Optimal Energy-Saving Investments and Jevons Paradox in Duopoly Markets

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Abstract

This study theoretically investigates energy-saving investment incentives in duopolies. First, we investigate a binary choice model in which each firm chooses whether to make an energy-saving investment and then they face Cournot competition. We focus on the incentive to become the leading firm by the investment, when the rival does not engage in this project. We find the private incentive to be insufficient for welfare (thereby requiring promotion through policies), if Pigouvian tax is imposed. However, this incentive can be excessive when the emission tax rate is lower than the Pigouvian level. Next, we investigate a model in which firms can choose energy-saving investment levels continuously. We find that the equilibrium investment can be (is not) excessive for welfare when the emission tax rate is lower than (equal to) the Pigouvian. These results suggest a risk of policy formation combining a low emission tax and subsidies for promoting energy-saving investments.

JEL classification codes: Q38, Q58, L13

Keywords: emission tax, investment subsidy, policy combination, energy-conservation, production substitution

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Highlights

We investigate firms' energy-saving investments in duopolies.

Emission tax internalizing the negative externality of energy consumption is crucial.

Equilibrium energy-saving investments can be excessive when emission tax rate is low.

Policy combination of a moderate emission tax and investment subsidy may harm welfare.

1 Introduction

In the wake of energy price spikes in 2022, the importance of energy savings is widely recognized among governments and enterprises. In addition, climate change and vulnerable fossil fuel supply enhance energy-saving investments by heavy energy-consuming enterprises. Most heavy energy-consuming enterprises operate in oligopolistic markets. Investigating the welfare impacts of energy-saving investments in oligopolies is thus increasingly important.

Enhancing energy efficiency through innovation is beneficial to the environment and can reduce energy consumption. However, improvements in energy efficiency do not necessarily reduce overall energy consumption, which was first pointed out by William Stanley Jevons in 1865 (Jevons paradox). Firms that invest in enhancing their energy efficiency often expand their production due to a decrease in marginal production costs. Subsequently, the existence of a rebound effect in various industries has also been debated extensively (Alcott, 2005). Amjadi et al. (2018) provide empirical evidence of rebound effects on the production side, and quantified the magnitude of the effect on fuel and electricity use in Sweden's four most energy-intensive sectors: pulp and paper, basic iron and steel, chemicals, and mining.[1]

This study focuses on oligopolistic firms' energy-saving investment incentives, which may lead to a rebound effect on energy consumption. We formulate a simple model where two firms choose whether to make large-scale energy-saving investments. Energy-saving investments reduce both energy use in production and emission tax payments, thereby reducing firms' marginal costs. However, they also increase the sunk costs of investments. We find that if energy consumption's negative externality is completely internalized by an environmental tax (i.e., a Pigouvian tax is imposed), the firm's incentive to invest is insufficient for welfare when its rival decides not to invest, regardless of the existence of the Jevons paradox. However, if the emission tax rate falls short of the energy consumption's

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[1] Similarly, a low carbon fuel standard (LCFS) may increase the total energy consumption (Holland et al., 2009).

marginal negative externality, the firm's incentive to invest can be excessive.[2] Our first result suggests that an investment subsidy for the firm, which is only one player engaging this energy-saving project, improves welfare when the emission tax rate equals the Pigouvian rate regardless of whether the Jevons paradox takes place. Nevertheless, our second result demonstrates that such an investment subsidy may harm welfare when the emission tax rate is lower than the Pigouvian rate.

These results may be counterintuitive. We naturally infer that an emission tax that is lower than Pigouvian reduces the incentive for energy-saving investments, making an investment subsidy more desirable with a lower emission tax rate. We show that this conjecture does not always hold, and an emission tax lower than the Pigouvian tax may provide excessive incentives for energy-saving investments. We also present a result suggesting a relationship between this counterintuitive result and the Jevons paradox.

We find that the firm has an excessive energy-saving investment incentive when the emission tax rate is substantially lower than the Pigouvian rate, the improvement of energy efficiency is moderate, and the Jevons paradox takes place, given the rival firm decides not to invest. The firm's energy-saving investments raises (reduces) its (the rival's) production level, which improves the industry's energy efficiency. Welfare always improves when the emission tax rate is equal to the Pigouvian rate, because the energy advanced firm's production level is not excessive. However, when the emission tax rate is substantially lower than the Pigouvian rate, and the improvement of energy efficiency is moderate, even the energy advanced firm's production level (and energy consumption level) can be excessive for welfare. In this case, an energy-saving investment worsens the firm's excessive production, thereby harming welfare. This suggests a risk of a policy combination of a low emission tax

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[2] The tax that internalizes the negative externality of emissions is known as a "Pigouvian tax." In imperfectly competitive markets, a Pigouvian tax is not optimal (Buchanan, 1969; Barnett, 1980; Misiolek, 1980; Baumol and Oates, 1988). For example, in a monopoly, the monopolist's production level falls below the optimal level. The emission tax rate should be lower than the Pigouvian rate to mitigate welfare losses owing to insufficient production levels due to the imperfect competition. Thus, investigating cases where the emission tax rate is lower than the Pigouvian rate in oligopolies is important.

and subsidies for non-drastic energy-saving investments.

