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Abstract

Mandatory refund policies have received a lot of attention from both policymakers and academics. Despite this, little is known about how sellers strategically respond to the policy and the resulting effects on competition. To address this, we analyze mandatory refund policies in a framework that flexibly accommodates the full competition spectrum. We show that the policy can benefit consumers in uncompetitive markets under certain conditions, despite reducing social welfare and profits. Nevertheless, we also demonstrate how the policy can be detrimental to consumers, even in very uncompetitive markets. Intuitively, while the policy always protects consumers from some bad outcomes post-purchase, sellers respond by increasing their prices and so consumers have less chance of obtaining a good deal pre-purchase.

Keywords: Refunds; Product Returns; Cooling-Off Periods; Returns Policy; Cancellation Rights

JEL Codes: D18; D21; M37

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1 Introduction

Consumer protection policy is becoming increasingly active in making markets work well.[1] Such policies can empower consumers either by helping them to identify and obtain the best deals before they buy, or by providing consumers with rights and resources to correct bad purchases (e.g. Armstrong and Vickers 2012; Grubb 2015a; Grubb 2015b; Krahmer and Strausz 2015; Armstrong and Zhou 2016, Rhodes and Wilson 2018; Heidhues et al. 2021). However, when implementing a consumer protection policy, it is important for policymakers to take into account how firms will strategically respond as this can impact upon market competition and lead to unintended consequences (Armstrong 2008; Spiegler 2015).

One common example of consumer protection is a mandatory refund policy. This dictates that, for a limited time after a transaction, sellers must offer consumers the right to return the product in exchange for a full refund. There has been a lot of attention on this policy from both policymakers and academics, especially within behavioral economics where it is seen as a leading example of soft paternalism (Camerer et al. 2003; Sunstein and Thaler 2003; Thaler and Sunstein 2008). However, in order to accommodate the modeling of behavioral biases, such justifications are often presented in simple monopoly contexts, with exogenous prices or overlooking important market characteristics. This focus has led to a lack of understanding about how firms respond strategically to the policy, especially in imperfectly competitive markets, even with rational consumers.

To help address this gap, this paper provides a theoretical analysis of how mandatory refund policies affect market outcomes and welfare across the full spectrum of competitive settings from (local) monopoly to Bertrand competition. In a model where sellers select prices and refund levels and where consumers are fully rational, we initially show that, a mandatory refund policy is detrimental to profits and total welfare but is weakly beneficial to consumers in a monopoly context. This formalizes a consumer protection case for mandatory refunds that does not rely on a behavioral mechanism. Then, away from the monopoly extreme, we find that these results will continue to apply in sufficiently uncompetitive markets provided consumers' costs of returning a product are small enough. Indeed, in such imperfectly competitive settings, it is possible for the policy to offer larger benefits to consumers than under monopoly. In contrast, we also demonstrate how a mandatory refund policy can be *detrimental* to consumers (as well

[1] See Waterson (2003), Vickers (2004), Armstrong (2008), Garrod et al. (2009), Fletcher (2017), Fletcher and Hansen (2020), and Fletcher et al. (2023).

as profits and welfare). Intuitively, the policy effectively forces sellers to make losses on returned products, which can lead to a strong strategic response, including raised prices and lower levels of advertising. This response is capable of dominating consumers' direct benefits of refunds in any competitive setting, even in those that are very uncompetitive, close to monopoly.

In more detail, we consider a market for an experience good in which each seller simultaneously chooses its price and refund level. Following any purchase, a consumer learns how much they value the product and decides whether to return it or not. A consumer incurs an effort cost if they choose to obtain a refund, whilst the seller receives a salvage value for any returned product. We compare market outcomes and welfare across two policy regimes. In the 'unregulated regime', each seller is free to choose any level of refund. In the 'regulated regime', there is a mandatory refund policy and so each seller is forced to offer a full refund equal to its price.

In the unregulated regime, sellers have two instruments to affect consumer utility because price and refund levels can differ. To overcome the complexities associated with sellers competing across two dimensions, we draw on the concept of competition in the utility space (Armstrong and Vickers 2001). Here, we show that sellers will always choose a refund level to maximize social welfare in any competitive setting. Sellers can then set their price to offer the required level of consumer surplus as determined by the level of competition. This extends the monopoly result of Matthews and Persico (2007) to any competitive setting. In the regulated regime, sellers only have one instrument to affect consumer utility because each seller's refund must equal its price. Here, we show that any profitable price-refund level will be higher than that in unregulated case because a mandatory refund policy raises a seller's average cost per consumer. This leads to a refund that is inefficiently high from a social perspective with a socially excessive level of returns. Further, if this effect is sufficiently large, then the policy can actually lead to the market becoming inactive with welfare losses for all parties.

When the market remains active, we show that a mandatory refund policy can improve consumer surplus under certain conditions despite always reducing social welfare. This can be understood in terms two effects, which we refer to as the 'return effect' and 'price effect'. The price effect is the impact on consumers from sellers' price response and this is adverse when prices increase. The return effect captures the benefits of the policy to consumers from being better insured against a bad outcome post-purchase. This effect raises consumer welfare for two reasons. First, in the event that a consumer would have returned the product even without the policy, the consumer would now receive an increased refund. Second, in the event that a consumer would have kept the product without the policy, a consumer may now find it optimal to return their product. Thus, a mandatory refund policy will strictly harm consumers if the price effect is sufficiently adverse that it dominates the beneficial return effect; otherwise it will be strictly beneficial to consumers.

To consider the size of these two effects, we then endogenize market prices across the full spectrum of competitive settings. Starting with monopoly, we show that the policy can never harm consumers and will strictly benefit consumers when their effort cost of returning the product is sufficiently small. In these cases, the beneficial return effect weakly dominates any potentially adverse price effect. We also demonstrate that the policy *strictly* reduces a monopolist's profits. Intuitively, this must arise because the monopolist in the unregulated regime can always earn at least as much than in the regulated regime by setting its refund equal to its price, but chooses not to.

Under Bertrand competition, at the other end of the spectrum, we demonstrate how a mandatory refund policy strictly *harms* consumers. Hence, a mandatory refund policy is unnecessary and detrimental to consumers in this setting as competition is already sufficient to safeguard consumer welfare. This follows because consumers appropriate all welfare under Bertrand and the policy leads to a reduction in total welfare due to the excessive level of returns. This implies that the beneficial return effect is dominated by an adverse price effect that arises from the policy raising sellers' average costs per consumer.

Having characterized the extremes of the competition spectrum, we are then able to draw some quite general conclusions about the effects in any model of imperfect competition that continuously links the competitive extreme (Bertrand) to monopoly. In particular, from continuity arguments, a mandatory refund policy will harm consumers when competition is close enough to the Bertrand extreme. However, when competition is sufficiently weak and close enough to monopoly, the policy will harm sellers, and strictly benefit consumers provided consumers' effort costs are low enough. Outside these cases, the effects of a mandatory refund policy are less certain as continuity arguments cannot apply.

To go further, we place more structure on the competition game. Specifically, we extend our framework to imperfect competition by developing an experience good version of the clearinghouse model (Baye and Morgan, 2001; Baye et al., 2006; Varian, 1980). This enables us to tractably encompass the entire spectrum of competition from (local) monopoly to Bertrand and illustrate a rich variety of policy outcomes due to its inclusion of two competition parameters: a proportion of captive consumers and a cost of advertising. Here, we find a range of results for imperfectly competitive markets. In particular, while the mandatory refund policy protects consumers from some bad outcomes post-purchase, we show that sellers' strategic response involves them advertising less and so the chance of consumers obtaining a good deal pre-purchase decreases. However, like under monopoly, it strictly decreases total seller profits as their costs of higher refunds always outweigh any benefit from reduced competition. Surprisingly, we also show that the policy can provide larger benefits to consumers away from the monopoly extreme. Thus, intervening in an imperfectly competitive market can lead to a range of outcomes including harming consumers or benefiting consumers to an extent greater than under monopoly. This highlights the benefits of our approach in analyzing the effects of the policy across the full spectrum of competition.

The rest of the paper is structured as follows. Section 2 discusses the policy background and related literature. After outlining the model in Section 3, Section 4 presents some preliminary results. Sections 5 and 6 then study the market equilibria across the different competitive settings and compare welfare measures between the two policy regimes. Section 7 concludes.

2 Background and Related Literature

Many governments require sellers to offer mandatory refunds for purchases of certain goods and services. These mandatory refund policies provide consumers with the right to return a product or cancel a contract for any reason in exchange for a full refund within a given time frame. Such regulations commonly cover transactions that occur away from the seller's place of business but there are exceptions for certain purchases made in-store. For instance, in the US, to combat high-pressure sales tactics associated with doorstep selling, the Federal Trade Commission's Cooling-Off Rule gives buyers three days to cancel certain sales made at the buyer's home, workplace, or at a seller's temporary location.[2] Similar regulations in the EU (Article 9 of Directive 2011/83/EU) and the UK (2013 Consumer Contracts Regulations) are broader in scope as they apply beyond 'off-premise' sales (that include doorstep selling) to additionally cover 'distance selling' that includes purchases made online or by mail order.[3]

[2] For more details, see https://consumer.ftc.gov/articles/buyers-remorse-ftcs-cooling-rule-may-help, accessed 13/09/24.

[3] See https://www.legislation.gov.uk/eudr/2011/83/body, accessed 13/09/24, and https://www.legislation.gov.uk/uksi/2013/3134/contents, accessed 13/09/24. The 2015 Consumer Rights

They also provide consumers with a wider time frame of fourteen days. Furthermore, the UK's Competition Commission has also imposed a mandatory refund policy as a consumer remedy for purchases occurring in-store (e.g. extended warranty schemes for electrical goods - see Fletcher (2016) for more details).

The relevant theoretical literature on consumer refunds for experience goods can be divided into three camps depending upon the nature of any (pre-purchase) private information regarding consumers' product valuations. In the first camp, neither side of the market has any private information. Here, the literature establishes how refunds can raise consumers' willingness-to-pay by acting as insurance against bad product outcomes (e.g. Davis et al. 1995; Che 1996; Davis et al. 1998). In the second camp, consumers possess (or have the opportunity to obtain) private information about their product value. Here, a seller can use refund options as a screening device to facilitate price discrimination, influence information gathering, or help extract surplus.[4] Finally, in the third camp, the sellers have some private information about product quality or consumers' tastes. Here, refunds can act as a signal or as a commitment device to provide credible advice about consumers' valuations (e.g. Grossman 1981; Moorthy and Srinivasan 1995; Inderst and Ottaviani 2013).[5]

Our paper falls squarely into the first camp because neither the firms or consumers have any private information. The previous literature within this first camp initially focused on understanding a monopolist's decision to offer a zero refund or a full refund (equal to its price). In contrast, in their base setting, Matthews and Persico (2007) allowed a monopolist to choose any level of refund and find that the monopolist will find it most profitable to offer a partial refund equal to the salvage value of a returned product. We follow this approach whilst permitting any competitive setting and aiming to understand the welfare effects of a mandatory refund policy where firms must offer refund levels equal to their price.

Only a small number of papers have previously analyzed the welfare effects of a mandatory refund policy. In the first camp, a mandatory refund policy is presented as a leading example of asymmetric or libertarian paternalism; where the policy can help insure behavioral consumers against 'bad' outcomes while not restricting the choices of others (Camerer et al. 2003; Sunstein

Act in the UK also guarantees buyers with a 30-day returns period for all purchases regardless of location, but only if the product is found to be faulty.

[4]See Courty and Hao (2000), Escobari and Jindapon (2014), Akan et al. (2015), Krahmer and Strausz (2015), and Hinnosaar and Kawai (2020) for price discrimination; Matthews and Persico (2007) and Janssen and Williams (2024) for information gathering; and Inderst and Tirosh (2015) for surplus extraction.

[5]For wider reviews of the refund literature, see Janakiraman et al. (2016), Abdulla et al. (2019) or Ambilkar et al. (2022).

and Thaler 2003; Thaler and Sunstein 2008). For instance, when consumers suffer from projection bias (Loewenstein et al. 2003), such a policy allows consumers to reverse their purchase decisions after overestimating the product's value, as recently formalized in a monopoly setting by Michel and Stenzel (2021). In contrast, our model does not rely on a behavioral justification. Instead, it focuses on sellers' strategic responses in a setting that extends beyond the monopoly case and includes other important market variables such as consumers' effort costs of making returns and sellers' salvage values of returned products. It explains how the policy will harm consumers when the beneficial return effect is dominated by a sufficiently large adverse price effect.

There a couple of other papers from the second camp that also study the effects of regulation. Krahmer and Strausz (2015) consider the impact of withdrawal rights in a setting where a monopolist uses a menu of sales contracts to screen heterogeneous consumers. The withdrawal rights are like mandated refunds as they enable consumers to be reimbursed for any payments and removing them from any future purchase obligations. The authors find that withdrawal rights always lower profits by limiting the menu of contracts and thereby hindering the monopolist's ability to screen consumers, but have an unclear effect on consumers welfare. In contrast, in our different setting, we are able to explain why and when consumer surplus will rise or fall under a mandatory refund policy. Furthermore, we allow for a full range of competitive settings in a way that enables us to investigate the potential tension between consumer protection policy and competition.

A more broadly related paper from the second camp is Janssen and Williams (2024). As an extension of their main results about the impact of refunds on consumer search markets, they consider the effects of a 'partial' refund policy where firms must offer a marginal increase in refund above the unregulated equilibrium refund level but below the subsequent equilibrium price. In contrast, within our different setting, we consider a mandatory refund policy where each seller's refund must always equal its price, both on and off the equilibrium path, as consistent with actual regulation.

Some of our analysis employs a clearinghouse model (Baye and Morgan, 2001; Baye et al., 2006; Varian, 1980). Versions of this model have increased in popularity recently to study issues including sales behavior and price comparison sites (e.g. Moraga-González and Wildenbeest 2012, Shelegia and Wilson 2021; Armstrong and Vickers 2022; Ronayne and Taylor 2022). Usefully, this provides us with a framework that encompasses the full competition spectrum while still remaining tractable enough to incorporate the effects of refunds. This is in contrast to the broader literature on refunds which has largely been restricted to monopoly apart from a few papers that allow for particular forms of competition.[6] As an additional advantage, the clearinghouse framework also enables us offer some novel results about how mandatory refund policies affect advertising behavior.

Finally, our paper contributes to a wider literature that studies how firms strategically respond to consumer protection policies. For instance, see Armstrong and Vickers (2012) and Grubb (2015a) for regulating contingent charges, Armstrong and Zhou (2016) and Rhodes and Wilson (2018) for deterring sellers from deceiving or pressurizing consumers, Grubb (2015b) for improving market transparency, and Heidhues et al. (2021) for a broad set of policies that limit ex post consumer harm.

3 Model

Consider a market with $n \geq 2$ symmetric sellers that each produce an experience good with marginal production cost, c. There is a unit mass of risk-neutral consumers who each have a unit demand and a zero outside option. Let consumer m value the good at seller $i = \{1, ..., n\}$ with an idiosyncratic match value or 'value', v_{mi}.[7] Both consumers and sellers are unaware of any match values pre-purchase, but after purchasing a given product, a consumer learns their match value for that product. We assume that each v_{mi} is drawn independently from a distribution, $F(v)$, on $[0, b]$ with $b > 0$ and where $E(v) \equiv \int_0^b vf(v)dv \in (0, b)$. Let $f(v) = F'(v)$ be strictly positive everywhere and non-decreasing to ensure that profits are concave.

We consider two policy regimes, $\kappa \in \{o, x\}$, where refunds are either unregulated, $\kappa = o$, or mandated, $\kappa = x$. Given the policy regime, κ, each seller i sets its price, $p_{i\kappa}$, and refund level, $r_{i\kappa}$. By setting a refund level, seller i offers to re-pay $r_{i\kappa}$ to any consumer that returns a product purchased at the seller. When refunds are unregulated, $\kappa = o$, each seller i is free to choose its refund, r_{io}, and its price, p_{io}. When refunds are mandated, $\kappa = x$, each seller i must offer a full refund, $r_{ix} = p_{ix}$.

