Automation, Human Task Innovation, and Labor Share: Unveiling the Role of Elasticity of Substitution

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Automation, Human Task Innovation, and Labor Share*

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Abstract

This study examines the declining trend in global labor share across countries and sectors, focusing on the roles of robotic innovation (RI) and human innovation (HI). To address potential endogeneity, we construct instrumental variables using US patent data and large language models, calculating similarity scores between patent descriptions and robot descriptions for RI, and between patent descriptions and O*NET occupation descriptions for HI. Employing a general equilibrium model to derive our regression formula, our empirical findings reveal that RI negatively affects labor share, while HI has a positive impact. We estimate the elasticity of substitution between non-robot capital and labor to be less than one, aligning with most literature but differing from some previous studies.
JEL Codes: D24, E24, E25, J23, O33, O57

Keywords: Human task innovation, Robotic innovation, Automation, Labor share, Elasticity of substitution

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*Replication data and code and the most recent version of paper:
https://github.com/jayjeo/public/blob/main/Laborshare/readme.md

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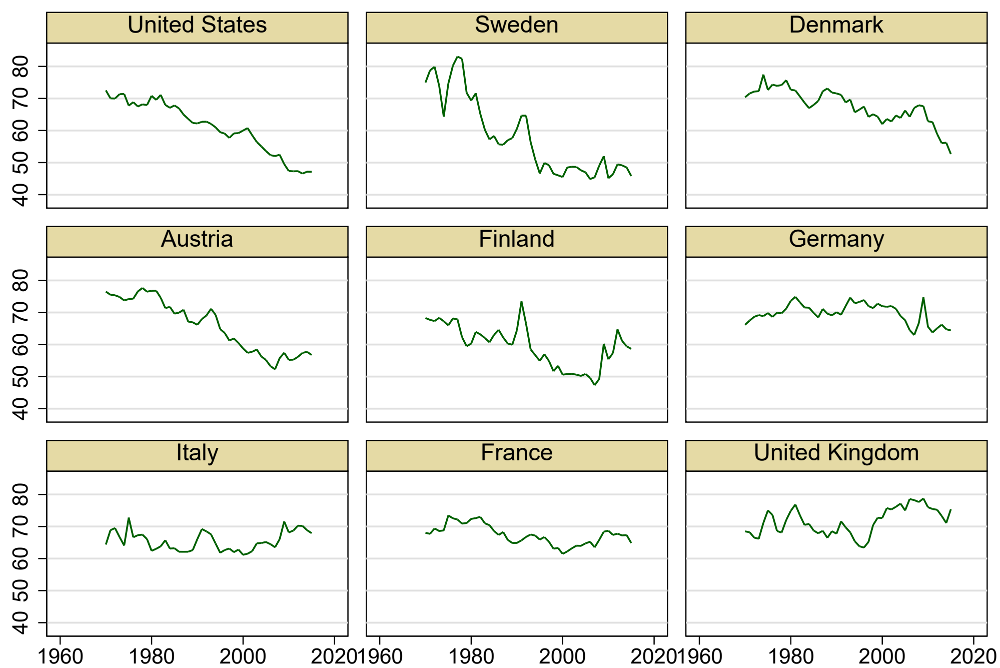
1 Introduction

The global labor share has exhibited a declining trend since the early 1980s, with an average decrease of approximately five percentage points, as observed by Karabarbounis and Neiman (2014) and Autor et al. (2020). Figure 1, based on data compiled by Gutiérrez and Piton (2020), illustrates a comparison of labor shares in the manufacturing sector between the United States and eight European Union countries analyzed in our study. While countries such as the USA, Sweden, Denmark, and Austria have experienced substantial declines, others report comparatively modest decreases. This discrepancy highlights the considerable heterogeneity in global labor share trends, further emphasizing the importance of our investigation into variations across countries and sectors to elucidate this decline.[1]

Figure 1: Labor shares



Although the precise cause of this decline remains a subject of debate, advancements in automation have emerged as a potential key driver. The urgency of addressing the diminishing labor share is intensified by the accelerated growth in automation and artificial intelligence technologies. For example, Tesla aims to deploy "genuinely useful humanoid robots," known as Optimus, in their factories by 2025. Additionally,

[1] In this context, our study aligns with Graetz and Michaels (2018), which assesses seventeen EU countries, although their focus is predominantly on productivity growth rather than the decrease in labor share.

the recent debut of Claude 3.5 Sonnet in June 2024, which builds upon the impressive performance of its predecessors in the Claude 3 family on standardized tests like the LSAT and GRE, further underscores the rapid evolution of AI systems.

The influence of automation on labor share continues to be a prominent topic in active research. Several studies, including those by Acemoglu and Restrepo (2020), Acemoglu et al. (2020), Dauth et al. (2021), and Martinez (2018), suggest that automation reduces labor share. Conversely, findings from research conducted by De Vries et al. (2020) and Gregory et al. (2016) propose that automation amplifies labor share. Moreover, studies by Humlum (2019) and Hubmer and Restrepo (2021) explore the diverse impacts of automation on various population groups and industry sectors.

Another factor potentially promoting labor share is 'human innovation' —innovative tasks beyond the capabilities of robots. Autor (2015) contends that the sustained relevance of human labor in the future will largely depend on the pace at which 'human innovation' outstrips the advancement of automation. To the best of our knowledge, Autor et al. (2024) represents the only study that empirically measures human innovations. They utilize *Census Alphabetical Index of Occupations and Industries* and patent information to produce a proxy for 'human innovation.'

However, few studies attempt to measure multiple factors within a unified framework (Bergholt et al., 2022). Bergholt points out that "while a large literature has discussed each of these four explanations in isolation, an empirical analysis including all of them in the context of the same model is lacking. Our aim is to fill this gap." Similarly, Grossman and Oberfield (2022) highlighted the importance of utilizing general equilibrium analysis, stating: "Many authors present different sides of the same coin ... Even if the various mechanisms are all active, it becomes difficult to gauge what part of the effect estimated in one study has already been accounted for elsewhere." To address this challenge, we adopt a general equilibrium model, an approach that represents a contribution to the existing literature.

To the best of our knowledge, no existing study within a general equilibrium framework has incorporated both robotic innovation (RI) and human innovation (HI).[2] Our study addresses this gap by integrating these concepts through the application of appropriate instruments. These instruments are derived from the semantic understanding feature of large language models, which we use to compare patent descriptions with robotic and human task descriptions. Through this approach, we meticulously examine how RI and HI influence labor share across countries and sectors. This comprehensive analysis constitutes our primary contribution to the literature.

[2]The study most akin to ours is that of Acemoglu and Restrepo (2022). They too utilize a general equilibrium model, though their main focus is on wage inequality rather than the decline in labor share. Our model is built on Acemoglu and Restrepo (2022) but is distinct in that it separately introduces both robot and non-robot capital as inputs for production. This model setup is important because it enables us to analyze how robot and non-robot capital differently affect the labor share in conjunction with four types of technological innovation.

Based on our theoretical framework, we derive a reduced-form regression equation. Our empirical estimation reveals that RI negatively affects labor share, while HI positively affects it. Other price factors —wage, robot price, and non-robot capital price— serve as control variables. Using the estimated coefficients of these price factors, we additionally calculate the elasticity of substitution between non-robot capital and labor as less than one. These results provide empirical evidence supporting the notion that the elasticity of substitution *between labor and capital* is less than one, a finding consistent with the majority of the literature, as noted by Chirinko (2008), Grossman and Oberfield (2022), and Glover and Short (2020). Our results differ from Karabarbounis and Neiman (2014) on this elasticity.

While our study is innovative, it is not without limitations. The primary concern is the endogeneity of price factors. Although RI and HI are instrumented by exogenous variables, other price factors inherently include endogeneity problems. This limitation underscores the need for further research and refinement of methodologies in this area.

In the following section, we provide key definitions used in this study. Section 3 presents our general equilibrium model, which forms the theoretical foundation of our analysis. Section 4 details the datasets employed in our research. Section 5 illustrates how we constructed instrumental variables (IVs) for RI and HI. Section 6 conducts the regression analysis, utilizing our model and data to examine the relationships between various factors and labor share. Section 7 performs various accounting exercises to ascertain which mechanisms predominantly explain labor share decline across different countries and industries. Section 8 offers various robustness checks to demonstrate that all our results remain stable across different specifications. Finally, Section 9 provides our concluding remarks.

2 Definitions

This section provides definitions for 'robot', 'robotic innovation (automation)', and 'human innovations' that will be used throughout this paper. We adhere to the definition of a robot as specified by ISO standard 8373:2012, which describes it as an "automatically controlled, reprogrammable, multipurpose manipulator programmable in three or more axes."[3] The International Federation of Robotics (IFR) also strictly adheres to this definition (Müller, 2022). We source our robot data from the IFR.

In Figure 2, Panel (a) depicts a robot. However, Panel (b) is not robot because this milling machine does not come with any type of hook-up to have it run automatically.

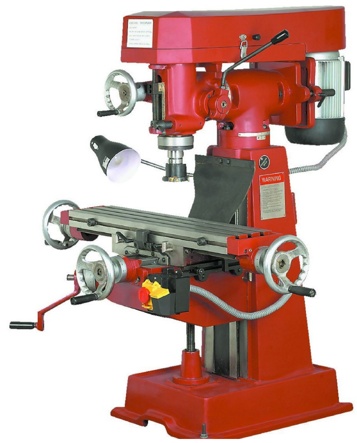
[3]Acemoglu and Restrepo (2020) also defines robots in a manner consistent with this description: "fully autonomous machines that do not need a human operator and can be programmed to perform several manual tasks ... This definition excludes other types of equipment."

[4]Vertical milling machine by harborfreight

Figure 2: Examples of Robot

(a) Robot

(b) Not robot[4]

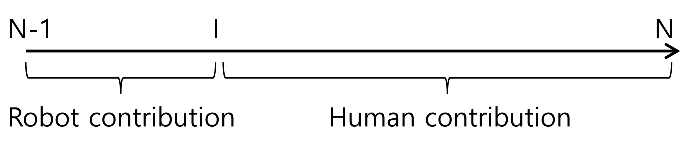


Therefore, it is neither reprogrammable nor automatically controlled. Additionally, it cannot be considered multipurpose, as it is designed solely for milling. Also, it does not operate on three or more axes. This example underscores the narrow definition of a robot.

We define 'automation' (or 'robotic innovation (RI)' in alternative terminology) as the enhancement of robots' capabilities, enabling them to perform tasks previously beyond their scope. This definition aligns with those proposed by Acemoglu and Restrepo (2018) and Acemoglu and Restrepo (2019).

We propose a novel definition of 'human innovations' (HI) as the expansion of tasks that human workers are expected to perform, specifically those beyond the current capabilities of robots. This concept is framed within a model where 'I' represents robot innovation in production, while 'N' denotes human innovation.

Figure 3: Conceptual Diagram



Unlike existing studies that define human innovation in terms of newly created occupations, tasks, or cognitive enhancements, we view it as the counterpart to automation (I) in our conceptual model. The development of new technologies that are not robots reflects human efforts to increase productivity and contribute to production processes performed by human beings.

The growing number of human-task-related patents serves as an exogenous shock in this framework, indicating a surge in human innovation. These patents demonstrate conscious efforts by workers and inventors to address complex problems and create novel solutions in areas where robots lack sufficient autonomy, creativity, or problem-solving skills. This trend illustrates that humans are not passive in the face of technological change. Rather, they are actively adapting and developing new methodologies to maintain their relevance in an increasingly automated world. Such proactive approaches challenge the notion of human obsolescence amid technological advancements.

It is important to note that our definition and empirical construction of HI does not inherently guarantee increases in wages, employment, or labor share; in fact, it may lead to decreases. This is consistent with other studies (Acemoglu and Restrepo, 2018; Autor et al., 2024). Specifically, Acemoglu and Restrepo (2018) demonstrates, through a proposition, that under certain parameters, HI indeed leads to increases in wages and employment. Similarly, Autor et al. (2024) finds empirical evidence that "employment and wage bills expand in occupations exposed to 'augmentation innovation' " (referred to as HI in our terminology).[5]

3 Model

Acemoglu and Restrepo (2018) propose a formal model that illustrates how RI and HI influence labor share. We have refined our model based on their static version, with our key contribution being the distinction between robots and other capital equipment —a delineation absent in their model. Subsequent research by Acemoglu and Restrepo (2020) found that advancements in robotics negatively impact wages and employment, while other forms of capital positively affect these variables. This distinction underscores that 'robots' and 'non-robot capital' can have divergent implications for labor demand.

Our model offers several advantages over existing literature, such as Berg et al. (2018) and DeCanio (2016), which also introduced robots as a distinct factor from traditional capital. Primarily, our model comprehensively incorporates multiple technological changes affecting labor share, most notably RI and HI, along with productivity enhancements in the manufacturing of both robotic and non-robotic capital, as well as wage dynamics. Second, the regression equation derived from our model allows us to estimate both the elasticity of substitution between labor and robot capital

[5] Meanwhile, Acemoglu and Restrepo (2019) employs a strong assumption that 'reinstatement' necessarily increases the labor share. Although they did not explicitly state, we surmise that their assumption might be supported by the propositions made by Acemoglu and Restrepo (2018). Utilizing this assumption, Acemoglu and Restrepo (2019) empirically infer the reinstatement through a decomposition of labor share. For a comprehensive explanation of their methodology, see Appendix A.

and the elasticity of substitution between labor and non-robot capital within a single framework. These advantages enable a more nuanced and thorough analysis of the interplay between different technological changes and their effects on labor share.

3.1 Firms

In our model, firms face monopolistic competition, which allows them to generate positive profits. For simplicity, we assume that the production function is the same for all firms[6]. Also, for brevity, we omit the time subscript.

Each firm utilizes a continuum of tasks, indexed between $N-1$ and N, in addition to capital, for production. As in Acemoglu and Restrepo (2018), N increases over time due to human innovations (HI), which can only be conducted by labor. Additionally, there is an index I that falls between $N-1$ and N. I is related to the possibility of automation (RI) and thus increases along with improvements in automation technology. Specifically, tasks below I in firm i can technically be conducted by either labor or robots, while tasks above I can only be performed by labor, as follows:

$$t_j(i) = m_j(i) + \gamma_j l_j(i) \ \text{ if } j \leq I \tag{1}$$

$$t_j(i) = \gamma_j l_j(i) \ \text{ if } j > I \tag{2}$$

, where $m_j(i)$ and $l_j(i)$ represent the number of robots and labor used for task j in firm i. γ_j represents the productivity of labor for task j. The productivity, γ_j, increases with a higher task index, j.