Two lines of research closely relate to our study. The first comprises literature on environmental investment under imperfect competition. A typical environmental investment is an end-of-pipe-abatement, which reduces emission after firms release pollutants (Ebert, 1991; Petrakis and Poyago-Theotoky, 2002; Xu et al., 2022). Montero (2002) and Amir et al. (2018) consider the situation, in which firms invest in abatement technologies to make the abatement at a lower cost and examines firms' R&D incentive under various environmental policies. These investments necessarily reduce the total emission, and thus, the rebound effect cannot be discussed. Papers closely related to our study include Katsoulacos and Xepapadeas (1996), Petrakis and Xepapadeas (1999), Chiou and Hu (2001), and McDonald and Poyago-Theotoky (2017), who consider investments in firms' production process that generates less pollution per unit of output. However, these studies focus on environmental policies that promote firms' emission-reducing investments, but do not discuss energy-saving investments. Moreover, none of these studies discusses the relationship between the Jevons paradox and possible excessive investment incentives.[3] We also portray that, even the front-runner who is only one player engaging in the investment with advanced technology can have excessive investment incentives, which has not been demonstrated in the literature.

The second line of research consists of literature on strategic cost-reducing R&D investments. Brander and Spencer (1983) show that the equilibrium investment level is efficient (equal to the second-best investment level) in their symmetric duopoly model with linear demand functions.[4] Our model differs from theirs in two respects. We consider a binary

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[3] The emission-reducing and energy-saving investments exhibit contrasting properties. In models discussing the aforementioned emission-reducing investments, firms have no incentive to invest without environmental policies (i.e., if the emission tax rate is zero). Therefore, without environmental policies, investment levels would be insufficient. By contrast, firms have an incentive for energy-saving investments because they reduce marginal costs even without an emission tax; thus, excessive investments can occur. Therefore, we show the possibility of overinvestment even without an environmental tax.

[4] Their strategic cost-reducing R&D model is extended to include spillover effects, entries, and risks in various ways. d'Aspremont and Jacquemin (1988), Okuno-Fujiwara and Suzumura (1993), and Kitahara and Matsumura (2006) among others, show possible insufficient and excessive investments.

choice model for investments and focus on an asymmetric equilibrium, in which only one firm invests. Investment by only one firm induces a welfare-improving production substitution discussed by Lahiri and Ono (1988). In our model, this substitution accentuates the suboptimality of the firm's investment. Nevertheless, we show that the firm's incentive to invest can be excessive, if the emission tax rate is lower than the Pigouvian level.[5] Moreover, we demonstrate the relationship between energy-saving investments and an environmental tax, which has not been discussed in the literature on strategic cost-reducing R&D investments.

The remainder of this paper is organized as follows: Section 2 formulates the model using binary investment choice. Section 3 discusses the equilibrium outcomes. Section 4 presents the welfare analysis and discusses policy implications. Section 5 considers an alternative model where two firms continuously choose energy-saving investment levels and discuss the welfare implications at the symmetric equilibrium. Finally, the conclusions are summarized in Section 6.

2 The model

We formulate a duopoly model where each firm i ($i = 1, 2$) chooses whether to make energy-saving investments. The demand function is $p = a - Q$, where p is the product price, a is a positive constant, and $Q = q_1 + q_2$ is the total output of both firms. The production of one unit of output consumes k (one) units of energy if a firm makes (does not make) the energy-saving investment, where $k \in [0, 1)$ is constant. One unit of energy consumption yields one unit of emissions, and one unit of emissions generates d units of damage for society. Thus, the total emissions are $E = k_1 q_1 + k_2 q_2$ where $k_i \in \{k, 1\}$ and the negative externality due to the emissions is dE.

Firms pay an emission tax for their emissions and the exogenous tax rate is $t \in [0, d]$.[6]

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[5] We discuss a symmetric duopoly with a continuous investment level in Section 4 to complete the analysis. We show that a similar result holds at the symmetric equilibrium where both firms choose the same investment level and thus equally energy efficient.

