The Paradox of Technological Progress, Growth, Distribution, and Employment in a Demand-led Framework

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12 August 2024

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Abstract

This study builds a Kaleckian model that incorporates endogenous technological progress and investigates how a change in a parameter that directly fosters technological progress affects growth and distribution. In this model, there is an optimal wage share that maximizes the technological progress rate. Accordingly, if the actual wage share can be moved to an optimal level, the economic growth rate will increase. This analysis reveals that a policy that directly promotes technological progress consequently decreases the long-run equilibrium value of the wage share, the capacity utilization rate, the employment rate, and the economic growth rate.

Keywords: endogenous technological progress; education; R&D; growth and distribution

JEL Classification: E11; E12; E24; E25; O33; O41

1 Introduction

The Kaleckian model belongs to the post-Keynesian growth model class. The Kaleckian model is useful for analyzing the relationship between growth and distribution.[1] If a rise in the wage share increases the economic growth rate, then this situation is called wage-led growth. On the other hand, if a rise in profit share increases the economic

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1) The contribution of Michał Kalecki is summarized in Kalecki (1971). For the conventional Kaleckian model, see Rowthorn (1981).

growth rate, this situation is called profit-led growth. Thus, the following inference arises: If the economy exhibits wage-led growth, the economic growth rate climbs as the wage share increases, implying that the wage share that maximizes the economic growth rate is unity; hence, the profit share is zero. Likewise, if the economy displays profit-led growth, the profit share that maximizes the economic growth rate is unity; as such, the wage share is zero. In either case, the income distribution that maximizes the economic growth rate is either zero or unity. This is extreme and unreasonable if we consider an economic policy that aims to increase the economic growth rate through a change in income distribution.

This study proposes a Kaleckian model that introduces endogenous technological progress. Specifically, I provide a Kaleckian model in which there is an optimal wage share that maximizes the technological progress rate and consequently the economic growth rate, which lies within the interval between zero and unity. Using this model, I investigate how an economic policy aimed at directly fostering technological progress affects growth and distribution.

To the best of my knowledge, only Lima (2004) has discussed the optimal wage share in the Kaleckian model. He proposes a technological progress function in which the growth rate of labor productivity is convex upward with respect to the wage share, $g_a(\omega) = \omega - \omega^2$, where g_a denotes the growth rate of labor productivity and ω is the wage share. In this specification, g_a is maximized when $\omega = 1/2$. Lima (2004) conducts both short- and long-run analyses. In the long-run equilibrium, the economic growth rate is equal to the natural growth rate, given by the sum of the technological progress rate and labor supply growth rate. Hence, the long-run economic growth rate is maximized when $\omega = 1/2$. Thus, the optimal wage share is 1/2. Lima (2004) explains this specification as follows: This simplified innovation function is intended to capture plausible non-linearity in the influence of the wage share on firms' propensity and ability to adopt labor-saving innovations, namely that the rate of innovation is lower for both low and high levels of the wage share and is higher for intermediate levels. At high profit share levels, the availability of funding for innovation is high, but the incentives to innovate are low; at low profit share levels, the incentives to innovate are high, but the availability of funding is low.

Similar to Lima (2004), I use a technological progress function that is convex upward with respect to the wage share. However, unlike Lima (2004), I incorporate certain parameters into the technological progress function. In the Lima model, there is no parameter in the technological progress function; hence, we cannot consider the effects of economic policies on growth and distribution. By contrast, in my model, a change in a parameter due to an economic policy directly affects the technological progress function, which enables us to examine the effect of economic policy on growth and distribution.

Let me now explain the technological progress function in detail. Suppose that the average labor productivity increases by two factors: workers' investment in education and firms' investment in research and development (R&D). Workers spend a fraction of their wages on education investment, which contributes to enhanced labor productivity. This implies that a rise in the wage share increases the growth rate of labor productivity. On the other hand, firms spend a fraction of their retained earnings on R&D investments, which fosters labor productivity growth. This denotes that a rise in the profit share increases the labor productivity growth rate. If we assume that the growth rate of average labor productivity takes a Cobb–Douglas function of workers' and firms' factors, it becomes a function convex upward with respect to the wage share. This indicates the existence of an optimal wage share that maximizes the technological progress rate and consequently the economic growth rate.

Some studies consider endogenous technological progress in the Kaleckian model. Dutt (2006), Flaschel and Skott (2006), and Sasaki (2010, 2013) use a specification in which the growth rate of labor productivity is an increasing function of the employment rate. Based on the concept of induced technological innovation, Tavani *et al.* (2011) employ a specification such that the growth rate of labor productivity is an increasing function of the wage share. Taylor *et al.* (2019) adopt a specification such that the growth rate of labor productivity is an increasing function of the capital stock per labor supply and a decreasing function of the profit rate.[2)] However, these studies do not explicitly consider innovation costs. Innovation cannot arise from nothing and requires resources. In addition, if some resources are used for innovation, the resources necessary for other economic activities are reduced, such as in the trade-off between innovation and other activities. A survey by Tavani and Zamparelli (2017) calls this "costly innovation."[3)] As previously stated, my specification considers costly innovation, which enables us to investigate the interactions between innovation and other economic activities.

For Kaleckian models that consider costly innovations, I refer to Lima *et al.* (2021), Serra (2023), and Carvalho *et al.* (2024). These studies assume that spending on educa-

2) For induced technical innovation, see also Hicks (1932), Kennedy (1964), Von Weizsäcker (1966), and Drandakis and Phelps (1966). The above-mentioned specification by Taylor *et al.* (2019) is a combination of the Kaldor–Verdoorn law and induced technical innovation. For the Kaldor–Verdoorn law, see Verdoorn (1949) and Kaldor (1966).

3) For specifications of innovation, see also Tavani and Zamparelli (2020, 2021).

tion contributes to human capital accumulation, which increases labor productivity.[4)] In Lima *et al.* (2021), the government imposes taxation on workers and capitalists and invests in education with the use of tax funds. In Serra (2023), households spend on education as a fraction of their income and borrowings. In Carvalho *et al.* (2024), workers spend a fraction of their wages on human capital accumulation. Among these three models, Serra (2023) and Carvalho *et al.* (2024) assume an exogenous wage share while Lima *et al.* (2021) endogenizes the wage share dynamics by using conflict theory.

This study incorporates costly innovation into the Kaleckian model and conducts short- and long-run analyses. In the short run, the capacity utilization rate becomes an adjustment variable, and short-run equilibrium is attained through a change in the capacity utilization rate when the price of the final goods is given. In the long run, assuming that a short-run equilibrium is always attained, the wage share and capital stock per effective labor supply are adjusted. In the long-run equilibrium, these two variables become constant.[5)] My model considers the dynamics of the wage share and is therefore close to Lima *et al*'s (2021) model.

The results of the analysis are as follows:

In the short-run equilibrium, the capacity utilization rate and economic growth rate are increasing functions of the wage share. This suggests that the short-run equilibrium exhibits wage-led demand and growth.

