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Abstract

This study investigates how the income redistribution policy affects economic growth, employment, income distribution, income inequality, and asset inequality. The income redistribution policy is defined as one that imposes capital taxation on capitalists and redistributes it to workers. Therefore, we constructed a Kaleckian model in which, in addition to capitalists, workers own capital stock through savings. Depending on the relative size of workers' and capitalists' saving rates, we obtained the Pasinetti equilibrium, in which both classes coexist, and the dual equilibrium, in which only workers own capital stock, whereas capitalists do not. In the Pasinetti equilibrium, raising the tax rate for capitalists drives an increase in workers' assets and income shares. Simultaneously, economic growth and employment rates increase when the short-run equilibrium is wage-led growth whereas they decrease when the short-run equilibrium is profit-led growth. Hence, the income redistribution policy is effective in reducing inequality and promoting economic growth and employment when the short-run equilibrium is wage-led.

Keywords: workers' saving, income equality, income redistribution policy, growth, employment

JEL Classification: E11; E12; E64; J31; J53

1 Introduction

Since Thomas Piketty's (2014) book *Capital in the Twenty-First Century* became a worldwide bestseller and Branko Milanovic's (2016) book *Global Inequality* became

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a hot topic, there has been increasing interest in inequality. Piketty (2014) reveals that, in developed countries, the income gap between the top and bottom groups has been expanding. Additionally, Milanovic (2016) extends the Kuznets inverted U-curve hypothesis such that as an economy develops, income inequality increases and then decreases, and suggests an elephant curve such that as the economy develops further, income inequality increases again. The Kaleckian model focuses on the relationship between income distribution and economic growth, and as such, is suitable for examining such issues. Piketty claims that capital taxation and income redistribution are progressive. Certainly, it seems reasonable to increase the tax burden on the top-income group and redistribute it to the lower-income group. However, the economic model developed in his book has some theoretical problems, as many economists state. Hence, we cannot conduct a rigorous analysis using it. Therefore, this study presents a long-run Kaleckian growth model to investigate how the income redistribution policy affects the income inequality between capitalists and workers.

As we attempt to examine wealth and income inequalities, we build a Kaleckian model in which both capitalists and workers own capital through savings. The debate between Pasinetti (1962) and Samuelson and Modigliani (1966) regarding workers' saving is important.[1)] Pasinetti (1962) proposes the Cambridge equation, such that even in an economy where workers save, the long-run profit rate is determined by the ratio of the natural growth rate to the saving rate of capitalists. This theory is known as the Pasinetti theorem. Conversely, Samuelson and Modigliani (1966) criticize Pasinetti and state that the Pasinetti theorem is true when the saving rate of capitalists is significantly higher than that of workers, but false when the saving rate of capitalists is insignificantly larger than that of workers. Moreover, they claim that unless Pasinetti's assumption holds, another equilibrium, that is, a dual equilibrium emerges, and the Cambridge equation does not hold. In summary, theoretically, both Pasinetti and dual equilibria exist depending on conditions.[2)] Our Kaleckian model follows this debate and uses the Pasinetti type saving function.

As in related studies that investigate inequality between capitalists and workers using a Kaleckian model with the Pasinetti type saving function, we refer to Ederer

1) In addition, Kaldor (1956) suggests a saving function in which the propensity to save from wages and from profit differ. However, Pasinetti (1962) criticizes Kaldor because his approach is a specification that a different saving rate corresponds to a different source of income, rather than a specification that a different saving rate corresponds to a different class. Unexpectedly, the Kaldorian saving function is used by neoclassical economists: Böhm and Kaas (2000), Klump and de La Grandville (2000), Klump and Preissler (2000), Dalgaard and Hansen (2005), and Irmen and Klump (2009).

2) Furuno (1970) introduces the Pasinetti type saving function into the Solow growth model, and calculates the speed of convergence toward the Pasinetti or Dual equilibrium. He reveals that 90% convergence from an initial value to the steady state requires hundreds or thousands of years.

and Rehm (2020), Kumar *et al.* (2018), Taylor (2014), and Taylor *et al.* (2019).[3] This study is part of a research series.

Ederer and Rehm (2020) presented a Kaleckian model with Marglin and Bhaduri's (1990) investment function. They assume that profit share is given exogenously. They presented two types of models. In the basic model, the long-run wealth shares are determined only by the saving rates and profit shares. In the extended model, capitalists and workers obtain wages. Moreover, they assume that workers and capitalists face different rates of return, which are given exogenously. In the long run, each class's wealth share becomes constant, which is determined by six parameters: workers' saving rate, capitalists' saving rate, profit share, wage distribution parameter between capitalists and workers, rate of return for workers, and rate of return for capitalists. They further estimated these parameters using data from different countries. Although they employed the Kaleckian model, they do not need to estimate the parameters of the investment function because investment demand does not affect each class's wealth distribution in the long run. It is important to note that the basic model's property that long-run wealth distribution is determined only by saving rates and profit share also holds in the neoclassical model. Hence, it is safe to conclude that their results are not based on the Kaleckian model. If we use the Kaleckian model, we must investigate the transitional dynamics toward the long-run and the long-run equilibrium.

Kumar *et al.* (2018) provided a Kalecki model in which the profit share is given exogenously, and the investment function is an increasing function of the profit rate. They estimated the parameters of the model utilizing Bayesian inference using data on the US economy during the period 1950–2015. Unlike Ederer and Rehm (2020), they estimated the investment function, presenting the transitional dynamics of each class's wealth share. However, they did not consider labor supply constraints. Therefore, their model was a short- or medium-run Kaleckian model. Accordingly, employment rate is not determined endogenously.[4]

Taylor's (2014) Kaleckian model uses an investment function in which a firm's planned investment is an increasing function of the profit rate. In contrast to the

3) We take Petach and Tavani (2020), Zamparelli (2017), Mattauch *et al.* (2016), and Sasaki (2022) as examples that investigate inequality between capitalists and workers using models other than the Kaleckian one. Petach and Tavani (2020) used the classical growth model, Zamparelli (2017) used the Solow model, Mattauch *et al.* (2016) and Sasaki (2022) used the hybrid Ramsey and overlapping generations model. Regarding the hybrid model, see Michl and Foley (2004), Michl (2009), Commendatore and Palmisani (2009), and Kurose (2022). Faria and Araujo (2004) and Góes and Teixeira (2022) present models in which both capitalists and workers follow the Ramsey type models.

4) For Kaleckian models that investigate the endogenous determination of the employment rate by considering the labor supply constraint, see Dutt (1992), Skott and Zipperer (2010), and Sasaki (2010, 2013).

abovementioned studies, he endogenizes the profit share, which is a decreasing function of the capacity utilization rate.[5] However, similar to Kumar *et al.* (2018), the employment rate is not determined endogenously. Moreover, he incorporated an income redistribution policy, as in this study, but did not investigate how this policy affects the economy.

The model by Taylor *et al.* (2019) is the closest to our model. The investment function is a Kaleckian type, in which a firm's planned investment is an increasing function of profit and capacity utilization rates. It also considers imports, exports, government expenditures, and taxation. The employment rate is determined endogenously by Okun's law, such that it is positively correlated with the capacity utilization rate. Therefore, the employment rate has a one-to-one relationship with the capacity utilization rate. Moreover, they endogenize the profit share using the Marx–Goodwin-type profit squeeze function, such that the profit rate is a decreasing function of the employment rate (i.e., the capacity utilization rate). They consider labor supply constraints and technological progress; hence, the long-run employment rate is constant. The technological progress rate, that is, the growth rate of labor productivity is an increasing and decreasing function of capital accumulation and profit rates, respectively. They present a numerical simulation of the US economy.

Compared with the aforementioned studies, our model has the following five characteristics:

First, regarding the investment function, we use the Marglin–Bhaduri type investment function to investigate how the income redistribution policy affects an economy, depending on whether the economy exhibits a wage-led or profit-led regime. Hence, a firm's planned investment is an increasing function of its capacity utilization rate and profit share.

Second, the profit share is endogenized by the reserve army effect á la Marx. Hence, profit share is a decreasing function of the employment rate. Our specification follows that of Ohno (2022).

Third, we introduce endogenous technological progress, such that the growth rate of labor productivity is an increasing function of the employment rate. Similar specifications were employed by Dutt (2006), Flaschel and Skott (2006), Sasaki (2010, 2013), and Lima *et al.* (2021). Sasaki (2013) refers to this specification as the reserve army creation effect. Given the output, an increase in labor productivity decreases employment, which increases the reserve army of labor, that is, unemployment.

5) For Kaleckian models that consider the endogenous determination of income distribution, see Dutt (1987) and Cassetti (2003, 2006).

