Effects of Minimum Wage Share and Wage Gap Reduction on Cyclical Fluctuation: A Goodwin Approach

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Effects of Minimum Wage Share and Wage Gap Reduction on Cyclical Fluctuation: A Goodwin Approach*

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Abstract

This study extends Goodwin's growth cycle model by considering low- and high-skilled workers. Using the parameters obtained from the Japanese economy data, we conduct numerical simulations to reproduce Japanese business cycles. We investigate how the introduction of the minimum wage share and reduction in the wage gap between low- and high-skilled workers affect the wage share and employment rates. The results reveal that introducing the minimum wage share diminishes the amplitude of the fluctuations in both the wage shares and employment rates of the two types of workers. Reducing the wage gap decreases the amplitude of fluctuations in the wage share and employment rate of high-skilled workers and increases the amplitude of fluctuations in the wage share and employment rate of low-skilled workers.

Keywords: growth cycles; low- and high-skilled workers; minimum wage share; wage gap
JEL Classification: E24; E25; E32

1. Introduction

This study presents Goodwin's growth cycle model that considers low- and high-skilled workers and theoretically and empirically investigates how the introduction of the minimum wage share and a reduction in the wage gap between the two types of workers affect economic fluctuations.

Many countries have introduced minimum wages to protect workers to prevent them from receiving extremely low wages. However, mainstream economists believe that minimum wages can have a negative effect on workers. First, even if unemployed workers want to work at a wage lower than the minimum wage, firms cannot hire them because this violates the minimum wage law, which consequently leads to a higher

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unemployment rate. Second, an increase in labor costs owing to high minimum wages diminishes the management of firms and increases the probability of bankruptcy. Simultaneously, firms decrease employment to reduce labor costs. Using data for the US economy, Brown (1988) demonstrates that an increase in minimum wages lowers employment rates. Card (1992) reveals that an increase in the minimum wage in New Jersey increases the employment rate. Card and Krueger (1994) extend Card's (1992) analysis to the US and conclude that an increase in the minimum wage has no definite effect on the employment rate.

The Goodwin model (Goodwin 1967) is based on Karl Marx's view of capitalist economies: business cycles arise from conflicts between workers and capitalists (Marx 1974 [1867]). For this purpose, the author uses the Lotka–Volterra equation developed in the field of mathematical biology. In the Goodwin model, business cycles arise owing to the following reasons. Suppose the employment rate rises for some reason, increasing the bargaining power of workers. The real wage rate increases through the reserve army effect that increases the wage share. This lowers the profit rate, and therefore, the capital accumulation rate declines. If the capital accumulation rate falls below the natural growth rate, unemployment increases. The bargaining power of workers and real wage rate decreases through the reserve army effect, decreasing the wage share. This effectuates an increase in the profit rate, and hence capital accumulation increases. If the capital accumulation rate exceeds the natural growth rate, the employment rate increases. This series of processes is repeated continuously. The Goodwin model shows that a capitalist economy necessarily creates endogenous perpetual business cycles by changing the bargaining positions of workers and capitalists.[1]

The Goodwin model is a model of growth cycles in which the equilibrium output grows at a constant rate; that is, an output trend exists and the actual output fluctuates around the trend. It is significant because it explains business cycles and economic growth simultaneously. In addition, as its structure is relatively simple, it can be extended easily.

Figure 1 shows the annual business cycle of the Japanese economy during 1989Q1–2020Q4. The horizontal and vertical axes correspond to the employment rate and wage share, respectively. Employment rate data are from the *Labour Force Survey* of the Statistics Bureau of Japan, and wage share data are from the *Financial Statements Statistics of Corporations by Industry* of the Ministry of Finance, Japan. Wage share is calculated by dividing labor costs by the sum of labor costs, operating surplus, and capital depreciation.

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[1] This study uses a continuous time version of the Goodwin model. For discrete time versions of the Goodwin model, see Pohjola (1981) and Foley Michl, and Tavani (2019: ch.6&7). Nikolaos, Persefoni, and Tsoulfidis (2022) present an endogenous growth and cycle model based on the idea of Karl Marx. They incorporate the interaction of profitability, investment, employment, technological change, and capital devaluation.

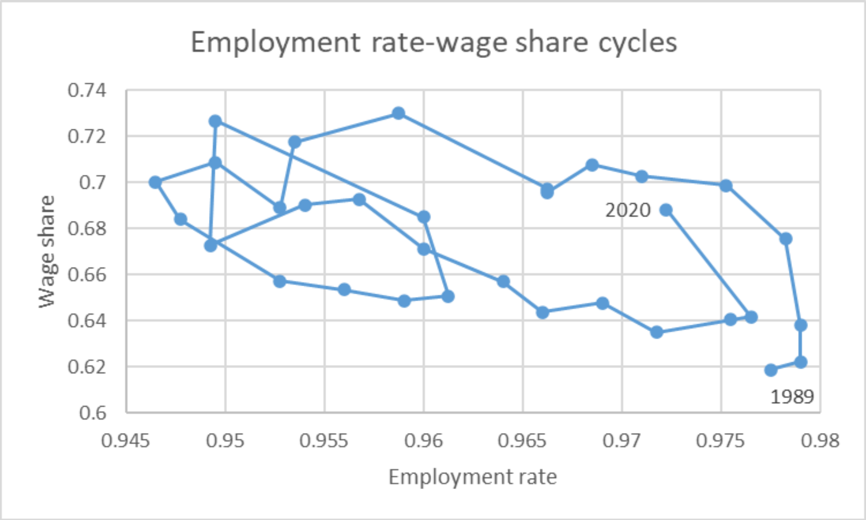

Figure 1: Fluctuations in the employment rate and wage share of the Japanese economy. Source: The *Labour Force Survey* and *Financial Statements Statistics of Corporations by Industry.*

Figure 1 shows that these variables rotate approximately counterclockwise, as suggested by the Goodwin model. These business cycle patterns have been observed in many developed countries. Using data for OECD countries, Zipperer and Skott (2011) show that the employment rate and wage share rotate counterclockwise. We use the Goodwin model to explain the actual business cycle pattern.

Some theoretical studies investigate the effect of the minimum wage on an economy using the Goodwin model. Flaschel and Greiner (2009) show theoretically that the introduction of a minimum wage can diminish the extent of the business cycle. Flaschel and Greiner (2011) introduce minimum wage into an extended Goodwin model that considers dual labor markets and demonstrate that it can diminish the size of the business cycle.[2] However, these studies are purely theoretical and hence cannot quantitatively determine the degree to which the size of business cycles can be diminished.

Some studies conduct empirical analysis based on the Goodwin model.[3]

Harvie (2000) conducts an empirical analysis using data for OECD countries for 1951–1994. The author estimates the growth rates of labor productivity and labor

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[2] Moreover, Flaschel et al. (2012) investigate the effect of the minimum wage on an economy using a Goodwin model.

[3] Ryzhenkov (2009) empirically analyzes the Italian economy by applying a Goodwin model. Duque Garcia (2022) evaluates the rate of profit and accumulation as the determinants of the growth rate in Colombia during 1967–2019 using a generalized vector autoregressive model. The author finds that both variables are statistically significant and, following Marxian theory predictions, positively affect the growth rate. Martin and Rugitsky (2021) empirically examine whether cyclical profit squeeze occurred between 2009 and 2014 in the Brazilian economy based on Goodwin's (1967) and Barbosa-Filho and Taylor's (2006) approaches.

supply, Phillips curve, and capital/output ratio. Then, the study obtains steady-state equilibrium values and fluctuations in the employment rate and wage share. It concludes that a closed orbit around the steady state produced by the theoretical model cannot be obtained; however, business cycles can be roughly captured.

Using quarterly data for the US during 1948–2004, Mohun and Veneziani (2008) investigate the possibility to observe the business cycle produced by the Goodwin model. They conclude that the Goodwin model cannot reproduce long-run business cycles but can reproduce short-run business cycles to some extent.

Using quarterly data for the US economy, Tarassow (2010) conducts an empirical analysis and concludes that the business cycle mechanism of the Goodwin model is appropriate.

Grasselli and Maheshwari (2018) perform an econometric test on a modified Goodwin model in which the savings rate of capitalists, which is equal to unity in the original Goodwin model, is less than unity. Furthermore, they address the methodological and reporting issues in Harvie (2000), which leads to remarkably better results. They conclude that, despite its simplicity and obvious limitations, the performance of the modified Goodwin model can be used as a reference for more sophisticated models of endogenous growth cycles.

Araujo, Dávila-Fernández, and Moreira (2019) present an extended Goodwin model in which the capacity utilization rate, employment rate, and wage share are endogenous variables and theoretically show that limit cycles occur. Additionally, using quarterly data for the US economy during 1948–2016, they estimate the parameters of the model, conduct numerical simulations, and show that different initial values converge to different limit cycles. This interesting result suggests path dependency.

However, these empirical studies do not analyze the effect of the minimum wage on business cycles.

Based on the above observations, this study uses an extended Goodwin model to theoretically and empirically investigate the effect of the minimum wage on growth cycles. The main characteristics of this study are as follows:

First, it classifies workers into two groups: low skilled and high skilled. It considers the dual labor market in the Japanese economy and interprets high- and low-skilled workers as employees in large-sized and small- and medium-sized enterprises. In reality, productivity differentials between enterprises cannot be attributed solely to differences in worker skills. Nevertheless, significant differences exist in labor productivity between large-sized and small- and medium-sized enterprises. To reflect this in our model, it considers two types of workers.

Second, it assumes that both types of workers save their wage income. When investigating the effect of the minimum wage, workers' savings should not be abstracted because it prevents from strictly analyzing the effect of the minimum wage.