In the long-run equilibrium, the capacity utilization rate, employment rate, economic growth rate, and wage share are all positively correlated. An increase in workers' propensity to spend on education investment decreases the capacity utilization rate, employment rate, economic growth rate, and wage share. An increase in a firm's propensity to invest in R&D decreases its capacity utilization rate, employment rate, economic growth rate, and wage share. An increase in firms' retained earnings rate decreases the capacity utilization rate, employment rate, economic growth rate, and wage share.

These results are consistent with those reported by Lima *et al.* (2021). They conclude that a rise in the tax rate to increase education investment decreases the wage share and capacity utilization rate, which decreases (increases) the economic growth rate if the economy has wage-led growth (profit-led growth). Moreover, they show that if the economy has wage-led growth (profit-led growth), an increase in the tax rate

4) Using the classical growth model, Dutt and Veneziani (2019, 2020) examine how human capital accumulation affects growth and distribution.

5) Dutt (1992) is the first to perform such short- and long-run analyses of the Kaleckian model. The capital stock per effective labor supply and the employment rate are interchangeable in long-run analysis.

has a negative (ambiguous) effect on the employment rate. In my model, an economic policy that directly fosters technological progress to increase the economic growth rate lowers the long-run economic growth rate. This outcome is opposite to the result obtained from the mainstream endogenous growth model, which emphasizes the supply side. The difference between my model and that of Lima *et al.* (2021) is that I consider firms' retained earnings and spending allocation from retained earnings to investments in R&D and equipment. This enables us to investigate how changes in firms' behavior affect their growth and distribution.

The remainder of this paper is organized as follows: Section 2 presents the basic framework of the proposed model. Section 3 contains a short-run analysis. Section 4 covers a long-run analysis. Section 5 outlines a comparative static analysis using numerical simulations. Finally, Section 6 concludes the study.

2 Model

Suppose an economy has three agents: workers, rentiers, and firms. Workers provide labor to firms and obtain wages. Rentiers own their equities and obtain profits through dividends. Firms produce a single final good with labor power and capital stock, allocate a fraction of their profits to rentiers as dividends, and save the remainder as retained earnings. A fraction of firms' retained earnings is devoted to R&D spending and the rest to equipment investment. Following the work of Kalecki, I assume that the final goods market is imperfectly competitive.

The production function is assumed to take the Leontief form.

$$Y = \min\{aE, uK\}, \tag{1}$$

where Y denotes the output, E refers to employment, and K indicates capital stock. From the firms' cost minimization behavior, I obtain $aE = uK$. In this case, $a = Y/E$ signifies labor productivity. I define the ratio of Y to the potential output Y^* as the capacity utilization rate u. I assume that the technologically given potential output-capital ratio is fixed at unity. Subsequently, I derive $u = Y/K$.

In the short run, the labor supply is unlimited; hence, firms can obtain as much labor as they need for production. In contrast, in the long run, labor supply is constrained, and labor supply N grows at a constant rate $n > 0$.

$$\frac{\dot{N}}{N} = n > 0. \tag{2}$$

Accordingly, I define the employment rate e as

$$e = \frac{E}{N}. \tag{3}$$

Because $E = uK/a$, I can decompose the employment rate into

$$e = u \cdot \underbrace{\frac{K}{aN}}_{k} = uk, \tag{4}$$

where k denotes capital stock per effective labor supply.

I specify the rates of change in nominal wages and prices using conflict theory. In conflict theory, income distribution is determined through labor-management negotiations. Rowthorn (1977) invented conflict theory, and Dutt (1987) and Cassetti (2003) adopted it in the Kaleckian model.

First, I assume that the nominal wage W changes according to the gap between the labor union's target wage share ω_w and actual wage share ω.

$$\frac{\dot{W}}{W} = \theta_w[\omega_w(e) - \omega], \quad \theta_w > 0, \ \omega_w'(e) > 0, \tag{5}$$

where θ_w indicates an adjustment parameter. This equation suggests that when the actual wage share is below the target level, labor unions demand a wage increase, which increases nominal wages. Moreover, I assume that the target level is an increasing function of the employment rate. When the employment rate is high, the bargaining power of labor unions is strong; hence, they set a higher target level. This specification captures the reserve army effect stressed by Marx (1867).

Second, I assume that the price P set by firms changes according to the gap between the actual and target wage shares of firms ω_f.[6)]

$$\frac{\dot{P}}{P} = \theta_f(\omega - \omega_f), \quad \theta_f > 0, \tag{6}$$

where θ_f is the adjustment parameter. This specification implies that when the actual wage share is higher than a firm's target level, firms set a higher price to obtain more profits. It is reasonable to assume that labor unions set a higher target wage share, whereas firms set a lower target wage share. Hence, it is assumed that $\omega_w > \omega_f$. For

6) In this study, ω_f is an exogenous variable. Lima (2004) assumes that ω_f is a decreasing function of the capacity utilization rate. If I adopt this specification, the long-run equilibrium is more likely to be stable.

the sake of convenience, I assume $\theta_w + \theta_f = 1$ and set $\theta_w = \theta \in (0,1)$ and $\theta_f = 1-\theta$.

I now specify the flow of incomes. A fraction τ of wages wE is allocated to an education investment, and the remaining $1-\tau$ is allocated to consumption. Here, $w = W/P$ denotes the real wage. A fraction s_f of profits rK is allocated to retained earnings $s_f rK$ and the remaining $(1-s_f)rK$ to dividends. Here, r refers to the profit rate, and s_f is the retained earnings rate.[7] A fraction σ of retained earnings is allocated to R&D investment and the remaining $1-\sigma$ is allocated to investment in equipment. Dividends are the source of rentiers' income; a fraction s_r of dividends is devoted to saving, and the remaining $1-s_r$ to consumption. Consequently, saving of the entire economy is the sum of the retained earnings (savings of firms) and savings of rentiers. Dividing the savings of the entire economy by capital stock and letting the resultant expression be g_s, I obtain

$$g_s = \underbrace{[s_f(1-\sigma) + s_r(1-s_f)]}_{s}(1-\omega)u \tag{7}$$

$$= s(1-\omega)u, \tag{8}$$

where s signifies the average savings rate of the entire economy and $0 < s < 1$. For s, the following relationship is obtained:

$$\frac{\partial s}{\partial \sigma} = -s_f < 0, \tag{9}$$

$$\frac{\partial s}{\partial s_f} = 1 - \sigma - s_r \gtrless 0. \tag{10}$$

An increase in the proportion of retained earnings allocated to R&D reduces the average savings rate. An increase in the retained earnings rate either increases or decreases the average savings rate.