Fourth, we rigorously examine the employment rate. First, the aforementioned studies, other than Taylor *et al.* (2019) do not consider labor supply constraints; hence, the employment rate is not constant in the long run. According to Sedgley and Elmslie (2004), the long-run employment rate is roughly constant in many countries. Accordingly, to conduct a long-run analysis, a growth model that considers labor supply constraints should be employed. Further, although Taylor *et al.* (2019) endogenize the employment rate, in their specification as stated above, the employment and capacity utilization rates necessarily move in the same direction. In contrast, we use the definition of the employment rate such that it is a product of the capacity utilization rate and capital stock per effective labor supply and obtain the dynamic equation of the employment rate. Therefore, in our model, the employment and capacity utilization rates do not necessarily move in the same direction. As the numerical simulation introduced later reveals, according to the regime an economy exhibits, the income redistribution policy has different effects on the employment and capacity utilization rates.

Subsequently, we investigate the effect of the income redistribution policy such that the government imposes taxation on capitalists and redistributes it to workers. However, this policy was not considered in the above-mentioned four studies.

Our results show that irrespective of the possible integrations of the demand and growth regimes, the long-run Pasinetti equilibrium is stable, provided that the reserve army and reserve army creation effects are relatively strong. In the long-run Pasinetti equilibrium, if we raise the tax rate for capitalists, workers' assets and income share increase. Simultaneously, the economic growth and employment rates increase when the short-run equilibrium is wage-led growth whereas they decrease when the short-run equilibrium is profit-led growth. Hence, the income redistribution policy is effective in reducing inequality and promoting economic growth and employment when the short-run equilibrium is wage-led growth.

This paper proceeds as follows. Section 2 explains the framework of the model. Section 3 presents a short-run analysis in which the capacity utilization rate is adjusted. Section 4 details a long-term analysis of capital accumulation, labor supply growth, and technological progress. Section 5 presents numerical simulations to examine how income redistribution policies affect growth, employment, and inequality. Section 6 further concludes the paper.

2 Model

Consider an economy with workers and capitalists. Capitalists own capital and obtain profits. Workers indirectly own capital through their savings; hence, they obtain profits and wages. The final good is produced using capital stock and labor power. The production function takes the Leontief function from:

$$Y = \min\{aE, uK\}. \tag{1}$$

Y denotes output, E denotes employment, and K denotes capital stock. From cost minimization, firms operate under $aE = uK$. From this, we obtain $Y = aE = uK$; hence, a and u drive labor productivity and the capacity utilization rate, respectively. Here, the potential output-capital ratio is normalized as unity. We assume that capital stock is constant at each time point; hence, output adjustment is conducted by employment and the capacity utilization rate. We let K_c and K_w denote the capital stocks of capitalists and workers, respectively. Therefore, we obtain $K = K_c + K_w$.

We define the employment rate e as

$$e = \frac{E}{N} = \frac{(uK/a)}{N} = uk, \quad k = \frac{K}{aN}. \tag{2}$$

N denotes labor supply and grows at a constant rate $n > 0$. Here, k is the capital stock per effective labor supply.

Here, we introduce the reserve army effect. According to Ohno (2022), we assume that profit share π is a decreasing function of the employment rate.

$$\pi = \pi(e) = \pi(uk), \quad \pi'(e) < 0. \tag{3}$$

This implies that as the bargaining power of labor unions escalates through an increase in the employment rate, the wage increase pressure rises, decreasing the profit share. We specify this as a linear function, as follows:

$$\pi(e) = \pi_0 - \pi_1 e = \pi_0 - \pi_1 uk, \quad \pi_0 > 0,\ \pi_1 > 0. \tag{4}$$

Capitalists save a fraction of their profit after taxation. Workers save a constant fraction of their total income, comprising wages, profits from capital holdings, and redistributed profits.

$$S_c = s_c(1-\tau)rK_c, \quad s_c \in (0,1),\ \tau \in (0,1), \tag{5}$$

$$S_w = s_w(wE + rK_w + \tau r K_c), \quad s_w \in (0,1). \tag{6}$$

S_c denotes capitalists' savings, S_w denotes workers' savings, s_c denotes capitalists' savings rate, s_w denotes workers' savings rate, r denotes the profit rate, w denotes the real wage rate, and τ denotes the capitalists' tax rate. We assume that $s_w < s_c$.

The total savings of the economy are given by $S = S_c + S_w$. Then, the savings per capita stock are given by

$$\frac{S}{K} \equiv g_s = [(s_c - s_w)(1-\tau)\pi k_c + s_w]u = su, \quad k_c = \frac{K_c}{K}. \tag{7}$$

Here, k_c denotes the share of capitalists' capital stock in the total capital stock. In the following, k_c represents capitalists' wealth share. Moreover, we define $s \equiv (s_c - s_w)(1-\tau)\pi k_c + s_w$. Therefore, we determine that $0 < s < 1$. Accordingly, we can regard s as the economy's average savings rate. Note that s depends on π and k_c. It is also important to note that $s = s_w$ when $k_c = 0$. Below, we examine the dual equilibrium in which the capitalists' wealth shares are zero. In this case, the average savings rate of the economy is equal to the workers' savings rate.

We assume that the investment function takes the Marglin–Bhaduri form; equip investment is an increasing function of the capacity utilization rate and profit share. We use the profit share after taxation $(1-\tau)\pi$ as a variable of the investment function.[6)] In this case, we specify the investment per capital stock g_d as follows:

$$\frac{I}{K} \equiv g_d = \gamma + \alpha u + \beta(1-\tau)\pi, \quad \gamma > 0,\ \alpha > 0,\ \beta > 0. \tag{8}$$

I denotes the planned investment, γ represents the parameter capturing animal spirits, and α and β parameters represent investment responses.

Considering the reserve army creation effect, we assume that the growth rate of labor productivity g_a is an increasing function of the employment rate.

$$g_a = g_a(e), \quad g_a'(e) > 0. \tag{9}$$

We specify it as follows:

$$g_a = \eta e^{\lambda} = \eta(uk)^{\lambda}, \quad \eta > 0,\ \lambda > 0. \tag{10}$$

6) Lima *et al.* (2021), who consider human capital investment by government taxation, also use the profit share after taxation as a variable of the investment function. In contrast, Taylor (2014), who considers capital taxation as in our study, uses profit rate pre-taxation.

Here, η represents the efficiency parameter, and the λ parameter governs the elasticity of technological progress with respect to the employment rate, which reflects the degree of the reserve army creation effect.

We specify the model, and proceed to the short- and long-run analyses.

3 Short-run analysis

Suppose that K, K_c, a, and N are constant in the short run. Then, k and k_c are constant in the short run. Quantity adjustments are prevalent in the goods market. Firms increase their output when the market is in excess demand and decrease it when the market is in excess supply.

$$\dot{u} = \phi\left(g_d - g_s\right), \quad \phi > 0. \tag{11}$$

Here, $\dot{u} = du/dt$ denotes the time derivative of the capacity-utilization rate. Parameter ϕ captures the adjustment speed of the goods market.

Consider the short-run equilibrium is a situation in which the capacity utilization rate is constant; that is, $\dot{u} = 0$ holds. From $\dot{u} = 0$, we obtain

$$su = \gamma + \alpha u + \beta(1-\tau)\pi(uk). \tag{12}$$

We note that π is a function of e because of the reserve army effect. Hence, π is a function of uk

By solving equation (12) for u, we ascertain that u is a function of k and k_c: Hence, we formulate it as

$$u^* = u(k, k_c). \tag{13}$$

This is the short-run equilibrium capacity utilization rate.

The short-run stability condition is given by $\partial\dot{u}/\partial u|_{u=u^*} < 0$, which induces

$$\Omega \equiv \underbrace{\alpha - s}_{\Gamma} + \underbrace{(1-\tau)k\pi'(u^*k)[\beta - (s_c - s_w)u^*k_c]}_{\Theta} < 0. \tag{14}$$

Γ summarizes the direct effect such that a change in the capacity utilization rate affects savings and investment. Θ summarizes the indirect effect such that a change in the capacity utilization rate affects savings and investment.

We assume $\Gamma < 0$, which corresponds to the Keynesian stability condition in the usual Kaleckian model; the direct effect of the capacity utilization rate on investment is smaller than that on savings. As stated above, we have $s = s_w$ when capitalists' wealth share is zero. Therefore, for $\Gamma < 0$ to hold even in such case, we assume $s_w > \alpha$.

The sign of Θ depends on the sign of $\beta - (s_c - s_w)u^* k_c$. Parameter β is a coefficient of the investment function, while $(s_c - s_w)u^* k_c$ is a coefficient of the saving function. We obtain $\Theta < 0$ when $\beta > (s_c - s_w)u^* k_c$, while $\Theta > 0$ when $\beta < (s_c - s_w)u^* k_c$.

For $\Theta < 0$, where $\Gamma < 0$, we obtain $\Omega < 0$. Therefore, the stability condition given by equation (14) is satisfied. In contrast, for $\Theta > 0$, we do not necessarily obtain $\Omega < 0$. Numerical simulations introduced later show that $\Theta > 0$. However, in these simulations, we obtain $\Omega < 0$.