Moreover, some workers engage in work around the minimum wage, whereas others do not. Hence, each group is affected differently by the minimum wage.[4]

Third, it conducts both theoretical and numerical analyses. Existing studies that incorporate the minimum wage into the Goodwin model theoretically show that the introduction of the minimum wage can diminish cyclical fluctuations; however, they cannot clarify the extent to which its introduction can diminish cyclical fluctuations. Using Japanese quarterly data during 1989Q1–2020Q4, we estimate the parameters of the Goodwin model, conduct numerical simulations to roughly reproduce business cycles, and quantitatively examine the extent to which business cycles diminish. In this study, the minimum wage corresponds to the minimum wage share. In our model, labor productivity increases at a constant rate. Hence, we must increase the minimum wage at the same rate as labor productivity to maintain the minimum wage share at the target level. Thus, the lower bound is set at the wage-share level.

Finally, our model includes an exogenously given wage gap between low- and high-skilled workers and quantitatively investigates the degree to which cyclical fluctuations diminish when the wage gap is reduced.

Let us compare our model with existing models. As stated above, Flaschel and Greiner (2009, 2011) introduce minimum wages into the Goodwin model. However, the former study does not introduce dual labor markets. The latter study considers two types of workers; however, the mechanism of business cycles in their model is different from that of the original Goodwin model because they use the neoclassical production function: an increase in the employment rate increases the real wage rate, which lowers the labor demand arising from firms' profit maximization and leads to a decline in the employment rate. By contrast, our model uses the Leontief production function. An increase in the employment rate increases the real wage rate, which causes a profit squeeze and thus decelerates capital accumulation and labor demand growth, leading to a decline in the employment rate. Accordingly, we can incorporate dual labor markets into the model, while inheriting Goodwin's original idea. Stamegna's (2024) study considers low- and high-skilled workers in the Goodwin model. Unlike our model, it introduces mark-up pricing over unit labor costs and endogenizes wage inequality between two types of workers. Hence, it assumes an imperfectly competitive goods market, whereas the original Goodwin model assumes a perfectly competitive market. Stamegna's (2024) study uses almost similar production function as our model. It assumes that the growth rate of the wages of low-skilled workers is constant, while the reserve army effect operates only in the labor

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[4] Pasinetti (1962) asserts that when workers save, not only capitalists but also workers own capital stock (see the debate between Pasinetti (1962) and Samuelson and Modigliani (1966)). For simplicity, we do not consider workers' capital accumulation. Van der Ploeg (1984) and Sordi (2001) consider workers' capital accumulation in the Goodwin model. For workers' capital accumulation in demand-led growth models, see Kumar, Schoder, and Radpour (2018), Taylor (2014), and Taylor, Foley, and Rezai (2019).

market of high-skilled workers. By contrast, we treat the wage gap between the two types of workers as a policy parameter. This corresponds to a gap in the bargaining power of workers in large-sized and small- and medium-sized enterprises in Japan. Accordingly, we can examine how the establishment of a cross-company labor management negotiation system, reflected in a decrease in that parameter, affects business cycles.

Using the above framework, we obtain the following two main results.

First, the introduction of the minimum wage share policy diminishes cyclical fluctuations in employment rates and wage shares for the entire economy and for both high- and low-skilled workers. Hence, the stabilizing effect of business cycle of the minimum wage policy is significant.

Second, a reduction in the wage gap between the two types of workers decreases cyclical fluctuations in employment rates and wage shares for the entire economy and high-skilled workers, while it increases fluctuations in the low-skilled workers' employment rate and wage share. However, the wage share of low-skilled workers increases on average, and wage gap reduction is favorable for low-skilled workers' wage share.

The remainder of this study is organized as follows: Section 2 describes the extended Goodwin model. Section 3 theoretically investigates the effects of introducing a minimum wage on business cycles. Section 4 examines how the introduction of endogenous technological progress affects dynamics. Section 5 presents numerical simulations and investigates how the introduction of the minimum wage and reduction in the wage gap between the two types of workers affect business cycles. Finally, Section 6 concludes the study.

2. Model

Let us suppose an economy in which low- and high-skilled workers and capitalists coexist. Firms produce a single good for both consumption and investment using low- and high-skilled workers and capital stock according to the following Leontief production function:

$$Y = \min\{aE_H, bE_L, \sigma K\}, \ \ \sigma > 0, \tag{1}$$

$$a = \bar{a}e^{\phi t}, \ \ \bar{a} > 1, \ \ \phi > 0, \tag{2}$$

$$b = e^{\phi t}, \tag{3}$$

where Y denotes the output, E_H is the employment of high-skilled workers, E_L is the employment of low-skilled workers, K is the capital stock, a is the labor productivity of high-skilled workers, b is the labor productivity of low-skilled workers, and σ is the capital productivity. Labor productivity of both groups increases at the same constant rate ϕ, which is exogenous. Inequality $\bar{a} > 1$ implies that the labor productivity level of high-skilled workers always exceeds that of low-skilled workers.

We assume that firms adopt cost-minimizing behaviors, and accordingly, from equation (1), they operate at a point where $aE_H = bE_L = \sigma K$. Let L denote the labor supply. Then, the employment rates of high-skilled workers, low-skilled workers, and the entire economy, respectively, are follows:

$$x_H = \frac{\sigma K}{aL}, \tag{4}$$

$$x_L = \frac{\sigma K}{bL} = \bar{a} x_H, \tag{5}$$

$$x = \frac{E}{L} = \frac{E_H + E_L}{L} = x_H + x_L = (1 + \bar{a}) x_H. \tag{6}$$

We assume that the labor supply L grows at a constant rate n, which is exogenous. From equations (4)–(6), the dynamics and values of x_L and x can be obtained from those of x_H. This property is used in the numerical simulations described below.

We assume that the growth rate of the real wage rate of high-skilled workers w_H is an increasing function of the employment rate of high-skilled workers x_H, specified as follows:

$$\frac{\dot{w}_H}{w_H} = -\alpha + \beta x_H, \quad \alpha > 0, \ \ \beta > 0, \tag{7}$$

where α denotes the constant term of the real wage Phillips curve and β is the response of the growth rate of the real wage rate to the employment rate of high-skilled workers. High-skilled workers belong to labor unions, and the real wage rate changes through negotiations between them and managers. When the employment rate is high, the bargaining power of labor unions is high, leading to a large change in the real wage rate. Conversely, when the employment rate is low, the bargaining power of labor unions is low, engendering a small change in the real wage rate.

Let us suppose that the real wage rate of high-skilled workers is higher than that of low-skilled workers by a constant factor:

$$w_H = \gamma w_L, \quad \gamma > 1, \tag{8}$$

where γ denotes the wage gap and is a positive constant. A similar specification is adopted by Lavoie (2009), Sasaki, Matsuyama, and Sako (2013), and Sasaki (2016). From equation (8), the real wage rate of the entire economy is given by

$$w = \frac{E_H}{E} w_H + \frac{E_L}{E} w_L = \frac{\bar{a} + \gamma}{(1 + \bar{a})\gamma} w_H. \tag{9}$$

Equation (9) shows that the real wage rate of the entire economy is proportional to that of the high-skilled workers. From equations (8) and (9), the wage shares of high-skilled workers, low-skilled workers, and the entire economy, respectively, are given by

$$y_H = \frac{w_H E_H}{Y} = \frac{w_H}{a}, \tag{10}$$

$$y_L = \frac{w_L E_L}{Y} = \frac{\bar{a}}{\gamma} y_H, \tag{11}$$

$$y = \frac{w_H E_H + w_L E_L}{Y} = y_L + y_H = \frac{\bar{a} + \gamma}{\gamma} y_H. \tag{12}$$

From equations (10)–(12), the dynamics and values of y_L and y can be obtained. This property is used for the numerical simulations introduced later in this study.

By definition, the profit rate is equal to capital productivity multiplied by profit share.

$$r = \sigma(1 - y) = \sigma\left(1 - \frac{\bar{a} + \gamma}{\gamma} y_H\right). \tag{13}$$

Let us suppose that low- and high-skilled workers save their wage incomes at constant rates s_w^L and s_w^H, respectively, and that capitalists save their profit incomes at a constant rate s_c. Thus, the savings/capital stock ratio is

$$\frac{S}{K} = \frac{s_w^L w_L E_L + s_w^H w_H E_H + s_c r K}{K} = s_w^L \sigma \frac{\bar{a}}{\gamma} y_H + s_w^H \sigma y_H + s_c \sigma \left(1 - \frac{\bar{a} + \gamma}{\gamma} y_H\right), \tag{14}$$

where we assume that $s_w^L < s_w^H < s_c$. This assumption is reasonable because capitalists' income is the highest, whereas low-skilled workers' incomes are the lowest.

Let δ denote the capital depreciation rate. The net investment $\dot{K}$ is equal to the gross investment I minus the capital depreciation δK. Hence, the capital accumulation rate is

$$\frac{\dot{K}}{K} = \frac{I}{K} - \delta, \quad 0 < \delta < 1. \tag{15}$$

Goods market clearing is attained when total saving is equal to total investment, that is, $S = I$. Taking the logarithms of equation (4), differentiating them with respect to time, and applying equations (14) and (15) to the resultant expressions, we obtain the rate of change in the employment rate of high-skilled workers as follows:

$$\frac{\dot{x}_H}{x_H} = \frac{I}{K} - \delta - \phi - n = s_w^L \sigma \frac{\bar{a}}{\gamma} y_H + s_w^H \sigma y_H + s_c \sigma \left(1 - \frac{\bar{a} + \gamma}{\gamma} y_H\right) - \delta - \phi - n. \tag{16}$$

Using equations (7) and (10), we obtain the rate of change in the wage share of high-skilled workers as follows:

$$\frac{\dot{y}_H}{y_H} = \frac{\dot{w}_H}{w_H} - \frac{\dot{a}}{a} = -\alpha + \beta x_H - \phi. \tag{17}$$

From equations (16) and (17), our model can be reduced to the following system of differential equations:

$$\dot{x}_H = \left[(s_c \sigma - \delta - \phi - n) - \sigma\left(\frac{\bar{a} + \gamma}{\gamma} s_c - s_w^H - \frac{\bar{a}}{\gamma} s_w^L\right) y_H\right] x_H, \tag{18}$$

$$\dot{y}_H = -[(\alpha + \phi) - \beta x_H] y_H. \tag{19}$$

The model comprising equations (18) and (19) has the same structure as that of Goodwin (1967), that is, the typical Lotka–Volterra equation. Therefore, the dynamics of the employment rate and wage share of high-skilled workers show closed orbits if a steady-state equilibrium exists in the first quadrant of the (x_H, y_H) plane.

