Licensing Probabilistic Patents and Liability Rules: The Duopoly Case

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LICENSING PROBABILISTIC PATENTS AND LIABILITY RULES: THE DUOPOLY CASE

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Abstract. In a market with two homogeneous firms that compete in quantities (Cournot), one firm gets a patented cost reduction innovation. Under this scenario the patent holder has the option to license or not this innovation to the other firm. On the other side, the incumbent firm (without the patent) could be continuing with the backstop technology, could be an infringer of the patent or if a license is offered for the patent holder could become a licensee.

When the property rights are probabilistic, injunctions and damage payments play a fundamental role in the interaction between the patent holder and the incumbent firm. Two damage rules are commonly used in the courts to determine the size of the damage payment: Lost Profits and Unjust Enrichment.

In this paper a dynamic game is developed to compare lost profits (LP) against unjust enrichment (UE), It's assumed that the lifetime of innovation is short enough for injunctions not to be important for the players (patent holder and incumbent firm).

The results show that UE is at least as good as the LP rule from the point of view of the welfare. Under UE *no licensing* is chosen by the patent holder and *infringement* is chosen by the incumbent firm . Under the LP rule and small innovations is observable the same situation as in UE however if the innovation is enough big, Licensing is chosen by the patent holder and the incumbent firm.

1. Introduction

Once a researcher gets an innovation and obtains a patent, this patent becomes in a right to sue infringers, under the charges of unauthorized use of a property right, as was noted in the early literature (Lemley and Shapiro 2005) patents are not ironclad rights, instead could considered them as probabilistic rights (Encauoa and Laffouni).

Is well known that sometimes the patent holder decides to share the property right with others (licensing), a license is a settlement that permits to a third part uses the innovation in exchange of a pecuniary payment, commonly are used: Fixed fees, royalty rates and auctions.

In this context is natural to ask :

(1) Which is the best mechanism used for the patent holder to give licenses? (fixed fees, royalty rates, autions, and so on).
(2) How many licenses the patent holder will offer? (maybe no one).
(3) How much the patent holder can ask for his innovation? (depends on the threat points)

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(4) How much this responses changes when the patent holder is a firm inside the market or just an inventor?
(5) How the industry's structure and the kind of competition affect the results? (monopolist/oligopolist/perfect competition)

There is a huge literature that have analyzed this questions under ironclad property rights, the common approach used has been game theory. In this approach the patent holder and one or several incumbent firms are players in a dynamic game of three stages: At the first stage of the game, the patent holder decides how much ask for the licenses and how many licenses he will offer. At the second stage the incumbents or potential licensees decide to get the license of continues using the backstop technology [1]. Finally in the last stage, firms choose quantities (Cournout competition) [2].

However, uncertainty over the property rights plays a important role in licensing, because under probabilistic property rights potential users of the innovation could decide infringe the patent if the patent strength is not so high. Also, this fact will change the threat points of the bargain between the patent holder and the other firms, when the patent holder wants to sell and the potential licensees want to buy a license.

If incumbent firms decide infringe the patent, the patent holder can enforce the property rights by using the legal system, in this arena the patent holder needs to prove *infringement* and if the patent holder is successful in to prove infringement, the court can authorize the pay of *damages*, in such way that the patent holder will be compensated by the infringement. The court could use also extra payments as three times damages i.e. or add injunction order[3].

Commonly are used two damage rules: Lost Profits (LP) and Unjust Enrichment (UE). Both of then try to compensate the patent holder using a profile scenario of no infringement, where the LP rule calculate damages comparing the profit of the patent holder and the UE rule makes the same but instead compare the profits of the infringer.

This article wants to study the relationship between damage rules and licensing, given that some recent articles have concluded that (LP) and (UE) rules do not deter infringement [4], licensing probabilistic patents clearly becomes different compared with the licensing of ironclad patents in the early literature.

This article is divided in several sections, in the section two is established the set up and is defined the licensing game, sections three, four and five are devoted to solve the game, section six gives a comparative welfare analysis between (LP) and (UE), the section seven gives conclusions and important remarks of this article and finally detailed proofs are showed in an appendix.

2. The set up

Lets consider a situation called two-supplier world in which there are two suppliers of a product: *firm* 1 and *firm* 2. Both firms produced the same good under a fixed marginal cost C, then the cost function is $f(Q_i) = CQ_i$ where $0 < C < A < \infty$ and Q_i is the quantity offered by the firm $i = 1, 2$.

[1] The best technology available without the use of the innovation
[2] see [4] and [5] for a survey about licensing games under ironclad rights
[3] In this case the infringer needs to stop the infringement action
[4] see [1] and [3]

The firm 1 (Patent Holder) gets a patented cost reduction innovation that reduces the cost from C to $C - \epsilon$, where $0 < \epsilon < C$ and the patent holder has the option to license this technology to the firm 2 (the incumbent firm), by using a fixed fee or a royalty rate.

A licensing game consists of three stages. In the first stage the firm 1 decides to license (l) or do not (n) the innovation, if decides licensing, it is established a fixed fee (F) or a royalty rate(R). In the second stage the firm 2 decides between three alternatives: 1) accept the offer of the patent holder ($\mathcal{L}$); 2) reject the offer and use the backstop technology ($\mathcal{N}$) and 3) Infringe the patent ($\mathcal{I}$) (see figure below). Finally the firms decide the quantities offered the market.

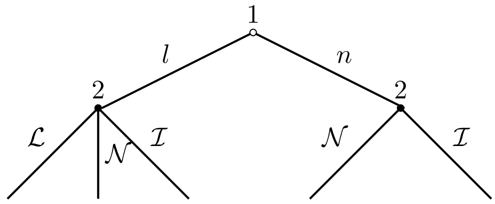

FIGURE 1. Extensive game form the first and second stages

In the last stage the two firms compete *ala Cournot* and face a inverse linear demand $P = A - Q$, where $Q = Q_1 + Q_2$ is the total quantity offered by the two firms,and Q_1, Q_2 are the quantities produced by the patent holder (firm 1) and the incumbent firm (firm 2) respectively.

This is a game of complete information, then if the firm 2 infringes the patent, the firm 1 will be aware of that and will try to enforces its rights in a court, where is assumed that the firm 1 has the common knowledge probability $\theta \in (0, 1)$ of success in the sue and if this happens the firm 2 will pay damages by D [5]. Is assumed also that when the court determines the pay of damages this payments are calculated always by the same rule (LP or UE). However in the case of licensing the patent holder will ask for a fixed fee $F > 0$ or a royalty rate R but not both of them, in consequence the case of $F < 0$ and $R > 0$ is not analyzed.