Proposition 1. *Consider the typical Keynesian stability condition holds. Moreover, consider the condition* $\beta - (s_c - s_w)u^* k_c > 0$ *holds. Therefore, the short-run equilibrium is asymptotically and locally stable.*

In our specification, the investment function becomes a straight line with a positive slope and intercept on the (u, g)-plane. The saving function becomes a convex upward parabola. When $u = 0$, g_s is positive; Additionally, when $u = 1$, g_s is positive. The value of g_s when $u = 0$ is greater than that when $u = 1$. Therefore, under appropriate conditions, as Figure 1 shows, both functions have two intersections: the smaller intersection is stable, whereas the larger one is unstable.

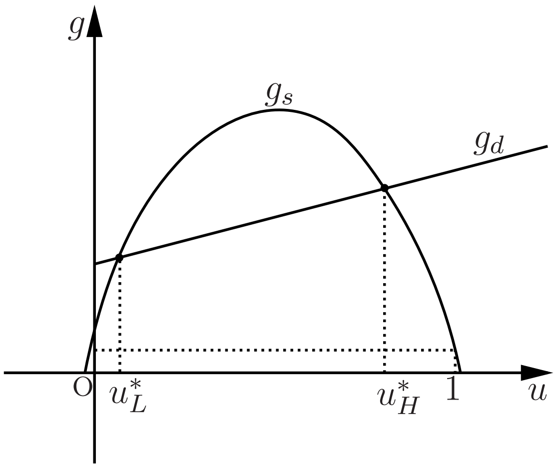


Figure 1: Short-run equilibrium

Specifically, the short-run equilibrium capacity utilization rate satisfies the following quadratic equation:

$$(s_c - s_w)(1-\tau)\pi_1 k k_c \cdot u^2 - [(s_c - s_w)(1-\tau)\pi_0 k_c + s_w - \alpha + \beta(1-\tau)\pi_1 k] \cdot u$$
$$+ \gamma + \beta(1-\tau)\pi_0 = 0. \tag{15}$$

When $s_w > \alpha$, the coefficient of u is negative. Accordingly, the left-hand side of this equation is a convex downward parabola, the axis of symmetry is positive, and the intercept through the vertical axis is positive. Accordingly, we find that this quadratic equation has two distinct real roots with positive signs u_L and u_H, of which the smaller one, u_L corresponds to a stable equilibrium.

When the short-run equilibrium is stable (i.e., when $\Omega < 0$, the effect of an increase in k_c on u^* is given by

$$\frac{du^*}{dk_c} = \frac{(s_c - s_w)(1-\tau)\pi(u^* k)u^*}{\Omega} < 0. \tag{16}$$

This negative sign follows from $\Omega < 0$. Similarly, the effect of an increase in k on u^* is given by:

$$\frac{du^*}{dk} = -\frac{(1-\tau)\pi'(u^* k)u^*[\beta - (s_c - s_w)k_c u^*]}{\Omega} = -\frac{\Theta}{\Omega}\frac{u^*}{k}. \tag{17}$$

The sign depends on the sign of Θ. We have $du^*/dk < 0$ when $\Theta < 0$ whereas $du^*/dk > 0$ when $\Theta > 0$.

From the above analysis, we obtain the following relationships:

$$u^* = u(k, k_c), \quad \frac{\partial u}{\partial k} \gtrless 0, \ \frac{\partial u}{\partial k_c} < 0. \tag{18}$$

Using equation (18), we obtain the effects of k and k_c on π and g_a as follows:

$$\frac{\partial \pi}{\partial k} = u\pi'(e)\,\frac{\Gamma}{\Omega} < 0, \tag{19}$$
$$\frac{\partial \pi}{\partial k_c} = k\pi'(e)\,\frac{\partial u}{\partial k_c} > 0, \tag{20}$$
$$\frac{\partial g_a}{\partial k} = ug_a'(e)\frac{\Gamma}{\Omega} > 0, \tag{21}$$
$$\frac{\partial g_a}{\partial k_c} = kg_a'(e)\frac{\partial u}{\partial k_c} < 0. \tag{22}$$

These relationships will be used in the long-run analysis.

3.1 Demand and growth regimes

In the following, we define the Pasinetti and Dual equilibria as situations in which $k_c > 0$ and $k_c = 0$, respectively. To focus on the economy in which capitalists and workers coexist, we mainly present an analysis of Pasinetti equilibirum, and that of the dual equilibrium is presented in the Appendix.

We consider the demand regime in the Pasinetti equilibrium. In the typical Kaleckian model, profit share is an exogenous variable. The demand regime is then defined by whether capacity utilization increases or decreases according to a change in profit share. In contrast, in our model, profit share is an endogenous variable in both the short and long run. Therefore, we regard profit share as an exogenous variable and define the demand regime. According to equation (12), the short-run Pasinetti equilibrium capacity utilization rate is given by

$$u_P^* = \frac{\gamma + \beta(1-\tau)\pi}{s(\pi) - \alpha}. \tag{23}$$

As s depends on π, we determine $s(\pi)$. By partially differentiating equation (23) with respect to π, we obtain

$$\frac{\partial u^*}{\partial \pi} = \frac{(1-\tau)[\beta(s_w - \alpha) - (s_c - s_w)\gamma k_c]}{(s-\alpha)^2}. \tag{24}$$

Equation (24) depends on k_c. Hence, to classify the regime in the long run, we must provide k_c that is fixed in the short run. In the numerical simulation introduced later, we assume that in the initial period, the economy is in a long-run equilibrium under the benchmark parameter setting. Therefore, we insert the long-run equilibirum values of k and k_c under the benchmark parameter setting into equation (24).

As we assume that $s_w > \alpha$, the sign of the right-hand side of equation (24) is ambiguous; hence, we obtain the wage-led demand (WLD) regime $\partial u^*/\partial \pi < 0$ or profit-led demand (PLD) regime $\partial u^*/\partial \pi > 0$. If we set $s_w = 0$,that is, if workers do not save, the sign on the right-hand side of equation (24) is negative; hence, we obtain the WLD regime. This suggests that we can obtain the PLD regime because workers save.

The sign on the right side of equation (24) depends on the sign of the numerator, which consequently depends on the following sign:

$$\beta \underbrace{(s_w - \alpha)}_{(+)} - (s_c - s_w)\gamma k_c. \tag{25}$$

This suggests that when animal spirits γ and capitalists' wealth shares k_c are relatively small, we obtain the PLD regime. In contrast, when they are relatively large, we obtain the WLD regime.

We further consider the growth regime. The short-run equilibrium value of g is given by substituting equation (23) into equation (7), which induces

$$g^* = \frac{s(\pi)[\gamma + \beta(1-\tau)\pi]}{s(\pi) - \alpha}. \tag{26}$$

By partially differentiating this equation with respect to π, we obtain

$$\frac{\partial g^*}{\partial \pi} = \frac{A(\pi)}{[s(\pi) - \alpha]^2}. \tag{27}$$

The sign of equation (27) depends on the sign of the numerator on the right-hand side. The term $A(\pi)$ is defined as

$$\begin{aligned} A(\pi) = \beta(1-\tau)^3(s_c - s_w)^2 k_c^2 \left[\pi + \frac{s_w - \alpha}{(1-\tau)(s_c - s_w)k_c}\right]^2 \\ + \alpha(1-\tau)[\beta(s_w - \alpha) - (s_c - s_w)\gamma k_c]. \end{aligned} \tag{28}$$

Most terms of the numerator on the right-hand side of equation (28) are positive. Then, if the term $\beta(s_w - \alpha) - (s_c - s_w)\gamma k_c$ is negative and its absolute value is relatively large, the numerator of equation (28) is negative. This condition is similar to that used to determine the demand regime. Accordingly, we are likely to find a wage-led growth (WLG) regime when we find a WLD regime. In contrast, we necessarily obtain a profit-led growth (PLG) regime when we obtain the PLD regime. This suggests that, in our model, we are more likely to obtain a PLG regime than WLG regime. As Blecker (2002) suggests, in the Kaleckian model, we are likely to obtain a PLG regime when we introduce workers' savings (strictly speaking, savings from wages). The same holds for our model.

Based on the above discussion, the possible integrations of the demand and growth regimes are as follows:

1. PLD and PLG

2. WLD and PLG

3. WLD and WLG

The subsequent numerical simulations show the parameter sets that reproduce these integrations of the demand and growth regimes.

From the analysis of the dual equilibirum in the Appendix, we obtain Table 1.

Table 1: Demand and growth regimes in short-run equilibrium

Pasinetti equilibrium	PLD & PLG	WLD & PLG	WLD & WLG
Dual equilibrium	PLD & PLG	n.a.	n.a.

3.2 Effects of capital taxation in the short run

We investigate the effect of an increase in the tax rate on the short-run equilibrium capacity utilization rate. By completely differentiating equation (12) with respect to u and τ, we obtain

$$\frac{du^*}{d\tau} = -\frac{[(s_c - s_w)k_c u^* - \beta]\pi}{\Omega}. \tag{29}$$

Note that $\Omega < 0$, The sign of the numerator depends on that of Θ. When $\Theta < 0$, we obtain $(s_c - s_w)k_c u^* - \beta < 0$, which induces $du^*/d\tau < 0$. Accordingly, strengthening the income redistribution policy decreases the capacity utilization rate. This finding implies that an increase in the tax rate depresses investment demand when the investment response to after-tax profits is sufficiently high. In contrast, when $\Theta > 0$, we obtain $du^*/d\tau > 0$. When the response of investment to after-tax profits is sufficiently low, an increase in the tax rate intensifies investment demand.