We define the steady state as the situation in which $\dot{x}_H = \dot{y}_H = 0$. When the right-hand sides of equations (18) and (19) are zero, we obtain the steady-state values of the employment rate and wage share of high-skilled workers as follows:

$$x_H^* = \frac{\alpha + \phi}{\beta}, \tag{20}$$

$$y_H^* = \frac{s_c\sigma - \delta - \phi - n}{\sigma\left(s_c\dfrac{\bar{a}+\gamma}{\gamma} - s_w^H - \dfrac{\bar{a}}{\gamma}s_w^L\right)} = \frac{\gamma(s_c\sigma - \delta - \phi - n)}{\sigma[(\bar{a}+\gamma)s_c - \bar{a}s_w^L - \gamma s_w^H]}. \tag{21}$$

Hereafter, a variable with "$*$" denotes the steady-state value of the variable. The origin $(x_H, y_H) = (0,0)$ is also in a trivial steady state. However, in this case, both the employment rate and wage share are zero; hence, this steady state is economically meaningless.

Using the steady-state values of x_H^* and y_H^*, we obtain the steady-state values of the other endogenous variables as follows:

$$x_L^* = \frac{\bar{a}(\alpha+\phi)}{\beta}, \tag{22}$$

$$x^* = (1+\bar{a})\frac{\alpha+\phi}{\beta}, \tag{23}$$

$$y_L^* = \frac{\bar{a}(s_c - \delta - \phi - n)}{\sigma[(\bar{a}+\gamma)s_c - \bar{a}s_w^L - \gamma s_w^H]}, \tag{24}$$

$$y^* = \frac{(\bar{a}+\gamma)(s_c\sigma - \delta - \phi - n)}{\sigma[(\bar{a}+\gamma)s_c - \bar{a}s_w^L - \gamma s_w^H]}. \tag{25}$$

All employment rates and wage shares must be greater than zero or less than unity. Hence, the following restrictions are required:

$$\frac{\beta}{\alpha+\phi} > 2, \tag{26}$$

$$s_c\sigma - \delta > \phi + n, \tag{27}$$

$$s_c > \frac{\bar{a}}{\bar{a}+\gamma}s_w^L + \frac{\gamma}{\bar{a}+\gamma}s_w^H, \tag{28}$$

$$(s_c\sigma - \delta) - (\phi + n) < \sigma\left(s_c - \frac{\bar{a}}{\bar{a}+\gamma}s_w^L - \frac{\gamma}{\bar{a}+\gamma}s_w^H\right). \tag{29}$$

Equation (26) indicates that the reserve army effect β is relatively large. Equation (27) shows that the warranted growth rate $s_c\sigma - \delta$ is larger than the natural growth rate $\phi + n$. Setting $y_H = 0$ in equation (14), the warranted growth rate corresponds to an extreme growth rate such that the wage share is zero and profit share is unity. In our model, full capacity utilization is imposed; hence, the warranted growth rate is the actual growth rate. This growth rate is higher than the natural growth rate, which causes an increase in the employment rate and, in turn, the wage rate, effecting an increase in the wage share from zero to positive. If the growth rate when the wage share is zero is lower than the natural growth rate, the employment rate declines further; hence, the wage share cannot increase from zero to positive. Therefore, equation (27) is a necessary condition for the equilibrium value of wage share to be positive. Similarly, for such a mechanism to work, a large reserve army effect is required, as reflected in equation (26). Equation (28) indicates that the savings rate of capitalists is higher than the average savings rate of workers. When equations (26) and (27) hold, both employment share and wage share continue to increase, and they sometimes explode unless other mechanisms work. In the Goodwin model, the profit squeeze owing to wage increases and the resultant deceleration of capital accumulation render stability to the dynamical system. As workers save in our model, for an increase in the wage share to lower capital accumulation, that is, to lower savings, the savings rate of capitalists must be higher than the average savings rate of workers. Therefore, equation (28) is necessary. Nevertheless, equation (28) is insufficient. For an increase in the wage share to lower the employment rate, the realized growth rate when the profit squeeze decreases capital accumulation must be lower than the natural growth rate so as to lower the employment rate. The negative effect of a change in the wage share on the actual growth rate is given by $\sigma \times (\text{saving rate of capitalists} - \text{average saving rate of high and low skilled workers})$. If this effect is stronger than $(\text{warranted growth rate} - \text{natural growth rate})$ in equation (27), an increase in wage share lowers the employment rate. This is an implication of equation (29).

Based on the above results, we derive a phase diagram of the employment rate and wage share of high-skilled workers (Figure 2). The horizontal and vertical axes correspond to the employment rate and wage share of high-skilled workers, respectively. Point E corresponds to the steady state. An economy starting from initial value S_1 moves along the corresponding closed orbit and returns to point S_1. This process is repeated endlessly. Additionally, an economy starting from the initial value S_2 moves along the corresponding closed orbit and returns to point S_2, implying that different closed orbits exist for different initial values. Thus, the employment rate and wage share of high-skilled workers continue to fluctuate periodically. When (x_H, y_H) moves along a closed orbit, (x_L, y_L) and (x, y) move along their corresponding closed orbits.

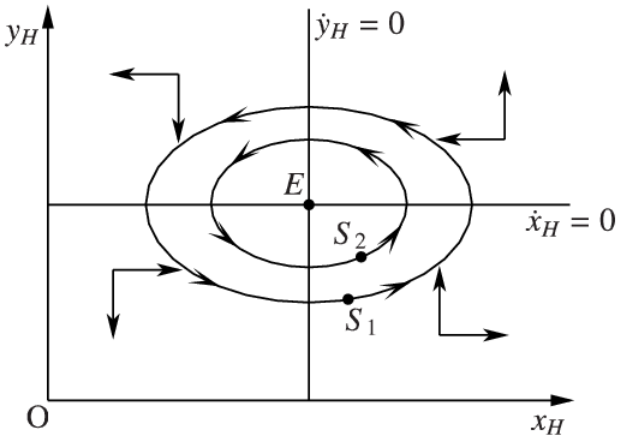

Figure 2: Closed orbits of employment rate and wage share of high-skilled workers

3. Introduction of minimum wage share

This section introduces the minimum wage into our model and investigates its effect on business cycles. Flaschel and Greiner (2009) theoretically show that the introduction of a minimum wage can diminish fluctuations in business cycles in the Goodwin model. In their model, labor productivity is constant and does not grow; hence, the minimum wage corresponds to the minimum wage share, restricting business cycles. However, in our model, labor productivity grows at a constant rate; therefore, we must assume that the minimum wage continues to increase at the same rate as the growth rate of labor productivity for the minimum wage to correspond to the minimum wage share. Based on this assumption, the lower bound is set to the wage share. If we set the minimum wage share to a value lower than the steady-state wage share (Figure 3), the economy starting from point S continues to move to the right from point P, reaches point Q, and then rides in a closed orbit smaller than the original orbit. This finding implies that introducing a minimum wage share can mitigate business cycles. However, if we set the minimum wage share to a value higher than the steady-state wage share (Figure 4), the economy converges to point Q, where the employment rate is zero. Therefore, a minimum wage-share policy should be implemented with caution.

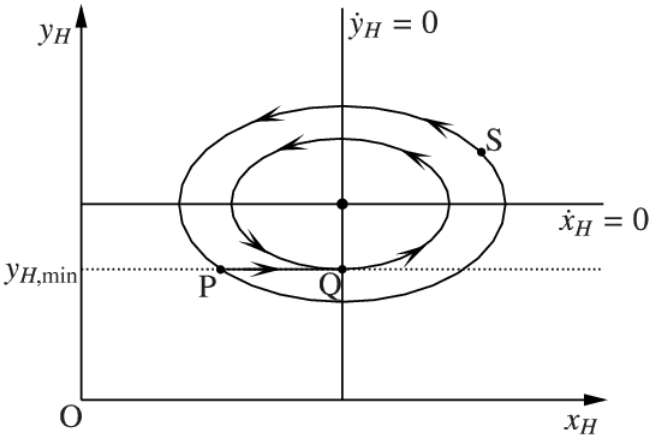

Figure 3: Dynamics of employment rate and wage share with appropriate minimum wage share

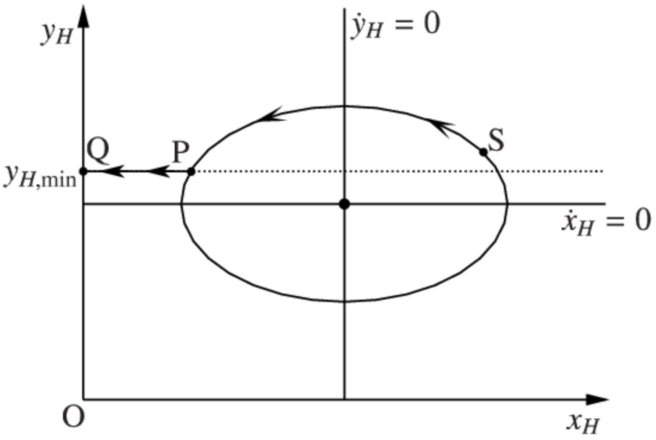

Figure 4: Dynamics of employment rate and wage share with inappropriate minimum wage share