[6] In settings closer to our unregulated regime where sellers select refund levels, Inderst and Tirosh (2015) examine a duopoly, whilst Janssen and Williams (2024) consider a unit mass of sellers. In contrast, Shulman et al. (2011) and Petrikaite (2018) consider duopolies too, but take a different direction by studying the effects of consumer effort costs under the assumption of full refunds (as closer to our regulated regime). None of these papers compare a fully regulated regime versus an unregulated regime.

[7] Our results also remain robust under the alternative assumption that each consumer's match value is the same across all sellers, v_m.

In either regime, whenever a good is returned, the consumer incurs an effort cost, $e \geq 0$, to obtain the refund, $r_{i\kappa}$, and the recipient seller obtains a salvage value, $s \geq 0$. To avoid the uninteresting case where returns are never socially desirable, we assume $e \leq s$. Further, we assume the salvage value is strictly lower than the marginal cost, $0 \leq s < c$, due to the associated costs of re-processing, repackaging or potential product damage. In addition, we assume that $c \leq E(v)$ to ensure that the market is always active in the unregulated regime. Hence, in summary, $0 \leq e \leq s < c \leq E(v) < b$.

The game is then structured as follows for a given policy regime, $\kappa \in \{o, x\}$. In Stage 1, each seller i simultaneously chooses $p_{i\kappa}$ and $r_{i\kappa}$. In Stage 2, each consumer selects which seller to buy from, if any. In Stage 3, any consumer m that purchased from seller i, then learns v_{mi}, before deciding whether to keep or return the good. We focus on symmetric equilibria.[8]

4 Preliminary Results

This section first derives some foundational results that apply throughout the paper. First, within a given policy regime, κ, Section 4.1 offers some general welfare measures for a given price and a given refund, $p_{i\kappa}$ and $r_{i\kappa}$. Then, Section 4.2 considers the unregulated regime for any given price. Among other results, it demonstrates that each firm will optimally set its refund level equal to the salvage value, $r_{io} = s$, within *any* competitive setting. Section 4.3 considers the regulated regime for any given price. It shows how the policy raises each seller's average cost per consumer and how, in extreme cases, it can render the market inactive. Finally, Section 4.4 identifies two fundamental effects of the policy, which we term as the return and price effects, before offering some general results about the impact of the policy on welfare under exogenous prices. Later, in Sections 5 and 6, we draw upon these results to analyze the effects of the policy while endogenizing the market prices across different competitive settings.

4.1 Welfare Measures

Consider any consumer m's purchase decision. If consumer m buys, it will buy from the seller with a price-refund pair that offers the highest expected utility. The consumer's net expected

[8]In Stage 2, we implicitly assume that buying from multiple sellers is prohibitively costly. This assumption is common within the literature (e.g. Inderst and Tirosh 2015) and trivially satisfied under monopoly. However, the Supplementary Appendix shows how this can be relaxed.

utility from a transaction with a given seller i equals (dropping subscript m):

$$u_{i\kappa} \equiv V(r_{i\kappa}) - p_{i\kappa} \tag{1}$$

where $V(r_{i\kappa})$ refers to the consumer's expected gross utility:

$$V(r_{i\kappa}) = \int_{max\{r_{i\kappa}-e,0\}}^{b} vf(v)dv + \int_{0}^{max\{r_{i\kappa}-e,0\}} (r_{i\kappa} - e)f(v)dv$$

$V(r_{i\kappa})$ has two components. The first derives from the possibility of the consumer keeping the good to receive v, which occurs when the value of keeping the good exceeds the net benefit of returning it, $v \geq r_{i\kappa} - e$. The second derives from the possibility of the consumer returning the good to receive $r_{i\kappa} - e$, which occurs when $v < r_{i\kappa} - e$.[9] Overall, relative to an outside option of no-purchase, the consumer will be willing to buy from i if $u_{i\kappa} \geq 0$ or equivalently, if $p_{i\kappa} \leq V(r_{i\kappa})$. Notice $V(r_{ik}) \geq E(v)$ given $r_{ik} \geq 0$, and that $V(r_{ik})$ is strictly increasing in the level of refund, r_{ik}, because the refund insures the consumer against low match values.

Next, consider seller i's profits. Specifically, given $p_{i\kappa} \leq V(r_{i\kappa})$, seller i's expected profits per transaction:

$$\pi_{i\kappa} \equiv \pi(p_{i\kappa}, r_{i\kappa}) = (p_{i\kappa} - c) - (r_{i\kappa} - s) \int_{0}^{max\{r_{i\kappa}-e,0\}} f(v)dv. \tag{2}$$

Intuitively, seller i receives an initial profit of $p_{i\kappa} - c$. However, with the probability that the consumer returns the product, seller i will have to refund $r_{i\kappa}$ while obtaining the salvage value, s.

Finally, consider the expected total welfare per transaction for a given $p_{i\kappa}$ and $r_{i\kappa}$, $w_{i\kappa}{=}u_{i\kappa}+\pi_{i\kappa}$. Summing (1) and (2) for any $u_{i\kappa} \geq 0$ and $\pi_{i\kappa} \geq 0$ yields:

$$w_{i\kappa} \equiv w(r_{i\kappa}) = \int_{max\{r_{i\kappa}-e,0\}}^{b} vf(v)\,dv + \int_{0}^{max\{r_{i\kappa}-e,0\}} (s-e)\,f(v)\,dv - c. \tag{3}$$

Intuitively, aside from the marginal cost, c, which is incurred for any transaction, (3) has two other components. The first derives from the possibility of the consumer keeping the good. The second derives from the possibility of the consumer returning the good, where the seller then obtains s from salvaging the good but the consumer incurs the effort cost, e.

[9] If a given consumer is indifferent over returning a product, we assume they keep it. Also note that the consumer will never return the good if $r_{i\kappa} - e < 0$ as the effort cost is larger than the refund.

For later, it is useful to consider the socially optimal outcome. From a social viewpoint, a product should be returned if and only if the consumer's value of keeping the product, v, is less than the net social value of returning it, $s-e$; otherwise, the product should be kept if $v \geq s-e$. Hence, for a given v, the total welfare from a transaction equals $\max\{v, s-e\} - c$. Given $s \geq e$, it will then clearly be socially optimal to set $r = s$ because a consumer will then only return the product iff $v < s-e$.

4.2 Unregulated Regime, $\kappa = o$

We now consider the welfare measures in the unregulated regime. Here, sellers are free to select any refund level and any price. To overcome the complexities associated with sellers competing across two dimensions, we draw on the concept of competition in the utility space (Armstrong and Vickers 2001). In particular, we can think of each seller i as having two decisions: how much utility to offer to each consumer, u_{io}, and how best to provide such a utility offer through its choice of price, p_{io}, and refund, r_{io}. For the latter decision, we can now state the following:

Proposition 1. *Suppose $\kappa = o$ and seller i wishes to offer each consumer a given utility $u \in [0, V(s)]$. Seller i will then always find it optimal to set a refund level equal to the salvage value, $r_{io} \equiv r_o^* = s$, and a price, $p_o^*(u) = V(s) - u \geq 0$.*

The previous literature has shown that it is profit-maximizing for a monopolist to set its refund equal to its salvage value (e.g. Matthews and Persico 2007). Here, we use a utility argument to show how this applies more generally across any competitive setting. In particular, seller i will provide a given utility, u, in the most profitable manner to maximize its profits per transaction, π_{io}. Given $w_{io} = \pi_{io} + u$, we can write $\pi_{io} = w_{io} - u$. Hence, for a given $u \in [0, V(s)]$, seller i will optimally select its refund level to maximize the total surplus per transaction, w_{io} in (3) such that $r_o^* = s$. Seller i can then use a price, $p_o^*(u) = V(s) - u$, to ensure that it offers utility $u = V(s) - p_o^*(u)$ as required, while retaining any remaining surplus.

Hence, for any given price, p_{io}, (1)-(3) reduce to the following given $b > c > r_{io} = s \geq e \geq 0$:

$$u_{io} \equiv V(s) - p_{io} = \int_{s-e}^{b} v f(v) dv + \int_{0}^{s-e} (s-e) f(v) dv - p_{io} \quad (4)$$

$$\pi_{io} \equiv \pi(p_{io}, s) = (p_{io} - c) - (r_{io} - s) \int_{0}^{s-e} f(v) dv = p_{io} - c \quad (5)$$

$$w_{io} \equiv w(s) = \int_{s-e}^{b} v f(v) dv + \int_{0}^{s-e} (s-e) f(v) dv - c \quad (6)$$

Three features are worth noting about the unregulated regime. First, the per-transaction profit function in (5) simply equals the mark-up, $p_{io} - c$, because each consumer has a unit demand and because the seller makes no direct losses on returned products given $r_o^* = s$. Second, total welfare in (6) is independent of price because consumers have unit demand and the refund equals s. Third, the market can always be active. For instance, seller i can always guarantee at least $\pi_{io} = 0$ by setting $p_{io} = c$ and $r_{io} = s$ to offer $u_{io} = w_{io} \geq 0$ as $V(s) \geq p_{io} = c$ given $V(s) \geq E(v) \geq c$.

4.3 Regulated Regime, $\kappa = x$

We now examine the welfare measures in the regulated regime. In this case, each seller must offer a full refund, $r_{ix} = p_{ix}$. Hence, (1)-(3) reduce to the following:

$$u_{ix} \equiv V(p_{ix}) - p_{ix} = \int_{max\{p_{ix}-e,0\}}^{b} vf(v)dv + \int_{0}^{max\{p_{ix}-e,0\}} (p_{ix} - e)f(v)dv - p_{ix} \quad (7)$$

$$\begin{aligned} \pi_{ix} \equiv \pi(p_{ix}, p_{ix}) &= (p_{ix} - c) - (p_{ix} - s)\int_{0}^{max\{p_{ix}-e,0\}} f(v)dv \\ &= (p_{ix} - s) \cdot (1 - F(max\{p_{ix} - e, 0\})) - (c - s) \quad (8) \end{aligned}$$

$$w_{ix} \equiv w(p_{ix}) = \int_{max\{p_{ix}-e,0\}}^{b} vf(v)\,dv + \int_{0}^{max\{p_{ix}-e,0\}} (s - e)\,f(v)\,dv - c \quad (9)$$

We now explain these equations while commenting on some initial impacts of the policy. Starting with (7), given $r_{ix} = p_{ix}$, a consumer will now return the product if the consumer's value of keeping it, v, is less than the consumer's value of returning it, $p_{ix} - e$ (provided $p_{ix} - e > 0$). Hence, contrary to the unregulated regime, total welfare per-transaction in (9) now becomes dependent upon price. The impact of the policy on the profits per-transaction in (8) is more subtle. To explore this further, observe that the profits per-transaction in (8) resembles a standard profit function where the price is p_{ix}, marginal cost is s, fixed costs are $(c - s)$ and $1 - F(p_{ix} - e)$ represents the demand curve for any $p_{ix} \in (e, V(p_{ix}))$. We can then see that the policy influences both demand and costs.[10]

First, consider how it influences demand. The demand curve, $1 - F(p_{ix} - e)$, is more precisely described as the probability of a consumer keeping the product (which occurs if $v > p_{ix} - e$). Consequently, a lower price (and thereby a lower refund) incentivizes more consumers to keep the product, and hence the seller faces a downward-sloping demand curve. Therefore,

[10] These effects arise provided i) a consumer strictly prefers to buy a given product, $p_{ix} < V(p_{ix})$, and ii) there is a strictly positive probability that the consumer returns the product, $r_{ix} = p_{ix} > e$. These conditions will always be satisfied in equilibrium given $c > s \geq e$.

related to the demand-rotating logic of Che (1996) and Johnson and Myatt (2006), a mandatory returns policy effectively prompts per-consumer demand to change from being box-shaped to downward-sloping.

Second, consider how the policy influences costs. The total costs per transaction is $c - s \cdot F(p_{ix} - e)$ for any $p_{ix} \in (e, V(p_{ix})]$. That is, for every transaction, a seller incurs the production cost, c, but also gains a salvage value, s, with the probability that the product is returned. By rewriting total costs in (8) as $(c - s) + s(1 - F(p_{ix} - e))$, we can see that $(c - s)$ is like a fixed cost, as it must be incurred regardless of whether the product is returned or not, and that s is like a marginal cost, as it is forgone only if the product is not returned. Thus, $s + \frac{c-s}{1-F(p_{ix}-e)} > c$ represents the average cost per non-returned product. Therefore, a mandatory refund policy leads to a downward sloping average cost function that converges to c as $1 - F(p_{ix} - e)$ tends to one. In contrast, the average cost function under the unregulated regime is horizontal and equal to c, and so the mandatory refund policy raises average costs by pivoting the average cost function clockwise. This leads to the following lemma.

Lemma 1. *In the regulated regime, seller i may be inactive for any $r_{ix} = p_{ix}$ if the marginal production cost, c, is sufficiently larger than the salvage value, s.*

From (8), note that seller i *cannot* remain profitable for any $p_{ix} \leq c$ (given $c > s \geq e$). Instead, the losses incurred on returned goods imply that seller i will be active only if $p_{ix} > c$. Indeed, these losses can become so large that there may not exist any $p_{ix} = r_{ix} > c$ at which seller i can profitably operate with $\pi_{ix} \geq 0$ and $u_{ix} \geq 0$.

Finally, for any active seller i with $\pi_{ix} \geq 0$, we now know that its price must be greater than its marginal production cost, c. As $p_{ix} = r_{ix}$, it now follows that $r_i = p_i > c > s$. Hence, the total welfare per-transaction is not maximized under regulation as the refund level will strictly exceed the salvage value.

4.4 Preliminary Policy Comparison

Here, we offer some initial welfare comparisons across the two regimes for given exogenous prices. First, as noted in Corollary 1, a mandatory refund policy can have a drastic impact in reducing the welfare of all parties when the policy leads the market to become inactive as per Lemma 1. (We will provide a formal condition for when this can occur under endogenous prices later in Section 5.)

Corollary 1. *If the marginal production cost, c, is sufficiently larger than the salvage value, s, such that the market is not active under a mandatory refund policy, then the policy reduces consumer surplus, industry profits and total welfare.*

From this point onwards, we focus on cases where the market always remains active. In such cases, the welfare effects from a mandatory refund policy are more nuanced. We now being to outline two fundamental effects of the policy - that we term as the return and price effect. As a first step, one can use (4)-(9) to express the difference between the welfare measures (per-transaction) under the two policies in (10)-(12) below. To help later intuition, we use the refund notation, r_{io} and r_{ix}, rather than their associated values, s and p_{ix}, and introduce the following notation: $\Delta p = p_{ix} - p_{io}$ and $\Delta r = r_{ix} - r_{io}$, where $\Delta r > 0$ given $p_{ix} = r_{ix} > r_{io} = s$.

$$\Delta u \equiv \quad u_{ix} - u_{io} = \quad \Delta r \int_0^{r_{io}-e} f(v)dv + \int_{r_{io}-e}^{r_{io}+\Delta r-e} (r_{io} + \Delta r - e - v) f(v)dv - \Delta p \tag{10}$$

$$\Delta \pi \equiv \quad \pi_{ix} - \pi_{io} = \quad \Delta p - \Delta r \int_0^{r_{io}-e} f(v)dv - (r_{io} + \Delta r - s) \int_{r_{io}-e}^{r_{io}+\Delta r-e} f(v)dv \tag{11}$$

$$\Delta w \equiv \quad w_{ix} - w_{io} = \quad - \int_{r_{io}-e}^{r_{io}+\Delta r-e} (v - (s-e)) f(v)\, dv < 0 \tag{12}$$

Consider the impact of the policy on consumer utility, (10). Let us denote the first and second terms as $R(\Delta r)$, such that $\Delta u = R(\Delta r) - \Delta p$, and refer to it as the return effect. This effect benefits consumers for two reasons given $\Delta r = r_{ix} - r_{io} > 0$. First, in the event of a low match value, $v < r_{io} - e$, that would have caused the consumer to return their product even without the policy, they now receive an increase in refund, $\Delta r = p_{ix} - s > 0$, (as shown in the first term of $R(\Delta r)$). Second, in the event of a medium match value, $v \in (r_{io} - e, r_{ix} - e)$, they now find it optimal to return their product to gain $r_{ix} - e = r_{io} + \Delta r - e$, rather than keep the product to obtain v (as shown in the second term of $R(\Delta r)$). However, as illustrated by the third term in (10), the policy also creates a price effect if the seller chooses to change its price, $\Delta p = p_{ix} - p_{io}$.