Tasks, $t_j(i)$, are aggregated using Constant Elasticity of Substitution (CES) aggregator, and both the aggregated tasks and capital are further combined using another CES function. Therefore, the production function is:

$$Y(i) = \left(T(i)^{\frac{\sigma-1}{\sigma}} + K(i)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{3}$$

$$T(i) = \left(\int_{N-1}^{N} t_j(i)^{\frac{\zeta-1}{\zeta}} dj\right)^{\frac{\zeta}{\zeta-1}} \tag{4}$$

, where $T(i)$ and $K(i)$ represent the number of aggregated tasks and capital used for the production of the final good i, denoted as $Y(i)$. Meanwhile, σ and ζ represent the elasticity of substitution between *aggregated tasks and non-robot capital*, and the elasticity of substitution between *tasks*, respectively.

Factor markets are assumed to be perfectly competitive. Additionally, since we focus on long-run change in labor share, it is reasonable to assume that factors are supplied elastically. For further simplicity, we assume that factors are supplied perfectly elastically at a given factor price at each period.

[6] Introducing heterogeneity in terms of Hicks-neutral productivity does not change our analysis.

3.2 Labor Share

Let us move the detailed elaboration of our model to Appendix C. Based on Equations (15) to (22) presented in this appendix, the labor share is derived as follows:

$$S_L = \frac{\eta - 1}{\eta} \frac{\int_I^N \left(\frac{W_j}{\gamma_j}\right)^{1-\zeta} dj}{P_T^{1-\zeta}} \frac{P_T^{1-\sigma}}{P_T^{1-\sigma} + R^{1-\sigma}} \tag{5}$$

$$\text{, where } P_T \equiv \left[(I - N + 1)\psi^{1-\zeta} + \int_I^N \left(\frac{W_j}{\gamma_j}\right)^{1-\zeta} dj \right]^{\frac{1}{1-\zeta}}$$

, where γ_j represents the productivity of labor for task j. The productivity, γ_j, increases with a higher task index, j. W_j, ψ, and R represent wage for labor conducting task j, robot price, and non-robot capital price, respectively.

It is worth mentioning that the term, $\frac{\eta-1}{\eta}$, is the inverse of the firm's mark-up. Since we focus on labor income as a fraction of total factor income, we denote it as S_L^f as follows:

$$S_L^f \equiv \frac{\eta}{\eta - 1} S_L = \frac{\int_I^N \left(\frac{W_j}{\gamma_j}\right)^{1-\zeta} dj}{P_T^{1-\zeta}} \frac{P_T^{1-\sigma}}{P_T^{1-\sigma} + R^{1-\sigma}} \tag{6}$$

By taking the natural log of Equation (6) and then computing the total derivative of the resulting equation with respect to the exogenous variables in the model (I, N, $\mathbb{W}$, ψ, R, and γ), we obtain Equation (7). This equation represents our final regression equation.

$$
\begin{aligned}
d\ln S_L^f =& \\
&\underbrace{\left[\underbrace{-\frac{\left(\frac{W_I}{\gamma_I}\right)^{1-\zeta}}{\int_I^N\left(\frac{W_j}{\gamma_j}\right)^{1-\zeta}dj}}_{\text{Ⓐ}} + \underbrace{\Big(-(1-\zeta)+S_K^f(1-\sigma)\Big)}_{\text{Ⓑ}}\underbrace{\frac{1}{1-\zeta}\frac{\psi^{1-\zeta}-\left(\frac{W_I}{\gamma_I}\right)^{1-\zeta}}{P_T^{1-\zeta}}}_{\text{Ⓒ}}\right]}_{\alpha_1} dI \\
&+\underbrace{\left[\underbrace{\frac{\left(\frac{W_N}{\gamma_N}\right)^{1-\zeta}}{\int_I^N\left(\frac{W_j}{\gamma_j}\right)^{1-\zeta}dj}}_{\text{Ⓓ}} + \underbrace{\Big(-(1-\zeta)+S_K^f(1-\sigma)\Big)}_{\text{Ⓑ}}\underbrace{\frac{1}{1-\zeta}\frac{-\psi^{1-\zeta}+\left(\frac{W_N}{\gamma_N}\right)^{1-\zeta}}{P_T^{1-\zeta}}}_{\text{Ⓔ}}\right]}_{\alpha_2} dN \\
&+\underbrace{\left[(1-\zeta)+\Big(-(1-\zeta)+S_K^f(1-\sigma)\Big)S_L^T\right]}_{\alpha_3} d\ln \mathbb{W} \\
&+\underbrace{\left[\Big(-(1-\zeta)+S_K^f(1-\sigma)\Big)S_M^T\right]}_{\alpha_4} d\ln \psi \\
&\underbrace{-\left[S_K^f(1-\sigma)\right]}_{\alpha_5} d\ln R. \\
&\underbrace{-\left[(1-\zeta)+\Big(-(1-\zeta)+S_K^f(1-\sigma)\Big)S_L^T\right]}_{\alpha_6} d\ln \gamma
\end{aligned} \tag{7}
$$

, where S_L^f represents labor share times markup, I is RI, N is HI, ψ is robot price, R is non-robot capital price, and γ is labor productivity. $\mathbb{W} \equiv \frac{\int_I^N\left(\frac{W_j}{\gamma_j}\right)^{1-\zeta}dj}{\int_I^N W_j^{-\zeta}\gamma_j^{\zeta-1}dj}$ is the average wage, and assume $d\ln\mathbb{W} = d\ln W_j$ for all j. Additionally, $d\ln\gamma$ represents the change in labor productivity. It is also assumed that $d\ln\gamma = d\ln\gamma_j$ for all j. S_K^f is the capital cost over total cost. By definition, $S_L^f + S_K^f = 1$.

S_M^T (S_L^T) represents the share of robot cost (labor cost) in the total combined task cost, which comprises both labor and robot costs. By definition, $S_M^T + S_L^T$ equals one. In detail, these are described mathematically as follows:

$$S_M^T = \frac{(I - N + 1)\psi^{1-\zeta}}{P_T^{1-\zeta}}$$

$$S_L^T = \frac{\int_I^N \left(\frac{W_j}{\gamma_j}\right)^{1-\zeta} dj}{P_T^{1-\zeta}}$$

$$\text{, where } P_T^{1-\zeta} = (I - N + 1)\psi^{1-\zeta} + \int_I^N \left(\frac{W_j}{\gamma_j}\right)^{1-\zeta} dj.$$

We acknowledge that price factors $-R$ (non-robot capital price), $\mathbb{W}$ (labor price), and ψ (robot price)— are not purely exogenous. We do not employ instrumental variables or other techniques to mitigate endogeneity. This limitation represents the most significant weakness of this paper. In the next section, we discuss the datasets used in this paper and the construction of the variables.

4 Data Collection and Variable Generation

For the purpose of assessing HI, we will use data from O*NET, which offers information on the number of new tasks in the USA, measured at the occupation-year level. To analyze RI, we will use data provided by the International Federation of Robotics (IFR), which gives us the number of automated machines at the country-industry-year level.

4.1 Human Innovations

To proxy dN in Equation (7), we utilize HI, which we elaborate on in this subsection. The Occupational Information Network (O*NET), managed and maintained by the United States Department of Labor, serves as a comprehensive database of occupational information (National Center for O*NET Development, 2023). For each Standard Occupational Classification (SOC),[7] O*NET consistently updates the spectrum of tasks that workers are expected to perform. For instance, in 2023, Automotive Engineers were assigned 25 responsibilities, including the calibration of vehicle systems, control algorithms, and other software systems. When new tasks, previously nonexistent, emerge, O*NET increases the number of tasks associated with the Automotive Engineering occupation.

[7] SOC is an acronym for Standard Occupational Classification employed by US agencies. The O*NET classification system (O*NET-code) is a subclassification of the SOC system, hence, every O*NET-code has a corresponding SOC. However, the O*NET-code does not align perfectly with the Occupational Classification Code (OCC).

Furthermore, O*NET periodically reports 'Emerging new tasks' approximately once or twice annually. These tasks have recently emerged but have not been extensively studied by the O*NET department; hence, these specific tasks are not included in the standard occupational list. We incorporate these 'Emerging new tasks' in addition to our base number of tasks provided by O*NET. This process completes our generation of 'Task scores' for each occupation.[8]

The 'Task scores' vary by Standard Occupational Classification (SOC) and year. Acemoglu and Restrepo (2019) (henceforth AR) translated this information into variations by industry and year using the US Census from IPUMS (Ruggles et al., 2020), a dataset comprising individual worker data with specific occupation codes.[9] After associating the 'Task score' with each individual, an average is calculated at the industry and year level. Subsequently, we compute the 5-year growth rate of this variable, which we denote as HI. This method can also be applied to European Union (EU) countries by using the EU Labor Force Survey (EU-LFS) instead of the US Census. It's important to note that even when calculating HI for EU countries, we still use the 'Task scores' from the US-based O*NET database.

The European Commission has recently initiated a project akin to O*NET, named 'European Skills, Competences, Qualifications, and Occupations' (ESCO). ESCO has disclosed the tasks required for workers only for two distinct years. In the absence of a European equivalent of the yearly 'Task scores', we depend on data from O*NET. A foundational assumption in the creation of the EU's HI is that the task requirements in the USA mirror similar trends in the EU. For instance, if the number of tasks required for Automotive Engineers surged in the USA in 2015, it is assumed that a similar trend occurred in the EU around the same period. Consequently, the variation for the EU stems from the differing composition of workers in each country, occupation, and year.

[8] Meanwhile, Acemoglu and Restrepo (2019) employs only 'Emerging new tasks' to construct the Task scores. We contend that our method of integrating both the 'base number of tasks' and 'Emerging new tasks' offers a more sophisticated approach than relying solely on Emerging new tasks, as AR does. Specifically, the 'base number of tasks' serves as a primary source of information for capturing new tasks that were nonexistent before, while 'Emerging new tasks' function as supplementary information.

[9] Our matching procedure from 'Task score' to the US Census is as follows: We use SOC as it is, instead of converting it to OCC as Acemoglu and Restrepo (2019) does. The US Census provides both SOC and OCC for occupational taxonomy, allowing us to simply use SOC to match the US Census with the 'Task score'.

Moreover, when matching 'Task score' to EU-LFS, using SOC is more advantageous than using OCC. EU-LFS uses ISCO for occupational taxonomy, and ISCO (4-digits) matches with SOC (6-digits). This granular level of crosswalk matching is made possible by the recent work of Frugoli and ESCO (2022). The excel file for the crosswalk between ISCO and SOC is in this link. This is publicly released by O*NET and ESCO.

4.2 Robot Innovations

The International Federation of Robotics (IFR) provides data on the number of automated robots (both flow and stock) at the country-industry-year level. Instead of using the raw data on the number of robots from the IFR, Acemoglu and Restrepo (2020) proposed utilizing the Adjusted Penetration of Robots (APR) to proxy automation. For a detailed explanation of APR, please refer to Appendix E.

One issue with APR is that it effectively represents $d(I - N + 1)$, not dI, which is the true measure of automation (RI). Our introduction of the proxy for dN, the HI, as explained in the previous section, enables us to address this issue in the following manner.

From Equation (7) in the Regression section,

$$d \ln S_L^f = \alpha_1 dI + \alpha_2 dN + \alpha_3 d \ln W + \alpha_4 d \ln \psi + \alpha_5 d \ln R$$

Therefore, on the right-hand side,

$$\begin{aligned} \alpha_1 dI + \alpha_2 dN &= \alpha_1 d(I - N + 1) + \alpha_1 dN + \alpha_2 dN \\ &= \alpha_1 \text{APR} + \alpha_1 \text{HI} + \alpha_2 \text{HI} \\ &= \alpha_1 (\text{APR} + \text{HI}) + \alpha_2 \text{HI} \\ &= \alpha_1 \text{RI} + \alpha_2 \text{HI} \end{aligned} \tag{8}$$

In short, we use the Robotics Innovation (RI) to proxy dI. RI is essentially a summation of APR and HI. One might wonder why we don't simply use dI from the beginning instead of using $d(I-N+1)+dN$. The issue here is that there is no effective alternative to proxy dI. As mentioned earlier, the number of robots used is the result of economic equilibrium and is not the abstract concept of dI. Should readers be curious about the outcomes if the regression had employed APR instead of RI, these results are provided in the Robustness Check section.

4.2.1 Variance Adjustment

Since APR and HI are constructed variables, they are not directly comparable. Given that RI is constructed by summing APR and HI (as shown in Equation (8)), ensuring comparability between APR and HI, especially in terms of variance, is important.

In Equation (9), the right-hand side represents the newly adjusted HI, whereas the left-hand side details the adjustment process. Since the variances of HI and APR are not directly comparable, we adjust by multiplying by $\frac{\sigma_{\text{APR}}}{\sigma_{\text{HI}}}$ to equate the variance of HI with that of APR. Here, σ represents the standard error.

$$\text{HI} \times \frac{\sigma_{\text{APR}}}{\sigma_{\text{HI}}} \times \frac{\sigma_{\text{inferred N}}}{\sigma_{\text{inferred (I-N)}}} \;\Rightarrow\; \text{HI} \tag{9}$$

We then multiply by $\frac{\sigma_{\text{inferred N}}}{\sigma_{\text{inferred (I-N)}}}$. Here, 'inferred N' (the inferred value of HI) and 'inferred I' (the inferred value of APR) are obtained by replicating the methodology of Acemoglu and Restrepo (2019), as detailed in Section 4.1 and Appendix A. We have extended this replication to nine countries and continued it through 2019.

While the variances of APR and HI were not directly comparable, those of 'inferred N' and 'inferred I' are. This comparability stems from the fact that Acemoglu and Restrepo (2019) inferred these values using the same set of variables, particularly focusing on the labor share. Our approach involves adjusting the variance of HI so that the difference in variance between HI and APR matches that between 'inferred N' and 'inferred I'.

According to our replication, the ratio $\frac{\sigma_{\text{inferred N}}}{\sigma_{\text{inferred (I-N)}}}$ equals $\frac{54.927}{17.923} = 3.435$. Throughout this paper, we will employ the variance-adjusted version of HI. To ensure robustness, we additionally provide regression tables in Section 8, using a ratio of $\frac{\sigma_{\text{inferred N}}}{\sigma_{\text{inferred (I-N)}}} = 1$ (i.e., with no adjustment). All other analyses remain unchanged even when using this value instead of 3.435.