4. Endogenous technological progress

This section introduces endogenous technological progress into the model. According to the theory of induced technical innovation (Drandakis and Phelps 1966), we assume that the growth rate of labor productivity ϕ is an increasing function of the wage share y as follows:[5]

$$\phi = \phi(y), \quad \phi'(y) > 0. \tag{30}$$

An increase in wage share means a rise in labor costs; hence, firms adopt more labor-saving techniques to reduce their labor costs, leading to an increase in labor productivity. Similar specifications are used in Shah and Desai (1981), Van der Ploeg (1987), Foley (2003), Julius (2005), and Tavani, Flaschel, and Taylor (2011).[6]

Using the relationship between y and y_H from equation (12), we obtain

$$\phi = \phi\left(\frac{\bar{a}+\gamma}{\gamma} y_H\right), \quad \frac{\partial \phi}{\partial y_H} = \frac{\bar{a}+\gamma}{\gamma}\phi'(y) > 0. \tag{31}$$

In this case, the elements of Jacobian matrix $\mathbf{J}$ corresponding to the extended Goodwin model are given by

$$J_{11} = \frac{\partial \dot{x}_H}{\partial x_H} = 0, \tag{32}$$

$$J_{12} = \frac{\partial \dot{x}_H}{\partial y_H} = -x_H^* \left[\frac{\bar{a}+\gamma}{\gamma}\phi'(y^*) + \sigma\left(\frac{\bar{a}+\gamma}{\gamma}s_c - s_w^H - \frac{\bar{a}}{\gamma}s_w^L\right)\right] < 0, \tag{33}$$

$$J_{21} = \frac{\partial \dot{y}_H}{\partial x_H} = \beta y_H^* > 0, \tag{34}$$

$$J_{22} = \frac{\partial \dot{y}_H}{\partial y_H} = -y_H^* \left[\frac{\bar{a}+\gamma}{\gamma}\phi'(y^*)\right] < 0. \tag{35}$$

The Routh–Hurwitz stability criterion states that the necessary and sufficient conditions for the local and asymptotic stability of the steady state are that the trace of $\mathbf{J}$ ($\mathrm{tr}\,\mathbf{J}$) is negative and determinant of $\mathbf{J}$ ($\det\mathbf{J}$) is positive. As both $\mathrm{tr}\,\mathbf{J} = J_{11} + J_{22} = J_{22} < 0$ and $\det\mathbf{J} = J_{11}J_{22} - J_{12}J_{21} = -J_{12}J_{21} > 0$ are satisfied, the steady state is locally and asymptotically stable.

The phase diagram for this case is shown in Figure 5. The employment rate and wage share of skilled workers rotate counterclockwise and converge to a steady state.

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[5] Specifically, we specify $a(t) = \bar{a}\exp[\int_0^t \phi(y(s))ds]$ and $b(t) = \exp[\int_0^t \phi(y(s))ds]$. From these, we obtain $d\ln a(t)/dt = d\ln b(t)/dt = d/dt \int_0^t \phi(y(s))ds = \phi(y(t))$. Hence, we can replace ϕ in equations (2) and (3) with $\phi(y)$.

[6] For micro-founded Goodwin models, see Tavani (2012, 2013) and Stamegna (2024).

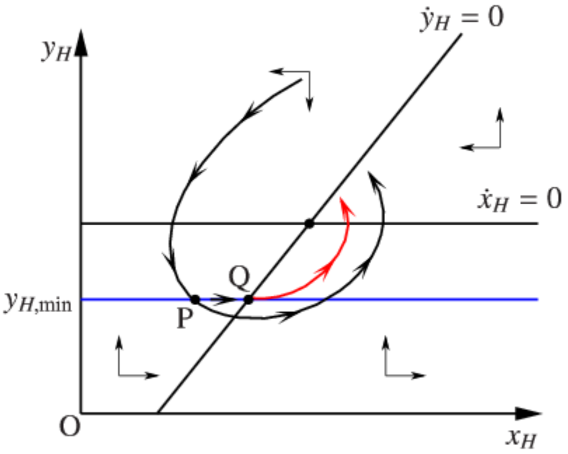

Figure 5: Phase diagram with endogenous technological progress

Boggio (2006) introduces the Kaldor–Verdoorn law into the Goodwin model to endogenize labor productivity growth. According to the Kaldor–Verdoorn law, the growth rate of labor productivity is an increasing function of the output growth rate. The author reveals that increasing returns in the form of the Kaldor–Verdoorn law cause instability.[7]

The difference lies in the sign of J_{22}, which is positive in Boggio (2006) and negative in our study. According to Boggio (2006), the growth rate of labor productivity increases with the capital accumulation rate, which is a decrease in the wage share. Overall, labor productivity growth decreases the wage share. Accordingly, an increase in the wage share decreases labor productivity growth, which increases the wage share. Hence, there is a positive feedback effect, effectuating destabilizing dynamics. In contrast, in our model, the growth rate of labor productivity increases the wage share. Accordingly, an increase in the wage share increases labor productivity growth, which decreases the wage share. Hence, there is a negative feedback effect leading to stabilizing dynamics.

If we introduce the minimum wage share into the abovementioned model, as in Figure 5, the employment rate and wage share of skilled workers reach the lower bound at point P, move along the boundary to the right, reach point Q, and then rotate counterclockwise, converging to the steady state. This means that the amplitude of the fluctuations diminishes to some extent when the minimum wage share is set to a level lower than the steady-state value.

The following section conducts some numerical simulations. For this purpose, we specify the function ϕ as follows:

$$\phi = \phi_0 + \phi_1 y = \phi_0 + \phi_1 \left(\frac{\bar{a}+\gamma}{\gamma}\right) y_H, \quad \phi_0 > 0, \ \ \phi_1 > 0. \tag{36}$$

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[7] Related to this, Aguiar-Conraria (2008) introduces both endogenous technological progress and capital-labor substitution into the Goodwin model. Then, the author shows that the stabilizing effect of smooth labor-capital substitution dominates the destabilizing effect of endogenous technological progress.

For ease of computation, we employ a linear function.

5. Numerical simulations of Goodwin growth cycles

5.1 Baseline simulation

We conduct numerical simulations using Japanese data to demonstrate that our model approximately reproduces the business cycles of the Japanese economy. We set 13 parameters: the three types of savings rates, capital productivity, coefficient and constant term of the real wage Phillips curve, wage gap between low- and high-skilled workers, growth rate of labor productivity, coefficient and constant term of the technological progress function, initial value of labor productivity of high-skilled workers, capital depreciation rate, and population growth rate.

First, we find the savings rates of two types of workers and capitalists. We use data from the *Survey of Households' Financial Behavior* conducted by the Bank of Japan from 2004 to 2020. This survey provides savings rates according to household income levels, classified into five income categories. We consider workers with income levels lower than 3 million yen as low-skilled workers and those with income levels between 3 million yen and 12 million yen as high-skilled workers and then obtain each class's average saving rate. For the savings rate of capitalists, we use the savings rate of households with income levels higher than 12 million yen. From the data, we obtain the annual average savings rates between 2004 and 2020 as follows: $s_w^L = 0.05$, $s_w^H = 0.11$, and $s_c = 0.17$. These savings rates satisfy the condition $s_w^L < s_w^H < s_c$.

Second, we obtain the capital productivity. Capital productivity is the inverse of the capital coefficient, which is the real capital stock/potential GDP ratio. Data on capital coefficients are obtained from the *National Accounts of Japan* from 1989 to 2020. The data on potential GDP are calculated using the real GDP provided by the *National Accounts of Japan* and GDP gap provided by the Bank of Japan. From this, we obtain $\sigma = 0.282$ as the annual average capital coefficient during 1989–2020.

Third, we estimate the real wage Phillips curve.[8] Details of the estimation are provided in the Appendix. We obtain $\alpha = 0.2$ and $\beta = 0.21$. However, $\alpha = 0.2$, that is, the intercept of the real wage Phillips curve is large to satisfy the constraint given by equation (26). We think that the coefficient β is important because it reflects the response of real wage growth with respect to the employment rate. Then, we use β as it is but set α flexibly so that the steady state values are approximately close to the average values of the Japanese economy. Therefore, we use $\alpha = 0.08$ and $\beta = 0.21$.

Fourth, we obtain the wage gap between high- and low-skilled workers. We regard university and high school graduates as high- and low-skilled workers, respectively. Using data of the *Basic Survey on Wage Structure*, we find that $\gamma = 1.34$ as the average value during 1996–2020.

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[8] This study estimates the real wage Phillips curve. However, Flaschel, Kauermann, and Semmler (2007) estimate the nominal wage Phillips curve and price Phillips curve separately.

Fifth, based on the *National Accounts of Japan*, the capital depreciation rate is set to $\delta = 0.02$ as the quarterly average value during 1994–2020.

Sixth, the quarterly average labor supply growth rate is set to $n = 0.001$. From the data of the *Labor Force Survey*, we calculate $n = \sqrt[124]{L_{2020}/L_{1989}} - 1 = 0.00087$. Accordingly, we use 0.001 as a value close to 0.00087.

Seventh, we set the technology gap between high- and low-skilled workers at $\bar{a} = 1.1$. From the production function, we obtain $\bar{a} = E_L/E_H$. Hence, if we obtain the data for E_L and E_H, we may calculate $\bar{a}$. From the *Labour Statistics Annual Report* by the Ministry of Health, Labour, and Welfare, we obtain the ratio of workers with high school education E_L to those with at least university education E_H. However, this calculation leads to $\bar{a} < 1$ that contradicts our assumptions. Therefore, we use $\bar{a} = 1.1$ to satisfy the constraints given by equations (28) and (29). The value $\bar{a} = 1.1$ means that the labor productivity of high-skilled workers is 10% higher than that of low-skilled workers.

Finally, for the growth rate of labor productivity, we use $\phi = 0.01$ to satisfy the constraints given by equations (26), (27), and (29). We calculated ϕ by dividing potential outputs by the number of employees times average working hours. However, the obtained value of ϕ is 0.004, which is small, not satisfying the parameter constraints. Hence, we use a relatively large value for ϕ. For endogenous technological progress case, we use $\phi_0 = \phi_1 = 0.005$ because we obtain $\phi = \phi_0 + \phi_1 y = 0.01$ when $y = 1$.

