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## **Macroeconomic Model of Generalized Feudal System**

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20 August 2024

Online at <https://mpra.ub.uni-muenchen.de/121771/>  
MPRA Paper No. 121771, posted 10 Sep 2024 06:26 UTC

# Macroeconomic Model of Generalized Feudal System

## Abstract

This paper presents an economic model that explores the optimisation of tax rates and reaching an equilibrium within a generalised feudal economy. This model examines the relationships between agricultural output, taxation and income distribution among the prominent societal classes of the feudal system. This societal class comprises the village headman, feudal lord, king, peasants, and merchants. Through the tools of Calculus methods, I try to derive the optimal conditions for maximising the tax rates of these societal class as per the agricultural production function. Additionally, I try to integrate a business cycle model which try to reflect economic fluctuations.

## 1 Introduction

Feudalism was the dominant socio-economic system in many parts of the world from Early medieveal period to early modern period. In India context, we see its rise from the Scynthian empire (Sharma,2006). It was characterized by the rigid social hierarchy, in which land ownership and agrarian production played a pivoted roles in defining power and redistribution of wealth. Since, during these times, majority of the economy was composed of agrarian structure. As of which, it is call Agrarian Economy.

In a typical feudal system, the king owned all the land nominally. He then granted large portions of these lands to feudal lords in his empire, who, in turn, allowed peasants to work the land in exchange for a share of their produce. They also supplied armies (consisted of these peasants), as and when the king requires. The village headman managed the local affairs , while merchants facilitated between various regions.

## 1.1 Theoretical Framework

In economic theory, the feudal system is often analyzed through the lens of rent-seeking behaviour, where landowners (feudal lords) extract economic rent from those who work on the land (peasants) (Botham, 2021). The system's stability hinges on the balance between the surplus produced by the peasants and the rent extracted by the landowners, through agricultural production. If the rent becomes too high, peasants may fall below subsistence levels, leading to social unrest or economic collapse. Conversely, if rent is too low, the landowners may lack the resources to maintain their power and protect their land, leading to political instability and break up of the feudal empire into smaller chieftains.

## 2 Feudal Economic Model

### 2.1 Assumptions

To make the this model more generalised , following the assumptions have been taken:

- Hierarchical Structure: The economy works in a stricter hierarchical framework. The descending order of hierarchy is: King, Feudal Lords, Village Headman and Peasants. Each level in the hierarchy has its own distinct roles and obligations in the economy.
- Land Ownership and control: In agrarian economy, land is the primary economic resource which is controlled by the kings, who grants it to feudal lords. The lords, in turn, grant portions of their land to vassals, and so forth. This system of land tenure and control is central to the functioning of the economy.
- The economy is agrarian economy. It drives both production and distribution which central for the functioning of the economy. The production function has been taken as Cobb- Douglas production function.
- Flow of Resources: The flow of resources is upwards in the hierarchy. Peasants provide agricultural produce or labour to the village headman, who then passes it to feudal lords and then to the king.

- Taxes and Obligations:
  - Taxes to Feudal Chiefs: Peasants pay a portion of their agricultural produce or labor in form of tax to feudal lords.
  - Feudal lords remit a portion of their gross income or resources to the king.
  - Taxes to Village Headman: Peasants also contributes to village headman’s resources.
- Economic Dependency: Each level of the hierarchy depends on the level below it for resources and services. This creates a level of mutual dependences as resources are transfered from lower tiers to higher tiers, vertically.
- Income: The tax is the only income for each of the classes in model.
- Tax Rates: For convinience, tax rates have been kept fixed and proportional to economic output or income at each level. This allows the clear representation of flow of the resources.
- Economic Stability: The model assumes a relatively stable economic environment where changes in agricultural productivity directly impact the economic well-being of all levels in the hierarchy.
- Limited Mobility: Economic Mobility is limited within the system. Peasants have restricted opportunities for their economic development due to rigid structure of land ownership and obligations.
- Economic cycles: It has been assumed that agricultural cycles influnces boom and recession affecting the entire feudal system.