I specify the firms' investment functions. I assume that firms' equity investment is an increasing function of retained earnings and the capacity utilization rate.

$$g_d = \gamma + \alpha(1-\sigma)s_f(1-\omega)u + \beta u, \quad \alpha, \beta, \gamma > 0, \tag{11}$$

where γ indicates the animal spirit of firms, α is the sensitivity of investment to retained earnings, and β is the sensitivity of investment to the capacity utilization rate. The specification that investment depends positively on retained earnings is supported

7) For Kaleckian models with retained earnings, see also Charles (2008) and Sasaki and Fujita (2012). They consider a situation in which rentiers make loans to firms; hence, firms hold debt. In contrast, I abstract firms' debt to clarify the analysis.

by empirical studies by Hayashi and Inoue (1991) and Hoshi *et al.* (1991), Fazzari *et al.* (1988), and Ndikumana (1999). The specification that investment depends positively on capacity utilization is generally used in the Kaleckian model.

I specify the technological progress function as follows: I assume that the growth rate of labor productivity $g_a \equiv \dot{a}/a$ takes the Cobb–Douglas form, which synthesizes the increase effect of labor productivity arising from workers' investment in education and firms' investment in R&D. As mentioned above, workers invest a proportion τ of their wages in education and firms invest a share σ of their retained earnings in R&D. I specify this as follows:

$$g_a = \phi_0 + \phi_1(\tau\omega)^{\psi}[\sigma s_f(1-\omega)]^{1-\psi}, \quad \phi_0, \phi_1 > 0,\ \psi \in [0,1]. \tag{12}$$

The restriction $\phi_0 > 0$ warrants $g_a > 0$ when $\omega = 0$ or $\omega = 1$. Parameter ϕ_1 shows the efficiency of education investment and R&D. Parameter ψ captures the weight of education investment in the technological progress function. I obtain $g_a'(\omega) < 0$ when $\psi = 0$ and $g_a'(\omega) > 0$ when $\psi = 1$. Subsequently, g_a is maximized at $\omega = \psi$.[8] Thus, the optimal wage share that maximizes the economic growth rate is $\omega_{\text{optimal}} = \psi$. Subsequently, I investigate how τ, σ, and s_f affect the model's main variables. Empirical analyses by Himmelberg and Petersen (1994) and Carpenter and Petersen (2002) show that R&D investments depend more heavily on internal funds than on external funds. Hence, the specifications of Equation (12) are reasonable.[9]

3 Short-run analysis

In the short run, labor productivity a, capital stock K, labor supply N, price level P, and nominal wage W are constant, and capacity utilization u becomes an endogenous variable for attaining a goods market equilibrium.

The aggregate demand Y_d is given by

$$Y_d = C + I = \underbrace{C_w + C_r}_{C} + I_d + I_p \tag{13}$$

$$= \underbrace{(1-\tau)wE}_{C_w} + \underbrace{(1-s_r)(1-s_f)rK}_{C_r} + I_d + \underbrace{(\tau wE + \sigma s_f rK)}_{I_p} \tag{14}$$

$$= wE + [(1-s_r)(1-s_f) + \sigma s_f]rK + I_d. \tag{15}$$

8) If I differentiate $g_a(\omega)$ with respect to ω, I obtain $g_a'(\omega) > 0$ for $0 < \omega < \psi$, $g_a'(\omega) = 0$ when $\omega = \psi$, and $g_a'(\omega) < 0$ when $\psi < \omega < 1$.

9) For R&D investment and its finance, see also Hall (2002).

where C_w denotes the consumption of workers, C_r is the consumption of rentiers, I_d is the investment of firms, and I_p is the R&D investment of firms. For equation (15), which is equal to national income $wE + rK$, I need

$$[s_f(1-\sigma) + s_r(1-s_f)](1-\omega)uK = I_d. \tag{16}$$

By dividing both sides of equation (16) by K, I obtain

$$g_s = g_d. \tag{17}$$

This is the short-run equilibrium condition for the goods market. By solving equation (17) for u, I obtain the following equation: [10)]

$$u^* = \frac{\gamma}{\varepsilon(1-\omega)-\beta}, \quad \varepsilon \equiv s - \alpha(1-\sigma)s_f > 0. \tag{18}$$

The superscript "*" refers to the short-run equilibrium value of a variable. Because ε is the sensitivity of savings to the profit rate s minus the sensitivity of investment to the profit rate $\alpha(1-\sigma)s_f$ and it can be rewritten as $\varepsilon = s_f(1-\sigma)(1-\alpha) + s_r(1-s_f)$, I find that $\varepsilon > 0$. This implies that the Robinsonian stability condition is satisfied in my model. Finally, I obtain

$$\frac{\partial \varepsilon}{\partial \sigma} = -s_f(1-\alpha) < 0, \tag{19}$$

$$\frac{\partial \varepsilon}{\partial s_f} = (1-\sigma)(1-\alpha) - s_r \gtreqless 0. \tag{20}$$

An increase in the proportion of the retained earnings devoted to R&D investments lowers ε. An increase in the retained earnings rate either increases or decreases ε.

For the positivity of the short-run equilibrium capacity utilization rate, I require

$$\varepsilon(1-\omega) - \beta > 0. \tag{21}$$

This is a Keynesian stability condition such that the quantity adjustment of the goods market is stable. For this constraint to be effective, I require $\varepsilon > \beta$, which I assume in the following analysis.[11)] Moreover, the capacity utilization rate must be less than

10) If $\beta = 0$ in the investment function, the short-run equilibrium growth rate is independent of the wage share. Hence, to investigate the relationship between growth and distribution, I need $\beta > 0$.
11) The Keynesian stability condition is criticized theoretically and empirically by Skott (2012).

unity, which requires the following conditions:

$$\gamma < \varepsilon(1-\omega) - \beta. \tag{22}$$

This equation can be rewritten as

$$\omega < 1 - \frac{\beta+\gamma}{\varepsilon} \equiv \omega_{\max}. \tag{23}$$

This corresponds to the upper limit of the capacity utilization rate. That is, the wage share must lie within the interval $\omega \in (0, \omega_{\max})$

As the short-run equilibrium capacity utilization rate u^* depends on ω, I can write it as $u^* = u(\omega)$. Moreover, if I use $u^* = u(\omega)$, the short-run equilibrium growth rate g^* also depends on ω. Hence, I can write $g^* = g(\omega)$, which leads to:

$$g(\omega) = \frac{s\gamma(1-\omega)}{\varepsilon(1-\omega)-\beta}. \tag{24}$$

By differentiating $u(\omega)$ and $g(\omega)$ with respect to ω, I derive the following relationship.

$$u'(\omega) = \frac{\gamma\varepsilon}{[\varepsilon(1-\omega)-\beta]^2} > 0, \tag{25}$$

$$g'(\omega) = \frac{s\gamma\beta}{[\varepsilon(1-\omega)-\beta]^2} > 0. \tag{26}$$

If other things are equal, a rise in the wage share increases the consumption of workers while decreasing the consumption of rentiers and the investment of firms. Since workers' propensity to consume is unity, rentiers' propensity is given by $1 - s_r < 1$ and it is less than unity, consumption of the entire economy increases. This rise in consumption outweighs the decline in firms' investment; hence, the aggregate demand increases. As such, both the short-run equilibrium capacity utilization rate and the growth rate increase.