Table 1 presents the estimates. These estimated parameter values satisfy all the parametric conditions given by equations (26)–(29).

Table 1: Estimates of 13 parameters

α	β	s_w^L	s_w^H	s_c	γ	δ	n	$\bar{a}$	σ	ϕ	ϕ_0	ϕ_1
0.08	0.21	0.05	0.11	0.17	1.34	0.02	0.001	1.1	0.282	0.01	0.005	0.005

Table 2 presents the equilibrium values of employment rates and wage shares using the values in Table 1.

Table 2: Estimated equilibrium values

	Entire economy	Low-skilled workers	High-skilled workers
Employment rate	0.90	0.471429	0.428571
Wage share	0.69008	0.311102	0.378979

The average employment rate and wage share of the entire economy between 1989Q1 and 2020Q4 are 0.96 and 0.67, respectively, and hence, our results are nearly consistent with the actual values.

Furthermore, initial values are required for the numerical simulations. The initial values of employment rates and wage shares are calculated as follows: First, from the 1989Q1 data, the employment rate and wage share for the entire economy are $x_{1989Q1} = 0.977$ and $y_{1989Q1} = 0.592$, respectively, which are used as the initial values of the employment rate and wage share. Numerical simulations are based on the dynamics of x_H and y_H, and the initial values of x_H and y_H are necessary. Then, we obtain $x_H(0) = x_{1989Q1}/(1+\bar{a}) = 0.465238$ and $y_H(0) = y_{1989Q1}\,\gamma/(\bar{a}+\gamma) = 0.325115$ from equations (4) and (10), respectively. In addition, we obtain $x_L(0) = \bar{a}x_H(0) = 0.511762$ and $y_L(0) = (\bar{a}/\gamma)y_H(0) = 0.266885$ from equations (5) and (11), respectively.

Using these estimated initial values and parameters, we conducted numerical simulations (Figures 6–8).[9] In Figures 6–8, the horizontal and vertical axes correspond to employment rates and wage shares, respectively. Figures 6, 7, and 8 show closed orbits for the entire economy, high-skilled workers, and low-skilled workers, respectively.

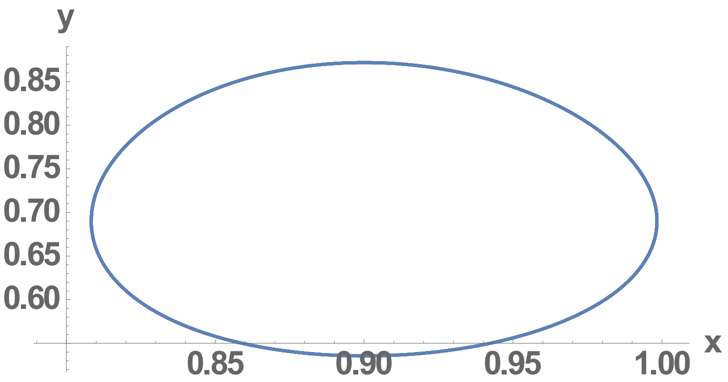
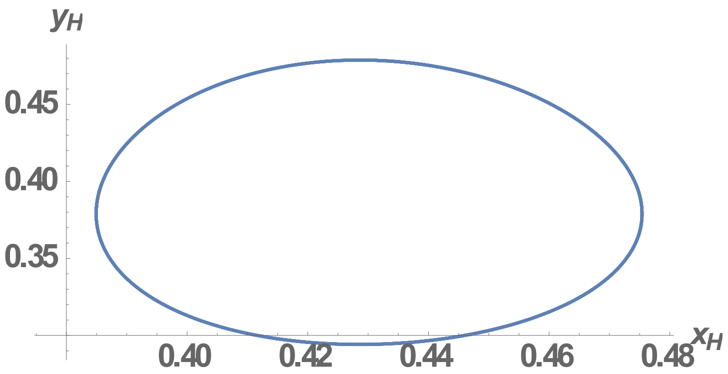


Figure 6: Estimated closed orbit for the entire economy



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[9] Numerical analysis of the differential equations is conducted with Mathematica 10. Our numerical simulations are based on the differential equations of x_H and y_H and their initial values $(x_H(0), y_H(0))$, from which we obtain the dynamics of (x_H, y_L) and (x, y). Hence, we do not use the differential equations of x_L, y_L, x, and y for numerical simulations. Alternatively, we can use the differential equations of all the endogenous variables and their initial values. Both methods produce the same results.

Figure 7: Estimated closed orbit for high-skilled workers

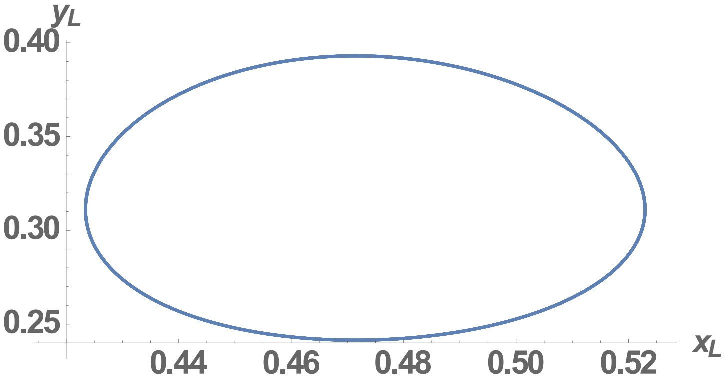

Figure 8: Estimated closed orbit for low-skilled workers

5.2 Minimum wage share

Using the obtained parameters, we numerically investigate the effect of the minimum wage share. In 2023, the weighted average of minimum wages across Japan is approximately 1000 yen. If we increase this by 20%, we have 1200 yen. From the data of the *Handbook of Labour Statistics* by the Ministry of Health, Labour, and Welfare, we have average working hours to obtain nominal monthly salaries for workers who receive minimum wages. Next, by dividing the monthly salary by the consumer price index, we obtain the real wage rate for minimum wage workers. Finally, using the method shown in the Appendix, we convert the resultant minimum wage into the minimum wage share, which leads to $y_{H,\min} = 0.278$. However, this minimum wage share is low to work as a binding constraint, because it is lower than the bottom of the closed orbit for high-skilled workers (Figure 7). Specifically, the present minimum wage in Japan is low, and consequently, it cannot stabilize business cycles. Hence, a significantly higher minimum wage is required. The Prime Minister of Japan, Kishida Fumio, has set a new goal of increasing the minimum wage by up to 1500 yen by the middle of the 2030s. Moreover, many labor unions in Japan require a minimum wage of 1500 yen. Therefore, we use 1500 yen as the minimum wage and obtain $y_{H,\min} = 0.348$ for the minimum wage share of high-skilled workers, which works as a binding constraint.

The results are shown in Figures 9–11. The blue lines correspond to the benchmark orbits, whereas the red lines correspond to the orbits with minimum wage share. Figure 9 illustrates the dynamics of the entire economy. With the minimum wage share for high-skilled workers set at 0.348, we use $0.348 \times (\bar{a} + \gamma)/\gamma = 0.633672$ for the minimum wage share for the entire economy. In Figure 10, after the combination of the employment rate and wage share reaches point P, it moves horizontally toward point Q, after which it rides on the new smaller closed orbit. Although not shown, these movements are similar in Figures 9 and 11.

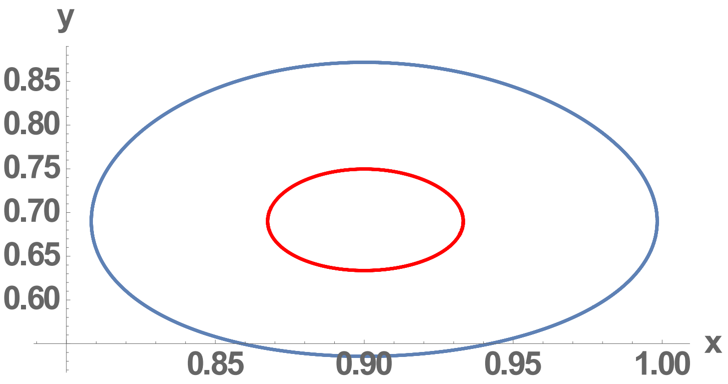

Figure 9: Dynamics for the entire economy with minimum wage share

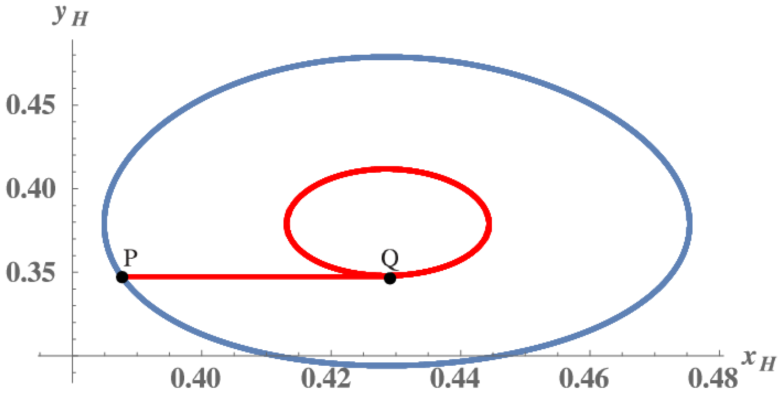

Figure 10: Dynamics for high-skilled workers with minimum wage share

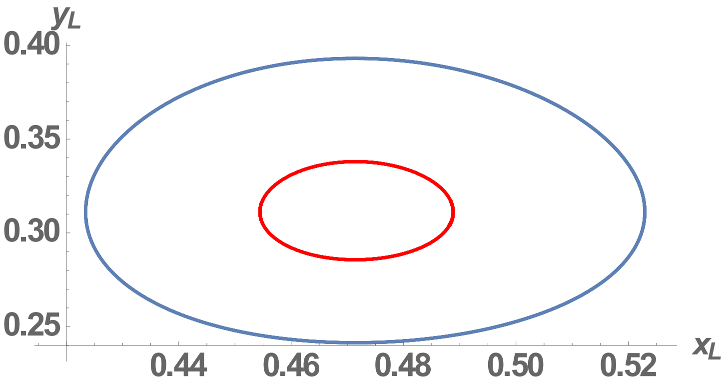

Figure 11: Dynamics for low-skilled workers with minimum wage share

Figures 9–11 show that the introduction of the minimum wage share considerably diminishes the extent of cyclical fluctuations. Nevertheless, the introduction of the minimum wage share requires a policymaker to increase the level of the minimum wage at the same rate as the growth rate of labor productivity, which in turn requires the policymaker to precisely measure the growth rate of labor productivity.