4.3 Robot Price

Unfortunately, the International Federation of Robotics (IFR) no longer provides information on the prices of robots. IFR provided robot prices in the form of an average unit price until 2009, and as a price index until 2005. Klump et al. (2021) and Jurkat et al. (2022) provide in-depth information on this topic.[10] An alternative method to obtain robot prices is by following the approach of Fernandez-Macias et al. (2021), which involves the use of UN Comtrade data.[11] We adopted this method, which illustrate in their Figures 3 and A1 that the robot price trends based on IFR and UN Comtrade data are similar. Furthermore, they demonstrate that the robot price has been steadily declining.[12]

4.4 Capital Price

In Figure 9, provided in Appendix L, we replicate the derivation of capital price following the approach used by Karabarbounis and Neiman (2014) (hereafter referred to as KN), utilizing the KLEMS data version. This ensures that the 'overall' capital price

[10]They noted, "Due to the considerable effort involved and owing to compliance issues, the IFR no longer continues to construct the price indices."

[11]https://comtradeplus.un.org/

[12]The data generation process is as follows: UN Comtrade provides annual import and export values in dollar for 'Machinaery and mechanical appliances; industrial robot, n.e.c. or included. (HS847950)' They also provide the quantity of these values for both imports and exports. Hence, we infer the robot prices by dividing the dollar values by their quantities.

variable is identical to that used by KN. Subsequently, we derive the non-robot capital price variable as detailed in Section 4.5. This non-robot capital price variable is then consistently utilized throughout Sections 6 and 7. Our data indicate that the prices of non-robot capital have generally increased over the past 15 years, as illustrated in Figure 9 in Appendix L. This observation might initially appear contradictory to the claims of KN, who reported a rapid global decline in capital prices (see Figure 7 of their paper). However, our Figure 9 is consistent with their findings, considering that capital prices began to rise from around year 2000. Furthermore, their figure aggregates data from all countries worldwide, whereas our analysis is more focused, presenting data at the country level for only 9 selected countries.

4.5 Non-robot Capital Price

Denote total capital that includes robot and non-robot as K. Also, denote robot capital and non-robot capital as M and R, respectively. Then it follows that

$$\text{gr\_Price}_K = \text{gr\_Price}_M \frac{\text{Cost}_M}{\text{Cost}_K} + \text{gr\_Price}_R \frac{\text{Cost}_R}{\text{Cost}_K}$$

, where 'gr' denotes the growth rate. The implication of this equation is that the level and scale of the prices do not matter in this growth rate relationship. The above equation can be rearranged to

$$\text{gr\_Price}_R = \frac{\text{gr\_Price}_K - \text{gr\_Price}_M \times \alpha}{1 - \alpha}$$

, where α is $\frac{\text{Cost}_M}{\text{Cost}_K}$. This completes the derivation of the growth rate of price for the non-robot capital.

For the capital price, gr_Price_K, we strictly adhere to the approach outlined by Karabarbounis and Neiman (2014) throughout this paper. For detailed explanations, please refer to Appendix F. We have values for Cost_K from KLEMS data. For further explanations regarding this, please refer to Appendix G.

We can estimate Cost_M by sector and country through two approaches. The first approach employs the value obtained using the approach introduced in Section 6.3. This approach yields the ratio $\frac{\text{Robot Cost}}{\text{Labor Cost}} = 2.813\%$, and labor cost information is available from the KLEMS dataset. Consequently, we can calculate Cost_M based on this information. However, this approach is contingent on labor cost values, raising concerns that the ratio $\frac{\text{Robot Cost}}{\text{Labor Cost}} = 2.813\%$ may vary significantly across sectors and countries. Therefore, we propose an alternative approach.

The alternative approach leverages information from the alternative method detailed in Appendix H.1. In this method, we have determined the cost ratio between OMach and robots to be $13.595 : 2.149$, where 'OMach' refers to the machinery and equipment in the KLEMS. Given that we possess detailed OMach cost data by sector and country, we can subsequently estimate Cost_M. This approach circumvents the need for labor cost data. By using this approach, we complete our derivation of the growth rate of non-robot capital price, which will be used in our regression analysis.

5 Instrumental Variables

In our general equilibrium model, multiple variables present challenges for applying instrumental variables (IVs) comprehensively. However, it is imperative to employ IVs at least for the key variables, RI and HI, to ensure exogeneity. To generate IVs for RI (henceforth RI-IV) and HI (henceforth HI-IV), we utilize the detailed descriptions of all U.S. granted patents from 2004 to 2019, which encompasses the entire span of our study. These detailed descriptions extend beyond abstracts, International Patent Classification (IPC), or Cooperative Patent Classification (CPC) information, providing comprehensive explanations of the patents. This approach constitutes one of our contributions, as most existing studies rely solely on abstracts, IPC, or CPC information.

Our critical assumption posits that patent invention represents an exogenous shock. To validate this premise, we argue that the development of an invention is a protracted process, not subject to high-frequency fluctuations in economy. Under this assumption, the growth rate of granted patents can serve as an IV, provided it demonstrates significant correlation with the endogenous variables.

Although U.S. patent data do not directly provide the country information of patent holders, they include company names and city locations. By leveraging the Google Maps API, we can infer the actual country of origin for each patent holder. Additionally, we can deduce the industrial sector of the patent. Lybbert et al. (2014) provide matching crosswalks between IPC codes and industrial sectors. Consequently, we construct a dataset comprising Patent ID, patent descriptions, patent holder's country, corresponding detailed industry in the manufacturing sector, and patent grant year.

Recent advancements in semantic embedding technology have led to significant improvements in natural language understanding. This technology enables the comprehension of semantic content within sentences. Unlike other studies, we utilized the most recently developed text-to-vector embedding software. One such software is 'sentence-transformers/all-mpnet-base-v2' developed by Microsoft, and the other is 'text-embedding-3-large' developed by OpenAI. To date, they represent one of the best-performing tools available (Harris et al., 2024).[13]

[13]While both OpenAI's 'text-embedding-3-large' and Microsoft's 'sentence-transformers/all-mpnet-base-v2' are among the best-performing tools available, they are not the only top performers. Other models like NVIDIA's 'NV-Embed' and Salesforce's 'SFR-Embedding' also demonstrate exceptional performance (Lee et al., 2024; Meng et al., 2024).

Both of these embedding software tools are unique in their ability to understand not only word-to-word similarity but also sentence-to-sentence similarity. If two sentences have completely different meanings, even if they use similar words, sentence embedding models will recognize them as different. In contrast, word embedding models will perceive the sentences as similar (Ul Haq et al., 2024; Zhang et al., 2024; Mandelbaum and Shalev, 2016; Li et al., 2015).

Baer and Purves (2023) demonstrates that the 'sentence-transformers/all-mpnet-base-v2' approach significantly outperforms TF-IDF in identifying similar documents, as judged by human annotators. Existing studies have predominantly relied on word embeddings. For instance, studies have utilized TF-IDF (Autor et al., 2024; Kogan et al., 2021; Webb, 2019) and BERT (Frugoli and ESCO, 2022). To the best of our knowledge, we are the first to apply sentence embedding technology in the field of economics.

To construct RI-IV, we compare each patent description with a curated list of vocabularies closely associated with robotics and automation technologies. The detailed vocabulary list is provided in the footnote.[14] By comparing each patent description with these automation-related terms, we derive a similarity score ranging from 0 to 1. We subsequently aggregate the scores by country, industry, and year. The growth rate of this aggregated value is what we define as RI-IV. For brevity, we present two scoring examples: one with a high score and another with a low score.

Patent Number: 10209063
Applicant: X Development LLC
City: Mountain View
Similarity Score: 0.61 (high)
Patent Description: (1) Robots may be programmed to perform a variety of tasks such as, for example, autonomous or semi-autonomous navigation, manipulating objects (e.g., repositioning an object, altering an object, and/or picking up an object and moving it to a different location), transporting objects (without necessarily manipulating those objects), monitoring environmental conditions, functioning as "video conferencing on wheels", and so forth. ...(Omitted to save space)... (3) The present disclosure is generally directed to using sensor-based observations from multiple agents (e.g., mobile robots and/or fixed sensors) in an environment to estimate the pose of an object in the environment at a target time and to estimate an uncertainty measure for that pose. The object for which the pose and uncertainty measure are estimated may be a non-agent object such as a pallet, a box, a product, etc. or may itself be an agent (e.g.,

[14] actuator, artificial intelligence, automation, autonomous, biomimetics, computer vision, cybernetics, human-machine interface (HMI), humanoid robots, industrial automation, industrial robot, kinematics, machine learning, machine perception, machine vision, motion control, Natural Language Processing (NLP), neural networks, object recognition, odometry, programmable, programmable logic controller, robot, Robot Operating System (ROS), robotic, robotic arm, robotic exoskeleton, robotic process automation (RPA), sensor fusion, servo motor, visual servoing, workflow automation.

a mobile robot). As used herein, "pose" of an object may reference a position of the object only (e.g., a multidimensional coordinate), or may reference both the position of the object and an orientation of the object (e.g., a pose in the SE(3) configuration space).

Patent Number: 10285908
Applicant: Sun Pharmaceutical Industries Limited
City: Mumbai
Similarity Score: 0.01 (low)
Patent Description: (1) The present invention relates to a dual-chamber pack for a multi-dose oral liquid pharmaceutical composition wherein the compositions of the first and second chambers are mixed at the time of first administration ...(Omitted to save space)... (6) The present invention provides an alternative pack for a multi-dose oral liquid pharmaceutical composition comprising of two chambers, wherein the pack is adaptable for low to high dose drugs. The pack allows the patient ease of dispensing with only a few simple steps required for reconstitution.

To construct HI-IV, the instrument for human innovation, we employ a comparable methodology, comparing patent descriptions with occupation descriptions provided by the Occupational Information Network (O*NET). The O*NET database contains approximately 800 distinct occupation descriptions. For each patent, we compute similarity scores against all of these occupation descriptions and select the highest score, which becomes the representative similarity score for that specific patent. We apply this process to all patents available from 2004 to 2019. Subsequently, we aggregate these similarity scores by country, industry, and year. The growth rate of this aggregated value constitutes what we define as the HI-IV.

Our conceptual framework posits that patents exhibiting a semantically close relationship to occupational descriptions are indicative of supporting human innovation, as they enhance the efficiency of human labor without reliance on robotic assistance. Fundamentally, we differentiate between autonomous robots and conventional machines or tools. Under this assumption, we infer that the aggregated similarity scores represent the evolution of human tasks. For the sake of brevity, we present two high-scoring examples.

Patent Number: 10300695
Applicant: Toshiba TEC Kabushiki Kaisha
City: Tokyo
Similarity Score: 0.66 (high)
Patent Description: An ink jet printer prints patterns according to an input signal corresponding to an image or text. The ink jet printer includes, for example, an ink jet head and an ink jet head control circuit that controls the ink jet head. The ink jet

head includes an actuator for ejecting ink and a driver integrated circuit (IC) that drives the actuator according to a control signal input from the ink jet head control circuit ...(Omitted to save space)... An ink jet head may include a non-volatile memory that stores unique information of the ink jet head, maintenance information, and the like. When a non-volatile memory is mounted on the ink jet head, the ink jet head control circuit also requires a connection terminal for accessing the non-volatile memory. However, adding such a terminal to the inkjet head control circuit and the inkjet head may increase costs.
O*NET Occupation Description: Printing Press Operators [SOC 51-5112] Set up and operate digital, letterpress, lithographic, flexographic, gravure, or other printing machines. Includes short-run offset printing presses.

Patent Number: 10513832
Applicant: Scott
City: Naples
Similarity Score: 0.68 (high)
Patent Description: A pile driver or piling hammer is a mechanical device used to drive piles, pilings or poles into the Earth to provide foundation support for docks, buildings or other structures. A conventional pile driver or piling includes a heavy weight that it is able to freely slide up and down in a single line, wherein the weight is placed above a pile, piling or pole. The weight is raised, and when the weight reaches its highest point, it is released and impacts the pile, piling or pole in order to drive it into the ground. ...(Omitted to save space)... Consequently, a need exists to overcome the problems with the prior art as discussed above, and particularly for improved and innovative pilings hammers.
O*NET Occupation Description: Pile Driver Operators [SOC 47-2072] Operate pile drivers mounted on skids, barges, crawler treads, or locomotive cranes to drive pilings for retaining walls, bulkheads, and foundations of structures such as buildings, bridges, and piers.

6 Regressions

6.1 Regression Equations

Based on the specification in Equation (7) shown in Section 3.2, we provide consistent regression equations as below:

$$\begin{aligned}\text{gr\_(laborshare} \times \text{markup)} =& \alpha_1 \text{RI} + \alpha_2 \text{HI} \\ &+ \alpha_3 \text{gr\_labor price} + \alpha_4 \text{gr\_robot price} \\ &+ \alpha_5 \text{gr\_non-robot capital price} \\ &+ \alpha_6 \text{gr\_labor productivity} \\ &+ \lambda_i + \lambda_j + \lambda_t + \lambda_{ij} + \varepsilon_{ijt}. \end{aligned} \tag{10}$$

gr indicates the variables are in a 5-year growth rate, and i, j, and t correspond to country, industry, and year, respectively. We exclude the notation of *gr* from RI and HI, as by definition, they already represent a 5-year growth rate.

6.2 Regression Results

We present IV regresion results (2SLS) in Table 1. Standard errors are clustered by country to account for serial correlation. To improve readability, both the coefficients and standard errors have been multiplied by 100.

Upon examination of Equation (7), it is evident that the sum of the coefficients for $d \ln \mathbb{W}$, $d \ln \psi$, and $d \ln R$ is equal to zero (i.e., $\alpha_3 + \alpha_4 + \alpha_5 = 0$). In the regression table, Column (1) does not incorporate this constraint, whereas Column (2) imposes it. The baseline model employed throughout this study is represented by Column (2), which includes this restriction.

In assessing the congruence between the regression results and the model's predictions, two findings are noteworthy. First, the model delineates the coefficient for robot price as α_4, with the term $S_M^T = 2.81\%$ included, which we estimated in Section 6.3. The model thus anticipates this coefficient to be of an insignificantly small value. In line with this prediction, the regression coefficient for robot price is not statistically significant, and the point estimate lacks precision.

Second, the IV regresion results maintain consistency in both magnitude and direction, regardless of whether the restriction is applied. Utilizing the regression without the restriction (as shown in Column (1)), we test the null hypothesis that the restriction is non-binding. The hypotheses is not rejected at the 0.05 significance level. This suggests an alignment between the data and the model's predictions. In subsequent analyses, we refer to the IV results from Column (2), the restricted version, as our baseline.