## 2.2 Agricultural Output

The total agricultural output  $A$  is modeled using the Cobb-Douglas production function:

$$A = L^\alpha P^\beta \tag{1}$$

where:

- $L$  represents land,

- $P$  represents labor,
- $\alpha$  and  $\beta$  are output elasticities of land and labor, respectively.

Assuming that land  $L$  is proportional to labor  $P$ , we have:

$$L = \gamma P \tag{2}$$

Substituting  $L$  into the production function:

$$A = (\gamma P)^\alpha P^\beta = \gamma^\alpha P^{\alpha+\beta} \tag{3}$$

Letting  $\lambda = \gamma^\alpha$ , we simplify to:

$$A = \lambda P^{\alpha+\beta} \tag{4}$$

## 2.3 Taxation and Income Distribution

Taxes are levied at three levels: village headman, feudal lord, and king. The tax for each hierarchy  $T_i$  is a fraction of the agricultural output  $A$ , where  $i$  stands for Village Headman, Feudal Lord, or King.

### 2.3.1 Village Headman's Tax

$$T_v = \tau_v A \tag{5}$$

### 2.3.2 Feudal Lord's Tax

$$T_f = \tau_f A \tag{6}$$

### 2.3.3 King's Tax

$$T_k = \tau_k A \tag{7}$$

## 2.4 Income of Each Player

### 2.4.1 Village Headman's Income

$$I_v = T_v = \tau_v \lambda P^{\alpha+\beta} \tag{8}$$

### 2.4.2 Feudal Lord's Income

$$I_f = T_f = \tau_f \lambda P^{\alpha+\beta} \quad (9)$$

### 2.4.3 King's Income

$$I_k = T_k = \tau_k \lambda P^{\alpha+\beta} \quad (10)$$

### 2.4.4 Peasant's Income

The peasant's income  $W$  is the remainder of the agricultural output after taxes:

$$W = A - (T_v + T_f + T_k) \quad (11)$$

Substituting for taxes:

$$W = (1 - \tau_v - \tau_f - \tau_k) \lambda P^{\alpha+\beta} \quad (12)$$

## 3 Business Cycle Model

To account for economic fluctuations, we introduce a business cycle model. The business cycle affects agricultural output  $A(t)$  and labor productivity  $P(t)$ .

### 3.1 Agricultural Output

The agricultural output as a function of time is given by:

$$A(t) = A_0 (1 + \phi \sin(\omega t)) \quad (13)$$

where:

- $A_0$  is the average agricultural output,
- $\phi$  is the amplitude of the cycle,
- $\omega$  is the angular frequency of the cycle,
- $t$  is time.

## 3.2 Labor Productivity

The labor productivity as a function of time is given by:

$$P(t) = P_0 (1 + \theta \sin(\omega t)) \quad (14)$$

Where:

- $P(t)$  = Labor productivity at time  $t$
- $P_0$  = Average labor productivity
- $\theta$  = Amplitude of seasonal variation in labor productivity
- $\omega$  = Frequency of seasonal cycles
- $t$  = Time

## 4 Optimization

### 4.1 Peasant's Income Optimization

The peasant's income  $W$  must be above the subsistence level  $W_{\min}$ :

$$W \geq W_{\min} \quad (15)$$

Substituting:

$$(1 - \tau_v - \tau_f - \tau_k)A(t) \geq W_{\min} \quad (16)$$

Using the equation for  $A(t)$ :

$$(1 - \tau_v - \tau_f - \tau_k)A_0 (1 + \phi \sin(\omega t)) \geq W_{\min} \quad (17)$$

### 4.2 Village Headman's Income Optimization

To maximize the village headman's income  $I_v$ :

$$I_v = \tau_v A(t) \quad (18)$$

Differentiate  $I_v$  with respect to  $\tau_v$ :

$$\frac{dI_v}{d\tau_v} = A(t) \quad (19)$$

Since the derivative is positive,  $I_v$  increases with  $\tau_v$ . The optimal  $\tau_v$  is:

$$\tau_v^* = 1 - \tau_f - \tau_k - \frac{W_{\min}}{A_0(1 + \phi \sin(\omega t))} \quad (20)$$