As the short-run equilibrium employment rate is given by $e^* = u^* k$ and k is fixed in the short run, the effect of an increase in τ on e^* is

$$\frac{de^*}{d\tau} = k \cdot \frac{du^*}{d\tau}. \tag{30}$$

Given that the effect of an increase in the tax rate on the capacity utilization rate is ambiguous, the effect on the employment rate is also ambiguous. Note that in the short run, both u^* and e^* move in the same direction.

The effect of an increase in the tax rate on profit share is given by

$$\frac{d\pi^*}{d\tau} = \pi'(e) \cdot k \cdot \frac{du^*}{d\tau}. \tag{31}$$

This sign is ambiguous: u^* and π^* move in the opposite directions.

The effect of an increase in the tax rate on the short-run capital accumulation rate is the sum of the direct effect through investment, indirect effect through a change in the capacity utilization rate, and indirect effect through a change in the profit share, which is given by

$$\frac{dg^*}{d\tau} = \frac{\partial g^*}{\partial \tau} + \frac{\partial g^*}{\partial u^*} \cdot \frac{du^*}{d\tau} + \frac{\partial g^*}{\partial \pi^*} \cdot \frac{d\pi^*}{d\tau}. \tag{32}$$

Further calculations yield the following:

$$\frac{dg^*}{d\tau} = -\beta\pi^* + \alpha \cdot \underset{(+/-)}{\frac{du^*}{d\tau}} + \frac{\overset{(+/-)}{A(\pi)}}{(s-\alpha)^2} \cdot \underset{(-/+)}{\frac{d\pi^*}{d\tau}} \tag{33}$$

$$= -\beta\pi^* + \underset{(+/-)}{\frac{du^*}{d\tau}} \left[\alpha - \pi_1 k \frac{\overset{(+/-)}{A(\pi)}}{(s-\alpha)^2} \right]. \tag{34}$$

The possible integrations are (i) $\Theta > 0$ and PLG, (ii) $\Theta > 0$ and WLG, (iii) $\Theta < 0$ and PLG, and (iv) $\Theta < 0$ and WLG. Note that $\Theta >$ and $\Theta < 0$ correspond to $du^*/d\tau > 0$ and $du^*/d\tau < 0$, respectively. Only in (iv) is the sign of equation (34) $dg^*/d\tau < 0$. In (i) () (iii), the sign of equation (34) is ambiguous. Nevertheless, considering the signs of the three terms on the right side of equation (34), it is safe to conclude that we are likely to have $dg^*/d\tau < 0$ in (i) and (iii), and $dg^*/d\tau > 0$ in (ii).

Finally, the sign of $dg_a^*/d\tau$ corresponds to that of $de^*/d\tau$ because $g_a(e)$ is an increasing function of e owing to the reserve army creation effect.

Summarizing the discussion above, we obtain Table 2.

Table 2: Effects of a rise in tax rate on short-run Pasinetti equilibrium

	u^*	e^*	π^*	g^*	g_a^*
τ	$+/-$	$+/-$	$-/+$	$+/-$	$+/-$

4 Long-run analysis

In the long run, the short-run equilibrium is always attained, K, K_c, a, and N change, and hence, k and k_c are adjusted variables.

By log-differentiating the definitions of k and k_c, we obtain:

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - g_a - n = \frac{S}{K} - g_a(e) - n, \tag{35}$$

$$\frac{\dot{k}_c}{k_c} = \frac{\dot{K}_c}{K_c} - \frac{\dot{K}}{K}. \tag{36}$$

Here, the rates of change of K and K_c are given by

$$\frac{\dot{K}}{K} = \frac{S}{K} = su, \tag{37}$$

$$\frac{\dot{K}_c}{K_c} = s_c(1-\tau)\pi u. \tag{38}$$

By substituting equations (37) and (38) into equation (63), we obtain the dynamic equations for k and k_c.

$$\dot{k} = \Bigg\{ \underbrace{[(s_c - s_w)(1-\tau)\pi(k,k_c)k_c + s_w]}_{\equiv s} u(k,k_c) - g_a(k,k_c) - n \Bigg\} k, \tag{39}$$

$$\dot{k}_c = \Bigg\{ (1-\tau)[s_c - (s_c - s_w)k_c]\pi(k,k_c) - s_w \Bigg\} u(k,k_c)k_c. \tag{40}$$

We note that the capacity utilization is a function of k and k_c. Additionally, because g_a and π are functions of e and $e = uk$, g_a and π become functions of k and k_c.

In the long-run equilibrium, $\dot{k} = \dot{k}_c = 0$ holds. As $k \neq 0$ and $u \neq 0$, the long-run equilibrium values of k and k_c satisfy the following system of equations.

$$\dot{k} = 0 \Rightarrow \Bigg[(s_c - s_w)(1-\tau)\pi(k,k_c)k_c + s_w \Bigg] u(k,k_c) - g_a(k,k_c) - n = 0, \tag{41}$$

$$\dot{k}_c = 0 \Rightarrow \Bigg\{ (1-\tau)[s_c - (s_c - s_w)k_c]\pi(k,k_c) - s_w \Bigg\} k_c = 0. \tag{42}$$

The cases $k_c \neq 0$ and $k_c = 0$ correspond to the Pasinetti and Dual equilibria, respectively.

4.1 Pasinetti equilibrium

When $k_c \neq 0$, equations (41) and (42) lead to

$$\Bigg[(s_c - s_w)(1-\tau)\pi(k,k_c)k_c + s_w \Bigg] u(k,k_c) - g_a(k,k_c) - n = 0, \tag{43}$$

$$(1-\tau)\Bigg[s_c - (s_c - s_w)k_c \Bigg]\pi(k,k_c) - s_w = 0. \tag{44}$$

From these equations, k^{**} and k_c^{**} are obtained. These equations suggest that both k^{**} and k_c^{**} depend on tax rate τ.

By rearranging equation (44), we obtain

$$(1-\tau)\pi s_c - s_w = (1-\tau)(s_c - s_w)\pi k_c. \tag{45}$$

As the right side is positive, the left side must also be positive. From this, we obtain

$$(1-\tau)\pi s_c > s_w. \tag{46}$$

If we set $\tau = 0$, we obtain the hidden assumption of Pasinetti (1962) pointed out by Samuelson and Modigliani (1966). In our model, the profit share π is an endogenous variable, and π depends on s_c and s_w.

From equation (45), we obtain the profit share in the long-run Pasinetti equilibrium.

$$\pi_P^{**} = \frac{s_w}{(1-\tau)\underbrace{[s_c - (s_c - s_w)k_c]}_{(+)}}. \tag{47}$$

Therefore, we find that the equilibrium profit share depends on the workers' savings rate. Additionally, from the reserve army effect $\pi = \pi(e)$, the equilibrium employment rate depends on workers' saving rate. Moreover, from the reserve army creation effect, $g_a = g_a(e)$, the growth rate of labor productivity depends on the workers' savings rate.

The long-run equilibrium profit rate is given by

$$r_P^{**} = \frac{g_{a,P}^{**} + n}{(1-\tau)s_c}. \tag{48}$$

The Cambridge equation is as follows:. However, g_a is determined endogenously, $g_{a,P}^{**}$ depends on the workers' savings rate, as stated above, and hence, r_P^{**} depends on the workers' savings rate. Pasinetti (1962) revealed that even in an economy in which workers save, the long-run equilibrium profit rate is determined by the ratio of the natural growth rate to the capitalists' savings rate, that is, the Pasinetti theorem; the long-run profit rate is independent of the workers' savings rate. However, in our model, this depends on workers' savings rates. By rearranging equation (48), we obtain $(1-\tau)r_P^{**} = (g_a + n)/s_c$. The profit rate after taxation becomes the usual Cambridge equation. This property was also observed by Faria and Teixeira (1999) and Taylor (2014).

We further examine the local stability of the Pasinetti equilibrium. The Jacobian matrix **J** corresponding to the system of equation comprising of equations (39) and

(40), is given by

$$J_{11} = \frac{\partial \dot{k}}{\partial k} = k \left\{ (s_c - s_w)(1-\tau)uk_c \underset{(-)}{\frac{\partial \pi}{\partial k}} + [(s_c - s_w)(1-\tau)\pi k_c + s_w] \underset{(+/-)}{\frac{\partial u}{\partial k}} - \underset{(+)}{\frac{\partial g_a}{\partial k}} \right\}, \tag{49}$$

$$J_{12} = \frac{\partial \dot{k}}{\partial k_c} = k \left\{ (s_c - s_w)(1-\tau)\pi u + k \underbrace{\left[(s_c - s_w)(1-\tau)k_c u \pi'(e) - g_a'(e) \right]}_{(-)} \underset{(-)}{\frac{\partial u}{\partial k_c}} + s \underset{(-)}{\frac{\partial u}{\partial k_c}} \right\}, \tag{50}$$

$$J_{21} = \frac{\partial \dot{k}_c}{\partial k} = k_c u(1-\tau)[(1-k_c)s_c + k_c s_w] \frac{\partial \pi}{\partial k} < 0, \tag{51}$$

$$J_{22} = -k_c u(1-\tau) \left\{ (s_c - s_w)\pi - \underbrace{[s_c - (s_c - s_w)k_c]}_{(+)} k \underset{(-)}{\pi'(e)} \underset{(-)}{\frac{\partial u}{\partial k_c}} \right\}. \tag{52}$$

Each element is evaluated at the long-run equilibrium values.