Table 1: IV Two-stage Regressions

Constraint	No	Yes
	(1)	(2)
α_1: RI	-0.437**	-0.380**
	(0.204)	(0.187)
α_2: HI	0.473**	0.360*
	(0.221)	(0.189)
α_3: gr_labor price	10.425***	10.669***
	(3.583)	(4.068)
α_4: gr_robot price	-0.293	-0.029
	(1.262)	(1.630)
α_5: gr_non robot capital price	-20.297***	-10.641***
	(4.035)	(3.745)
N	839	839
R^2	0.430	0.445

The coefficients and the standard errors have been multiplied by 100 for better readability.
Standard errors in parenthesis are clustered by country.
$^* p < 0.10$, $^{**} p < 0.05$, $^{***} p < 0.01$

For Column (1), the first-stage F-values are 39.27 (RI-IV) and 68.83 (HI-IV). For Column (2), the first-stage F-values are 13.83 (RI-IV) and 33.18 (HI-IV). The rule of thumb suggests that an F-value above 10 indicates a sufficient instrument. Therefore, both RI-IV and HI-IV meet the criteria for appropriate instrumental variables.

6.3 Estimation of S_M^T

S_M^T represents the share of robot cost in the total combined task cost, which comprises both labor and robot costs. This metric is vital for our analysis in the Regression section. Unfortunately, no official data is available that directly quantifies this value, requiring us to rely on multiple sources for an accurate estimation.

For a detailed explanation of how we estimated S_M^T, please refer to Appendix H. By synthesizing all available information, we estimate S_M^T to be 2.813% for the total manufacturing sectors. An alternative method detailed in Appendix H.1 estimates the S_M^T value at 2.104%. However, we consider the method outlined in this section to be more accurate and reliable, leading us to conclude that the S_M^T value is 2.813%.

6.4 Estimation of σ and ζ

By utilizing Equation (7) along with the regression results, we estimate the values of σ and ζ. σ represents the elasticity of substitution between *the aggregate task and non-robot capital.* Notably, labor costs account for 97.2% of the aggregate task cost, while non-robot capital accounts for 91.1% of the 'overall' capital cost. Thus, σ serves as a close proxy for the elasticity of substitution between *labor and overall capital*.

We detail the methodology for estimating these two elasticities, σ and ζ in Appendix I. Our results are as follows: first, we calculate $\sigma = 0.527$, with a 90% confidence interval for σ of $(0.253, 0.801)$. σ differs from the elasticity of substitution between *labor and non robot-capital*, but as mentioned, σ serves as a close proxy of this elasticity. In Appendix J, we provide a formal estimation of the elasticity of substitution between *labor and non-robot capital* using the estimation of σ. This measure closely aligns with the measures used by Karabarbounis and Neiman (2014) and Glover and Short (2020), and our estimate ranges between 0.518 and 0.574. Thus, this result contributes to literature by providing additional empirical evidence that the elasticity of substitution between labor and non-robot capital is less than one, indicating a gross complementary relationship between the two. This is supported by most literature, as suggested by Chirinko (2008), Grossman and Oberfield (2022), and Glover and Short (2020).

We estimate $\zeta = 0.833$, with a 90% confidence interval of (-0.071, 1.838). DeCanio (2016) proposed a ζ value of 1.9. A ζ value greater than one implies that improvements in robot productivity —as reflected by a decrease in robot price— significantly affect the labor share. Unfortunately, due to the inconclusive nature of our ζ estimation, we are unable to draw further conclusions on this matter.

6.5 Effects of Price Factors on Labor Share

6.5.1 Labor Price

The regression findings provide important insights into the relationship between factor prices and labor share. Our analysis reveals a positive correlation between the labor price (wage) and labor share. This relationship can be understood through the concept of gross complementarity between labor and non-robot capital, as indicated by $\sigma < 1$ in our model.

The mechanism underlying this relationship can be explained as follows: When the wage increases, the usage of labor does not decrease proportionally to the price increase. This disproportionate response leads to an overall increase in the cost attributed to labor. Consequently, a larger portion of the cost is allocated to labor, resulting in a rise in labor share.

Technically speaking, the robot cost share, denoted by S_M^T, is a very small value, specifically 0.028. This indicates that when wages change, substitution between labor and robots does not have a significant effect, and substitution between labor and non-robot capital plays a more important role, as demonstrated below. In essence, the condition that determines $\alpha_3 > 0$ is fundamentally $\sigma < 1$, from a technical perspective.

$$\begin{aligned}
\alpha_3 =&(1-\zeta) + \Big(-(1-\zeta) + S_K^f(1-\sigma)\Big) S_L^T \\
=&(1-\zeta)(1-S_L^T) + S_K^f(1-\sigma)S_L^T \\
=&(1-\zeta)(S_M^T) + S_K^f(1-\sigma)S_L^T \\
=&0.00328 + S_K^f(1-\sigma)S_L^T \\
\approx&\ S_K^f(1-\sigma)S_L^T = 0.10341 > 0.
\end{aligned}$$

6.5.2 Non-robot Capital Price

The underlying principle is analogous to the labor price scenario. An increase in the price of non-robotic capital does not elicit a proportional decrease in its utilization. This disproportionate response engenders an overall increase in the costs associated with non-robotic capital, consequently leading to a reduction in the relative costs attributed to labor. As a result, a diminished proportion of total costs is allocated to labor, precipitating a decline in the labor share. From a technical perspective, the fundamental reason for $\alpha_5 < 0$ is essentially that $\sigma < 1$.

$$\alpha_5 = -\Big[S_K^f(1-\sigma)\Big] < 0 \tag{11}$$

6.5.3 Robot Price

The regression results indicate a negative, albeit small, association between robot price and labor share. This insignificance is attributed to the low share of the robot cost ($S_M^T = 2.8\%$). This means that even if robot prices change, their impact on labor share will inevitably be small.

$$\alpha_4 = \Big(-(1-\zeta) + S_K^f(1-\sigma)\Big) S_M^T < 0 \tag{12}$$

In the future, we anticipate that the coefficient for robot price will become more significant, yielding a stronger association as the proportion of robots in society increases. This expectation is attributable to the term S_M^T, which represents the share of robot costs and is projected to be larger in the future.

Our analysis of the robot price factor reveals a negative correlation with labor share. We demonstrated, ζ has a 90% confidence interval of (-0.071, 1.838), yielding inconclusive results. Furthermore, the term $\Big(-(1-\zeta) + S_K^f(1-\sigma)\Big)$ has confidence interval of (-0.963, 0.943) with a point estimate of -0.010. Consequently, interpreting this negative coefficient in terms of the elasticity of substitution between robot capital and labor lacks statistical significance.

7 Accounting Exercise

Our primary research objective in this paper is to elucidate the factors influencing labor share, both in terms of magnitude and direction. This fundamental inquiry drives our investigation. The Accounting section of our study directly addresses this core research question. By employing a comprehensive accounting framework, we quantitatively assess the relative contributions of various factors to changes in labor share across different countries.

Based on the main regression results from Column (2) in Table 1, we have generated Figures 4 and 5. In this paper, we exclusively focus on country-level variation to maintain brevity. Accordingly, the values in these figures are derived by aggregating data at the country level. During this aggregation process, 'Average variables' are consolidated by weighting the value-added in each sector and year. Intuitively, the values illustrated in Figures 4 and 5 quantify the extent to which each factor influences the growth rate of the markup-adjusted labor share (S_L^f).[15]

Figure 4: Labor shares

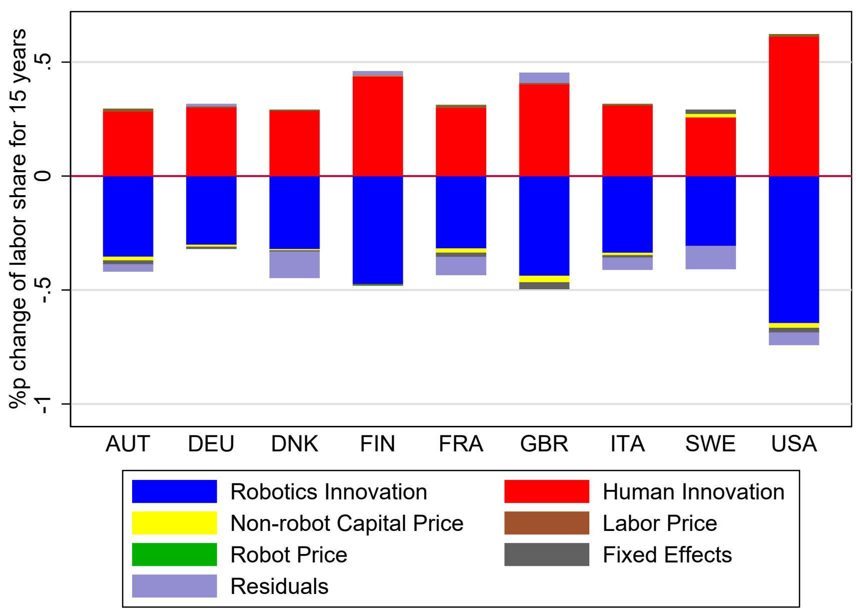
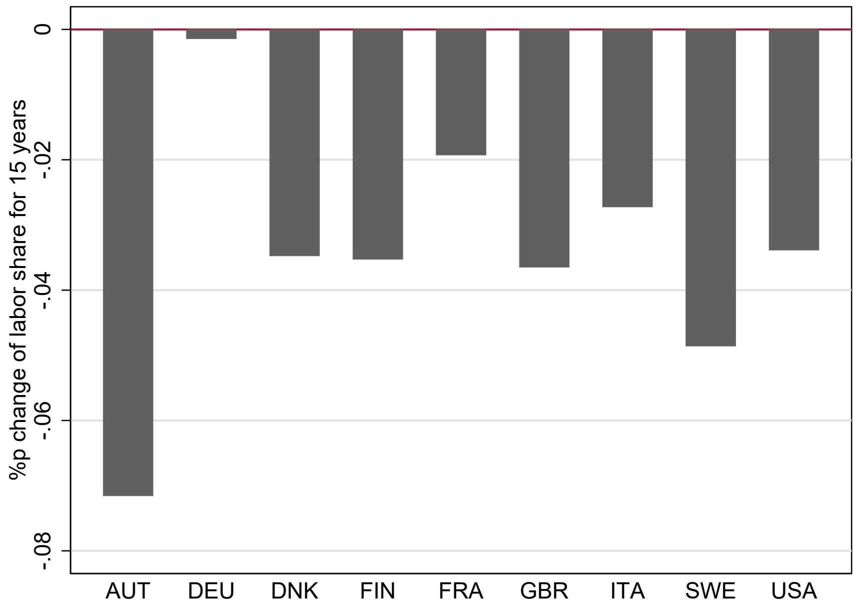


Figure 4 presents the results of our accounting analysis, demonstrating that RI and HI are significant factors influencing labor share. Specifically, an increase in RI corresponds to a decrease in labor share, while an increase in HI leads to an increase. These findings corroborate the argument put forth by Autor (2015), who posited that

[15] S_L^f is defined in Equation (6) in the Model section.

Figure 5: Labor shares (NET of RI and HI)



"the sustained relevance of human labor in the future will largely depend on the pace at which 'human innovations' outstrips the advancement of automation."

Our data reveal a general upward trend in non-robotic capital prices over the past 15 years, as illustrated in Figure 9 in Appendix L. The negative coefficient associated with the price of non-robotic capital suggests a consequent decline in labor share. This observation stands in direct contrast to the argument presented by Karabarbounis and Neiman (2014), who contended that the decline in capital prices has led to a drop in labor share. Our findings yield the opposite result. Meanwhile, Figure 4 incorporates a component representing fixed effects, albeit small, accounting for unobserved time-sector-country invariant factors.

Figure 5 provides an alternative perspective by netting out the effects of RI and HI, effectively canceling out their opposing directional impacts. Despite the substantial offsetting effects between these two factors, the aggregate result indicates that the negative effect of automation (RI) marginally surpasses the positive impact of human innovations (HI) on labor share. This nuanced analysis underscores the intricate balance between technological advancement and human innovation in shaping labor market outcomes.

8 Robustness Check

As we promised in previous sections, we provide several robustness information. First, we use APR and HI as an explantory variable instead of using RI and HI.

Table 2: IV Two-stage Regressions using APR

Constraint	No	Yes
	(1)	(2)
α_1: APR	-0.437**	-0.380**
	(0.204)	(0.187)
α_2: HI	0.035	-0.020
	(0.051)	(0.047)
α_3: gr_labor price	10.425***	10.669***
	(3.583)	(4.068)
α_4: gr_robot price	-0.293	-0.029
	(1.262)	(1.630)
α_5: gr_non robot capital price	-20.297***	-10.641***
	(4.035)	(3.745)
N	839	839
R^2	0.430	0.445

The coefficients and the standard errors have been multiplied by 100 for better readability.
Standard errors in parenthesis are clustered by country.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

In our baseline analysis presented in Table 1, we employed a variance adjustment value of 3.435, as elucidated in Section 4.2.1. To assess the robustness of our findings, we subsequently conducted an alternative analysis using a variance adjustment value of 1. The results of this analysis, while slightly different, prove to be remarkably consistent with those presented in our baseline table.

9 Concluding Remarks

This paper has investigated the declining trend in global labor share during 2004 to 2019, with a particular focus on the roles of robotic innovation (RI) and human innovation (HI) in shaping this trend. We have developed a general equilibrium model that incorporates both RI and HI, addressing a gap in the existing literature and providing a unified framework for analyzing multiple factors influencing labor share.

Our primary contribution lies the development of novel instrumental variables for RI and HI. Leveraging recent advancements in natural language processing, specifically sentence embedding technology, we constructed these instruments by analyzing the semantic content of patent descriptions. This approach represents a significant

Table 3: IV Two-stage Regressions using without variance adjustment

Constraint	No	Yes
	(1)	(2)
α_1: RI	-0.437**	-0.380**
	(0.204)	(0.187)
α_2: HI	0.559*	0.312
	(0.297)	(0.238)
α_3: gr_labor price	10.425***	10.669***
	(3.583)	(4.068)
α_4: gr_robot price	-0.293	-0.029
	(1.262)	(1.630)
α_5: gr_non robot capital price	-20.297***	-10.641***
	(4.035)	(3.745)
N	839	839
R^2	0.430	0.445

The coefficients and the standard errors have been multiplied by 100 for better readability.
Standard errors in parenthesis are clustered by country.
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

methodological advancement over previous studies that relied primarily on word embeddings or more limited patent information.