Without loss of generality, when Π_i is denoted the profit on the firm $i = 1, 2$ the expression $\pi_i = \Pi_i/(A - C)^2$ is the profit normalized by $(A - C)^2$ in analog way $q_i = Q_i/(A - C)$, $\gamma = \epsilon/(A - C)$, $f = F/(A - C)^2$ and $r = R/(A - C)$. This article will use extensive the terms π_i, q_i , γ, f and r, this way could be considered equivalent to set $A = 1$ and $C = 0$ also.

The concept used for solve this game will be Sub-Game Perfect Nash Equilibrium (SPNE), that is a backward solution method where the complete solution is a Nash Equilibrium (NE) at each stage of the game, the next section calculates different Nash Equilibriums in the quantity competition stage.

3. Competition stage

Given a defined rule for the calculations of damages (LP or UE), a level of technology chosen by the incumbent firm ($\mathcal{N}$, $\mathcal{B}$, $\mathcal{L}$) and a licensing policy defined

[5]An injunction order could be considered, but we assume that the lifetime of innovation is enough small for injunction orders have no impact in the payoffs of the players

by the patent holder (to offer or not a license to the incumbent firm by using a fixed fee or royalty rate), both firms compete by choosing quantities.

At the case when the firm 2 decides to use the backstop technology ($\mathcal{N}$), the NE is

$$(1) \quad q_1^{\mathcal{N}} = \begin{cases} \dfrac{1+2\gamma}{3} & \text{if } 0<\gamma<1 \\ \dfrac{1+\gamma}{2} & \text{if } 1\leq\gamma \end{cases} ; q_2^{\mathcal{N}} = \begin{cases} \dfrac{1-\gamma}{3} & \text{if } 0<\gamma<1 \\ 0 & \text{if } 1\leq\gamma \end{cases}$$

and the following payoffs are obtaining for the firms 1 and 2.

$$(2) \quad \pi_1^{\mathcal{N}} = \begin{cases} \left(\dfrac{1+2\gamma}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\dfrac{1+\gamma}{2}\right)^2 & \text{if } 1\leq\gamma \end{cases} ; \pi_2^{\mathcal{N}} = \begin{cases} \left(\dfrac{1-\gamma}{3}\right)^2 & \text{if } 0<\gamma<1 \\ 0 & \text{if } 1\leq\gamma \end{cases}$$

As was noted for the early literature [2] big innovations could permit to the patent holder to reduce the price till levels below the competitive prices, meaning that just the patent holder can remain in the market, this kind of innovations are called drastic. In this particular setup an innovation is non drastic if $0 \leq \gamma < 1$ and is defined drastic if $\gamma \geq 1$. By looking in the eq. (1) is straightforward to understand why if $\gamma \geq 1$, this cost reduction is drastic enough for convert a duopoly in a monopoly situation.

A more complex situation emerges when the firm 2 infringes the patent, given that this is a game of perfect information once infringement is played the patent holder will try to enforce the property rights by suing the incumbent firm. It is assumed too that if the patent holder is successful in the court, the court will ask just for damages [6], in this case the profit functions are structurally as

$$(3) \quad \pi_1 = (1-q_1-q_2+\gamma)q_1 + \theta d(q_1,q_2)\,;\pi_2 = (1-q_1-q_2+\gamma)q_2 - \theta d(q_1,q_2)$$

notice that the profit is based in two components (see eq.(3)), the first is the part gained by the sales and the second part are the damage payments[7], damages could be calculated in different ways, the most common way to do it is using the LP rule or the UE rule [8]. LP and UE are based in a profile scenario, the scenario of no infringement, the idea behind LP is to compensate the share of profit loss by the patent holder caused by the infringement, and in the case of UE transfer the share of the incumbent's profit that is above its profit under no infringement to the patent holder.

In this article $\pi_i^{\mathcal{N}}$ are used as the profile profits, then in the case of LP the damage payments are calculated as the loss profit of the firm 1 compared as the situation of no infringement, then

$$(4) \qquad d^{LP} = \max\left\{\pi_1^{\mathcal{N}} - (1-q_1-q_2+\gamma)q_1, 0\right\}$$

[6] The case when the court choose a injunction order and no damage payments is broadly studied in Shapiro (2008)

[7] Damages without the normalization by the factor are denoted as D and $d = D/(A-C)^2$

[8] see Cotter et al for a complete analysis about damage rules.

and in the case of UE, the damages are calculated in base to excess of profit for the firm 2 respect to its position without infringement, then

$$d^{UE} = (1 - q_1 - q_2 + \gamma)q_2 - \pi_2^{\mathcal{N}} \tag{5}$$

The NE when damages are calculated by using the LP rule, and when the incumbent firm decides to infringe the patent deserve a special treatment and here its just showed some relevant results[9], in equilibrium the firm 2 will offer the quantity

$$q_2^{\mathcal{I},\mathcal{LP}} = \begin{cases} q_2^{\mathcal{N}} & \text{if } 0 < \gamma \le \theta/(3-2\theta) \\ (1-\theta)\dfrac{1+\gamma}{3-\theta} & \text{if } \theta/(3-2\theta) < \gamma \end{cases} \tag{6}$$

A careful view in the profit expression of the firm 1 eq.(eqdam) shows that the response functions for the patent holder is exactly the same as when the firm 2 uses the backstop technology ($\mathcal{N}$), then when when $q_2^{\mathcal{I},\mathcal{LP}} = q_2^{\mathcal{N}}$ eq. ([**?**]), the patent holder gets the same profit that in the situation of no infringement if $0 < \gamma \le \theta/(3-2\theta)$, in the words of Anton and Yao [1] the infringer acts as a *Passive Infringer*, because the firm 1 cannot see the effects of the infringement cause on its profit has not changed, and when $q_2^{\mathcal{I},\mathcal{LP}} > q_2^{\mathcal{N}}$ the firm 2 acts as an *Active Infringer* just when $\gamma > \theta/(3-2\theta)$, then by using the last result is straightforward to get the payoffs for the case of damages calculated using the LP rule

$$\begin{aligned} \pi_1^{\mathcal{I},\mathcal{LP}} &= \begin{cases} \left(\dfrac{1+2\gamma}{3}\right)^2 & \text{if } 0 < \gamma \le \theta/(3-2\theta) \\ (1-\theta)\left(\dfrac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\dfrac{1+2\gamma}{3}\right)^2 & \text{if } \theta/(3-2\theta) < \gamma < 1 \\ (1-\theta)\left(\dfrac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\dfrac{1+\gamma}{2}\right)^2 & \text{if } 1 \le \gamma \end{cases} \\ \pi_2^{\mathcal{I},\mathcal{LP}} &= \begin{cases} \left(\dfrac{1+2\gamma}{3}\right)\left(\dfrac{1-\gamma}{3}\right) & \text{if } 0 < \gamma \le \theta/(3-2\theta) \\ \left(\dfrac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\dfrac{1+2\gamma}{3}\right)^2 & \text{if } \theta/(3-2\theta) < \gamma < 1 \\ \left(\dfrac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\dfrac{1+\gamma}{2}\right)^2 & \text{if } 1 \le \gamma \end{cases} \end{aligned} \tag{7}$$

and in the case when damage payments are calculated by using the UE rule the payoffs become