[6] This assumption implies that the emission tax is beyond the regulator's control. For instance, taxation

If $t = d$, we say that the negative externality is fully internalized (Pigouvian tax). Let v be the pre-tax energy price and F, the investment cost for energy savings. Both v and F are positive constants. Firm i's profit is $\pi_i = (p-(v+t)k)q_i - F$ if firm i makes an energy-saving investment and $\pi_i = (p-(v+t))q_i$, otherwise. Welfare is defined as

$$W = \int_0^Q p(q)dq - pQ + \pi_1 + \pi_2 - dE + tE.$$

We define $w := v+t$ and $r := d-t$, where w denotes the after-tax energy price for the firm and r denotes the degree of negative externality that is not covered by tax. Consequently, we can rewrite firm i's profit as $\pi_i = (p-kw)q_i - F$ ($\pi_i = (p-w)q_i$) if firm i makes (does not make) energy-saving investments, and welfare as

$$W = \int_0^Q p(q)dq - pQ + \pi_1 + \pi_2 - rE.$$

The game proceeds as follows: In the first stage, each firm i independently chooses whether to make energy-saving investments. After observing the first-stage choices, each firm i independently chooses q_i in the second stage (Cournot competition).

3 Equilibrium analysis

We use the subgame perfect Nash equilibrium as the equilibrium concept and thus, solve the game using backward induction. In the second stage, given the investment decisions, each firm i independently chooses q_i. The first-order condition for firm i is

$$a - (q_i + q_j) - q_i - c_i = 0 \ (i, j = 1, 2, \ i \neq j), \tag{1}$$

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measures can be implemented at the supranational level, such as in the European Union (Petrakis and Poyago-Theotoky, 2002). Another interpretation relates to the emissions permit program where firms are not allowed a quota to emit free of charge and have to pay a price t on all emissions across industries. Industry specific tax rates and programs are not allowed. Alternatively, it might be reasonable to assume that the government chooses welfare-maximizing emission tax (Ino and Matsumura, 2021a; Xu et al., 2022). However, the government may not be able to set such rate because of the lobbying by the firms (Hirose et al., 2024).

where c_i denotes firm i's marginal cost and $c_i = kw$ ($c_i = w$) if firm i makes (does not make) energy-saving investments.

From these first-order conditions, we obtain the following reaction functions at the second stage:

$$R_i(q_j) = \frac{a - c_i - q_j}{2}. \tag{2}$$

From the reaction functions of the two firms, we derive the following equilibrium outputs in the second stage:[7]

$$q_i = \frac{a - 2c_i + c_j}{3}. \tag{3}$$

To ensure interior solutions (positive outputs for both firms), we assume $a > (2-k)w$.[8]

Let $\pi(I, I)$ be the equilibrium payoff for each firm when both firms invest, $\pi(N, N)$ be the equilibrium payoff for each firm when no firm invests, $\pi(N, I)$ ($\pi(I, N)$) be the equilibrium payoff for the non-investing (investing) firm when the rival invests (does not invest). From (3), we determine the following equilibrium payoffs for four subgames:

$$\begin{aligned}
\pi(I, I) &= \frac{(a - kw)^2}{9} - F, && (4)\\
\pi(N, N) &= \frac{((a - w)^2}{9}, && (5)\\
\pi(I, N) &= \frac{(a + (1-2k)w)^2}{9} - F, && (6)\\
\pi(N, I) &= \frac{(a - (2-k)w)^2}{9}. && (7)
\end{aligned}$$

Table 1: Payoff matrix for the first stage

		Firm 2	
		I	N
Firm 1	I	$\pi(I, I), \pi(I, I)$	$\pi(I, N), \pi(N, I)$
	N	$\pi(N, I), \pi(I, N)$	$\pi(N, N), \pi(N, N)$

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[7] The second-order conditions are satisfied throughout this study.

[8] If $a \leq (2-k)w$, the less energy-efficient firm does not produce and the market is monopolized by the energy efficient firm in the asymmetric case.

In the first stage, each firm independently chooses whether to invest. There are three possible pure strategy equilibrium patterns: the equilibrium in which both firms invest (we call this pattern the energy-efficient equilibrium) exists if and only if $\pi(I,I) \geq \pi(N,I)$; the equilibrium in which no firm invests (we call this the energy-inefficient equilibrium) exists if and only if $\pi(N,N) \geq \pi(I,N)$; and the equilibrium in which only one firm invests (we call this the asymmetric equilibrium) exists if and only if $\pi(I,N) \geq \pi(N,N)$ and $\pi(N,I) \geq \pi(I,I)$.

The following lemma characterizes the equilibrium pattern.

Lemma 1 *(i) The energy-efficient equilibrium exists if and only if*

$$F \leq F_I := \frac{4(a-w)(1-k)w}{9}.$$

(ii) The energy-inefficient equilibrium exists if and only if

$$F \geq F_N := \frac{4(a-kw)(1-k)w}{9}$$

and $F_N > F_I$ *holds.*

(iii) The asymmetric equilibrium exists if and only if $F \in [F_I, F_N]$.

Proof See Appendix.

Lemma 1 is intuitive. When the investment cost F is sufficiently low, both firms invest. When F is high, no firm invests. When F is moderate, only one firm invests.