Next consider the impact of the policy on per-transaction profits, (11). The first term originates from the price effect. The second and third term describe how the return effect damages profits. Specifically, the return effect reduces profits per-transaction for two reasons. First, in the event of a low match value that would have caused the consumer to return their product even without the policy, the seller now has to pay out a higher refund (as $r_{ix} = p_{ix} > s$). Second, in the event of a medium match value, a consumer now finds optimal to return their product and so in that case, the seller loses $r_{ix} - s > 0$.

Finally, consider the impact of the policy on total welfare per-transaction, (12). Here, there is no role for the price effect as Δp is simply a transfer between a consumer and seller. Instead, the change in total welfare only involves the return effect. In the event of a low match value, the increased refund level also only represents a transfer between a consumer and seller. However, the increased refund level also prompts a consumer with a moderate match value, $v \in (r_{io} - e, r_{ix} - e)$, to return the product, despite it being socially inefficient, $v > r_{io} - e = s - e$, creating a welfare loss. Proposition 2 now follows:

Proposition 2. *Suppose prices are exogenous with p_{io} and p_{ix} and assume the market is active under both policy regimes such that $p_{ix} > c$. Then, for a given transaction, a mandatory refund policy i) always strictly decreases total welfare, $\Delta w < 0$, ii) damages profits, $\Delta \pi < 0$, if it benefits consumers, $\Delta u > 0$, and iii) strictly harms consumers, $\Delta u < 0$, only if there is a sufficiently large adverse price effect, $\Delta p > R(\Delta r) > 0$.*

The derivation is straightforward as follows. Result i) follows immediately from (12). Given this, result ii) follows from rearranging $\Delta w = \Delta u + \Delta \pi < 0$. Result iii) then follows from rearranging $\Delta u = R(\Delta r) - \Delta p < 0$ as the return effect is always beneficial, $R(\Delta r) > 0$ given $p_{ix} = r_{ix} > c > s$.

5 Monopoly and Bertrand Competition

Building on Section 4, the next two sections now derive the market equilibria under endogenous prices for each policy regime and compares the associated welfare measures. We start by considering the extremes of the competition spectrum, with monopoly in Section 5.1 and Bertrand competition in Section 5.2. Section 6 then examines the full competition spectrum by analyzing imperfect competition more broadly.

5.1 Monopoly

The monopoly equilibrium under each policy regime can be stated as follows (where we denote the equilibrium price and refund level for a given $\kappa \in \{o, x\}$ as p_κ^m and r_κ^m):

Proposition 3. *Under monopoly, there exists a unique equilibrium:*

a) In the unregulated regime, $r_o^m = s$ and $p_o^m = V(s) \geq E(v)$.

b) In the regulated regime, provided the market is active, $p_x^m = r_x^m$ *i) uniquely satisfies*

$$\frac{\partial \pi\left(p_{ix}^m, p_{ix}^m\right)}{\partial p_{ix}} = 1 - F\left(p_{ix} - e\right) - \left(p_{ix} - s\right) f\left(p_{ix} - e\right) = 0 \tag{13}$$

with $p_x^m = r_x^m \in \left(s, V\left(p_x^m\right)\right]$ *if* e *is sufficiently small, and ii) uniquely satisfies* $p_x^m = V\left(p_x^m\right)$*, otherwise.*

The unregulated equilibrium is straightforward and consistent with the literature (e.g. Matthews and Persico 2007). Intuitively, the monopolist maximizes welfare by setting $r_o^m = s$ and then extracts all of the rents via $p_o^m = V(s)$.

Now consider the regulated regime when the market is active. From the unregulated results, we know that if it were allowed, the monopolist would like to set $r = s$ and then extract the remaining rents with a high price. However, now that the monopolist's refund is tied to its price, it must trade-off the benefits from setting a low refund close to s with the benefits from setting a higher price. For situations where the effort cost of returning the product is sufficiently small, as in bi) of Proposition 3, the FOC in (13) suggests that the monopolist's optimal price-refund, $p_x^m = r_x^m > s$, balances the increase in profit that results from charging a higher price to a consumer when they are expected to keep the good, $1 - F\left(p_x - e\right)$, with the decrease in profit due to a greater probability of a return, $-\left(p_x - s\right) f\left(p_x - e\right)$. In contrast, for higher effort costs in line with bii), the equilibrium price is constrained by the consumers' *ex ante* valuation and so the monopolist sets $p_x^m = V\left(p_x^m\right)$ to ensure that the consumer buys.

In either case of low or high effort costs, the monopolist will be active in the regulated market if $\pi_x^m \geq 0$. Note in Proposition 3, p_x^m is independent of marginal cost, c, in both cases bi) and bii) from equations (13) and the definition of $V\left(r_{i\kappa}\right)$ below (1). Hence, when combined with (1), we know that the regulated market will be active provided c is sufficiently small with $c < \hat{c}(e, s)$ where

$$\hat{c}(e, s) \equiv \left(p_x^m - s\right) \cdot \left(1 - F(p_x^m - e)\right) + s \tag{14}$$

Indeed, this condition on c will be enough to ensure that the regulated market is active for all other competitive settings. Therefore, henceforth, we assume $c < \hat{c}(e, s)$ to allow the regulated monopoly profits to be strictly positive.

We now consider the welfare impact of the policy. Given a unit mass of consumers, we denote the equilibrium values of consumer surplus, seller profits and total welfare under monopoly for each policy regime κ as $u^m_\kappa = V(r^m_\kappa) - p^m_\kappa$, $\pi^m_\kappa = \pi\left(p^m_\kappa, r^m_\kappa\right)$, and $w^m_\kappa = w\left(r^m_\kappa\right)$ respectively. To compare the welfare measures across the two regimes, we build on the measures developed in Section 4.4 but evaluate them at the equilibrium levels of price and refund with $\Delta u^m \equiv u^m_x - u^m_o$, $\Delta\pi^m \equiv \pi^m_x - \pi^m_o$ and $\Delta w^m \equiv w^m_x - w^m_o$, where $\Delta p^m \equiv p^m_x - p^m_o$ and $\Delta r^m \equiv r^m_x - r^m_o = p^m_x - s$.

Proposition 4. *Under monopoly, provided the market remains active, a mandatory refund policy i) (weakly) increases consumer surplus, $\Delta u^m \geq 0$, and ii) strictly decreases profits and total welfare, $\Delta\pi^m < 0$ and $\Delta w^m < 0$.*

Proposition 4 states that the policy is weakly beneficial for consumers. Specifically, when the effort cost is relatively small, the policy strictly increases consumer surplus, $\Delta u^m > 0$. From Proposition 2, we know that this is because the beneficial return effect dominates any potentially adverse price effect, $R(\Delta r^m) > \Delta p^m$. However, when the effort cost is relatively larger, the monopolist is able to extract all the consumer surplus in both regimes such that $\Delta u^m = 0$. This implies that the beneficial return effect must equal the adverse price effect, $R(\Delta r^m) = \Delta p^m$.

Proposition 4 also demonstrates that the policy *strictly* reduces monopoly profits. Intuitively, this must arise because the monopolist in the unregulated regime can always earn at least as much profits than in the regulated regime by setting its refund equal to its price, but chooses not to. More specifically, when the effort cost is relatively large, the seller can extract the entire consumer surplus in the regulated regime, but this surplus is strictly smaller than without regulation due to the inefficiencies related to excessive returns given $r_{ix} > s$. When the effort cost is smaller, the monopolist cannot even extract the entire surplus from a consumer and so its profits under regulation are further hindered.

It is worthwhile to further discuss the price effect. For high levels of the effort cost, there is always an adverse price effect with $\Delta p^m > 0$. We know this because the monopolist extracts the entire surplus through its price under both regimes, but the surplus is strictly lower under regulation. However, understanding the sign of the price effect is more difficult when e is low. Nevertheless, under a uniform match value distribution with $F(v) = \frac{v}{b}$, one can demonstrate that the price effect remains adverse with $\Delta p^m \equiv \frac{(s+e)}{2} - \frac{(s-e)^2}{2b} \geq 0$ for low e.[11]

[11] Specifically, this applies when $e < 3b + s - \sqrt{8b(b+s)}$ which is true for all $e \leq s$ when $s \leq b/8$.

5.2 Bertrand Competition

We now analyze the other end of the competition spectrum by considering a Bertrand setting with $n \geq 2$ sellers in which all consumers are able to buy from the seller offering the largest expected utility. To ensure that the regulated market is always active, we continue to assume $c \leq \hat{c}(e,s)$ (such that the profits per-transaction under monopoly would be non-negative, $\pi_x^m \geq 0$). We can then state the equilibrium as follows (where we denote the equilibrium price and refund level for a given κ as p_κ^B and r_κ^B):

Proposition 5. *Under Bertrand competition, there exists a unique symmetric equilibrium:*
a) In the unregulated regime, $r_o^B = s$ and $p_o^B = c$.
b) In the regulated regime, $p_x^B = r_x^B$ uniquely satisfies $\pi(p_x^B, r_x^B) = 0$ in (8) such that $p_x^B = r_x^B > c > s$.

This can be understood as follows. In the unregulated regime, we know the equilibrium refund must equal the salvage value, $r_o^B = s$, via Proposition 1. Then, from standard Bertrand reasoning, the equilibrium price must be bid down towards marginal cost, $p_o^B = c$, such that each seller's profits per-transaction in (5) equal zero, with $\pi_o^B \equiv p_o^B - c = 0$. However, the equilibrium in the regulated regime is more subtle as the price and refund are tied together by policy. Nevertheless, competition will force the price-refund level down until the price where each seller's per-transaction profits in (8) equal zero, with $\pi_x^B \equiv (p_x^B - s)\cdot(1 - F(p_x^B - e)) - (c - s) = 0$. Such a price exists for all $p_x^B \leq p_x^m$ given $c \leq \hat{c}(e,s)$.

Now note that the policy can create an 'adverse price effect', $\Delta p^B \equiv p_x^B - p_o^B > 0$. This follows as we can rewrite the Bertrand profits as $\pi_x^B \equiv (1 - F(p_x^B - e))\left[p_x^B - \left(s + \frac{(c-s)}{1-F(p_x^B - e)}\right)\right] = 0$. Hence, due to the policy raising sellers' average costs to $s + \frac{(c-s)}{1-F(p_x^B - e)} > c$, the price-refund cannot be driven to marginal cost, c. Therefore, the equilibrium price must be strictly above marginal cost and a mandatory refund policy always raises price relative to the unregulated regime.

We can now compare the equilibrium welfare measures across the two regimes. Given a unit mass of consumers, we denote the equilibrium values of consumer surplus, seller profits and total welfare under Bertrand for each policy regime κ as $u_\kappa^B = V(r_\kappa^B) - p_\kappa^B$, $\pi_\kappa^B = \pi\left(p_\kappa^B, r_\kappa^B\right)$, and $w_\kappa^B = w\left(r_\kappa^B\right)$ respectively. We further define $\Delta u^B \equiv u_x^B - u_o^B$, $\Delta \pi^B \equiv \pi_x^B - \pi_o^B$ and $\Delta w^B \equiv w_x^B - w_o^B$, where $\Delta r^B \equiv r_x^B - r_o^B$.

Proposition 6. *Under Bertrand competition, a mandatory refund policy i) strictly decreases consumer surplus and total welfare,* $\Delta u^B = \Delta w^B < 0$*, and ii) has no effect on profits,* $\Delta\pi = 0$.

In contrast to our results in the monopoly setting, Proposition 6 demonstrates how a mandatory refund policy strictly *harms* consumers under Bertrand competition. Intuitively, this must arise because the adverse price effect dominates the beneficial returns effect, $\Delta p^B > R(\Delta r^B)$. To see this, note $\Delta\pi^B = 0$ because π_κ^B always equals zero for both regimes and so $\Delta u^B = \Delta w^B = R(\Delta r^B) - \Delta p^B$ and $\Delta w^B < 0$ from Proposition 2.

6 Imperfect Competition

Having examined the two extreme situations of monopoly and Bertrand competition, we now analyze the competition spectrum more broadly by examining imperfect competition. Since we know from Section 4.4, that mandated refunds will always decrease total welfare, $\Delta w \leq 0$, this section focuses on the impact of the policy on profits, $\Delta\pi$, and consumer surplus, Δu. To begin, let us discuss the extent to which our existing results will apply across *any* model of imperfect competition that encompasses the whole competition spectrum from monopoly to Bertrand. Formally, suppose $\Delta\pi$ and Δu are continuous in a competition index, $\psi \in [0, 1]$, where $\psi = 0$ corresponds to the monopoly comparison, $\Delta\pi = \Delta\pi^m$ and $\Delta u = \Delta u^m$, and $\psi = 1$ corresponds to the Bertrand comparison, $\Delta\pi = \Delta\pi^B$ and $\Delta u = \Delta u^B$.

Proposition 7. *For any model of imperfect competition where* $\Delta\pi$ and Δu *are continuous in the competition index,* ψ*, a mandatory refund policy will have the following effects: a) when competition is sufficiently strong (with* ψ *close to one), it will harm consumers,* $\Delta u < 0$*, but b) when competition is sufficiently weak (with* ψ *close to zero), it will benefit consumers,* $\Delta u > 0$*, if* e *is sufficiently low such that* $\Delta u^m > 0$*, and harm sellers,* $\Delta\pi < 0$.

The proof follows immediately by using simple continuity arguments with Propositions 4 and 6. For example, given $\Delta u^m > 0$ (when $\psi = 0$) if the effort cost is low and $\Delta u^B < 0$ (when $\psi = 1$), it follows that consumers will benefit in sufficiently uncompetitive markets but be harmed in sufficiently competitive markets. However, when the effort cost is high, such that $\Delta u^m = 0$, it remains an open question whether consumers benefit or not in markets close to monopoly because Δu could be positive or negative as one moves away from the monopoly extreme. Similarly, we can use continuity arguments to state that profits will be harmed in sufficiently uncompetitive markets as $\Delta\pi^m < 0$. However, we cannot use such arguments to analyze the impact on profits when competition is close to Bertrand as $\Delta\pi^B = 0$.

To go further in understanding the policy impact under imperfect competition, we need to use a framework with more structure. We do this by using a version of a clearinghouse model with experience goods (Baye and Morgan, 2001; Baye et al., 2006; Varian, 1980). This model is particularly well-suited to our purposes for two reasons. First, it enables us to tractably encompass the entire spectrum of competition from (local) monopoly to Bertrand. Second, it is capable of illustrating a rich variety of different policy impacts due to its inclusion of two competition parameters: an exogenous cost of advertising and an exogenous proportion of captive consumers. We now proceed by outlining the model assumptions in Section 6.1, before Section 6.2 derives the equilibrium and Section 6.3 then compares the two policy regimes.