Proposition 1. *The short-run equilibrium exhibits wage-led demand and growth regimes.*

Bhaduri and Marglin (1990) and Marglin and Bhaduri (1990) show that both wage- and profit-led regimes are obtained by specifying the investment function as an increasing function of the profit share and capacity utilization rate.[12)] I use a different investment function; hence, I obtain only a wage-led demand/growth regime.

12) Blecker (2002) is an excellent study that clearly explains the classification of regimes in the Kaleckian model.

Finally, the employment rate can be rewritten as

$$e = u(\omega)k. \tag{27}$$

In the long-run analysis, both ω and k are adjustment variables, and the employment rate shifts as these variables change.

4 Long-run analysis

In the long run, labor productivity a, capital stock K, labor supply N, price level P, and nominal wage W change; hence, wage share $\omega = (W/P)/a$ and capital stock per effective labor supply $k = K/(aN)$ become endogenous variables. Because ω and k are positive, I can take their logarithms. By differentiating them with respect to time, I obtain

$$\frac{\dot{\omega}}{\omega} = \frac{\dot{W}}{W} - \frac{\dot{P}}{P} - \frac{\dot{a}}{a}, \tag{28}$$

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{a}}{a} - \frac{\dot{N}}{N}. \tag{29}$$

Substituting the rates of change of W, P, a, K, and N into the above equations, I derive a system of differential equations for ω and k:

$$\dot{\omega} = \omega\Big\{\theta_w[\omega_w(u(\omega)k) - \omega] - \theta_f(\omega - \omega_f) - g_a(\omega)\Big\}, \tag{30}$$

$$\dot{k} = k\Big[g(\omega) - g_a(\omega) - n\Big]. \tag{31}$$

I define the long-run equilibrium as a situation in which ω and k are constant; that is, $\dot{\omega} = \dot{k} = 0$. With $k \neq 0$, from $\dot{k} = 0$, I obtain

$$g(\omega^{**}) = g_a(\omega^{**}) + n. \tag{32}$$

The superscript "$**$" denotes the long-run equilibrium value of a variable. Equation (32) determines the long-run equilibrium wage share: Figure 1 illustrates the determination of ω^{**}. The graph of $g(\omega)$ is upward sloping because the short-run equilibrium is wage-led growth. The graph of $g_a(\omega) + n$ is convex upward because $g_a(\omega)$ is convex. The value $\omega_{\min}$ in Figure 1 is explained later. As Figure 1 shows, the number of the long-run equilibrium value of ω is 0, 1, or 2, which depends on the values of $g(\omega_{\min})$, $g_a(\omega_{\min}) + n$, $g(\omega_{\max})$, and $g_a(\omega_{\max}) + n$, and the shapes of $g(\omega)$ and $g_a(\omega) + n$.

Equation (32) indicates that the economic growth rate is equal to the natural growth rate in the long run. As such, the employment rate in the long-run equilibrium becomes constant. Substituting ω^{**} into the equation $\dot{\omega} = 0$, I can solve it for the long-run equilibrium value k^{**}.

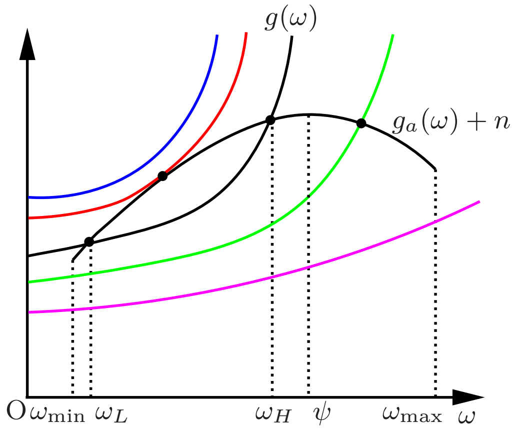

Figure 1: Determining the long-run equilibrium value of ω

For the numerical simulations that I conduct later, I specify labor unions' target wage share as a linear function:

$$\omega_w(e) = \delta_0 + \delta_1 \underbrace{u(\omega)k}_{e}, \quad \delta_0 > 0,\ \delta_1 > 0. \tag{33}$$

Parameter δ_1 captures the degree of the reserve army effect. With this specification, the relationship between k and ω and that between e and ω are given by

$$k = \frac{[\varepsilon(1-\omega) - \beta]\Big\{\phi_0 + \phi_1(\tau\omega)^{\psi}[\sigma s_f(1-\omega)]^{1-\psi} + \omega - [\theta\delta_0 + (1-\theta)\omega_f]\Big\}}{\theta\delta_1 u(\omega)}, \tag{34}$$

$$e = \frac{\phi_0 + \phi_1(\tau\omega)^{\psi}[\sigma s_f(1-\omega)]^{1-\psi} + \omega - [\theta\delta_0 + (1-\theta)\omega_f]}{\theta\delta_1}. \tag{35}$$

By substituting ω^{**} into these equations, I obtain k^{**} and e^{**}.

Each element of the Jacobian matrix $\mathbf{J}$ corresponding to the system of differential equations is given by

$$J_{11} = \frac{\partial\dot{\omega}}{\partial\omega} = \omega^{**}\Big[\theta\delta_1 k^{**} \underset{(+)}{u'(\omega^{**})} - 1 - \underset{(+/-)}{g_a'(\omega^{**})}\Big], \tag{36}$$

$$J_{12} = \frac{\partial\dot{\omega}}{\partial k} = \omega^{**}\theta\delta_1 u(\omega^{**}) > 0, \tag{37}$$

$$J_{21} = \frac{\partial \dot{k}}{\partial \omega} = k^{**}\Big[\underset{(+)}{g'(\omega^{**})} - \underset{(+/-)}{g'_a(\omega^{**})}\Big], \tag{38}$$

$$J_{22} = \frac{\partial \dot{k}}{\partial k} = 0. \tag{39}$$

J_{11} shows the impact of a change in the wage share on the wage share itself. This impact comprises the reserve army effect $\delta_1 > 0$, the wage-led demand run $u'(\omega) > 0$, and the response of technological progress to the wage share $g'_a(\omega) \gtreqless 0$. J_{11} is positive if the reserve army effect is strong, the wage-led demand effect is strong, and the technological progress response is negative. In contrast, if the reserve army effect is weak, the wage-led demand effect is weak, and the response of technological progress is positive; J_{11} will be negative. When J_{11} is negative, the long-run equilibrium is likely stable.

J_{12} displays the impact of a change in capital stock per effective labor supply on the wage share. Because the reserve army effect works ($\delta_1 > 0$), J_{12} becomes positive.

J_{21} presents the impact of a change in wage share on capital stock per effective labor supply. This effect comprises the wage-led growth run $g'(\omega) > 0$ and the response of technological progress to the wage share $g'_a(\omega) \gtreqless 0$. If this response to technological progress is negative, J_{21} is positive. By contrast, J_{21} can be negative if this response is positive.

J_{22} portrays the impact of a change in capital stock per effective labor supply on the capital stock per effective labor supply. This self-feedback effect is zero near the long-run equilibrium.