4 Welfare analysis and policy implications

We discuss the welfare implications. Let $W(I,I)$ ($E(I,I)$) be the welfare (emissions) when both firms invest, $W(N,N)$ ($E(N,N)$) be the welfare (emissions) when no firm invests, and $W(I,N)$ ($E(I,N)$) be the welfare (emissions) when only one firm invests. We obtain the following result.

Lemma 2 *(i)* $W(I,I) \geq \max\{W(I,N), W(N,N)\}$ *if and only if*

$$F \leq F_I^s := \frac{(1-k)(a(6r+8w) - w(12r + (11-3k)w))}{18}.$$

(ii) $W(N,N) \geq \max\{W(I,I), W(I,N)\}$ *if and only if*

$$F \geq F_N^s := \frac{(1-k)(2a(3r+4w) - w(12kr - (3-11k)w))}{18}$$

and $F_N^s > F_I^s$ *holds.*

(iii) $W(I,N) \geq \max\{W(I,I), W(N,N)\}$ *if and only if* $F \in [F_I^s, F_N^s]$.

Proof See Appendix.

Lemma 2 is also intuitive. When investment cost F is sufficiently low (sufficiently high, intermediate), energy-efficient (energy-inefficient, asymmetric) equilibrium is best for welfare.

We now compare F_N^s with F_N. If $F_N < F_N^s$, it is possible that no firm invests in equilibrium, but one firm should invest from the welfare viewpoint, whereas the reverse never occurs. In other words, the incentive for the energy-saving investment is insufficient for welfare, given that the rival does not invest. In contrast, if $F_N > F_N^s$, it is possible that no firm should not invest from the welfare viewpoint but one firm invests in equilibrium, whereas the reverse never occurs. In other words, the incentive for the energy-saving investment is excessive for welfare given that the rival does not invest. Therefore, the comparison of F_N^s and F_N has important policy and welfare implications. We present a condition for this comparison of F_N^s and F_N.

Lemma 3 $F_N < (\geq) F_N^s$ *if and only if* $\Theta_1 := 2(a - 2kw)r + (1-k)w^2 > (\leq) 0$.

Proof See Appendix.

Henceforth, we state that the front-runner's incentive is insufficient (excessive) for welfare if $F_N < (\geq) F_N^s$ holds. If one firm invests, it becomes the front-runner of energy efficiency given that its rival decides not to invest, and $F_N < F_N^s$ implies that one firm's incentive to become the front runner of energy efficiency is insufficient for welfare. Therefore, we use the word front-runner in this context.

We now present the main findings.

Proposition 1 *(i)* $\Theta_1 > 0$ *holds and thus* $F_N < (\geq) F_N^s$ *holds if and only if* $k < k^W$ *where* $k^W := (2ar + w^2)/w(4r + w)$*. In other words, the front-runner's incentive is insufficient for welfare if the improvement of energy-saving is sufficiently large (if* k *is sufficiently small).*
(ii) $k^W = 1$ *when* $r = 0$*. In other words, the front-runner's incentive is always insufficient for welfare under Pigouvian tax.*
(iii) $k^W < 1$ *if and only if* $a < 2w$ *and* $r > 0$*. In other words, the front-runner's incentive can be excessive if the tax rate is lower than Pigouvian and the before-tax energy price is sufficiently large.*
(iv) k^W *is decreasing in* r *when* $a < 2w$*. In other words, when the before-tax energy price is sufficiently large, the front-runner's incentive is more likely excessive when the emission tax rate is lower.*[9]
Proof See Appendix.

When the negative externality of energy consumption is fully internalized by the environmental tax, private incentive for energy-saving investments is insufficient for welfare (Proposition 1(ii)). We explain this result by the general principle of welfare-improving production substitution by Lahiri and Ono (1988). Suppose that firm 2 decides not to invest. Firm 1's investment reduces firm 1's marginal cost, which increases firm 1's output. This reduces firm 2's output through strategic interactions. In other words, the switching from (N, N) to (I, N) induces production substitution from firm 2 to firm 1. This reduces the industry's average production costs and improves welfare. Firm 1 chooses whether to invest without considering this welfare-improving effect, and thus, the incentive for investment is

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[9] Throughout the paper, we use "more likely" to express that certain result holds for a broader range of parameter values.

insufficient.

If $r > 0$, uninternalized negative externalities of energy consumption exist. Thus, we naturally expect private incentive for energy-saving investments to be further insufficient for welfare. However, Proposition 1(iii) states that this conjecture is incorrect, because Proposition 1(iii) implies that there exists (r, k, a, w) such that $F_N > F_N^s$. Proposition 1(iv) implies that if the incentive for energy-saving investments is excessive when $r = r'$, then the incentive for energy-saving investments is excessive for any $r \geq r'$.

Proposition 2 helps us understand the intuition behind these results, especially Proposition 1(i).