The instrumental variables we developed for RI and HI demonstrate the potential of applying cutting-edge language models to economic research. By utilizing tools such as 'sentence-transformers/all-mpnet-base-v2' and 'text-embedding-3-large', we were able to capture the nuanced relationships between patent descriptions and both automation-related terms and occupation descriptions. This method allowed us to more accurately identify patents related to robotic and human innovations, respectively.

Our empirical findings, based on data from nine countries, confirm the significant and opposing effects of RI and HI on labor share. Specifically, we find that increases in RI are associated with decreases in labor share, while increases in HI correspond to increases in labor share. These results provide empirical support for the theoretical arguments put forth by scholars such as Autor (2015) regarding the importance of human innovation in maintaining the relevance of human labor in the face of advancing automation.

The accounting exercise we conducted reveals that while RI and HI have substantial offsetting effects, the negative impact of automation slightly outweighs the positive effect of human innovation on labor share. This nuanced finding underscores the complex interplay between technological advancement and human capital development in shaping labor market outcomes.

Furthermore, our analysis yields an estimated elasticity of substitution between labor and non-robot capital that is less than one, consistent with the majority of the literature. This finding suggests a gross complementary relationship between labor and non-robot capital, contributing to the ongoing debate on factor substitutability.

While our study makes significant contributions to the understanding of labor share dynamics, it is not without limitations. The primary challenge lies in the endogeneity of price factors, which we were unable to fully address through instrumental variables. This limitation underscores the need for further research and refinement of methodologies in this area. In conclusion, our research provides valuable insights into the factors driving changes in labor share, particularly the roles of robotic and human innovation.

A Appendix: Human Innovations by Acemoglu and Restrepo (2019)

Acemoglu and Restrepo (2019) (henceforth referred to as AR) presents a tool for inferring automation and human innovation (henceforth, HI). This tool utilizes a relatively small set of variables: labor compensation, employee count, value-added, wage, and investment price. The AR framework enables the inference of automation and HI. Fundamentally, the AR framework operates under the assumption that if there is an observed *increase* in labor share, it must be attributed to HI. Conversely, if there is a *decrease*, it is attributable to automation. This principle is clearly articulated in Figure 1 of their paper.

The online appendix of the AR paper elaborates on this framework. For ease of reference, we include it in our Appendix B. In this appendix, Term (AR4) represents the percentage change in labor share, which can be broken down into Terms (AR6) and (AR7). The former represents the percentage change in substitution effects, while the latter shows the percentage change in 'task contents.' A positive (negative) result in Term (AR7) is interpreted as indicative of HI (automation). Given that the percentage change in substitution effects (Term AR6) is usually minimal, the percentage change in 'task contents' (Term AR7) virtually mirrors the percent change in labor share (Term AR4).

To summarize, AR's inference of automation and HI is largely based on the percent change in labor share. However, using these inferred variables in our primary analysis presents a challenge due to the expected high correlation with labor share, which could lead to reverse causality. Furthermore, there is no certainty that the inferred variables accurately represent the real-world values of automation and HI. Consequently, we require variables obtained through direct measurement.

B Appendix: Acemoglu and Restrepo (2019)

Let me first introduce their notations in Table 4.

Table 4

Notation	Meaning
i	Industry sector
P_i	The price of the goods produced by sector i
Y_i	Output (value added) of sector i
$Y = \sum_i P_i Y_i$	Total value added (GDP) in the economy
$\chi_i = \frac{P_i Y_i}{Y} = \frac{P_i Y_i}{\sum_i P_i Y_i} = \frac{\text{GDP}_i}{\text{GDP}}$	The share of sector i's GDP
W_i	Wage per worker in sector i
L_i	Number of workers in sector i
$W_i L_i$	Total wage bill in sector i
$WL = \sum_i W_i L_i$	Total wage bill in the economy
$\ell_i = \frac{W_i L_i}{WL}$	The share of the wage bill in sector i
$s_i^L = \frac{W_i L_i}{P_i Y_i} = \frac{\text{Total wage bill}_i}{\text{GDP}_i}$	The labor share in sector i
$s^L = \frac{WL}{Y} = \frac{\text{Total wage bill}}{\text{GDP}}$	The labor share in the economy
$\Gamma_i = \Gamma(N_i, I_i)$	The task content of production with regards to labor in sector i
γ_i^L	The comparative advantage schedules for labor in sector i
γ_i^K	The comparative advantage schedules for captial in sector i

The decomposition starts from the percent change in the wage bill normalized by population (Equation (AR1)). Since $\ln\left(\frac{W_t L_t}{N_t}\right)$ can be expressed as $\ln\left(Y_t \sum_i \chi_{it} s_{it}^L\right)$, Equation (AR1) can be decomposed as Equation (AR2);

$$
\begin{aligned}
&\ln\left(\frac{W_t L_t}{N_t}\right) - \ln\left(\frac{W_{t0} L_{t0}}{N_{t0}}\right) && \text{(AR1)}\\
&= \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t0}}{N_{t0}}\right) && \text{(AR2)}\\
&+ \ln\left(\sum_i \chi_{it} s^L_{it}\right) - \ln\left(\sum_i \chi_{it0} s^L_{it0}\right)\\
&= \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t0}}{N_{t0}}\right)\\
&+ \ln\left(\sum_i \chi_{it} s^L_{it}\right) - \ln\left(\sum_i \chi_{it0} s^L_{it}\right)\\
&+ \ln\left(\sum_i \chi_{it0} s^L_{it}\right) - \ln\left(\sum_i \chi_{it0} s^L_{it0}\right)\\
&\approx \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t0}}{N_{t0}}\right)\\
&+ \sum_i \frac{s^L_{it}}{\sum_j \chi_{jt0} s^L_{jt}} \left(\chi_{it} - \chi_{it0}\right)\\
&+ \ln\left(\sum_i \chi_{it0} s^L_{it}\right) - \ln\left(\sum_i \chi_{it0} s^L_{it0}\right)\\
&\approx \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t0}}{N_{t0}}\right) && \text{(AR3)}\\
&+ \sum_i \frac{s^L_{it}}{\sum_j \chi_{jt0} s^L_{jt}} \left(\chi_{it} - \chi_{it0}\right)\\
&+ \sum_i \ell_{it0} (\ln s^L_{it} - \ln s^L_{it0}) && \text{(AR4)}
\end{aligned}
$$

The first-order Taylor expansion of Term (AR4) yields Terms (AR6) and (AR7); Denote $(1-\sigma)(1-s^L_{it0})\left(\ln \frac{W_{it}}{W_{it0}} - \ln \frac{R_{it}}{R_{it0}} - g^A_{i,t0,t}\right)$ as Substitution$_{i,t0,t}$, we can rewrite Equation (AR5) as (AR8); Denote $\left(\ln s^L_{it} - \ln s^L_{it0}\right) -$ Substitution$_{i,t0,t}$ as ChangeTaskContent$_{i,t0,t}$, we can rewrite Equation (AR8) as (AR9).

$$
\begin{aligned}
&\approx \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t0}}{N_{t0}}\right) && \text{(AR5)}\\
&+ \sum_i \frac{s_{it}^L}{\sum_j \chi_{jt0} s_{jt}^L}\left(\chi_{it} - \chi_{it0}\right)\\
&+ \sum_i \ell_{it0}\Bigg[(1-\sigma)(1-s_{it0}^L)\bigg(\ln\frac{W_{it}}{W_{it0}} - \ln\frac{R_{it}}{R_{it0}} - g_{i,t0,t}^A\bigg) && \text{(AR6)}\\
&\qquad + \frac{1-s_{it0}^L}{1-\Gamma_{it0}}(\ln\Gamma_{it} - \ln\Gamma_{it0})\Bigg] && \text{(AR7)}\\
&\approx \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t0}}{N_{t0}}\right)\\
&+ \sum_i \frac{s_{it}^L}{\sum_j \chi_{jt0} s_{jt}^L}\left(\chi_{it} - \chi_{it0}\right)\\
&+ \sum_i \ell_{it0}\Bigg[\text{Substitution}_{i,t0,t}\\
&\qquad + \frac{1-s_{it0}^L}{1-\Gamma_{it0}}(\ln\Gamma_{it} - \ln\Gamma_{it0})\Bigg]\\
&\approx \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t0}}{N_{t0}}\right) && \text{(AR8)}\\
&+ \sum_i \frac{s_{it}^L}{\sum_j \chi_{jt0} s_{jt}^L}\left(\chi_{it} - \chi_{it0}\right)\\
&+ \sum_i \ell_{it0}\Bigg[\text{Substitution}_{i,t0,t}\\
&\qquad + \left(\ln s_{it}^L - \ln s_{it0}^L\right) - \text{Substitution}_{i,t0,t}\Bigg]\\
&\approx \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t0}}{N_{t0}}\right) && \text{(AR9)}\\
&+ \sum_i \frac{s_{it}^L}{\sum_j \chi_{jt0} s_{jt}^L}\left(\chi_{it} - \chi_{it0}\right)\\
&+ \sum_i \ell_{it0}\Bigg[\text{Substitution}_{i,t0,t}\\
&\qquad + \text{ChangeTaskContent}_{i,t0,t}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&\approx \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t0}}{N_{t0}}\right) \\
&+ \sum_i \frac{s_{it}^L}{\sum_j \chi_{jt0} s_{jt}^L} (\chi_{it} - \chi_{it0}) \\
&+ \text{Substitution}_{t0,t} \\
&+ \sum_i \ell_{it0} \left[\text{ChangeTaskContent}_{i,t0,t} \right]
\end{aligned}
$$

$\sum_i \ell_{it0}[\text{ChangeTaskContent}_{i,t0,t}]$ can be decomposed again into Equation (AR10), assuming that over five-year windows, an industry engages in either automation or the creation of new tasks but not in both activities.

$$
\begin{aligned}
\text{Displacement}_{t-1,t} &= \sum_{i \in \mathcal{I}} \ell_{i,t0} \min\left\{0, \frac{1}{5} \sum_{\gamma=t-2}^{t+2} \text{ChangeTaskContent}_{i,\gamma-1,\gamma}\right\} \\
\text{Reinstatement}_{t-1,t} &= \sum_{i \in \mathcal{I}} \ell_{i,t0} \max\left\{0, \frac{1}{5} \sum_{\gamma=t-2}^{t+2} \text{ChangeTaskContent}_{i,\gamma-1,\gamma}\right\}
\end{aligned}
\tag{AR10}
$$

To sum up, starting from Equation (AR1), it can be decomposed into 1) productivity, 2) composition, 3) substitution, 4) displacement, and 5) reinstatement effects.

$$
\begin{aligned}
&\ln\left(\frac{W_t L_t}{N_t}\right) - \ln\left(\frac{W_{t0} L_{t0}}{N_{t0}}\right) && \text{[Wage bill per capita]} \\
&\approx \ln\left(\frac{Y_t}{N_t}\right) - \ln\left(\frac{Y_{t0}}{N_{t0}}\right) && \text{[Productivity effect]} \\
&+ \sum_i \frac{s_{it}^L}{\sum_j \chi_{jt0} s_{jt}^L} (\chi_{it} - \chi_{it0}) && \text{[Composition effect]} \\
&+ \text{Substitution}_{t0,t} && \text{[Substitution effect]} \\
&+ \text{Displacement}_{t0,t} && \text{[Displacement effect (Automation)]} \\
&+ \text{Reinstatement}_{t0,t} && \text{[Reinstatement effect (New tasks)]}
\end{aligned}
\tag{AR11}
$$

C Appendix: Model

C.1 Households

The representative consumer consumes an aggregated continuum of final goods, with the mass of final goods assumed to be one for simplicity. It's also assumed that there is no disutility from the supply of labor. The utility function of the representative consumer takes the following form:

$$U = \left(\int_0^1 Y(i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}} \tag{13}$$

, where η represents the elasticity of substitution between final goods.

The representative consumer's budget constraint is as follows:

$$\int_0^1 P(i)Y(i)di = \int_0^1 \left(\int_{N-1}^{N} W_j l_j(i)dj + \int_{N-1}^{N} \psi m_j(i)dj + RK_i + \Pi_i \right) di \tag{14}$$

, where W_j, ψ, and R represent wage for labor conducting task j, robot price, and capital price, respectively.

C.2 Labor Share

A step-by-step process for this section is provided in Appendix D. We set an assumption related to robot and labor productivity for simple algebra in deriving the equilibrium in the model.

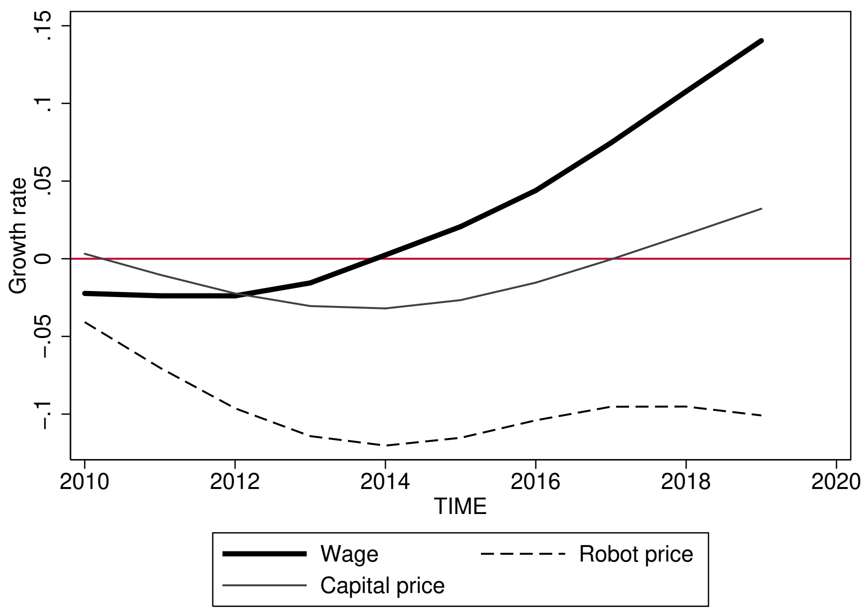
Assumption 1. $\psi < \frac{W_I}{\gamma_I}$

The above assumption implies that it is efficient to use a robot for task j below I. In other words, whenever firms have the technological capability to substitute labor with a robot, they would be inclined to do so. This is a reasonable assumption, especially considering that robot prices have significantly declined, while wages have seen a steady increase. Figure 6 illustrates these trends by depicting the 5-year growth rates of the respective prices.