From the Routh–Hurwitz stability criterion, the necessary and sufficient condition for local and asymptotical stability of the long-run equilibrium is that the trace of $\mathbf{J}$ is negative and the determinant of $\mathbf{J}$ is positive. The trace and determinant are given by

$$\mathrm{tr}\,\mathbf{J} = \underset{(+/-)}{J_{11}}, \tag{40}$$

$$\det \mathbf{J} = -\underset{(+)}{J_{12}}\,\underset{(+/-)}{J_{21}}. \tag{41}$$

When $g'_a(\omega^{**}) < 0$, $J_{21} > 0$, which yields that $\det \mathbf{J} < 0$. In this case, the long-run equilibrium is unstable. When $g'_a(\omega^{**}) > 0$, which is fairly large, I have $J_{11} < 0$ and $J_{21} < 0$. In this case, the long-run equilibrium is stable.

Proposition 2. *Suppose that the technological progress function is decreasing in the wage share around the long-run equilibrium. Hence, the long-run equilibrium is unstable. On the other hand, suppose that the technological progress function is increasing* *in the wage share around the long-run equilibrium. The long-run equilibrium is stable if the response of technological progress to the wage share is relatively strong, whereas it is unstable if the response is relatively small.*

I draw a phase diagram to analyze the transitional dynamics. The equation $\dot{k} = 0$ contains only ω. At most, two ω values exist that satisfy $\dot{k} = 0$. The locus $\dot{\omega} = 0$ is a convex upward curve. The equation $\dot{\omega} = 0$ can be rewritten as $k = k(\omega)$. I have $k'(0) > 0$ when $\omega = 0$ and $k'(1) < 0$ when $\omega = 1$. Accordingly, there is an ω such that $k'(\omega) = 0$ for $\omega \in (0, 1)$. The reason this locus is convex upward is unrelated to the fact that $g_a(\omega)$ is convex upward. Specifically, when $\psi = 0$, $g_a(\omega)$ becomes a monotonically decreasing function of ω and when $\psi = 1$, it becomes a monotonically increasing function of ω. However, in either case, the locus $\dot{\omega} = 0$ is convex upward. Moreover, in the equation $\dot{\omega} = 0$, there is an ω such that $k = 0$, which I define $\omega_{\min}$. Therefore, interval ω is given by $\omega \in [\omega_{\min}, \omega_{\max}]$:

From the above discussion, I obtain Figures 2–5: four kinds of phase diagrams arise.

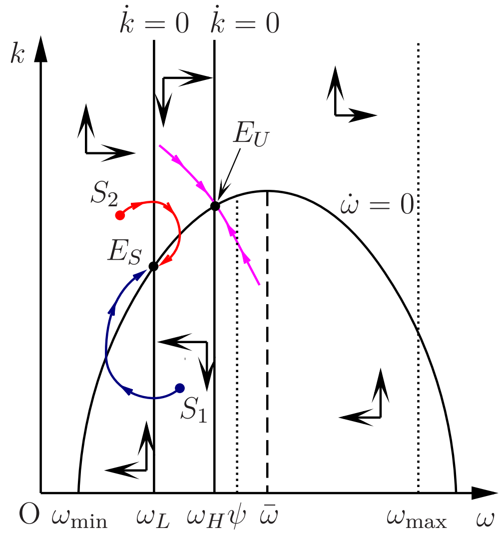

Figure 2: Phase diagram in Case 1

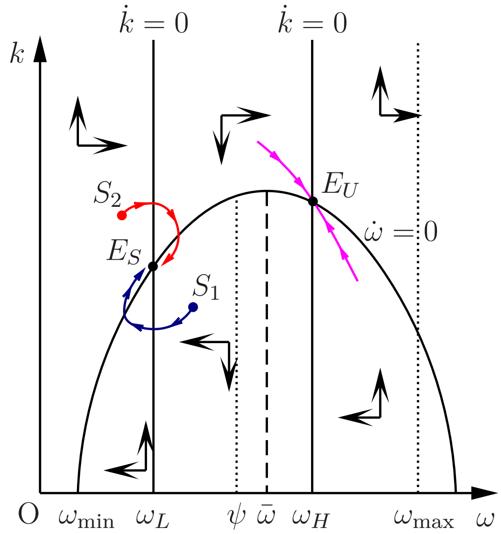

Figure 3: Phase diagram in Case 2

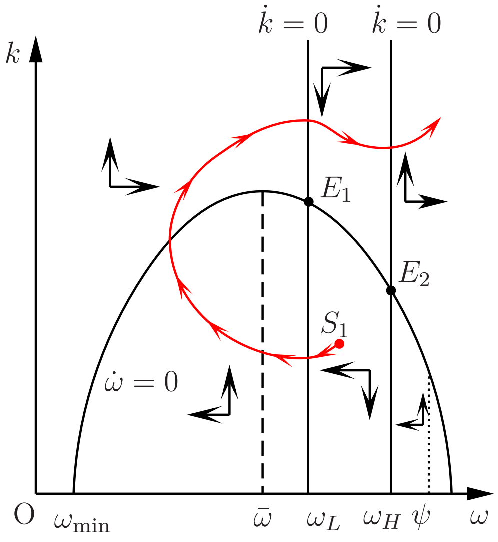

Figure 4: Phase diagram in Case 3

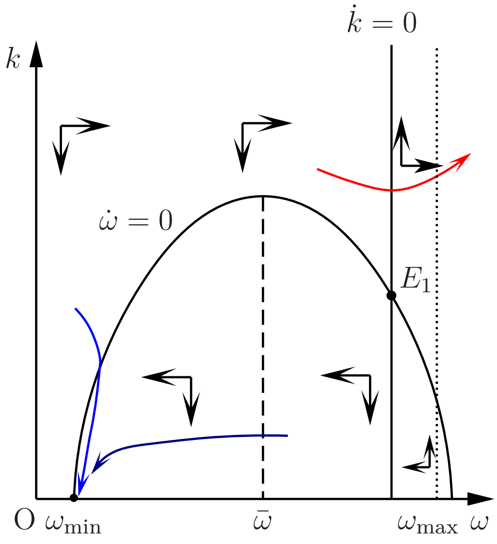

Figure 5: Phase diagram Case 4

In Case 1, two loci of $\dot{k} = 0$ are located to the left of the top of the locus of $\dot{\omega} = 0$ (Figure 2). E_S and E_U denote the stable and unstable equilibria, respectively. If the economy starts at S_1 or S_2, then it converges toward E_S. E_U is a saddle point; if the economy starts to the right of the pink saddle path, it explodes, whereas if it starts to the left of the saddle path, it converges toward E_S. As previously stated, if $g_a'(\omega^{**}) < 0$, the long-run equilibrium is unstable. E_U in Figure 3 corresponds to such a case. By contrast, if $g_a'(\omega^{**}) > 0$ and its degree is relatively small, the long-run equilibrium is unstable. Comparing E_S and E_U in Figure 2, I find that $0 < g_a'(\omega_U) < g_a'(\omega_S)$, which means that E_S is stable because the degree of technological progress response to the wage share is relatively large at E_S. As explained in Section 2, the wage share that maximizes the technological progress rate is $\omega_{\text{optimal}} = \psi$. Note that the wage share that gives the top of the locus $\dot{\omega} = 0$; that is, $\bar{\omega}$ is not ψ. In Figures 2 and 3, $\bar{\omega}$ exceeds ω_{optimal}.