6.1 Model Assumptions

We now assume that consumers are initially uninformed of sellers' prices and refunds. However, sellers have the opportunity to inform consumers by advertising for an exogenous cost, $A > 0$. Specifically, in Stage 1 of the game, each seller i simultaneously chooses its price, $p_{i\kappa}$, and refund, $r_{i\kappa}$, and whether to inform consumers of $\{p_{i\kappa}, r_{i\kappa}\}$ through informative advertising (denoted by $a_i = 1$), or not ($a_i = 0$). In Stage 2, consumers observe any advertised prices and refunds before making their purchase decisions. Stage 3 then continues as before - any consumers that made a purchase, learn their match value and decide whether to return their good or not.[12]

The consumers are decomposed into two types, $t \in \{NS, S\}$, with proportions, $1 - \sigma$ and σ respectively, where $\sigma \in (0, 1)$. 'Non-shopper' consumers, $t = NS$, only ever consider buying from their designated 'local' seller. Each seller has a symmetric share of non-shopper consumers, $(1 - \sigma)/n$. The remaining 'shopper' consumers, $t = S$, have no such loyalty. Instead, they compare all advertised offers. Specifically, as consistent with Baye and Morgan (2001), we assume the shoppers make their purchase decisions as follows. If no seller advertises such an offer, then shoppers visit a seller at random and buy (provided the utility offer is above the outside option of zero). If one or more sellers advertise a utility offer (weakly) above zero, then all shoppers buy from the seller that advertises the highest expected utility.[13]

[12] Alternatively, our results are identical in a different setting where refunds are unobservable and cannot be advertised. Intuitively, i) in the regulated regime, a seller's refund can be fully inferred from its advertised price, and ii) in the unregulated regime, consumers will continue to expect refund levels to be equal to the salvage value as a seller will have no incentive to deviate from this value given it maximizes profits.

[13] In the event of a tie where the shoppers are indifferent between some set of sellers, we assume that they

We focus on symmetric equilibria. However, to allow for sellers' potentially mixed strategies, we define $\alpha_{i\kappa} \equiv Pr(a_i = 1) \in [0, 1]$ as seller i's probability of advertising for a given policy regime, κ, and $H^A_{i\kappa}(p)$ and $H^N_{i\kappa}(p)$ as seller i's price distribution when $a_i = 1$ and $a_i = 0$, respectively. Finally, for a given policy regime, note that the total expected advertising expenditure in equilibrium equals $\phi_\kappa = nA\alpha_\kappa$. When measuring total welfare, we assume that any expenditure on advertising represents a transfer to some (unmodeled) advertising channel so that it does not form a dead-weight loss. This ensures consistency with Sections 4 and 5 where there was no advertising.[14]

6.2 Equilibrium

We now present the equilibrium for a given policy regime, κ. To proceed, it is useful to denote $\tilde{A}_\kappa \equiv \sigma\left(\frac{n-1}{n}\right) \cdot \pi^m_\kappa > 0$ and

$$\alpha_\kappa = \max\left\{0, 1 - \left(\frac{A}{\tilde{A}_\kappa}\right)^{\frac{1}{n-1}}\right\} \tag{15}$$

Lemma 2. *For any given policy regime, κ, and associated equilibrium refund, $r^*_{i\kappa}$ (with $r^*_{io} = s$ and $r^*_{ix} = p_{ix}$), a unique symmetric equilibrium exists:*

a) If advertising costs are high, $A \geq \tilde{A}_\kappa$, each seller earns $\overline{\pi}_\kappa = \pi^m_\kappa / n$ by setting p^m_κ and $r^*_{i\kappa}$ and never advertising, $\alpha_\kappa = 0$.

b) If advertising costs are low, $A < \tilde{A}_\kappa$, each seller earns $\overline{\pi}_\kappa = \left(\frac{1-\sigma}{n}\right)\pi^m_\kappa + \frac{A}{n-1}$ by i) setting p^m_κ and not advertising with probability $1 - \alpha_\kappa \in (0,1)$, and ii) advertising a price-refund pair, $\{p_{i\kappa}, r^*_{i\kappa}\}$, with probability $\alpha_\kappa \in (0,1)$ where $p_{i\kappa}$ is selected from $[\underline{p}_\kappa, p^m_\kappa]$ using price distribution

$$H^A_\kappa(p_{i\kappa}) = \frac{1}{\alpha_\kappa}\left(1 - \left(\frac{\frac{(1-\sigma)}{n}[\pi^m_\kappa - \pi_{i\kappa}(p_{i\kappa}; r^*_{i\kappa})] + (\frac{n}{n-1})A}{\sigma \cdot \pi_{i\kappa}(p_{i\kappa}; r^*_{i\kappa})}\right)^{\frac{1}{n-1}}\right)$$

and where $\underline{p}_\kappa$ satisfies $\pi_{i\kappa}\left(\underline{p}_\kappa; r^*_{i\kappa}\right)[\frac{1-\sigma}{n} + \sigma] = \left(\frac{1-\sigma}{n}\right)\pi^m_\kappa + \frac{A}{n-1}$ with $\underline{p}_\kappa > c > s$.

The proof follows straightforwardly from Baye and Morgan (2001) and Baye et al. (2006) while accounting for a given refund, $r^*_{i\kappa}$. Intuitively, we know from Proposition 1 that each seller will

simply randomize between the tied sellers with equal probability.

[14] Whilst more complicated, our results also remain robust if one assumes that advertising costs are a dead-weight loss. In that case, instead of being added to industry profits, the advertising expenditure would be deducted from total welfare and so the documented losses in welfare would be even greater.

always have a unique, optimal refund level for any price, $r^*_{i\kappa}$: under no regulation, $r^*_{io} = s$, and under regulation, $r^*_{ix} = p_{ix}$.

To understand the equilibrium, consider two special parameter cases. For the first special parameter case, suppose $A \to 0$ such that the base model reduces to Varian (1980). Here, the sellers effectively always advertise, $\alpha_\kappa \to 1$, but face a tension between pricing high to their non-shopper consumers, and pricing lower in an effort to sell to the shoppers. This tension is resolved through the use of a price distribution, $H^A_\kappa(p_{i\kappa})$. When almost all consumer are non-shoppers, $\sigma \to 0$, this distribution converges to the monopoly price, p^m_κ, but when almost all consumers are non-shoppers, $\sigma \to 1$, this distribution converges to the Bertrand price, p^B_κ.

For the second special parameter case, suppose that advertising costs are non-trivial, but almost all consumers are shoppers, $\sigma \to 1$. When advertising costs are sufficiently large, $A \geq \tilde{A}_\kappa$, the sellers simply refrain from advertising and price at the monopoly level. However, when $A < \tilde{A}_\kappa$, each seller faces a tension between i) advertising a low price to attract the shoppers, and ii) pricing high and not advertising in the hope that its rivals do the same. This tension is resolved by the sellers mixing over advertising with a probability $\alpha_\kappa \in (0, 1)$ and mixing over advertised prices using $H^A_\kappa(p_{i\kappa})$. When advertising costs are almost zero, $A \to 0$, each firm almost advertises for sure, $\alpha \to 1$, and the advertised price distribution converges to the Bertrand price, p^B_κ.

6.3 Policy Comparisons

We can now compare the two regimes. To begin to see the interaction between consumer protection and competition, Section 6.3.1 first examines how regulation impacts competition through advertising behavior. This will provide a useful foundation to then analyze the effects of the policy on profits and consumer surplus in Sections 6.3.2 and 6.3.3. Throughout this section, we focus on advertising costs, $A < \tilde{A}_o$. We discard any situations with $A \geq \tilde{A}_o$ because the sellers then act as (local) monopolists under both regimes and so the policy comparison coincides exactly with our prior monopoly analysis in Section 5.1.

6.3.1 Advertising

This subsection examines the impact of the policy on advertising behavior.

Proposition 8. *For any $A \in (0, \tilde{A}_o)$ and $\sigma \in (0,1)$, a mandatory refund policy strictly reduces the probability that a seller advertises, $\Delta\alpha = \alpha_x - \alpha_o < 0$.*

This implies that the policy limits competition in the sense that sellers become less likely to advertise and so more likely to price at the monopoly level. Intuitively, by raising average costs, the policy lowers the gains from attracting a shopper. The proof is straightforward. First, note there are two important intervals within $A \in (0, \tilde{A}_o)$ given $\tilde{A}_x < \tilde{A}_o$. Then observe that when advertising costs are sufficiently high, $A \in [\tilde{A}_x, \tilde{A}_o)$, advertising only occurs in unregulated regime, such that $\Delta\alpha < 0$. Alternatively, when advertising costs are low, $A < \tilde{A}_x$, then advertising occurs in both regimes, $\alpha_x \in (0,1)$ and $\alpha_o \in (0,1)$, but the probability of advertising is lower in the regulated regime as $\pi_x^m < \pi_o^m$.

6.3.2 Profits

We can now analyze how the policy impacts expected industry profits which we define as the sum of expected total seller profits, $n\overline{\pi}_\kappa$, and advertiser profits, $\phi_\kappa = nA\alpha_\kappa$. We denote the change in expected total seller profits as $\Delta n\overline{\pi} = n(\overline{\pi}_x - \overline{\pi}_o)$, the change in expected advertiser profits as $\Delta\phi = nA\Delta\alpha$ and the change in expected industry profits as $\Delta\pi = \Delta\phi + \Delta n\overline{\pi}$.

Proposition 9. *For any $A \in (0, \tilde{A}_o)$ and $\sigma \in (0,1)$, a mandatory refund policy strictly reduces expected industry profits, $\Delta\pi < 0$ expected total seller profits, $\Delta n\overline{\pi} = n(\overline{\pi}_x - \overline{\pi}_o) < 0$, and expected advertiser profits, $\Delta\phi = nA\Delta\alpha < 0$.*

Intuitively, the policy reduces advertiser profits by lowering the probability of advertising in equilibrium, $\Delta\alpha < 0$. Despite this reduction in advertising and the associated reduction in competition, Proposition 9 also states that total seller profits still strictly decrease. The reason is that the policy restricts sellers' ability to extract surplus from consumers when not advertising and this reduction in profits dominates the reduction in advertising expenditure.

6.3.3 Consumer Surplus

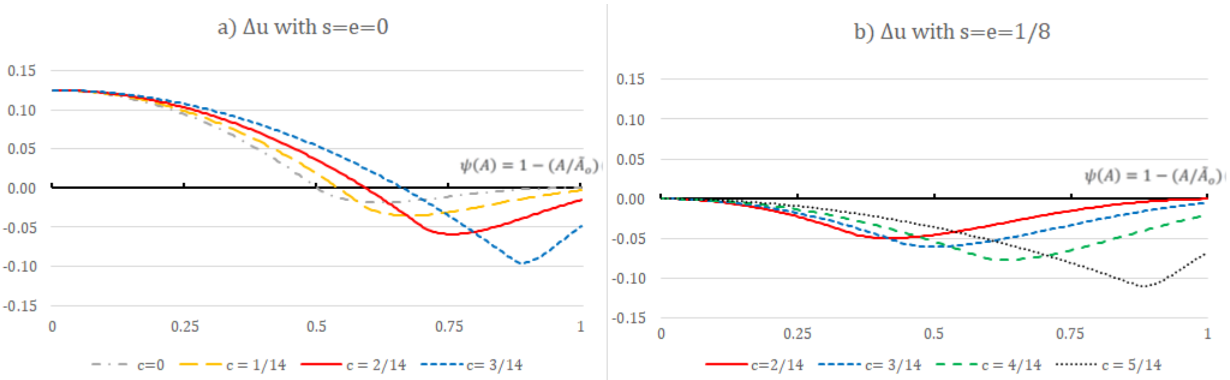
This section considers the impact of the policy on consumer surplus, Δu. The scope for obtaining general results is limited because the shape of Δu between the extremes of the competition spectrum is heavily dependent upon parameters. However, we can state the following analytical results presented in Propositions 10 and 11. (The proofs for these results are provided separately in Appendix B.)

Proposition 10. *Under imperfect competition, a mandatory refund policy can i) harm consumers, $\Delta u < 0$, even in very uncompetitive markets, and ii) harm consumers by more than under Bertrand, $\Delta u < \Delta u^B < 0$.*

Proposition 10 implies that there may not be a consumer protection case for a mandated refund policy even in markets that are very uncompetitive, and the damage from such a policy can be at its largest in the interior of the competition spectrum away from the Bertrand extreme.

Before explaining the intuition, Figure 1 provides a duopoly example to help visualize the result. It illustrates how Δu varies with the competition index, ψ, when all consumers are shoppers, $\sigma \to 1$. In this case, ψ is a function of the advertising cost, A. For instance, in Figure 1, we set $\psi(A) \equiv 1 - A/\tilde{A}_o \in [0, 1]$ so that in both regimes $\psi(\tilde{A}_o) = 0$ corresponds to monopoly and $\psi(0) = 1$ corresponds to Bertrand competition. In Figure 1a effort costs are low (with $s = e = 0$ such that $\Delta u^m > 0$), whilst in Figure 1b effort costs are high (with $s = e = 1/8$ such that $\Delta u^m = 0$). In both panels, marginal cost varies with $s = e < c < \hat{c}(e, s)$ and the match value distribution is uniform, $F(v) = v$.

Figure 1: The Impact of a Mandatory Refund Policy on Consumers, Δu, when $\sigma \to 1$ where $n = 2$ and $F(v) = v$



Consistent with our earlier general Proposition 7, Figure 1a shows that when the effort cost is low, the policy benefits consumers in less competitive markets (low $\psi(A)$) but harms consumers in more competitive markets (high $\psi(A)$). In contrast, Figure 1b shows that when the effort cost is high, the policy harms consumers for all levels of competitiveness, $\psi(A)$.

To understand the intuition of Proposition 10, first consider why the policy can harm consumers even in very uncompetitive markets when the effort cost is high (in line with Figure 1b). At $\psi(\tilde{A}_o) = 0$, where the sellers act as local monopolists in both regimes, we know from Section 5.1 that $\Delta u^m = 0$ as the beneficial return effect equals the adverse price effect. However, for an advertising cost marginally below $\tilde{A}_o$ (and so above $\tilde{A}_x$), the policy harms consumers, $\Delta u < 0$, because the adverse price effect becomes relatively stronger while the beneficial return effect stays constant. The adverse price effect becomes stronger, because the expected market price under the regulated regime remains constant at the monopoly level given $A > \tilde{A}_x$, whilst the expected market price under the unregulated regime falls due to an increase in advertising. In contrast, the beneficial return effect stays unchanged under both regimes because the refund levels remain at the monopoly price and the salvage value, respectively.

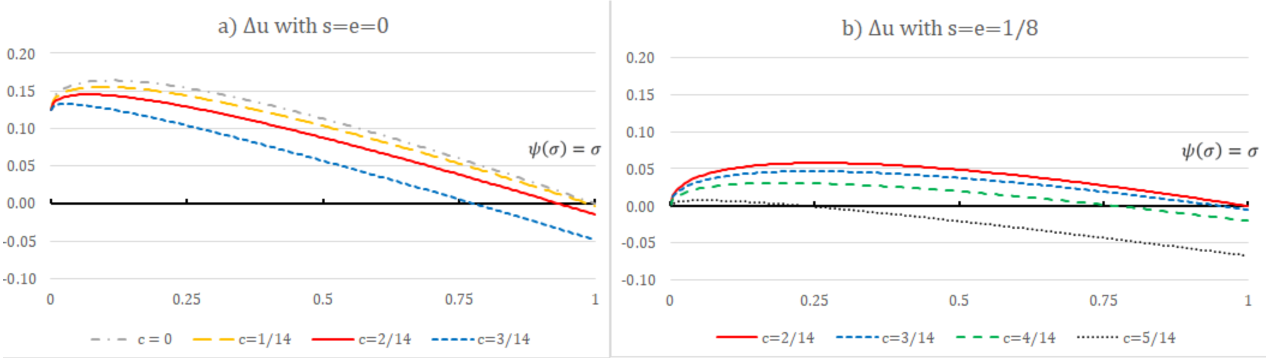
Now consider why the policy can actually harm consumers by more than it does under Bertrand, $\Delta u < \Delta u^B < 0$ (in line with Figures 1a and 1b). At $\psi(0) = 1$, where the market is fully competitive in both regimes, we know from Section 5.2 that $\Delta u^B < 0$ as the beneficial return effect is outweighed by the adverse price effect. However, for an advertising cost marginally above zero (and so below $\tilde{A}_x$), such that the market becomes slightly less competitive in both regimes, the policy increases the harm to consumers. In this case, the beneficial return effect gets larger because the unregulated refund stays constant at the salvage value but the expected refund under the regulated regime increases in line with the expected market price. However, the increase in the beneficial return effect is dominated by a stronger adverse price effect.

