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Guerrazzi, Marco

Department of Economics - DIEC, University of Genoa

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Private and Social Welfare Gains in the Diamond-Dybvig Model: A Rationale for the Existence of Banks*

Marco Guerrazzi†

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Abstract

In this note, I evaluate the private and the social welfare gains that in the Diamond-Dybvig model of bank runs characterize the switch from a decentralized to a centralized equilibrium that may hold even in an atomistic environment with banking intermediation. Specifically, relying on logarithmic preferences, I show that such a social welfare gain is an increasing function of the discount rate of more patient agents. Moreover, I show that for each level of the discount rate of patient agents, there is an optimal value of the proportion of these agents in the economy that maximizes the social welfare gain.

Keywords: Bank runs; Private and social welfare gains; Banking intermediation; Bernoulli distribution.

JEL Classification: D02, E02, E44, G21.

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†Author contacts: Department of Economics – DIEC, University of Genoa, via F. Vivaldi, n. 5, 16126 Genoa (Italy), Phone (+39) 010 2095702, Fax (+39) 010 2095269, E-mail: guerrazzi@economia.unige.it.

1 Introduction

In a renowned contribution, Diamond and Dybvig (1983) present a theoretical framework in which they show how demand deposit contracts offered by a bank can provide the public with superior allocations compared to those available in a decentralized environment where banking intermediation is absent. Within this setting, they also show that when atomistic depositors face a privately observed risk that leads to a sudden demand for liquidity, the bank can still attract deposits, but it is vulnerable to runs.[1]

After its publication, the seminal work by Diamond and Dybvig (1983) has been developed in many directions and it continues to be widely cited as providing a definitive theoretical setting for the design of financial-stability policies against runs. For instance, Gertler and Kiyotaki (2015) extend the Diamond-Dybvig (1983) model to a general-equilibrium setting with many banks by showing that anticipations of a run have harmful effects on the economy even if the run does not occur. Shell and Zhang (2018) introduce extrinsic (sunspot) uncertainty in the original setting by Diamond and Dybivid (1983) by distinguishing between pre-deposit and post-deposit incentives. More recently, Gu et al. (2023) embed the Diamond-Dybvig (1983) framework into an infinite-horizon environment to highlight the role of banks' reputation in preventing runs.

In this brief note, I dig into an aspect of the Diamond-Dybvig (1983) model of bank runs that – to the best of my knowledge – remained unexplored, i.e., the strength of the incentives that may lead the society to the adoption of banking intermediation among individual agents. Specifically, relying on logarithmic preferences, I assess the private and the social welfare gains that characterize the switch from a decentralized to a centralized equilibrium that can be alternatively implemented by an omniscient social planner or – alternatively – by a bank on which depositors may decide to run. Carrying out that theoretical exploration, I show that the welfare gain experienced by the society as whole by switching to a centralized solution is an increasing function of the discount rate of more patient agents. Moreover, I show that for each level of the discount rate of agents who can afford to postpone consumption, there is an optimal value of the proportion of this kind of agents in the economy that maximizes such a social welfare gain.

The note is arranged as follows. Section 2 propaedeutically outlines the backbone of the Diamond-Dybvig (1983) model. Section 3 analyses private and social welfare gains. Finally, Section 4 concludes.

[1] In the original contribution the desire for liquidity arises because depositors that wants to consume. Similar arguments, however, also apply when depositors are presented with a favorable investment opportunity (cf. Holmström and Tirole, 1998).

2 The model

Diamond and Dybvig (1983) consider a three-period framework ($T = 0, 1, 2$) in which there is there is a continuum of economic agents whose mass is normalized to 1. In period 0, each agent does not consume but she/he is endowed with one unit of the unique consumption good that can be invested in a risk-free project. Specifically, whenever the unit of the consumption good is invested until period 2, the agent receives $R > 1$ units of the consumption good by getting a positive return. Otherwise, i.e., whenever the investment is stopped in period 1, the investor receives back the same unit of the consumption good invested in period 0 by realizing no return.