Based on the Assumption 1 and by solving the firm's cost minimization problem, factor demands, the price for the aggregated task, and the marginal cost of firm i are derived as follows:

$$l_j(i) = 0, \quad \text{if } j \leq I \tag{15}$$

Figure 6: Prices in a 5-year growth rate



$$l_j(i) = \gamma_j^{\zeta-1} \left(\frac{W_j}{P_T}\right)^{-\zeta} T(i), \text{ if } j > I \tag{16}$$

$$m_j(i) = \left(\frac{\psi}{P_T}\right)^{-\zeta} T(i), \text{ if } j \leq I \tag{17}$$

$$m_j(i) = 0, \text{ if } j > I \tag{18}$$

$$T(i) = \left(\frac{P_T}{MC(i)}\right)^{-\sigma} Y(i) \tag{19}$$

$$K(i) = \left(\frac{R}{MC(i)}\right)^{-\sigma} Y(i) \tag{20}$$

$$P_T = \left[(I - N + 1)\psi^{1-\zeta} + \int_I^N \left(\frac{W_j}{\gamma_j}\right)^{1-\zeta} dj\right]^{\frac{1}{1-\zeta}} \tag{21}$$

$$MC(i) = \left[P_T^{1-\sigma} + R^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \tag{22}$$

$$W_j l_j(i) = \left(\frac{W_j}{\gamma_j}\right)^{1-\zeta} \cdot P_T^{\zeta} \cdot T_i \tag{23}$$

, where P_T and MC_i represent the price for the aggregated task and marginal cost of firm i, respectively.

D Appendix: Detailed Model Derivations

D.1 Environment

There is a representative household with utility function in Equation (24):

$$U = \left(\int_0^1 Y(k)^{\frac{\eta-1}{\eta}} dk \right)^{\frac{\eta}{\eta-1}}. \tag{24}$$

There are infinite number of identical firms i with production functions in Equation (27) and (28):

$$t_j(i) = m_j(i) + \gamma_j l_j(i) \ \text{ if } j \leq I \tag{25}$$
$$t_j(i) = \gamma_j l_j(i) \ \text{ if } j > I \tag{26}$$
$$T(i) = \left(\int_{N-1}^{N} t_j(i)^{\frac{\zeta-1}{\zeta}} dj \right)^{\frac{\zeta}{\zeta-1}} \tag{27}$$
$$Y(i) = \left(T(i)^{\frac{\sigma-1}{\sigma}} + K(i)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{28}$$

By Assumption 1, Equation (25) simplifies to Equation (29). Without this assumption, the algebra becomes too complex to yield a closed-form solution. The implication of this assumption is that whenever robot operation is technically feasible, firms opt for robots over labor. This is because, according to Assumption 1, the cost of using a robot is lower than the cost of labor for unit of production.

$$t_j(i) = m_j(i) \ \text{ if } j \leq I \tag{29}$$

D.2 Step 1: derive P_T, and optimal inputs for robot* and labor*

We derive P_T, the price for an aggregated task, $T(i)$, by solving the cost minimization problem. We assume perfectly competitive market.

$$\min \text{cost(i) for } T(i) \text{ s.t. Equation(29), (26), and (27)}$$

$$\Rightarrow \min \int_{N-1}^{I} \psi m_j dj + \int_{I}^{N} w_j l_j dj \ \text{ s.t. } \left(\int_{N-1}^{I} m_j^{\frac{\zeta-1}{\zeta}} dj + \int_{I}^{N} (\gamma_j l_j)^{\frac{\zeta-1}{\zeta}} dj \right)^{\frac{\zeta}{\zeta-1}} = T(i)$$

$\Rightarrow$ This finds optimal inputs for robot* and labor* to produce T(i)

$\Rightarrow$ Specifically, letting T(i)=1 means the minimization solution is the price for T(i), P_T :

$$\Rightarrow P_T = \left[(I - N + 1)\psi^{1-\zeta} + \int_{I}^{N} \left(\frac{w_j}{\gamma_j} \right)^{1-\zeta} dj \right]^{\frac{1}{1-\zeta}} \tag{30}$$

D.3 Step 2: find optimal inputs for $T(i)$ and $K(i)$

Next, we find optimal inputs for $T(i)$ and $K(i)$ to produce $Y(i)$.

$$
\begin{aligned}
&\min \text{cost(i) for } Y(i) \text{ s.t. Equation(28)} \\
\Leftrightarrow &\min P_T \cdot T(i) + R \cdot K(i) \text{ s.t. Equation(28)} \\
\Rightarrow &\text{This finds optimal inputs for T(i)* and K(i)* to produce Y(i)} \\
\Rightarrow &\text{Specifically, the minimization solution is the minimum cost for producing } Y(i) \\
\Rightarrow &\begin{cases}
T(i)^* = Y(i)P_T^{-\sigma} \\
K(i)^* = Y(i)R^{-\sigma} \\
\text{Cost for } Y(i) = Y(i)\left[P_T^{1-\sigma} + R^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \\
\qquad\quad = Y(i) \times \text{AC} \\
\qquad\quad = Y(i)
\end{cases}
\end{aligned}
$$

We let $\left[P_T^{1-\sigma} + R^{1-\sigma}\right]^{\frac{1}{1-\sigma}} = 1$ as a numeraire. This numeraire significantly simplifies the algebraic complexity. Since we let AC$= 1$, MC is also one.

D.4 Step 3: find a demand function for $Y(i)$

Next, we find a demand function for $Y(i)$ by minimizing consumption cost.

$$
\begin{aligned}
&\min \text{cost for consumption s.t. Equation(24)} \\
\Leftrightarrow &\min \int_0^1 P(i)Y(i)di \text{ s.t. Equation(24)} \\
\Rightarrow &\text{Specifically, this yields a demand function for } Y(i) \\
\Leftrightarrow &Y(i) = \left(\frac{P(i)}{\mathbb{P}}\right)^{-\eta} \text{, where } \mathbb{P} \equiv \left[\int_0^1 P(i)^{1-\eta}di\right]^{\frac{1}{1-\eta}}
\end{aligned}
$$

D.5 Step 4: find firm(i)'s profit

The final goods market is the monopolistic competition that allows firms' positive profit. Until now, we know two things: (1) a demand function for $Y(i)$, and (2) the minimum cost for producing $Y(i)$. Firm's profit maximization problem yields:

$$
\begin{aligned}
P(i)^* &= \frac{\eta}{\eta - 1} \\
\Rightarrow \Pi(i) &= \frac{1}{\eta - 1}Y(i)^*
\end{aligned}
$$

Meanwhile, we naturally get optimal $Y(i)$ as below, but this is redundant for this paper.

$$Y(i)^* = \left(\frac{\eta}{(\eta-1)\mathbb{P}}\right)^{-\eta}, \text{ where } \mathbb{P} \equiv \left[\int_0^1 P(i)^{1-\eta} di\right]^{\frac{1}{1-\eta}}$$

D.6 Step 5: derive the labor cost for producing optimal $Y(i)$

In Step 1, we already found optimal inputs of $l_j(i)$ to produce $T(i)$. Therefore we can also know the optimal labor cost at task j for firm i to produce $T(i)$.

$$l_j(i)^* = \left(\frac{W_j(i)}{\gamma_j P_T}\right)^{-\zeta} \gamma_j^{-1} T(i) \tag{31}$$

$$\Rightarrow W_j(i) l_j(i)^* = \left(\frac{W_j(i)}{\gamma_j}\right)^{1-\zeta} P_T^{\zeta} T(i)$$

And we also derived optimal $T(i)$ while in Step 2: $T(i)^* = Y(i) P_T^{-\sigma}$. Plugging in this to the equation above,

$$W_j(i) l_j(i)^* = \left(\frac{W_j(i)}{\gamma_j}\right)^{1-\zeta} P_T^{\zeta-\sigma} Y(i)$$

Therefore, the optimal labor cost for firm i to produce $Y(i)$ by using every task from I to N is:

$$\begin{aligned}\int_I^N W_j(i) l_j(i)^* dj &= \int_I^N \left(\frac{W_j(i)}{\gamma_j}\right)^{1-\zeta} P_T^{\zeta-\sigma} Y(i) dj \\ &= \int_I^N \left(\frac{W_j(i)}{\gamma_j}\right)^{1-\zeta} dj \cdot P_T^{\zeta-\sigma} Y(i)\end{aligned}$$

D.7 Step 6: derive an expression for labor share

Until now, we have figured out (1) labor cost, (2) total cost, and (3) profit. Putting all together, we find labor share. Since we prefer not to focus on $\frac{\eta-1}{\eta}$, we move this term to the left-hand side.

$$
\begin{aligned}
S_L(i) &= \frac{\text{Labor cost(i)}}{\text{Total cost(i)} + \text{Profit(i)}} = \frac{\text{Labor cost(i)}}{Y(i) + \frac{1}{\eta-1}Y(i)} \\
&= \frac{\eta-1}{\eta}\frac{\text{Labor cost(i)}}{\text{Total cost(i)}} \\
\Leftrightarrow \frac{\eta}{\eta-1}S_L(i) &= \frac{\text{Labor cost(i)}}{\text{Total cost(i)}} \\
&\equiv S_L^f(i)
\end{aligned}
$$

After substituting the expressions for Labor cost(i) and Total cost(i) that we derived earlier, we finally construct a detailed expression for $S_L^f(i)$.

$$
\begin{aligned}
S_L^f(i) &= \frac{\text{Labor cost(i)}}{\text{Total cost(i)}} \\
&= \frac{\int_I^N W_j(i)l_j(i)dj}{Y(i)} \\
&= \frac{\int_I^N W_j(i)l_j(i)dj}{P_T T(i) + RK(i)} \\
&= \frac{\int_I^N \left(\frac{W_j(i)}{\gamma_j}\right)^{1-\zeta} dj \cdot P_T^{\zeta-\sigma} Y(i)}{P_T^{1-\sigma}Y(i) + R^{1-\sigma}Y(i)} \\
&= \frac{\int_I^N \left(\frac{W_j(i)}{\gamma_j}\right)^{1-\zeta} dj}{P_T^{1-\zeta}} \frac{P_T^{1-\sigma}}{P_T^{1-\sigma} + R^{1-\sigma}} \\
&\text{, where } P_T \equiv \left[(I-N+1)\psi^{1-\zeta} + \int_I^N \left(\frac{W_j}{\gamma_j}\right)^{1-\zeta} dj\right]^{\frac{1}{1-\zeta}}
\end{aligned}
$$

E Appendix: Adjusted Penetration of Robots

APR is defined as in Equation (32):

$$
\begin{aligned}
\text{APR}_{i,(t5,t1)} &\equiv \frac{M_{i,t5} - M_{i,t1}}{L_{i,2005}} - \frac{Y_{i,t5} - Y_{i,t1}}{Y_{i,t1}} \frac{M_{i,t1}}{L_{i,2005}} && (32) \\
&= \Big(\frac{M_{i,t5} - M_{i,t1}}{M_{i,t1}} - \frac{Y_{i,t5} - Y_{i,t1}}{Y_{i,t1}}\Big) \frac{M_{i,t1}}{L_{i,2005}} && (33) \\
&= \big(g_M - g_Y\big) \frac{M_{i,t1}}{L_{i,2005}} && (34)
\end{aligned}
$$

, where i is the industry sector (country $\times$ industry in our case), and t5 is 5-year after t1. M is the number of robots (stock), L is the number of employees, Y is value-added (in real terms).

Acemoglu and Restrepo (2020) employs APR as a proxy for $d(I - N + 1)$ primarily because the term dI encapsulates the theoretical concept of a 'pure direction of automation,' which is abstract and not directly observable in empirical settings. The observable growth rate of the number of robots is not a suitable proxy for dI since it reflects an equilibrium outcome in real-world scenarios. Given this, Acemoglu and Restrepo (2020) proposes APR to effectively serve as a proxy for $d(I - N + 1)$.

The second term in Equation (34), $-g_Y$, serves to measure the 'penetration' of robots. In other words, if the growth rate of robots exceeds that of value-added, they interpret this as a positive penetration. This penetration equates to $I - N + 1$ in their terminology, which represents the length between $N - 1$ and I. The inclusion of the second term, (34), $-g_Y$, in Equation (34) is necessary for the following reason: Suppose there is an economic boom. In such a scenario, the growth rate of robot adoption would likely surge, while $d(I - N + 1)$ remains unchanged. Therefore, they adjust the growth rate of robot adoption by subtracting the growth rate of value-added, g_Y.

The APR represents the 5-year growth rate of robots adjusted by labor input and the value-added within a given sector. Multiplication by $\frac{M_{i,t1}}{L_{i,2005}}$ is necessary as the raw number of robots does not adequately represent their definition of automation. Consider, for instance, that the IFR began collecting data in many countries starting in 2004. A change from 1 robot to 100 robots between 2004 and 2005 would represent a growth rate of 9900%, whereas an increase from 100 to 200 robots between 2005 and 2006 would only reflect a 100% growth rate. These rates are not useful because the number of machines increased by the same amount (100) in both cases. The term $\frac{M_{i,t1}}{L_{i,2005}}$ is introduced to adjust for this discrepancy. Suppose $L_{i,2005} = 100$. In 2005, $g_M \times \frac{M_{i,t1}}{L_{i,2005}}$ equals 99%, and in 2006, it amounts to 100%, which makes them comparable. The underlying idea is that the 5-year difference in the number of machines across countries and industries is not directly comparable; they needed to normalize it by dividing by the number of employees.[16]

F Appendix: Capital Price

In our paper, we utilize the replicated values for capital price from Karabarbounis and Neiman (2014) (hereinafter KN). To calculate this, we initially require the investment price, which the KLEMS data provides, including industry variations.

[16]Instead of dividing by $L_{i,2005}$, dividing by 'quantity' would be more accurate, but it will not change the results significantly.

It's important to note that we don't directly observe the capital price, which represents the *usage* cost of one unit of capital. We do, however, observe the investment price, which signifies the *purchase* cost of one unit of capital. In accordance with the theory of investment by Jorgenson (1963), we can calculate the capital price as follows:

$$R_t = \xi_{t-1}(1 + i_t) - \xi_t(1 - \delta_t) \tag{35}$$

$$R_t = \xi_t \left(\frac{1}{\beta} - 1 + \delta \right) \tag{36}$$

In this Equation (35), R represents the capital price, ξ is the investment price, i is the interest rate, and δ is the depreciation rate. All values are expressed in real terms. This equation signifies that investors are indifferent between paying a *usage* cost for capital (R_t) and *purchasing* capital, paying interest, and then selling the depreciated capital at a later date.