[9]interested readers cold see Anton and Yao [1]

$$\pi_1^{\mathcal{I},\mathcal{UE}} = \begin{cases} \left(\frac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\frac{1-\gamma}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\frac{1+\gamma}{3-\theta}\right)^2 & \text{if } 1\leq\gamma \end{cases} \tag{8}$$

$$\pi_2^{\mathcal{I},\mathcal{UE}} = \begin{cases} (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1-\gamma}{3}\right)^2 & \text{if } 0<\gamma<1 \\ (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 & \text{if } 1\leq\gamma \end{cases}$$

When the patent holder decides offer a license by using a fixed fee (F) or a royalty rate (R) and the incumbent firm decides to buy the license, the following payoffs are obtained

$$\pi_1^{\mathcal{L},\mathcal{F}} = \left(\frac{1+\gamma}{3}\right)^2 + f\,; \pi_2^{\mathcal{L},\mathcal{F}} = \left(\frac{1+\gamma}{3}\right)^2 - f \tag{9}$$

and

$$\pi_1^{\mathcal{L},\mathcal{R}} = \left(\frac{1+\gamma+r}{3}\right)^2 + r\frac{1+\gamma-2r}{3}\,; \pi_2^{\mathcal{L},\mathcal{R}} = \left(\frac{1+\gamma-2r}{3}\right)^2 \tag{10}$$

where $f = F/(A-C)^2$ and $r = R/(A-C)$.

4. Technology stage

In the second stage of the game the firm 2 have to decide among the technology that will be used, three situations arise, the first one use the backstop technology $\mathcal{N}$, the second one is to use the new cost reduction innovation trough a licensee $\mathcal{L}$ and at last to infringe the patent $\mathcal{I}$.

In this stage of the game the firm will take its decision by direct comparison of the expected payoffs in each situation $\pi_2^{\mathcal{N}}$, $\pi_2^{\mathcal{L}}$ and $\pi_2^{\mathcal{I}}$, is expectable too that $\pi_2^{\mathcal{L}}$ and $\pi_2^{\mathcal{I}}$ differ depending on the rule used to calculate damages payments, the first important result is summarized in the Lemma 1 below.

Lemma 1. *If the courts calculates damages using the* LP *rule or the* UE *rule* $\pi_2^{\mathcal{I}} > \pi_2^{\mathcal{N}}$.

The Lemma 1 says that that in any situation, infringement is always best than uses the backstop technology for the incumbent firm, then in this case the liability rules do not deter infringement, in the case of both rules (LP and UE) such result is expected, because the damages are calculated as the loss profit of the patent holder or the excess of profit of the infringer compared with them under a situation of no infringement. Then if the patent holder is a neutral risk agent easily will realizes that in expectation infringement gives at least the same profit as not infringe and use the backstop technology, a more general proof of this fact could be find it in Anton and Yao [1] for the case of Lost Profits rule and non-drastic innovations.

The Lemma 1 makes irrelevant the action $\mathcal{N}$ for the firm 2, but even due we cannot order completely the actions of the firm 2, because we need to determine the values of the profits under the license option that depends on a fixed fee or on a royalty rate, that for now are given. By using (9) and (10), is easy to see that for a fixed fee the value that make license as good as infringe is

$$\underline{f} = \left(\frac{1+\gamma}{3}\right)^2 - \pi_2^I \tag{11}$$

, notice too that if $\underline{f}$ is negative there is no positive fixed fee or royalty that makes the license option as good as infringe. Then is straightforward to obtain that in the case of lost profits

$$\underline{f}^{\mathcal{LP}} = \begin{cases} \left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+2\gamma}{3}\right)\left(\frac{1-\gamma}{3}\right) & \text{if } 0 < \gamma \leq \theta/(3-2\theta) \\ \left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1+2\gamma}{3}\right)^2 & \text{if } \theta/(3-2\theta) < \gamma < 1 \\ \left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1+\gamma}{2}\right)^2 & \text{if } 1 \leq \gamma \end{cases} \tag{12}$$

and in the case of UE

$$\underline{f}^{\mathcal{UE}} = \begin{cases} \left(\frac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\frac{1-\gamma}{3}\right)^2 & \text{if } 0 < \gamma < 1 \\ \left(\frac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 & \text{if } 1 \leq \gamma \end{cases} \tag{13}$$

Unfortunately this expression is sometimes lower than zero, in particular

Lemma 2. $\underline{f}^{\mathcal{UE}} > 0$ *if* $\gamma_1 = \dfrac{12 - 5\theta + \theta^2 - 2\sqrt{(3-\theta)^2(3+\theta)}}{6 - 7\theta + \theta^2} < \gamma$, *where* $\gamma_1 < 1$

Notice too that that is possible also define a minimum positive royalty rate that makes license as good as infringement for the incumbent firm

$$\underline{r} = (1 + \gamma - 3\sqrt{\pi_2^{\mathcal{I}}})/2 \tag{14}$$

in consequence when $\underline{r} \geq 0$ if and only if $\underline{f} \geq 0$, then we can stablish

Lemma 3. *In the case of LP always exists a fixed fee or royalty rate that makes the action $\mathcal{L}$ as good as other actions for the incumbent firm, but in the case of UE there is this fixed fee or royalty rate just if $\gamma > \gamma_1$.*

5. Licensing stage

It is supposed in the model that the patent holder gives a take it or leave it offer to the incumbent firm, in the case of the fixed fee the patent holder gets $\pi_1^{\mathcal{L},\mathcal{F}} = \left(\frac{1+\gamma}{3}\right)^2 + f$ then the maximum fixed fee that the patent holder can ask is $f = \max\{\underline{f}, 0\}$, by using the payoffs when the patent holder licenses by using a fixed fee and when the incumbent firm buys the license are

(15)

$$\pi_1^{\mathcal{L},\mathcal{F},\mathcal{UE}} = \begin{cases} 2\left(\frac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\frac{1-\gamma}{3}\right)^2 & \text{if } \gamma_1 < \gamma < 1 \\ 2\left(\frac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 & \text{if } \gamma \geq 1 \end{cases}$$

for the case of UE rule and in the case of lost profits

$$\pi_1^{\mathcal{L},\mathcal{F},\mathcal{LP}} = \begin{cases} 2\left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+2\gamma}{3}\right)\left(\frac{1-\gamma}{3}\right) & \text{if } \gamma \leq \frac{\theta}{3-2\theta} \\ 2\left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1+2\gamma}{3}\right)^2 & \text{if } \gamma > \frac{\theta}{3-2\theta} \\ 2\left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1+\gamma}{2}\right)^2 & \text{if } \gamma \geq 1 \end{cases} \quad (16)$$

When the patent holder licenses trough a royalty rate, the royalty rate is chosen in a way that maximizes the patent holder's profits

$$\pi_1^{\mathcal{L},\mathcal{R}} = \left(\frac{1+\gamma+r}{3}\right)^2 + r\frac{1+\gamma-2r}{3}$$

, if the patent holder hasn't any restriction the equilibrium royalty rate will be $r = (1+\gamma)/2$, however the patent holder just can choose a r inside the interval $[0, \underline{r}]$, where $\underline{r} = (1+\gamma-3\sqrt{\pi_1^{\mathcal{I}}})/2$ the in a consequence that in probabilistic patents $\pi_1^{\mathcal{I}} \geq 0$, the patent holder will choose $r = \max\{\underline{r}, 0\}$ [10].

