_{A_g}\right]. \tag{41}$$

As shown above, the employment shares of private and public services do not approach unity; hence, the growth rate of per capita real GDP depends on every sector's productivity even in the long run. For this reason, in some cases, g_y decreases with time and approaches the value given by equation (41), whereas in other cases, g_y increases with time and approaches it. This suggests that in two- or three-good models, we necessarily obtain Baumol's growth disease whereas not necessarily in the four-good model.

As we explained in equation (28), in the three-good model, the long-run value of g_y is a decreasing function of the tax rate. In contrast, from equation (41), we find that in the four-good model, the long-run value of g_y becomes either an increasing or decreasing function of the tax rate.

4.1 Laffer curve

This subsection investigates whether there exists a tax rate that maximizes government tax revenue. If it exists, we call it a Laffer tax rate. Tax revenue in the model is given by $w\tau(1-\ell)$. Without loss of generality, let the wage rate be numéraire and $w = 1$. The tax revenue R becomes a function of τ, $R(\tau)$. Then, $R(\tau)$ leads to

$$R(\tau) = \frac{1}{\alpha+\beta+\gamma}\left[\gamma \frac{\tau}{1-\tau}\left(\frac{\bar{c}_m}{A_m} - \frac{\bar{c}_s}{A_s}\right) + (\alpha+\beta)\tau\right]. \tag{42}$$

Note that when $\gamma = 0$, $R(\tau)$ becomes a monotonic increasing function of τ, and hence, the tax rate that maximizes tax revenue is unity.

Differentiating equation (42) with respect to τ, we obtain

$$\frac{dR(\tau)}{d\tau} = \frac{1}{\alpha + \beta + \gamma} \left[-\gamma \underbrace{\left(\frac{\bar{c}_s}{A_s} - \frac{\bar{c}_m}{A_m} \right)}_{(+)} \frac{1}{(1-\tau)^2} + (\alpha + \beta) \right]. \tag{43}$$

If we assume $\bar{c}_m/A_m < \bar{c}_s/A_s$, the first and second terms in the square brackets are negative and positive, respectively. Then, there exists a τ such that $R'(\tau) = 0$. In addition, we obtain $R''(\tau) < 0$. Therefore, a τ such that $R'(\tau) = 0$ provides the tax rate that maximizes tax revenue. We define this tax rate as τ_{Laffer}.

Since $\tau \in [0, 1]$, τ_{Laffer} is given by

$$\tau_{\text{Laffer}} = 1 - \sqrt{\frac{\gamma}{\alpha + \beta} \left(\frac{\bar{c}_s}{A_s} - \frac{\bar{c}_m}{A_m} \right)}. \tag{44}$$

The Laffer tax rate depends on the productivities of manufacturing and private services though not on that of public services. If we assume that the productivity of private services is constant whereas that of manufacturing increases, τ_{Laffer} approaches the following value:

$$\lim_{A_m \to \infty} \tau_{\text{Laffer}} = 1 - \sqrt{\frac{\gamma}{\alpha + \beta} \frac{\bar{c}_s}{A_s}}. \tag{45}$$

This value is smaller than unity. However, if the productivity of private services increases, τ_{Laffer} approaches unity. Therefore, in the general case where the productivity of private services increases, the Laffer tax rate is unity.

Related to this, Mann and Pecorino (2023) present a model in which the government provides a public good subject to Baumol's cost disease that is financed by income taxes. If this public good is a poor substitute for private goods, then the tax rate rises monotonically up to the revenue-maximizing level (the top of the Laffer curve). In summary, in their framework, the optimal and Laffer tax rates coincide at the limit.

In contrast, as the following numerical simulations show, the optimal tax rate that maximizes utility approaches a constant value smaller than unity when the productivities of all sectors increase, while τ_{Laffer} approaches unity. Hence, the optimal tax rate and τ_{Laffer} are different in our framework.

4.2 Numerical examples

We now conduct some numerical simulations of the four-good model for the Japanese economy. The purposes of this experiment are (i) to show that the optimal tax rate approaches a constant value, and (ii) to clarify that the time paths generated by the model are roughly consistent with actual time paths.

Data and parameter values are as follows:

- We assume that the growth rate of each sector's productivity is equal to that of total factor productivity (TFP). We use the Japan Industrial Productivity Database 2023 (JIP) of the Research Institute of Economy, Trade and Industry (RIETI), and obtain the TFP growth rates for manufacturing, private services, and public services between 2000 and 2020.