To simplify Equation (35) into the form presented in Equation (36), we follow a specific process. This involves the assumption of a constant interest rate, i, and approximating $1 + i$ as $\frac{1}{\beta}$. Equation (36), as employed by KN in their KLEMS version of the capital price variable, assumes a depreciation rate of 10%. This rate aligns closely with the 10.8% rate assumed by Stehrer et al. (2019), an official KLEMS document. Throughout this paper, we strictly adhere to the approach by KN.[17]

G Appendix: KLEMS Data and Capital Cost

G.1 KLEMS Data

Aside from the IFR dataset, the O*NET dataset, and Robot Price, we will use data from KLEMS.[18] All nominal values are converted to real values through division by the chain-linked price index provided by KLEMS (VA_PI), following the methodology implemented by Karabarbounis and Neiman (2014).

KLEMS comes in two different versions: one follows national accounts, and the other follows growth accounts. The main difference between these versions is that the national accounts allow room for a markup greater than one, while the growth accounts do not. The latter assumes that the sum of labor cost and capital cost equals

[17] It is important to note that KN employed a β value of 0.909 (corresponding to an interest rate, $i = 0.100$), reflecting the high real interest rates prevalent in the 1970s. In contrast, our study adopts a β of 0.988 (equivalent to $i = 0.012$), derived from averaging the real interest rates from 2005 to 2019 across ten countries. However, the specific value of β does not influence the regression outcomes in our analysis, as we focus on the growth rate of the capital price, which effectively cancels out the impact of β.

[18] KLEMS: EU level analysis of capital (K), labour (L), energy (E), materials (M) and service (S) inputs.

the value-added, implying that the markup is exactly one. As allowing for a markup is critical for our analysis, we use the national accounts when using KLEMS.

KLEMS shares similar characteristics with OECD STAN in terms of many national account variables at a country-industry-year level. Table 5 presents descriptive statistics. Predominantly, the values for OECD STAN and KLEMS are comparable, albeit not identical. In some instances, the values are in fact identical. This alignment is a result of collaborative projects aimed at fostering more consistent values between the two.

Table 5: Descriptive Statistics

Country	WL (labor comp)		RK (capital comp)		Value added		Labor Share	
	STAN	KLEMS	STAN	KLEMS	STAN	KLEMS	STAN	KLEMS
USA	867,789	851,834	292,456	308,662	1,647,140	1,593,719	52.85	53.60
DEU	366,787	366,806	104,117	104,034	569,189	570,196	64.67	64.57
SWE	256,507	256,540	115,040	124,370	502,728	502,728	51.17	51.18
DNK	219,076	226,496	199,337	220,713	410,478	426,533	55.33	54.87
ITA	140,568	140,568	57,107	54,924	253,368	253,353	55.60	55.60
FRA	135,093	135,098	52,379	41,244	226,181	226,181	59.74	59.74
GBR	110,603	109,347	26,230	25,535	171,778	170,498	64.45	64.19
AUT	28,106	29,959	9,427	12,090	51,011	54,254	55.22	55.31
FIN	17,100	17,979	7,512	7,204	33,112	34,848	51.91	51.85
PRT	11,537	12,897	3,166	3,166	20,575	23,030	56.06	55.99
Total	215,317	214,753	86,677	90,194	388,556	385,534	56.75	56.69

G.2 Capital Cost

The KLEMS data has one limitation: it lacks RK (rental cost for capital stock) and profit (operating surplus and mixed income). If either RK or Profit were available, we could deduce the other because Value-added is calculated as WL + RK + Profit. Regrettably, the absence of both presents a challenge. This issue is addressed by utilizing OECD STAN data.

In particular, the KLEMS dataset lacks RK. It does include I_GFCF (Investment in Gross Fixed Capital Formation) and K_GFCF (Capital Stock of Gross Fixed Capital Formation), but these do not provide the necessary RK information. I_GFCF represents the net investment in fixed assets —a flow metric indicating capital goods investment. K_GFCF, on the other hand, denotes the total value of all fixed assets available for production —a stock variable. Consequently, although RK can be estimated based on K_GFCF, this method lacks precision. This is because K_GFCF represents the purchase cost, not the rental cost. To convert the purchase cost into rental cost, the real interest rate and depreciation rate as shown in Equation (35) are required. Notably, the depreciation rate requires numerous assumptions, and we lack this information.

A pertinent question arises: why not use OECD STAN initially, instead of KLEMS? The response lies in the fact that OECD STAN does not contain R (capital price) data. Therefore, we resort to using R obtained from KLEMS. However, integrating this with other data from OECD STAN, particularly wage variables, poses complications. Furthermore, STAN does not provide industry-specific Producer Price Index (PPI). To enhance the accuracy of our analysis, we prefer to use industry-specific PPI, specifically the VA_PI variable from KLEMS.

Hence, an alternative approach is to employ RK from OECD STAN. This is feasible because the value-added and WL (labor compensation) figures are nearly identical in both STAN and KLEMS datasets (as illustrated in Figures 8 in Appendix L). Consequently, it is highly probable that RK, along with operating surplus and mixed income, are consistent across both KLEMS and STAN. Therefore, in this paper, we assume that the markups in KLEMS and STAN are identical, denoted by $\frac{\text{Value-added}}{\text{WL+RK}}$. Based on this assumption, we are able to recover RK for KLEMS as below:

$$\frac{\text{Value-added}_{\text{STAN}}}{\text{WL}_{\text{STAN}} + \text{RK}_{\text{STAN}}} = \frac{\text{Value-added}_{\text{KLEMS}}}{\text{WL}_{\text{KLEMS}} + \mathbf{RK}_{\mathbf{KLEMS}}}.$$

H Appendix: Estimation of S_M^T

Denote Ψ, M, W, and L as robot price, number of robots, wage, and employment, respectively. Then S_M^T can be expressed as follows:

$$\begin{aligned} S_M^T &= \frac{\Psi M}{\Psi M + WL} \\ &= \frac{1}{1 + \frac{WL}{\Psi M}} \\ &= \frac{1}{1 + \left(\frac{M}{L}\right)^{-1} \frac{W}{\Psi}} \end{aligned}$$

Unfortunately, the International Federation of Robotics (IFR) provided robot prices in the form of an average unit price until 2009 and discontinued this practice thereafter. Access to robot price information prior to 2009 is also restricted for those who have purchased IFR data after this point. Nonetheless, Fernandez-Macias et al. (2021) offers a comprehensive method to approximate the missing price information from the IFR dataset. Specifically, they provide values for M/L as well as Ψ. We supplement these data with wage information from the OECD STAN database to complete the S_M^T value in the equation above.

It is important to note that the equipment cost for robots is estimated to constitute around 33.04% of the total robot costs[19], covering elements like operation, training, software, maintenance, and disposal (Zhao et al., 2021). The figures provided by Fernandez-Macias et al. (2021) pertain only to equipment cost. Therefore, we have accounted for this information accordingly.

H.1 An Alternative Approach to Estimating the S_M^T

Let's assume labor cost to be 100 without loss of generality. According to KLEMS data, the rental cost for OMach is recorded as 13.595. But it's important to note that OMach encompasses not just robots but also a range of other items, including equipment, machinery, engines, and turbines (Stehrer et al., 2019; Gouma and Timmer, 2013). Therefore, the challenge is to determine the share of robots within the broader category of OMach. The most reliable approach we can consider involves utilizing UN Comtrade data, which offers information about import and export values by detailed commodity categories. By calculating the total export values of commodities corresponding to OMach,[20] and separately calculating the total export values of HS Code 8479 (which pertains to robots),[21] we find that the ratio between these values is 13.595 : 0.71. In brief, the ratio between labor cost, OMach cost, and robot cost is 100 : 13.595 : 0.71.

The equipment cost for robots is estimated to be around 33.04% of the total robot costs (Zhao et al., 2021), and the UN Comtrade estimate of 0.71 corresponds to the equipment cost. Therefore, the total cost of the robot amounts to $0.71/0.33 = 2.149$. Hence, S_M^T is estimated to be 2.104%.[22]

I Appendix: Estimation of σ and ζ

Given that $S_K^f > 0$ and the coefficient for $d \ln R$ is negative, we can infer that $\sigma < 1$. Further, by substituting the value $S_K^f = 0.225$ that we obtained from the data, we calculate $\sigma = 0.527$, as illustrated in Equation (37). We conduct a Wald test on the null hypothesis that $\sigma = 0$ and find that it can be rejected at the 0.05 significance level. The confidence interval for σ is $(0.253, 0.801)$. Consequently, we can conclude with

[19] $33.04\% = 35.73\% \times (1 - 0.075)$, where 0.075 represents taxes, transactions, and after-sales fees. The cost share of robot equipment accounts for 35.73% of the total cost for using robots, as estimated by Zhao et al. (2021).

[20] HS Classification 84 excluding 8401, 8402, 8403, 8404, 8405, 8429, 8440, 8443, 8470, 8471, and 8472.

[21] Machinery and mechanical appliances; having individual functions, n.e.c. in this chapter.

[22] $2.104\% = \frac{2.149}{2.149+100}$

confidence that σ lies within the range of 0 to 1 (gross complement).

$$-\underbrace{S_K^f}_{0.225}(1-\sigma) = \underbrace{\alpha_5}_{-0.00029} \tag{37}$$

$$\Rightarrow \sigma = 1 + \frac{\alpha_5}{S_K^f} \tag{Sigma}$$

The derivation of the value for ζ proceeds as follows. From Equation (7), utilizing coefficients α_3 and α_5, we arrive at Equation (Zeta).

$$\zeta = 1 - \frac{\alpha_3 + \alpha_5 S_L^T}{1 - S_L^T} \tag{Zeta}$$

As demonstrated earlier in Section 6.3, we estimate S_L^T to be 0.972. Upon substituting $S_L^T = 0.972$ into Equation (Zeta), we obtain an estimate for ζ of 0.833. We then conduct a Wald test on the null hypothesis that $\zeta = 0$ and find it cannot be rejected at the 0.05 significance level. Specifically, the confidence interval is from -0.071 to 1.838. Consequently, we cannot draw any definitive conclusions about ζ.

J Appendix: Estimation of the Elasticity of Substitution between Labor and Non-robot Capital

The condition $\sigma < 1$ indirectly confirms that capital and labor are gross complementary, a result that aligns with the findings reported by Glover and Short (2020). Conversely, this result contradicts the hypothesis of gross substitutability ($\sigma > 1$) posited by Karabarbounis and Neiman (2014) (henceforth KN). We clarify that the term σ in our general equilibrium model does not align exactly with the definition of σ in the work of KN as well as Glover and Short (2020). The divergence stems from our model's distinction between robots and non-robot capital. Specifically, in our model, σ represents the elasticity of substitution between 'non-robot capital' and 'aggregated tasks', where the latter encompasses both robot and labor inputs.

Hence, in this subsection, we introduce the elasticity of substitution between labor and non-robot capital, denoted by μ, a measure that closely aligns with the findings of both KN and Glover and Short (2020). The solution for μ is given in Equation (38), and its derivation can be found in Appendix K.

$$\mu \equiv \frac{d\left(\frac{L}{K}\right)}{d\left(\frac{R}{\mathbb{W}}\right)} \frac{\frac{R}{\mathbb{W}}}{\frac{L}{K}} \text{ , where} \tag{38}$$

$$d\left(\frac{L}{K}\right) = \left(\frac{\mathbb{W}_1}{R_1}\right)^{-\sigma} \left[\frac{S_M^T}{1 - S_M^T}\left(\frac{\mathbb{W}_0}{\mathbb{W}_1}\right)^{1-\zeta} + 1\right]^{\frac{\zeta-\sigma}{1-\zeta}} - \left(\frac{\mathbb{W}_0}{R_0}\right)^{-\sigma} \left[\frac{S_M^T}{1 - S_M^T} + 1\right]^{\frac{\zeta-\sigma}{1-\zeta}}$$

$$\frac{L}{K} = \left(\frac{\mathbb{W}_0}{R_0}\right)^{-\sigma} \left[\frac{S_M^T}{1 - S_M^T} + 1\right]^{\frac{\zeta-\sigma}{1-\zeta}}$$

$$\Rightarrow \ \mu = \sigma \ \text{ if } S_M^T = 0.$$

Differentiating Equation (38) is infeasible. However, we can employ numerical approximation to estimate μ. We use actual $\mathbb{W}$ and R values from the dataset (all possible combinations of these), along with $\sigma = 0.527$. We introduce small random variations to each $\mathbb{W}$ and R and consider scenarios where $|\Delta \frac{R}{\mathbb{W}}|$ is approximately 0.01. These values are then plugged into Equation (38) to obtain an approximated μ.

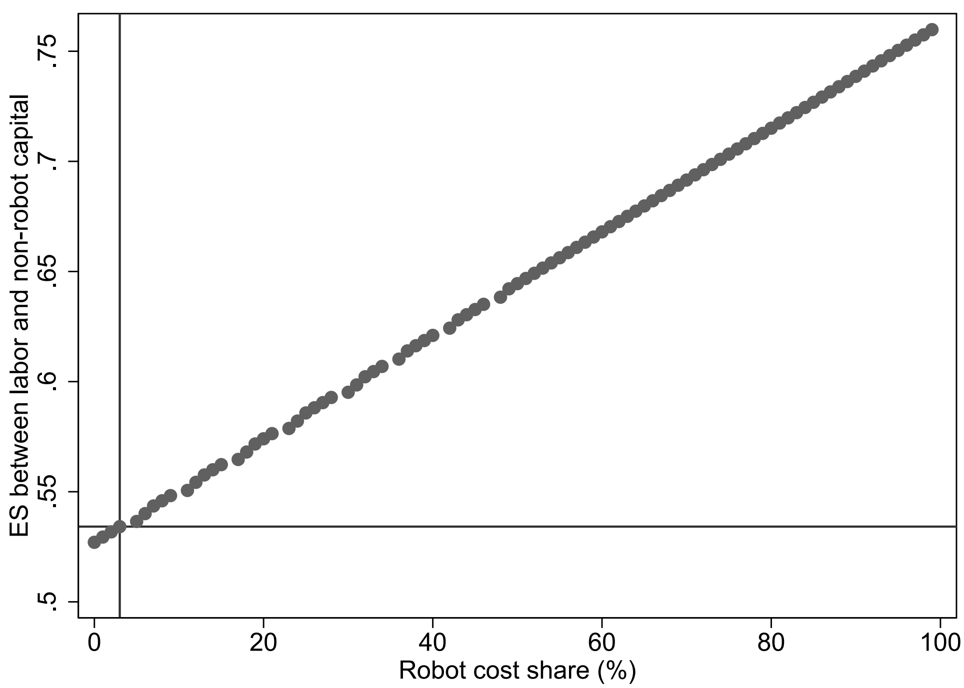
Panel (a) of Figure 7 displays the approximation results. When S_M^T is zero, we find that $\mu = \sigma = 0.527$. This stage indicates a complete absence of robot tasks, with all tasks being performed by labor. When $S_M^T = 2.813\%$, which corresponds to our estimate presented in Section 6.3, we obtain $\mu = 0.534$. Even if $S_M^T = 100\%$, μ does not exceed one. Consequently, we argue that in the context of the KN model, the elasticity of substitution between labor and non-robot capital closely approximates σ. Our analysis suggests that μ ranges between 0.527 and 0.574, supporting the idea of a gross complementary relationship between the two. In the future, as automated robots assume a greater share of tasks, the elasticity of substitution between labor and non-robot capital may rise. However, making accurate predictions about this trend necessitates more comprehensive research.