Proposition 1. *Licensing by a royalty rate is preferred than licensing by a fixed fee for the patent holder for any positive fixed fee,* $\pi_1^{\mathcal{L},\mathcal{R}} - \pi_1^{L,F} > 0$.

The proposition above is very important because shows that the patent holder in our particular setup will always license via royalty rates, its important also remark that this result is obtained just in the case where licensing is affordable (Lemma 3), in consequence is necessary now to know when the patent holder will offer a license, this could be obtained for a direct comparison between the profit under licensing $\pi_1^{\mathcal{L},\mathcal{R}}$ and the profit under no licensing $\pi_1^{\mathcal{I}}$.

When the UE rule is used by the court $\pi_1^{\mathcal{L},\mathcal{R},\mathcal{UE}} < \pi_1^{\mathcal{I},\mathcal{UE}}$ meaning that there is not enough incentives for the patent holder for to offer a license, a similar situation is obtained when LP rule is used but when the innovation is enough big licensing is profitable for the patent holder, in particular when $\gamma > \dfrac{\theta(3-2\theta)+3\sqrt{(3-\theta)^2(2-\theta)}}{18-15\theta+4\theta^2} = \gamma_2$, $\pi_1^{\mathcal{L},\mathcal{R},\mathcal{LP}} > \pi_1^{\mathcal{I},\mathcal{LP}}$.

Proposition 2. *The patent holder will never license under UE (drastic or not) a similar situation is obtained when LP rule is used but when the innovation is enough big* $(1 < \gamma_2 < \gamma)$ *licensing is more profitable for the patent holder.*

In the following graph is observable in dark the areas where the patent holder offers a license payed by a royalty rate, notice that only big innovations and licensed by using royalty rates when LP rule is used in the calculation of damage payments.

[10] The expressions for $\pi_1^{\mathcal{L},\mathcal{R}}$ and $\pi_1^{\mathcal{L},\mathcal{R}}$ are complex for be written explicitly, instead, implicit expression where enough for get the results of this study

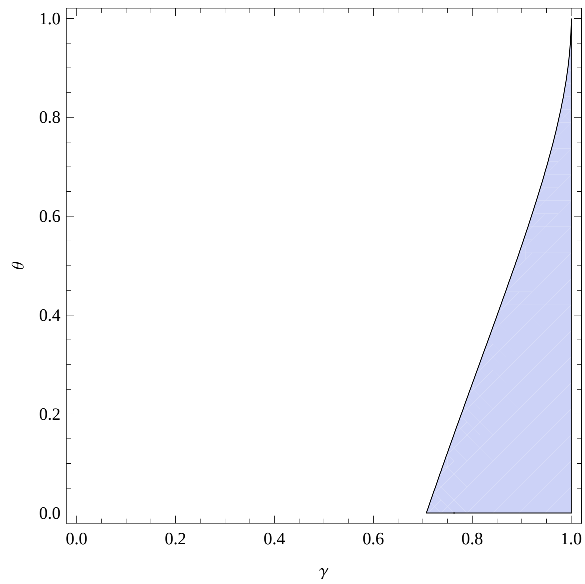

FIGURE 2. $\pi_1^{\mathcal{L},\mathcal{R},\mathcal{LP}} - \pi_1^{\mathcal{I},\mathcal{LP}} > 0$

6. Conclusions and remarks

One work that is related with this work is the work of Wang [6], in this work he analyzes the ironclad case, his results are sumarizes in three points:

(1) Under fixed-fee licensing, firm 1 will license its innovation to firm 2 if and only if $\gamma < 2/3$. In particular, firm 1 will become a monopoly when the innovation is drastic.
(2) Under royalty licensing, firm 1 will license its innovation to firm 2 if the innovation is non-drastic. In the case of a drastic innovation, firm 1 will become a monopoly.
(3) With either a non-drastic or a drastic innovation, licensing by means of a royalty is at least as good as licensing by means of a fee for the patent-holding firm (firm 1), and licensing by means of a fee is at least as good as licensing by means of a royalty for consumers

our results show that

(1) Under fixed-fee licensing, firm 1 will never license its innovation to firm 2.
(2) Under royalty licensing, firm 1 will license its innovation to firm 2 if the innovation is greater and just if the liability rule is LP.

The reason for this differences is consequence that liability rules and probabilistic patents changes the threats points in the bargaining for licenses, in fact under probabilistic rights the incumbent obtain a greater profit without licensing in comparison with the case of ironclad patents and no licensing.

For another part, the liability rules becomes a transfer system between the infringer and the patent holder making in this way a more profitable infringement for the infringer and for the patent holder.

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Appendix

Result 1. $\Pi_1 = (A - C - Q_1 - Q_2 + \epsilon)Q_1$ and $\Pi_2 = (A - C - Q_1 - Q_2)Q_2$ after solving the f.o.c. of the problem we get the reaction functions defined as

(17)
$$\rho_1^{\mathcal{N}}(Q_2,\epsilon) = \begin{cases} 0 & \text{if } Q_2 > A - C + \epsilon \\ \dfrac{A - C - Q_2 + \epsilon}{2} & \text{if } 0 < \gamma < 1 \\ \dfrac{A - C + \epsilon}{2} & \text{if } 1 \leq \gamma \end{cases} ; \rho_2^{\mathcal{N}}(Q_2,\epsilon) = \begin{cases} 0 & \text{if } Q_1 > A - C \\ \dfrac{A - C - Q_1}{2} & \text{if } 0 < \gamma < 1 \\ 0 & \text{if } 1 \leq \gamma \end{cases}$$

with that we get the following

$$q_1^{\mathcal{N}} = \begin{cases} \dfrac{1 + 2\gamma}{3} & \text{if } 0 < \gamma < 1 \\ \dfrac{1+\gamma}{2} & \text{if } 1 \leq \gamma \end{cases} ; q_2^{\mathcal{N}} = \begin{cases} \dfrac{1-\gamma}{3} & \text{if } 0 < \gamma < 1 \\ 0 & \text{if } 1 \leq \gamma \end{cases} \tag{18}$$