In Case 2, one locus of $\dot{k} = 0$ is located to the left of the top of the $\dot{\omega} = 0$ locus, whereas the other locus of $\dot{k} = 0$ is located to the right of the top of the locus of $\dot{\omega} = 0$ (Figure 3). As in Case 1, the stable and unstable equilibria coexist.

In Case 3, two loci of $\dot{k} = 0$ are located to the right of the top of the locus of $\dot{\omega} = 0$ (Figure 4). The two long-run equilibria E_1 and E_2 are both unstable. This implies that if $\bar{\omega} < \omega_L$, no stable equilibrium exists.

In Case 4, only one locus, $\dot{k} = 0$ exists (Figure 5). In this case, the long-run equilibrium E_1 is the saddle point. The pair (ω, k) explodes over time or converges toward $(\omega_{\min}, 0)$. In the latter case, $k = 0$ at the equilibrium, which has no economic meaning.

The above discussion can be described as follows:

- If ω^{**} is larger than ω_{optimal}, it is unstable because $g_a'(\omega) < 0$ holds at the equilibrium, which is a condition for instability. Moreover, from the analysis of the phase diagram, the long-run equilibrium is unstable when it is located on the right of $\bar{\omega}$, even if it is smaller than ω_{optimal}.

- In Case 1, ω_L is stable whereas ω_H is unstable. Moreover, ω_L is smaller than ω_{optimal}. Hence, if we can increase ω_L, economic growth also increases.

- In Case 2, I obtain $\omega_L < \bar{\omega} < \omega_H$ and $\omega_L < \omega_{\text{optimal}} < \omega_H$. Since $\omega_L < \omega_{\text{optimal}}$, the economic growth rate rises if we can increase ω_L.

- In Case 3, I obtain $\bar{\omega} < \omega_L < \omega_H$ and both equilibria are located on the left of $\bar{\omega}$. In this case, I have $g_a'(\omega_L) > 0$ and $g_a'(\omega_H) > 0$, which implies that the long-run equilibrium can be stable. However, since $\bar{\omega} < \omega_L < \omega_H$, both long-run equilibria are unstable.

- In Case 4, only one long-run equilibrium exists, and $\omega_{\text{optimal}} < \bar{\omega} < \omega_H$. Accordingly, the equilibrium is unstable.

The wage share that maximizes the technological progress rate and economic growth rate is given by $\omega_{\text{optimal}} = \psi$. In this case, $g_a'(\psi) = 0$, resulting in $J_{21} > 0$. Subsequently, I obtain $\det \mathbf{J} < 0$. Hence, the long-run equilibrium in which $\omega = \psi$ is unstable. This suggests that if we can obtain $\omega = \psi$ through an economic policy, such a state cannot last. For $\omega = \psi$, from equation (32), the following equation holds.

$$\frac{s\gamma(1-\psi)}{\varepsilon(1-\psi)-\beta} = \phi_0 + \phi_1(\tau\psi)^{\psi}[\sigma s_f(1-\psi)]^{1-\psi}. \tag{42}$$

It is unlikely that combinations of many parameters satisfy this equation. From the above discussion, the effective interval of the wage share is $\omega_{\min} < \omega < \min\{\psi, \bar{\omega}\}$ under the condition that the long-run equilibrium is stable.

Proposition 3. *Suppose the long-run equilibrium lies within the interval* $\omega_{\min} < \omega^{**} < \min\{\psi, \bar{\omega}\}$. *The economic growth rate rises if we increase the long-run equilibrium wage share.*

This proposition implies that the effective wage share must lie within a specific interval and that we cannot increase the economic growth rate indefinitely by increasing the wage share, even if the short-run equilibrium exhibits a wage-led growth regime.

5 Comparative static analysis in the long-run equilibrium

This section presents a comparative static analysis of long-run equilibrium. To proceed, I require a stable long-run equilibrium, which I assume in the following analysis.

From equation (32), in the long-run equilibrium, the following relationship holds:

$$\underbrace{g(\omega; \sigma, s_f)}_{\text{Demand growth rate}} = \underbrace{g_a(\omega; \tau, \sigma, s_f) + n}_{\text{Supply growth rate}}. \tag{43}$$

The left-hand side is determined by the principle of effective demand; hence, we call it the "demand growth rate." The right-hand side is the sum of the technological progress rate and labor supply growth rate; as such, I call it the "supply growth rate." When a parameter changes, both the left- and right-hand sides change; accordingly, the wage share must change to equalize both sides of the equation (43). The propensity to spend on education investment τ is included only on the right side, whereas the propensity to spend on R&D investment σ and the retained earnings rate s_f are included on both sides. Many Kaleckian models stress the principle of effective demand and underestimate the impact of supply. However, as Dutt (2006) points out, it is reasonable that in the long run, the interaction between the demand and supply sides determines the economic growth rate.

An increase in τ does not affect $g(\omega)$ but shifts the graph of $g_a(\omega)$ upward. As such, at a stable equilibrium, ω declines, which also decreases the economic growth rate. In this case, the long-run equilibrium values of capacity utilization and the economic growth rate also fall.

An increase in σ shifts the graph of $g_a(\omega)$ upward. However, the manner in which the graph of $g(\omega)$ shifts remains unclear. The same holds for an increase in s_f.

This discussion is examined in detail. Let the right-hand sides of the differential equations for ω and k be $F_1(\omega, k; z)$ and $F_2(\omega, k; z)$, respectively. Here, z denotes one of the parameters τ, σ, or s_f. In this case, the long-run equilibrium conditions are

$$F_1(\omega^{**}, k^{**}; z) = 0, \tag{44}$$

$$F_2(\omega^{**}, k^{**}; z) = 0. \tag{45}$$

I completely differentiate both sides of the above equation. From this, I obtain a Jacobian matrix whose elements are equations (36)–(39). Because the long-run equilibrium is stable, $\det \mathbf{J} > 0$. Accordingly, I can use the implicit function theorem to obtain the following equation:

$$\frac{d\omega^{**}}{dz} = \frac{1}{\det \mathbf{J}} \left(-\frac{\partial F_1}{\partial z} J_{22} + \frac{\partial F_2}{\partial z} J_{12} \right), \tag{46}$$

$$\frac{dk^{**}}{dz} = \frac{1}{\det \mathbf{J}} \left(-\frac{\partial F_2}{\partial z} J_{11} + \frac{\partial F_1}{\partial z} J_{21} \right). \tag{47}$$

First, we obtain $J_{22} = 0$. Second, for the long-run equilibrium to be stable, I require $g_a'(\omega^{**}) > 0$. Moreover, when the long-run equilibrium is stable, I obtain $J_{11} < 0$, $J_{12} > 0$, and $J_{21} < 0$.