J_{11} shows the self-feedback effect such that an increase in capital stock per effective labor supply affects its dynamics. When J_{11} is negative (i.e., when the self-feedback effect is negative), the long-run equilibrium is likely to be stable. This is achieved when the absolute value of the reserve army effect $\pi'(e) < 0$ and the reserve army creation effect $g_a'(e) > 0$ are relatively large.

J_{12} shows the effect of a change in the capitalists' wealth share on the dynamics of capital stock per effective labor supply. Similar to J_{11}, when the absolute value of the reserve army effect and the reserve army creation effect are large, J_{12} is likely to be negative.

J_{21} shows the effect of a change in capital stock per effective labor supply on the dynamics of the capitalists' wealth share. This sign is negative. If J_{12} is negative, an increase in k_c escalates k, and decreases k_c when $J_{12} < 0$. Hence, this exerts an indirect negative feedback effect on k_c, inducing stability in the long-run equilibrium.

J_{22} shows the self-feedback effect, such that an increase in the capitalists' wealth share affects its dynamics. When J_{22} is negative (i.e., when the self-feedback effect is negative), the long-run equilibirum is likely to be stable. This is attained when the absolute value of the reserve army effect is relatively large.

According to the Routh–Hurwitz stability criterion, the necessary and sufficient conditions for the steady state to be asymptotically and locally stable are that the determinant and trace of $\mathbf{J}$ must be positive and negative, respectively.

The determinant is given by:

$$\det \mathbf{J} = \underset{(+/-)}{J_{11}} \underset{(+/-)}{J_{22}} - \underset{(+/-)}{J_{12}} \underset{-}{J_{21}}. \tag{53}$$

The sign of $\det \mathbf{J}$ is ambiguous. Subsequently, the trace is given by

$$\operatorname{tr} \mathbf{J} = \underset{(+/-)}{J_{11}} + \underset{(+/-)}{J_{22}}. \tag{54}$$

The signs of $\operatorname{tr} \mathbf{J}$ are ambiguous. Nevertheless, we obtain the following proposition.

Proposition 2. *Consider that the reserve army and reserve army creation effects are relatively large. Therefore, the long-run Pasinetti equilibrium is asymptotically and locally stable.*

The above discussion on J_{ij} $(i, j = 1, 2)$ shows that if the prerequisites of Proposition 2 are satisfied, we obtain $J_{11} < 0$, $J_{12} > 0$, and $J_{22} < 0$. In this case, the Routh–Hurwitz conditions are satisfied; hence, the long-run Pasinetti equilibrium is stable.

In the subsequent numerical examples, we have $J_{11} < 0$, $J_{12} > 0$, and $J_{22} < 0$ in all cases.

4.2 Effects of capital taxation in the long run

We investigate the effect of an increase in the tax rate on the economy in the long-run Pasinetti equilibrium.

We determine $\dot{k} = 0$ and $\dot{k}_c = 0$ be $F_1 = 0$ and $F_2 = 0$, respectively. The long-run equilibrium conditions are then reformulated as

$$F_1(k, k_c; \tau) = 0, \tag{55}$$

$$F_2(k, k_c; \tau) = 0. \tag{56}$$

By totally differentiating these equations and expressing the resultant expressions in matrix form, we obtain

$$\begin{pmatrix} \frac{\partial F_1}{\partial k} & \frac{\partial F_1}{\partial k_c} \\ \frac{\partial F_2}{\partial k} & \frac{\partial F_2}{\partial k_c} \end{pmatrix} \begin{pmatrix} \frac{dk}{d\tau} \\ \frac{dk_c}{d\tau} \end{pmatrix} = - \begin{pmatrix} \frac{\partial F_1}{\partial \tau} \\ \frac{\partial F_2}{\partial \tau} \end{pmatrix}. \tag{57}$$

For example, because $\partial F_1/\partial k = \partial \dot{k}/\partial k = J_{11}$, the matrix on the left is a Jacobian matrix $\mathbf{J}$. Consider the long-run equilibrium is stable. Then, $\det \mathbf{J} > 0$. As $\det \mathbf{J} \neq 0$, we can use the implicit function theorem, and locally solve the above equations for the column vector $(dk/d\tau, dk_c/d/\tau)'$. The resulting expressions can be differentiated continuously with respect to τ.

Using Cramer's rule, the equations above can be solved as follows:

$$\frac{dk}{d\tau} = \frac{1}{\det \mathbf{J}} \begin{vmatrix} -\frac{\partial F_1}{\partial \tau} & J_{12} \\ -\frac{\partial F_2}{\partial \tau} & J_{22} \end{vmatrix} = \frac{1}{\det \mathbf{J}} \left(-\frac{\partial F_1}{\partial \tau} J_{22} + \frac{\partial F_2}{\partial \tau} J_{12} \right), \tag{58}$$

$$\frac{dk_c}{d\tau} = \frac{1}{\det \mathbf{J}} \begin{vmatrix} J_{11} & -\frac{\partial F_1}{\partial \tau} \\ J_{21} & -\frac{\partial F_2}{\partial \tau} \end{vmatrix} = \frac{1}{\det \mathbf{J}} \left(-\frac{\partial F_2}{\partial \tau} J_{11} + \frac{\partial F_1}{\partial \tau} J_{21} \right). \tag{59}$$

Here, we obtain

$$\frac{\partial F_1}{\partial \tau} = -(s_c - s_w)\pi u k_c k < 0, \tag{60}$$

$$\frac{\partial F_2}{\partial \tau} = -[s_c - (s_c - s_w)k_c]\pi u k_c < 0. \tag{61}$$

Accordingly, we obtain

$$\frac{dk^{**}}{d\tau} = \frac{\pi u k_c \left\{ k(s_c - s_w) \overset{(+/-)}{J_{22}} - \overbrace{[s_c - (s_c - s_w)k_c]}^{(+)} \overset{(+/-)}{J_{12}} \right\}}{\underset{(+)}{\det \mathbf{J}}}, \tag{62}$$

$$\frac{dk_c^{**}}{d\tau} = \frac{\pi u k_c \left\{ \overbrace{[s_c - (s_c - s_w)k_c]}^{(+)} \overset{(+/-)}{J_{11}} - k(s_c - s_w) \overset{(-)}{J_{21}} \right\}}{\underset{(+)}{\det \mathbf{J}}}. \tag{63}$$

When the long-run Pasinetti equilibrium is stable, we obtain $J_{11} < 0$, $J_{12} > 0$, $J_{21} < 0$, and $J_{22} < 0$. In this case, the sign of equation (62) is determined as $dk^{**}/d\tau < 0$. Hence, an increase in the tax rate decreases capital stock per effective labor supply. However, even in this case, the sign of equation (63) is ambiguous.

Regarding the capacity utilization rate, we obtain:

$$\frac{du^{**}}{d\tau} = \underset{(+/-)}{\frac{\partial u}{\partial \tau}} + \underset{(+/-)}{\frac{\partial u}{\partial k}} \cdot \underset{(-)}{\frac{dk^{**}}{d\tau}} + \underset{(-)}{\frac{\partial u}{\partial k_c}} \cdot \underset{(+/-)}{\frac{dk_c^{**}}{d\tau}}. \tag{64}$$

Hence, this sign is ambiguous.

Regarding the employment rate, we obtain

$$\frac{\partial e}{\partial \tau} = k \cdot \frac{\partial u}{\partial \tau} \gtrless 0, \tag{65}$$
$$\frac{\partial e}{\partial k} = k \cdot \frac{\partial u}{\partial k} + u \gtrless 0, \tag{66}$$
$$\frac{\partial e}{\partial k_c} = k \cdot \frac{\partial u}{\partial k_c} < 0. \tag{67}$$

Using these relationships, we obtain

$$\frac{de^{**}}{d\tau} = \underset{(+/-)}{\frac{\partial e}{\partial \tau}} + \underset{(+/-)}{\frac{\partial e}{\partial k}} \cdot \underset{(-)}{\frac{dk}{d\tau}} + \underset{(-)}{\frac{\partial e}{\partial k_c}} \cdot \underset{(+/-)}{\frac{dk_c}{d\tau}}. \tag{68}$$

Hence, this sign is ambiguous.

The profit share is a decreasing function of the employment rate; hence, the profit share changes in the direction opposite to the employment rate.

$$\text{sgn}\frac{d\pi^{**}}{d\tau} = -\text{sgn}\frac{de^{**}}{d\tau}. \tag{69}$$

As the sign of $de^{**}/d\tau$ is ambiguous, the sign of $d\pi^{**}/d\tau$ is also ambiguous.