- For the classification of manufacturing, we use the JIP's "204 Manufacturing sector" unmodified.

- For the classification of private services, we use the JIP's "206 Non-manufacturing sectors (only market economy, excluding housing and activities not elsewhere classified)" but we subtract "1 Agriculture," "2 Agricultural services," "3 Forestry," "4 Fisheries," and "5 Mining."

- For the classification of public services, we assume that the JIP's "91 Public administration," "92 Education," "93 Medical service, health and hygiene," "94 Social insurance and social welfare," and "95 Nursing care," correspond to public services. We obtain the TFP growth rate of public services by calculating the weighted average TFP growth rate of these five sectors with each weight being the corresponding value added share.

- The average growth rates of TFP between 2000 and 2020 are 1.40%, 0.05%, and 0.60% for manufacturing, private services, and public services, respectively.

- As for each sector's employment share, we use the JIP and divide the number of workers by sector by the total number of workers. However, some sectors are excluded, and hence, the sum of the three sectors is slightly less than unity.

- Since we obtain the initial value of each sector's employment share (i.e., the values in 2000), by assuming that the parameters of the utility function, α, β, and $1-\alpha-\beta-\gamma$ correspond to $L_m(0)$, $L_s(0)$, and $L_g(0)$, respectively, we set $\alpha = 0.2$, $\beta = 0.6$, and $1-\alpha-\beta-\gamma = 0.13$ ($\gamma = 0.07$).

- As for productivity growth rates, based on the TFP growth rate data, we set $g_{A_m} = 0.014$, $g_{A_s} = 0.0005$, and $g_{A_g} = 0.006$.
- As for initial levels of the productivities, we use $A_m(0) = 2$, $A_s(0) = 1$, and $A_g(0) = 1$.
- As for the parameters of the utility function, $\bar{c}_m$ and $\bar{c}_s$, previous studies use various values, and estimation is also difficult. Accordingly, for the employment shares of the model to be close to the actual values, we use $\bar{c}_m = 0.1$ and $\bar{c}_s = 0.1$.[10)]
- As for the tax rate, we use the result of the Statistical Survey of Actual Status for Salary in the Private Sector by National Tax Agency of Japan. In 2020, the average annual salary in Japan is 4.58 million yen, the corresponding staturoty income tax rate is 20%, and accordingly, we set $\tau = 0.2$.[11)]

Suppose that the productivities of all sectors are fixed at the initial levels. Then, the tax rate that maximizes instantaneous utility is $\tau^* = 0.147312$. If all sectors' productivities increase, the optimal tax rate approaches $\tau^* = 0.139785$. Therefore, under Baumol's cost disease, we do not necessarily need an unacceptable tax rate.

Suppose that all sectors' productivities increase. If we draw the graph of the relationship between the tax rate and lifetime utility W, we obtain Figure 5. Here, W is defined as follows:

$$W = \int_0^\infty u e^{-\rho t} \, dt, \quad \rho > 0, \tag{46}$$

where ρ denotes the rate of time preference, and we use $\rho = 0.01$. Figure 5 shows the existence of a tax rate that maximizes lifetime utility. Since we set the average income tax rate to 0.2, we find that the optimal tax rate is less than 0.2. Note that lifetime utility depends on the rate of time preference. Accordingly, if the rate of time preference changes, the optimal tax rate also changes. Moreover, from this fact, we find that the optimal tax rates that maximize instantaneous and lifetime utility are not generally equal.

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10) Our parameter setting satisfies the inequality $\bar{c}_m/A_m < \bar{c}_s/A_s$ given in Section 4.1. Since $\bar{c}_m = \bar{c}_s$, $A_m(0) > A_s(0)$, and the growth rate of A_m is larger than that of A_s, this inequality continues to hold.
11) Rogerson (2008) uses 0.26 for the US tax rate in 2003 to conduct numerical simulations of structural change in industries.