5.1 An increase in τ

By partially differentiating F_1 and F_2 with respect to τ, I obtain

$$\frac{\partial F_1}{\partial \tau} = -\omega^{**} \underset{(+)}{\frac{\partial g_a}{\partial \tau}} < 0, \tag{48}$$

$$\frac{\partial F_2}{\partial \tau} = -k^{**} \underset{(+)}{\frac{\partial g_a}{\partial \tau}} < 0. \tag{49}$$

From this, I obtain the following equation:

$$\frac{d\omega^{**}}{d\tau} = \underset{(+)}{\frac{1}{\det \mathbf{J}}} \left(\underset{(-)}{\frac{\partial F_2}{\partial \tau}} \underset{(+)}{J_{12}} \right) < 0, \tag{50}$$

$$\frac{dk^{**}}{d\tau} = \underset{(+)}{\frac{1}{\det \mathbf{J}}} \left(-\underset{(-)}{\frac{\partial F_2}{\partial \tau}} \underset{(-)}{J_{11}} + \underset{(-)}{\frac{\partial F_1}{\partial \tau}} \underset{(-)}{J_{21}} \right). \tag{51}$$

Therefore, an increase in τ reduces ω^{**}. However, the effect of increasing τ on k^{**} remains unclear.

For the other endogenous variables, I obtain

$$\frac{du^{**}}{d\tau} = \underset{(+)}{\frac{\partial u}{\partial \omega}} \cdot \underset{(-)}{\frac{d\omega^{**}}{d\tau}} < 0, \tag{52}$$

$$\frac{dg^{**}}{d\tau} = \underset{(+)}{\frac{\partial g}{\partial \omega}} \cdot \underset{(-)}{\frac{d\omega^{**}}{d\tau}} < 0, \tag{53}$$

$$\frac{de^{**}}{d\tau} = k^{**} \underset{(-)}{\frac{du^{**}}{d\tau}} + u^{**} \frac{dk^{**}}{d\tau}, \tag{54}$$

$$\frac{dg_a^{**}}{d\tau} = \frac{dg^{**}}{d\tau} < 0. \tag{55}$$

5.2 An increase in σ

By partially differentiating F_1 and F_2 with respect to σ, I obtain

$$\frac{\partial F_1}{\partial \sigma} = \omega^{**} \left(\theta \underset{(+)}{\frac{\partial \omega_w}{\partial \sigma}} - \underset{(+)}{\frac{\partial g_a}{\partial \sigma}} \right), \tag{56}$$

$$\frac{\partial F_2}{\partial \sigma} = k^{**} \left(\underset{(+/-)}{\frac{\partial g}{\partial \sigma}} - \underset{(+)}{\frac{\partial g_a}{\partial \sigma}} \right). \tag{57}$$

Here, I have

$$\frac{\partial \omega_w}{\partial \sigma} = \delta_1 k^{**} \frac{\partial u(\omega)}{\partial \sigma} > 0. \tag{58}$$

The parameter σ appears in ε in $u(\omega)$, and $\partial\varepsilon/\partial\sigma < 0$. Furthermore, $u(\omega)$ is a decreasing function of ε. Accordingly, $\partial u(\omega)/\partial\sigma > 0$, leading to $\partial\omega_w/\partial\sigma > 0$.

The effect of an increase in σ on g is given by:

$$\frac{\partial g}{\partial \sigma} = \gamma(1-\omega)\frac{s_f\Big\{\beta - (1-\omega)[\alpha s_r(1-s_f)]\Big\}}{[\varepsilon(1-\omega)-\beta]^2} \gtrless 0. \tag{59}$$

From this, I find that

$$\omega^{**} > 1 - \frac{\beta}{\alpha s_r(1-s_f)} \Longrightarrow \frac{\partial g}{\partial \sigma} > 0, \tag{60}$$

$$\omega^{**} < 1 - \frac{\beta}{\alpha s_r(1-s_f)} \Longrightarrow \frac{\partial g}{\partial \sigma} < 0. \tag{61}$$

For the other endogenous variables, I derive the following relationship:

$$\frac{du^{**}}{d\sigma} = \frac{\partial u}{\partial \sigma} + \underset{(+)}{\frac{\partial u}{\partial \omega}} \cdot \frac{d\omega^{**}}{d\sigma}, \tag{62}$$

$$\frac{dg^{**}}{d\sigma} = \underset{(+/-)}{\frac{\partial g}{\partial \sigma}} + \underset{(+)}{\frac{\partial g}{\partial \omega}} \cdot \frac{d\omega^{**}}{d\sigma}, \tag{63}$$

$$\frac{de^{**}}{d\sigma} = k^{**}\frac{du^{**}}{d\sigma} + u^{**}\frac{dk^{**}}{d\sigma}, \tag{64}$$

$$\frac{dg_a^{**}}{d\sigma} = \frac{dg^{**}}{d\sigma}. \tag{65}$$

The way in which an increase in σ shifts the graph of $g(\omega)$ is ambiguous. If the graph shifts upward, g^{**} either increases or decreases, depending on the size of the shift.

5.3 An increase in s_f

By partially differentiating F_1 and F_2 with respect to s_f, I obtain

$$\frac{\partial F_1}{\partial s_f} = \omega^{**}\left(\theta\underset{(+)}{\frac{\partial \omega_w}{\partial s_f}} - \underset{(+)}{\frac{\partial g_a}{\partial s_f}}\right), \tag{66}$$

$$\frac{\partial F_2}{\partial s_f} = k^{**}\left(\underset{(+/-)}{\frac{\partial g}{\partial s_f}} - \underset{(+)}{\frac{\partial g_a}{\partial s_f}}\right). \tag{67}$$

Here, I have

$$\frac{\partial \omega_w}{\partial s_f} = \delta_1 k^{**}\frac{\partial u(\omega)}{\partial s_f} > 0. \tag{68}$$

With $\partial\varepsilon/\partial s_f < 0$, $\partial u(\omega)/\partial s_f > 0$ holds

The effect of an increase in s_f on g is given by

$$\frac{\partial g}{\partial s_f} = \gamma(1-\omega)\frac{\alpha s_r(1-\sigma)(1-\omega) - (1-\sigma-s_r)\beta}{[\varepsilon(1-\omega)-\beta]^2} \gtreqless 0. \tag{69}$$

From this, I find that

$$\omega^{**} < 1 - \frac{\beta(1-\sigma-s_r)}{\alpha(1-\sigma)s_r} \Longrightarrow \frac{\partial g}{\partial s_f} > 0, \tag{70}$$

$$\omega^{**} > 1 - \frac{\beta(1-\sigma-s_r)}{\alpha(1-\sigma)s_r} \Longrightarrow \frac{\partial g}{\partial s_f} < 0 \tag{71}$$

For the other endogenous variables, I obtain

$$\frac{du^{**}}{ds_f} = \frac{\partial u}{\partial s_f} + \underset{(+)}{\frac{\partial u}{\partial \omega}}\cdot\frac{d\omega^{**}}{ds_f}, \tag{72}$$

$$\frac{dg^{**}}{ds_f} = \underset{(+/-)}{\frac{\partial g}{\partial s_f}} + \underset{(+)}{\frac{\partial g}{\partial \omega}}\cdot\frac{d\omega^{**}}{ds_f}, \tag{73}$$

$$\frac{de^{**}}{ds_f} = k^{**}\frac{du^{**}}{ds_f} + u^{**}\frac{dk^{**}}{ds_f}, \tag{74}$$

$$\frac{dg_a^{**}}{ds_f} = \frac{dg^{**}}{ds_f}. \tag{75}$$

The way in which an increase in s_f shifts the graph of $g(\omega)$ is ambiguous. If the graph shifts upward, g^{**} either increases or decreases, depending on the size of the shift.