The growth rate of labor productivity is an increasing function of the employment rate; hence, it changes in the same direction as the employment rate.

$$\text{sgn}\frac{dg_a^{**}}{d\tau} = \text{sgn}\frac{de^{**}}{d\tau}. \tag{70}$$

Finally, regarding the economic growth rate, we obtain:

$$\frac{dg^{**}}{d\tau} = -\beta\pi^{**} + \underset{(+/-)}{\frac{du^{**}}{d\tau}} \left[\alpha - \pi_1 k^{**} \frac{\overset{(+/-)}{A(\pi^{**})}}{(s-\alpha)^2} \right]. \tag{71}$$

This sign is ambiguous.

In summary, we cannot analytically obtain definite answers regarding the effect of the income redistribution policy. Accordingly, we conduct the numerical simulation in the next section.

5 Numerical simulations

In this section, we conduct numerical simulations to examine the stability of the dynamics and the effects of the income redistribution policy. This exercise is aimed at(1) reproducing every case, (2) showing that the long-run equilibrium in every case is stable, and (3) examining the effect of the income redistribution policy, which is ambiguous in the analytical method. Therefore, our exercises might sacrifice the reproducibility of reality because the obtained long-run equilibrium values deviate from the corresponding actual values. This is a limitation of numerical simulations.

5.1 Pasinetti equilibrium

Consider that at the initial time, the economy is in Pasinetti equilibrium. Then, according to the integration of the parameters, we obtain the following three cases:

Case 1 PLD and PLG regimes

Case 2 WLD and PLG regimes

Case 3 WLD and WLG regimes

Table 3 presents the parameter sets that reproduce these three cases. As shown in equation (25), when the capitalists' savings rate is relatively large, equation (25) is likely to be negative, which makes equation (24) negative and induces a WLD regime. In Cases 2 and 3, where the WLD regime was obtained, we used a larger s_c than in Case 1, where the PLD regime was obtained. Moreover, as equation (28) shows, when the coefficient of the investment function β is relatively large, we are likely to obtain the PLG regime. In Cases 1 and 2, where the PLG regime is obtained, we use a larger β than in Case 3, where the WLG regime is obtained.

Table 3: Parameter settings

	Case 1 (PLD, PLG)	Case 2 (WLD, PLG)	Case 3 (WLD, WLG)
s_w	0.15	0.12	0.12
s_c	0.4	0.5	0.8
γ	0.1	0.13	0.22
α	0.02	0.02	0.06
β	0.15	0.17	0.05
π_0	0.6	0.6	0.6
π_1	0.2	0.2	0.2
η	0.4	0.4	0.4
λ	1	1	1
n	0.01	0.01	0.01
τ	0.02	0.02	0.02

A stable long-run equilibrium is required to conduct a comparative static analysis. Therefore, $\operatorname{tr}\mathbf{J} = J_{11} + J_{22} < 0$ and $\det\mathbf{J} = J_{11}J_{22} - J_{12}J_{21} > 0$ are necessary.

The signs of each element, J_{ij}, $\operatorname{tr}\mathbf{J}$, and $\det\mathbf{J}$ are presented in Table 4.

Table 4: Elements of Jacobian matrix

$\tau = 0.02$	Case 1 (PLD, PLG)	Case 2 (WLD, PLG)	Case 3 (WLD, WLG)
J_{11}	-0.192885	-0.24742	-0.353918
J_{12}	0.0568736	0.10855	0.270927
J_{21}	-0.0215662	-0.0309542	-0.0341099
J_{22}	-0.0408117	-0.100631	-0.1638
$\operatorname{tr}\mathbf{J}$	-0.233697	-0.348051	-0.517719
$\det\mathbf{J}$	0.00909852	0.0282583	0.0672132
Γ	-0.179103	-0.219902	-0.300257
Θ	-0.00497812	0.00791998	0.0649321
$\Omega = \Gamma + \Theta$	-0.184081	-0.211982	-0.235325

As Table 4 shows, in every case, we have $J_{11} < 0$, $J_{12} > 0$, and $J_{22} < 0$, and hence, we obtain both $\operatorname{tr}\mathbf{J} = J_{11} + J_{22} < 0$ and $\det\mathbf{J} = J_{11}J_{22} - J_{12}J_{21} > 0$. In Cases 2 and 3, $\Theta > 0$, but in this case, the long-run equilibrium is stable. Therefore, we obtain $\Omega < 0$ for all the cases.

For confirmation, we calculate the values after the income redistribution policy, as shown in Table 5. The signs do not change after a change in the tax rate. Furthermore, the signs of the regimes do not change after tax rate changes, although we do not offer them here.

Table 5: Elements of Jacobian matrix after tax rate change

$\tau = 0.025$	Case 1 (PLD, PLG)	Case 2 (WLD, PLG)	Case 3 (WLD, WLG)
J_{11}	-0.192438	-0.246874	-0.353978
J_{12}	0.0564586	0.107783	0.269589
J_{21}	-0.0213731	-0.0309931	-0.0344234
J_{22}	-0.0401264	-0.0999629	-0.163247
$\mathrm{tr}\,\mathbf{J}$	-0.232564	-0.346837	-0.517225
$\det\mathbf{J}$	0.00892854	0.0280188	0.0670659
Γ	-0.178145	-0.218757	-0.298354
Θ	-0.00502995	0.00780315	0.0644947
$\Omega = \Gamma + \Theta$	-0.183175	-0.210954	-0.233859

After confirming that the long-run equilibrium is stable, we proceed with a comparative static analysis. Consider that, initially, the economy is in a long-run Pasinetti equilibrium under the benchmark parameter set. Additionally, consider that the government raises the tax rate for capitalists. Subsequently, a new long-run Pasinetti equilibrium emerges. We increase the tax rate from $\tau = 0.02$ to $\tau = 0.025$. Accordingly, we compare the steady-state values of the benchmark case with those obtained after the shock. We consider eight endogenous variables: capitalists' wealth share, capacity utilization rate, employment rate, profit share, economic growth rate, labor productivity growth rate, capitalists' income share, and workers' income share. Here, the capitalists' income share σ_c and workers' income share σ_w are defined as follows:

$$\sigma_c = \frac{(1-\tau)rK_c}{Y} = (1-\tau)\pi(k,k_c)k_c, \tag{72}$$

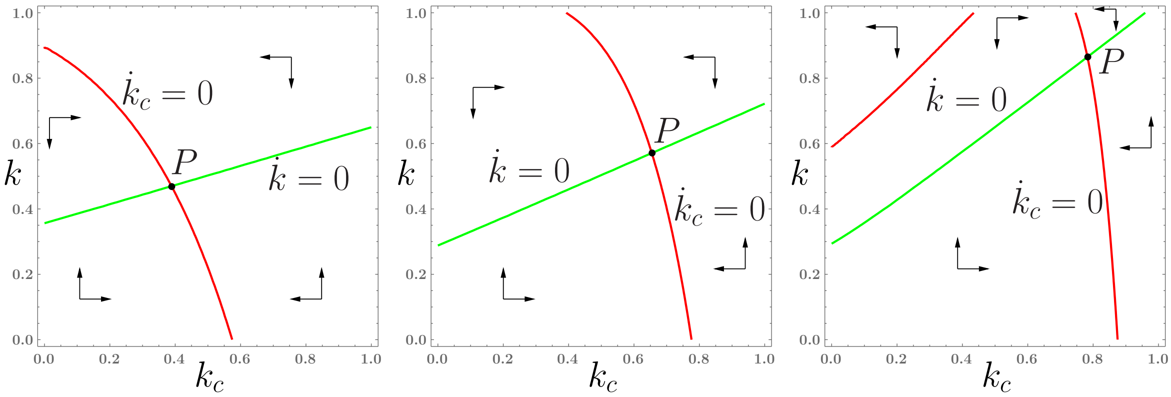
$$\sigma_w = \frac{wE + r(K_w + \tau K_c)}{Y} = 1 - (1-\tau)\pi(k,k_c)k_c. \tag{73}$$

Therefore, each income share depends on k and k_c. The effect of an increase in the tax rate on the capitalists' income share is given by

$$\frac{d\sigma_c}{d\tau} = \underbrace{-\pi(k,k_c)k_c}_{(-)} + (1-\tau)\underset{(-)}{\frac{\partial \pi}{\partial k}} \cdot \underset{(-)}{\frac{dk}{d\tau}} \cdot k + (1-\tau)\left[\underset{(+)}{\frac{\partial \pi}{\partial k_c}} k_c + \pi(k,k_c)\right]\underset{(+)/(-)}{\frac{dk_c}{d\tau}}. \tag{74}$$

When the long-run Pasinetti equilibrium is stable, the last term $dk_c/d\tau$ is negative. In this case, the right-hand side of (74) is likely to be negative. Hence, income redistribution from capitalists to workers is likely to reduce income inequality between the two classes.