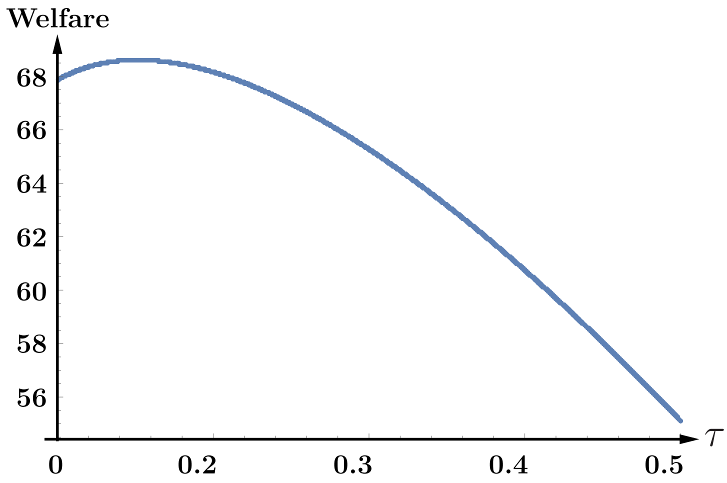

Figure 5: Relationship between tax rate and life time utility

The time path of each sector's employment share is shown in Figures 6–8. Manufacturing employment share monotonically decreases and approaches a constant value. Private services employment share monotonically increases and approaches a constant value. Public services employment share first decreases, then increases, and approaches a constant value. This means that even under Baumol's cost disease, public services employment share does not necessarily continue to expand. Furthermore, these time paths are roughly consistent with the actual time paths in Figures 1 given in the Introduction.