then

(19)
$$Q_1^{\mathcal{N}} = \begin{cases} \dfrac{A - C + 2\epsilon}{3} & \text{if } 0 < \gamma < 1 \\ \dfrac{A - C + \epsilon}{2} & \text{if } 1 \leq \gamma \end{cases} ; Q_2^{\mathcal{N}} = \begin{cases} \dfrac{A - C - \epsilon}{3} & \text{if } 0 < \gamma < 1 \\ 0 & \text{if } 1 \leq \gamma \end{cases}$$

and with that

$$Q^{\mathcal{N}} = \begin{cases} \dfrac{2A - 2C + \epsilon}{3} & \text{if } 0 < \gamma < 1 \\ \dfrac{A - C + \epsilon}{2} & \text{if } 1 \leq \gamma \end{cases} \tag{20}$$

$$P^{\mathcal{N}} = \begin{cases} \dfrac{A + 2C - \epsilon}{3} & \text{if } 0 < \gamma < 1 \\ \dfrac{A + C - \epsilon}{2} & \text{if } 1 \leq \gamma \end{cases} \tag{21}$$

then

(22)

$$\Pi_1^{\mathcal{N}} = \begin{cases} \left(\dfrac{A-C+2\epsilon}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\dfrac{A-C+\epsilon}{2}\right)^2 & \text{if } 1\le\gamma \end{cases} ; \Pi_2^{\mathcal{N}} = \begin{cases} \left(\dfrac{A-C-\epsilon}{3}\right)^2 & \text{if } 0<\gamma<1 \\ 0 & \text{if } 1\le\gamma \end{cases}$$

then the Total profit will be

$$T^{\mathcal{N}} = \begin{cases} \left(\dfrac{A-C+2\epsilon}{3}\right)^2 + \left(\dfrac{A-C-\epsilon}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\dfrac{A-C+\epsilon}{2}\right)^2 & \text{if } 1\le\gamma \end{cases} \tag{23}$$

because the demand is linear we get that the consumer surplus is $CSP = AQ - Q^2/2 - PQ = Q^2/2$ then

$$CSP^{\mathcal{N}} = \begin{cases} \left(\dfrac{2A-2C+\epsilon}{3}\right)^2/2 & \text{if } 0<\gamma<1 \\ \left(\dfrac{A-C+\epsilon}{2}\right)^2/2 & \text{if } 1\le\gamma \end{cases} \tag{24}$$

then the welfare will be

(25)

$$TW^{\mathcal{N}} = \begin{cases} \left(\dfrac{2A-2C+\epsilon}{3}\right)^2/2 + \left(\dfrac{A-C+2\epsilon}{3}\right)^2 + \left(\dfrac{A-C-\epsilon}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\dfrac{A-C+\epsilon}{2}\right)^2/2 + \left(\dfrac{A-C+\epsilon}{2}\right)^2 & \text{if } 1\le\gamma \end{cases}$$

□

Result 2. We will begin with the LP case $\Pi_1 = (A-C-Q_1-Q_2+\epsilon)Q_1 + \theta D(Q_1,Q_2)$ and $\Pi_2 = (A-C-Q_1-Q_2+\epsilon)Q_2 - \theta D(Q_1,Q_2)$ in the case of lost profits $D(Q_1,Q_2)^{\mathcal{LP}} = \max\left\{\Pi_1^{\mathcal{I},\mathcal{LP}} - (A-C-Q_1-Q_2+\epsilon)Q_1, 0\right\}$ at first if we assume that $D^{\mathcal{LP}} > 0$ in this case we get that

$$\begin{aligned} \Pi_1^{\mathcal{LP}} &= \begin{cases} (1-\theta)(A-C-Q_1-Q_2+\epsilon)Q_1 + \theta\Pi^{\mathcal{I},\mathcal{LP}} & \text{if } D^{\mathcal{LP}}>0 \\ (A-C-Q_1-Q_2+\epsilon)Q_1 & \text{if } D^{\mathcal{LP}}=0 \end{cases} \\ \Pi_2^{\mathcal{LP}} &= \begin{cases} (A-C-Q_1-Q_2+\epsilon)(Q_2+\theta Q_1) - \theta\Pi^{\mathcal{I},\mathcal{LP}} & \text{if } D^{\mathcal{LP}}>0 \\ (A-C-Q_1-Q_2+\epsilon)Q_2 & \text{if } D^{\mathcal{LP}}=0 \end{cases} \end{aligned} \tag{26}$$

then the reaction functions are more complex in this case then we just jump to the equilibrium solution(see Anton and Yao for a complete study of this case)

(27)

$$Q_1^{\mathcal{I},\mathcal{LP}} = \begin{cases} \dfrac{A-C+2\epsilon}{3} & \text{if } 0<\gamma<\theta/(3-2\theta) \\ \dfrac{A-C+\epsilon}{3-\theta} & \text{if } \gamma\ge\theta/(3-2\theta) \end{cases} ; Q_2^{\mathcal{I},\mathcal{LP}} = \begin{cases} \dfrac{A-C-\epsilon}{3} & \text{if } 0<\gamma<\theta/(3-2\theta \\ (1-\theta)\dfrac{A-C+\epsilon}{3-\theta} & \text{if } 1\le\gamma \end{cases}$$

and with that

$$Q^{\mathcal{I},\mathcal{LP}} = \begin{cases} \dfrac{2A-2C+\epsilon}{3} & \text{if } 0<\gamma<\theta/(3-2\theta) \\ (2-\theta)\dfrac{A-C+\epsilon}{3-\theta} & \text{if } \gamma \geq \theta/(3-2\theta) \end{cases} \tag{28}$$

$$P^{\mathcal{I},\mathcal{LP}} = \begin{cases} \dfrac{A+2C-\epsilon}{3} & \text{if } 0<\gamma<\theta/(3-\theta) \\ \dfrac{A+(2-\theta)(C-\epsilon)}{3-\theta} & \text{if } \gamma \geq \theta/(3-\theta) \end{cases} \tag{29}$$

then

(30)

$$\Pi_1^{\mathcal{I},\mathcal{LP}} = \begin{cases} \left(\dfrac{A-C+2\epsilon}{3}\right)^2 & \text{if } 0<\gamma<\theta/(3-\theta) \\ (1-\theta)\left(\dfrac{A-C+\epsilon}{3-\theta}\right)^2 + \theta\left(\dfrac{A-C+2\epsilon}{3}\right)^2 & \text{if } \theta/(3-\theta)\leq\gamma<1 \\ (1-\theta)\left(\dfrac{A-C+\epsilon}{3-\theta}\right)^2 + \theta\left(\dfrac{A-C+\epsilon}{2}\right)^2 & \text{if } \gamma\geq 1 \end{cases}$$