Figure 2 shows the phase diagrams for Cases 1–3. Point P denotes the long-term Pasinetti equilibrium. When the economy starts near the equilibrium, it converges into the long-run equilibirum.



(a) Case 1 (PLD, PLG)

(b) Case 2 (WLD, PLG)

(c) Case 3 (WLD, WLG)

Figure 2: Long-run phase diagram in Cases 1–3

The results of the comparative static analysis of Cases 1–3 are shown in Table 6. In each case, capitalists' assets and income shares decrease, whereas workers' assets and income shares increase. Therefore, this income redistribution policy improves the inequality between the two classes. However, its effects on growth and employment differ among cases. In Cases 1 and 2, in which the short-run equilibrium exhibits a profit-led growth regime, an increase in the tax rate for capitalists decreases both the economic growth rate and employment. In contrast, in Case 3, where the short-run equilibrium exhibits a wage-led growth regime, it increases both the economic growth rate and employment. Therefore, income redistribution policy improves inequality between the two classes but negatively affects growth and employment in a profit-led growth economy, whereas it positively affects growth and employment in a wage-led growth economy.

In Cases 1-3, the income redistribution policy decreases the capitalists' income share σ_c and increases the workers' income share σ_w, which consequently decreases the capitalists' asset share k_c.

In Case 1, where the short-run equilibrium exhibits a profit-led demand regime and a growth regime, an increase in the tax rate for capitalists decreases both the capital

Table 6: Results of comparative static analysis in Cases 1–3

(a) Case 1 (PLD, PLG)				(b) Case 2 (WLD, PLG)				(c) Case 3 (WLD, WLG)			
	$\tau = 0.02$	$\tau = 0.025$			$\tau = 0.02$	$\tau = 0.025$			$\tau = 0.02$	$\tau = 0.025$	
k	0.472122	0.469811	$\downarrow$	k	0.57377	0.570995	$\downarrow$	k	0.86969	0.865113	$\downarrow$
k_c	0.394594	0.388764	$\downarrow$	k_c	0.657626	0.654469	$\downarrow$	k_c	0.784593	0.782512	$\downarrow$
u	0.975213	0.978442	$\uparrow$	u	0.962096	0.965352	$\uparrow$	u	0.807696	0.812448	$\uparrow$
π	0.507916	0.508063	$\uparrow$	π	0.489596	0.489758	$\uparrow$	π	0.459511	0.459428	$\downarrow$
e	0.46042	0.459683	$\downarrow$	e	0.552021	0.551211	$\downarrow$	e	0.702444	0.70286	$\uparrow$
g	0.194168	0.193873	$\downarrow$	g	0.230809	0.230484	$\downarrow$	g	0.290978	0.291144	$\uparrow$
σ_c	0.196412	0.192579	$\downarrow$	σ_c	0.315531	0.312518	$\downarrow$	σ_c	0.353319	0.35052	$\downarrow$
σ_w	0.803588	0.807421	$\uparrow$	σ_w	0.684469	0.687482	$\uparrow$	σ_w	0.646681	0.64948	$\uparrow$
g_a	0.184168	0.183873	$\downarrow$	g_a	0.220809	0.220484	$\downarrow$	g_a	0.280978	0.281144	$\uparrow$

accumulation rate g and capital stock per effective labor supply k. A slowdown in capital accumulation decreases the employment rate e. A decline in the employment rate increases profit share π through the reserve army effect and decreases the growth rate of labor productivity g_a. As the short-run equilibrium exhibits a profit-led demand regime, an increase in profit share increases the capacity utilization rate u.

In Case 2, where the short-run equilibrium exhibits a wage-led demand regime and a profit-led growth regime, an increase in the tax rate for capitalists decreases both the capital accumulation rate g and capital stock per effective labor supply k. A slowdown in capital accumulation decreases the employment rate e. A decline in the employment rate increases profit share π through the reserve army effect and decreases the growth rate of labor productivity g_a. As the short-run equilibrium exhibits a wage-led demand regime, an increase in the profit share negatively affects the capacity utilization rate u. However, an increase in workers' profit income through an increase in workers' asset share increases consumption demand and positively affects the capacity utilization. As the latter positive effect is larger than the former negative effect, the capacity utilization rate increases in the long-run equilibrium.

In other words, Cases 1 and 2 imply scenarios in which the income redistribution policy causes a decrease in economic growth and employment rates when the short-run equilibrium exhibits a profit-led growth regime, resulting in a trade-off between inequality and growth.

In Case 3, in which the short-run equilibrium exhibits a wage-led demand regime and a wage-led growth regime, as in Cases 1 and 2, an increase in the tax rate on capitalists negatively affects the capital accumulation rate g, the capital stock per effective labor supply k, and the employment rate e. However, because the short-run equilibrium exhibits both a wage-led demand regime and a wage-led growth regime, an increase in workers' profit income through an increase in workers' asset share strongly positively affects demand and capital accumulation, resulting in an increase in the capital accumulation rate g and employment rate e in the long-run equilibrium. An increase in the employment rate decreases profit share π through the reserve army effect, and increases the growth rate of labor productivity g_a. As the short-run equilibrium exhibits a wage-led demand regime, a decrease in profit share increases the capacity utilization rate u.

In other words, Case 3 implies a scenario in which the income redistribution policy causes an increase in economic growth and employment rates when the short-run equilibrium exhibits a wage-led growth regime, resulting in overcoming the trade-off between inequality and growth.

We compared our results with those of related studies.

Zamparelli (2017) constructed a Solow growth model with a Pasinetti type savings function, and investigated the effect of capital taxation on the economy. His main contribution was the introduction of a constant elasticity of substitution (CES) production function. According to the elasticity of substitution between capital and labor, he shows two cases. In one case, elasticity is relatively small, and the economy converges to a steady state. In the other case, elasticity is relatively large, and endogenous growth occurs. In the first case, an increase in the tax rate decreases the capitalists' wealth share in the steady state and increases the workers' wealth share. Therefore, the income redistribution policy reduces the inequality between workers and capitalists. In this case, the economic growth rate is equal to the natural growth rate, which is given exogenously; hence, a tradeoff between growth and inequality never occurs. However, steady-state per capita output declines; hence, a tradeoff between output and inequality occurs. In the second case, the income redistribution policy increases workers' wealth share and the endogenously determined economic growth rate declines. Therefore, a tradeoff between growth and inequality occurs.

Petach and Tavani (2020)conduct a similar analysis. They constructed a classical growth model incorporating a Pasinetti type saving function, and investigated the effect of capital taxation on the economy. They endogenize the growth rates of labor and capital productivity using the theory of induced innovation, and wage share dynamics based on Goodwin (1967). An increase in the tax rate in the steady state increases the workers' wealth share but decreases the economic growth rate; a trade-off between growth and inequality occurs.

In our model, there is a tradeoff between growth and inequality in the profit-led growth case, similar to Zamparelli (2017) and Petach and Tavani (2020). However, in the case of wage-led growth, there is no trade-off between growth and inequality. Hence, in a wage-led growth economy, an income redistribution policy is favorable.

According to the numerical simulation, in the PLG regime (Cases 1 and 2), the profit share and economic growth rate move in opposite directions: an increase in the tax rate increases the profit share and decreases the economic growth rate. This implies an apparent WLG regime. What should we think about this?

We determine the growth regime by substituting the long-run equilibrium values (k^{**}, k_c^{**}) under the benchmark parameter setting into equation (27).

In our model, the profit share is an endogenous variable in both the short and long run; therefore, it is natural that the result obtained from the discriminant given by equation (27) treats the profit share as an exogenous variable, and that obtained from the analysis of the simultaneous changes in the profit share and economic growth that arise in the tax rate change differ.

As explained in the long-run comparative static analyses of Cases 1 and 2, an increase in the tax rate decreases the employment rate, which increases the profit share through the reserve army effect. However, the economic growth rate declines, because the direct negative profit squeeze effect of taxation is large. Thus, when considering the effect of a change in the tax rate on economic growth, we must consider the direct and indirect effects of a change in the profit share. The discriminant of the growth regime given by equation (27) captures only the former direct effect. Hence, the final results deviate from the sign of the growth regime discriminant.

6 Conclusions

This study presents an extended Kaleckian model and investigates how an income redistribution policy based on taxation of capitalists affects capacity utilization, economic growth rate, employment rate, wealth share, and income share. Similar to many related studies that consider workers' saving rates, according to the relative size of workers' and capitalists' saving rates, we obtain the Pasinetti equilibirum and dual equilibria.

Focusing on the Pasinetti equilibrium that is realistic, we obtain the following three possible integrations of demand and growth regimes: Case 1: PLD and PLG; Case 2: WLD and PLG; and Case 3: WLD and WLG. According to the numerical simulation, the long-run equilibrium is stable in every case.

Therefore, to resolve the inequality between workers and capitalists, we consider an income redistribution policy in which the government imposes taxation on capitalists' profits and redistributes it to workers. If the government increases the tax rate and hence, reinforces the policy, then in every case, workers' wealth and income shares increase, whereas capitalists' wealth and income shares decrease. Moreover, this policy decreases the economic growth rate if the economy exhibits a profit-led growth regime, and increases it if the economy exhibits a wage-led growth regime. Hence, the effectiveness of this income redistribution policy differs depending on the type of regime.