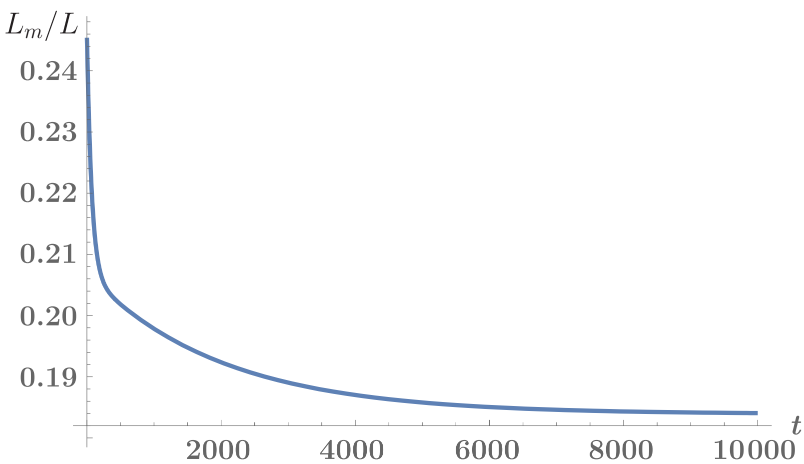

Figure 6: Employment share of manufacturing

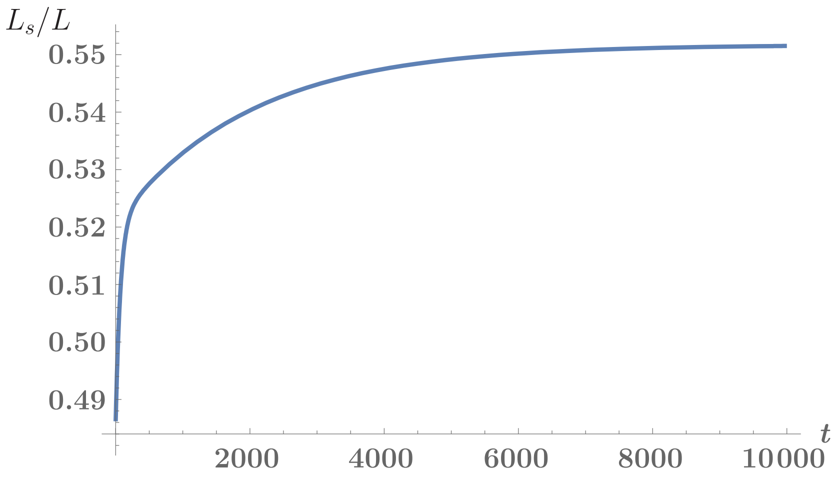

Figure 7: Employment share of private services

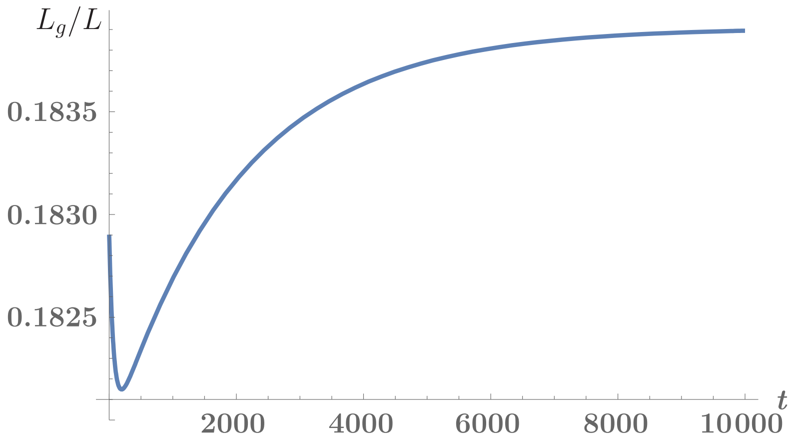

Figure 8: Employment share of public services

If we change the tax rate, the time paths of the employment share are shown in Figures 9–11. In Figures 9 and 10, we change the tax rate from 0.1 to 0.5 with an interval of 0.1. The top blue line corresponds to $\tau = 0.1$ and the bottom purple line corresponds to $\tau = 0.5$. In both manufacturing and private services, the employment shares decrease as the tax rate increases. In Figure 11, we change the tax rate from 0.18 to 0.2 with an interval of 0.01. The bottom blue line corresponds to $\tau = 0.18$ and the top green line corresponds to $\tau = 0.2$. Hence, the employment share of public services increases alongside the tax rate.