Figure 12 illustrates the dynamics of high-skilled workers in endogenous technological progress. The combination of employment and wage share reaches a minimum of 0.348 at point P, moves horizontally toward point Q, and then rides along a smaller green-colored cyclical path. Finally, it converges to a steady state.

Accordingly, fluctuations in business cycles diminish, as in the case of exogenous technological progress. The same holds true for the dynamics of low-skilled workers and entire economy.

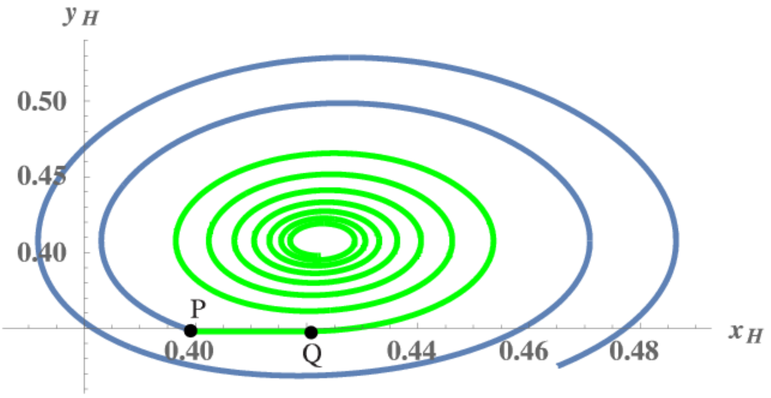

Figure 12: Dynamics for high-skilled workers with minimum wage share in case of endogenous technological progress

5.3 Wage gap reduction

Sections 3 and 5.2 show that the appropriate introduction of the minimum wage share diminish fluctuations in business cycles. In this section, as an alternative policy, we investigate the effect of reducing the wage gap between high- and low-skilled workers on business cycles. A reduction in the wage gap is similar to “equal pay for equal work” that the Japanese government pursues.

If the wage gap γ decreases, y_H^* and y^* decrease but y_L^* increases. However, x_H^*, x_L^*, and x^* do not change. Therefore, the loci of $\dot{x}_H = 0$ and $\dot{x} = 0$ shift downward, whereas the locus of $\dot{x}_L = 0$ shifts upward. However, loci of $\dot{y}_H = 0$, $\dot{y}_L = 0$, and $\dot{y} = 0$ do not change. The shifts at $\dot{x}_H = 0$, $\dot{x} = 0$, and $\dot{x}_L = 0$, respectively, are shown in Figures 13–15. However, it is uncertain whether the amplitudes of the fluctuations decreases or increases. Accordingly, we conduct numerical simulations to examine how a reduction in the wage gap affects business cycles.

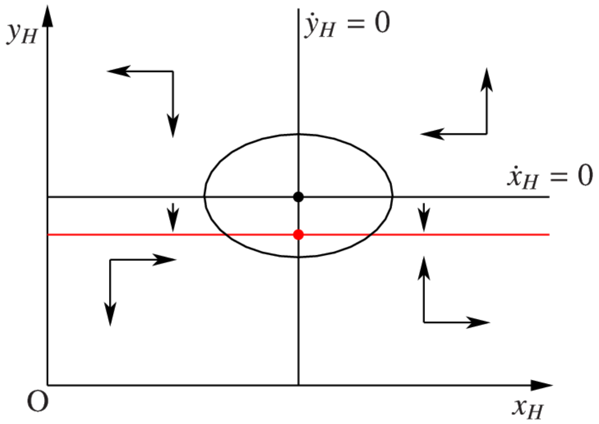

Figure 13: Effect of wage gap reduction on phase diagram of (x_H, y_H)

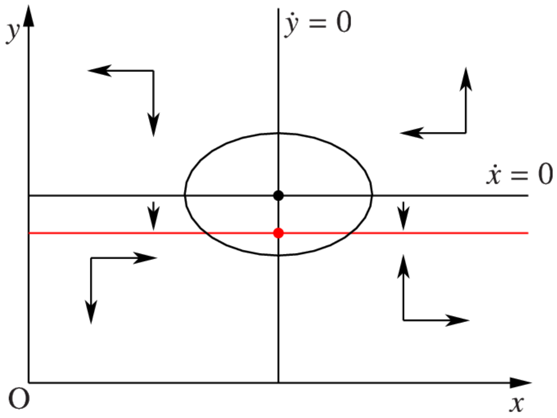

Figure 14: Effect of wage gap reduction on phase diagram of (x, y)

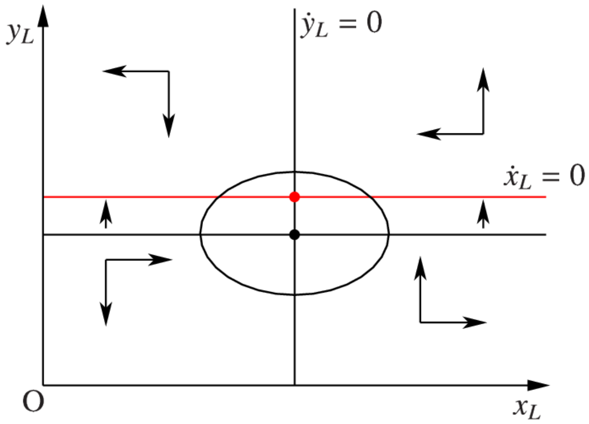

Figure 15: Effect of wage gap reduction on phase diagram of (x_L, y_L)

Figures 16–18 show business cycles when the wage gap is reduced from $\gamma = 1.34$ to $\gamma = 1$. In Figures 16–18, the blue lines correspond to pre-reduction business cycles, and the red lines correspond to post-reduction business cycles.[10] The solution path of the Goodwin model is a closed orbit, and different initial values correspond to different closed orbits. This means that different fluctuations appear with different initial values even if parameters of the model do not change. Accordingly, if initial values change with a reduction of the wage gap, we cannot discriminate whether a different closed orbit appears because of the change in initial value or decrease in wage gap. Hence, it is difficult to compare solution paths with different initial values. Therefore, we conduct numerical simulations under the assumption that initial values do not change when the wage gap decreases. The results show that the reduction of the wage gap decreases the fluctuations for high-skilled workers and entire economy and increases that for low-skilled workers.

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[10] The period of a cycle is given by $T = 2\pi/\sqrt{(\alpha + \phi)(s_c\sigma - \delta - \phi - n)}$, which is independent of the wage gap γ. Therefore, a decrease in the wage gap does not affect the period of a cycle.

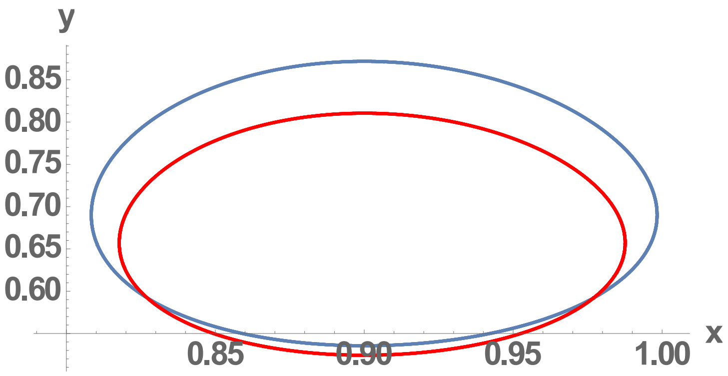

Figure 16: Effect of reduction in wage gap on the entire economy

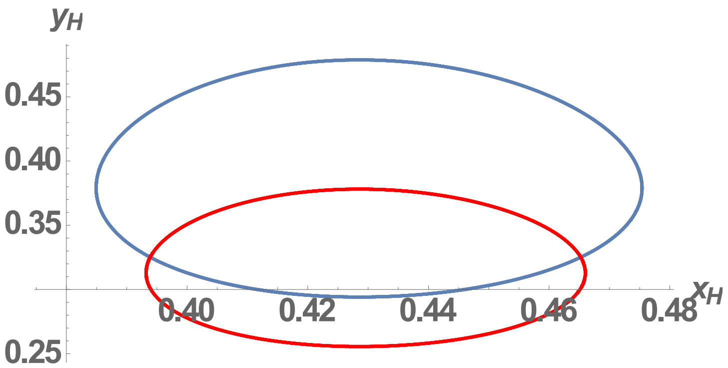

Figure 17: Effect of reduction in wage gap on high-skilled workers

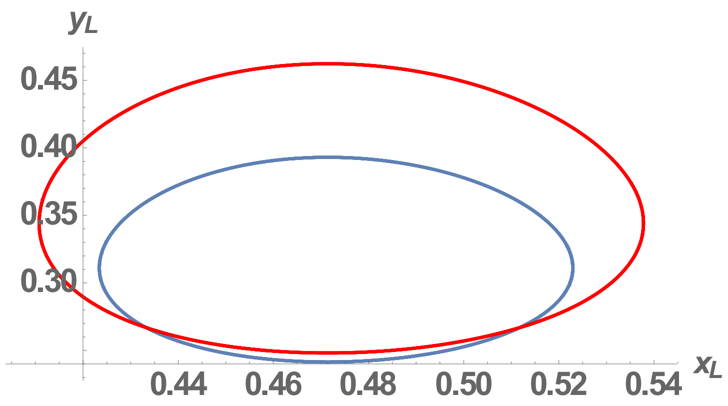

Figure 18: Effect of reduction in wage gap on low-skilled workers

Figure 19 shows the effect of a decrease in the wage gap on steady-state values with endogenous technological progress. A decrease in γ shifts the locus of $\dot{x}_H = 0$ downward and rotates the locus of $\dot{y}_H = 0$ clockwise around point P. The steady-state value of y_H always decreases, whereas that of x_H either increases or decreases according to the size of the shift and rotation, as indicated by the arrows A and B in Figure 19, respectively.