$$\Pi_2^{\mathcal{I},\mathcal{LP}} = \begin{cases} \left(\dfrac{A-C+2\epsilon}{3}\right)\left(\dfrac{A-C-\epsilon}{3}\right) & \text{if } 0<\gamma<\theta/(3-\theta) \\ \left(\dfrac{A-C+\epsilon}{3-\theta}\right)^2 - \theta\left(\dfrac{A-C+2\epsilon}{3}\right)^2 & \text{if } \theta/(3-\theta)\leq\gamma<1 \\ \left(\dfrac{A-C+\epsilon}{3-\theta}\right)^2 - \theta\left(\dfrac{A-C+\epsilon}{2}\right)^2 & \text{if } \gamma\geq 1 \end{cases}$$

then the Total profit will be

(31)

$$T^{\mathcal{I},\mathcal{LP}} = \begin{cases} \left(\dfrac{A-C+2\epsilon}{3}\right)^2 + \left(\dfrac{A-C+2\epsilon}{3}\right)\left(\dfrac{A-C-\epsilon}{3}\right) & \text{if } 0<\gamma<\theta/(3-\theta) \\ (2-\theta)\left(\dfrac{A-C+\epsilon}{3-\theta}\right)^2 & \text{if } \theta/(3-\theta)\leq\gamma<1 \\ (2-\theta)\left(\dfrac{A-C+\epsilon}{3-\theta}\right)^2 & \text{if } \gamma\geq 1 \end{cases}$$

then

$$CSP^{\mathcal{I},\mathcal{LP}} = \begin{cases} \left(\dfrac{2A-2C+\epsilon}{3}\right)^2/2 & \text{if } 0<\gamma<\theta/(3-2\theta) \\ \left((2-\theta)\dfrac{A-C+\epsilon}{3-\theta}\right)^2/2 & \text{if } \gamma\geq\theta/(3-2\theta) \end{cases} \tag{32}$$

then the welfare will be

(33)
$$TW^{\mathcal{I},\mathcal{LP}} = \begin{cases} \left(\frac{2A-2C+\epsilon}{3}\right)^2/2 + \left(\frac{A-C+2\epsilon}{3}\right)^2 + \left(\frac{A-C+2\epsilon}{3}\right)\left(\frac{A-C-\epsilon}{3}\right) & \text{if } 0<\gamma<\theta/(3- \\ \left((2-\theta)\frac{A-C+\epsilon}{3-\theta}\right)^2/2 + (2-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 & \text{if } \theta/(3-\theta)\leq\gamma< \\ \left((2-\theta)\frac{A-C+\epsilon}{3-\theta}\right)^2/2 + (2-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 & \text{if } \gamma\geq 1 \end{cases}$$

□

Result 3. We will begin with the LP case $\Pi_1 = (A-C-Q_1-Q_2+\epsilon)Q_1+\theta D(Q_1,Q_2)$ and $\Pi_2 = (A-C-Q_1-Q_2+\epsilon)Q_2-\theta D(Q_1,Q_2)$ in the case of unjust enrichment $D(Q_1,Q_2)^{\mathcal{UE}} = (A-C-Q_1-Q_2+\epsilon)Q_2 - \Pi_2^{\mathcal{N}}$

then in equilibrium

(34)
$$\begin{aligned} Q_1^{\mathcal{I},\mathcal{UE}} &= (1-\theta)\frac{A-C+\epsilon}{3-\theta} \\ Q_2^{\mathcal{I},\mathcal{UE}} &= \frac{A-C+\epsilon}{3-\theta} \end{aligned}$$

and with that

(35)
$$Q^{\mathcal{I},\mathcal{UE}} = (2-\theta)\frac{A-C+\epsilon}{3-\theta}$$

(36)
$$P^{\mathcal{I},\mathcal{UE}} = \frac{A+(2-\theta)(C-\epsilon)}{3-\theta}$$

then

(37)
$$\Pi_1^{\mathcal{I},\mathcal{UE}} = \begin{cases} \left(\frac{A-C+\epsilon}{3-\theta}\right)^2 - \theta\left(\frac{A-C-\epsilon}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\frac{A-C+\epsilon}{3-\theta}\right)^2 & \text{if } \gamma\geq 1 \end{cases}$$
$$\Pi_1^{\mathcal{I},\mathcal{UE}} = \begin{cases} (1-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 + \theta\left(\frac{A-C-\epsilon}{3}\right)^2 & \text{if } 0<\gamma<1 \\ (1-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 & \text{if } \gamma\geq 1 \end{cases}$$

then the Total profit will be

(38)
$$T^{\mathcal{I},\mathcal{UE}} = (2-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2$$

then

(39)
$$CSP^{\mathcal{I},\mathcal{UE}} = \left((2-\theta)\frac{A-C+\epsilon}{3-\theta}\right)^2/2$$

then the welfare will be

$$TW^{\mathcal{I},\mathcal{UE}} = \left((2-\theta)\frac{A-C+\epsilon}{3-\theta}\right)^2/2 + (2-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 \tag{40}$$

□

Result 4. In the case of a given Fixed fee we get that $\Pi_1 = (A - C - Q_1 - Q_2 + \epsilon)Q_1 + F$ and $\Pi_2 = (A - C - Q_1 - Q_2 + \epsilon)Q_2 - F$ then in equilibrium

$$\begin{aligned} Q_1^{\mathcal{L},\mathcal{F}} &= \frac{A-C+\epsilon}{3} \\ Q_2^{\mathcal{L},\mathcal{F}} &= \frac{A-C+\epsilon}{3} \end{aligned} \tag{41}$$

and

$$Q^{\mathcal{L},\mathcal{F}} = 2\frac{A-C+\epsilon}{3} \tag{42}$$

$$P^{\mathcal{L},\mathcal{F}} = \frac{A+2(C-\epsilon)}{3} \tag{43}$$

$$\begin{aligned} \Pi_1^{\mathcal{L},\mathcal{F}} &= \left(\frac{A-C+\epsilon}{3}\right)^2 + F \\ \Pi_2^{\mathcal{L},\mathcal{F}} &= \left(\frac{A-C+\epsilon}{3}\right)^2 - F \end{aligned} \tag{44}$$

then the Total profit will be

$$T^{\mathcal{L},\mathcal{F}} = 2\left(\frac{A-C+\epsilon}{3}\right)^2 \tag{45}$$

then

$$CSP^{\mathcal{L},\mathcal{F}} = 2\left(\frac{A-C+\epsilon}{3}\right)^2 \tag{46}$$

then the welfare will be

$$TW^{\mathcal{L},\mathcal{F}} = 4\left(\frac{A-C+\epsilon}{3}\right)^2 \tag{47}$$