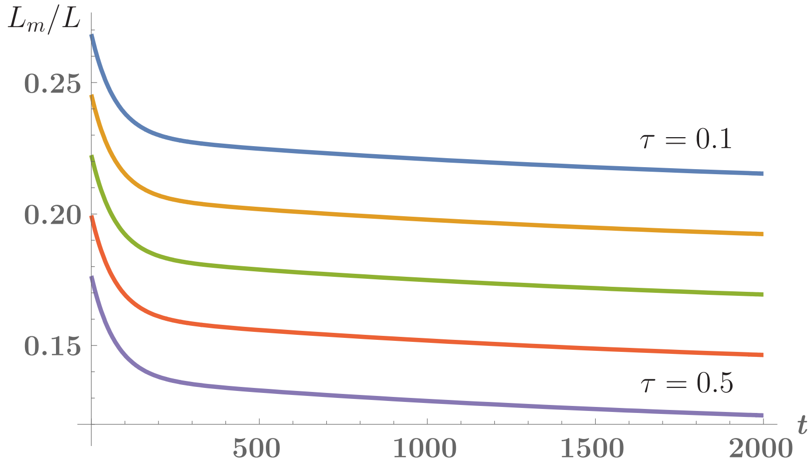

Figure 9: Employment share of manufacturing for different tax rates

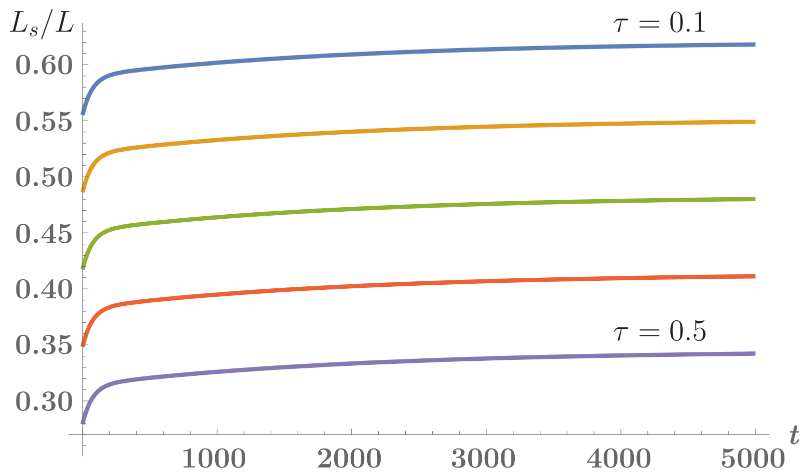

Figure 10: Employment share of private services for different tax rates

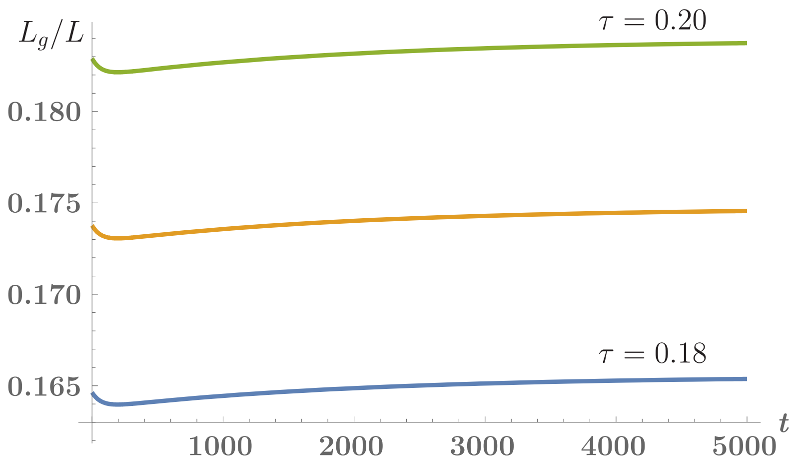

Figure 11: Employment share of public services for different tax rates

The time path of the growth rate of per capita real GDP is shown in Figure 12. We change the tax rate from 0.1 to 0.5 with an interval of 0.1. The bottom blue line corresponds to $\tau = 0.1$ and the top purple line corresponds to $\tau = 0.5$. In every case, the growth rate decreases with time, and approaches a constant value. Therefore, we observe Baumol's growth disease. In this example, the growth rate increases alongside the tax rate.

We set the benchmark tax rate to $\tau = 0.2$, which corresponds to the yellow line second from the bottom. In this case, the per capita growth rate converges to about 0.4%. The annual average TFP growth rate of the whole economy obtained from JIPD is about 0.6% between 2000 and 2020, and the annual average growth rate of per capita real GDP obtained from the System of National Accounts in Japan between 2000 and 2020 is about 0.5%. Therefore, our numerical examples are roughly consistent with the actual data.

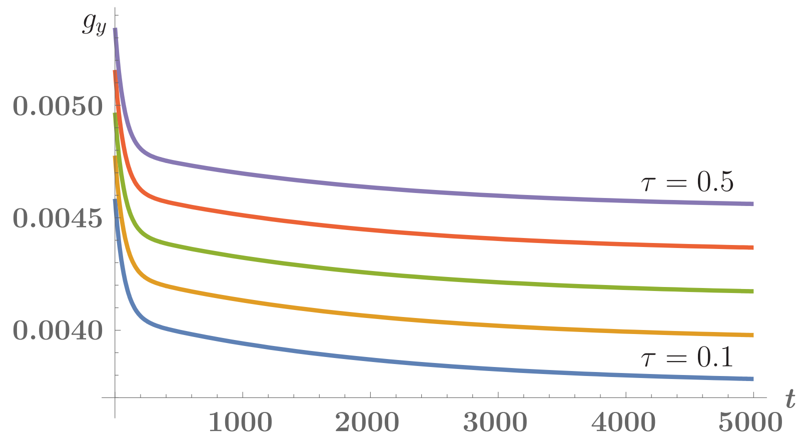

Figure 12: Growth rate of per capita real GDP

The growth rate of instantaneous utility is shown in Figure 13. We change the tax rate from 0.1 to 0.5 with an interval of 0.1. A change in the tax rate does not change the time path very much. In every case, the growth rate first declines, then increases and approaches a constant value. This growth rate is positive, and hence, even under Baumol's cost disease, instantaneous utility increases over time. This is a benefit of productivity improvements.

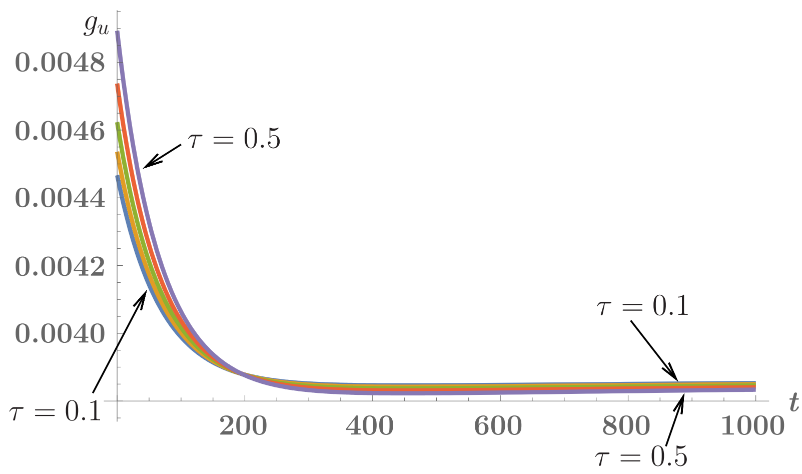

Figure 13: Growth rate of instantaneous utility

4.3 Change in parameters

Let us examine how time paths change if we change parameters.