In period 0, all the agents feel to be alike because they cannot forecast their attitude towards consumption in the subsequent periods. However, in period 1 – consistently with a Bernulli distribution – a fraction $t \in (0, 1)$ of agents realizes that they are impatient and that they want to consume only in period 1 (type-1 agents), whereas the remaining $1 - t$ agents – despite some discounting – are more patient because they are realize to be indifferent between consuming in period 1 or in period 2 (type-2 agents). Denoting by c_T^i the level of consumption in period T by type-i agent, the preferences over consumption of the two types of agents are assumed to be given by

$$\mathcal{U}^1 = u(c_1^1) \quad \text{and} \quad \mathcal{U}^2 = \rho u(c_1^2 + c_2^2) \tag{1}$$

where $\mathcal{U}^1$ ($\mathcal{U}^2$) is the utility function of type-1 (type-2) agents, $\rho \in (0, 1)$ with $\rho R > 1$, whereas $u(\cdot)$ is a continuous, concave twice-differentiable function with a relative-risk-aversion index higher or equal to 1.[2]

In a decentralized equilibrium in which each agent manages its period-0 endowment in isolation, type-1 agents liquidate their investment in period 1 and consume the proceeds, whereas type-2 agents wait until period 2 to consume. Consequently, in that situation, the equilibrium allocations of individual consumption are given by

$$\begin{aligned} c_1^1 &= 1 \\ c_2^1 &= 0 \\ c_1^2 &= 0 \\ c_2^2 &= R \end{aligned} \tag{2}$$

The levels of consumption in (2) imply that in a decentralized equilibrium the social welfare amounts to the following expression:

$$tu(1) + (1 - t)\, \rho u\,(R) \tag{3}$$

A centralized equilibrium can be framed instead as the solution of a social planner problem in which the planner is able to observe the type of each agent, and it is also aware of the investment

[2] According to the Keynes-Ramsey rule, the inequality $\rho R > 1$ recalls that type-2 agents should find profitable to increase their consumption (cf. Cass, 1965).

opportunities available in the economy. As in the decentralized equilibrium described in the array in (2), the social planner will fix to zero the level of consumption of type-1 (type-2) agents in period 2 (1), so that $c_2^{1*} = c_1^{2*} = 0$. Thereafter, the social planner will choose c_1^1 and c_2^2 by maximizing the social welfare of the economy subject to the constraint that the yield on the investment projects which are not liquidated in period 1 by type-1 agents that want to consume, i.e., $(1 - tc_1^1)\,R$, has to be equally divided among the $1-t$ type-2 agents in period 2. Consequently, the social planner problem can be written as

$$\begin{array}{c} \max\limits_{c_1^1, c_2^2} tu(c_1^1) + (1-t)\,\rho u(c_2^2) \\ \text{s.to} \\ c_2^2 = \frac{\left(1-tc_1^1\right)R}{1-t} \end{array} \tag{4}$$

Whenever the relative-risk-aversion index of $u(\cdot)$ is constant and equal to $\gamma \geqslant 1$, i.e., whenever $u(\cdot) = \frac{1}{1-\gamma}\left(c_T^j\right)^{1-\gamma}$ with $T, j = 1, 2$, the solution of (4) is given by the following pair:

$$\begin{array}{l} c_1^{1*} = \frac{1}{t+(1-t)\rho^{\frac{1}{\gamma}}R^{\frac{1-\gamma}{\gamma}}} \\ c_2^{2*} = \frac{(\rho R)^{\frac{1}{\gamma}}}{t+(1-t)\rho^{\frac{1}{\gamma}}R^{\frac{1-\gamma}{\gamma}}} \end{array} \tag{5}$$

The expressions in (2) and (5) imply that in a centralized equilibrium the individual level of consumption realized by type-1 (type-2) agents in period 1 (2) is higher (lower) than the one obtained by these agents in a decentralized equilibrium. Consequently, the implementation of a centralized equilibrium that increases the welfare of the society as a whole with respect to the level achieved by the expression in (3) requires to transfer some resources from type-2 agents to type-1 agents. Diamond and Dybvig (1983) shows that in a decentralized setting such a transfer can be effectively implemented by a bank that – by exploiting the risk-free project available in the economy – offers a demand deposit contract that promises to pay c_1^{1*} to all the depositors that withdraw their funds in period 1 by leaving all the remaining resources to those who wait to withdraw until period 2.