We now introduce the second Proposition:

Proposition 11. *Under imperfect competition, in markets that are sufficiently uncompetitive, a mandatory refund policy can benefit consumers by more than under monopoly,* $\Delta u > \Delta u^m \geq 0$.

Surprisingly, Proposition 11 implies that the consumer protection case for a mandated refund policy can be stronger under imperfect competition than in the monopoly extreme. Before explaining the intuition, Figure 2 provides a different duopoly example to help visualize the result. It illustrates how Δu varies with the competition index, ψ, when advertising costs are zero, $A \to 0$. Here, ψ is a function of the proportion of shoppers, σ. For instance, in Figure 2, we set $\psi(\sigma) \equiv \sigma$, so that in both regimes, $\psi(0) = 0$ corresponds to monopoly and $\psi(1) = 1$ corresponds to Bertrand competition. All other parameters are the same as in Figure 1 where Figure 2a and 2b present the case of low and high effort costs, respectively.

Figure 2: The Impact of a Mandatory Refund Policy on Consumers, Δu, where $n = 2$ and $F(v) = v$



Consistent with our earlier general Proposition 7, Figures 2a and 2b both show that the policy benefits consumers in less competitive markets (low $\psi(\sigma)$) but harms them in more competitive markets (high $\psi(\sigma)$). However, in contrast to Figure 1 and as consistent with Proposition 11, they show how the policy can be at its strongest away from the monopoly extreme, $\Delta u > \Delta u^m$.

Now consider the intuition of Proposition 11. At $\psi(0) = 0$, where the market is formed of local monopolies in both regimes, we know from Section 4 that $\Delta u^m \geq 0$ because the beneficial return effect (weakly) dominates any adverse price effect. However, for a proportion of shoppers marginally above zero, such that the market becomes slightly more competitive in both regimes, the policy increases the benefit to consumers. Intuitively, the beneficial return effect gets smaller because the unregulated refund level remains constant at the salvage value, whilst the expected refund under the regulated regime decreases in line with the expected market price. However, despite this reduction in the beneficial return effect, the price effect becomes less adverse and so consumers' expected utility rises.

7 Conclusions

Mandatory refund policies have received a lot of attention from both policymakers and academics. However, previous theoretical studies are often presented in simple monopoly contexts, with exogenous prices or neglecting important market characteristics. Consequently, there has been a lack of understanding about how sellers strategically respond to the policy, especially in imperfectly competitive markets. This has led to the tension between protecting consumers and promoting competition being overlooked.

To help address this gap, this paper has provided a theoretical analysis of how mandatory refund policies affect market outcomes and welfare across the full spectrum of competitive settings from (local) monopoly to Bertrand competition. We have shown how the mandatory refund policy raises a seller's average cost per consumer. If this effect is sufficiently large, then the policy can actually lead to the market becoming inactive, with welfare losses for all parties. However, when the market remains active, we have demonstrated how the policy will always reduce total welfare and profits, but has the potential to raise or lower consumer surplus depending upon the relative sizes of the return and price effects. In particular, while the mandatory refund policy always protects consumers from some bad outcomes post-purchase, sellers' strategic responses involving advertising and prices can harm consumers.

Our results indicate that policymakers should carefully consider the competitiveness of a market before imposing consumer protection remedies. In particular, our model suggests that the consumer protection case for a mandatory refund policy is strongest when the market is sufficiently uncompetitive and the consumers' effort cost of returning purchased products is small. Under such conditions, the beneficial return effect of the policy will dominate any adverse effect on prices. Surprisingly, the case for the policy can be at its strongest under imperfect competition away from the monopoly extreme. However, we find that the policy will harm consumers when the market is sufficiently competitive; here, the policy is counter-productive as as competition is already sufficient to safeguard consumer welfare. Furthermore, the policy can also harm consumers, even in very uncompetitive markets, if consumers' effort cost are not small. In either case, the policy damages consumers by causing an adverse price effect that dominates the benefits of the return effect. These results highlight the importance of understanding how mandatory refund policies interact with competition.

References

Abdulla, H., M. Ketzenberg, and J. D. Abbey (2019): "Taking Stock of Consumer Returns: A Review and Classification of the Literature," *Journal of Operations Management*, 65, 560–605.

Akan, M., B. Ata, and J. D. Dana Jr (2015): "Revenue Management by Sequential Screening," *Journal of Economic Theory*, 159, 728–774.

Ambilkar, P., V. Dohale, A. Gunasekaran, and V. Bilolikar (2022): "Product Returns Management: A Comprehensive Review and Future Research Agenda," *International Journal of Production Research*, 60, 3920–3944.

Armstrong, M. (2008): "Interactions Between Competition and Consumer Policy," *Competition Policy International*, 4, 97–147.

Armstrong, M. and J. Vickers (2001): "Competitive Price Discrimination," *RAND Journal of Economics*, 32, 579–605.

——— (2012): "Consumer Protection and Contingent Charges," *Journal of Economic Literature*, 50, 477–93.

——— (2022): "Patterns of Competitive Interaction," *Econometrica*, 90, 153–191.

Armstrong, M. and J. Zhou (2016): "Search Deterrence," *Review of Economic Studies*, 83, 26–57.

Baye, M. R. and J. Morgan (2001): "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets," *American Economic Review*, 91, 454–474.

Baye, M. R., J. Morgan, and P. Scholten (2006): "Information, Search, and Price Dispersion," *Handbook on Economics and Information Systems*, 1, 323–375.

Camerer, C., S. Issacharoff, G. Loewenstein, T. O'Donoghue, and M. Rabin (2003): "Regulation for Conservatives: Behavioral Economics and the Case for 'Asymmetric Paternalism'," *University of Pennsylvania Law Review*, 151, 1211.

Che, Y. K. (1996): "Customer Return Policies for Experience Goods," *Journal of Industrial Economics*, 44, 17–24.

Courty, P. and L. Hao (2000): "Sequential Screening," *Review of Economic Studies*, 67, 697–717.

Davis, S., E. Gerstner, and M. Hagerty (1995): "Money Back Guarantees in Retailing: Matching Products to Consumer Tastes," *Journal of Retailing*, 71, 7.

Davis, S., M. Hagerty, and E. Gerstner (1998): "Return Policies and the Optimal Level of Hassle," *Journal of Economics and Business*, 50, 445–460.

Escobari, D. and P. Jindapon (2014): "Price Discrimination Through Refund Contracts in Airlines," *International Journal of Industrial Organization*, 34, 1–8.

Fletcher, A. (2016): *The Role of Demand-Side Remedies in Driving Effective Competition, a Review for Which?*, Centre for Competition Policy at University of East Anglia.

——— (2017): "Exploitation of Consumer Decision-Making and How to Address It: Lessons from Past Demand-Side Interventions," *Journal of European Competition Law and Practice*, 8, 1–7.

Fletcher, A., G. S. Crawford, J. Crémer, D. Dinielli, P. Heidhues, M. Luca, T. Salz, M. Schnitzer, F. M. S. Morton, K. Seim, et al. (2023): "Consumer Protection for Online Markets and Large Digital Platforms," *Yale Journal on Regulation*, 40, 875.

Fletcher, A. and D. Hansen (2020): "The Role of Demand Side Remedies in Resolving Competition Concerns," in *Remedies in EU Competition Law: Substance, Process and Policy*, ed. by A. Komninos and D. Gerard, Wolters Kluwer (UK) Ltd, 17–38.

Garrod, L., M. Hviid, G. Loomes, and C. W. Price (2009): "Competition Remedies in Consumer Markets," *Loyala Consumer Law Review*, 21, 439–495.

Grossman, S. J. (1981): "The Informational Role of Warranties and Private Disclosure about Product Quality," *Journal of Law and Economics*, 24, 461–483.

Grubb, M. D. (2015a): "Consumer Inattention and Bill-Shock Regulation," *Review of Economic Studies*, 82, 219–257.

——— (2015b): "Failing to Choose the Best Price: Theory, Evidence, and Policy," *Review of Industrial Organization*, 47, 303–340.

Heidhues, P., J. Johnen, and B. Kőszegi (2021): "Browsing versus Studying: A Pro-Market Case for Regulation," *Review of Economic Studies*, 88, 708–729.

Hinnosaar, T. and K. Kawai (2020): "Robust Pricing With Refunds," *RAND Journal of Economics*, 51, 1014–1036.

Inderst, R. and M. Ottaviani (2013): "Sales Talk, Cancellation Terms and the Role of Consumer Protection," *Review of Economic Studies*, 80, 1002–1026.

Inderst, R. and G. Tirosh (2015): "Refunds and Returns in a Vertically Differentiated Industry," *International Journal of Industrial Organization*, 38, 44–51.

Janakiraman, N., H. A. Syrdal, and R. Freling (2016): "The Effect of Return Policy Leniency on Consumer Purchase and Return Decisions: A Meta-Analytic Review," *Journal of Retailing*, 92, 226–235.

Janssen, M. and C. Williams (2024): "Consumer Search and Product Returns in E- Commerce," *American Economic Journal: Microeconomics*, 16, 387–419.

Johnson, J. P. and D. P. Myatt (2006): "On the Simple Economics of Advertising, Marketing, and Product Design," *American Economic Review*, 96, 756–784.

Krahmer, D. and R. Strausz (2015): "Optimal Sales Contracts with Withdrawal Rights," *Review of Economic Studies*, 82, 762–790.

Loewenstein, G., T. O'Donoghue, and M. Rabin (2003): "Projection Bias in Predicting Future Utility," *The Quarterly Journal of Economics*, 118, 1209–1248.

Matthews, S. A. and N. Persico (2007): "Information Acquisition and Refunds for Returns," *PIER Working Paper 07-021.*

Michel, C. and A. Stenzel (2021): "Model-Based Evaluation of Cooling-Off Policies," *Games and Economic Behavior*, 129, 270–293.

Moorthy, S. and K. Srinivasan (1995): "Signaling Quality with a Money-Back Guarantee: The Role of Transaction Costs," *Marketing Science*, 14, 442–466.

Moraga-González, J. L. and M. Wildenbeest (2012): "Comparison Sites," in *The Oxford Handbook of the Digital Economy*, ed. by M. Peitz and J. Waldfogel, Oxford University Press, 224–53.

Morgan, J., H. Orzen, and M. Sefton (2006a): "An Experimental Study of Price Dispersion," *Games and Economic Behavior*, 54, 134–158.

——— (2006b): "A Laboratory Study of Advertising and Price Competition," *European Economic Review*, 50, 323–347.

Petrikaite, V. (2018): "A Search Model of Costly Product Returns," *International Journal of Industrial Organization*, 58, 236–251.

Rhodes, A. and C. M. Wilson (2018): "False Advertising," *RAND Journal of Economics*, 49, 348–369.

Ronayne, D. and G. Taylor (2022): "Competing Sales Channels with Captive Consumers," *The Economic Journal*, 132, 741–766.

Shelegia, S. and C. M. Wilson (2021): "A Generalized Model of Advertised Sales," *American Economic Journal: Microeconomics*, 13, 195–223.

Shulman, J. D., A. T. Coughlan, and R. C. Savaskan (2011): "Managing Consumer Returns in a Competitive Environment," *Management Science*, 57, 347–362.

Spiegler, R. (2015): "On the Equilibrium Effects of Nudging," *The Journal of Legal Studies*, 44, 389–416.

Sunstein, C. R. and R. H. Thaler (2003): "Libertarian Paternalism is not an Oxymoron," *University of Chicago Law Review*, 70, 1159–1202.

Thaler, R. and C. Sunstein (2008): *Nudge: The Gentle Power of Choice Architecture*, New Haven, Conn.: Yale.

Varian, H. (1980): "A Model of Sales," *American Economic Review*, 70, 651–659.

Vickers, J. (2004): "Economics for Consumer Policy," *Proceedings of the British Academy*, 125, 287–310.

Waterson, M. (2003): "The Role of Consumers in Competition and Competition Policy," *International Journal of Industrial Organization*, 21, 129–150.

Appendix A: Main Proofs

Proof of Proposition 1. Let $\kappa = o$. Any seller i must choose $\{p_{io}, r_{io}\}$ to maximize $\pi\left(p_{io}, r_{io}\right)$ subject to $u_i \equiv V\left(r_{io}\right) - p_{io} = u' \in [0, V(s)]$ where $\pi\left(p_{io}, r_{io}\right) = (p_{io} - c) - (r_{io} - s)\int_0^{max\{r_{io}-e,0\}} f(v)dv$ from (2) and $V\left(r_{io}\right)$ is given by (1). Rearranging (1) in terms of p_{io} and substituting transforms the problem to $\text{Max}_{r_{io}} \int_{max\{r_{io}-e,0\}}^{b}(v-(s-e))f(v)dv + (s-e) - c - u'$ or equivalently with use of (3), $\text{Max}_{r_{io}}\, w\left(r_{io}\right) - u'$. We now show that $w\left(r_{io}\right)$ is maximized at $r_{io} \equiv r_o^* = s$. First, note that when $r \leq e$ $w\left(r_{io}\right) = E(v) - c$ as no returns take place; $w\left(r_{io}\right)$ is independent of r.

Second, when $r \geq b + e$, $w(r_{io}) = s - e - c < E(v) - c$ as returns always take place; $w(r_{io})$ is also independent of r. Third, when $r \in (e, b + e)$, note $w'(r_{io}) = (s - r_{io})f(r_{io} - e)$ such that $w(r_{io})$ is single-peaked with $w'(r) > 0$ for $r \in (e, s)$, $w'(r) = 0$ for $r = s$, and $w'(r) < 0$ for $r \in (s, b + e)$. Hence, if $s > e$, then the increasing region, $r \in (e, s)$, exists and so $r_{io} \equiv r_o^*$ is uniquely defined at s. Alternatively, if $s = e$, then $r_{io} \equiv r_o^*$ is not unique - any $r_{io} \leq s = e$ prompts no returns to occur in equilibrium such that $w(r_{io}) = E(v) - c$. Given this, i can then set $p_o^*(u') = V(s) - u'$ to ensure that it offers $u' = V(s) - p_o^*(u')$. □

Proof of Lemma 1. For the market to be active given $r_{ix} = p_{ix}$, we require $u_{ix} \geq 0$ and $\pi_{ix} \geq 0$. We already know that any $r_{ix} = p_{ix} \leq c$ cannot be profitable as (8) becomes strictly negative. Hence, from hereon consider $r_{ix} = p_{ix} > c$. By manipulating (7), we can state $(p_{ix} - s)\int_0^{p_{ix}-e} f(v)dv = u_{ix} + p_{ix} - \int_{p_{ix}-e}^{b} vf(v)dv - (s - e)\int_0^{p_{ix}-e} vf(v)dv$. Then, by inserting this into (8), it follows that $\pi_{ix} = -c - u_{ix} + \int_{p_{ix}-e}^{b} vf(v)dv + \int_0^{p_{ix}-e}(s - e)f(v)dv$. Hence, for seller i to have any potential to be active given $r_{ix} = p_{ix}$, we require $\pi_{ix} \geq 0$ for some $u_{ix} \geq 0$. Given $s \geq e$, there then exists no $p_{ix} = r_{ix} > c$ such that $\pi_{ix} \geq 0$ for $u_{ix} \geq 0$ if $c - s$ is sufficiently large because i) $\pi_{ix} < 0$ is strictly negative when $c - s$ is at its largest value with $c = E(v)$ and $s = e$, for any relevant $p_{ix} = r_{ix} > c$, and ii) π_{ix} is weakly decreasing in $c - s$. □

Proof of Proposition 3. a) We know $r_o^m = s$ from Proposition 1. Then we can use (5) to show that $\frac{\partial \pi(p_{io}, s)}{\partial p_{io}} > 0$ until $u_{io} = 0$. Hence, p_o^m must be uniquely defined with $p_o^m = V(s)$. From inspection of (4), it is clear that $p_o^m = V(s) \geq E(v)$ given $s \geq e$. Therefore, from (5), $\pi(p_o^m, s) = p_o^m - c \geq 0$ given $E(v) \geq c$.

b) Provided the market is active, the monopolist will set $p_x^m = r_x^m$ to maximize (8) subject to consumers being willing to purchase, $u_{ix} \geq 0$ in (7). The proof now proceeds as follows. First, we characterize the unique unconstrained maximizer. Second, we characterize the constrained maximizer. Third, we show when the constrained maximizer applies in terms of e. Finally, we show that whenever the constrained maximizer applies, it is unique.