Proposition 2 *(i)* $E(I, N) > E(N, N)$ *if and only if* $2kw - a > 0$*. In other words, Jevons paradox takes place if* k *and* w *are large. (ii)* $2kw - a > 0$ *holds if* $\Theta_1 < 0$*. In other words, the front-runner's investment incentive is excessive only when the one firm's energy-saving investment induces Jevons paradox.*

Proof See Appendix.

Proposition 2(i) implies that one firm's energy-saving investment can increase the total energy consumption in the industry (i.e., the Jevons paradox can occur), regardless of r. This proposition also suggests that the Jevons paradox is more likely to occur when the emission tax rate is higher. Note that w (the after tax fuel price) increases with t. Propositions 1(i,iii,iv) and 2(ii) imply that the front-runner's incentive is excessive for welfare only when the emission tax rate is lower than Pigouvian rate, the improvement of energy efficiency is moderate, and the Jevons paradox takes place. We explain the intuition behind this result.

One firm's energy-saving investment raises (reduces) its (the rival's) production level, which improves the energy efficiency in the industry. This always improves welfare when the emission tax rate is equal to Pigouvian rate because in the presence of Pigouvian tax, the firm's production level is effectively restricted by the emission tax. Thus the investment incentive is never excessive for welfare, even when Jevons paradox takes place. However, when the emission tax rate is substantially lower than Pigouvian rate and the improvement of energy efficiency is moderate, even the front-runner's production level (and thus energy consumption level) can be excessive for welfare, because the improvement of energy efficiency may worsen excessive emissions. An energy-saving investment exacerbates the welfare loss owing to the investing firm's overproduction, and thus, harms welfare. We show that this welfare-reducing energy saving investment occurs only when Jevons paradox occurs (Proposition 2(ii)).

Our result has important policy implications. When the negative externality of energy consumption is fully internalized, the front-runner's investment should be subsidized. In fact, such subsidy policies are widely implemented.[10] However, they can be harmful to welfare if the negative externality of energy consumption is not fully internalized. Therefore, such subsidy policies should be carefully implemented. If k is sufficiently small, however, subsidy policies improve welfare because the Jevons paradox never occurs (Proposition 1(i), Proposition 2(i)). Thus, the government should focus on drastic innovation when it introduces subsidy policies for energy-saving investments.

We now discuss the lagged (less-energy efficient) firm's incentive for investment. We compare the asymmetric case (I, N) and the energy-efficient case (I, I) and discuss the incentive of the lagged firm to catch up with the more energy-efficient rival. Similar to the analysis of the incentive for the front runner, whether $F_I < F_N^s$ or $F_I > F_I^s$ is crucial. If $F_I < F_I^s$, it is possible that only one firm invests in equilibrium, but both firms should invest from the welfare viewpoint, whereas the reverse never occurs. In other words, the lagged firm's investment incentive is insufficient for welfare. In contrast, if $F_I > F_I^s$, it is possible that only one firm should invest from the welfare viewpoint but both firms invest in equilibrium, whereas the reverse never occurs. In other words, the lagged firm's investment incentive is excessive for welfare. We present a condition for this comparison of F_I^s and F_I.

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[10] Policies for promoting energy conservation investments exist globally (Matsumura and Yamagishi, 2017).

Lemma 4 *$F_I < (\geq) F_I^s$ if and only if $\Theta_2 := 2(a-2w)r - (1-k)w^2 > 0$.*

Proof See Appendix.

We now present supplementary results to Propositions 1 and 2.

Proposition 3 *(i) $\Theta_2 < 0$ if r is sufficiently small. (ii) There exists (r, k, a, w) such that $\Theta_2 > 0$. (iii) $\Theta_2 < \Theta_1$ always holds.*

Proof See Appendix.

Proposition 4 *(i) $E(I, I) > E(I, N)$ if and only if $2w - a > 0$. In other words, Jevons paradox takes place if w is large. (ii) $2w - a < 0$ holds if $\Theta_2 > 0$. In other words, the lagged firm's investment incentive is excessive only when the firm's investment does not induce Jevons paradox.*

Proof See Appendix.

Suppose that firm 1 invests. Firm 2's private incentive for energy-saving investments is excessive for welfare if $r = 0$ (Proposition 3(i)). This is a mirror result of Proposition 1 because firm 2's investment induces welfare-reducing production substitution. However, if $r > 0$, it is possible that the private incentive for the lagged firm is insufficient from the welfare viewpoint (Proposition 3(ii)). This may be more intuitive than Proposition 1(iii). $r > 0$ implies that a negative externality uncovered by the emission tax exists, and firm 2 does not consider this effect when deciding whether to invest. This may lead to firm 2's insufficient incentive for energy-saving investments, when the Jevons paradox does not occur.