If we set the minimum level of manufacturing consumption to $\bar{c}_m = 0$, we obtain the following results:

- The time path of the growth rate of instantaneous utility is different. As Figure 14 shows, it increases and approaches a constant value.
- The limit of the tax rate that maximizes instantaneous utility is the same as that in the benchmark case with $\bar{c}_m > 0$.
- The time paths of the employment shares change relatively less compared with the case of $\bar{c}_m > 0$.
- The time path of the growth rate of per capita real GDP changes relatively less compared with the case of $\bar{c}_m > 0$.

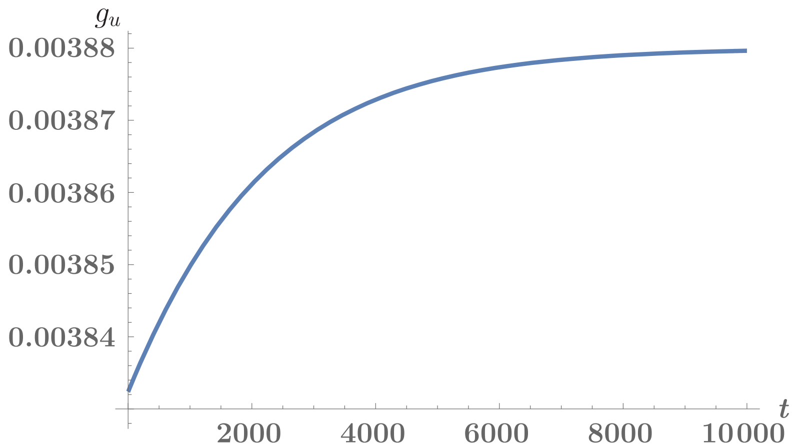

Figure 14: Growth rate of instantaneous utility when $\bar{c}_m = 0$

If we increase the growth rate of the productivity of private services by ten times, that is, $g_{A_s} = 0.005$, we obtain the following results:

- The growth rate of per capita real GDP increases alongside the tax rate with $g_{A_s} = 0.0005$; however, as Figure 15 shows, it decreases as the tax rate increases with $g_{A_s} = 0.005$.
- Besides this difference, the other time paths do not change much.

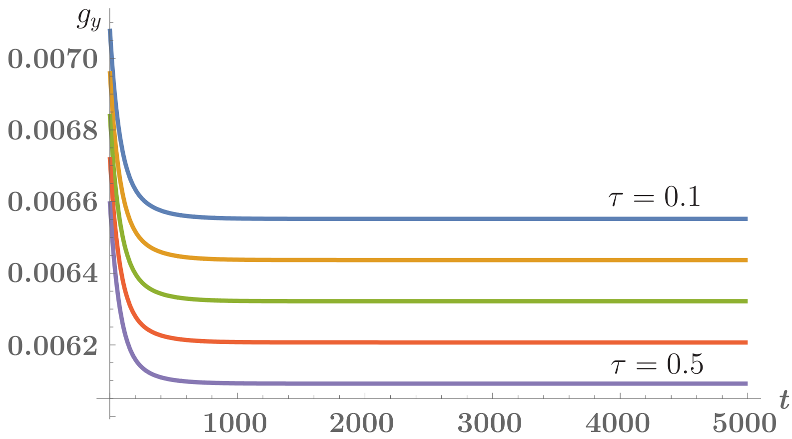

Figure 15: Growth rate of per capita real GDP

5 Conclusion

This study provides a three-sector model with manufacturing, private services, and public services, and examines structural changes in these sectors. In the four-good model incorporating leisure, we obtain the following results:

1. The employment share of public services increases in the long run but approaches a certain level.

2. There is a tax rate that maximizes instantaneous utility; this tax rate changes over time but converges to a certain level.

3. In the general case, a tax rate that maximizes government tax revenue approaches unity.

4. There is a tax rate that maximizes lifetime utility.

5. The growth rate of per capita real GDP continues to decrease or increase and approaches a certain level. Accordingly, Baumol's growth disease does not necessarily apply.

6. The growth rate of instantaneous utility continues to decrease or increase with time and approaches a certain level.

7. The tax rate has a positive or negative correlation with the growth rate of per capita real GDP.

Consequently, even if a government can control the income tax rate to maximize welfare, the optimal tax rate does not necessarily increase endlessly. Moreover, the tax rate that maximizes lifetime utility is constant even though the productivities of all sectors continue to increase through time. In this sense, the welfare state is sustainable.

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