□

Result 5. In the case of a given Royalty rate R we get that $\Pi_1 = (A - C - Q_1 - Q_2 + \epsilon)Q_1 + RQ_2$ and $\Pi_2 = (A - C - Q_1 - Q_2 + \epsilon - R)Q_2$ then in equilibrium

$$\begin{aligned} Q_1^{\mathcal{L},\mathcal{R}} &= \frac{A-C+\epsilon+R}{3} \\ Q_2^{\mathcal{L},\mathcal{R}} &= \frac{A-C+\epsilon-2R}{3} \end{aligned} \tag{48}$$

and

$$Q^{\mathcal{L},\mathcal{R}} = \frac{2(A-C+\epsilon)-R}{3} \tag{49}$$

$$P^{\mathcal{L},\mathcal{R}} = \frac{A+2(C-\epsilon)+R}{3} \tag{50}$$

$$\begin{aligned} \Pi_1^{\mathcal{L},\mathcal{R}} &= \left(\frac{A-C+\epsilon+R}{3}\right)^2 + R\frac{A-C+\epsilon-2R}{3} \\ \Pi_2^{\mathcal{L},\mathcal{R}} &= \left(\frac{A-C+\epsilon-2R}{3}\right)^2 \end{aligned} \tag{51}$$

then the Total profit will be

$$T^{\mathcal{L},\mathcal{R}} = \left(\frac{A-C+\epsilon+R}{3}\right)^2 + R\frac{A-C+\epsilon-2R}{3} + \left(\frac{A-C+\epsilon-2R}{3}\right)^2 \tag{52}$$

then

$$CSP^{\mathcal{L},\mathcal{R}} = \left(\frac{2(A-C+\epsilon)-R}{3}\right)^2 /2 \tag{53}$$

then the welfare will be

(54)

$$TW^{\mathcal{L},\mathcal{R}} = \left(\frac{2(A-C+\epsilon)-R}{3}\right)^2 /2 + \left(\frac{A-C+\epsilon+R}{3}\right)^2 + R\frac{A-C+\epsilon-2R}{3} + \left(\frac{A-C+\epsilon-2R}{3}\right)^2$$

□

Proof Lemma 1. For the case under the LP rule, by using the equations (2) and (7) and when $\gamma \leq \theta/(3-2\theta)$, $\pi_2^{\mathcal{I},\mathcal{LP}} = \left(\frac{1+2\gamma}{3}\right)\left(\frac{1-\gamma}{3}\right) > \left(\frac{1-\gamma}{3}\right)^2 = \pi_2^{\mathcal{N}}$

When $\theta/(3-2\theta) \leq \gamma < 1$,

$$G(\gamma,\theta) = \pi_2^{\mathcal{I},\mathcal{LP}} - \pi_2^{\mathcal{N}} = \left(\frac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\frac{1+2\gamma}{3}\right)^2 - \left(\frac{1-\gamma}{3}\right)^2$$

, now notice that $G_{11} = \left(\frac{1}{3-\theta}\right)^2 - \frac{4\theta+1}{9}$, because at $\theta = 0$ $G_{11} = 0$ and because $dG_{11}/d\theta = 2(3-\theta)^{-3} - 4/9 < (2)^{-2} - 4/9 < 0$, $G_{11} < 0$ for $\theta \in (0,1)$, then G is concave in γ for $\theta \in (0,1)$.

$G(1,\theta) = \left(\frac{2}{3-\theta}\right)^2 - \theta$, moreover $G_2(1,\theta) = 8(3-\theta)^{-3} - 1 < 0$ for $\theta \in (0,1)$, $G(1,0) = \left(\frac{2}{3}\right)^2$ and $G(1,1) = 0$ then by continuity $G(1,\theta) > 0$ for $\theta \in (0,1)$.

$$G(\theta/(3-2\theta),\theta) = \left(\frac{1}{3-2\theta}\right)^2 - \theta\left(\frac{1}{3-2\theta}\right)^2 - \left(\frac{1-\theta}{3-2\theta}\right)^2 = \frac{\theta(1-\theta)}{(3-2\theta)^2} > 0$$

because G is concave in γ and $G(\theta/(3-2\theta),\theta), G(1,\theta) > 0$, we conclude that $G > 0$ for $\gamma > \theta/(3-2\theta)$ and $\theta \in (0,1)$

Now in the case when $\gamma > 1$ is straightforward to prove that $\pi_2^{\mathcal{I},\mathcal{LP}} >= 0$.

In the UE case by using (8) and (2) for $\gamma < 1$ we get that

$\pi_2^{\mathcal{I},\mathcal{UE}} = (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1-\gamma}{3}\right)^2 > \left(\frac{1-\gamma}{3}\right)^2 = \pi_2^{\mathcal{N}}$

and in the case $\gamma > 1$ we get that $\pi_2^{\mathcal{I},\mathcal{UE}} > 0$

meaning that under the UE the incumbent firm always will prefer infringe the patent over use the backstop technology.

□

Lemma 2. $\underline{f} = \left(\frac{1+\gamma}{3}\right)^2 - \pi_2^I$, notice too that if $\underline{f}$ is negative there is no positive fixed fee or royalty that makes the license option as good as infringe.

Then in the case of UE we get that

(55)
$$\underline{F}^{\mathcal{UE}} = \begin{cases} \left(\frac{A-C+\epsilon}{3}\right)^2 - (1-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 - \theta\left(\frac{A-C-\epsilon}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\frac{A-C+\epsilon}{3}\right)^2 - (1-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 & \text{if } 1 \le \gamma \end{cases}$$

then

$$\underline{f}^{\mathcal{UE}} = \begin{cases} \left(\frac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\frac{1-\gamma}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\frac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 & \text{if } 1 \le \gamma \end{cases} \tag{56}$$

Unfortunately this expression is not greater that zero in all the cases, but is straightforward to prove that $\underline{f}^{\mathcal{UE}} > 0$ if

$$\gamma > \frac{12-5\theta+\theta^2-2\sqrt{(3-\theta)^2(3+\theta)}}{6-7\theta+\theta^2}$$

Now in the case of lost profits we get that

(57)
$$\underline{f}^{\mathcal{LP}} = \begin{cases} \left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+2\gamma}{3}\right)\left(\frac{1-\gamma}{3}\right) & \text{if } 0<\gamma\le\theta/(3-2\theta) \\ \left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1+2\gamma}{3}\right)^2 & \text{if } \theta/(3-2\theta)<\gamma<1 \\ \left(\frac{1+\gamma}{3}\right)^2 - \left(\frac{1+\gamma}{3-\theta}\right)^2 + \theta\left(\frac{1+\gamma}{2}\right)^2 & \text{if } 1 \le \gamma \end{cases}$$