5.4 Numerical simulations

In this section, I describe the numerical simulations. The purpose of this simulation is to clarify the effect of a change in parameter on the long-run equilibrium values in each case. Reproducing the actual economy is not its purpose; hence, some long-run equilibrium values deviate from actual ones. I set the parameters producing Cases 1 and 2 as listed in Table1. Cases 1 and 2 differ in the settings of ϕ_0, ω_f, and δ_0.

Table 1: Parameter sets in Cases 1 and 2

	Case 1	Case 2
α	0.05	0.05
β	0.05	0.05
γ	0.053	0.053
σ	0.1	0.1
s_r	0.5	0.5
s_f	0.3	0.3
τ	0.1	0.1
ϕ_0	0.015	0.02
ϕ_1	1.5	1.5
ψ	0.5	0.5
θ	0.6	0.6
ω_f	0.6	0.5
δ_0	0.1	0.05
δ_1	0.9	0.9
n	0.008	0.008

The numerical simulation outcomes are summarized in Tables 2 and 3. In these tables, $\lambda_r \equiv (1-s_f)(1-\omega)$ and $\lambda_f \equiv s_f(1-\omega)$ denote the income shares of rentiers and firms, respectively. The workers' income share is their wage share.

From Tables 2 and 3, we obtain the following results. First, an increase in workers' propensity to spend on education investment decreases the long-run equilibrium values of capacity utilization, the employment rate, the economic growth rate, and the wage share. Second, an increase in a firm's propensity to invest in R&D decreases the long-run equilibrium values of capacity utilization, the employment rate, the economic growth rate, and the wage share. Third, an increase in firms' retained earnings ratios decreases the long-run equilibrium values of capacity utilization, the employment rate, the economic growth rate, and the wage share. Fourth, an increase in the labor supply growth rate decreases the long-run equilibrium values of capacity utilization, the employment rate, the economic growth rate, and the wage share.

An increase in τ has a direct positive effect on technological progress but decreases the consumption of workers. This decline in workers' consumption lowers effective demand and consequently lowers the wage share. This drop in the wage share lowers the technological progress rate and in turn the economic growth rate. Therefore, an increase in workers' propensity to invest in education negatively affects growth and distribution.

An increase in σ has a direct positive effect on technological progress but a direct negative effect on firms' equipment investment. A decrease in equipment investment leads to a decline in effective demand, which consequently lowers the wage share. This decline in the wage share decreases the technological progress rate. Overall, the negative effect outweighs the positive effect; hence, an increase in firms' propensity to spend on R&D worsens growth and distribution.

An increase in s_f has a direct positive effect on technological progress and a firm's investment in equipment. An increase in the retained earnings rate lowers the dividends for rentiers, which decreases consumption of rentiers because their income declines. A positive direct effect on firms' equipment investments leads to an increase in effective demand, whereas a decline of consumption among rentiers leads to a decrease in effective demand. Overall, the negative effect outweighs the positive one; hence, effective demand falls, which produces a drop in the wage share, leading to a decline in the technological progress rate and economic growth rate. As such, an increase in the retained earnings rate negatively affects growth and distribution.

Table 2: Results of numerical simulations in Case 1

		τ	σ	s_f	n
	Benchmark	0.101	0.101	0.301	0.0081
ω	0.3396	0.3275	0.3277	0.3352	0.3330
u	0.1512	0.1481	0.1482	0.1500	0.1495
e	0.1731	0.1505	0.1508	0.1649	0.1607
k	1.1452	1.0160	1.0175	1.0996	1.0746
g_a	0.0539	0.0537	0.0538	0.0538	0.0537
g	0.0619	0.0617	0.0618	0.0618	0.0618
λ_r	0.4623	0.4707	0.4706	0.4647	0.4669
λ_f	0.1981	0.2017	0.2017	0.2001	0.2001

Table 3: Results of numerical simulations in Case 2

		τ	σ	s_f	n
	Benchmark	0.101	0.101	0.301	0.0081
ω	0.1916	0.1887	0.1888	0.1906	0.1898
u	0.1204	0.1199	0.1200	0.1201	0.1201
e	0.0257	0.0204	0.0205	0.0239	0.0222
k	0.2138	0.1704	0.1710	0.1992	0.1852
g_a	0.0523	0.0523	0.0523	0.0523	0.0522
g	0.0603	0.0603	0.0603	0.0603	0.0603
λ_r	0.5659	0.5679	0.5679	0.5658	0.5671
λ_f	0.2425	0.2434	0.2434	0.2436	0.2431

6 Conclusions

This study presents a Kaleckian model with endogenous technological progress and conducts short- and long-run analyses. Assuming that technological progress arises from the combination of workers' education investment and firms' R&D investment, the technological progress function becomes convex upward with respect to the wage share, which assures the existence of an optimal wage share that maximizes the technological progress rate.

In the short run, capacity utilization becomes an adjusted variable and the long-run equilibrium exhibits wage-led demand and growth regimes. In the long run, the wage share and capital stock per the effective labor supply become adjustment variables. For the long-run equilibrium to be stable, the technological progress function must increase the wage share around the long-run equilibrium. Moreover, the wage share that is effective in the long run must lie within a specific interval, and a larger wage share does not necessarily increase economic growth, even if the short-run equilibrium is a wage-led growth regime. Furthermore, I obtain four types of long-run phase diagrams: Two of the four cases produce a stable long-run equilibrium whereas the remaining two cases produce an unstable long-run equilibrium.

The comparative static analysis with numerical simulations shows that in every stable case, a parameter change that directly promotes technological progress negatively affects the long-run equilibrium values of the capacity utilization, employment, technological progress, and economic growth rates through a decline in the wage share. This suggests that an economic policy aimed at fostering technological progress in a demand-constrained economy lowers the rates of technological progress and economic growth; this is called the "technological progress paradox."

I obtained my results purely for theoretical analysis. To examine whether the paradox of technological progress arises in an actual economy, I need to perform an empirical analysis based on economic data, which will be left for future research.

Acknowledgement

I would like to thank Editage (www.editage.jp) for English language editing.

Disclosure statement

The author declare no conflicts of interest.

Funding

Not available.

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