### 4.3 Feudal Lord's Income Optimization

To maximize the feudal lord's income  $I_f$ :

$$I_f = \tau_f A(t) \quad (21)$$

Differentiate  $I_f$  with respect to  $\tau_f$ :

$$\frac{dI_f}{d\tau_f} = A(t) \quad (22)$$

The optimal  $\tau_f$  is:

$$\tau_f^* = 1 - \tau_v^* - \tau_k - \frac{W_{\min}}{A_0(1 + \phi \sin(\omega t))} \quad (23)$$

### 4.4 King's Income Optimization

To maximize the king's income  $I_k$ :

$$I_k = \tau_k A(t) \quad (24)$$

Differentiate  $I_k$  with respect to  $\tau_k$ :

$$\frac{dI_k}{d\tau_k} = A(t) \quad (25)$$

The optimal  $\tau_k$  is:

$$\tau_k^* = 1 - \tau_v^* - \tau_f^* - \frac{W_{\min}}{A_0(1 + \phi \sin(\omega t))} \quad (26)$$



## 4.5 Business Cycle

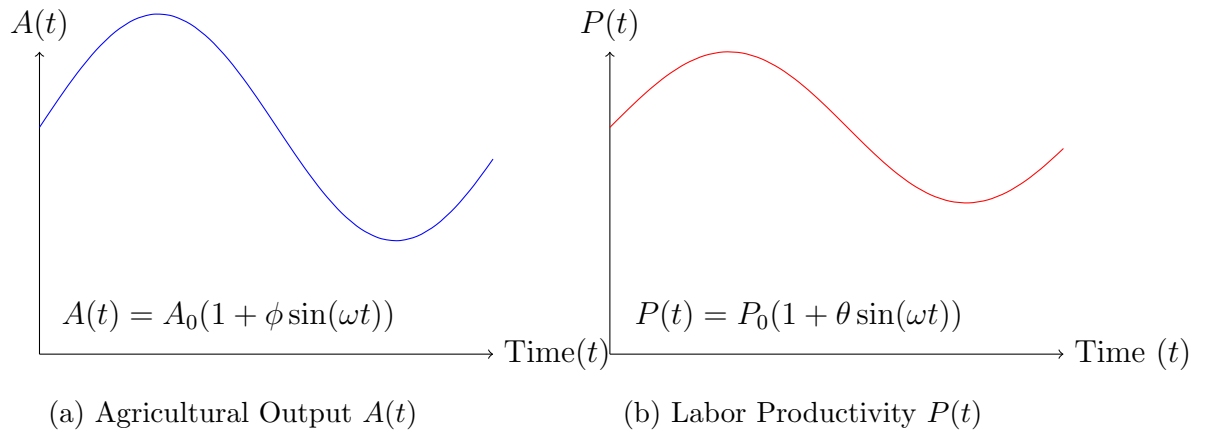


Figure 1: Business Cycle Models

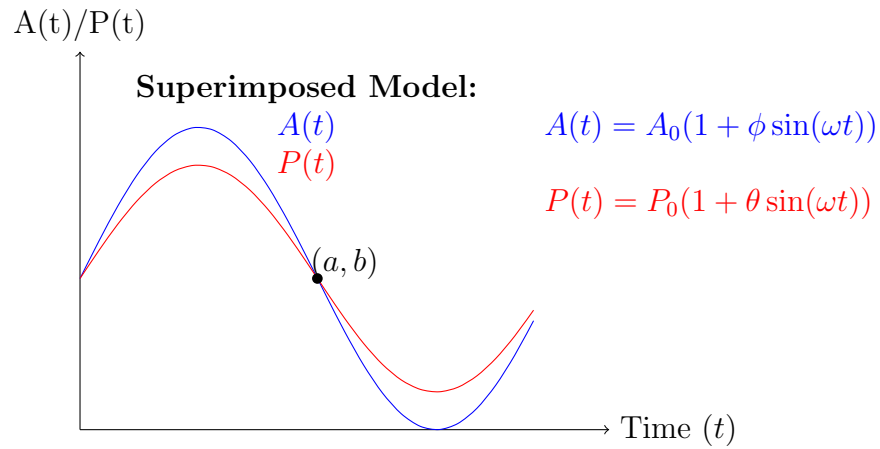


Figure 2: Superimposed Graphs  
Axis Notations:

- Horizontal Axis: Time ( $t$ )
- Vertical Axis for Agricultural Output  $A(t)$ : Output Level
- Vertical Axis for Labor Productivity  $P(t)$ : Productivity Level

### 4.5.1 Intersection Points

**Agricultural Output Function:**

$$A(t) = A_0(1 + \phi \sin(\omega t)) \quad (27)$$

**Labor Productivity Function:**

$$P(t) = P_0(1 + \theta \sin(\omega t)) \quad (28)$$

**Intersection Condition:** The intersection of  $A(t)$  and  $P(t)$  occurs when:

$$A(t) = P(t) \quad (29)$$

Substituting the functions:

$$A_0(1 + \phi \sin(\omega t)) = P_0(1 + \theta \sin(\omega t)) \quad (30)$$

**Solving for Time  $t$  at Intersection:** Rearranging the above equation:

$$A_0 + A_0\phi \sin(\omega t) = P_0 + P_0\theta \sin(\omega t) \quad (31)$$

Simplify to:

$$A_0 - P_0 = (P_0\theta - A_0\phi) \sin(\omega t) \quad (32)$$

Solving for  $\sin(\omega t)$ :

$$\sin(\omega t) = \frac{A_0 - P_0}{P_0\theta - A_0\phi} \quad (33)$$

Thus, the intersection occurs at:

$$\omega t = \arcsin\left(\frac{A_0 - P_0}{P_0\theta - A_0\phi}\right) \quad (34)$$

### 4.5.2 Interpretation

- **Intersection Point:** The intersection point  $t$  represents the time when agricultural output  $A(t)$  and labor productivity  $P(t)$  are equal. This indicates a moment in the economic cycle when the benefits from increased agricultural output match the productivity levels of labor, reflecting a balance in the economic system.

- **Economic Cycles:** The seasonal variations captured by  $\sin(\omega t)$  show how agricultural output and labor productivity fluctuate over time. When both functions intersect, it highlights a specific phase in the economic cycle where both outputs are balanced. This might correspond to a period of economic equilibrium where fluctuations in productivity and output are synchronized.
- **Implications for Planning:** Understanding these intersections helps in planning and managing resources. For instance, if a policy aims to boost productivity or output, knowing the timing of these intersections can aid in aligning agricultural and labor productivity strategies to maximize economic benefits.

## 5 Discussion

This model reveals how taxation impacts the income distribution among various players in a feudal system. The village headman's, feudal lord's, and king's incomes are directly influenced by their respective tax rates. Higher taxes increase their income, but they also reduce the peasants' surplus, which must meet or exceed a minimum subsistence level to avoid economic collapse.

The integration of the business cycle model shows that economic fluctuations influence agricultural output and labor productivity. During boom periods, the agricultural output and labor productivity increase, enhancing the income of all players. Conversely, during recessions, output and productivity decline, potentially reducing income and increasing economic vulnerability.

In essence, the model demonstrates the intricate balance required to maintain a stable feudal economy. The interplay between taxation and income distribution highlights the need for careful policy formulation to ensure that all societal classes can thrive. The inclusion of business cycle effects underscores the importance of adapting to economic fluctuations to sustain long-term economic stability and growth.

## 6 Conclusion

The model provides a comprehensive framework for understanding how taxation and economic cycles affect income distribution and overall stability in a feudal system. By optimizing tax rates and considering business cycle impacts, we can ensure the well-being of all players in the system while maintaining economic equilibrium.

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