Finally, our numerical simulation was based on parameters that were set ad hoc to reproduce each case. To address how the actual economic policy should be, there is need to estimate the parameters based on actual data. This issue should be addressed in future studies.

Appendix: Analysis of dual equilibrium

In this appendix, we investigate the dual equilibrium where $k_c = 0$.

A–1: Short-run analysis

When the short-run equilibrium is a dual equilibrium, we substitute $k_c = 0$ into $s \equiv (s_c - s_a)(1-\tau)\pi k_c s_w$, and obtain $s = s_w$.

First, to determine demand and growth regimes, we consider a situation in which the profit share is fixed. In this case, the short-run equilibrium values of the capacity utilization and capital accumulation rates are given by

$$u_D^* = \frac{\gamma + \beta(1-\tau)\pi}{s_w - \alpha}, \tag{75}$$

$$g_D^* = \frac{s_w[\gamma + \beta(1-\tau)\pi]}{s_w - \alpha}. \tag{76}$$

For these values to be positive, we require $s_w > \alpha$. In this case, an increase in π increases both u^* and g^*. In the following, we assume $s_w > \alpha$. Therefore, the short-run equilibrium exhibits the PLD and PLG regimes.

Further, we consider a case in which profit share is endogenized. From $\dot{u} = 0$, we obtain

$$s_w u = \gamma + \alpha u + \beta(1-\tau)(\pi_0 - \pi_1 uk). \tag{77}$$

From equation (77), we obtain the short-run equilibrium capacity utilization rate and the capital accumulation rate.

$$u_D^* = \frac{\gamma + \beta(1-\tau)\pi_0}{(s_w - \alpha) + \beta(1-\tau)\pi_1 k}, \tag{78}$$

$$g_D^* = \frac{s_w[\gamma + \beta(1-\tau)\pi_0]}{(s_w - \alpha) + \beta(1-\tau)\pi_1 k}. \tag{79}$$

From equation (11), the short-run stability condition is given by

$$\frac{\partial \dot{u}}{\partial u} = -\phi[\underbrace{s_w - \alpha}_{(+)} + \beta(1-\tau)\pi_1 k] < 0. \tag{80}$$

Assuming $s_w > \alpha$, this stability condition is satisfied. Therefore, the short-run dual equilibrium is stable.

The effects of an increase in k on the shot-run equilibrium values are as follows.

$$\frac{\partial u}{\partial k} = -\frac{\beta\pi_1(1-\tau)[\gamma + \beta(1-\tau)\pi_0]}{[(s_w - \alpha) + \beta(1-\tau)\pi_1 k]^2} < 0, \tag{81}$$

$$\frac{\partial e}{\partial k} = \frac{(s_w - \alpha)[\gamma + \beta(1-\tau)\pi_0]}{[(s_w - \alpha) + \beta(1-\tau)\pi_1 k]^2} > 0, \tag{82}$$

$$\frac{\partial \pi}{\partial k} = -\frac{\pi_1(s_w - \alpha)[\gamma + \beta(1-\tau)\pi_0]}{[(s_w - \alpha) + \beta(1-\tau)\pi_1 k]^2} < 0, \tag{83}$$

$$\frac{\partial g_a}{\partial k} = \frac{g_a'(e)(s_w - \alpha)[\gamma + \beta(1-\tau)\pi_0]}{[(s_w - \alpha) + \beta(1-\tau)\pi_1 k]^2} > 0. \tag{84}$$

Hence, all signs are determined.

The effects of an increase in τ on short-run equilibrium values are as follows:

$$\frac{du}{d\tau} = -\frac{\beta[\pi_0(s_w - \alpha) - \pi_1\gamma k]}{[(s_w - \alpha) + \beta(1-\tau)\pi_1 k]^2}, \tag{85}$$

$$\frac{de}{d\tau} = k\,\frac{du}{d\tau}, \tag{86}$$

$$\frac{d\pi}{d\tau} = -\pi_1 k\,\frac{du}{d\tau}, \tag{87}$$

$$\frac{dg}{d\tau} = s_w\,\frac{du}{d\tau}, \tag{88}$$

$$\frac{dg_a}{d\tau} = g_a'(e)k\,\frac{du}{d\tau}. \tag{89}$$

If the sign of $du/d\tau$ is determined, then all the other signs are determined. The sign of $du/d\tau$ depends on that of $\pi_0(s_w - \alpha) - \pi_1\gamma k$. As we assume $s_w > \alpha$, this sign is ambiguous.

A–2: Long-run analysis

We consider the case in which $(1-\tau)\pi s_c < s_w$. This inequality is consistent for $s_w < s_c$. Only when $k_c = 0$, we obtain $\dot{k}_c = 0$.

From equation (41), we obtain

$$\underbrace{s_w u(k;\tau)}_{g} - g_a(k;\tau) - (n+\delta) = 0. \tag{90}$$

By solving equation (90), we obtain the long-run dual equilibrium value of k, which indicates that $k = k_w$ depends on τ.

The long-run Dual equilibirum capacity utilization rate is given by:

$$u_D^{**} = \frac{g_{a,D}^{**} + n + \delta}{s_w}. \tag{91}$$

This finding is consistent with that of Samuelson and Modigliani (1996).

We examine the local stability of the long-run dual equilibrium. Each element of the Jacobian matrix $\mathbf{J}$ is given by:

$$J_{11} = k \left(s_w \underset{(-)}{\frac{\partial u}{\partial k}} - \underset{(+)}{\frac{\partial g_a}{\partial k}} \right) < 0, \tag{92}$$

$$J_{12} = k \left[(s_c - s_w)(1-\tau)\pi u + [s_w - k \underset{(+)}{g_a'(e)}] \underset{(-)}{\frac{\partial u}{\partial k}} \right], \tag{93}$$

$$J_{21} = 0, \tag{94}$$

$$J_{22} = [(1-\tau)\pi s_c - s_w]u < 0. \tag{95}$$

From the dual equilibrium condition $(1-\tau)\pi s_c < s_w$, we obtain $J_{22} < 0$. Then, we obtain both $\operatorname{tr}\mathbf{J} < 0$ and $\det \mathbf{J} > 0$. Therefore, from the Routh–Hurwitz stability criterion, we find that the long-run dual equilibrium is asymptotically and locally stable.

As k depends on τ, u also depends on τ. When $e = uk$, e depends on τ, From $\pi = \pi(e)$, π depends on τ. Moreover, as $g_a = g_a(e)$, g_a depends on τ. Accordingly, considering that u and g_a are functions of τ, we rewrite the long-run equilibrium condition given by equation (90) as

$$\underbrace{s_w u(k,\tau)}_{g} - g_a(k,\tau) - (n+\delta) = 0. \tag{96}$$

By totally differentiating this equation, we obtain

$$\frac{dk}{d\tau} = -k\frac{s_w \frac{\partial u}{\partial \tau} - \frac{\partial g_a}{\partial \tau}}{J_{11}}. \tag{97}$$

The denominator on the right side of Equation (97) is necessarily negative. However, the sign of the numerator was ambiguous. The numerator can be further rewritten as

$$s_w \frac{\partial u}{\partial \tau} - \frac{\partial g_a}{\partial \tau} = [s_w - kg_a'(e)] \frac{\partial u}{\partial \tau}. \tag{98}$$

Accordingly, if the signs of $s_w - kg_a'(e)$ and $\partial u/\partial \tau$ are determined, the sign of Equation (97) is also determined.

The other derivatives are given by:

$$\frac{du}{d\tau} = \frac{\partial u}{\partial \tau} + \frac{\partial u}{\partial k} \cdot \frac{dk}{d\tau}, \tag{99}$$

$$\frac{de}{d\tau} = \frac{\partial e}{\partial \tau} + \frac{\partial e}{\partial k} \cdot \frac{dk}{d\tau}, \tag{100}$$

$$\frac{d\pi}{d\tau} = \pi'(e) \frac{de}{d\tau}, \tag{101}$$

$$\frac{dg}{d\tau} = s_w \frac{du}{d\tau}, \tag{102}$$

$$\frac{dg_a}{d\tau} = g_a'(e) \frac{de}{d\tau}. \tag{103}$$

The results of the comparative static analysis of the long-run dual equilibirum are as follows. An increase in the tax rate decreases the capacity utilization rate, profit share, and economic growth rate but increases the employment rate and growth rate of labor productivity. As the short-run analysis shows, the dual equilibrium exhibits PLD and PLG regimes. Therefore, profit share, capacity utilization rate, and economic growth rate move in the same direction. Moreover, labor productivity growth is an increasing function of the employment rate; hence, these two variables move in the same direction.

At the Dual equilibrium, capitalists are in euthanasia and workers also serve as capitalists; hence, the economy becomes a one-class economy. In this case, the workers' income share is unity, whereas the capitalists' income share is zero. Therefore, we could not investigate the effect of income redistribution policies on inequality.

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