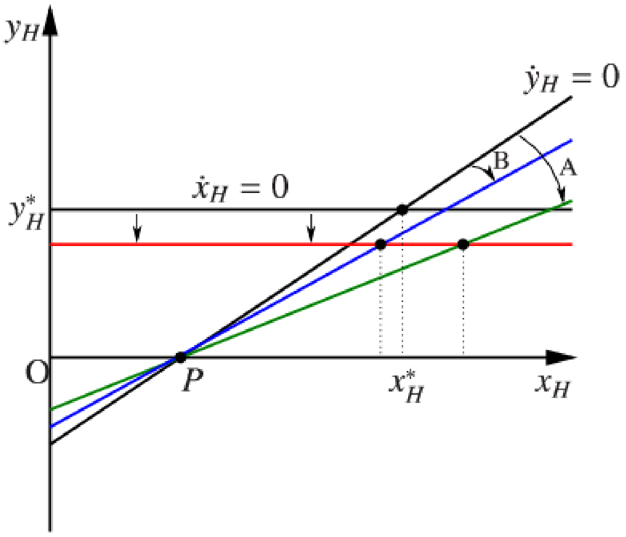

Figure 19: Effect of wage gap reduction in case of endogenous technological progress

The effects of the wage gap reduction on cyclical fluctuations are shown in Figures 20–22. The blue and red lines correspond to the benchmark and post-reduction scenarios, respectively.

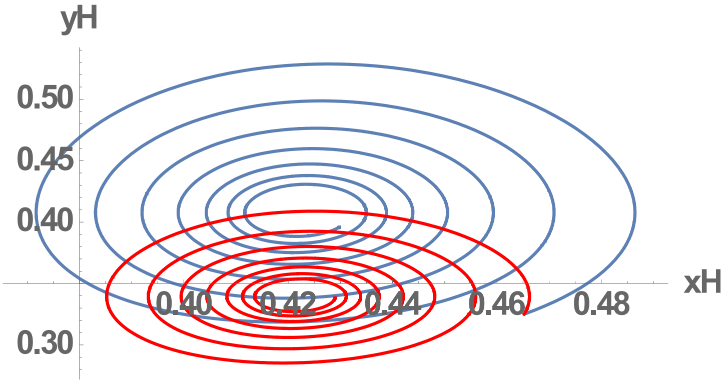

Figure 20: Effect of reduction in wage gap on high-skilled workers in case of endogenous technological progress

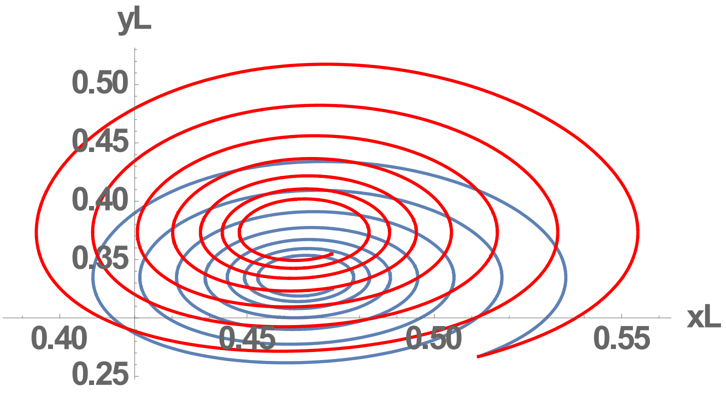

Figure 21: Effect of reduction in wage gap on low-skilled workers in case of endogenous technological progress

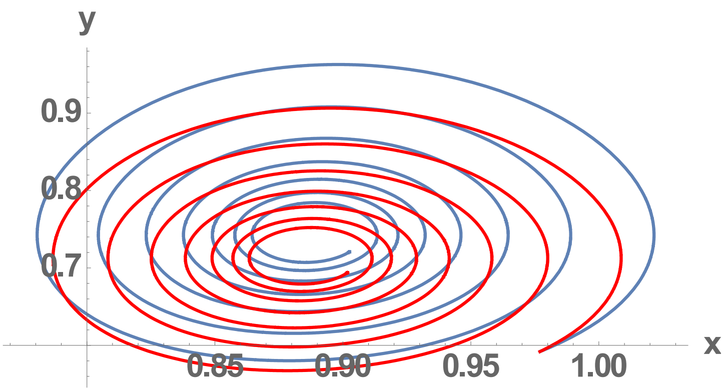

Figure 22: Effect of reduction in wage gap on the entire economy in case of endogenous technological progress (period: $t = 500 - 1000$)

When technological progress is endogenized, the fluctuation for high-skilled workers diminishes significantly, that for low-skilled workers increases significantly, and that for the entire economy diminishes slightly. This is because the patterns of fluctuations differ for high- and low-skilled workers, and the peaks and troughs for high-skilled workers do not correspond to those for low-skilled workers. Hence, the fluctuation for the entire economy does not change substantially.

Fluctuations in skilled workers and the entire economy diminish in the case of exogenous technological progress, because the reserve army effect acting on the real wage rate of high-skilled workers spreads strongly to that of low-skilled workers. When the real wage rate of high-skilled workers increases (decreases) due to an increase (decrease) in the employment rate of high-skilled workers, the wage share of the entire economy increases (decreases) as the wage gap decreases, which squeezes (recovers) the profit share. For this reason, a decrease (increase) in the employment share due to the profit squeeze (recovery) also occurs quickly, and consequently, the magnitude of fluctuations decreases. This is desirable concerning the stability of economic fluctuations.

However, for low-skilled workers, as stated above, the magnitude of the fluctuations increases because the reserve army effect for high-skilled workers largely spreads to low-skilled workers. Nevertheless, as Figure 18 shows, the wage share of low-skilled workers rotates at higher levels owing to the wage gap reduction, and their wage share in a recession is also raised. This is a desirable result for low-skilled workers. However, the magnitude of fluctuations in the employment rate of low-skilled workers increases slightly, which is unfavorable for low-skilled workers. Therefore, there is a weak trade-off between wage share and employment rate for low-skilled workers.

6. Conclusions

We present an extended version of the Goodwin model that considers low- and high-skilled workers and their savings. Using the parameters obtained from the data of the Japanese economy, we conduct numerical simulations to reproduce Japanese business cycles. Moreover, we investigate the effects of the minimum wage share and reduction in the wage gap between the two types of workers on business cycles.

The minimum wage in Japan is approximately 1000 yen, and the calculated minimum wage share is low to stabilize economic fluctuations. The minimum wage of 1500 yen discussed currently effectuates a reasonable minimum wage share that stabilizes economic fluctuations. Hence, raising the minimum wage to 1500 yen is meaningful in terms of stabilizing the economy.

Reducing the wage gap between high- and low-skilled workers does not significantly affect the fluctuations in the employment rates of both workers, whereas it significantly affects the fluctuations in the wage shares of both workers. The fluctuation in the wage share of high-skilled workers decreases, whereas that of low-skilled workers increases. Nevertheless, the average wage share of low-skilled workers has increased. Hence, low-skilled workers obtain a higher wage share on average.

Accordingly, if we can raise the minimum wage and simultaneously reduce the wage gap, we can achieve both the stabilization of the economy and a more equal distribution. For this purpose, the government must pursue minimum wage increases, and simultaneously, labor and management must establish a cross-enterprise labor-management bargaining system contributing to wage gap reduction.

A limitation of this study is that, although most parameters for the numerical simulation are obtained from quarterly data, some parameters are obtained from annual data owing to data constraints or set arbitrarily while considering the constraints of the model. Hence, not all the parameters are obtained from actual data. However, using numerical simulations with plausible parameter values, we analyze the effect of the minimum wage share, which has not been considered in previous empirical studies of the Goodwin model. This is one of the main contributions of this study.

Declarations

Funding: Not applicable

Conflicts of interest: The authors declare that they have no conflicts of interest.

Availability of data and material (data transparency): The datasets analyzed during this study are available from the corresponding author upon reasonable request.

Code availability (software application or custom code): The authors declare that they used EViews 12 for estimation and Mathematica 10 for numerical simulations.

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Appendix: Estimation of real wage Phillips curve

Using quarterly data on the employment rate of the entire economy x and rate of change in real wages $\hat{w}$, we estimate the real wage Phillips curve and convert it into a real wage Phillips curve for high-skilled workers (equation 7).

Estimation period

1989Q1−2020Q4

Data

- Employment rate x_t is given by $1 - \text{unemployment rate}$. Unemployment rates are quarterly averages of seasonally adjusted monthly unemployment rates obtained from the *Labour Force Survey* of the Statistics Bureau of Japan.
- Rate of change in real wage $\hat{w}_t$ is given by the rate of change in nominal wage divided by the consumer price index. Nominal wage is given by total wage divided by the number of workers times average hours of work. Total wage is obtained from the *Financial Statements Statistics of Corporations by Industry*, and average hours of work are obtained from the *Monthly Labour Survey*. The consumer price

index is obtained from the Statistics Bureau of Japan and based on 2020 standards, which is seasonally adjusted by X12-ARIMA.