Unfortunately, in the decentralized environment in which the bank is called in to manage the endowments of the two types of agents by creating liquidity, the welfare-enhancing equilibrium described in (5) is only one of two distinct Nash equilibria. Specifically, whenever each type-2 agent irrationally anticipates that all of her/his pairs will behave as a type-1 agent, there another Nash equilibrium in which all the depositors withdraw their funds in period 1 and the bank fails.

3 Private and social welfare gains

Suppose that $u(\cdot)$ is a logarithmic function, so that $\gamma = 1$ for all c_T^j, with $T, j = 1, 2$. In this case, recalling the expressions in (2), (3) and (5), switching from a decentralized to a centralized solution, each type-1 agent face an increase in her/his consumption by experiencing a welfare gain equal to

$$-\ln\left(t+(1-t)\,\rho\right) \tag{6}$$

On the other side, each type-2 agent faces a reduction in her/his consumption, and she/he suffers a loss that amounts to

$$\rho\ln\frac{\rho}{t+(1-t)\,\rho} \tag{7}$$

Eq.s (6) and (7) imply that the social gain experienced by the society depends on two parameters with finite upper and lower bounds; indeed, the algebraic sum of (6) and (7) is given by the following expression:

$$(1-t)\,\rho\ln\rho-\left(t+(1-t)\,\rho\right)\ln\left(t+(1-t)\,\rho\right) \tag{8}$$

As illustrated in the three-dimensional diagram of Figure 1, for all the eligible values of t and ρ the social welfare gain from switching from a decentralized to a centralized solution is always positive by corroborating the superiority of the allocations that are possible in an atomistic environment with banking intermediation. Obviously, this means that the welfare gain experienced by all the type-1's agents always exceeds the loss suffered by all the type-2's agents.