First, denote $\widehat{p}$ as the unconstrained maximizer of (8) which sets $\frac{\partial \pi(p_{ix}, p_{ix})}{\partial p_{ix}} = 1 - F(p_{ix} - e) - (p_{ix} - s) f(p_{ix} - e) = 0$. This is unique because the profit function is strictly concave over the relevant range, $\frac{\partial^2 \pi(p_{ix}, p_{ix})}{\partial p_{ix}^2} = -[2f(p_{ix} - e) + (p_{ix} - s) f'(p_{ix} - e)] < 0$ $\forall p_{ix} \in [s, b + e]$ given $f'(v) \geq 0$ for $v \in [a, b]$. From here, it also follows that $\widehat{p} \in (s, b + e)$. This interval is non-empty given $b > E(v) \geq c > s \geq e$. $\widehat{p}$ lies within this range because i) $\frac{\partial \pi(s,s)}{\partial p_{ix}} = 1 - F(s-e) > 0$, ii) $\frac{\partial \pi(b+e,b+e)}{\partial p_{ix}} = -(b+e-s) f(b) < 0$, and iii) $\frac{\partial^2 \pi(p_{ix},p_{ix})}{\partial p_{ix}^2} < 0 \; \forall p_{ix} \in [s, b+e]$.

Second, this unconstrained maximizer, $\widehat{p}$, will be the equilibrium monopoly price, $p_x^m = r_x^m = \widehat{p}$, if $u_{ix} = V(\widehat{p}) - \widehat{p} \geq 0$. If not, with $\widehat{p} > V(\widehat{p})$, then p_x^m will become constrained. Denote the constrained maximizer as $\overline{p} < \widehat{p}$ where $V(\overline{p}) - \overline{p} = 0$.

Third, we show that the unconstrained maximizer applies only if e is sufficiently small. Specifically, given $b > E(v) \geq c > s \geq e$, we now show there exists a unique $\overline{e} \in (0, E(v))$ that is the level of e that sets $V(\widehat{p}) - \widehat{p} = 0$ where $V(\widehat{p}) - \widehat{p} \geq 0$ if and only if $e \leq \overline{e}$. First, from (7), note that when $e = 0$, $V(\widehat{p}) - \widehat{p} = \int_{\widehat{p}}^{b} (v - \widehat{p}) f(v)\, dv > 0$ given $b > \widehat{p} > s$. Second, when $e = E(v) = \int_0^b v f(v) dv$, $V(\widehat{p}) - \widehat{p} = -\int_0^{\widehat{p}-e} v f(v) dv - \int_{\widehat{p}-e}^{b} (\widehat{p} - e) f(v) dv < 0$ given $b + e > \widehat{p} > s \geq e$. Third, totally differentiating $V(\widehat{p}) - \widehat{p}$ in (7) with respect to e yields $\frac{d}{de}(V(\widehat{p}) - \widehat{p}) = -\left(1 - \frac{\partial \widehat{p}}{\partial e}\right) F(\widehat{p} - e) - \frac{\partial \widehat{p}}{\partial e}$. This is strictly negative given $b + e > \widehat{p} > s \geq e$ and $\frac{\partial \widehat{p}}{\partial e} \in (0,1)$, where the latter can be shown from the FOC above:

$$\frac{\partial \widehat{p}}{\partial e} = -\frac{1}{\frac{\partial^2 \pi(p_{ix},p_{ix})}{\partial p_{ix}}} \frac{\partial^2 \pi(p_{ix}, p_{ix})}{\partial p_{ix} \partial e} = \frac{f(p_{ix} - e) + (p_{ix} - s) f'(p_{ix} - e)}{2 f(p_{ix} - e) + (p_{ix} - s) f'(p_{ix} - e)} \in (0,1)$$

Finally, we show that the constrained maximizer, $\overline{p} < \widehat{p}$, is unique whenever it applies for $e \in (\overline{e}, E(v))$. Recall that $\overline{p}$ is defined as the level of $\overline{p}$ such that $V(\overline{p}) - \overline{p} = 0$. To begin, note that i) $V(e) - e = \int_0^b v f(v)\, dv - e = E(v) - e > 0 \; \forall e < E(v)$; ii) $\frac{\partial}{\partial p_{i,x}}(V(p_{i,x}) - p_{i,x}) = -(1 - F(p_{i,x} - e)) < 0$ given $e \leq s < p_{ix} < \widehat{p} < b + e$; and iii) $V(\widehat{p}) - \widehat{p} < 0 \; \forall e > \overline{e}$ from above. □

Proof of Proposition 4. Given $u_o^m = V(s) - p_o^m = 0$, it follows that $\Delta u^m = u_x^m = V(p_x^m) - p_x^m$. Hence, using Proposition 3, we know $\Delta u^m > 0$ if $e < \overline{e}$ but $\Delta u^m = 0$ for $e \geq \overline{e}$. Using the identity, $\Delta w^m = \Delta \pi^m + \Delta u^m$ we can then state $\Delta \pi^m = \Delta w^m - \Delta u^m$. Using (12) and evaluating at p_x^m yields $\Delta \pi^m = -\int_{s-e}^{p_x^m - e} (v - (s-e)) f(v) dv - \Delta u^m$. It then follows that both $\Delta \pi^m$ and Δw^m are strictly negative given $p_x^m > s \geq e$. □

Proof of Proposition 5. a) We know $r_o^B = s$ from Proposition 1. Given this, competition then forces profits, π_{io} in (5), down to zero such that $p_o^B = c$. From inspection of (4), it is clear that $V(s) \geq E(v) > c$ such that $u_o^B > 0$. b) From (7), we know $u_{ix} = \int_{p_{ix}-e}^{b} (v - (p_{ix} - e)) f(v)\, dv - e$. This is decreasing in $p_{ix} = r_{ix}$ for the relevant range as

$\partial u_{ix}/\partial p_{ix} = -\int_{p_{ix}-e}^{b} f(v)\,dv = -(1-F(p_{ix}-e)) < 0$ for all $p \in [0,b)$ and $e \geq 0$. Thus, sellers will compete by lowering $p_{ix} = r_{ix}$ until the point where $\pi_{ix} = 0$. From the proof of Proposition 3, we know π_{ix} is strictly concave. Therefore, if $\pi_x^m \geq 0$, there exists a unique $p_x^B \in (c, p_x^m]$ that sets $\pi_{ix} \equiv (p_x^B - s) \cdot (1 - F(p_x^B - e)) - (c-s) = 0$. □

Proof of Proposition 6. Since $\Delta\pi^B = 0$, it follows from the identity, $\Delta u^B = \Delta w^B - \Delta\pi^B$, and (12) that $\Delta u^B = \Delta w^B = -\int_{s-e}^{p_x^B - e}(v-(s-e))f(v)\,dv$. This is negative given $p_x^B > s$. □

Proof of Lemma 2. The equilibrium follows immediately from Baye and Morgan (2001) for a given refund, $r_{i\kappa}^* \in \{s, p_{i\kappa}\}$, after adjusting for the different per-consumer profit function. One can verify that $H_\kappa^A(p_{i\kappa})$ is well-behaved with $H_\kappa^A(\underline{p}_\kappa) = 0$, $H_\kappa^A(p_\kappa^m) = 1$, and $H_\kappa^{A\prime}(p_{i\kappa}) \geq 0$ for $p_{i\kappa} \in [\underline{p}_\kappa, p_\kappa^m]$ given that $\pi_{i\kappa}(p_{i\kappa}; r_{i\kappa}^*)$ is strictly concave. Finally, to show that $\underline{p}_\kappa \geq c > s$, note that $\underline{p}_\kappa$ will be at its lowest when $A \to 0$ and $\sigma \to 1$ such that $\pi_{i\kappa}\left(\underline{p}_\kappa; r_{i\kappa}^*\right) \to 0$. But this then implies that $\underline{p}_\kappa \to p_\kappa^B$ which we know is weakly greater than c and strictly greater than s as $p_x^B > p_o^B = c > s$ from Proposition 5. □

Proof of Proposition 9. First, recall that $0 < \tilde{A}_x < \tilde{A}_o$. Second, note that the change in total seller profits $\Delta n\overline{\pi} \equiv (1-\sigma)\,\Delta\pi^m < 0$ for $A \in (0, \tilde{A}_x]$ as total seller profits then equal $(1-\sigma)\,\pi_\kappa^m + \frac{nA}{n-1}$ in each regimes. However, for $A \in (\tilde{A}_x, \tilde{A}_o)$, $\Delta n\overline{\pi} = \pi_x^m - (1-\sigma)\,\pi_o^m - \frac{nA}{n-1}$ as $n\overline{\pi}_x = \pi_x^m$. Given $\tilde{A}_x = \sigma\left(\frac{n-1}{n}\right)\pi_x^m$, this can be rewritten as $\Delta n\overline{\pi} = (1-\sigma)\,\Delta\pi^m + \frac{n}{n-1}(\tilde{A}_x - A)$ which is strictly negative for all σ as $A > \tilde{A}_x$. Third, note that the change in advertiser profits equals $\Delta\phi = nA\Delta\alpha$. We then know a) $\Delta\phi = -nA\alpha_o < 0$ if $A \in (\tilde{A}_x, \tilde{A}_o)$ as $\alpha_x = 0$ and $\alpha_o \in (0,1)$, and b) $\Delta\phi = nA(\alpha_x - \alpha_o) = -nA^{\frac{n}{n-1}}\left(\tilde{A}_x^{-\frac{1}{n-1}} - \tilde{A}_o^{-\frac{1}{n-1}}\right) < 0$ if $A \in (0, \tilde{A}_x]$ because $0 < \tilde{A}_x < \tilde{A}_o$. □

Appendix B: Proofs of Propositions 10 and 11

Proof of Proposition 10

Suppose $\sigma \to 1$. In this case, for a given policy regime, all the consumers are shoppers and will optimally expect to pay the lowest price in the market which we denote as $E(minp_\kappa)$. We know from existing results based on the Baye and Morgan framework, such as Morgan et al.

(2006b), that $E(minp_\kappa) = \underline{p}_\kappa + \int_{\underline{p}_\kappa}^{p_\kappa^m}(1-\alpha_\kappa H_\kappa^A(p))^n dp$. For what follows, it is useful to derive some features of $E(minp_\kappa)$. (The proof is relegated to the end of this appendix).

Lemma 3. *Suppose $\sigma \to 1$. For any given policy regime, κ, in equilibrium, the equilibrium expected minimum price, $E(minp_\kappa)$, lies in the range, $(p_\kappa^B, p_\kappa^m]$, where a) $lim_{A\to 0}E(minp_\kappa) = p_\kappa^B$, b) $E(minp_\kappa)|_{A\geq\tilde{A}_\kappa} = p_\kappa^m$, and c) $\frac{\partial E(minp_\kappa)}{\partial A} = \frac{n\left(E(minp_x)-\underline{p}_x\right)}{(n-1)A} > 0$ for $A \in (0, \tilde{A}_\kappa)$ where $lim_{A\to 0}\frac{\partial E(minp_x)}{\partial A} = \frac{n^2}{(n-1)\pi_{ix}'(p_x^B; r_x^*)} > 0$.*

Under regulation, given consumers expect to pay the expected minimum price, then they will also expect a refund equal to the expected minimum price, $r_x^* = E(minp_x)$. (The refund under no regulation remains equal to $r_o^* = s$.) It then follows from (1) that the expected consumer surplus per consumer equals

$$u_{i\kappa} = \int_{r_\kappa^*-e}^{b} vf(v)dv + \int_0^{r_\kappa^*-e}(r_\kappa^*-e)f(v)dv - E(minp_\kappa) \tag{16}$$

where $r_\kappa^* \in \{s, E(minp_x)\}$. Total welfare per consumer remains equal to

$$w_{i\kappa} = \int_{r_\kappa^*-e}^{b} vf\,(v)\,dv + \int_0^{r_\kappa^*-e}(s-e)\,f\,(v)\,dv - c \tag{17}$$

We are now in a position to prove results i) and ii) of Proposition 10. To do so, it is useful to employ the identity, $\Delta u = \Delta w - \Delta n\overline{\pi} - \Delta\phi$.

We begin with result i). Suppose $A \in (\tilde{A}_x, \tilde{A}_o)$. For this interval of A, we know from Lemma 2 that $\Delta w = -\int_{s-e}^{p_x^m-e}(v-(s-e))f\,(v)\,dv < 0$ as $E(minp_x) = p_x^m$, $\Delta n\overline{\pi} = \pi_x^m - \frac{nA}{n-1} < 0$ as $n\overline{\pi}_x = \pi_x^m$ and $\sigma \to 1$, and $\Delta\phi = nA\Delta\alpha = -nA\alpha_o < 0$ as $\alpha_x = 0$. By differentiating Δu, one obtains $\frac{\partial\Delta u}{\partial A} = \frac{\partial\Delta w}{\partial A} - \frac{\partial\Delta n\overline{\pi}}{\partial A} - \frac{\partial\Delta\phi}{\partial A}$ where $\frac{\partial\Delta w}{\partial A} = 0$, $-\frac{\partial\Delta n\overline{\pi}}{\partial A} = \frac{n}{n-1} > 0$ and $-\frac{\partial\Delta\phi}{\partial A} = n[A\frac{\partial\alpha_o}{\partial A} + \alpha_o] = n[1-(\frac{n}{n-1})\left(\frac{A}{\tilde{A}_o}\right)^{\frac{1}{n-1}}]$. Hence, $\frac{\partial\Delta u}{\partial A} = (\frac{n}{n-1}) + n[1-(\frac{n}{n-1})\left(\frac{A}{\tilde{A}_o}\right)^{\frac{1}{n-1}}]$. Then note $lim_{A\to\tilde{A}_o}\frac{\partial\Delta u}{\partial A} = (\frac{n}{n-1}) - (\frac{n}{n-1}) = 0$ and $\frac{\partial^2\Delta u}{\partial A^2} = -(\frac{n}{n-1})^2\left(\frac{A}{\tilde{A}_o}\right)^{\frac{1}{n-1}}\cdot\frac{1}{A} < 0$. Therefore, at $A = \tilde{A}_o$, Δu is at a maximum point and thus, for $A \in (\tilde{A}_x, \tilde{A}_o)$, $\frac{\partial\Delta u}{\partial A} > 0$. Then, at $A = \tilde{A}_o$ we know from Proposition 4 that $\Delta u = \Delta u^m = 0$ for any $e \geq \overline{e}$, and so it follows that $\Delta u < \Delta u^m = 0$ for all $A \in (\tilde{A}_x, \tilde{A}_o)$ if $e \geq \overline{e}$.

Next, we move onto result ii). Suppose $A \in (0, \tilde{A}_x)$. Given $\sigma \to 1$, we immediately know that $\Delta u = \Delta w - \Delta\phi$ as $\Delta n\overline{\pi} = 0$ from Proposition 9. Therefore,

$$\Delta u = \Delta w - \Delta\phi = -\int_{s-e}^{E(minp_x)-e}(v-(s-e))f\,(v)\,dv - nA\Delta\alpha$$

We now show that $\Delta u < \Delta u^B < 0$ holds as A marginally increases away from zero.