The above estimation of μ is contingent upon the value of $\zeta = 0.883$, which is our point estimate as derived in Section 6.4. However, the confidence interval for ζ varies: it spans from -0.071 to 1.838. To demonstrate the robustness of our μ estimate, we examine its sensitivity across a wide range of ζ values. This analysis is presented in Panel (b) of Figure 7. Within the ζ range of 0 to 1.838, μ varies between 0.518 and 0.551, confirming the robustness of our μ estimation.

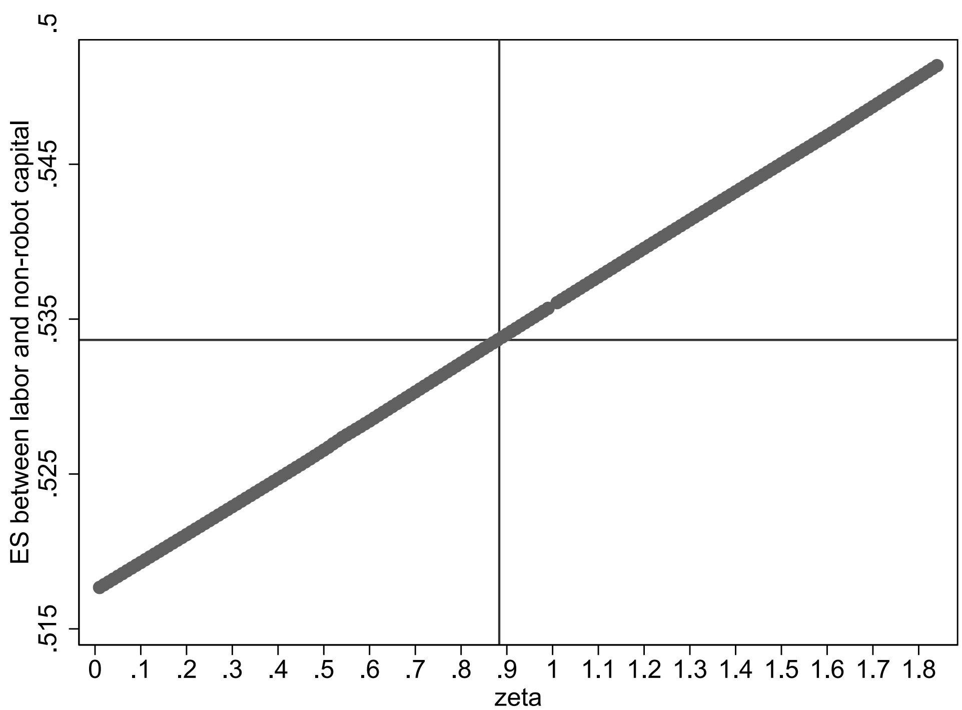
Recent research underscores the importance of quantifying this elasticity of substitution between labor and capital, as highlighted by Martinez (2018), Oberfield and Raval (2021), and Zhang (2023). Many studies report an elasticity less than one, endorsing the concept of gross complementarity. However, Piketty and Zucman (2014) suggest the potential for gross substitutability. They observed an escalating capital-output ratio and argued that this trend could consistently account for the declining

Figure 7: Elasticity of Substitution between Labor and Non-robot Capital

(a) Fixing ζ to be 0.883; Moving S_M^T



(b) Fixing S_M^T to be 2.813%; Moving ζ



labor share if the elasticity of substitution between labor and capital exceeds one —a claim our estimates do not corroborate.

Our finding also does not support the hypothesis proposed by Karabarbounis and Neiman (2014), who argue that the falling price of capital accounts for half of the recent decline in labor share. For their argument to hold, the elasticity of substitution between labor and capital must be greater than one (gross substitutes). They directly measured the correlation between the trend of capital price and labor share without using instrumental variables.

In contrast, Glover and Short (2020) reached a different conclusion, that of gross complements, by using cross-country variation with instrumental variables. They argue that correcting for bias is critical when estimating the correlation between the capital price and labor share. Our paper addresses omitted variable bias using a control function approach. We regress automation, the emergence of new tasks, wages, and robot price, along with capital price, on labor share, believing that this approach corrects for omitted variable bias. Our study supports Glover and Short (2020).

K Appendix: Derivation of μ

Let μ denote the elasticity of substitution between labor and non-robot capital. The concept of elasticity of substitution formally defines μ as follows:

$$\mu \equiv \frac{d\left(\frac{L}{K}\right)}{d\left(\frac{R}{\mathbb{W}}\right)} \frac{\frac{R}{\mathbb{W}}}{\frac{L}{K}}. \tag{39}$$

To proceed, we must express L and K in terms of W and R, respectively. Equation (31), derived in Appendix D.6, provides the formulation for L as follows:

$$\begin{aligned} l_j(i)^* &= \left(\frac{W_j(i)}{\gamma_j P_T}\right)^{-\zeta} \gamma_j^{-1} T(i) \\ \Rightarrow L &= \int_I^N l_j(i)^* dj \\ &= \int_I^N \left(\frac{W_j(i)}{\gamma_j P_T}\right)^{-\zeta} \gamma_j^{-1} T(i) dj. \end{aligned} \tag{40}$$

We introduce a parameter β_j to serve as a weight for the wage distribution corresponding to each worker, indexed by j. Utilizing β_j enables us to establish a representative measure for wages, $\mathbb{W}$.

$$W_j \equiv \beta_j \mathbb{W} \tag{41}$$

Consequently, Equation (40) can be restructured to yield Equation (42). To streamline the notation, we define $A = \int_I^N \gamma_j^{\zeta-1}\beta_j^{-\zeta}dj$.

$$L = \int_I^N \gamma_j^{\zeta-1}\beta_j^{-\zeta}dj \ \cdot T(i)\Big(\frac{\mathbb{W}}{P_T}\Big)^{-\zeta} \tag{42}$$

$$= A \cdot T(i)\Big(\frac{\mathbb{W}}{P_T}\Big)^{-\zeta} \tag{43}$$

We have derived $T(i)$ in Appendix D.3 and P_T in Appendix D.2. For the sake of clarity, we restate these formulations here:

$$T(i) = Y(i)P_T^{-\sigma}$$

$$P_T = \left[(I - N + 1)\psi^{1-\zeta} + \int_I^N \left(\frac{w_j}{\gamma_j}\right)^{1-\zeta} dj\right]^{\frac{1}{1-\zeta}}$$

By substituting $T(i)$ and P_T into Equation (43),

$$\begin{aligned} L =& A \cdot Y(i)P_T^{-\sigma}\Big(\frac{\mathbb{W}}{P_T}\Big)^{-\zeta} \\ =& A \cdot Y(i)P_T^{\zeta-\sigma}\mathbb{W}^{-\zeta} \\ =& A \cdot Y(i)\left[(I - N + 1)\psi^{1-\zeta} + \int_I^N \left(\frac{w_j}{\gamma_j}\right)^{1-\zeta} dj\right]^{\frac{\zeta-\sigma}{1-\zeta}} \mathbb{W}^{-\zeta}. \end{aligned}$$

$(I - N + 1)\psi^{1-\zeta}$ and $\int_I^N \left(\frac{w_j}{\gamma_j}\right)^{1-\zeta} dj$ correspond to the cost share of robots and human labor, respectively. Consequently, we can reformulate these expressions as follows:

$$(I - N + 1)\psi^{1-\zeta} \equiv S_M^T$$

$$\int_I^N \left(\frac{w_j}{\gamma_j}\right)^{1-\zeta} dj \equiv S_L^T$$

Therefore, L can be reformulated as follows:

$$L = A \cdot Y(i)\Big[S_M^T + S_L^T\Big]^{\frac{\zeta-\sigma}{1-\zeta}}\mathbb{W}^{-\zeta}$$

$$= A \cdot Y(i)\left[\frac{S_M^T}{S_L^T} + 1\right]^{\frac{\zeta-\sigma}{1-\zeta}}\mathbb{W}^{-\zeta} \tag{44}$$

We derived the optimal value of K in Appendix D.3, given by $K = Y(i)R^{-\sigma}$. Consequently, we complete our derivation of $\frac{L}{K}$ as follows:

$$\begin{aligned}
\frac{L}{K} =& \frac{A \cdot Y(i)\left[\frac{S_M^T}{S_L^T}+1\right]^{\frac{\zeta-\sigma}{1-\zeta}} \mathbb{W}^{-\zeta}}{Y(i)R^{-\sigma}} \\
=& \frac{A \cdot \left[\frac{S_M^T}{S_L^T}+1\right]^{\frac{\zeta-\sigma}{1-\zeta}} \mathbb{W}^{-\zeta}}{R^{-\sigma}}
\end{aligned}$$

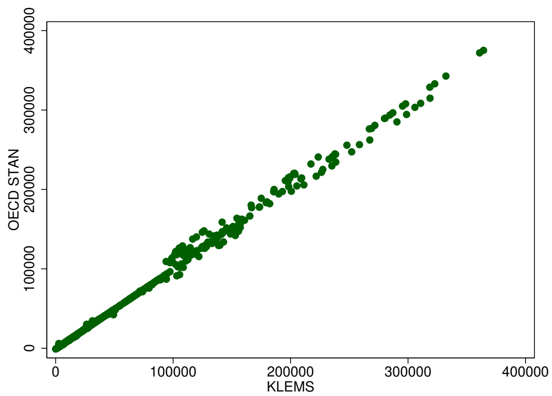
Thus, the expression for $d\left(\frac{L}{K}\right)/\frac{L}{K}$ is given below. This concludes our derivation of μ.

$$\frac{d\left(\frac{L}{K}\right)}{\frac{L}{K}} = \frac{\left(\frac{\mathbb{W}_1}{R_1}\right)^{-\sigma}\left[\frac{S_M^T}{1-S_M^T}\left(\frac{\mathbb{W}_0}{\mathbb{W}_1}\right)^{1-\zeta}+1\right]^{\frac{\zeta-\sigma}{1-\zeta}} - \left(\frac{\mathbb{W}_0}{R_0}\right)^{-\sigma}\left[\frac{S_M^T}{1-S_M^T}+1\right]^{\frac{\zeta-\sigma}{1-\zeta}}}{\left(\frac{\mathbb{W}_0}{R_0}\right)^{-\sigma}\left[\frac{S_M^T}{1-S_M^T}+1\right]^{\frac{\zeta-\sigma}{1-\zeta}}}$$

L Appendix: Tables and Figures

Figure 8: Values by Country, Sector, and Year

(a) Value-added



(b) Labor Compensation[23]

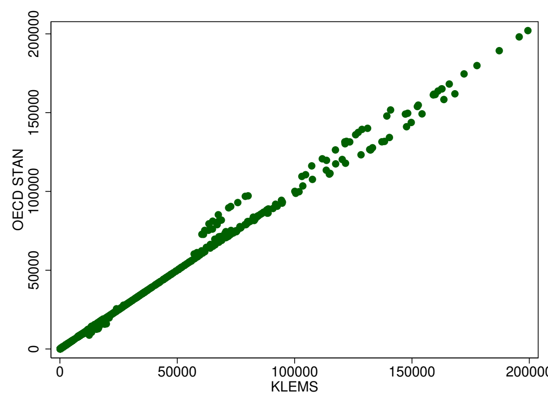
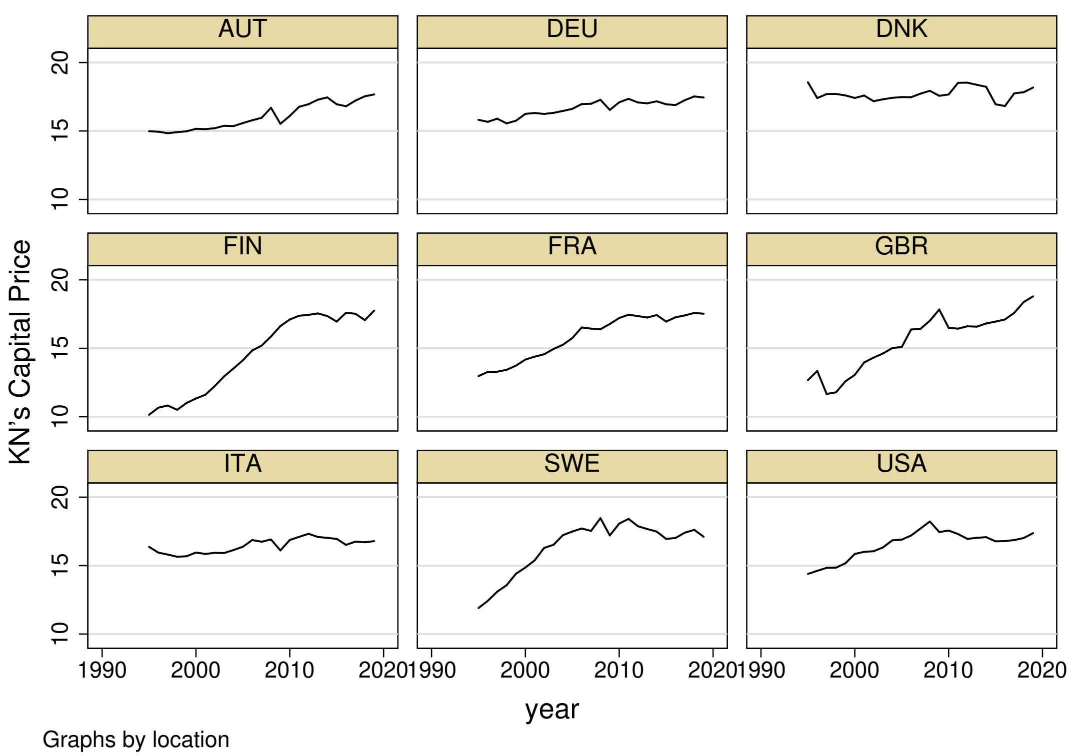


Figure 9: KN's Capital Prices



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