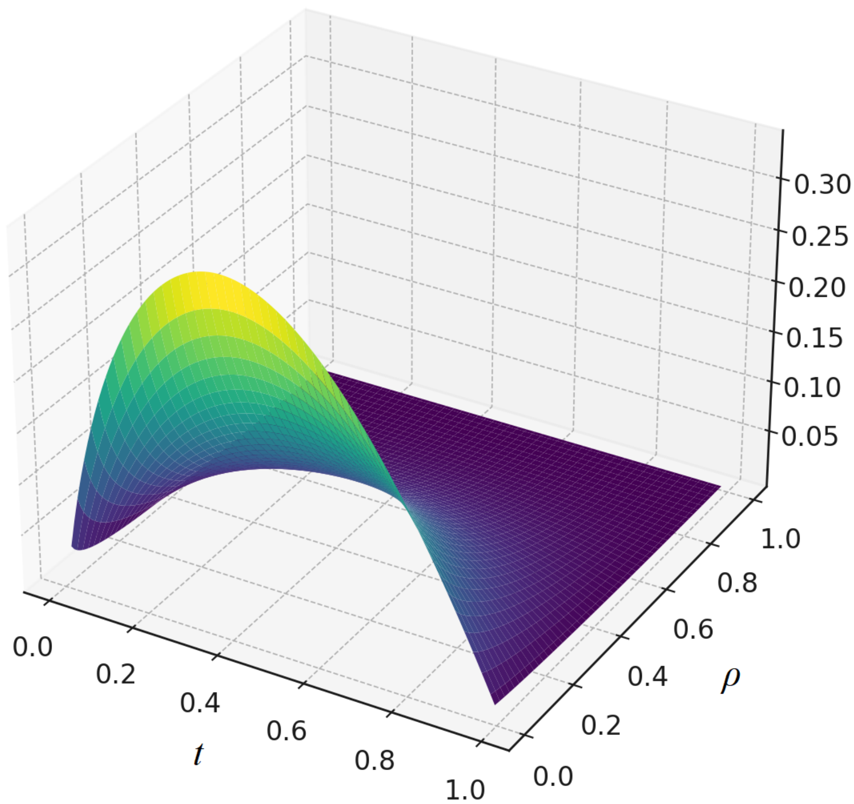

Figure 1: Social welfare gain

The expression in eq. (8) has two additional intriguing properties. On the one hand, the social welfare gain is an increasing function of the discount rate of type-2's agents and such a social welfare gain vanishes as ρ tends to unity as well as when t is equal to 1 or to 0. This result is far from surprising; indeed, when the discount factor tends to unity and – a fortiori – when there is no heterogeneity among the two types of agents, the transfer which is required to shift from a decentralized to a centralized solution becomes pointless and the same holds for banking intermediation. On the other hand, for each value of ρ, there is a corresponding value of t that maximizes the social welfare gain. Such an optimal share of type-1's agents is equal to

$$\widehat{t} = \frac{\exp\left(\frac{\rho(1-\ln\rho)-1}{1-\rho}\right) - \rho}{1-\rho} \tag{9}$$

Some interesting considerations about the expression in eq. (9) come from its evaluation at the extreme values of ρ. On the one hand, when ρ tends to unity the maximum value of the social welfare gain is achieved for a value of t equal to $1/2$, i.e., when the proportions of the two types of agents are equal. Such a critical level of $\widehat{t}$ is the value that maximizes the variance the Bernoulli distribution of the two types of agents in the economy. On the other hand, when ρ tends to zero, i.e., when the discount rate of type-2's agents tends to infinity, $\widehat{t}$ amounts to $\exp(-1) \simeq 0.367\,8$ and the social welfare gain achieves its maximum possible level.

4 Concluding remarks

In this note, drawing on the Diamond-Dybvig (1983) model of bank runs, I evaluated the private and the social welfare gains arising from switching from a decentralized to a centralized equilibrium that can be also achieved through banking intermediation. Assuming logarithmic preferences, I showed that the implied social welfare gain is an increasing function of the discount rate of more patient agents and for each value of that discount rate there is an optimal level of the proportion of these agents for which the social welfare gain achieves its maximum. These findings suggest that the implementation of banking intermediation is more likely in economic environments characterized by substantial discount rates and a certain degree of heterogeneity in liquidity needs.

When the function $u(\cdot)$ is a constant-relative-risk-aversion (CRRA) function with a value of γ strictly higher than 1, the social welfare gain arising from switching to a centralized equilibrium depends also on the unbounded values of R and γ. In this more general scenario, following the same procedure implemented above, it would be possible to show that for given values of t, ρ and R there is an optimal value of γ – say $\widehat{\gamma}$ – that maximizes the social welfare gain arising from switching to a centralized equilibrium; indeed, for values of γ higher than $\widehat{\gamma}$ the social welfare gain tends to vanish, whereas for values of γ lower than $\widehat{\gamma}$ the social welfare gain shrinks by tending to the level achieved with logarithmic preferences. Moreover, for given values for given values of t, ρ and γ, the social welfare gain increases with R by tending to a positive upper bound equal to $t/\left(1-\gamma\right)\left(t^{\gamma-1}-1\right)$.[3]

The CRRA patterns described above suggest a positive observation and a normative issue. On the one hand, as a far as the behaviour of the social welfare gain with respect to γ is concerned, it is interesting to notice that whenever this parameter tends to infinity the two types of agents end up to consume the same amount of resources in the centralized equilibrium, i.e., $c_1^{1*} = c_2^{2*}$. Therefore, it is not surprising that in this case there is no gain to leave the decentralized equilibrium since the loss suffered by type-2 agents is exactly equal to the gain experienced by type-1 agents. On the other hand, given the preferences of depositors, the behaviour of the social welfare gain with respect to R suggests that over a certain range there may be a role to play for the central bank – when it sets interest rates – to influence the incentives to switch to an institutional setting in which banking intermediation is allowed.

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[3] Interestingly, while the welfare gain of type-1 agents is a monotonically increasing function of R, the loss of type-2's agents displays a u-shaped pattern with respect to that parameter. Consequently, for given values of t, ρ and γ, there is a value of R that maximizes the loss of type-2's agents.