First, note that $\lim_{A\to 0}\Delta u = \lim_{A\to 0}\Delta w$ because $\lim_{A\to 0}\Delta\phi = 0$ given $\Delta\phi = nA\Delta\alpha = -nA^{\frac{n}{n-1}}\left(\tilde{A}_x^{-\frac{1}{n-1}} - \tilde{A}_o^{-\frac{1}{n-1}}\right)$.

Second, as $\lim_{A\to 0}E(minp_x) = p_x^B$, it follows that $\lim_{A\to 0}\Delta u = \Delta w^B = \Delta u^B < 0$ from Proposition 6.

Third, as A increases away from zero, it follows that

$$\frac{\partial\Delta u}{\partial A} = \frac{\partial\Delta w}{\partial A} - \frac{\partial\Delta\phi}{\partial A} = -(E(minp_x) - s)\cdot f\left(E(minp_x) - e\right)\cdot\frac{\partial E(minp_x)}{\partial A} - \frac{\partial nA\Delta\alpha}{\partial A} \tag{18}$$

where $\frac{\partial\Delta w}{\partial A} = -(E(minp_x) - s)f\left(E(minp_x) - e\right)\frac{\partial E(minp_x)}{\partial A} < 0$ using Lemma 3, and where $-\frac{\partial\Delta\phi}{\partial A} = -\frac{\partial nA\Delta\alpha}{\partial A} = n(\frac{n}{n-1})A^{\frac{1}{n-1}}\left(\tilde{A}_x^{-\frac{1}{n-1}} - \tilde{A}_o^{-\frac{1}{n-1}}\right) > 0$ for all $A \in (0, \tilde{A}_o)$ using Proposition 8.

Fourth, $\lim_{A\to 0}\frac{\partial\Delta u}{\partial A}$=$\lim_{A\to 0}\frac{\partial\Delta w}{\partial A}$ because $\lim_{A\to 0}\frac{\partial\Delta\phi}{\partial A} = 0$ and so by using (18), the sign of $\lim_{A\to 0}\frac{\partial\Delta u}{\partial A}$ hinges on the sign of $\lim_{A\to 0}\frac{\partial E(minp_x)}{\partial A}$ given $E(minp_x) > s \geq e$.

Fifth, using Lemma 3, we know $lim_{A\to 0}\frac{\partial E(minp_x)}{\partial A} = \frac{n^2}{(n-1)\pi'_{ix}(p_x^B;r_x^*)} > 0$. Thus, $lim_{A\to 0}\frac{\partial E(minp_x)}{\partial A} > 0$ such that $\lim_{A\to 0}\frac{\partial\Delta u}{\partial A} < 0$. Therefore, $\Delta u < \Delta u^B < 0$ for some A close to zero as required.

Proof of Proposition 11

Suppose $A \to 0$ such that $\lim_{A\to 0}\alpha_\kappa = 1$. In this case, for a given policy regime, note that shopper consumers (who optimally buy from the seller offering the lowest price) expect to pay the expected minimum price on the market, whereas non-shoppers (who buy from their local seller) expect to pay a price equal to the mean price. This implies that the two consumer types expect to pay different prices. The different prices are described in (19) and (20) where the latter versions of the expressions follow from integration by parts (see Morgan et al. 2006a).

$$E(p_\kappa) = \underline{p}_\kappa + \int_{\underline{p}_\kappa}^{p_\kappa^m}(1 - H_\kappa^A(p))dp \tag{19}$$

$$E(minp_\kappa) = \underline{p}_\kappa + \int_{\underline{p}_\kappa}^{p_\kappa^m}(1 - H_\kappa^A(p))^n dp \tag{20}$$

For what follows it is useful to derive some features of $E(p_\kappa)$ in the following lemma. (The proof is relegated to the end of this appendix. Note $E(minp_\kappa)$ behaves very similarly to $E(p_\kappa)$ but it is not necessary to detail at this point.)

Lemma 4. *Suppose $A \to 0$. For any given policy regime, κ, in equilibrium, the equilibrium expected price, $E(p_\kappa)$, lies in the range, $(p_\kappa^B, p_\kappa^m]$, where a) $lim_{\sigma\to 1}E(p_\kappa) = p_\kappa^B$, b) $lim_{\sigma\to 0}E(p_\kappa) = p_\kappa^m$, and c) $\frac{\partial E(p_\kappa)}{\partial \sigma} = -\frac{\left(E(p_x)-\underline{p}_x\right)}{(n-1)(1-\sigma)\sigma} < 0$ for $\sigma \in (0,1)$ where $lim_{\sigma\to 0}\frac{\partial E(p_x)}{\partial \sigma} = -\frac{\pi_x^m}{\pi_{ix}'(p_x^m; r_x^*)} < 0$.*

We can now form the associated welfare measures. The welfare measures are more involved in this setting because of the different consumer types and the equilibrium price distribution. Specifically, under either policy regime, each consumer type expects to pay different prices, $E(p_\kappa)$ and $E(minp_\kappa)$. Furthermore, under a mandatory refund policy, in addition to paying different prices, the consumer types also expect different refund levels. Hence, we can denote u_κ^{NS} and u_κ^S as the expected utility per-consumer of non-shoppers (NS) and shoppers (S) as below, where $r_\kappa^{NS} \in \{s, E(p_o)\}$ and $r_\kappa^S \in \{s, E(minp_o)\}$. It then follows that overall expected consumer surplus equals $u_\kappa = (1-\sigma)u_\kappa^{NS} + \sigma u_\kappa^S$.

$$u_\kappa^{NS} = \int_{r_\kappa^{NS}-e}^{b} vf(v)dv + \int_0^{r_\kappa^{NS}-e} (r_\kappa^{NS} - e)f(v)dv - E(p_\kappa) \tag{21}$$

$$u_\kappa^{S} = \int_{r_\kappa^{S}-e}^{b} vf(v)dv + \int_0^{r_\kappa^{S}-e} (r_\kappa^{S} - e)f(v)dv - E(minp_\kappa) \tag{22}$$

Finally, total welfare per transaction can be expressed as before in (6) because $r_\kappa^{NS} = r_\kappa^S = s$. However, under the refund policy, the consumer types expect different refunds and so we denote w_κ^{NS} and w_κ^S as the expected total welfare per transaction for non-shoppers (NS) and shoppers (S) as below using (9), where overall expected welfare per transaction equals $w_\kappa = (1-\sigma)w_\kappa^{NS} + \sigma w_\kappa^S$.

$$w_\kappa^{NS} \equiv \int_{r_\kappa^{NS}-e}^{b} vf(v)\,dv + \int_0^{r_\kappa^{NS}-e} (s-e)\,f(v)\,dv - c \tag{23}$$

$$w_\kappa^{S} \equiv \int_{r_\kappa^{S}-e}^{b} vf(v)\,dv + \int_0^{r_\kappa^{S}-e} (s-e)\,f(v)\,dv - c \tag{24}$$

We are now in a position to prove Proposition 11. To do so, it is useful to employ the identity, $\Delta u = \Delta w - \Delta n\overline{\pi}$ (as advertiser profits are not applicable, $\Delta\phi = 0$, given $A \to 0$). Therefore, using $n\overline{\pi} = (1-\sigma)\,\pi_\kappa^m$ from above given $A \to 0$, we can state

$$\Delta u = \Delta w - (1-\sigma)\Delta\pi^m.$$

We next show that $\Delta u > \Delta u^m$ as σ marginally increases away from zero.

First, note that $\lim_{\sigma\to 0}\Delta u = \Delta w^m - \Delta\pi^m = \Delta u^m \geq 0$. This follows from Proposition 4 because $\lim_{\sigma\to 0}\Delta w = \Delta w^m$ as $\lim_{\sigma\to 0} w_\kappa = w_\kappa^{NS} = \int_{r_\kappa^{NS}-e}^{b} vf(v)\,dv + \int_0^{r_\kappa^{NS}-e}(s-e)f(v)\,dv - c$ for $\kappa \in \{o, x\}$ where $r_o^{NS} = s$, $r_x^{NS} = E(p_x)$ and $\lim_{\sigma\to 0}E(p_x) = p_x^m$.

Second, as σ increases, it follows that

$$\frac{\partial\Delta u}{\partial\sigma} = \frac{\partial\Delta w}{\partial\sigma} - \frac{\partial(1-\sigma)\Delta\pi^m}{\partial\sigma} \equiv \frac{\partial w_x}{\partial\sigma} + \Delta\pi^m \tag{25}$$

as $\frac{\partial w_o}{\partial\sigma} = 0$. Then we know $\Delta\pi^m < 0$ from Proposition 4. Further, $\frac{\partial w_x}{\partial\sigma} > 0$ as $\frac{\partial w_x}{\partial\sigma} = (w_x^S - w_x^{NS}) + (1-\sigma)\frac{\partial w_x^{NS}}{\partial\sigma} + \sigma\frac{\partial w_x^S}{\partial\sigma}$ where $w_x^S - w_x^{NS} = \int_{E(minp_x)-e}^{E(p_x)-e}(v-(s-e))f(v)\,dv > 0$, and

$$\frac{\partial w_x^{NS}}{\partial\sigma} = -\left[E(p_\kappa) - s\right]\cdot f\left(E(p_\kappa) - e\right)\cdot\frac{\partial E(p_\kappa)}{\partial\sigma} \tag{26}$$

$$\frac{\partial w_x^S}{\partial\sigma} = -\left[E(minp_\kappa) - s\right]\cdot f\left(E(minp_\kappa) - e\right)\cdot\frac{\partial E(minp_\kappa)}{\partial\sigma} \tag{27}$$

Third, we evaluate (25) at the limit, $\sigma \to 0$. Note

$$lim_{\sigma\to 0}\frac{\partial\Delta u}{\partial\sigma} = lim_{\sigma\to 0}\left(\frac{\partial w_x}{\partial\sigma} + \Delta\pi^m\right) \tag{28}$$

where

$$lim_{\sigma\to 0}\frac{\partial w_x}{\partial\sigma} = lim_{\sigma\to 0}\left((w_x^S - w_x^{NS}) + \frac{\partial w_x^{NS}}{\partial\sigma}\right) = lim_{\sigma\to 0}\left(\frac{\partial w_x^{NS}}{\partial\sigma}\right)$$

because $lim_{\sigma\to 0}(w_x^S - w_x^{NS}) = lim_{\sigma\to 0}\int_{E(minp_x)-e}^{E(p_x)-e}(v-(s-e))f(v)\,dv = 0$ as $E(p_x)$ and $E(minp_x)$ tend to p_x^m via (versions of) Lemma 4.

From (26), we then know that the magnitude of $lim_{\sigma\to 0}\left(\frac{\partial w_x^{NS}}{\partial\sigma}\right)$ depends upon $lim_{\sigma\to 0}\frac{\partial E(p_x)}{\partial\sigma}$. Using Lemma 4, recall that $lim_{\sigma\to 0}\frac{\partial E(p_x)}{\partial\sigma} = -\frac{\pi_x^m}{\pi'_{ix}(p_x^m; r_{ix}^*)}$ and consider two cases that depend upon the level of e.

First, if $e \leq \overline{e}$, then we know from Proposition 3 $\pi'_{ix}(p_x^m; r_{ix}^*) = 0$ as a monopolist can optimally set the unconstrained maximizer, $p_x^m = \widehat{p}$. Therefore, $lim_{\sigma\to 0}\frac{\partial E(p_x)}{\partial\sigma} = -\infty$ and $lim_{\sigma\to 0}\left(\frac{\partial w_x^{NS}}{\partial\sigma}\right) = \infty$. Hence, from (23) and (28), it must be that $lim_{\sigma\to 0}\frac{\partial\Delta u}{\partial\sigma} = \infty$ given $\Delta\pi^m$ is a (negative) finite number and so $\Delta u > \Delta u^m > 0$ at $\sigma \to 0$. Such a level of $e \leq \overline{e}$ exists because we know from the proof of Proposition 3 that $\overline{e} > 0$ and is independent of σ.

Second, if $e > \overline{e}$, then we know from Proposition 3 that $\pi'_{ix}(p_x^m; r_{ix}^*) > 0$ as a monopolist becomes constrained with $p_x^m = \overline{p} < \widehat{p}$. Here, $lim_{\sigma \to 0} \frac{\partial \Delta u}{\partial \sigma}$ will still be positive if $e \downarrow \overline{e}$ such that $\pi'_{ix}(p_x^m; r_{ix}^*)$ is sufficiently close to zero and so $\Delta u > \Delta u^m = 0$ at $\sigma \to 0$. However, as $c \to \hat{c}(e,s)$ such that $\pi_x^m \to 0$, then $lim_{\sigma \to 0} \frac{\partial \Delta u}{\partial \sigma}$ can be negative because $lim_{\sigma \to 0} \frac{\partial E(p_x)}{\partial \sigma}$ becomes sufficiently close to zero to leave $lim_{\sigma \to 0} \frac{\partial \Delta u}{\partial \sigma} = lim_{\sigma \to 0} \left(\frac{\partial w_x}{\partial \sigma} + \Delta \pi^m \right)$ such that $\Delta u < 0 = \Delta u^m$ at $\sigma \to 0$.

Finally, we show that such a level of $e > \overline{e}$ can exist with $c \to \hat{c}(e,s)$. For example, we demonstrate that when $s \to c$, $\overline{e} \to 0$ such that $e > \overline{e}$ always applies given $e > 0$. As $s \to c$, we know from (14) that $\hat{c}(e,s) \equiv (p_x^m - c) \cdot (1 - F(p_x^m - e)) + c$. Given $s \to c$, this implies that $lim_{c \to \hat{c}(e,s)} \pi_x^m = (p_x^m - c) \cdot (1 - F(p_x^m - e)) = 0$ such that $\hat{c}(e,s) = p_x^m$. We now show that at $s \to c \to \hat{c}(e,s) = p_x^m$, it must be that p_x^m equals the constrained maximizer, $\overline{p} < \widehat{p}$, such that $V(\overline{p}) - \overline{p} = 0$. This follows by contradiction. Suppose instead that $p_x^m = \widehat{p}$. Then it follows from the FOC, $\frac{\partial \pi(p_{ix}, p_{ix})}{\partial p_{ix}} = 1 - F(p_{ix} - e) - (p_{ix} - s) f(p_{ix} - e) = 0$, that $\widehat{p} = b + e$ given $c = s = p_{ix}$. At $\widehat{p} = b + e$, we know $\widehat{p} > V(\widehat{p}) = \int_{\widehat{p}-e}^{b} v f(v) dv + \int_0^{\widehat{p}-e} (\widehat{p} - e) f(v) dv$ will always apply such that p_x^m is always constrained, $p_x^m = \overline{p}$. To derive this note that $\widehat{p} > V(\widehat{p})$ becomes $b + e > b \int_0^b f(v) dv$ when evaluated at $\widehat{p} = b + e$ and this condition always holds for any $e > 0$. Therefore, as the monopoly price is always constrained, $p_x^m = \overline{p}$, it must be that $\bar{e} = 0$ at $s \to c \to \hat{c}(e,s) = p_x^m = \overline{p}$ such that $e > \overline{e}$ always applies.