Proposition 3(iii) also has an important policy implication. It implies that when the incentive for the investment is insufficient given that the rival invests, the incentive is also insufficient given that the rival does not invest, whereas the reverse is not true. Thus, a subsidy policy for a lagged firm is rationalized only when the front-runner is also subsidized.

Comparing Proposition 2(i) and Proposition 4(i), we find that the switch from (I, N) to (N, N) is more likely to induce Jevons paradox than the stich from (N, N) to (I, N). This is because the latter yields production substitution from non-investing firm to investing firm, which itself reduces emissions. Nevertheless, as we discussed above, Proposition 2(i) implies that the switch from (I, N) to (N, N) can yield Jevons paradox.

5 Alternative model: a symmetric duopoly with continuous investments

In this section, we formulate an alternative model in which each firm chooses its energy-saving investment level continuously. Each firm i's energy efficiency k_i depends on I_i. All other elements such as demand, cost, and emission functions are the same as those in the model formulated in Section 2. Firm i's profit is given by $\pi_i = (a-(q_i+q_j))q_i - wk(I_i)q_i - I_i$.

In the first stage, each firm i independently chooses I_i that affects energy efficiency k_i. We assume $k_i(I_i) : \mathbb{R}_+ \to [0,1]$, twice continuously differentiable, decreasing and concave, and $k_i(0) = 1$. Moreover, we assume that k'' is sufficiently large to satisfy the second-order condition in the first stage. In the second stage, firms face Cournot competition.

In the second stage, firm i chooses q_i to maximize its profit. The first-order condition is

$$a - (q_i + q_j) - q_i - wk_i = 0. \tag{8}$$

The equilibrium output is

$$q_i(k_i, k_j) = \frac{a - 2wk_i + wk_j}{3} \ (i = 1, 2, \ i \neq j). \tag{9}$$

To ensure the nonnegative outputs for both firms, we assume $a - 2wk_i + wk_j > 0$.

In the first stage, each firm chooses I_i to maximize

$$\pi_i(k_i(I_i), k_j(I_j); I_i) = \frac{(a - 2wk_i(I_i) + wk_j(I_j))^2}{9} - I_i.$$

The first-order condition is:

$$\frac{d\pi_i}{dI_i} = -\frac{4(a - 2wk_i + wk_j)}{9}wk_i' - 1 = 0. \tag{10}$$

Focusing on the symmetric equilibrium ($I_i = I_j = I$), we obtain[11]

$$-\frac{4\,(a - wk(I))}{9} wk'(I) - 1 = 0. \tag{11}$$

Let the superscript $*$ denote the equilibrium level, and I^* be the equilibrium investment level satisfying (11).

We discuss the relationship between k and total emission $E = 2kq(k)$. We obtain

$$\frac{dE}{dk} = \frac{d\,(2kq(k))}{dk} = \frac{2}{3}(a - 2wk). \tag{12}$$

(12) implies that an increase in I (a decrease in k) reduces total emission if and only if $a > 2wk$.

We compare this equilibrium investment level with the second-best investment level. Welfare is given

$$W = \int_0^Q p(q)dq - pQ + \pi_1 + \pi_2 - r(k_i q_i + k_j q_j). \tag{13}$$

The social planner chooses a common investment level $I_1 = I_2 = I$ to maximize welfare. Because of the symmetry, welfare can be written as

$$W(I) = \frac{4(a - wk(I))^2}{9} - \frac{2rk(a - wk(I))}{3} - 2I. \tag{14}$$

We then obtain

$$\frac{dW}{dI} = -\frac{8wk'(a - k(I))}{9} - \frac{2rk'(a - 2wk(I))}{3} - 2. \tag{15}$$

Evaluating this at the equilibrium investment level I^*, we obtain

$$\left.\frac{dW}{dI}\right|_{I=I^*} = -\frac{2rk'}{3}(a - 2wk), \tag{16}$$

where we use (11).

The equilibrium investment level exceeds the second-best investment level if and only if (16) is strictly negative, and (16) is strictly negative if and only if $r > 0$ and $a < 2wk$. This

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[11] This symmetric equilibrium is a unique equilibrium (i.e., no asymmetric equilibrium exists) under the assumption that k'' is sufficiently large.

yields the following proposition:

Proposition 5 *In the continuous investment model, market competition yields excessive energy-saving investments if and only if* $r > 0$ *and* $a < 2wk$.