Estimation method

- We follow the method provided by Grasselli and Maheshwari (2018) (GM, hereafter). To estimate the long-run relationship, that is, cointegration, between explanatory and dependent variables, we check whether the long-run relationship holds after we estimate our model based on the autoregressive distributive lag (ARDL) estimator (equation (25) of GM).
- When the orders of the unit roots are different between the two variables, we must use the bounds testing procedure based on the error correction model estimator (equations (26) and (27) of GM).
- If we can observe the long-run relationship between the two variables by the abovementioned procedures, we can estimate our equation in "level forms" using original series of data (equation (28) of GM).
- We must check the possibility of structural change during the estimation period using the cumulative sum of recursive residuals (CUSUM) and cumulative sum of recursive squared residuals (CUCUMSQ) tests (Figure A3 of GM).

For the estimation, we used EView 12. The names of the series are as follows.

- Employment rate: EMPLOYMENT_RATE
- Rate of change in real wage compared to the previous period: REALWAGE_D1
- Rate of change in real wages compared to the same period in the previous year: REALWAGE_D4.

Unit root test

The results of the unit root test are as follows:

- EMPLOYMENT_RATE (level)

	t-Statistic	Prob.*
Augmented Dickey–Fuller test statistic	-1.450443	0.5554

- EMPLOYMENT_RATE (1st difference)

	t-Statistic	Prob.*
Augmented Dickey–Fuller test statistic	-7.674510	0.0000

- REALWAGE_D1 (level)

	t-Statistic	Prob.*
Augmented Dickey–Fuller test statistic	-16.01325	0.0000

- REALWAGE_D4 (level)

	t-Statistic	Prob.*
Augmented Dickey–Fuller test statistic	-6.264308	0.0000

- For the employment rate, we cannot reject the unit roots in the original series. However, we can reject the unit roots in the first difference series. Accordingly, the employment rates are integrated in order one, $I(1)$.
- For the rate of change in real wages, we can reject the unit roots in both series at the 1% significance level. Accordingly, the rate of change in real wages is integrated in order one, $I(0)$.
- The orders of the unit roots are different between the explanatory and dependent variables; hence, we have to conduct the bounds testing procedure when checking the validity of the ARDL model.

- **ARDL model**

Dependent Variable: REALWAGE_D1

Method: ARDL

Sample (adjusted): 1989Q2–2020Q4

Included observations: 127 after adjustments

Maximum dependent lags: 4 (automatic selection)

Model selection method: Akaike information criterion

Dynamic regressors (4 lags, automatic): EMPLOYMENT_RATE

Fixed regressors: C

Number of models evaluated: 20

Selected Model: ARDL(1, 0)

Note: Final equation sample is larger than selection sample

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
REALWAGE_D1(-1)	-0.374356	0.083189	-4.500043	0.0000
EMPLOYMENT_RATE	0.288663	0.117001	2.467172	0.0150
C	-0.275216	0.112668	-2.442727	0.0160

- **ARDL Long Run Form and Bounds test**

Dependent Variable: D(REALWAGE_D1)

F-Bounds Test		Null Hypothesis: No levels relationship		
Test Statistic	Value	Signif.	I(0)	I(1)
			Asymptotic: n=1000	
F-statistic	90.98326	10%	3.02	3.51
k	1	5%	3.62	4.16
		2.5%	4.18	4.79
		1%	4.94	5.58

Dependent Variable: REALWAGE_D4
Method: ARDL
Sample (adjusted): 1989Q3 2020Q4
Included observations: 126 after adjustments
Maximum dependent lags: 4 (Automatic selection)
Model selection method: Akaike info criterion
Dynamic regressors (4 lags, automatic): EMPLOYMENT_RATE
Fixed regressors: C
Number of models evaluated: 20
Selected Model: ARDL(2, 0)
Note: Final equation sample is larger than selection sample

Variable	Coefficient	Std. Error	t-Statistic	Prob.*
REALWAGE_D4(-1)	0.386827	0.088873	4.352590	0.0000
REALWAGE_D4(-2)	0.166414	0.087657	1.898457	0.0600
EMPLOYMENT_RATE	0.339061	0.160800	2.108594	0.0370
C	-0.324334	0.154658	-2.097102	0.0380

- **ARDL Long Run Form and Bounds Test**

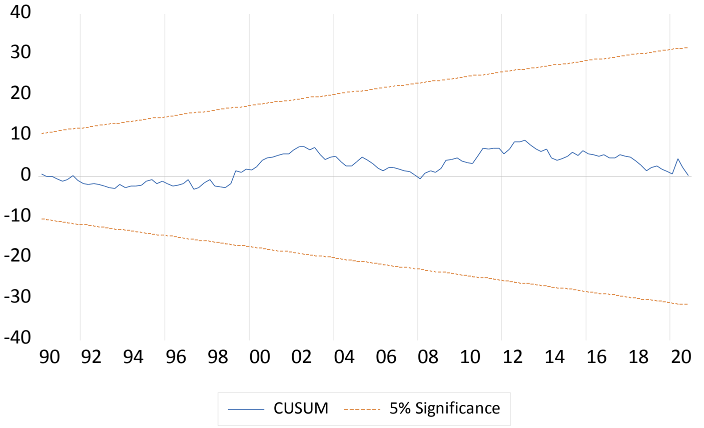
Dependent Variable: D(REALWAGE_D4)

F-Bounds Test			Null Hypothesis: No levels relationship	
Test Statistic	Value	Signif.	I(0)	I(1)
			Asymptotic: n=1000	
F-statistic	8.188847	10%	3.02	3.51
k	1	5%	3.62	4.16
		2.5%	4.18	4.79
		1%	4.94	5.58

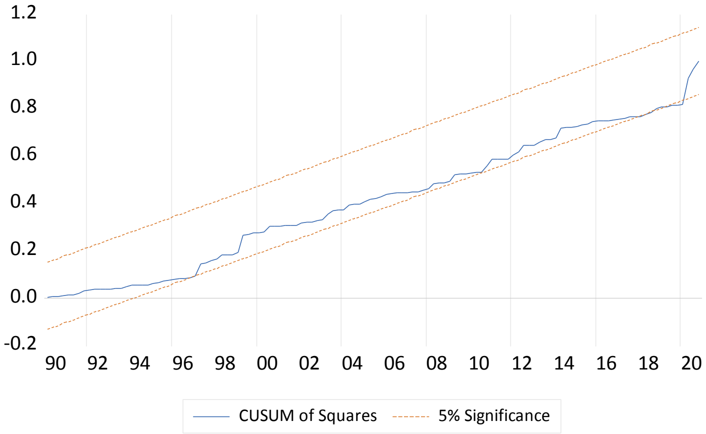
We reject the null hypothesis that there is no long-term relationship between the employment rate and rate of change in real wages at the 1% significance level. Accordingly, we can say that the relationship $\hat{w}_t = -\alpha + \beta x_t$ holds between x_t and $\hat{w}_t$ in the long run.

Test for structural change

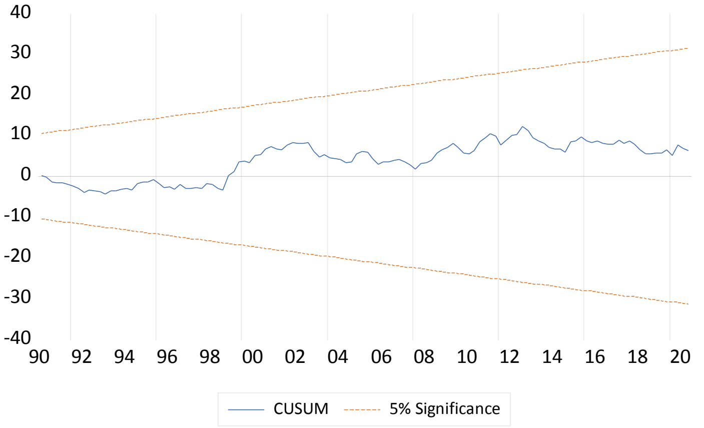
- **CUSUM（REALWAGE_D1）**



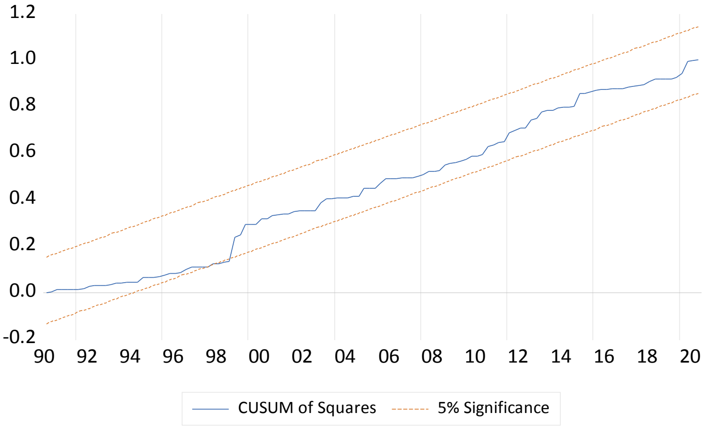
- **CUSUMSQ（REALWAGE_D1）**



- **CUSUM（REALWAGE_D4）**



- **CUSUMSQ（REALWAGE_D4）**



From CUSUM and CUSUMSQ tests, the null hypothesis that there are no structural changes during the estimation period cannot be rejected. Accordingly, we estimate the real wage Phillips curve under judgment that the shape of the real wage Phillips curve does not change throughout the estimation period.

- **Long-run relationship**

REALWAGE_D1

Levels Equation
Case 2: Restricted Constant and No Trend

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EMPLOYMENT_RATE	0.210035	0.084251	2.492973	0.0140
C	-0.200251	0.081148	-2.467726	0.0150

REALWAGE_D4

Levels Equation
Case 2: Restricted Constant and No Trend

Variable	Coefficient	Std. Error	t-Statistic	Prob.
EMPLOYMENT_RATE	0.758934	0.328294	2.311749	0.0225
C	-0.725970	0.316169	-2.296141	0.0234

From the above estimations, we find that the real wage Phillips curve takes the

following form:
REALWAGE_D1: $\widehat{w}_t = -0.200 + 0.210x_t$
REALWAGE_D4: $\widehat{w}_t = -0.726 + 0.759x_t$