Notice too that is straightforward to prove that $\underline{f} > 0$ in this case, and in consequence of the past results and because

$$\underline{r} = (1+\gamma-3\sqrt{\pi_2^{\mathcal{I}}})/2$$

, we got the lemma □

Proof Result 5. By remembering that

$$\begin{aligned} \Pi_1^{\mathcal{L},\mathcal{F}} &= \left(\frac{A-C+\epsilon}{3}\right)^2 + F \\ \Pi_2^{\mathcal{L},\mathcal{F}} &= \left(\frac{A-C+\epsilon}{3}\right)^2 - F \end{aligned} \tag{58}$$

and replacing

(59)

$$\Pi_1^{\mathcal{L},\mathcal{F},\mathcal{UE}} = \left(\frac{A-C+\epsilon}{3}\right)^2 + \begin{cases} \left(\frac{A-C+\epsilon}{3}\right)^2 - (1-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 - \theta\left(\frac{A-C-\epsilon}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\frac{A-C+\epsilon}{3}\right)^2 + \left(\frac{A-C+\epsilon}{3}\right)^2 - (1-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 & \text{if } 1\le\gamma \end{cases}$$

and

(60)

$$\Pi_2^{\mathcal{L},\mathcal{F},\mathcal{UE}} = \left(\frac{A-C+\epsilon}{3}\right)^2 - \begin{cases} \left(\frac{A-C+\epsilon}{3}\right)^2 + (1-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 + \theta\left(\frac{A-C-\epsilon}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\frac{A-C+\epsilon}{3}\right)^2 - \left(\frac{A-C+\epsilon}{3}\right)^2 + (1-\theta)\left(\frac{A-C+\epsilon}{3-\theta}\right)^2 & \text{if } 1\le\gamma \end{cases}$$

$$\underline{f}^{\mathcal{UE}} = \begin{cases} \left(\frac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 - \theta\left(\frac{1-\gamma}{3}\right)^2 & \text{if } 0<\gamma<1 \\ \left(\frac{1+\gamma}{3}\right)^2 - (1-\theta)\left(\frac{1+\gamma}{3-\theta}\right)^2 & \text{if } 1\le\gamma \end{cases} \tag{61}$$

□

Proof Proposition 1. In the case of royalties (see eq 10)we get that $\pi_1^{\mathcal{L},\mathcal{R}} = \left(\frac{1+\gamma+r}{3}\right)^2 + r\frac{1+\gamma-2r}{3}$ this function reach the maximum when r is equal to $(1+\gamma)/2$, but because $\underline{r} = (1+\gamma-3\sqrt{\pi_1^{\mathcal{I}}})/2$ and $\pi_1^{\mathcal{I},\mathcal{LP}}$ and $\pi_1^{\mathcal{I},\mathcal{UE}}$ are greater than 0, we get that

$$r = \underline{r}$$

.

now because

$$r = (1+\gamma-3\sqrt{\pi_1^{\mathcal{I}}})/2$$

we get that

$$\pi_1^{\mathcal{L},\mathcal{R}} = \left((1+\gamma)/2 - \sqrt{\pi_2^{I}}/2\right)^2 + ((1+\gamma)/2 - 3\sqrt{\pi_2^{I}}/2)\sqrt{\pi_2^{I}}$$

then

$$\pi_1^{\mathcal{L},\mathcal{R}} = ((1+\gamma)/2)^2 - 5 * \pi_2^{I}/4$$

we know that

$$\pi_2^{I} = ((1+\gamma)/3)^2 - F$$

then

$$\pi_1^{\mathcal{L},\mathcal{R}} = ((1+\gamma)/2)^2 - 5/4(((1+\gamma)/3)^2 - F)$$

then

$$\pi_1^{\mathcal{L},\mathcal{R}} = ((1+\gamma)/2)^2 + 5/4(-2*((1+\gamma)/3)^2 + \pi_1^{L,F})$$

then we obtain that

$$\pi_1^{\mathcal{L},\mathcal{R}} - \pi_1^{L,F} = ((1+\gamma)/2)^2 - 5/2((1+\gamma)/3)^2 + \pi_1^{L,F}/4$$

then in the case where $f > 0$

$$\pi_1^{\mathcal{L},\mathcal{R}} - \pi_1^{L,F} > ((1+\gamma)/2)^2 - 5/2((1+\gamma)/3)^2 + 1/4((1+\gamma)/3)^2 = 0$$

□

Proof Proposition 2. first as we know

$$\pi_1^{\mathcal{L},\mathcal{R}} = ((1+\gamma)/2)^2 - 5/4(((1+\gamma)/3)^2 - f)$$

then

$$\pi_1^{\mathcal{L},\mathcal{R}} = ((1+\gamma)/3)^2 + 5/4f$$

Now we first analyze the case of LP rule, then for the case $0 < \gamma \leq \theta/(3-2\theta)$ we get that $f^{LP} = (1+\gamma)^2/9 - (1+2\gamma)(1-\gamma)/9$

then

$$\pi_1^{\mathcal{L},\mathcal{R},\mathcal{LP}} = 9\left((1+\gamma)/3\right)^2/4 - 5/4((1+2\gamma)(1-\gamma)/9)$$

then

$$\pi_1^{\mathcal{L},\mathcal{R},\mathcal{LP}} - \pi_1^{\mathcal{I},\mathcal{LP}} = 9\left((1+\gamma)/3\right)^2/4 - 5/4((1+2\gamma)(1-\gamma)/9) - (1+2\gamma)^2/9$$

then after some algebra

$$\pi_1^{\mathcal{L},\mathcal{R},\mathcal{LP}} - \pi_1^{\mathcal{I},\mathcal{LP}} = -(3\gamma + 17\gamma^2)/(3^2 2^2) < 0$$

for the case $0 < \gamma \leq \theta/(3-2\theta)$ we get that $f^{LP} = (1+\gamma)^2/9 - (1+2\gamma)(1-\gamma)/9$

for the case $\gamma \geq 1$ we get that $f^{LP} = (1+\gamma)^2/9 - (1-\gamma)^2/(3-\theta)^2 + \theta(1+\gamma)^2/4$

then

$$\pi_1^{\mathcal{L},\mathcal{R},\mathcal{LP}} = \frac{4+5\theta}{4}\left(\frac{1+\gamma}{2}\right)^2 - 5\left((1+\gamma)/(3-\theta)\right)^2/4$$

then

$$\pi_1^{\mathcal{L},\mathcal{R},\mathcal{LP}} - \pi_1^{\mathcal{I},\mathcal{LP}} = \frac{4+5\theta}{4}\left(\frac{1+\gamma}{2}\right)^2 - 5\left((1+\gamma)/(3-\theta)\right)^2/4 - (1-\theta)\left((1+\gamma)/(3-\theta)\right)^2 - \theta\left(\frac{1+\gamma}{2}\right)^2$$

then

$$\pi_1^{\mathcal{L},\mathcal{R},\mathcal{LP}} - \pi_1^{\mathcal{I},\mathcal{LP}} = \left(\frac{1+\gamma}{2}\right)^2 \frac{\theta(\theta^2 - 2\theta + 9)}{2^2(3-\theta)^2} > 0$$

□