Proposition 5 states that when the negative externality of emissions is fully internalized by the emission tax, the emission tax provides an appropriate incentive for energy-saving investments. In other words, the competition leads to the efficient investment level for welfare. We naturally conjecture that when the emission tax is not fully internalizes the negative externality, incentives for energy-saving investments are insufficient; then, a subsidy for promoting energy-saving investments improve welfare. Proposition 5 states that this conjecture does not always hold true. When the negative externality is not fully internalized by the emission tax, energy-saving investments can be excessive for welfare. This is because energy-saving investment may induce the Jevons paradox, which harms welfare in the presence of insufficient emission tax. In other words, energy-saving investments may lead to an excessive expansion of energy consumption (and thus emission) under the emission tax that is lower than Pigouvian, which may harm welfare.[12] Proposition 4, thus, again suggests a risk of policy combination of a low emission tax and subsidies for energy-saving investments.[13]

6 Conclusion

This study investigates energy-saving investments in duopoly markets. We investigate the relationship between the emissions tax rate and investment incentives. Naturally, we expect investment incentives to be sufficient when the negative externality of energy consumption is internalized by the emission tax because firms have a strong incentive to save energy in the

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[12] Note that energy-saving investments may increase the total emissions even when the Pigouvian tax is imposed. However, in the presence of Pigouvian tax, the output expansions caused by energy-saving investments are restricted to the efficient level by the emission tax, and thus, do not reduce welfare.

[13] However, we should note that the investment incentives are insufficient if $r > 0$ and k is small. Thus, drastic innovation that yields small k should be subsidized.

presence of tax. We also naturally conjecture that if the emissions tax rate falls short of the negative externality, firms' incentives for energy-saving investments are insufficient; therefore, such investments should be promoted through additional policies. In fact, subsidies for energy-saving investments are prevalent globally.

However, our study demonstrates that this conjecture does not always hold. We show that competition may lead to excessive investments when the emission tax rate is low (falls short of the negative externality), although that never occurs when the emission tax is Pigouvian. This is because energy-saving investments may increase emission due to the output expansion (Jevons paradox), which yields serious welfare loss when the emission tax rate is low. Our results suggest a risk of the combination of low emission taxes and subsidies to compensate for low emission tax rates. We also demonstrate that an investment subsidy is desirable if the investment sufficiently improves energy efficiency (i.e., the energy-saving investment project is innovative) because such investments do not result in the Jevons paradox.

Nonetheless, there is a need for extensive future research due to the following limitations. In this study, we focus on energy-saving investments. Firms may make other investments, such as switching fuels (Hirose and Matsumura, 2023). Incorporating various firm activities remains a topic for future research.[14] We also focus on emissions taxes. Many other environmental policies exist, such as emission intensity regulations and green portfolio standards (Montero, 2002; Lahiri and Ono, 2007; Holland, 2012; Fowlie et al., 2016; Ino and Matsumura, 2019, 2021a,b; Hirose and Matsumura, 2020). The relationship between these policies and the optimality of energy-saving investments is also worth investigation. Moreover, we assume that the firms are profit maximizers. However, firms may be concerned about corporate social responsibility and may deviate from a profit-maximizing behavior (Lee and Park, 2019; Fukuda and Ouchida, 2020, 2023; Hirose et al., 2020; Hirose and Mat-

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[14] For discussions on green innovation, see Montero, 2002; Schiederig et al., 2012; Lambertini et al., 2017; McDonald and Poyago-Theotoky, 2017; Poyago-Theotoky and Yong, 2019.

sumura, 2022,2023; Xu et al., 2022). Extending our analysis to the non-profit-maximizing case remains a topic for future research.[15]

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[15] For a discussion on the relationship between non-profit-maximizing objectives and cost-reducing investments, see Matsumura et al. (2013) and López and Vives (2019).

Appendix

Proof of Lemma 1

(i) From (4) and (7), we obtain

$$\pi(I,I) - \pi(N,I) = \frac{4(a-w)(1-k)w}{9} - F.$$

Thus, $\pi(I,I) \geq \pi(N,I)$ holds if and only if

$$\frac{4(a-w)(1-k)w}{9}(:= F_I) \geq F.$$

(ii) From (5) and (6), we obtain

$$\pi(N,N) - \pi(I,N) = -\frac{4(a-kw)(1-k)w}{9} + F.$$

Thus, $\pi(N,N) \geq \pi(I,N)$ holds if and only if

$$F > \frac{4(a-kw)(1-k)w}{9}(:= F_N).$$

Comparing F_I and F_N, we also obtain

$$F_N - F_I = \frac{4(1-k)^2w^2}{9} > 0.$$

(iii) From Lemma 1(i) and (ii), we know that $\pi(I,N) \geq \pi(N,N)$ and $\pi(I,N) \geq \pi(I,I)$ holds if and only if $F_I \leq F \leq F_N$. ■

Proof of Lemma 2

(i,ii) From (3), we obtain

$$\begin{aligned} W(I,I) &= \frac{2(2a-k(3r+2w))(a-kw)}{9} - 2F, \\ W(N,N) &= \frac{2(2a-3r-2w)(a-w)}{9}, \\ W(I,N) &= \frac{8a^2 - 2a(k+1)(3r+4w) + w\left(12\left(k^2-k+1\right)r + \left(11k^2-14k+11\right)w\right)}{18} - F. \end{aligned}$$