Proof of Lemma 3. b) If $A \geq \tilde{A}_\kappa$, $E(minp_\kappa) = p_\kappa^m$ as the sellers always price at p_κ^m. To show a) and c), let $z_\kappa(p_{i\kappa}, A) = \frac{n}{n-1} \frac{A}{\pi_{i\kappa}(p_{i\kappa}; r_{i\kappa}^*)}$ such that $H_\kappa^A(p_{i\kappa}) = \frac{1}{\alpha_\kappa^*} \left(1 - z_\kappa(p_{i\kappa}, A)^{\frac{1}{n-1}} \right)$ and $E(minp_\kappa) = \underline{p}_\kappa + \int_{\underline{p}_\kappa}^{p_\kappa^m} z_\kappa(p, A)^{\frac{n}{n-1}} dp$. To prove a) $\lim_{A \to 0} E(minp_\kappa) = p_\kappa^B$, note that $\lim_{A \to 0} E(minp_\kappa) = \underline{p}_\kappa$ and $\lim_{A \to 0} \underline{p}_\kappa^* = p_\kappa^B$ from the definition of $\underline{p}_\kappa^*$: $\pi_{i\kappa}\left(\underline{p}_\kappa; r_{i\kappa}^*\right) = \frac{nA}{n-1}$. To show b), use the Leibniz integral rule to differentiate $E(minp_\kappa)$ with respect to A to obtain $\int_{\underline{p}_\kappa}^{p_\kappa^m} \frac{n}{n-1} z_\kappa(p, A)^{\frac{1}{n-1}} \cdot \frac{\partial z_\kappa(p, A)}{\partial A} dp$ where $\frac{\partial z_\kappa(p_{i\kappa}, A)}{\partial A} = \frac{n}{n-1} \frac{1}{\pi_{i\kappa}(p_{i\kappa}; r_{i\kappa}^*)} = \frac{z_\kappa(p_{i\kappa}, A)}{A}$. Therefore, $\frac{\partial E(minp_\kappa)}{\partial A} = \frac{n}{A(n-1)} \int_{\underline{p}_\kappa^*}^{p_\kappa^m} z_\kappa(p, A)^{\frac{n}{n-1}} dp = \frac{n\left(E(minp_\kappa) - \underline{p}_\kappa\right)}{(n-1)A}$. This is strictly positive as $E(minp_\kappa) > \underline{p}_\kappa$ for $A \in (0, \tilde{A}_\kappa)$. At $A = 0$, $\frac{\partial E(minp_\kappa)}{\partial A}$ is an indeterminate form because both the numerator and denominator equal zero (as $E(minp_x)$ and $\underline{p}_x$ equal p_x^B). However, by applying L'Hôpital's rule, we know that $lim_{A \to 0} \frac{\partial E(minp_x)}{\partial A} = lim_{A \to 0} \frac{n\left(\frac{\partial E(minp_x)}{\partial A} - \frac{\partial \underline{p}_x}{\partial A}\right)}{(n-1)}$ which, after rearranging, gives $lim_{A \to 0} \frac{\partial E(minp_x)}{\partial A} = lim_{A \to 0} n \frac{\partial \underline{p}_x}{\partial A}$. From using the implicit function theorem and Lemma 2, we know $\frac{\partial \underline{p}_x}{\partial A} = \frac{n}{(n-1)\pi'_{ix}\left(\underline{p}_x; r_{ix}^*\right)}$. Hence, $lim_{A \to 0} \frac{\partial E(minp_x)}{\partial A} = \frac{n^2}{(n-1)\pi'_{ix}\left(p_x^B; r_{ix}^*\right)} > 0$ as $\underline{p}_x$ tends to $p_x^B < p_x^m$. □

Proof of Lemma 4. Let $z_\kappa(p_{i\kappa},\sigma) = \frac{(1-\sigma)\left[\pi_\kappa^m - \pi(p_{i\kappa}, r_{i\kappa}^*)\right]}{\sigma n \pi(p_{i\kappa}, r_{i\kappa}^*)} \geq 0$ such that $H_\kappa^A(p_{i\kappa}) = \left(1 - z_\kappa(p_{i\kappa},\sigma)^{\frac{1}{n-1}}\right)$ and $E(p_\kappa) = \underline{p}_\kappa + \int_{\underline{p}_\kappa}^{p_\kappa^m} z_\kappa(p,\sigma)^{\frac{1}{n-1}} dp$. To show a) $\lim_{\sigma\to 1} E(p_\kappa) = p_\kappa^B$ note that $\lim_{\sigma\to 1} E(p_\kappa) = \underline{p}_\kappa$ and $\lim_{\sigma\to 1}\underline{p}_\kappa^* = p_\kappa^B$ from the definition of $\underline{p}_\kappa$: $\pi(\underline{p}_\kappa, r_{i\kappa}^*) = \frac{(1-\sigma)\pi_\kappa^m}{[(1-\sigma)+\sigma n]}$. For b) $\lim_{\sigma\to 0} E(p_\kappa) = p_\kappa^m$ because $\lim_{\sigma\to 0}\underline{p}_\kappa = p_\kappa^m$. To show c), use the Leibniz integral rule to differentiate $E(p_\kappa) = \underline{p}_\kappa + \int_{\underline{p}_\kappa}^{p_\kappa^m} z_\kappa(p,\sigma)^{\frac{1}{n-1}} dp$ with respect to σ to obtain $\int_{\underline{p}_\kappa}^{p_\kappa^m} \frac{1}{n-1} z_\kappa(p,\sigma)^{\frac{1}{n-1}-1} \cdot \frac{\partial z_\kappa(p,\sigma)}{\partial\sigma} dp$. As $\frac{\partial z_\kappa(p_{i\kappa},\sigma)}{\partial\sigma} = \frac{-\left[\pi_\kappa^m - \pi(p_{i\kappa}, r_{i\kappa}^*)\right]}{\sigma^2 n \pi(p_{i\kappa}, r_{i\kappa}^*)} = -\frac{1}{\sigma(1-\sigma)} z_\kappa(p_{i\kappa},\sigma) \leq 0$, $\frac{\partial E(p_x)}{\partial\sigma} = -\frac{1}{\sigma(1-\sigma)(n-1)} \int_{\underline{p}_\kappa^*}^{p_\kappa^m} z_x(p,\sigma)^{\frac{1}{n-1}} dp = -\frac{\left(E(p_x) - \underline{p}_x\right)}{(n-1)(1-\sigma)\sigma}$. This is strictly negative for $\sigma \in (0,1)$ as $\underline{p}_\kappa < p_\kappa^m$. At $\sigma = 0$, $\frac{\partial E(p_x)}{\partial\sigma}$ is an indeterminate form because both the numerator and denominator equal zero (as $E(p_x)$ and $\underline{p}_x$ equal p_x^m). However, by applying L'Hôpital's rule, we know that $lim_{\sigma\to 0}\frac{\partial E(p_x)}{\partial\sigma} = lim_{\sigma\to 0} - \frac{\left(\frac{\partial E(p_x)}{\partial\sigma} - \frac{\partial \underline{p}_x}{\partial\sigma}\right)}{(n-1)}$ which, after rearranging, gives $lim_{\sigma\to 0}\frac{\partial E(p_x)}{\partial\sigma} = lim_{\sigma\to 0}\frac{1}{n}\frac{\partial \underline{p}_x}{\partial\sigma}$. From using the implicit function theorem and Lemma 2, we know $\frac{\partial \underline{p}_x}{\partial\sigma} = -\frac{(n-1)\pi_{ix}\left(\underline{p}_x; r_{ix}^*\right) + \pi_x^m}{\pi'_{ix}\left(\underline{p}_x; r_{ix}^*\right)\cdot(1-\sigma+n\sigma)}$. Therefore, $lim_{\sigma\to 0}\frac{\partial E(p_x)}{\partial\sigma} = lim_{\sigma\to 0}\frac{1}{n}\frac{\partial \underline{p}_x}{\partial\sigma} = -\frac{\pi_x^m}{\pi'_{ix}\left(p_x^m; r_{ix}^*\right)} < 0$ as $\underline{p}_x$ tends to p_x^m and $p_x^m \leq \widehat{p}$ such that $\pi'_{ix}(p_x^m; r_{ix}^*) \geq 0$. □

Supplementary Appendix: Buying from Multiple Sellers

In the context of unit demand, the main model assumed that it was prohibitively costly for consumers to buy from more than one seller. Although this assumption is common within the literature (e.g. Inderst and Tirosh 2015) and trivially satisfied under monopoly, consumers can sometimes buy from multiple sellers in practice before returning some unwanted purchases. In this section, we show how our results can remain robust when one relaxes this assumption to allow consumers to buy from more than one seller.

To do so, we make a few modifications to the main model (under the assumption there are $n \geq 2$ sellers). First, we focus on the case where the sellers sell identical experience goods such that a given consumer's match value is now the same across all sellers, v_m. Specifically, any given consumer's match value continues to be drawn from $F(v)$ and is still unknown until after a purchase is made, but the match value is now only consumer specific and does not vary across sellers. Second, contrary to the main model, after observing the sellers' prices and refunds, we allow each consumer to simultaneously purchase from any number of sellers, $k \leq n$, in Stage 2. Then, in Stage 3, any consumer m that purchased from $k \geq 1$ sellers, then learns v_m, before deciding how many, and which, of their purchases to return. For any consumer m, this Stage 3 decision reduces to a choice between returning $k-1$ or all k of their purchases as the consumer will never wish to keep more than one product given their unit demand. If a consumer returns multiple goods, we assume they incur the effort cost, $e \in [0, s)$, on each returned product as each product has to be returned to a different seller. In line with the main model, we assume that $r_i \geq s > e$ for any relevant seller i such that consumers are not completely deterred from making returns and sellers do not make losses from returns.

Under these conditions, we now show that all the results from the main model remain robust. This follows because, despite being able to buy from more than one seller, Proposition 12 demonstrates that consumer will never find it optimal to do so, even if sellers are allowed to have asymmetric prices and refunds.

Proposition 12. *Suppose the sellers sell identical experience goods. Then, even if consumers can simultaneously purchase from any $k \leq n$ sellers, they will never find it strictly optimal to buy from more than one seller.*

At first glance, this result may seem obvious because the purchase of an additional identical product may seem pointless. However, as we now explain, this is not so straightforward - the purchase of an additional product with a different refund level can generate subtle, qualitatively important effects on a consumer's return decisions.

To derive the result, suppose without loss of generality that the sellers' offers can be ranked in terms of their ex-ante expected net utility, $u_{i\kappa} \equiv V(r_{i\kappa}) - p_{i\kappa}$, such that $u_{1\kappa} \geq u_{2\kappa} \geq ... \geq u_{n\kappa}$. Further suppose that at least two sellers' offers exceed the zero outside option, $u_{2\kappa} > 0$.

If any consumer was going to buy only from a single seller, they would optimally buy from the seller with the highest ex-ante utility, seller 1. Following the logic of (1), the consumer would then keep the product only if $v \geq r_{1\kappa} - e$ and so gain an expected payoff equal to

$$u_{1\kappa} = -p_{1\kappa} + V(r_{1\kappa}) = -p_{1\kappa} + \int_{r_{1\kappa}-e}^{b} vf(v)dv + \int_{0}^{r_{1\kappa}-e} (r_{i\kappa} - e)f(v)dv \tag{29}$$

We now consider whether any consumer would ever find it optimal to also buy from any additional seller, $j \neq 1$. The expected utility of buying from both seller 1 and j can be expressed as follows.

$$u_{1j\kappa} = -p_{1\kappa} - p_{j\kappa} + \max\{r_{1\kappa}, r_{j\kappa}\} - e + V(\min\{r_{1\kappa}, r_{j\kappa}\}) \tag{30}$$

Intuitively, if the consumer buys from both sellers 1 and j, the consumer will always want to return at least one product because they only have a unit demand and because $r_{i\kappa} > e$ $\forall i$. To decide which product to return for sure, the consumer need only compare $r_{1\kappa} - e$ versus $r_{j\kappa} - e$ because the two products have a common value, v. Hence, aside from paying for both products, the consumer will always return the product with the highest refund to obtain $\max\{r_{1\kappa}, r_{j\kappa}\} - e$. The consumer then faces a decision of whether to keep the remaining product or to return that one as well. Following the logic of (1), the consumer will then keep the remaining product only if $v \geq \min\{r_{1\kappa}, r_{j\kappa}\} - e$ and this product will offer an expected gross utility of $V(\min\{r_{1\kappa}, r_{j\kappa}\}) = \int_{\min\{r_{1\kappa},r_{j\kappa}\}-e}^{b} vf(v)dv + \int_{0}^{\min\{r_{1\kappa},r_{j\kappa}\}-e} (\min\{r_{1\kappa}, r_{j\kappa}\} - e)f(v)dv$.

Given this, the consumer will prefer to buy from only seller 1 rather than buying from both sellers 1 and j if $u_{1j\kappa} - u_{1\kappa} \leq 0$. To examine $u_{1j\kappa} - u_{1\kappa}$, it is useful to consider two exhaustive cases depending on whether $r_{1\kappa} \leq r_{j\kappa}$ or $r_{1\kappa} > r_{j\kappa}$. Using (29) and (30), we can write

$$(u_{1j\kappa} - u_{1\kappa})|_{r_{1\kappa} \leq r_{j\kappa}} = -p_{1\kappa} - p_{j\kappa} + r_{j\kappa} - e + V(r_{1\kappa}) + p_{1\kappa} - V(r_{1\kappa}) = -p_{j\kappa} + r_{j\kappa} - e \tag{31}$$

$$(u_{1j\kappa} - u_{1\kappa})|_{r_{1\kappa} > r_{j\kappa}} = -p_{1\kappa} - p_{j\kappa} + r_{1\kappa} - e + V(r_{j\kappa}) + p_{1\kappa} - V(r_{1\kappa}) = -p_{j\kappa} + r_{1\kappa} - e + V(r_{j\kappa}) - V(r_{1\kappa}) \tag{32}$$

These expressions can be understood as follows. First, consider the simpler case in (31) where $r_{1\kappa} \leq r_{j\kappa}$. Here, the consumer knows that they will always return product j if they buy both products. The consumer's key decision of whether to keep the remaining product 1 is then identical to the return decision that the consumer would have faced had they only bought product 1. Therefore, the net benefit of additionally buying product j in (31) boils down to a simple comparison between product j's price, $p_{j\kappa}$, and its net return value, $r_{j\kappa} - e$. Hence, there can never be a strict net benefit from buying an additional product because $-p_{j\kappa} + r_{j\kappa} - e \leq 0$ given $r_{j\kappa} \leq p_{j\kappa}$ and $e \geq 0$.

Second, consider the more subtle case where $r_{1\kappa} > r_{j\kappa}$ in (32). Here, the intuition is more involved because if the consumer buys both products, they know that they will now always return product 1, rather than product j. Thus, the net benefit of additionally buying product j in (32) firstly involves a comparison between the extra price of buying product j, $p_{j\kappa}$, with the net return value of product 1, $r_{1\kappa} - e$. However, unlike the first case, the consumer's key decision of whether to keep the remaining product j now differs to the return decision that the consumer would have faced had they only bought product 1. Hence, the expression in (32) also involves an evaluation of the gross expected value of buying product j, $V(r_{j\kappa})$, with the gross expected value of buying product 1, $V(r_{1\kappa})$. To then evaluate the sign of $(u_{1j\kappa} - u_{1\kappa})|_{r_{1\kappa} > r_{j\kappa}}$, one can use $u_{1\kappa} \geq u_{j\kappa}$ to bound $p_{j\kappa} \geq V(r_{j\kappa}) - V(r_{1\kappa}) + p_{1\kappa}$. By then substituting this into (32), it follows that $(u_{1j\kappa} - u_{1\kappa})|_{r_{1\kappa} > r_{j\kappa}} \leq r_{1\kappa} - (p_{1\kappa} + e)$. This implies $(u_{1j\kappa} - u_{1\kappa})|_{r_{1\kappa} > r_{j\kappa}} \leq 0$ as $r_{1\kappa} - (p_{1\kappa} + e) \leq 0$ given $r_{1k} \leq p_{1\kappa}$ and $e \geq 0$.

Therefore, $(u_{1j\kappa} - u_{1\kappa}) \leq 0$ for $r_{1\kappa} \leq r_{j\kappa}$ or $r_{1\kappa} > r_{j\kappa}$. Hence, a consumer will never find it optimal to buy more than one product and all the results from the main paper remain robust.