Comparing these three values, we obtain

$$\begin{aligned} W(I,I)-W(I,N) &= \frac{(1-k)(a(6r+8w)-w(12r+(11-3k)w))}{18}-F, \\ W(N,N)-W(I,N) &= \frac{(1-k)(2a(3r+4w)-w(12kr-(3-11k)w))}{18}+F. \end{aligned}$$

Thus, $W(I,I) \geq W(I,N)$ holds if and only if

$$\frac{(1-k)(a(6r+8w)-w(12r+(11-3k)w))}{18} (:= F_I^s) > F.$$

$W(N,N) \geq W(I,N)$ holds if and only if

$$F > \frac{(1-k)(2a(3r+4w)-w(12kr-(3-11k)w))}{18} (:= F_N^s).$$

Comparing F_I^s and F_N^s, we obtain

$$F_N^s - F_I^s = \frac{(1-k)^2 w(6r+7w)}{9} > 0.$$

From these conditions, we obtain $W(I,I) \geq W(I,N) \geq W(N,N)$ if and only if $F \leq F_N^s$.

Using a similar procedure, we have $W(N,N) \geq W(I,N) \geq W(I,I)$ if and only if $F \geq F_N^s$.

(iii) From Lemma 2(i,ii), $W(I,N) \geq W(N,N)$ and $W(I,N) \geq W(I,I)$ hold if and only if $F_I^s \leq F \leq F_N^s$. ■

Proof of Lemma 3

From Lemmas 1 and 2, we obtain

$$\begin{aligned} F_N^s - F_N &= \frac{(1-k)(2a(3r+4w)-w(12kr-(3-11k)))}{18} - \frac{4(a-wk)(1-k)w}{9} \\ &= \frac{(1-k)\left(2(a-2kw)r+(1-k)w^2\right)}{6}. \end{aligned}$$

This is positive if and only if $\Theta_1 > 0$. ■

Proof of Proposition 1

(i) By rearranging, we can verify that $\Theta_1 > 0$ is satisfied if and only if

$$k < \frac{2ar + w^2}{w(4r + w)} := k^W.$$

(ii) When $r = 0$, we have $k^W = 1$.

(iii) From Proposition 1(iii), $k^W = 1$ when $r = 0$. Suppose $r > 0$. Then, rearranging $k^W < 1$ gives us $2r(2w - a) > 0$. Thus, $k^W < 1$ if and only if $2w > a$ and $r > 0$.

(iv) Differentiating k^W with r, we obtain

$$\frac{dk^W}{dr} = \frac{2(a - 2w)}{(4r + w)^2}.$$

Thus, dk^W/dr is negative if $a < 2w$. ■

Proof of Proposition 2

(i) From (3), we obtain

$$E(N, N) = \frac{2(a - w)}{3}, \quad E(I, N) = \frac{a(1 + k) - 2(1 - k + k^2)w}{3}. \tag{17}$$

Thus, we obtain

$$E(I, N) - E(N, N) = \frac{(1 - k)(2kw - a)}{3}. \tag{18}$$

This is positive if and only if $2kw - a > 0$.

(ii) As we show in the Proof of Proposition 1(iii), $2kw - a > 0$ is a necessary condition for $\Theta_1 < 0$. ■

Proof of Lemma 4

From Lemmas 1 and 2, we obtain

$$\begin{aligned} F_I^s - F_I &= \frac{(1 - k)(a(6r + 8w) - w(12r + (11 - 3k)w))}{18} - \frac{4(a - w)(1 - k)w}{9} \\ &= \frac{(1 - k)\left(2(a - 2w)r - (1 - k)w^2\right)}{6}. \end{aligned}$$

This is positive if and only if $\Theta_2 > 0$. ■

Proof of Proposition 3

(i) If r is sufficiently small such that

$$r < \bar{r} = \frac{(1-k)w^2}{2(a-2w)}, \tag{19}$$

$\Theta_2 < 0$ holds. Note that $\bar{r}$ is negative if $a - 2w < 0$, which implies $\Theta_2 < 0$ for any r.

(ii) $\Theta_2 > 0$ is satisfied if $a > 2w$ and $r > \bar{r}$.

(iii) From Propositions 1(i) and 3(i), $\Theta_1 - \Theta_2 = 2(1-k)w(2r + w) > 0$. ■

Proof of Proposition 4

(i) From (3), we obtain

$$E(I, I) = \frac{2(a - kw)k}{3}. \tag{20}$$

Thus, we obtain

$$E(I, I) - E(I, N) = \frac{(1-k)(2k - a)}{3}. \tag{21}$$

(ii) As shown Proposition 3 (ii), $\Theta_2 < 0$ is satisfied if $a < 2w$ and $r < \bar{r}$. ■

Declaration of interest

The authors declare that they have no conflicts of interest and that there are no financial or personal relationships with other individuals or organizations that could inappropriately influence our work.

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